# New effects in precision <br> Brout-Englert-Higgs physics 

Axel Maas
$1^{\text {st }}$ of December 2022 Ljubljana
 Slovenia

NAWI Graz
Natural Sciences


Der Wissenschaftsfonds. @ @axelmaas@sciencemastodon.com

## What is this talk about?

- Gauge invariance and the Brout-Englert Higgs effect
- Physical states
- Deviations and signals at experiments
- Implications beyond the standard model


## What's the deal? <br> Gauge symmetry

A toy model

A toy model: Higgs sector of the SM

## A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar


## A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c}
\end{gathered}
$$

- Ws $W_{\mu}^{a}$ W
- Coupling $g$ and some numbers $f^{a b c}$


## A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

- Ws $W_{\mu}^{a}$ W
- Higgs $h_{i}$ h
- Coupling $g$ and some numbers $f^{a b c}$ and $t_{a}^{i j}$


## A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k}+\lambda\left(h^{a} h_{a}^{+}-v^{2}\right)^{2} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

- Ws $W_{\mu}^{a}$ W
- Higgs $h_{i}$ h
- Couplings $g, v, \lambda$ and some numbers $f^{a b c}$ and $t_{a}^{i j}$


## A toy model: Higgs sector of the SM

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k}+\lambda\left(h^{a} h_{a}^{+}-v^{2}\right)^{2} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

- Ws $W_{\mu}^{a}$ W
- Higgs $h_{i}$ h
- Couplings $g, v, \lambda$ and some numbers $f^{a b c}$ and $t_{a}^{i j}$
- Parameters selected for a BEH effect


## A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k}+\lambda\left(h^{a} h_{a}^{+}-v^{2}\right)^{2} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{v} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

## A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k}+\lambda\left(h^{a} h_{a}^{+}-v^{2}\right)^{2} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

- Local $\operatorname{SU}(2)$ gauge symmetry $W_{\mu}^{a} \rightarrow W_{\mu}^{a}+\left(\delta_{b}^{a} \partial_{\mu}-g f_{b c}^{a} W_{\mu}^{c}\right) \phi^{b}$ $h_{i} \rightarrow h_{i}+g t_{a}^{i j} \phi^{a} h_{j}$


## A toy model: Symmetries

- Consider an SU(2) with a single fundamental scalar
- Essentially the standard model Higgs

$$
\begin{gathered}
L=-\frac{1}{4} W_{\mu \nu}^{a} W_{a}^{\mu \nu}+\left(D_{\mu}^{i j} h^{j}\right)^{+} D_{i k}^{\mu} h_{k}+\lambda\left(h^{a} h_{a}^{+}-v^{2}\right)^{2} \\
W_{\mu \nu}^{a}=\partial_{\mu} W_{v}^{a}-\partial_{\nu} W_{\mu}^{a}+g f_{b c}^{a} W_{\mu}^{b} W_{v}^{c} \\
D_{\mu}^{i j}=\delta^{i j} \partial_{\mu}-i g W_{\mu}^{a} t_{a}^{i j}
\end{gathered}
$$

- Local $\operatorname{SU}(2)$ gauge symmetry
$W_{\mu}^{a} \rightarrow W_{\mu}^{a}+\left(\delta_{b}^{a} \partial_{\mu}-g f_{b c}^{a} W_{\mu}^{c}\right) \phi^{b}$ $h_{i} \rightarrow h_{i}+g t_{a}^{i j} \phi^{a} h_{j}$
- Global SU(2) custodial (flavor) symmetry
- Acts as (right-)transformation on the scalar field only $W_{\mu}^{a} \rightarrow W_{\mu}^{a}$ $h \rightarrow h \Omega$


## Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect


## Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action


## Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain 'spontaenous gauge symmetry breaking': SU(2) $\rightarrow 1$


## Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain 'spontaenous gauge symmetry breaking': SU(2) $\rightarrow 1$
- Get masses and degeneracies at treelevel


## Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect
- Minimize the classical action
- Choose a suitable gauge and obtain 'spontaenous gauge symmetry breaking': SU(2) $\rightarrow 1$
- Get masses and degeneracies at treelevel
- Perform perturbation theory


## Physical spectrum

Perturbation theory
$0 \quad$ Mass

# Physical spectrum 

Perturbation theory
Scalar
$\backsim \Delta$ fixed charge

Custodial singlet

# Physical spectrum 

Perturbation theory

## Scalar Vector

$\backsim \wedge$ fixed charge gauge triplet

- Both custodial singlets


# The origin of the problem 

- Elementary fields are gauge-dependent


# The origin of the problem 

- Elementary fields are gauge-dependent
- Change under a gauge transformation


# The origin of the problem 

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice


# The origin of the problem 

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice...
- ...and gauge-symmetry breaking is not there


# The origin of the problem 

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice...
- ...and gauge-symmetry breaking is not there
- Just a figure of speech
- Actually just ordinary gauge-fixing


## The origin of the problem

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice...
- ...and gauge-symmetry breaking is not there
- Just a figure of speech
- Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities


## The origin of the problem

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice...
- ...and gauge-symmetry breaking is not there
- Just a figure of speech
- Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities
- And this includes non-perturbative aspects


## The origin of the problem

- Elementary fields are gauge-dependent
- Change under a gauge transformation
- Gauge transformations are a human choice...
- ...and gauge-symmetry breaking is not there
- Just a figure of speech
- Actually just ordinary gauge-fixing
- Physics has to be expressed in terms of manifestly gauge-invariant quantities
- And this includes non-perturbative aspects...
- ...even at weak coupling [Gribov'7, Singeri7, fujikawa'82]


## Physical states

- Need physical, gauge-invariant particles


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W
(W) W


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.



## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

- Has nothing to do with weak coupling


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

- Has nothing to do with weak coupling
- Think QED (hydrogen atom!)


## Physical states

- Need physical, gauge-invariant particles
- Cannot be the elementary particles
- Non-Abelian nature is relevant
- Need more than one particle: Composite particles
- Higgs-Higgs, W-W, Higgs-Higgs-W etc.

- Has nothing to do with weak coupling
- Think QED (hydrogen atom!)
- Can this matter?


## How to make predictions

- JPC and custodial charge only quantum numbers


## How to make predictions

- J ${ }^{\text {PC }}$ and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods!


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods! - Lattice


## Lattice calculations

- Take a finite volume - usually a hypercube



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integral
- Can be done numerically
- Uses Monte-Carlo methods



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integral
- Can be done numerically
- Uses Monte-Carlo methods
- Artifacts
- Finite volume/discretization



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integral
- Can be done numerically
- Uses Monte-Carlo methods
- Artifacts
- Finite volume/discretization
- Masses vs. wave-lengths



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integral
- Can be done numerically
- Uses Monte-Carlo methods
- Artifacts
- Finite volume/discretization
- Masses vs. wave-lengths



## Lattice calculations

- Take a finite volume - usually a hypercube
- Discretize it, and get a finite, hypercubic lattice
- Calculate observables using path integral
- Can be done numerically
- Uses Monte-Carlo methods
- Artifacts
- Finite volume/discretization
- Masses vs. wave-lengths
- Euclidean formulation


Masses from Euclidean propagators

## Masses from Euclidean propagators

$$
D(p)=\left\langle O^{+}(p) O(-p)\right\rangle
$$

- Masses can be inferred from propagators


## Masses from Euclidean propagators

$$
D(p)=\left\langle O^{+}(p) O(-p)\right\rangle \sim \frac{1}{p^{2}+m^{2}}
$$

- Masses can be inferred from propagators


## Masses from Euclidean propagators

$$
\begin{gathered}
D(p)=\left\langle O^{+}(p) O(-p)\right\rangle \sim \frac{1}{p^{2}+m^{2}} \\
C(t)=\left\langle O^{+}(x) O(y)\right\rangle \sim \exp (-m \Delta t)
\end{gathered}
$$

- Masses can be inferred from propagators


## Masses from Euclidean propagators

$$
\begin{gathered}
D(p)=\left\langle O^{+}(p) O(-p)\right\rangle \sim \sum \frac{a_{i}}{p^{2}+m_{i}^{2}} \\
C(t)=\left\langle O^{+}(x) O(y)\right\rangle \sim \sum a_{i} \exp \left(-m_{i} \Delta t\right) \\
\sum a_{i}=1 \wedge m_{0}<m_{1}<\ldots
\end{gathered}
$$

- Masses can be inferred from propagators
- Long-time behavior relevant
- No exact results on time-like momenta


## Masses from Euclidean propagators



- Masses can be inferred from propagators
- Long-time behavior relevant
- No exact results on time-like momenta


## Masses from Euclidean propagators

Propagator

- Masses can be inferred from propagators
- Long-time behavior relevant
- No exact results on time-like momenta


## Masses from Euclidean propagators



- Masses can be inferred from propagators
- Long-time behavior relevant
- No exact results on time-like momenta


# Physical spectrum 

Perturbation theory

## Scalar Vector

$\backsim$ « fixed charge gauge triplet

- Both custodial singlets

Experiment tells that somehow the left is correct

Physical spectrum
Perturbation theory
Composite (bound) states
n ${ }^{\wedge}$ fixed charge gauge triplet


Experiment tells that somehow the left is correct Theory say the right is correct

Physical spectrum
Perturbation theory
Composite (bound) states
$\backsim \wedge$ fixed charge gauge triplet

| Scalar | Vector |
| :---: | :---: |
| $\sim \Delta$ fixed charge | gauge triplet |



Experiment tells that somehow the left is correct Theory say the right is correct There must exist a relation that both are correct

# Physical spectrum 

Perturbation theory
Scalar Vector
n ${ }^{\wedge}$ fixed charge gauge triplet
Mass

- Both custodial singlets

$$
h(x)^{+} h(x) \quad \text { h }
$$

# Physical spectrum 

Perturbation theory
Scalar Vector
n ${ }^{\wedge}$ fixed charge gauge triplet

## Gauge-invariant

 Scalar singlet- Both custodial singlets Custodial singlet

$$
h(x)^{+} h(x) \text { h }
$$

# Physical spectrum 

Perturbation theory
Scalar Vector
n $\sqrt{\text { dixed charge gauge triplet }}$

Gauge-invariant
Scalar singlet

- Both custodial singlets Custodial singlet


# Physical spectrum 



Both custodial singlets Custodial singlet

# Physical spectrum 

Perturbation theory
Scalar Vector
n ${ }^{\wedge}$ fixed charge gauge triplet

## Gauge-invariant

 Scalar singletVector
singlet

- Both custodial singlets Custodial singlet

$$
\operatorname{trt}^{a} \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}
$$



# Physical spectrum 

Perturbation theory
Scalar Vector
n ${ }^{\wedge}$ fixed charge gauge triplet

## Gauge-invariant

 Scalar singletVector
singlet

- Both custodial singlets Custodial singlet Triplet

$$
\operatorname{tr} @ \frac{h^{+}}{\sqrt{h^{+} h}} D_{\mu} \frac{h}{\sqrt{h^{+} h}}
$$



# Physical spectrum 

Perturbation theory
Scalar Vector
n $\sqrt{\text { d }}$ fixed charge gauge triplet

Gauge-invariant
Scalar singlet

Equal!

Custodial singlet Triplet
Vector
singlet

Both custodial singlets

# A microscopic mechanism 

Why on-shell is important

## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods?


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods?
- But coupling is still weak and there is a BEH


## How to make predictions

- JPC and custodial charge only quantum numbers
- Different from perturbation theory
- Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
- Bound state structure - non-perturbative methods?
- But coupling is still weak and there is a BEH
- Perform double expansion ${ }_{\text {FFroblich etal: } 80, \text { Mas }{ }^{122]}}$
- Vacuum expectation value (FMS mechanism)
- Standard expansion in couplings
- Together: Augmented perturbation theory


## Augmented perturbation theory

1) Formulate gauge-invariant operator

## Augmented perturbation theory

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$

Higgs field

## Augmented perturbation theory

1) Formulate gauge-invariant operator

$$
0^{+} \text {singlet: }\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle
$$

## (h) $n$

## Augmented perturbation theory

1) Formulate gauge-invariant operator

$$
0^{+} \text {singlet: }\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle
$$

2) Expand Higgs field around fluctuations $h=v+\eta$

## Augmented perturbation theory

1) Formulate gauge-invariant operator

$$
0^{+} \text {singlet: }\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle
$$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

## Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator

$$
0^{+} \text {singlet: }\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle
$$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

3) Standard perturbation theory

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \boldsymbol{\eta}(y)\right\rangle \\
+\left\langle\boldsymbol{\eta}^{+}(x) \boldsymbol{\eta}(y)\right\rangle\left\langle\eta^{+}(x) \boldsymbol{\eta}(y)\right\rangle+O(g, \lambda)
\end{gathered}
$$

## Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

3) Standard perturbation theory

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+\left\langle\eta^{+}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \boldsymbol{\eta}(y)\right\rangle+O(g, \lambda)
\end{gathered}
$$

4) Compare poles on both sides

## Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

3) Standard perturbation theory

Bound state

$$
\begin{aligned}
& \left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
& +\left\langle\eta^{+}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)
\end{aligned}
$$

4) Compare poles on both sides

## Augmented perturbation theory

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

3) Standard perturbation theory Bound state mass

$$
\frac{\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle}{\left.\frac{\gamma \eta}{}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)}
$$

Trivial two-particle state
4) Compare poles on both sides

## Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

3) Standard perturbation theory

Bound
state mass

$$
\begin{aligned}
& \frac{\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle}{+\left\langle\eta^{+}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)} v^{2} \eta^{+}(x) \eta(y)
\end{aligned}
$$

Higgs mass
4) Compare poles on both sides

## Augmented perturbation theory

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{aligned}
& \left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
& \quad+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle \quad \text { Standard }
\end{aligned}
$$

Perturbation Theory
3) Standard perturbation theory Bound state mass

$$
\frac{\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle}{+\left\langle\eta^{+}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)}
$$

4) Compare poles on both sides

## Augmented perturbation theory

Mrohlich et al.'80,'81
Maas \& Sond

1) Formulate gauge-invariant operator $0^{+}$singlet: $\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{aligned}
& \left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
& +v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{aligned}
$$

What about this?
3) Standard perturbation theory

$$
\begin{aligned}
& \left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
& +\left\langle\eta^{+}(x) \eta(y)\right\rangle\left\langle\eta^{+}(x) \eta(y)\right\rangle+O(g, \lambda)
\end{aligned}
$$

4) Compare poles on both sides

## Consequences: The Higgs

$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

## Consequences: The Higgs



$$
\begin{gathered}
\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{gathered}
$$

## Consequences: The Higgs



$$
\begin{aligned}
&\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle \\
&+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle
\end{aligned}
$$

## Consequences: The Higgs


$\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle$
$+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle$

## Consequences: The Higgs


$\left\langle\left(h^{+} h\right)(x)\left(h^{+} h\right)(y)\right\rangle=v^{2}\left\langle\eta^{+}(x) \eta(y)\right\rangle$
$+v\left\langle\eta^{+} \eta^{2}+\eta^{+2} \eta\right\rangle+\left\langle\eta^{+2} \eta^{2}\right\rangle$

## Consequences: The Higgs



# Consequences: The Higgs 



## Consequences: The Higgs





## Consequences: The Higgs






## What about the vector?

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$

## What about the vector?

1) Formulate gauge-invariant operator

$$
1^{-} \text {triplet: }\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle
$$

2) Expand Higgs field around fluctuations $h=v+\eta$

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\left\langle\left(\tau^{i} h^{+} D_{\sharp} h\right)(x)\left(\tau^{j} h^{+} D_{\sharp} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\sharp}^{a}(x) W^{b}(y)^{u}\right\rangle+\ldots
$$

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\mu}^{a}(x) W^{b}(y)^{u}\right\rangle+\ldots
$$

Matrix from group structure

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\mu}^{a}(x) W^{b}(y)^{u}\right\rangle+\ldots \\
=v^{2}\left\langle W_{\mu}^{i} W_{u}^{j}\right\rangle+\ldots
\end{gathered}
$$

Matrix from group structure

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\begin{gathered}
\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\mu}^{a}(x) W^{b}(y)^{\mu}\right\rangle+\ldots \\
=v^{2}\left\langle W_{\mu}^{i} W_{\mu}^{j}\right\rangle+\ldots
\end{gathered}
$$

Matrix from group structure
c projects custodial states to gauge states

## What about the vector?

1) Formulate gauge-invariant operator 1- triplet: $\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle$
2) Expand Higgs field around fluctuations $h=v+\eta$

$$
\left\langle\left(\tau^{i} h^{+} D_{\mu} h\right)(x)\left(\tau^{j} h^{+} D_{\mu} h\right)(y)\right\rangle=v^{2} c_{i j}^{a b}\left\langle W_{\mu}^{a}(x) W^{b}(y)^{\mu}\right\rangle+\ldots
$$

c projects custodial states to gauge states

Exactly one gauge boson for every physical state

## Phenomenological Implications

Can we measure this?

## Bound states as extended objects

- Bound states have an extension
- Can it be measured?


## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor $F\left(q^{2}, q^{2}, q^{2}\right)$



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor

$$
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots
$$



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor

$$
\begin{gathered}
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots \\
=F_{W W W}\left(q^{2}, q^{2}, q^{2}\right)+\ldots
\end{gathered}
$$



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor

$$
\begin{gathered}
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots \\
=F_{W W W}\left(q^{2}, q^{2}, q^{2}\right)+\ldots \\
=\frac{1}{q^{2}-m^{2}}+\ldots
\end{gathered}
$$



## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor

$$
\begin{gathered}
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots \\
\quad=F_{W W W}\left(q^{2}, q^{2}, q^{2}\right)+\ldots
\end{gathered}
$$



$$
=\frac{1}{q^{2}-m^{2}}+\ldots
$$

- Comparison proton: mr~5


## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor


$$
\begin{gathered}
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots \\
=F_{W W W}\left(q^{2}, q^{2}, q^{2}\right)+\ldots \\
=\frac{1}{q^{2}-m^{2}}+\ldots
\end{gathered}
$$



- Comparison proton: mr~5 - Here: Lattice


## Bound states as extended objects

- Bound states have an extension
- Can it be measured?
- Example: Vector
- Measure the form factor


$$
\begin{gathered}
F\left(q^{2}, q^{2}, q^{2}\right)=1-\frac{q^{2}\left\langle r^{2}\right\rangle}{6}+\ldots \\
=F_{W W W}\left(q^{2}, q^{2}, q^{2}\right)+\ldots \\
=\frac{1}{q^{2}-m^{2}}+\ldots
\end{gathered}
$$



- Comparison proton: mr~5 - Here: Lattice
- Experimentally hard, but possible


## Bound states as extended objects

[Maas,Raubitzke,Törek'18]

## Vector form factor



## Bound states as extended objects

[Maas,Raubitzke,Törek'18]

## Vector form factor



## Bound states as extended objects

[Maas,Raubitzke,Törek'18]

## Vector form factor



## Bound states as extended objects

[Maas,Raubitzke,Törek'18]
Vector form factor


## Bound states as extended objects

[Maas,Raubitzke,Törek'18]

## Vector form factor



- Gauge-dependent W has mr~0.5i


## Bound states as extended objects

[Maas,Raubitzke,Törek'18]
Vector form factor


- Gauge-dependent W has mr~0.5i


## Bound states as extended objects

Vector form factor


- Gauge-dependent W has mr~0.5i


## Bound states as extended objects

Vector form factor


- Physical $m r \sim 2$ while gauge-dependent W has $m r \sim 0.5 i$


## Measuring the radius

- Two standard possibilities


## Measuring the radius

- Two standard possibilities
- Form factor
- Difficult
- Higgs and Z need to be both produced in the same process


## Measuring the radius

- Two standard possibilities
- Form factor
- Difficult
- Higgs and Z need to be both produced in the same process
- Elastic scattering
- Standard vector boson scattering process at low energies
- Use this one


## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system


## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis


## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

$$
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2}
$$

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

Cross section

$$
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2}
$$

Matrix element

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

Matrix element $\quad d \Omega=\frac{1}{64 \pi^{2} s}$

$$
-M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta)
$$

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

Matrix element

$$
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2}, \begin{aligned}
& \text { Partial wave } \\
& \text { amplitude }
\end{aligned}
$$

$$
-M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta)
$$

Legendre polynom

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

$$
\begin{gathered}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2} \\
M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta)
\end{gathered}
$$

Partial wave

$$
-f_{J}(s)=e^{i \delta_{J}(s)} \sin \left(\delta_{J}(s)\right)
$$



## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

$$
\begin{gathered}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2} \\
M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta)
\end{gathered}
$$

Partial wave

$$
-f_{J}(s)=e^{i \delta_{J}(s)} \sin \left(\delta_{J}(s)\right)
$$

Phase shift

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

$$
\begin{gathered}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2} \\
M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta) \\
f_{J}(s)=e^{i \delta_{J}(s)} \sin \left(\delta_{J}(s)\right) \\
a_{0} \stackrel{4 m_{W}^{2}}{=} \tan \left(\delta_{J}\right) / \sqrt{s-4 m_{W}^{2}}
\end{gathered}
$$

Phase shift

## Radius from elastic scattering in VBS

- Elastic region: $160 / 180 \mathrm{GeV} \leqslant \sqrt{s} \leqslant 250 \mathrm{GeV}$
- $s$ is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

$$
\begin{gathered}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2} s}|M|^{2} \\
M(s, \Omega)=16 \pi \sum_{J}(2 J+1) f_{J}(s) P_{J}(\cos \theta) \\
f_{J}(s)=e^{i \delta \delta_{J}(s)} \sin \left(\delta_{J}(s)\right) \\
s \rightarrow 4 m_{w}^{2} \\
a_{0} \stackrel{\operatorname{lan}}{ }=\tan \left(\delta_{J}\right)!\sqrt{s-4 m_{W}^{2}}
\end{gathered}
$$

Scattering length~"size"
Phase shift

## Impact of a finite size of the Higgs

Consider the Higgs: $J=0$

## Impact of a finite size of the Higgs



- Consider the Higgs: J=0


## Impact of a finite size of the Higgs



Contribution from finite size

- Consider the Higgs: $J=0$
- Mock-up effect
- Scattering length $1 /(40 \mathrm{GeV})$


# Impact of a finite size of the Higgs 



- Consider the Higgs: J=0
- Mock-up effect
- Scattering length $1 /(40 \mathrm{GeV})$


## Excited states on the lattice

- Each quantum number channel has a spectrum
- Discreet in a finite volume



## Excited states on the lattice

- Each quantum number channel has a spectrum
- Discreet in a finite volume
- States can be either stable, excited states,


Elastic
Excited state

Ground state

## Excited states on the lattice

- Each quantum number channel has a spectrum
- Discreet in a finite volume
- States can be either stable, excited states, resonances

』



Inelastic

Elastic
Excited state

Ground state

## Excited states on the lattice

- Each quantum number channel has a spectrum
- Discreet in a finite volume
- States can be either stable, excited states, resonances or scattering states
$\Delta$

$------\quad$ Inelastic

Resonances or scattering states

Elastic
Excited state

Ground state

# Excited states on the lattice 

# Excited states on the lattice 

- Polynominal (inverse) volume dependence
- Width and nature from phase shifts below the inelastic threshold

Elastic
Excited state

- if stable against decays into other channels

Inelastic

Exponential volume dependency
Resonances or scattering states
$\qquad$

Ground state

# Excited states on the lattice 

Above inelastic threshold still complicated

- Polynominal (inverse) volume dependence
- Width and nature from phase shifts below the inelastic threshold


## Inelastic <br> Resonances or scattering states

Elastic
Excited state

Ground state

## Excited states on the lattice

## Spectrum



## Excited states on the lattice

## Spectrum



## Excited states on the lattice

## Spectrum



Inelastic threshold: H->2H

Elastic threshold: $\mathrm{H}->2 \mathrm{~W}$
Ground state

# Excited states on the lattice 

```
Spectrum
```

Scattering states


## Excited states on the lattice

```
Spectrum
```

Scattering states


Inelastic threshold: $\mathrm{H}->2 \mathrm{H}$
Avoided level crossing Identification and widths from phase shifts

Elastic threshold: H->2W
Ground state
Measuring these levels as a function of volume allows the extraction of the phase shift in each quantum number channel

## Excited states on the lattice

## Spectrum

Scattering states


Inelastic threshold: H->2H
Avoided level crossing Identification and widths from phase shifts

Elastic threshold: H->2W
Ground state
Measuring these levels as a function of volume allows the extraction of the phase shift in each quantum number channel

$$
\tan \delta(E)=\frac{L \pi^{3 / 2} \sqrt{E^{2} / 4-m^{2}}}{2 \pi a Z\left(1, E^{2} / 4-m^{2}\right)}
$$

## Excited states on the lattice

## Spectrum

Scattering states

Inelastic threshold: $\mathrm{H}->2 \mathrm{H}$
Avoided level crossing Identification and widths from phase shifts
Elastic threshold: H->2W
Ground state
Measuring these levels as a function of volume allows the extraction of the phase shift in each quantum number channel

$$
\begin{aligned}
& \tan \delta(E)=\frac{L \pi^{3 / 2} \sqrt{E^{2} / 4-m^{2}}}{2 \pi a Z\left(1, E^{2} / 4-m^{2}\right)} \\
& \text { Geometry }
\end{aligned}
$$

## Impact on the radius of the Higgs

- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy ( 145 GeV ) and too strong weak coupling
- Qualitatively but not quantitatively


## Impact on the radius of the Higgs



- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy ( 145 GeV ) and too strong weak coupling
- Qualitatively but not quantitatively


## Impact on the radius of the Higgs



- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy ( 145 GeV ) and too strong weak coupling
- Qualitatively but not quantitatively


## Impact on the radius of the Higgs



- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy ( 145 GeV ) and too strong weak coupling
- Qualitatively but not quantitatively


## Impact on the radius of the Higgs



- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy (145 GeV) and too strong weak coupling
- Qualitatively but not quantitatively


## Impact on the radius of the Higgs



- Reduced SM: Only W/Z and the Higgs
- Parameters slightly different
- Higgs too heavy ( 145 GeV ) and too strong weak coupling
- Qualitatively but not quantitatively

Generic behavior: DIS-like


Generic behavior: DIS-like


Generic behavior: DIS-like


## Generic behavior: DIS-like



## Generic behavior: DIS-like



## Generic behavior: DIS-like



## Summary

- Field theory requires composite states


## Summary

- Field theory requires composite states
- Confirmed by lattice
- Analytically treatable with FMS
- Can have measurable impact


## Summary

- Field theory requires composite states
- Confirmed by lattice
- Analytically treatable with FMS
- Can have measurable impact
- Unaccounted-for SM background


## Summary

- Field theory requires composite states
- Confirmed by lattice
- Analytically treatable with FMS
- Can have measurable impact
- Unaccounted-for SM background
- Or: Guaranteed discovery of the effect in the SM or a serious theoretical problem


## Summary

- Field theory requires composite states
- Confirmed by lattice
- Analytically treatable with FMS
- Can have measurable impact
- Unaccounted-for SM background
- Or: Guaranteed discovery of the effect in the SM or a serious theoretical problem
- Invalidates many new physics scenarios


## Summary

- Field theory requires composite states
- Confirmed by lattice
- Analytically treatable with FMS
- Can have measurable impact
- Unaccounted-for SM background
- Or: Guaranteed discovery of the effect in the SM or a serious theoretical problem
- Invalidates many new physics scenarios
- FMS applicable to many theories
- MSSM, Quantum gravity, supergravity,...

