

A new approach to Observables in Quantum Gravity

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Setup

- QFT setting – no strings or other non-QFT structures

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- Diffeomorphism is like a gauge symmetry [Hehl et al.'76]
 - Arbitrary local choices of coordinates do not affect observables – pure passive formulation
 - Physical observables must be manifestly invariant [Fröhlich et al.'80]

Dynamical formulation

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 - Other choices (e.g. vierbein) possible

Dynamical formulation

Standard gravity coupling Standard gravity

Other fields

$$Z = \int_{\Omega} Dg_{\mu\nu} \overrightarrow{D} \phi^a e^{iS[\phi, e] + iS_{EH}[e]}$$

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 - Other choices (e.g. vierbein) possible
- Otherwise standard
 - E.g. Asymptotic safety for ultraviolet stability

Dynamical formulation

$$\langle O \rangle = \int_{\Omega} Dg_{\mu\nu} D\phi^a O e^{iS[\phi, e] + iS_{EH}[e]}$$

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Needs to be invariant

- Locally under Diffeomorphism
- Locally under Lorentz transformation
- Locally under gauge transformation
- Globally under custodial, ... transformation

to be non-zero

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 - Invariants identify the particular type

Simplest object: Scalar

- Consider a scalar particle
 - E.g. described by a scalar field
 - Completely invariant

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Completely scalar: Invariant under all symmetries

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Argument is the event, not the coordinate

Result depends on events

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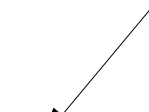
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 - Events not a useful argument

Simplest object: Scalar

[Schaden'15,
Ambjorn et al.'12]

Some distance function

$$\langle O(x)O(y) \rangle = D(\vec{r}(x, y))$$


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- Distance is a quantum object: Expectation value
 - Needs a diff-invariant formulation

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Maas'19]

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$$\langle O(x)O(y) \rangle = D(r(x, y)) \quad \text{Separate calculation}$$

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- Generalization of flat-space arguments

Fröhlich-Morchio-Strocchi mechanism

[Fröhlich et al.'80'81]

- Horrible complicated calculation

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 - Split **after** gauge-fixing fields such that they become classical fields plus quantum corrections
 - Calculate order-by-order in quantum corrections

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- FMS prescription:
 - Chose a gauge compatible with the desired classical behavior
 - Split **after** gauge-fixing fields such that they become classical fields plus quantum corrections
 - Calculate order-by-order in quantum corrections
- Works very well in particle physics [Review: Maas'19a]

Applying FMS

- Our universe is well-approximated by a classical metric
 - Due to the parameter values – special!
 - Small quantum fluctuations at large scales
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 - Due to the parameter values – special!
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 - Empirical result
- FMS split after (convenient) gauge fixing
 - $g_{\mu\nu} = g_{\mu\nu}^c + \gamma_{\mu\nu}$
 - Classical part g^c is a metric, chosen to give exact (observed) curvature
 - Quantum part is assumed small
 - Haywood gauge convenient

Distance

[Maas'19]

$$r(x, y) = \left\langle \min_z \int_x^y d\lambda g_{\mu\nu} \frac{dz^\mu}{d\lambda} \frac{dz^\nu}{d\lambda} \right\rangle$$

- Application to distance between two events

Distance

$$\begin{aligned}
 r(x, y) &= \left\langle \min_z \int_x^y d\lambda g_{\mu\nu} \frac{dz^\mu}{d\lambda} \frac{dz^\nu}{d\lambda} \right\rangle \\
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Classical geodesic
distance

- Application to distance between two events
 - Yields to leading order classical distance

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Quantum corrections

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 - Yields at leading-order classical space-time
 - Quantum corrections depends on events

Propagators

[Maas'19]

$$\langle O(x)O(y) \rangle$$

Propagators

$$\langle O(x)O(y) \rangle = D_c(r^c) + \sum (\delta r)^n \partial_r^n D_c(r) + \langle O(x)O(y) \rangle_y$$

- Double expansion

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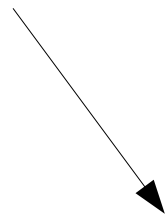
Leading term is
flat space propagator

$$D_c = \langle O(x)O(y) \rangle_{g^c}$$

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Propagators

Corrections from quantum
distance effects



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 - Quantum fluctuations in the argument

Corrections from
metric fluctuations

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- Double expansion
 - Quantum fluctuations in the argument and action
 - Consistent with EDT results [Dai'22]
- Reduces to QFT at vanishing gravity
 - Higgs and W/Z mass in quantum gravity calculated

Non-trivial geon

- Pure gravity excitation: Curvature-curvature correlator

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$$\langle R(x)R(y) \rangle$$

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Graviton propagator

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Differential operator

Graviton propagator

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- Flat space: Better divergence properties

Predictions for CDT

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 - Allows reconstruction of metric in a fixed gauge on every configuration
- deSitter structure observed in CDT
 - Metric fluctuations per configuration should be small compared to de Sitter metric
- Geon propagator should behave as contracted metric propagator
 - As a function of the geodesic distance

Summary

- Full invariance necessary for physical observables in path integrals
- FMS mechanism allows estimates of quantum effects in a systematic expansion
- Gives a new perspective on quantum gravity testable by simulations