Gauge invariant spectra and FMS mechanism for gauge theories with BEH effect

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University of Jena, 08.10.2020

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Outline and Motivation

Outline:

- Motivation: GI formulation of Higgs theories.
- Introduction to FMS mechanism.
- Examples of spectroscopy with FMS.
- Composite fermion bound states lattice spectroscopy.

Motivation: Gauge invariant formulation of Higgs theories

Standard Higgs approach

- Gauge Higgs theory prototype:
  $\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) + \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi + \lambda(\phi^\dagger\phi - v^2)^2.$

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- This construction is gauge dependent.

Main motivation

- $\langle \phi \rangle$ is dependent on the gauge.
- There are gauges in which $\langle \phi \rangle = 0$.
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- Why PT works so well for the Standard Model?
- Is PT always reliable then?
- Answers lie in a gauge-invariant formulation of the theory.

GI description of SM and FMS mechanism

Gauge invariant formulation of SM observables

- Elementary fields are treated as observable in PT, even if not gauge invariant.
- The real physical objects must be described by gauge invariant composite operators.
  [Fröhlich,Morchio,Strocchi-Nucl.Phys.B190(1981)553-582]

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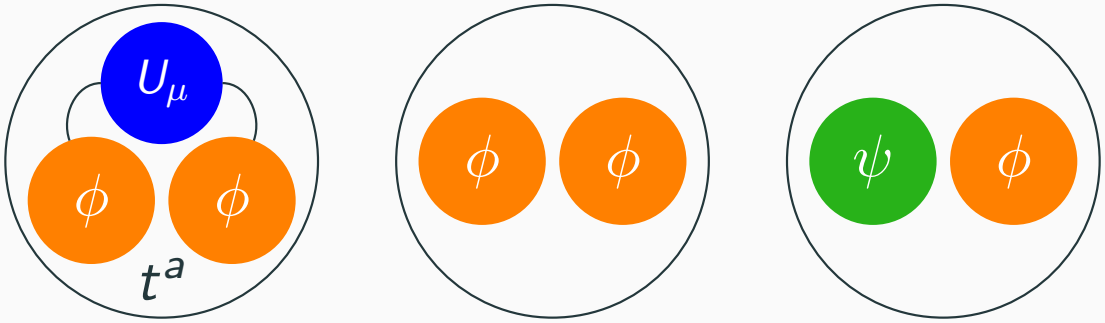
- One must not identify elementary fields in the lagrangian with physical particles, but use composite operators:
- Elementary Higgs $\phi(x) \to$ Physical Higgs $(\phi^\dagger\phi)(x)$
- Elementary Fermion $\psi(x) \to$ Physical fermion $(\phi^\dagger\psi)(x)$
- Elementary Vector $W^a_\mu(x) \to$ Physical Vector $(t^a\phi^\dagger D^\mu\phi)(x)$



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- Higgs is then split in a suitable fixed gauge $\phi(x) = vn + \varphi(x)$.
- One expand the n-point functions over fluctuations of the Higgs.
- Right-hand side can be computed with standard perturbation theory techniques.
- Importance of contributions are ordered by powers of the Higgs vev as in a expansion.

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- Expand the correlator

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- Poles are found at the same places on the left-hand side and the right-hand side of the equation.

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- FMS can be used on EW sector of Standard Model.
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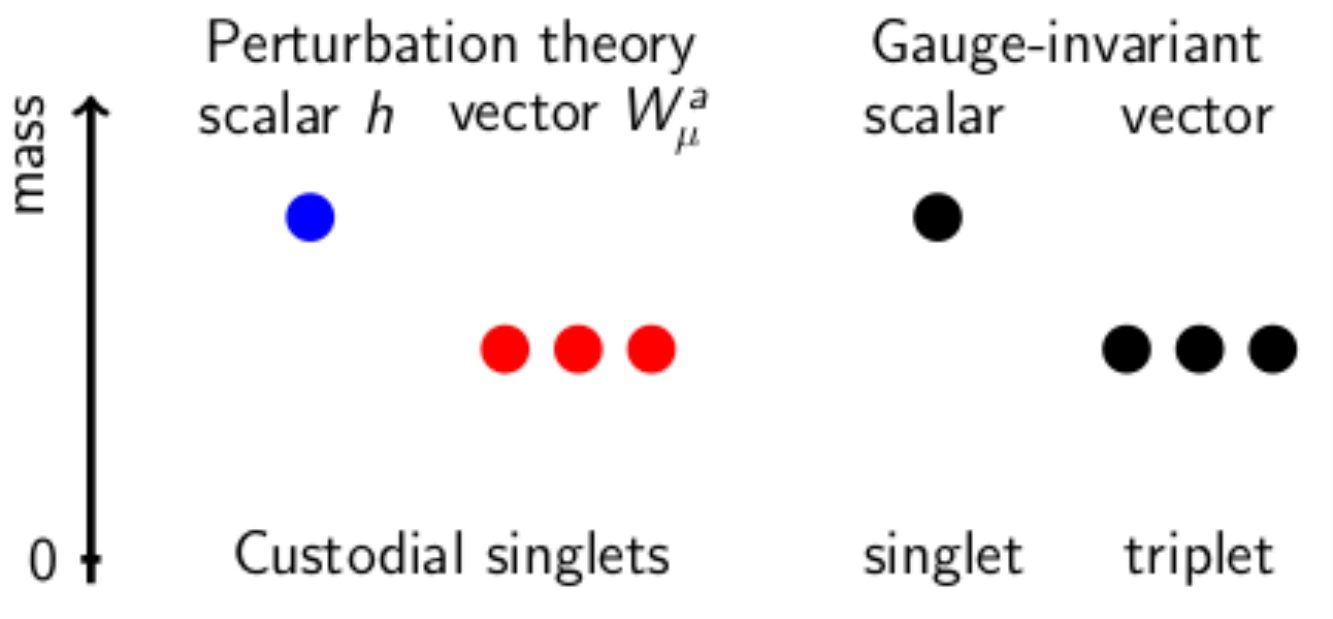

Review: [Maas-1712.04721(hep-ph)]

Other applications of FMS

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- QED: Dirac dressing $\rightarrow$ no appreciable deviations.
- QCD: Quark-hadron duality $\rightarrow$ FMS can still hold in principle.

- FMS explains why PT in SM is successfull.
- Does it work also for BSM theories?

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- FMS: $O_\mu(x) = i(\phi^\dagger D_\mu \phi)(x)$,
  $\langle O_\mu(x) O^{\dagger\mu}(y)\rangle = v^4 \langle W^H_\mu(x) W^{H\,\mu}(y)\rangle + \dots$
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- 1 massive state
- Contradiction in the vector channel

[Maas,Sondenheimer,Török-1709.07477(hep-ph)]

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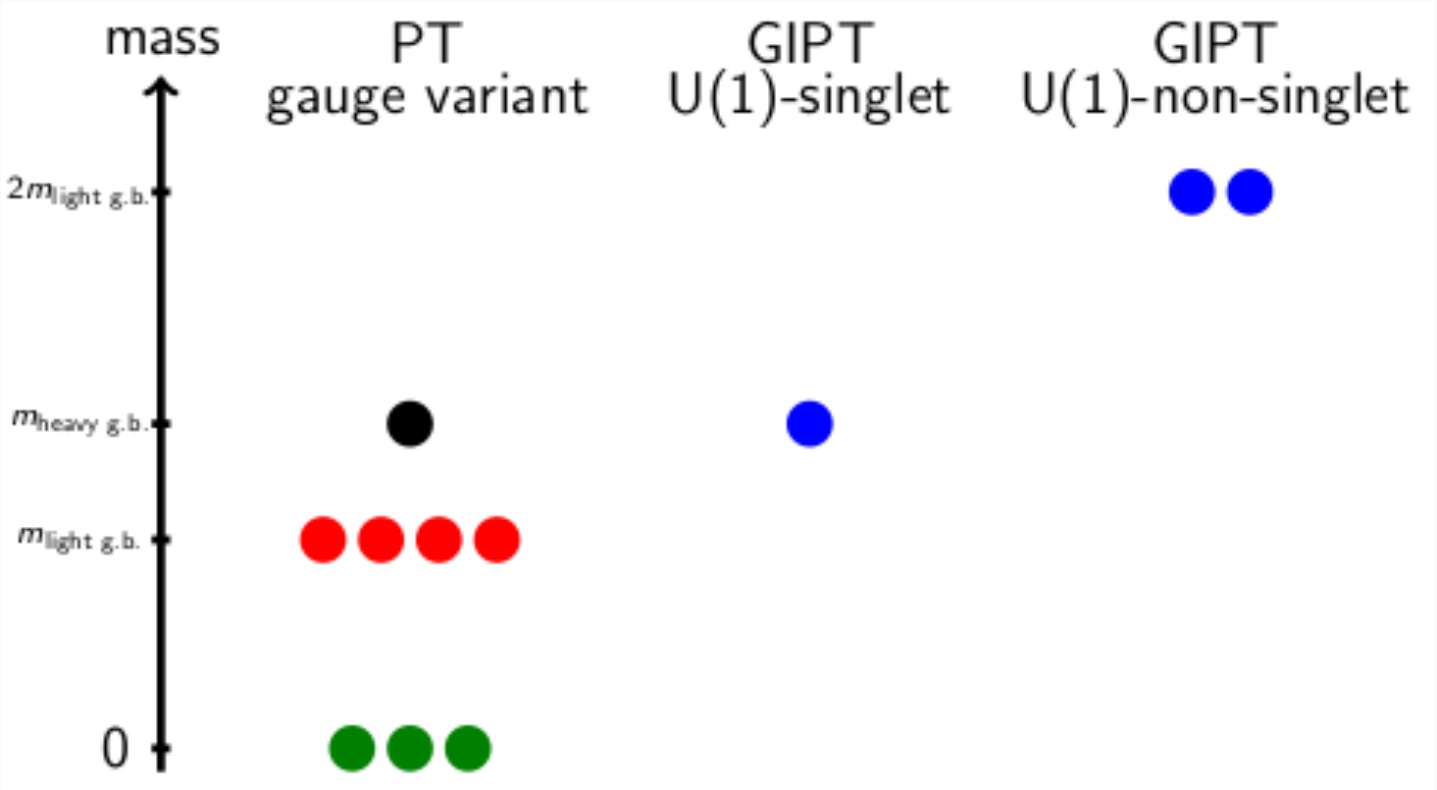

- Lattice spectrum support FMS predictions.

[Maas,Török-1804.04453(hep-lat)]

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- The fermion sector of the SM, as GI bound states, has never been analyzed nonperturbatively.
- Parity violation is not accessible on the lattice.

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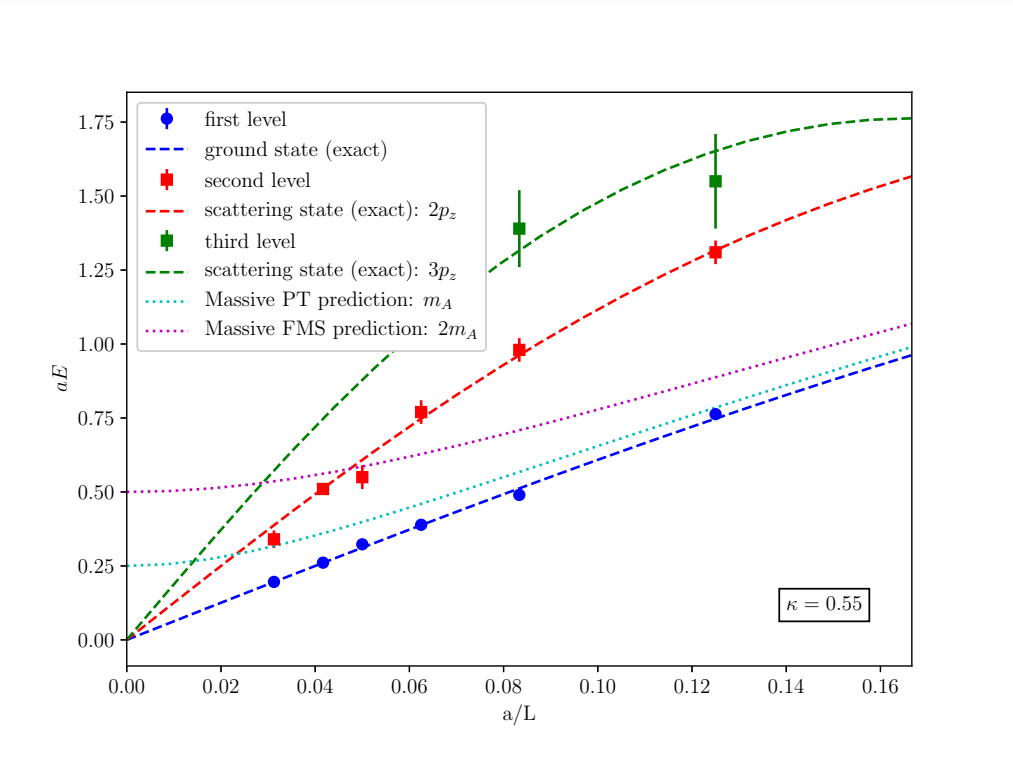
Massless composite vector in SU(2) + Adjoint Higgs

- A massless vector composite particle is predicted in a SU(2) theory coupled with an adjoint Higgs.



- Lattice simulations confirm the hypothesis.
- Toy model of a GUT with a photon.

[Afferrante, Maas,Török-2002.08221(hep-lat)]

Gauge invariant fermion spectrum

FMS Mechanism for fermions in SM

- Left handed fermions in SM are not GI $\rightarrow$ they can be treated with the FMS mechanism.
- We can employ fermionic GI bound states $\Psi(x) = \phi^\dagger(x)\psi(x)$, but never proven.

$$\langle \Psi(x)\overline{\Psi}(y)\rangle = \frac{v^2}{2}\langle \psi_2(x)\overline{\psi_2}(y)\rangle + O(\varphi)$$

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- If the FMS construction holds, the mass of the bound state should be close to the elementary one.
- The main goal of this work is to analyze this hypothesis on the lattice.

FMS for fermions

- Hypothesis of bound state scattering has been explored preliminarily

  [Egger,Maas,Sondenheimer-1701.02881(hep-ph)]

- Lattice chiral fermions formulation is a longstanding problem

  $\rightarrow$ Vectorial fermions.

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- Lattice chiral fermions formulation is a longstanding problem $\rightarrow$ Vectorial fermions.
- One vectorial fermion ψ which is gauged(L), one ungauged fermionic doublet χ (ν_R, l_R).

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) + \overline{\psi} \left(\mathrm{i} \not{D} - m_\psi \right) \psi + \overline{\chi}_{\bar{k}} \left(\mathrm{i} \not{\partial} - m_\psi \right) \chi_{\bar{k}} - h(\overline{\psi} \tilde{\phi} \chi_1 + \overline{\chi}_1 \tilde{\phi}^\dagger \psi) - h(\overline{\psi} \phi \chi_2 + \overline{\chi}_2 \phi^\dagger \psi) - V(\phi^\dagger \phi) \Big] .$$

Elementary mass spectrum

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- Tree level mass matrix for the fermion fields with the convention $(\psi_1\ \psi_2\ \chi_1\ \chi_2)^{\mathrm{T}}$:

$$M = \begin{pmatrix} m & 0 & \frac{v}{\sqrt{2}}h & 0 \\ 0 & m & 0 & \frac{v}{\sqrt{2}}h \\ \frac{v}{\sqrt{2}}h & 0 & m & 0 \\ 0 & \frac{v}{\sqrt{2}}h & 0 & m \end{pmatrix} .$$

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- The ψ and χ doublet are degenerate at a tree level.

Lattice setup

- Fermion propagator obtained by inversion of the Dirac operator, quenched setting

$$\begin{pmatrix} \overline{\psi} & \overline{\chi} \end{pmatrix} D \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \begin{pmatrix} \overline{\psi} & \overline{\chi} \end{pmatrix} \begin{pmatrix} D^{\overline{\psi}\psi} & D^{\overline{\psi}\chi} \\ D^{\overline{\chi}\psi} & D^{\overline{\chi}\chi} \end{pmatrix} \begin{pmatrix} \psi \\ \chi \end{pmatrix} ,$$

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- Standard Wilson-Dirac operator

$$D^{\overline{\psi}\psi}(x|y)_{ij} = \mathbb{1}\delta_{ij} - \kappa_\psi \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu)\, U_\mu(x)_{ij}\, \delta_{x+\hat{\mu},y} ,$$

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- The second block is the free Wilson-Dirac operator.
- Non-diagonal blocks are the Yukawa couplings.

Lattice Setup

- Non-diagonal blocks are the Yukawa couplings

$$D^{\overline{\psi}\chi}_{i\bar{j}} = h_1 \tilde{\phi}_i \delta_{1\bar{j}} + h_2 \phi_i \delta_{2\bar{j}} \quad , \quad D^{\overline{\chi}\psi}_{\bar{i}j} = h_1 \tilde{\phi}^\star_j \delta_{\bar{i}1} + h_2 \phi^\star_j \delta_{\bar{i}2} \, .$$

- BiCGStab method has been implemented for inversion of Dirac operator.
- Inversion of full Wilson-Yukawa has been checked on the free analytical case ($U_\mu(x) = \mathbb{1}$) with static Higgs field.

FMS observables

- Interesting GI observables are

$$\phi^\dagger\psi \overset{\text{FMS}}{\propto} \psi_2 + \cdots \quad , \quad \tilde{\phi}^\dagger\psi \overset{\text{FMS}}{\propto} \psi_1 + \cdots \quad , \quad \chi_1 \quad , \quad \chi_2 \, .$$

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- Computing the correlators by using Wick contractions give

$$\begin{aligned}
\left\langle (\phi^\dagger\psi_\alpha)(x)\, \overline{(\phi^\dagger\psi_\beta)}(y) \right\rangle &= \left\langle \phi_i^\star(x)\, D^{-1}(x|y)_{ij,\alpha\beta}\, \phi_j(y) \right\rangle \, , \\
\left\langle (\tilde{\phi}^\dagger\psi_\alpha)(x)\, \overline{(\tilde{\phi}^\dagger\psi_\beta)}(y) \right\rangle &= \left\langle \epsilon_{ij}\phi_j(x)\, D^{-1}(x|y)_{ik,\alpha\beta}\, \epsilon_{kl}\phi_l^\star(y) \right\rangle \, , \\
\left\langle \chi_{1,\alpha}(x)\overline{\chi}_{1,\beta}(y) \right\rangle &= \left\langle D^{-1}(x|y)_{22,\alpha\beta} \right\rangle \, , \\
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\end{aligned}$$

- We add to the base the two gauge variant component ψ_1, ψ_2, which are evaluated on a smaller subset of gauge fixed configurations.

Mass Eigenstates

- The ψ_i are not mass eigenstates.
- Expectation at leading order

$$\begin{aligned}
\left\langle \bar{\Psi}\Psi \right\rangle &= \alpha \left\langle \bar{\zeta}^{+}\zeta^{+} \right\rangle + \sqrt{1-|\alpha|^2} \left\langle \bar{\zeta}^{-}\zeta^{-} \right\rangle && (1)\\
\langle \bar{\chi}\chi \rangle &= \beta \left\langle \bar{\zeta}^{+}\zeta^{+} \right\rangle + \sqrt{1-|\beta|^2} \left\langle \bar{\zeta}^{-}\zeta^{-} \right\rangle && (2)\\
\left\langle \bar{\psi}\psi \right\rangle &= \gamma \left\langle \bar{\zeta}^{+}\zeta^{+} \right\rangle + \sqrt{1-|\gamma|^2} \left\langle \bar{\zeta}^{-}\zeta^{-} \right\rangle && (3)
\end{aligned}$$

- At tree level $\psi_i = (\zeta_i^{+} + \zeta_i^{-})/\sqrt{2}$ yields

$$\Psi = |v|\psi + \mathcal{O}(\eta) = \frac{|v|}{\sqrt{2}}\left(\zeta^{+} + \zeta^{-}\right) + \mathcal{O}(\eta).$$

NLO masses eigenstates

- We can add the NLO correction to TL masses

$$
\begin{aligned}
m_{\psi}^{(1)} &= m + \alpha_W a_\psi + y^2 b_\psi \\
m_{\chi_i}^{(1)} &= m + y^2 b_\chi,
\end{aligned}
$$

- NLO eigenmasses

$$
\begin{aligned}
M^{\pm} &= \frac{1}{2}\left(m_{\psi}^{(1)} + m_{\chi_i}^{(1)} \pm \sqrt{\left(m_{\psi}^{(1)} + m_{\chi_i}^{(1)}\right)^2 + 2vy^2}\right) \\
&= m + \frac{1}{2}\left(\alpha_W a_\psi + y^2(b_\psi + b_\chi)\right) \\
&\quad \pm \frac{1}{2}\sqrt{2vy^2 + (\alpha_W a_\psi - y^2(b_\psi + b_\chi))^2}.
\end{aligned} \tag{4}
$$

- Behaviour with respect to y is not linear.

Quantum corrections to mixing

- Expected mixing

$$
\begin{aligned}
\zeta_i^\beta &= \cos\beta\psi_i + \sin\beta\chi_i \\
\zeta_i^\gamma &= \sin\gamma\psi_i + \cos\gamma\chi_i \\
\sin\beta &= \frac{1}{\sqrt{1+\frac{\delta-\Delta}{2vy^2}}} \\
\sin\gamma &= \frac{1}{\sqrt{1+\frac{\delta+\Delta}{2vy^2}}} \\
\Delta &= \sqrt{\delta^2+2vy^2} \\
\delta &= \alpha_W a_\psi + y^2(b_\psi - b_\chi)
\end{aligned} \tag{5}
$$

- At small y, β and γ will be close to zero.

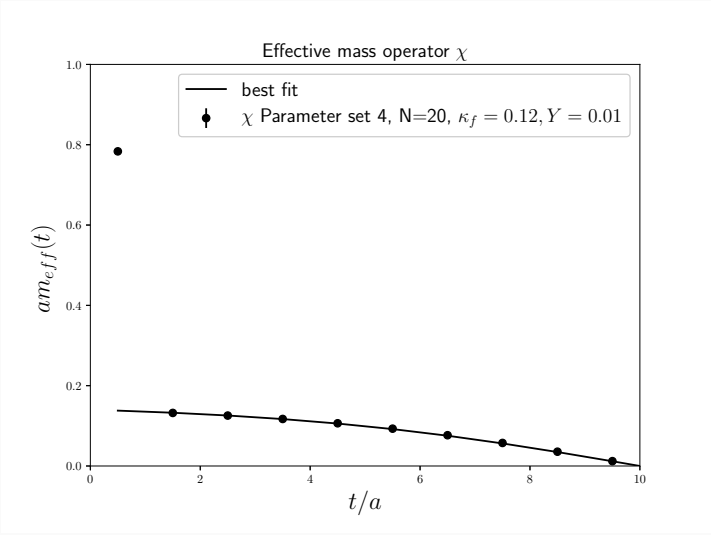
Spectroscopic results

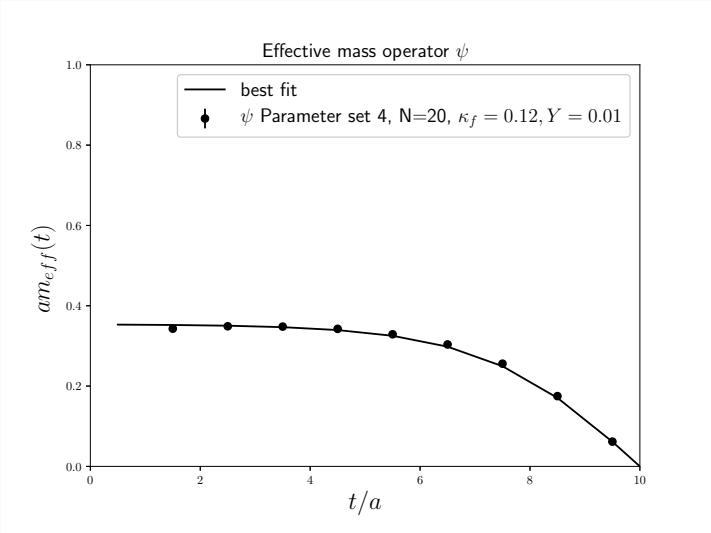
- Results are compatible with $\alpha = 1, \beta = 0$.
- Propagator of $\chi \rightarrow$ First mass eigenvalue M^-.
- Propagator of $\psi(GF) \rightarrow$ Second mass eigenvalue M^+.
- The factor γ depends on the Yukawa coupling.
- Bound state is a combination of the two.
- Fit for bound state

$$\frac{C(t)}{C(N_t/2)} = \frac{1}{1+r}[\cosh(M^-(t - N_t/2)) + r\cosh(M^+(t - N_t/2))]$$

ψ and χ masses

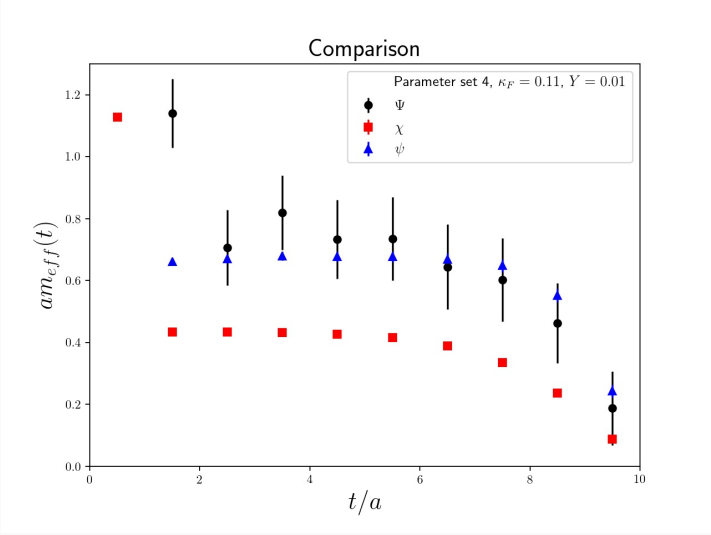
- Both correlators show a good plateau.
- The two eigenmasses are obtained.

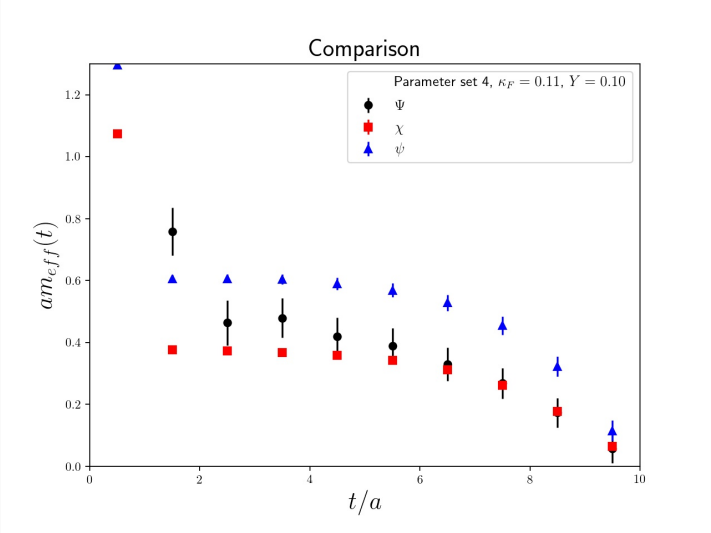




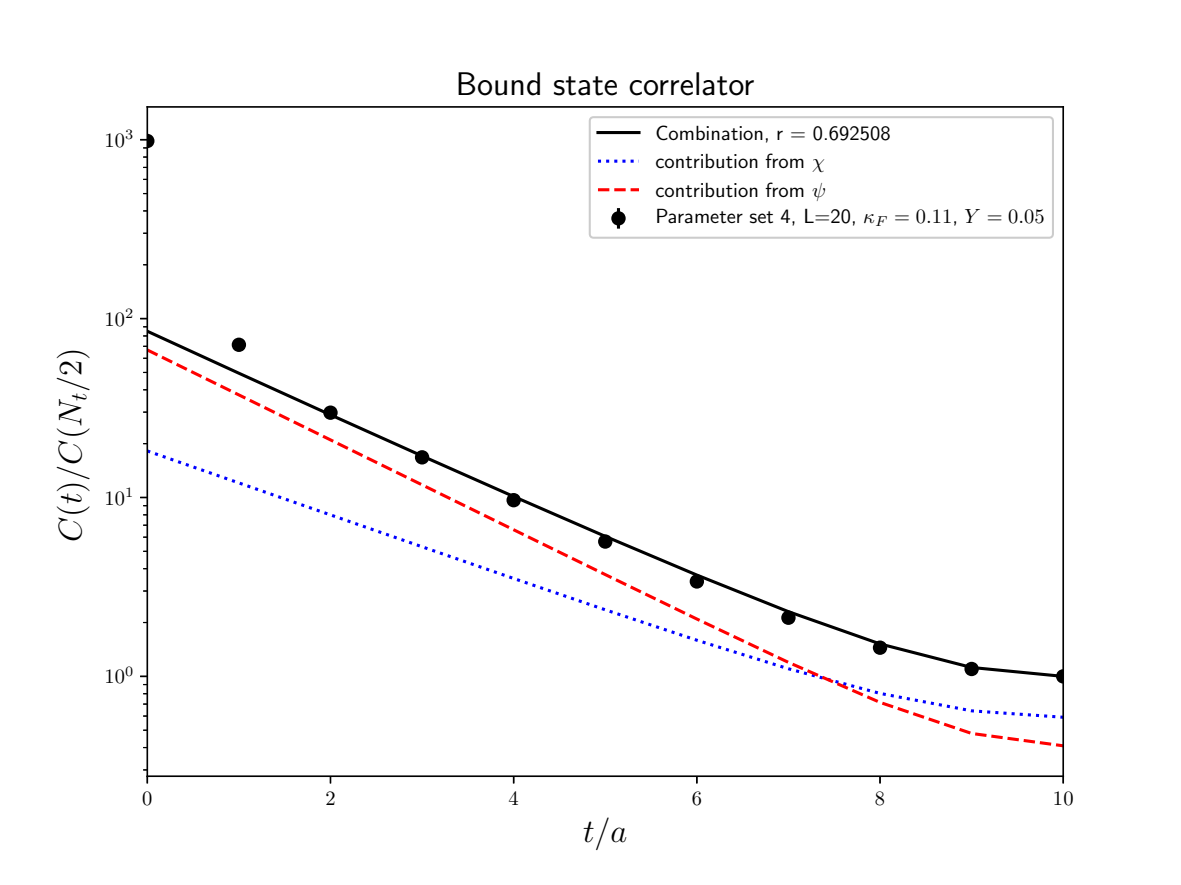
Comparison of bound state

- Behaviour of Ψ depends on Yukawa.
- Small Yukawa: compatibility with ψ.
- Large Yukawa: compatibility with χ.



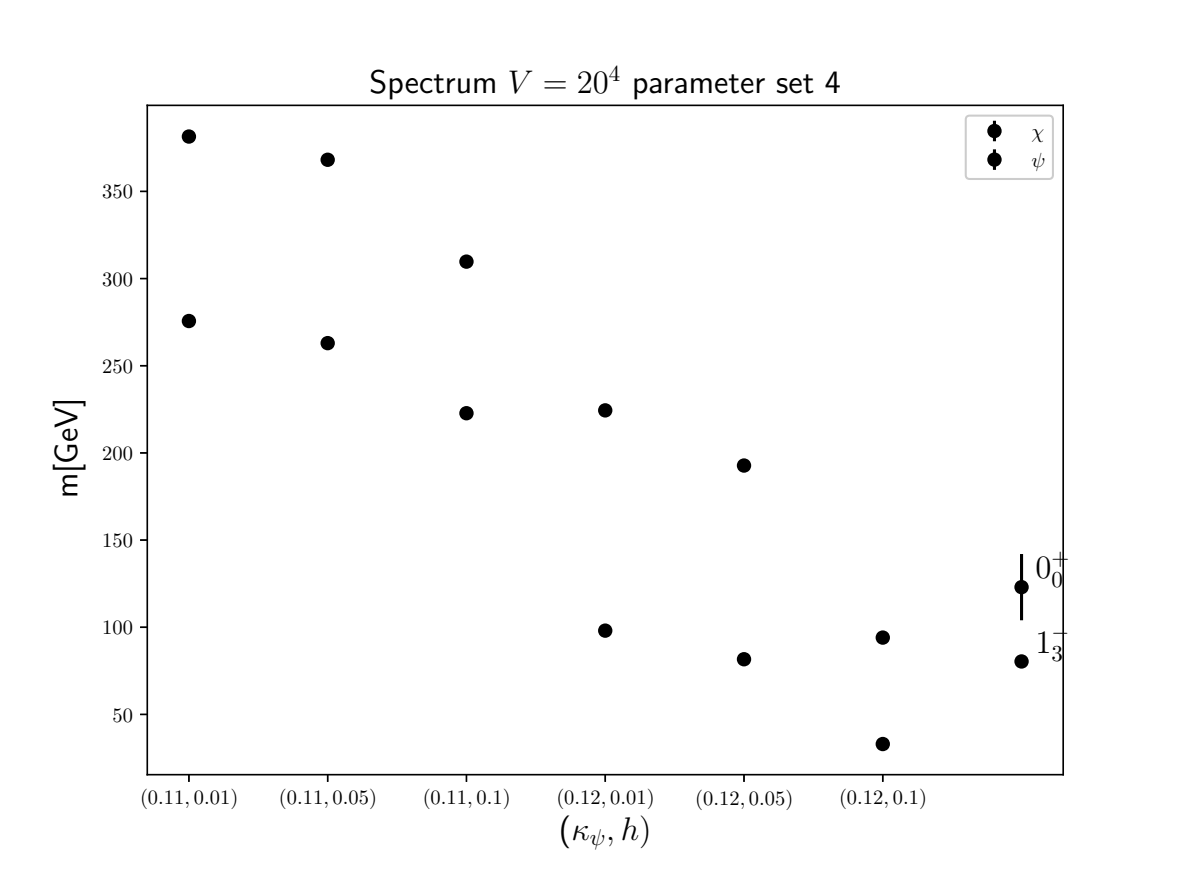


Bound state mixing



- Intermediate Yukawa: combination of ψ and χ gives a good fit.

Spectroscopic results



- The operators shows the expected degeneracies.
- It is possible to have, with fine tuning, very light and very heavy fermions.

Conclusions

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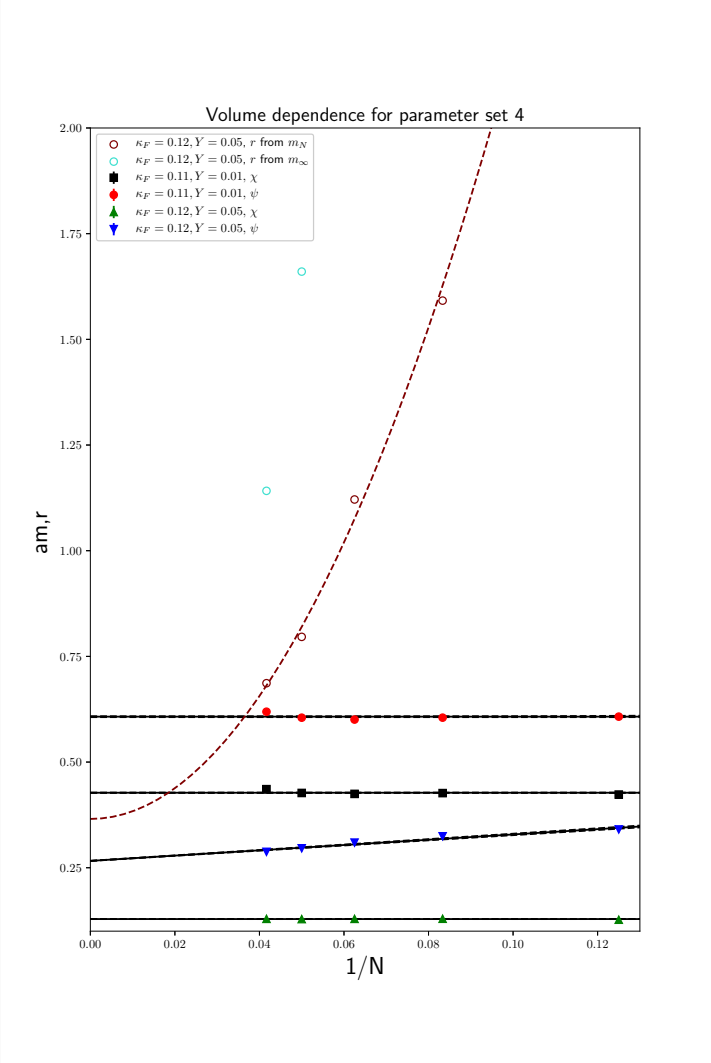
Thanks!

Backup slides

Infinite volume behaviour

Masses behaves exponentially with respect to infinite volume limit.



Full results

We examined 5 different parameters set for the bosonic sector.

#	β	κ	λ	a^{-1} [GeV]	$m_{0_0^+}$ [GeV]	$\alpha_W(200\text{ Gev})$	$v(200\text{ GeV}) = \frac{m_{1_3^-}}{\sqrt{\pi\alpha_W}}$ [GeV]
1	2.7984	0.2954	1.328	384	118(9)	0.544	39
2	2.7984	0.2978	1.317	326	129(12)	0.495	64
3	3.9	0.2679	1	509	116(19)	0.140	121
4	5.082	0.249	0.7	636	123(19)	0.170	110
5	5.082	0.2552	0.7	427	131(5)	0.0794	161

#	κ_F	Y	aM^-	aM^+	r
1	0.12	0.01	$0.421^{+0.001}_{-0.008}$	0.817(3)	1.9
1	0.12	0.05	0.407(6)	0.77(3)	0.4
1	0.12	0.1	0.353(9)	0.54(1)	0.2
1	0.11	0.01	0.137(1)	0.58(1)	1.4
1	0.11	0.05	0.111(1)	0.45(1)	0.2
1	0.11	0.1	0.044(5)	0.21(1)	0.1
2	0.12	0.01	0.422(3)	0.810(4)	1.4
2	0.12	0.05	0.406(3)	0.75(2)	0.6
2	0.12	0.1	0.352(2)	0.62(3)	0.4
2	0.11	0.01	0.136(1)	0.583(4)	1.4
2	0.11	0.05	0.103(1)	0.49(2)	0.3
2	0.11	0.1	0.032(2)	0.17(1)	0.2
3	0.12	0.01	$0.422^{+0.001}_{-0.006}$	0.674(3)	6.5
3	0.12	0.05	0.407(5)	0.645(2)	0.3
3	0.12	0.1	0.357(3)	0.574(4)	0.09
3	0.11	0.01	0.136(1)	0.426(5)	3.0
3	0.11	0.05	$0.112^{+0.004}_{-0.002}$	0.385(2)	0.8
3	0.11	0.1	0.043(1)	0.24(1)	0.2
4	0.12	0.01	0.422(1)	0.604(2)	11.5
4	0.12	0.05	0.402(2)	0.54(1)	0.6
4	0.12	0.10	0.331(7)	0.43(1)	0.1
4	0.11	0.01	0.136(3)	0.346(2)	2.4
4	0.11	0.05	0.098(1)	0.27(2)	1.0
4	0.11	0.10	0.036(9)	0.09(1)	0.4
5	0.12	0.01	0.422(5)	0.599(2)	7.1
5	0.12	0.05	0.39(1)	0.51(1)	1.0
5	0.12	0.1	0.305(5)	0.35(1)	0.4
5	0.11	0.01	0.126(4)	0.347(6)	7.1
5	0.11	0.05	0.086(2)	0.22(1)	1.0
5	0.11	0.1	0.03(2)	$0.1^{+0.09}_{-0.05}$	0.1