# New effects in precision **Brout-Englert-Higgs** physics

**Axel Maas** 

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#### **NAWI Graz**

Natural Sciences

Der Wissenschaftsfonds

FШF

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#### What is this talk about?

- Gauge invariance and the Brout-Englert Higgs effect
- Physical states
- Deviations and signals at experiments
- Implications beyond the standard model

Review: 1712.04721

# What's the deal? -Gauge symmetry

#### A toy model

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only  $W^a_{\mu} \rightarrow W^a_{\mu}$   $h \rightarrow h \Omega$

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- Choose a suitable gauge and obtain 'spontaenous gauge symmetry breaking': SU(2) → 1
- Get masses and degeneracies at treelevel
- Perform perturbation theory

#### **Physical spectrum**

Perturbation theory



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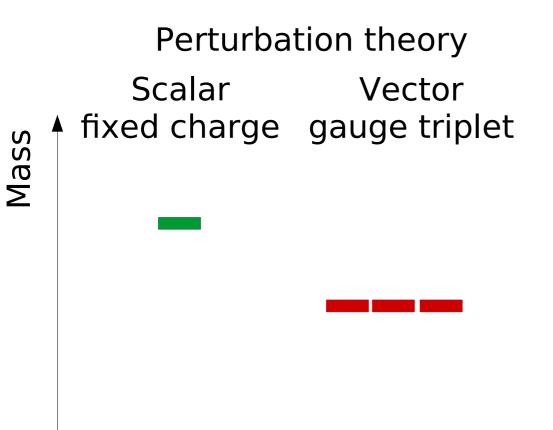
#### **Physical spectrum**

Perturbation theory Scalar fixed charge

• Custodial singlet

Mass

#### **Physical spectrum**



Both custodial singlets

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  - ...even at weak coupling [Gribov'78,Singer'78,Fujikawa'82]

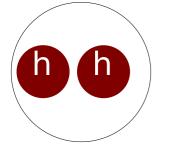
[Fröhlich et al.'80, Banks et al.'79]

• Need physical, gauge-invariant particles

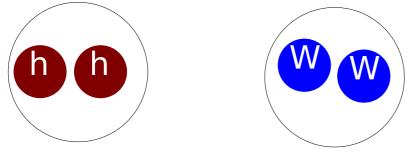
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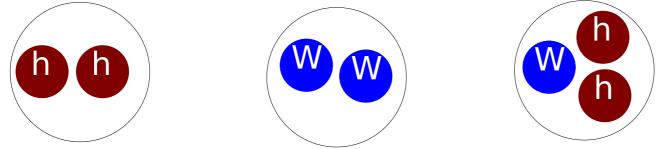
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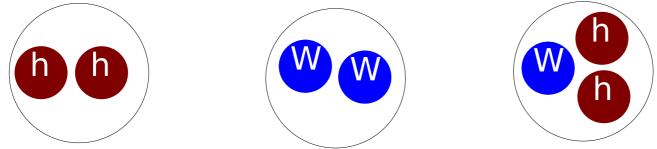
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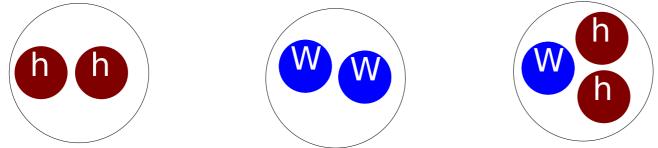
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# **Physical states**

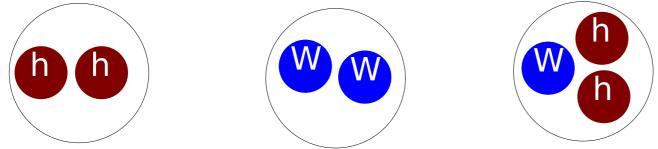
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  - Think QED (hydrogen atom!)
- Can this matter?

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17]

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- J<sup>PC</sup> and custodial charge only quantum numbers
  - Different from perturbation theory
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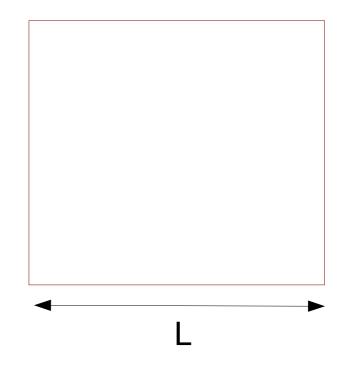
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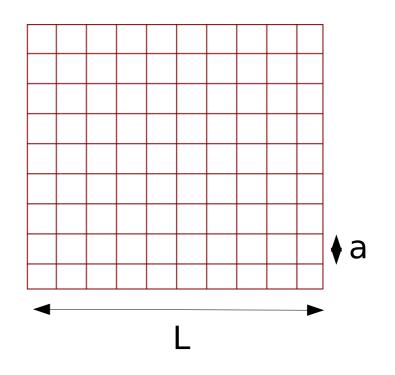
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  - Bound state structure non-perturbative methods! - Lattice

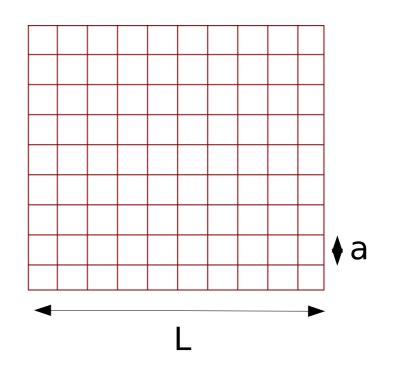
• Take a finite volume – usually a hypercube



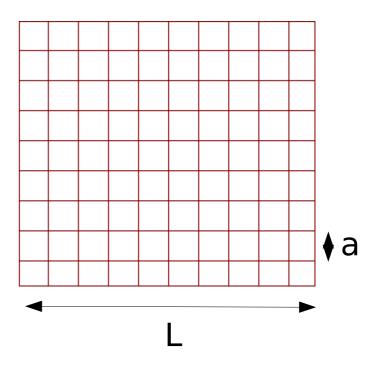
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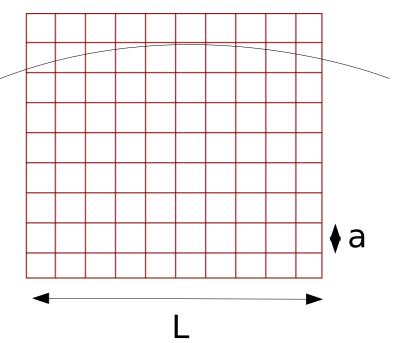
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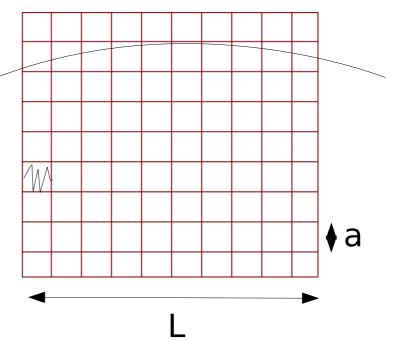
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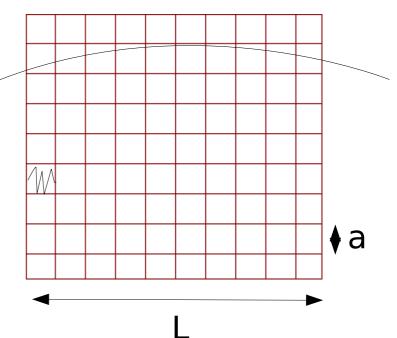
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# $D(p) = \langle O^+(p)O(-p) \rangle$

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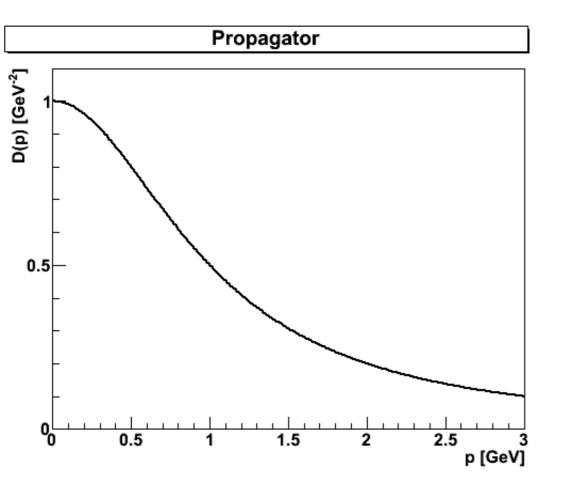
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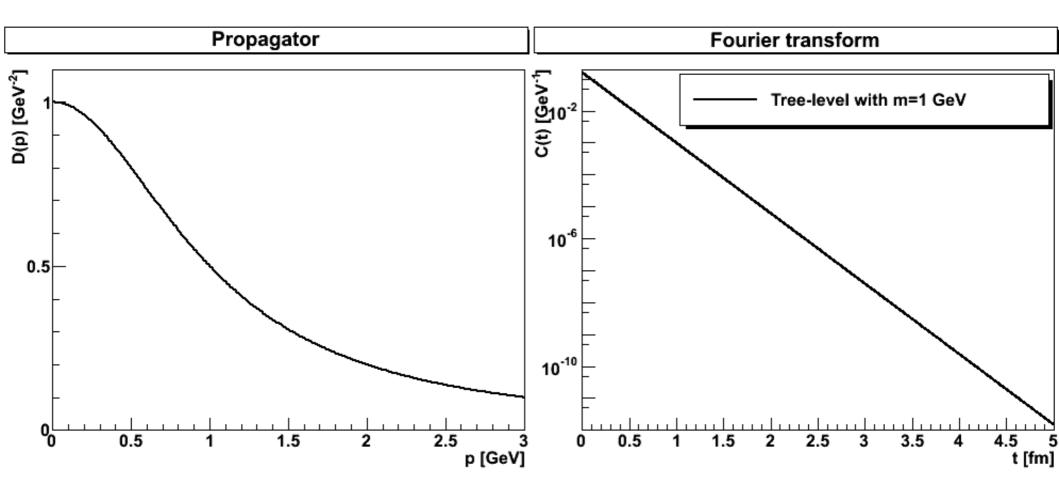
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$$\sum a_i = 1 \wedge m_0 < m_1 < \dots$$

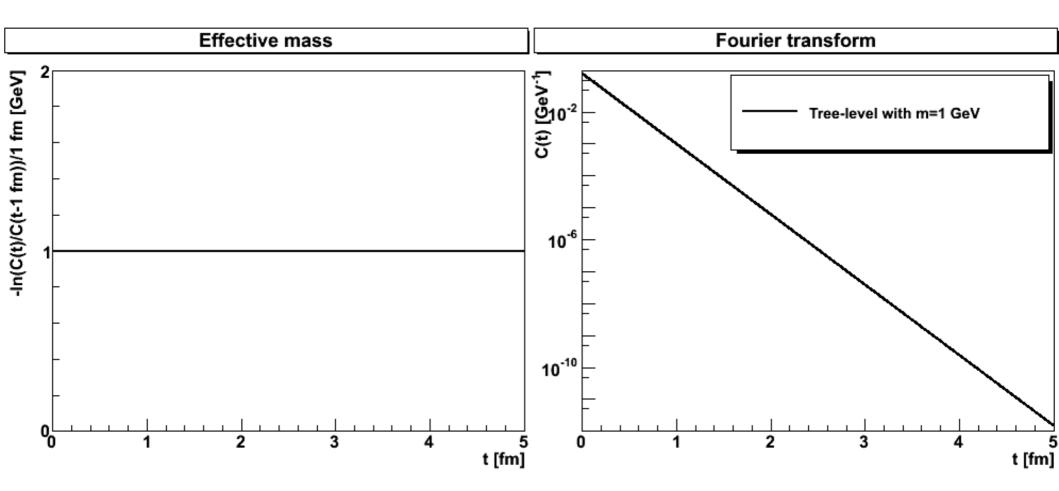
- Masses can be inferred from propagators
- Long-time behavior relevant
  - No exact results on time-like momenta



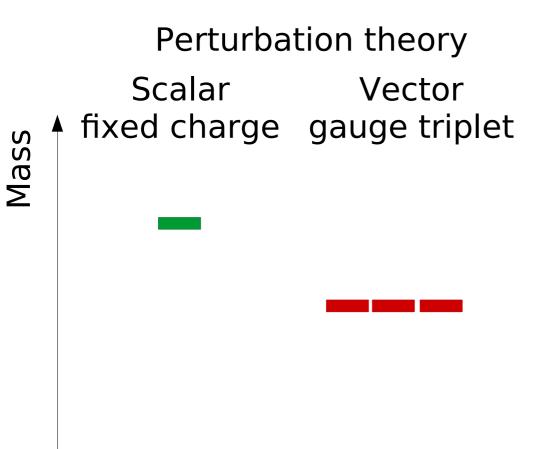
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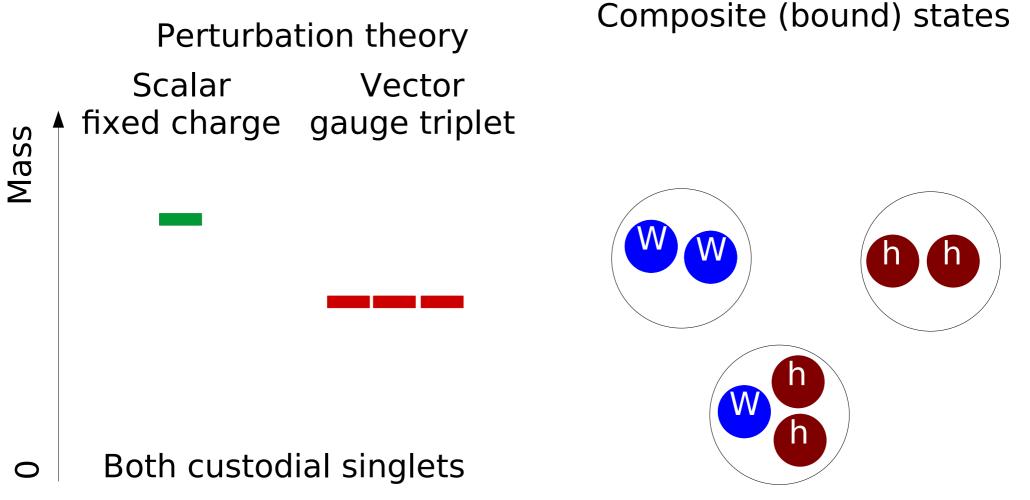


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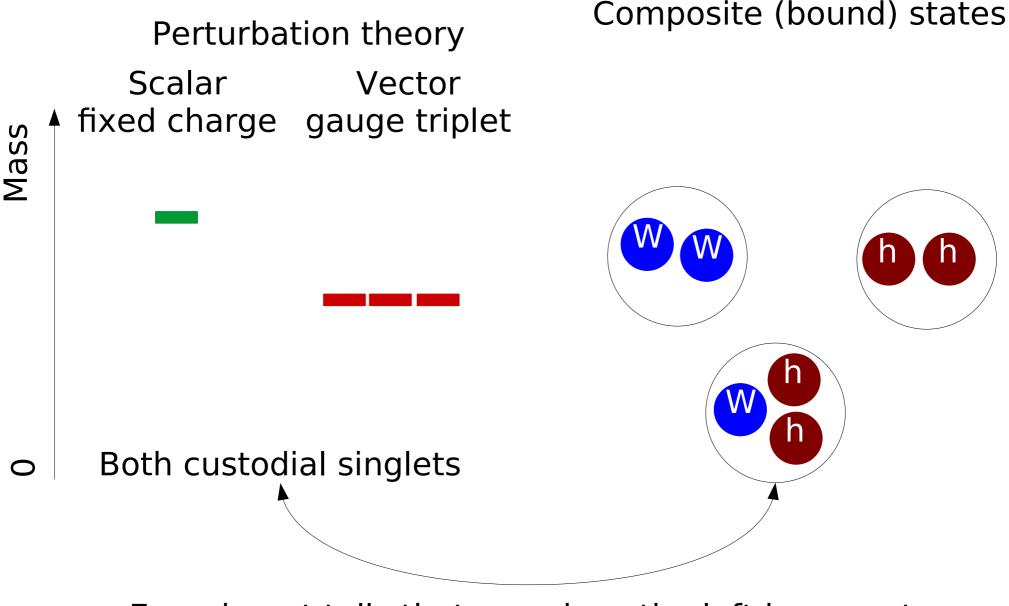


Both custodial singlets

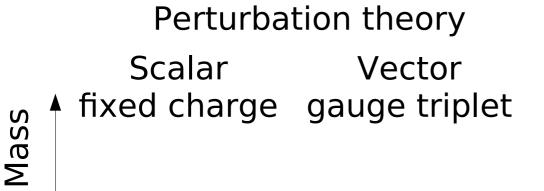
Experiment tells that somehow the left is correct



Experiment tells that somehow the left is correct Theory say the right is correct



Experiment tells that somehow the left is correct Theory say the right is correct There must exist a relation that both are correct



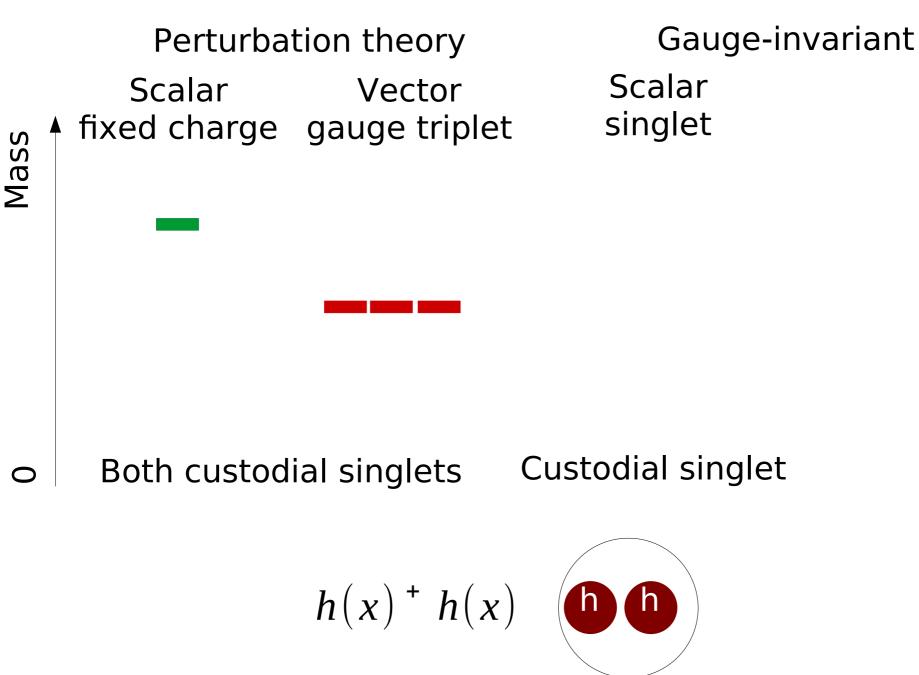
Gauge-invariant

Scalar singlet

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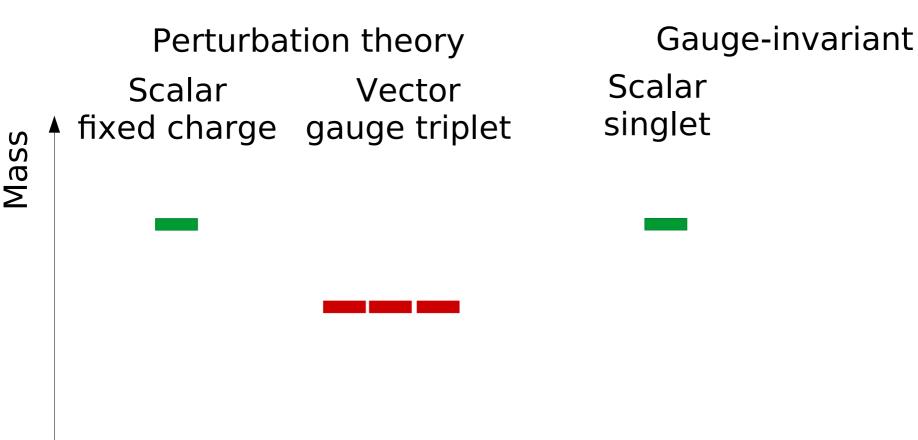
$$h(x) + h(x)$$



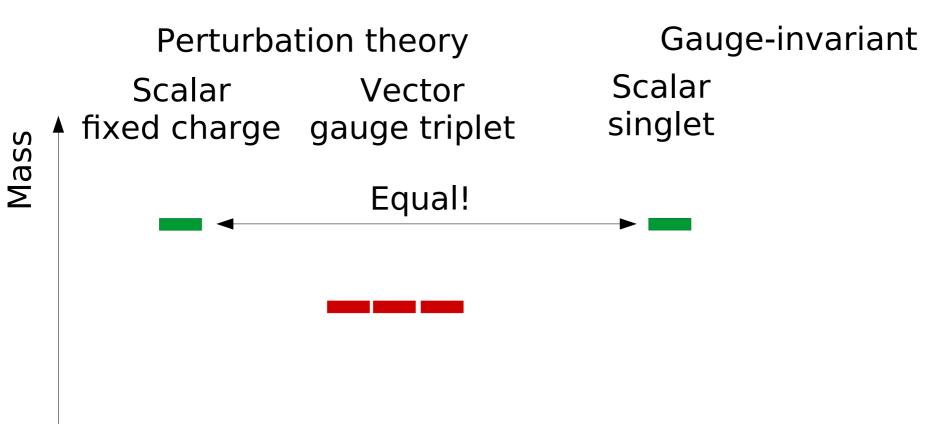


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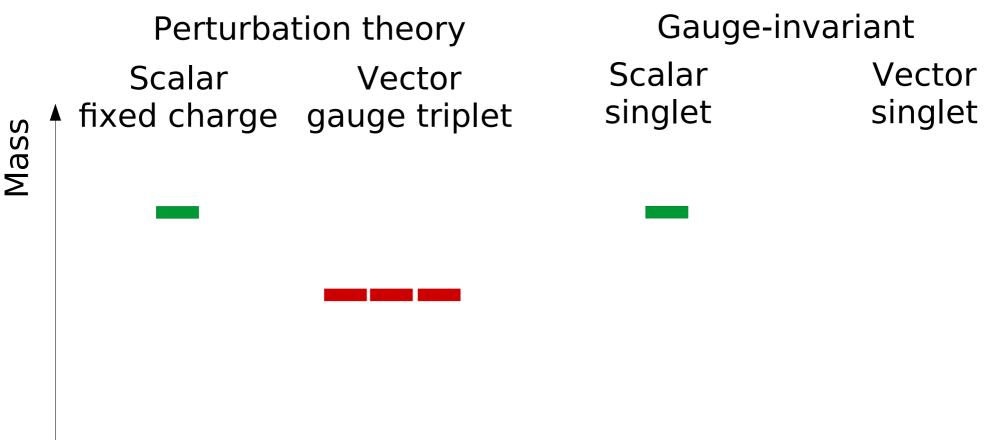
 $\square$ 



**Custodial singlet** 



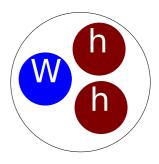
#### • Both custodial singlets Custodial singlet

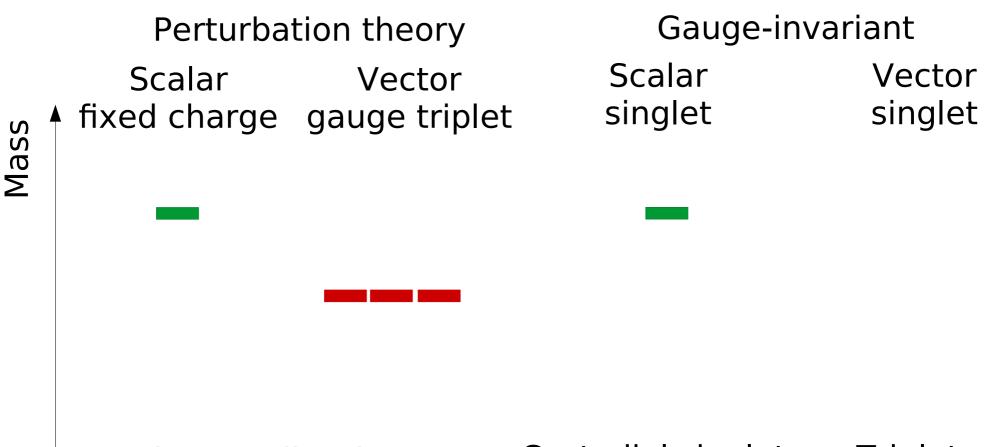


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**Custodial singlet** 

$$tr t^{a} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}$$



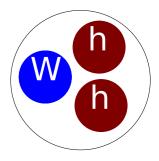


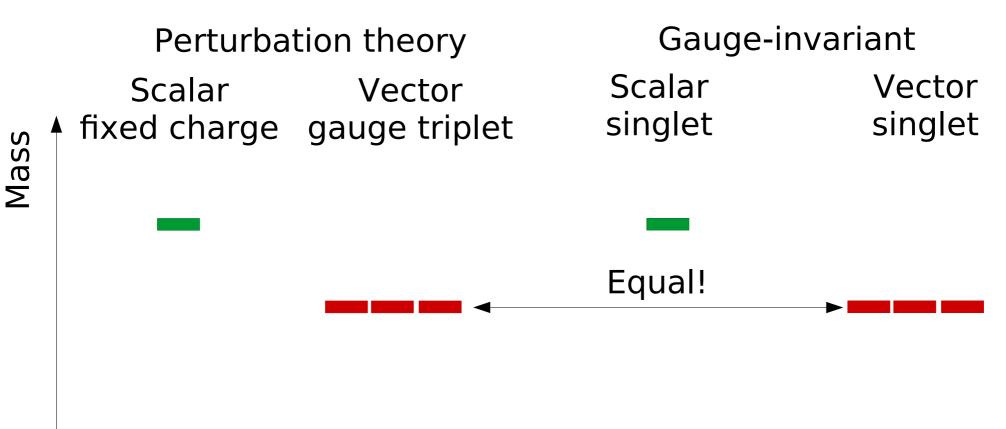
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#### Custodial singlet

Triplet

$$tr \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}$$





#### • Both custodial singlets Custodial singlet Triplet

# A microscopic mechanism -Why on-shell is important

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17 Maas & Sondenheimer '20]

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  - Different from perturbation theory
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# How to make predictions

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17 Maas & Sondenheimer '20]

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  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas'12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

[Fröhlich et al.'80,'81 Maas'12,'17]

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Higgs field

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$$\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle \\ + v \langle \eta^+ \eta^2 + \eta^{+2} \eta \rangle + \langle \eta^{+2} \eta^2 \rangle$$

3) Standard perturbation theory

Bound state  $\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle$ mass  $+ \langle \eta^+ (x)\eta(y) \rangle \langle \eta^+ (x)\eta(y) \rangle + O(g,\lambda)$ 

```
[Fröhlich et al.'80,'81
Maas'12,'17]
```

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Trivial two-particle state

```
[Fröhlich et al.'80,'81
Maas'12,'17]
```

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Bound state  $\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle$ mass  $+ \langle \eta^+ (x)\eta(y) \rangle \langle \eta^+ (x)\eta(y) \rangle + O(g,\lambda)$  Higgs mass

[Fröhlich et al.'80,'81 Maas'12,'17]

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0<sup>+</sup> singlet:  $\langle (h^+ h)(x)(h^+ h)(y) \rangle$ 

2) Expand Higgs field around fluctuations  $h=v+\eta$ 

$$\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle$$
  
+  $v \langle \eta^+ \eta^2 + \eta^{+2} \eta \rangle + \langle \eta^{+2} \eta^2 \rangle$ 

3) Standard perturbation theory

Standard Perturbation Theory

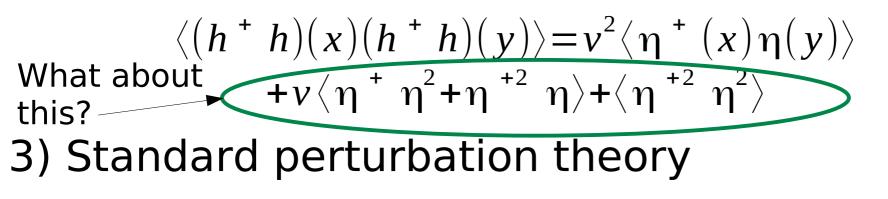
Bound state  $\langle (h^+ h)(x)(h^+ h)(y) = v^2 \eta^+ (x)\eta(y) \rangle$ mass  $+ \langle \eta^+ (x)\eta(y) \rangle \langle \eta^+ (x)\eta(y) \rangle + O(g,\lambda)$ 

[Fröhlich et al.'80,'81 Maas'12,'17 Maas & Sondenheimer'20]

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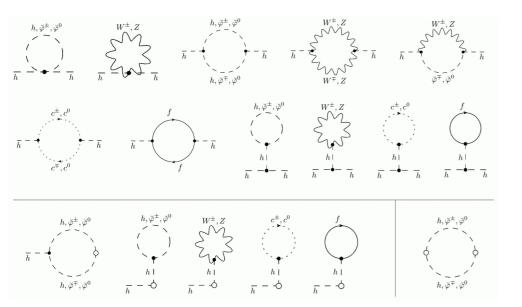
2) Expand Higgs field around fluctuations  $h=v+\eta$ 



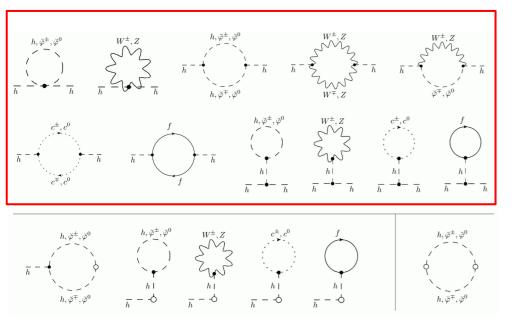
$$\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle + \langle \eta^+ (x)\eta(y) \rangle \langle \eta^+ (x)\eta(y) \rangle + O(g,\lambda)$$

[Maas'12,'17 Maas & Sondenheimer'20]

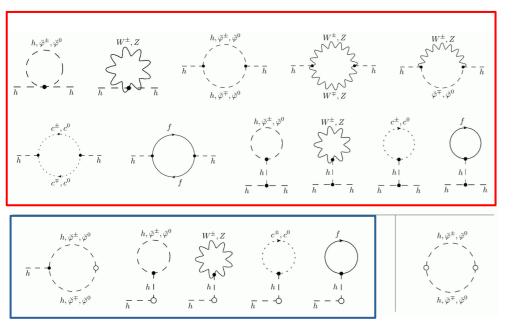
## $\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle$ $+ v \langle \eta^+ \eta^2 + \eta^{+2} \eta \rangle + \langle \eta^{+2} \eta^2 \rangle$



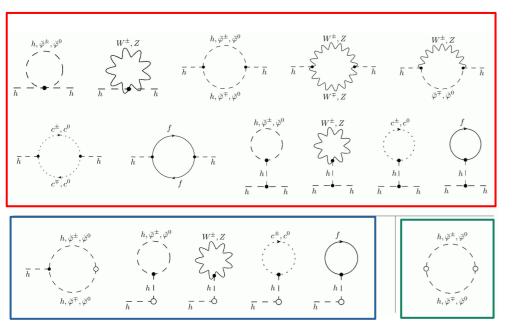
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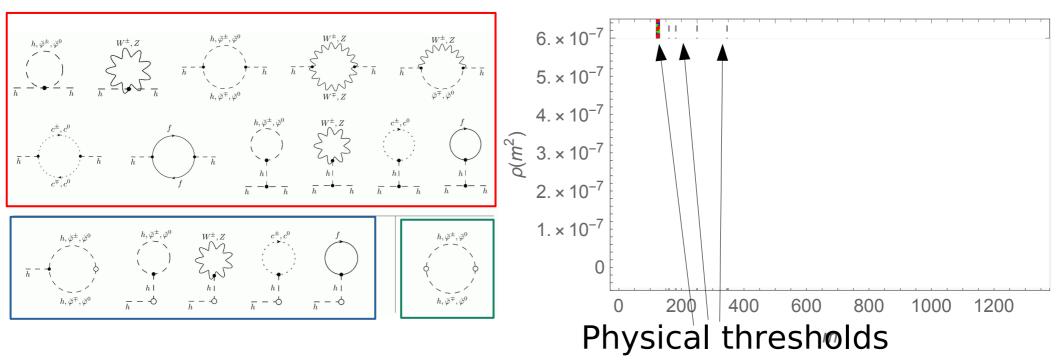
 $\langle (h^+ h)(x)(h^+ h)(y) \rangle = v^2 \langle \eta^+ (x)\eta(y) \rangle$ + $v\langle \eta^{+} \eta^{2} + \eta^{+2} \eta \rangle$  +  $\langle \eta^{+2} \eta^{2} \rangle$ 

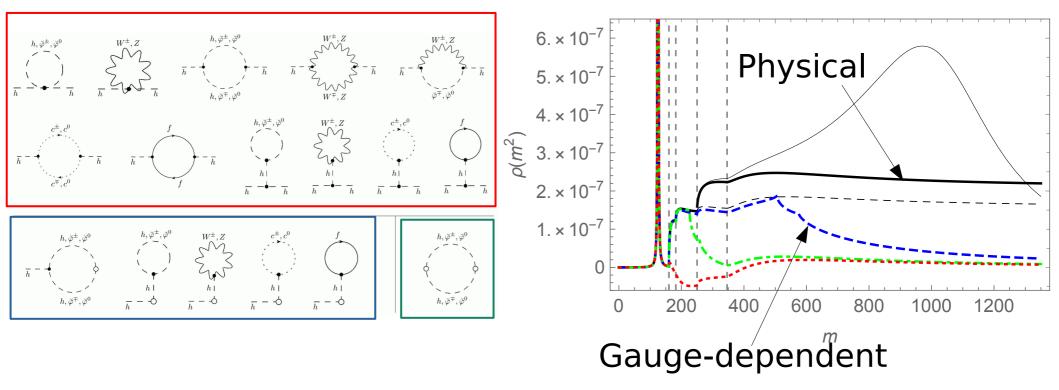


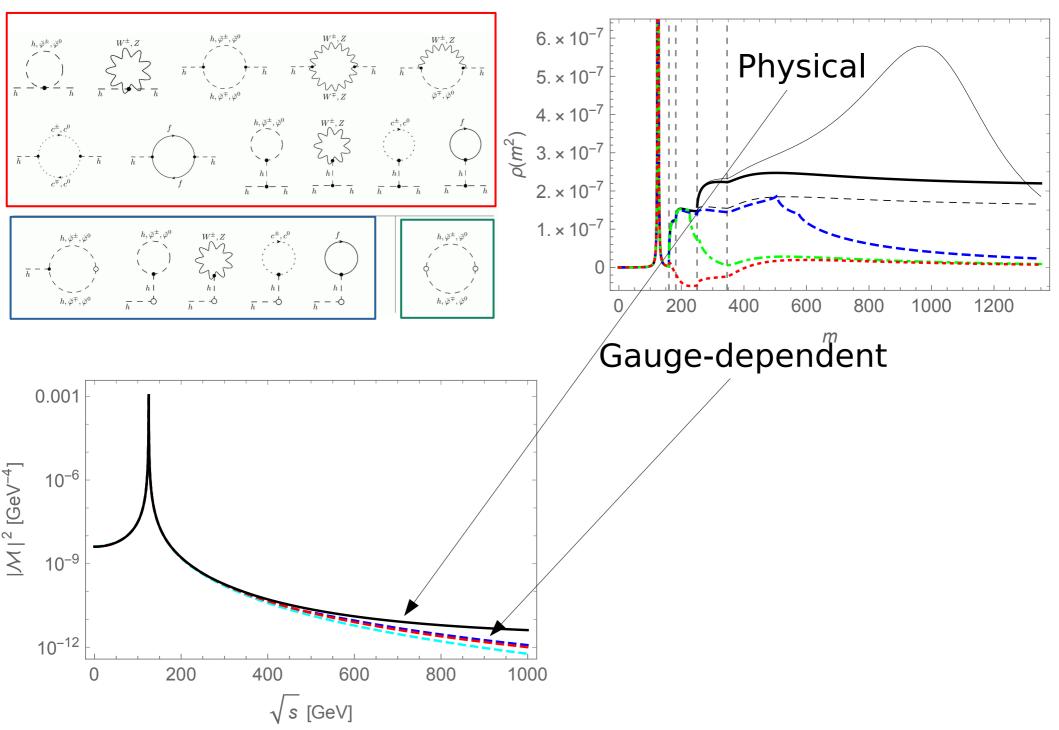
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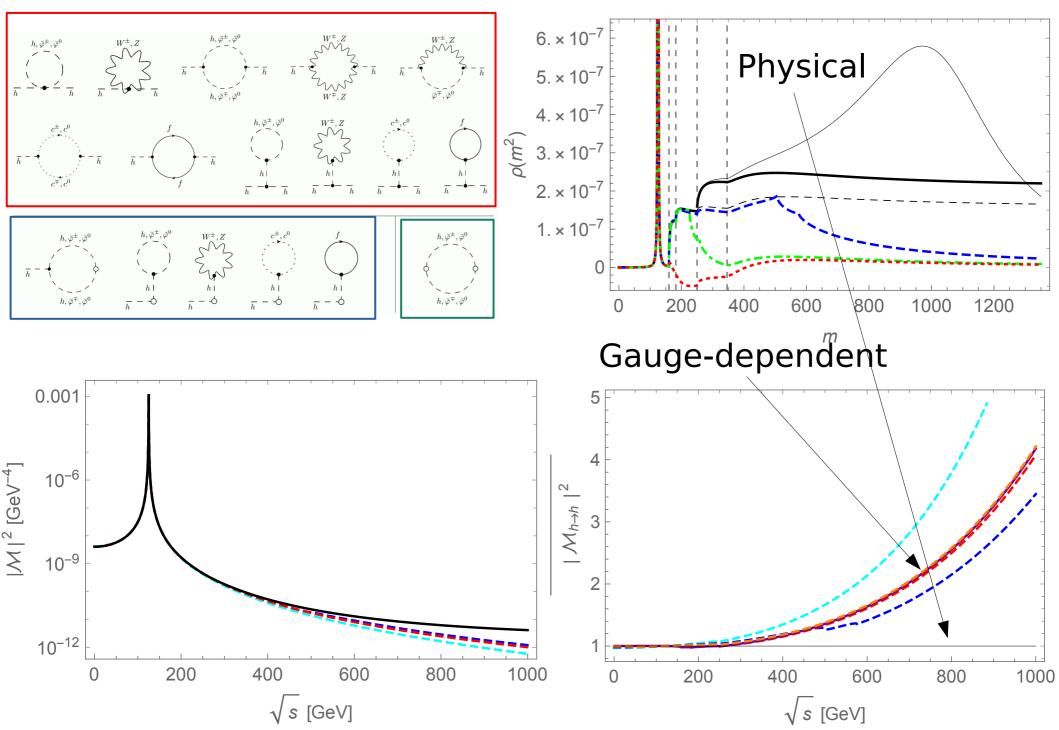


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[Fröhlich et al.'80,'81 Maas'12]

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Matrix from group structure

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$$= v^2 \langle W^i_{\mu} W^j_{\mu} \rangle + \dots$$

Matrix from group structure

- **1)** Formulate gauge-invariant operator  $1^{-}$  triplet:  $\langle (\tau^{i}h^{+}D_{\mu}h)(x)(\tau^{j}h^{+}D_{\mu}h)(y) \rangle$
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Matrix from group structure

*c* projects custodial states to gauge states

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*c* projects custodial states to gauge states

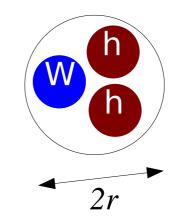
Exactly one gauge boson for every physical state

# Phenomenological Implications

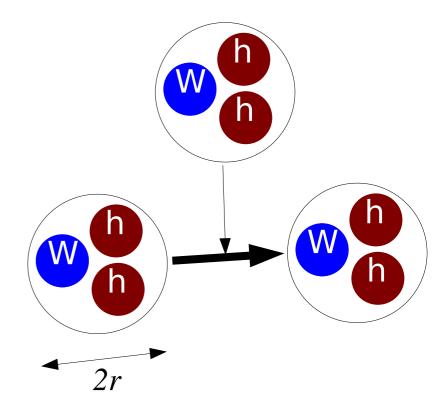
Can we measure this?

- Bound states have an extension
  - Can it be measured?

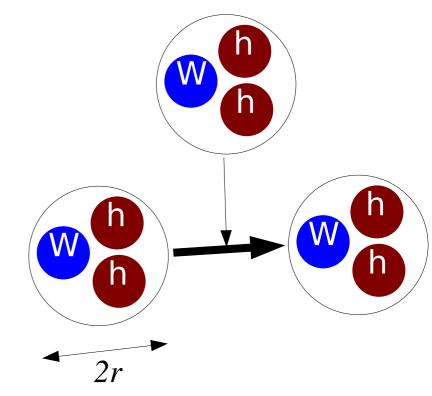
- Bound states have an extension
  - Can it be measured?
  - Example: Vector



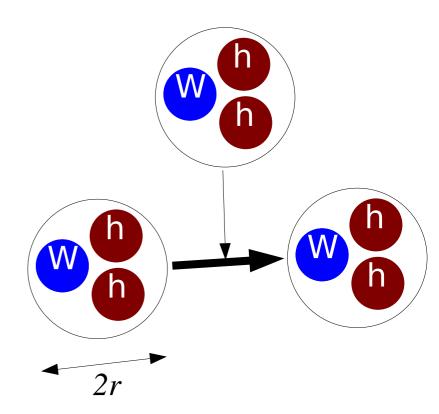
- Bound states have an extension
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  - Measure the form factor



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  - Measure the form factor  $F(q^2, q^2, q^2)$

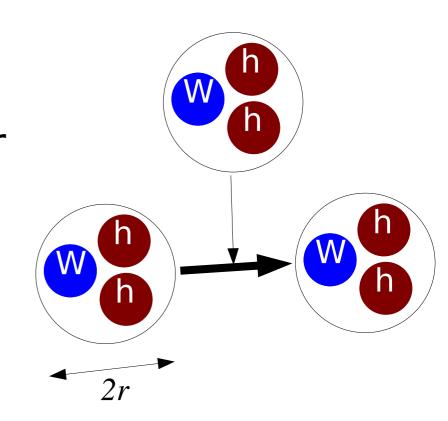


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  - Measure the form factor  $F(q^2, q^2, q^2) = 1 - \frac{q^2 \langle r^2 \rangle}{6} + ...$



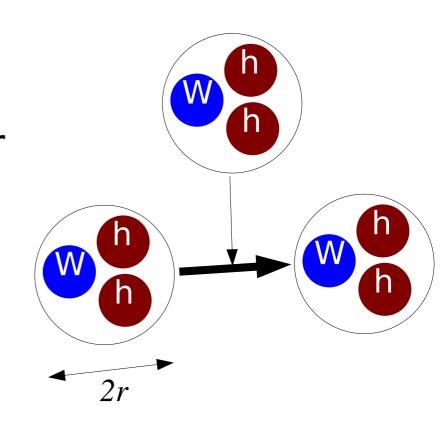
[Maas,Raubitzke,Törek'18]

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[Maas,Raubitzke,Törek'18]

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Maas,Raubitzke,Törek'18]

2r

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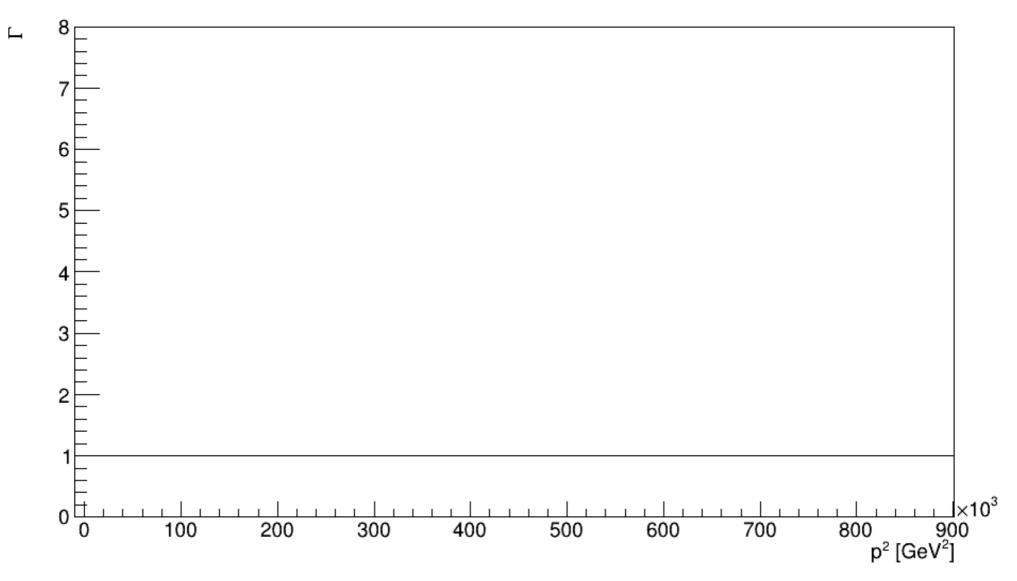
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- $=\frac{1}{q^2-m^2}+...$  Comparison proton:  $mr\sim5$  Here: Lattice

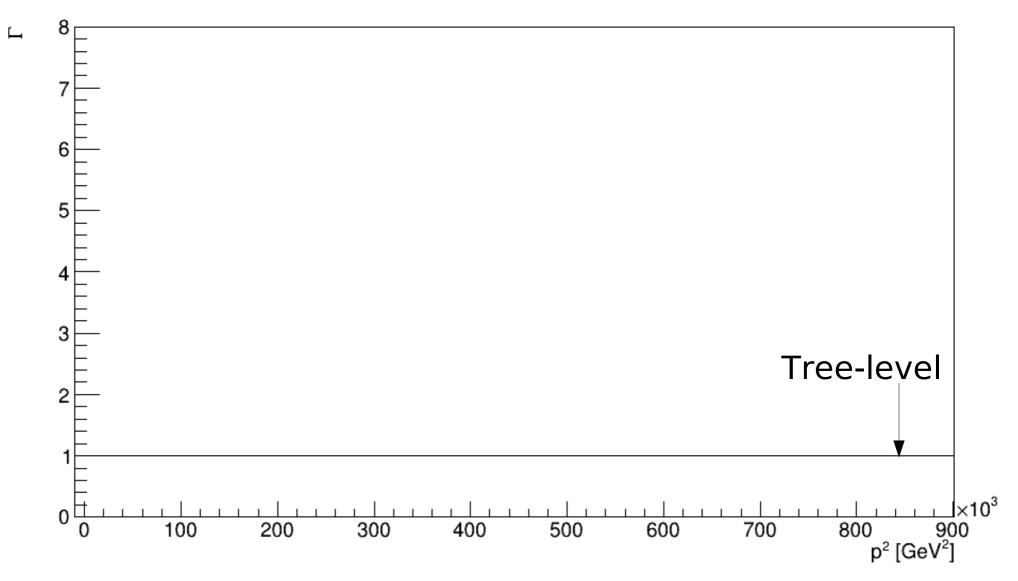
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- $=\frac{1}{q^2-m^2}+...$  Comparison proton:  $mr\sim5$  Here: Lattice
  - Experimentally hard, but possible

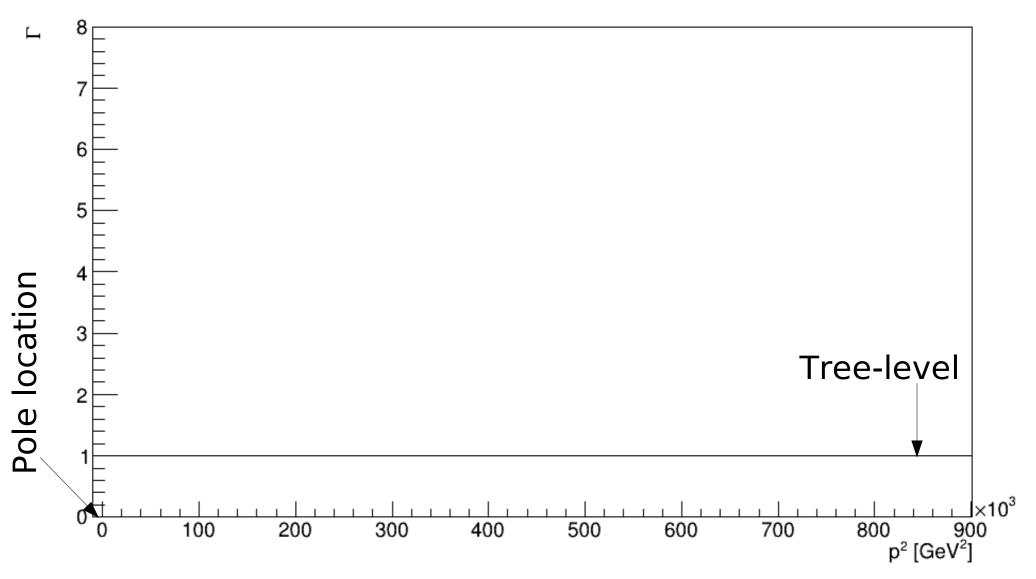
[Maas,Raubitzke,Törek'18]



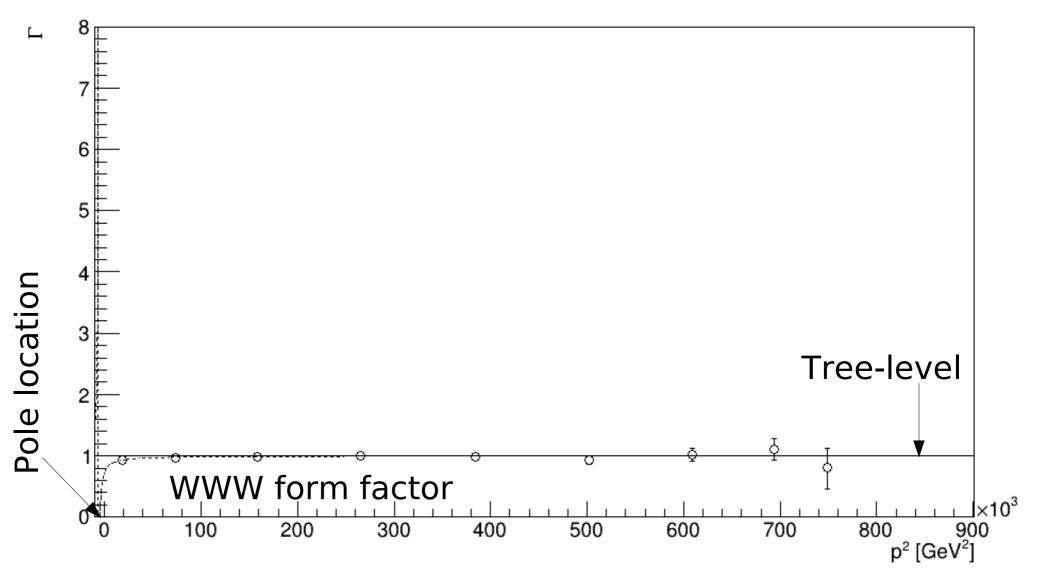
[Maas,Raubitzke,Törek'18]



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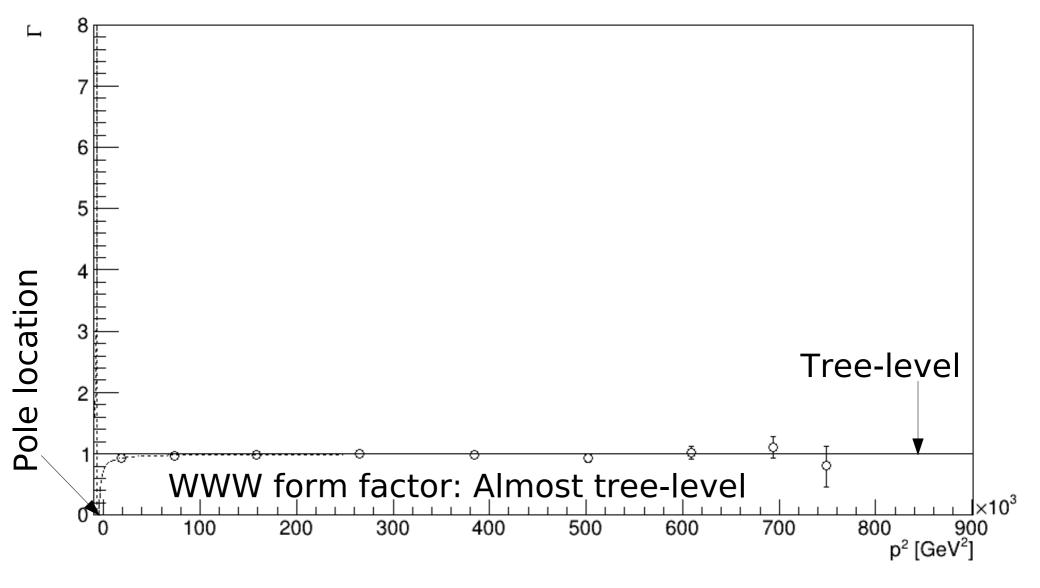


[Maas,Raubitzke,Törek'18]



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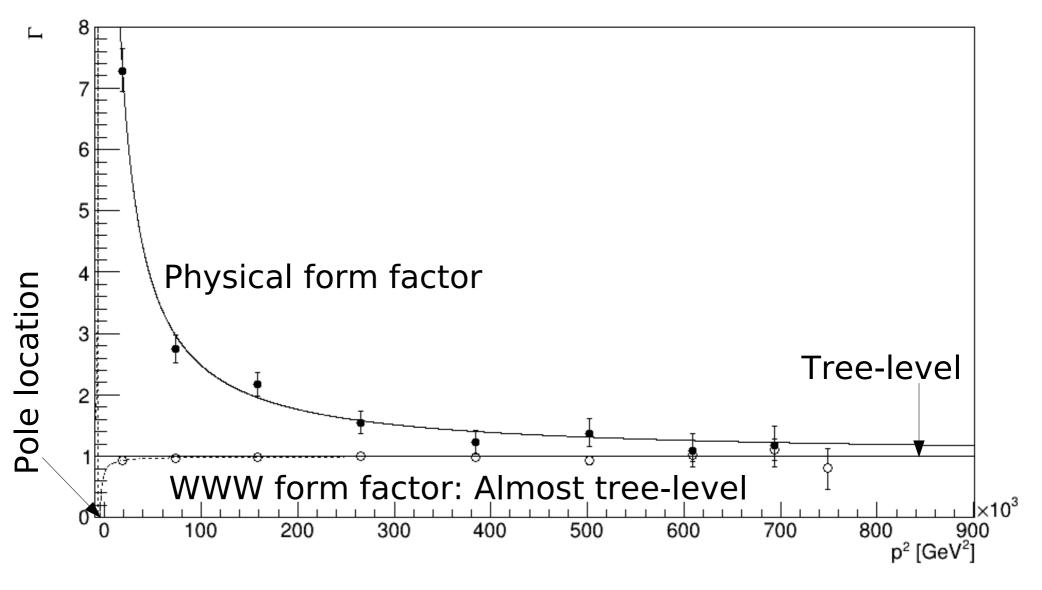
Vector form factor



• Gauge-dependent W has mr~0.5i

[Maas,Raubitzke,Törek'18]

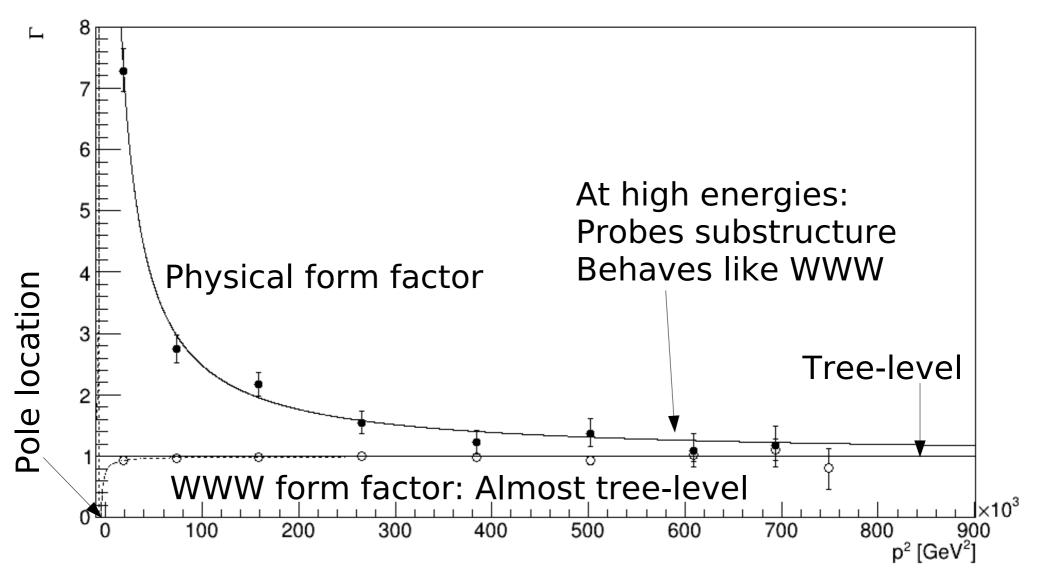
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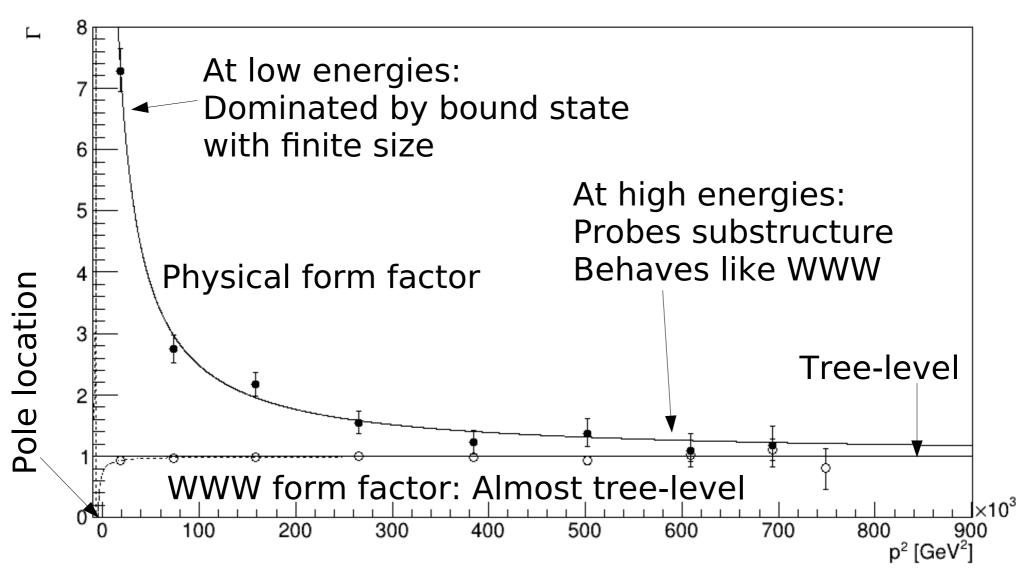
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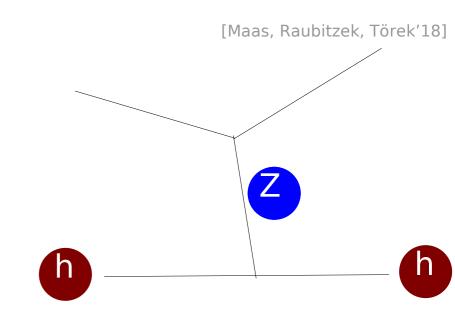
• Physical mr~2 while gauge-dependent W has mr~0.5i

## Measuring the radius

Two standard possibilities

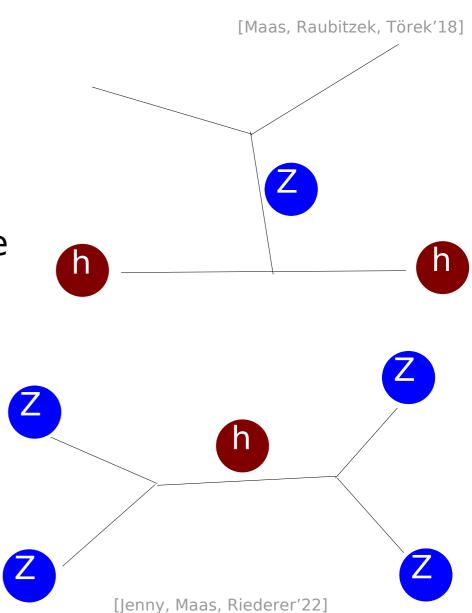
# Measuring the radius

- Two standard possibilities
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process



# Measuring the radius

- Two standard possibilities
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process
  - Elastic scattering
    - Standard vector boson scattering process at low energies
    - Use this one



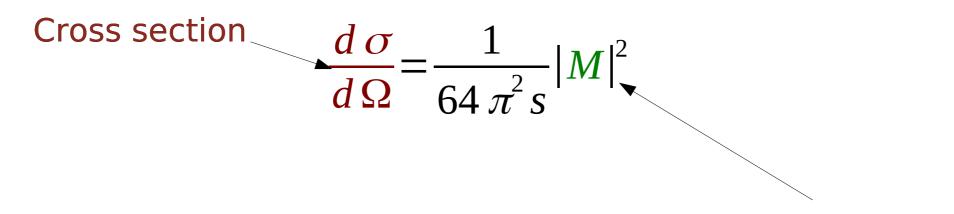
- Elastic region:  $160/180 \, GeV \leq \sqrt{s} \leq 250 \, GeV$ 
  - s is the CMS energy in the initial/final ZZ/WW system

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Matrix element

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$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |M|^2$$
  
M(s,  $\Omega$ ) = 16 $\pi \sum_{J} (2J+1) f_J(s) P_J(\cos\theta)$ 

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Matrix element  

$$\frac{d \sigma}{d \Omega} = \frac{1}{64 \pi^2 s} |M|^2 \quad \text{Partial wave amplitude}$$

$$M(s, \Omega) = 16 \pi \sum_{J} (2J+1) f_{J}(s) P_{J}(\cos \theta)$$
Legendre polynom

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amplitude

Phase shift

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$$f_J(s) = e^{i \delta_J(s)} \sin(\delta_J(s))$$

$$s \to 4m_W^2$$

$$a_0 = \tan(\delta_J) / \sqrt{s - 4m_W^2}$$
Phase shift

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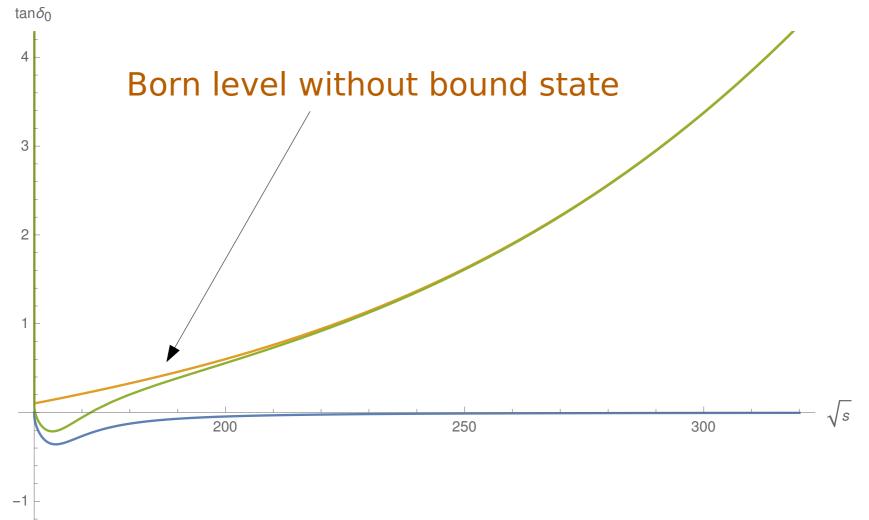
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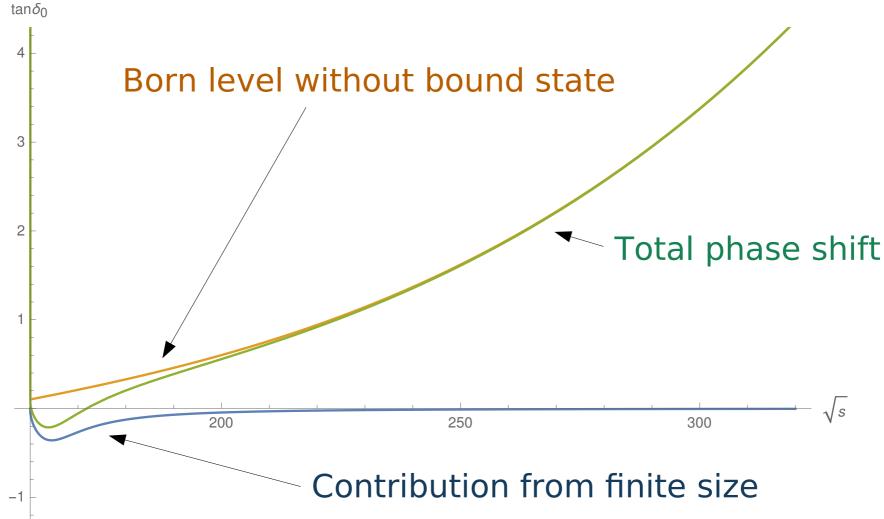
$$f_J(s) = e^{i \delta_J(s)} \sin(\delta_J(s))$$

$$s \to 4m_W^2 \tan(\delta_J) / \sqrt{s - 4m_W^2}$$
Scattering length~"size" Phase shift

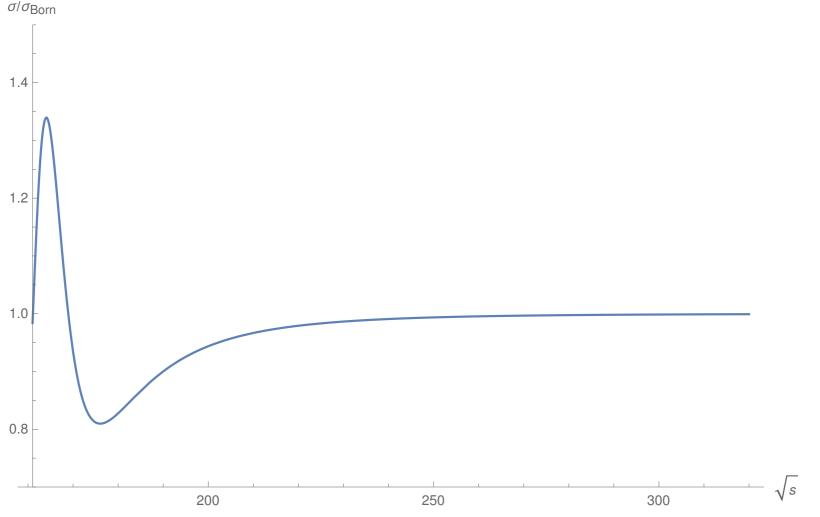
• Consider the Higgs: *J*=0



• Consider the Higgs: J=0

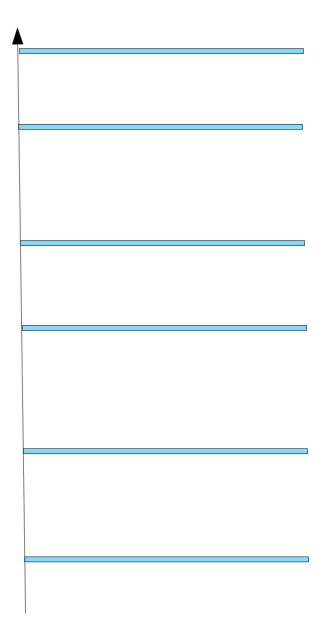


- Consider the Higgs: *J*=0
- Mock-up effect
  - Scattering length 1/(40 GeV)

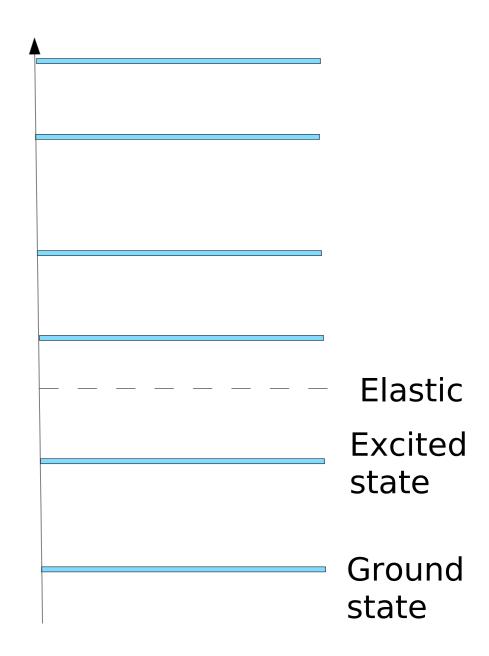


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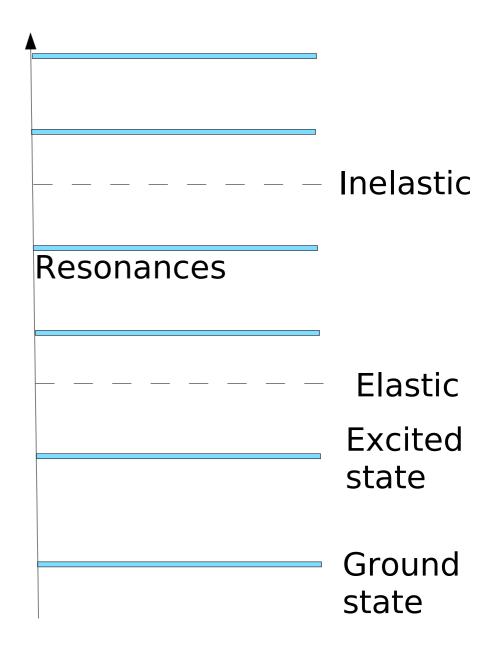
- Each quantum number channel has a spectrum
  - Discreet in a finite volume



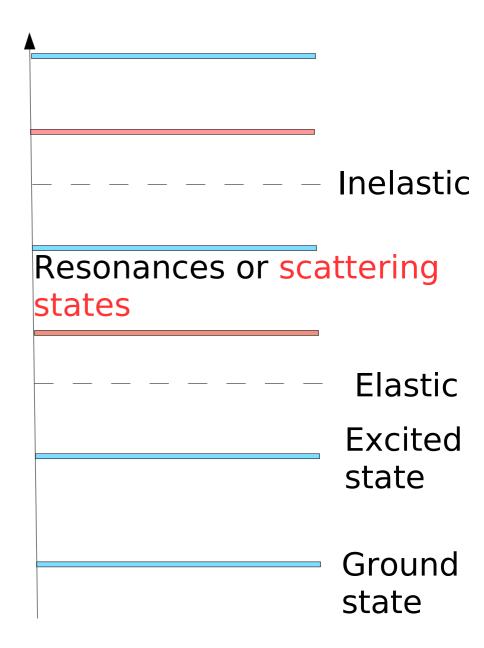
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  - Discreet in a finite volume
- States can be either stable, excited states,



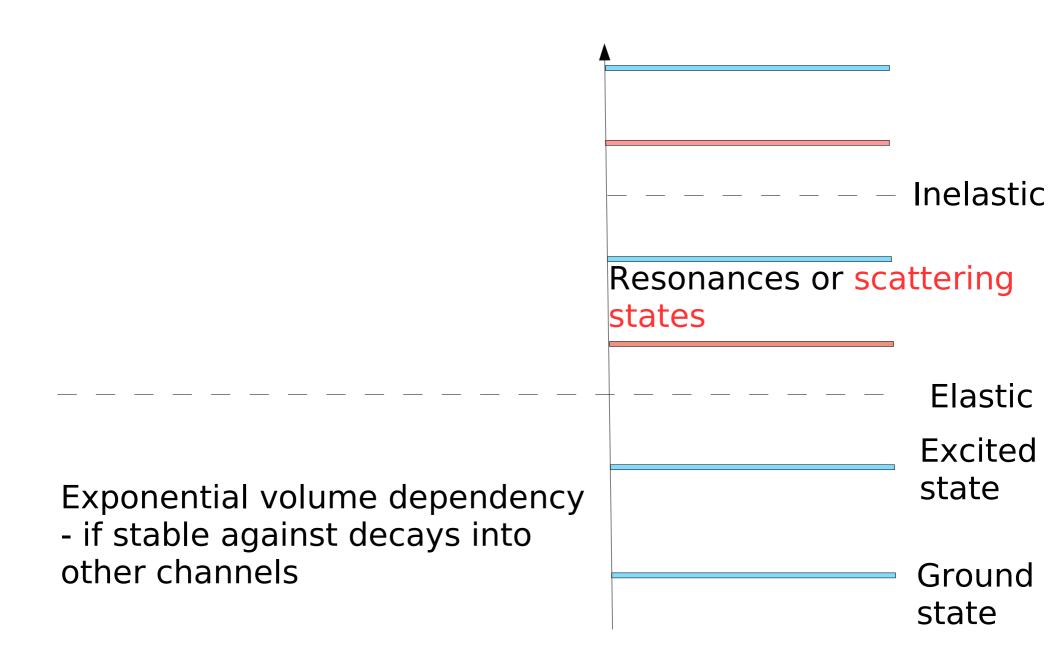
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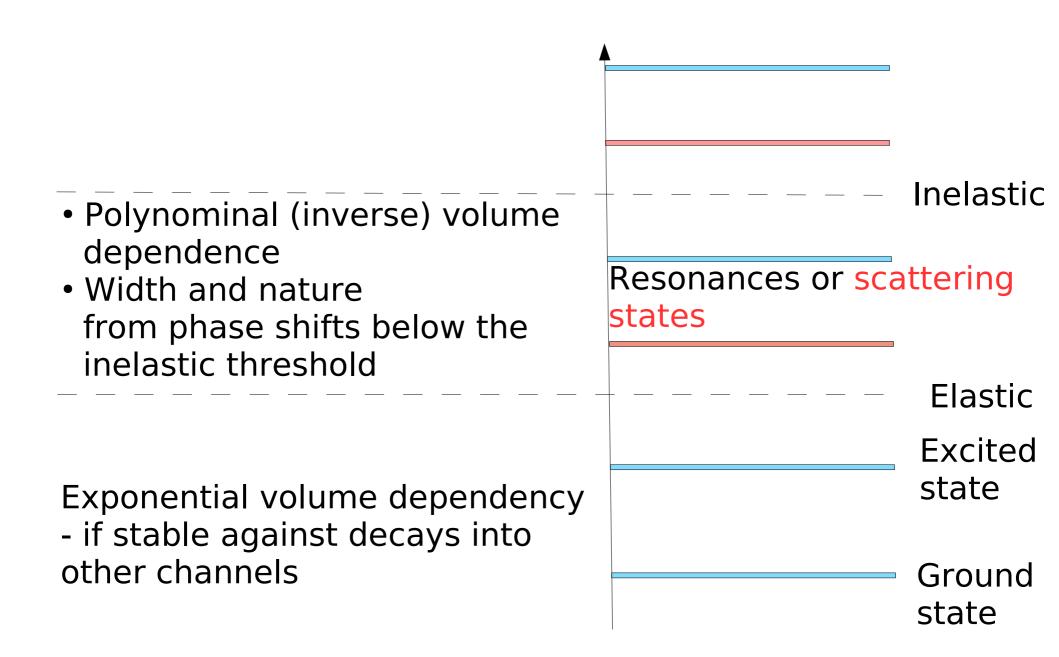
- Each quantum number channel has a spectrum
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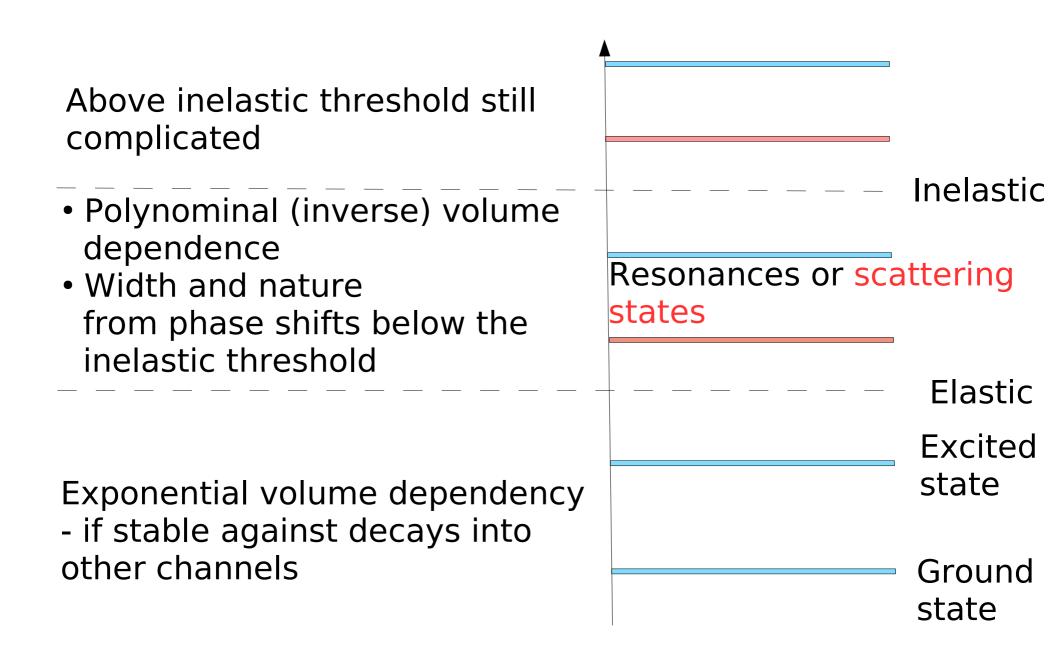


#### [Luescher'85,'86,'90,'91]

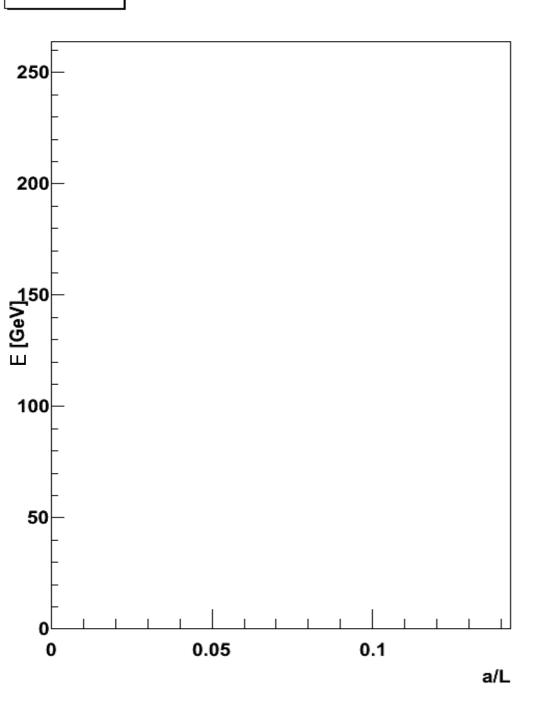


#### [Luescher'85,'86,'90,'91]

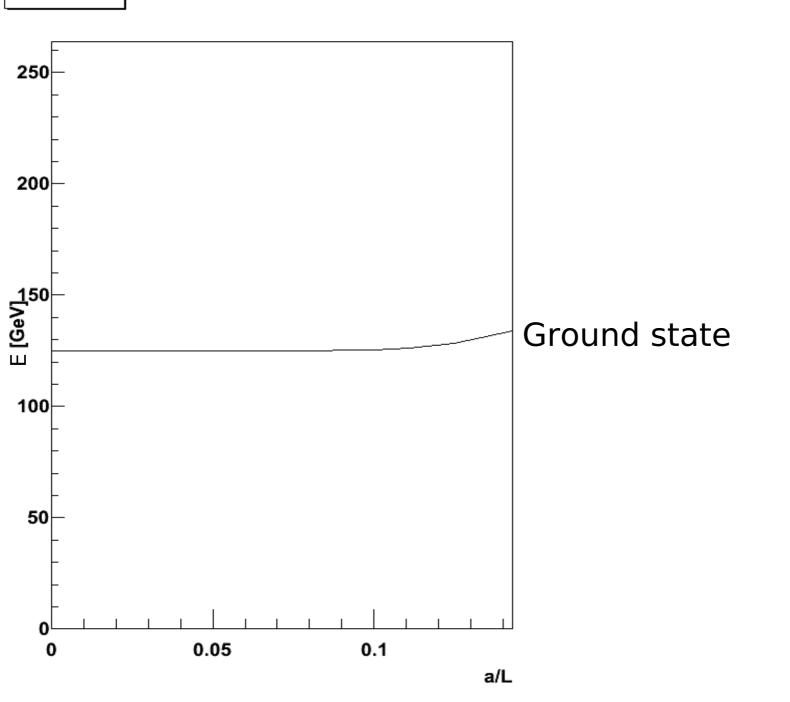




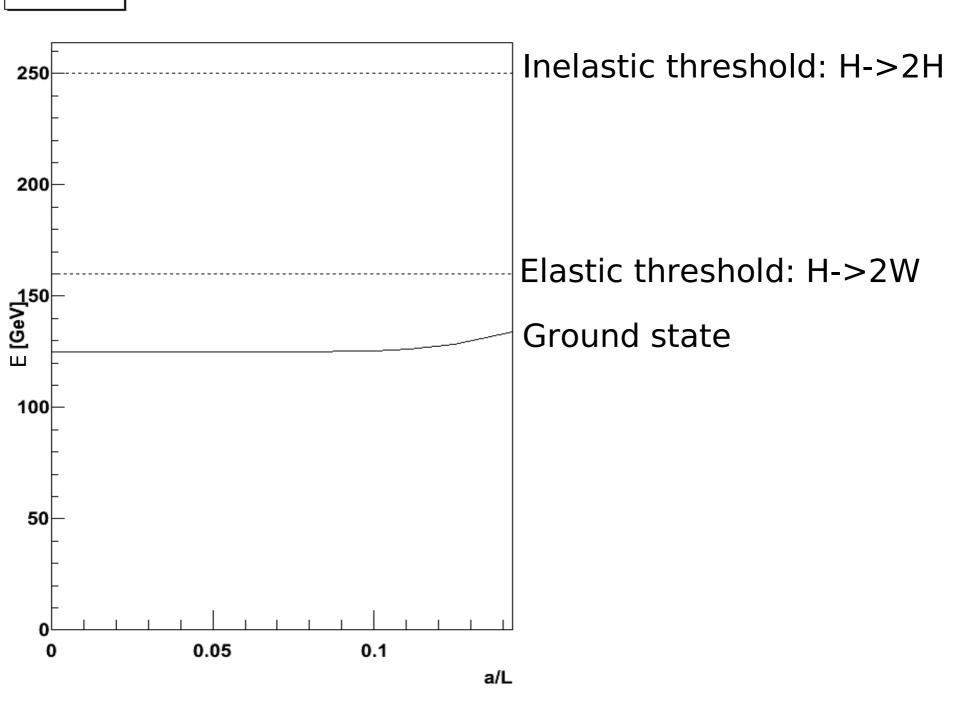
Spectrum

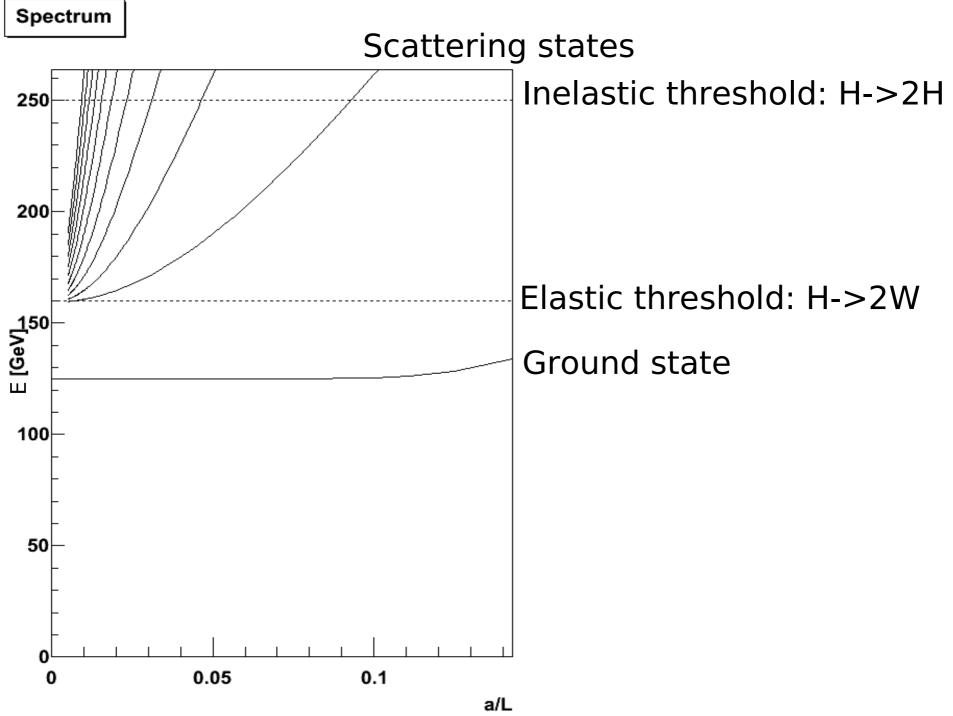


Spectrum

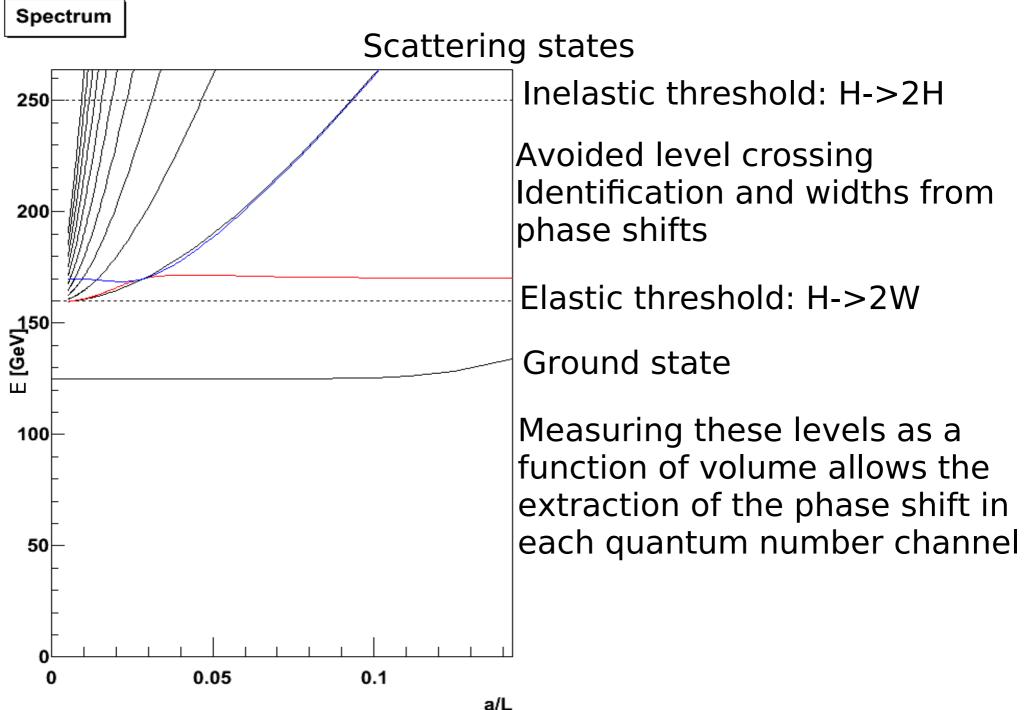


Spectrum









250

200

150 [Ve0] ]

100

50

0

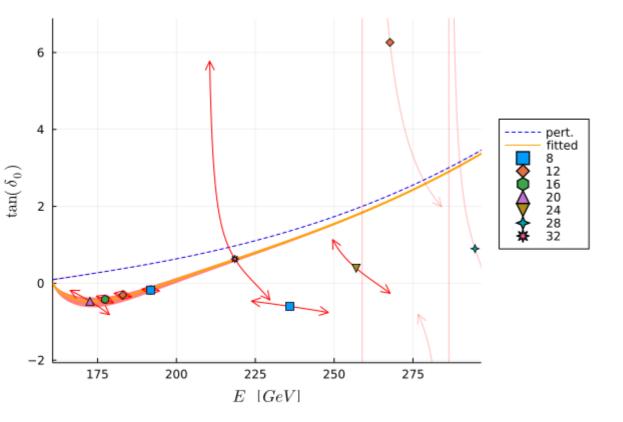
#### [Luescher'85,'86,'90,'91] Spectrum Scattering states Inelastic threshold: H->2H Avoided level crossing Identification and widths from phase shifts Elastic threshold: H->2W Ground state Measuring these levels as a function of volume allows the extraction of the phase shift in each quantum number channel $\tan \delta(E) = \frac{L \pi^{3/2} \sqrt{E^2/4 - m^2}}{2 \pi a Z (1, E^2/4 - m^2)}$ 0.1 0.05

a/L

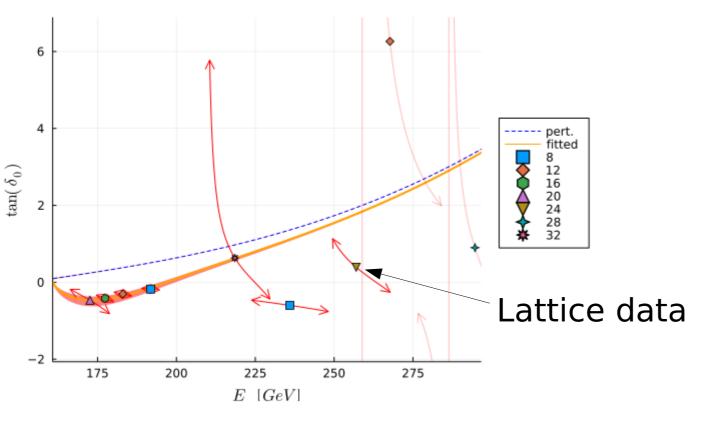
#### Spectrum Scattering states Inelastic threshold: H->2H 250 Avoided level crossing Identification and widths from 200 phase shifts Elastic threshold: H->2W 150<sup>′</sup> [780] [067] Ground state Measuring these levels as a 100 function of volume allows the extraction of the phase shift in each quantum number channel 50 $\tan \delta(E) = \frac{L \pi^{3/2} \sqrt{E^2/4 - m^2}}{2 \pi g Z (1, E^2/4 - m^2)}$ 0.1 0.05 0 a/L Geometry

[Luescher'85,'86,'90,'91]

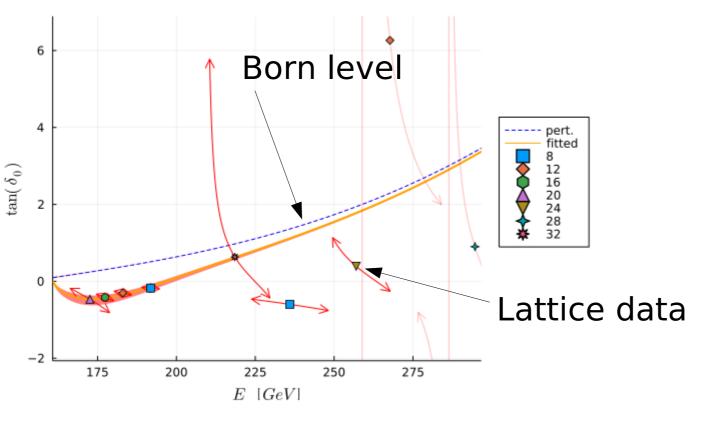
- Reduced SM: Only W/Z and the Higgs
  - Parameters slightly different
    - Higgs too heavy (145 GeV) and too strong weak coupling
  - Qualitatively but not quantitatively



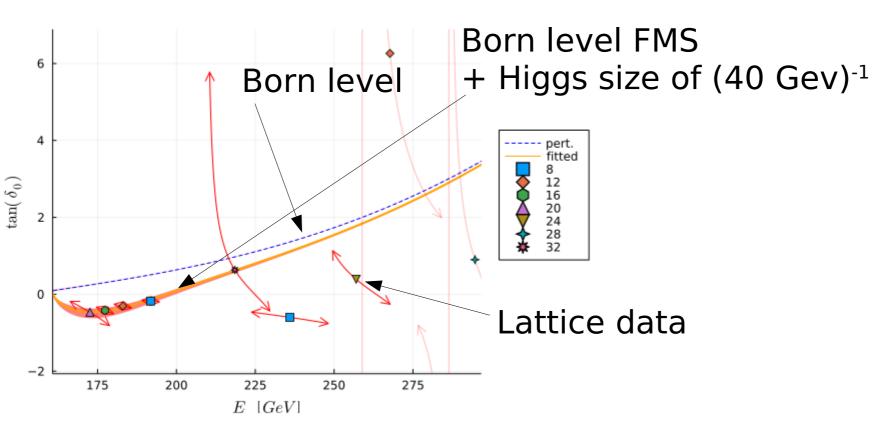
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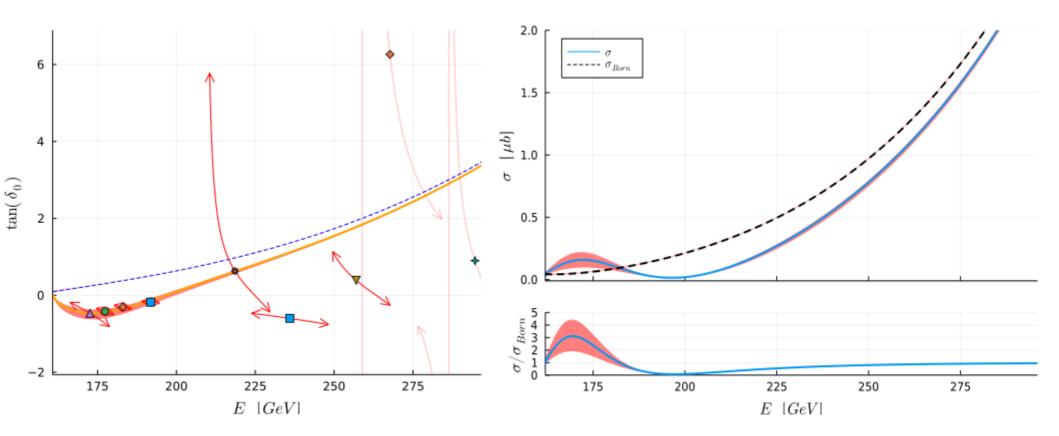
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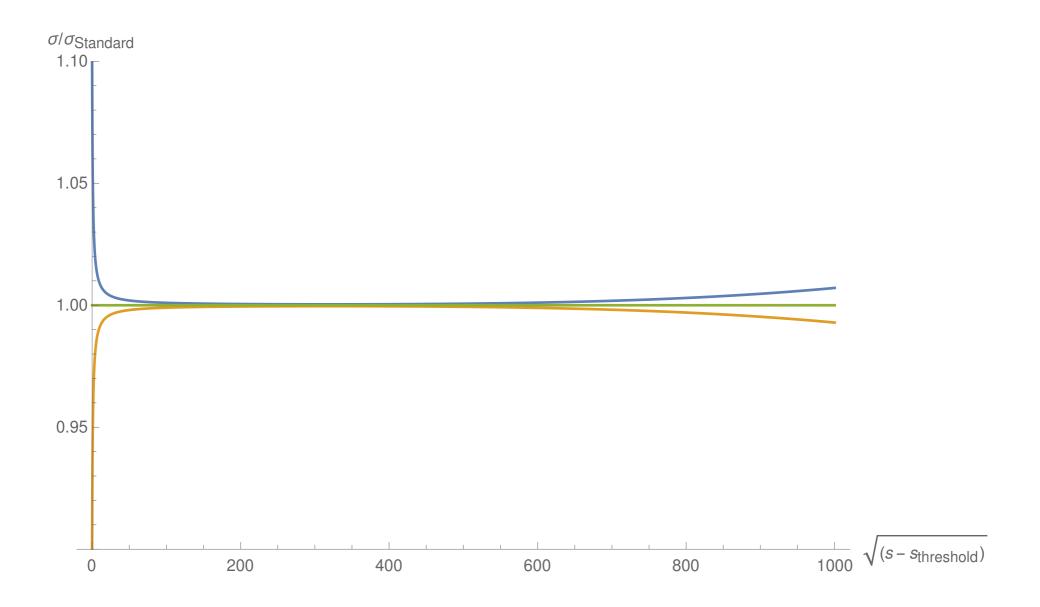
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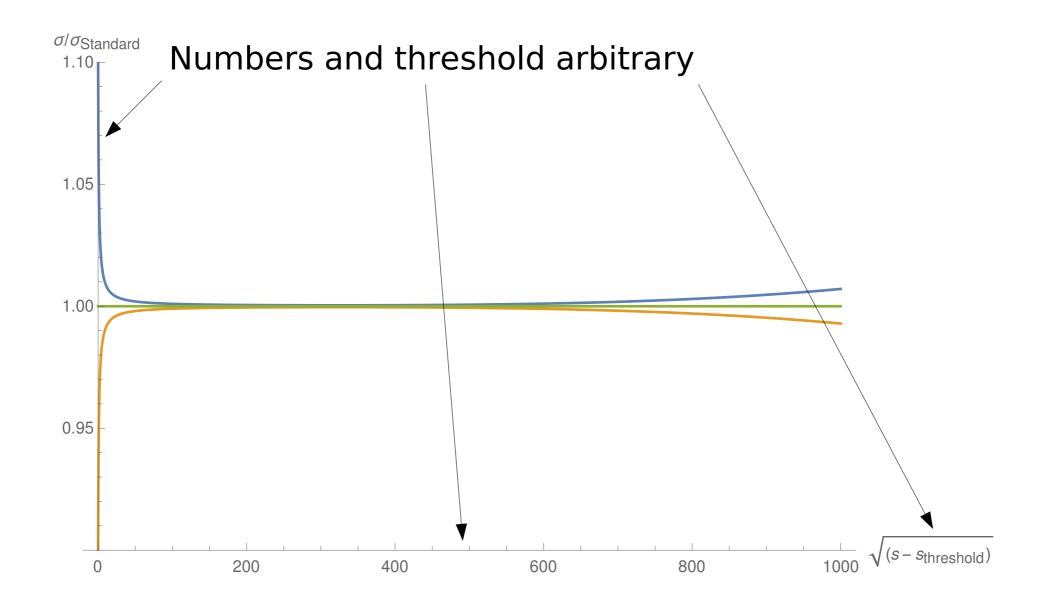


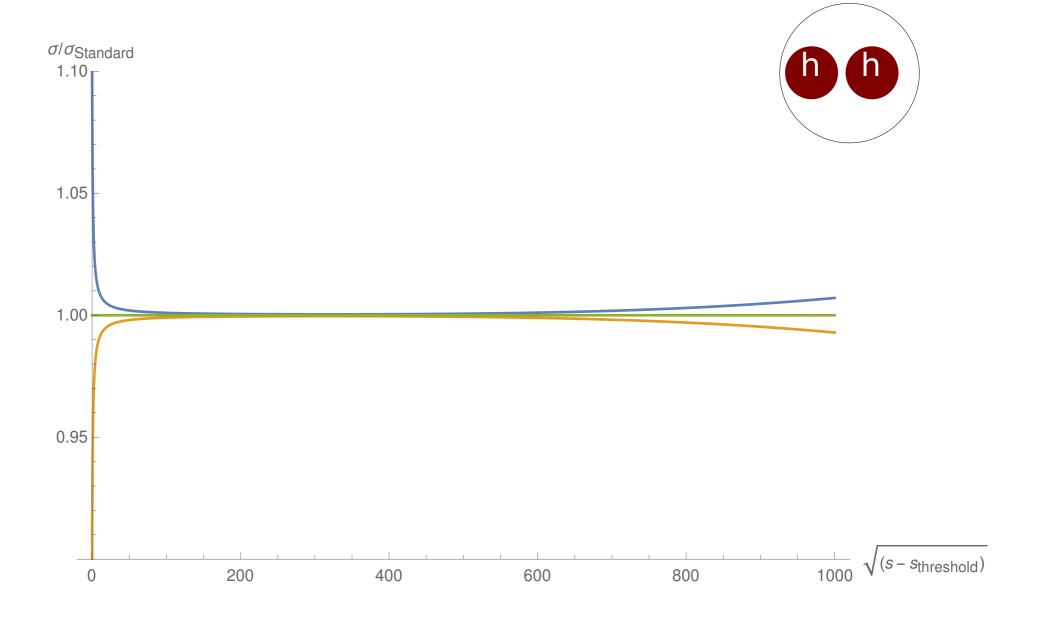
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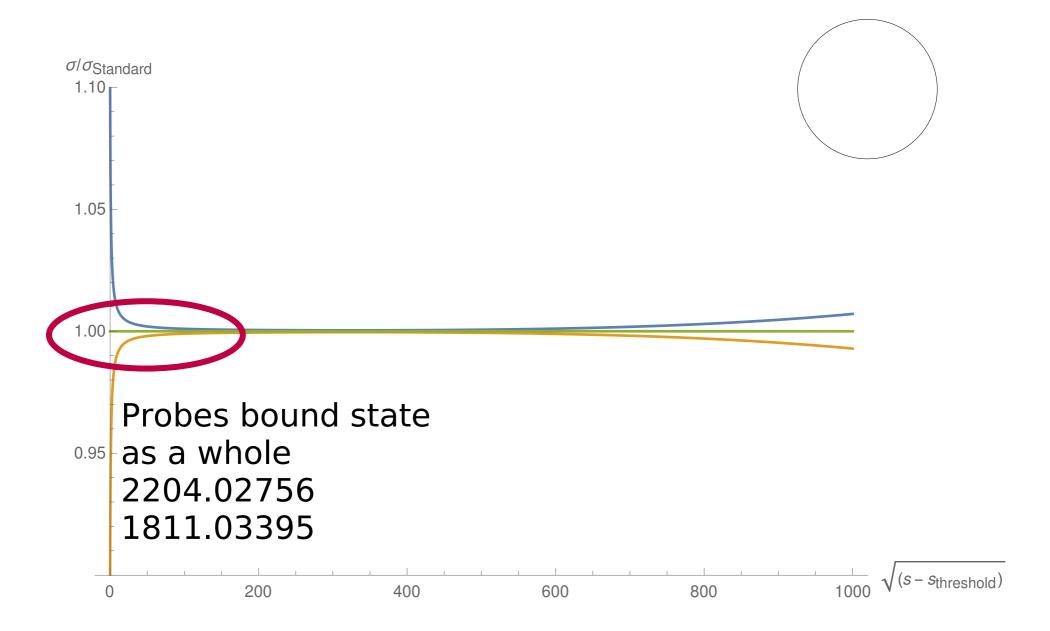


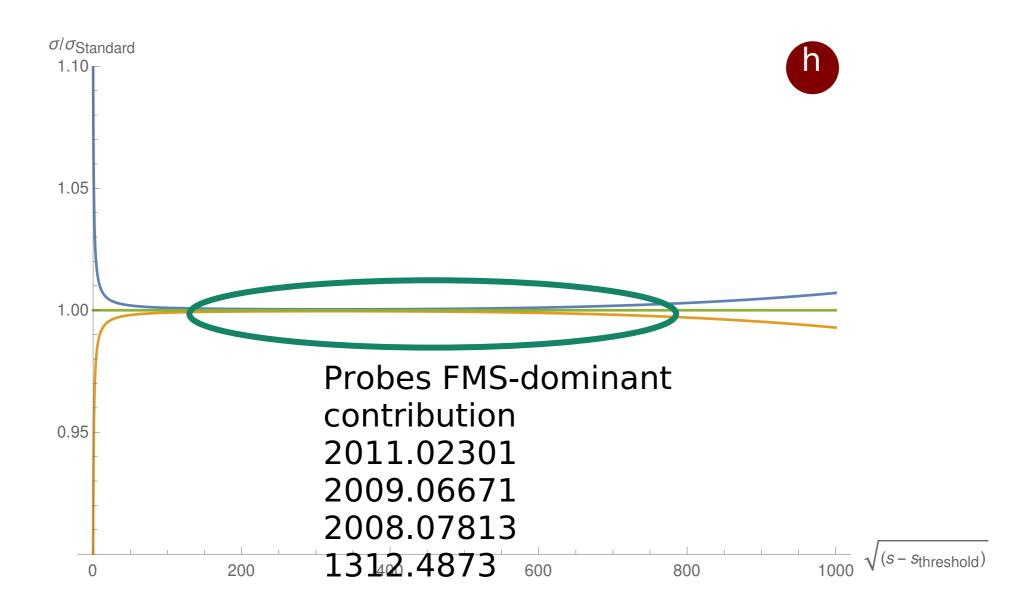
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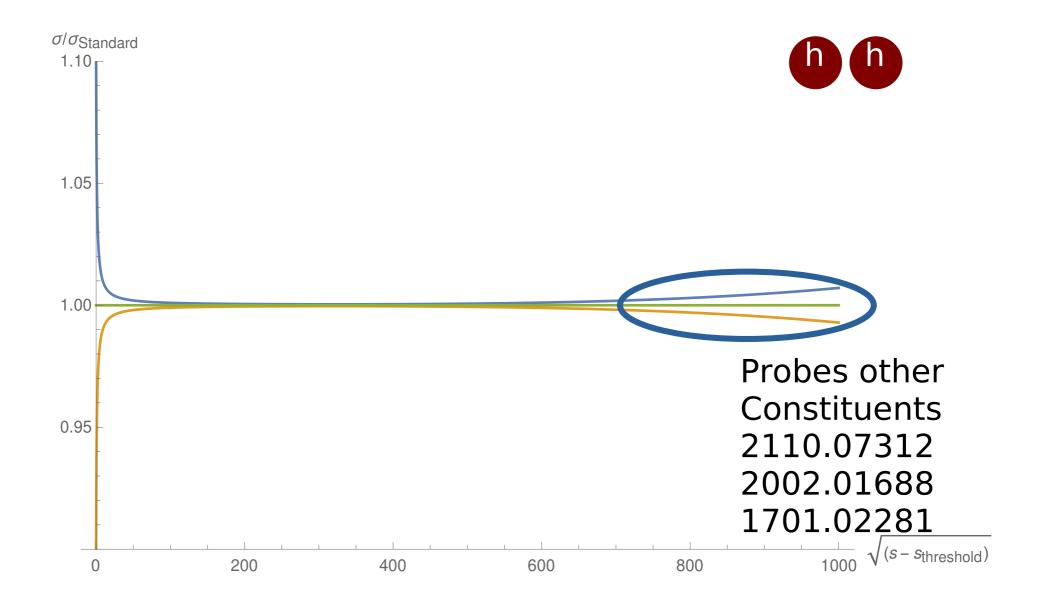












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- Analytically treatable with FMS
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- Invalidates many new physics scenarios
- FMS applicable to many theories
  - MSSM, Quantum gravity, supergravity,...

🔰 @axelmaas