$G(2) - QCD$ Neutron Star

Ouraman Hajizadeh

in collaboration with Axel Maas

Cakovec
October 7, 2016

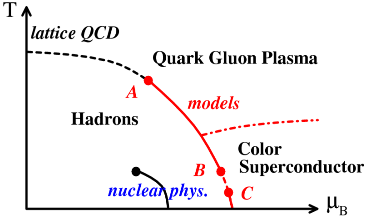
Der Wissenschaftsfonds.

Why Neutron Stars?

- Neutron Stars: Laboratory of Strong Interaction
- Dense Objects: Study of strong interaction at high densities
- Different phases: Explanation of phase transition based on the fundamental theory
- Macroscopic objects: Show large scale effects of a non-Abelian gauge theory.

Why G(2)?

- $SU(3) QCD$ Theory of Strong interaction



- $SU(3)$ Lattice : Sign Problem at finite density
- $G(2)$: a QCD like theory without the sign problem, with Neutrons
- Equation of state of $G(2)$ neutron star from Lattice simulation
- Mass-radius of $G(2)$ neutron star from e.o.s.
  - Qualitative insight to real neutron star
  - Theoretical information about large scale consequences of a non-abelian interaction

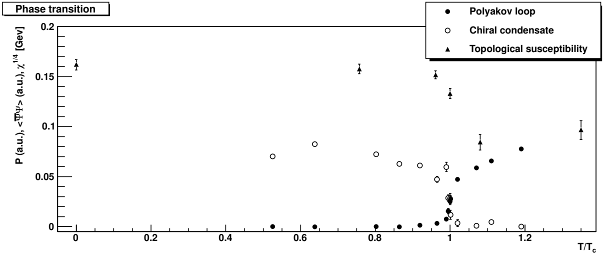
$G(2)$ Group Structure Facts

- $G(2)$ has 14 generators$\Rightarrow$ 14 Gluon
  Fundamental representation = Quark number = 7
- All the representations are real : No sign problem
- Diquarks are color singlets:
  $7 \otimes 7 = 1 \oplus \ldots$
  qq: Diquark
  qqq: Neutron like $7 \otimes 7 \otimes 7 = 1 \oplus \ldots$
  qggg: Hybrid $7 \otimes 14 \otimes 14 \otimes 14 = 1 \oplus \ldots$
- Rank 2 : like $SU(3)$

Why $G(2)$?

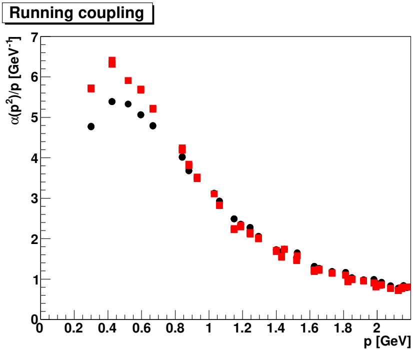
Shares many feauture with $SU(3) - QCD$

- Chiral and deconfinement transition coincide. (J. Danzer, C. Gattringer, A. Maas JHEP 2009; G. Cossu, M. D'Elia, A. Di Giacomo, B. Lucini, C. Pica JHEP 2007, E.M. Ilgenfritz, A. Maas 2012)



A decisive result for further study.

Running Coupling in 3 dimensions (A. Maas, S. Olejnik, JHEP 2008)

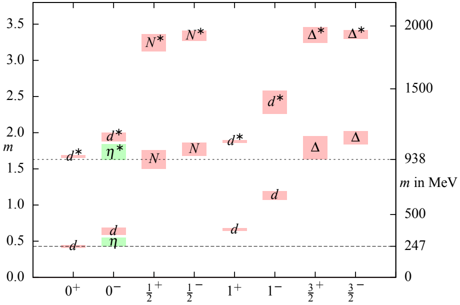


$SU(3)$ (black) and G_2 (red) YM agree qualitatively for low and quantitatively for high momenta.

Spectrum of $G(2) - QCD$

- Mass of proton is set equal to the QCD proton.
- Chiral symmetry breaking manifests itself in the spectrum.

  (B. H. Wellegehausen et. al., Phys. Rev. D 89)



Matter and gravity ~ Mass-radius relation

- Heavy, compact stars: general relativity is applied
- Neutron star ~ nuclear matter ~ ideal fluid
- Einstein euations quantify the relation between matter and space-time: $G(g^{\mu\nu}) \sim T^{\mu\nu}$
- Metric tensor: $g^{\mu\nu}$ represents gravity
- Energy-momentum tensor: $T^{\mu\nu}$ represents matter
- For a static star:

$$T^{\mu}_{\mu} = -p(r) \sim \text{pressure} \qquad \mu = 1, 2, 3$$

$$T^{0}_{0} = \epsilon(r) \sim \text{energy density}$$

Oppenheimer-Volkoff Equation

Solving Einstein equations for static, spherical symmetric metric, which approximate metric of the neutron star, gives Oppenheimer-Volkoff (O-V) equation:

$$\frac{dp(r)}{dr} = -\frac{[p(r)+\epsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r\,[r-2M(r)]}$$

($M(r)$ is the mass-energy enclosed in radius r: $M(r) = 4\pi \int_0^r \epsilon(r')\, r'^2\, dr'$)

- O-V equation together with the **equation of state** $\epsilon(p)$ give an ordinary differential equation for $\epsilon(r)$ with initial conditions $\epsilon(0) = \epsilon_0$, $M(0) = 0$
- Result: mass-radius relation of the star $M(r)$ corresponding to each central energy.

Limit on the mass

- O-V equation leads to a limit on the mass, that a star can get for each equation of state.
- Repulsive nuclear interaction increases the mass limit
- The value of mass limit is determined having the equation of state
- The upper bound on the limit, is set by stability and causality arguments, around $3.14M_{\odot}$
- We find the limit on the mass of a G(2)-QCD neutron star by obtaining M(r) for different values of central chemical potential relevant for neutron star.

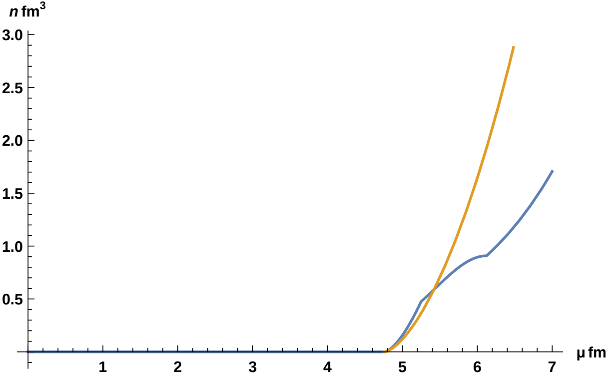
Mass-Radius relation: M(r)

- To find M(r) for G(2)-QCD neutron star we need equation of state
  - E.o.s given by $n(\mu)$ and $p = \mu n(\mu) - \epsilon$
  - And cold matter approximation for energy density: $\epsilon = mn(\mu)$
- At zero temperature: $\frac{dp}{d\mu} = n$
- We write Oppenheimer-Volkoff equation in terms of μ

$$\frac{d\mu(r)}{dr} = -\frac{[\mu(r)]\,[M(r) + 4\pi r^3 p(r)]}{r\,[r - 2M(r)]}$$

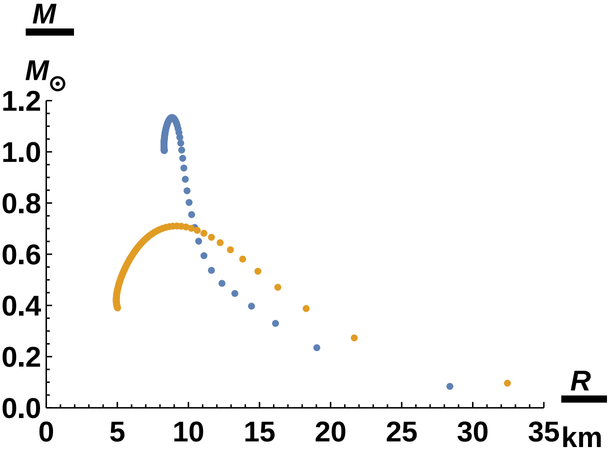
- Next step is to solve Oppenheimer-Volkoff equation for this e.o.s after modifying the result of interpolation of lattice data for low density regime aacording to normal matter condition: $n(\mu) = \alpha(\mu^2 - m^2)^{\frac{3}{2}}$, α is set by the continuity condition of e.o.s

G(2)-QCD versus free Fermi gas E.o.S:



baryon number density in terms of nucleon chemical potential:
Orange curve: Free Fermi gas, Blue curve: G(2)-QCD

Mass-radius curves



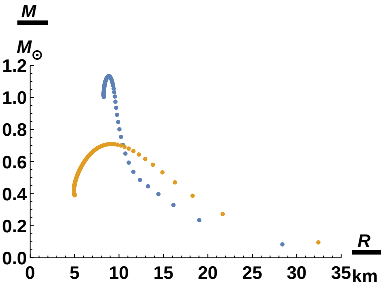
Blue: G(2)-QCD, Orange: free Fermi gas

(O. Hajizadeh, A. Maas, arXiv:1609.06979)

Observations:

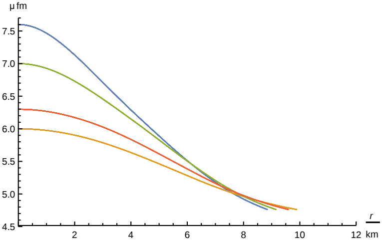
Comparing to free Fermi gas G(2)-QCD neutron stars show

- Sudden, steeper rise in mass
- Sharper maximum
- Dense population of stars close to the maximum
- Smaller masse for lower region of chemical potential
- Negative slop in both cases, as a generic feauture.

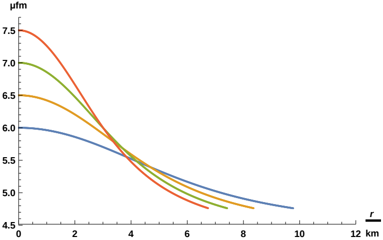


Explanation

- Interplay of O-V equation and e.o.s
- $\mu(r)$ inside the stars is an informative quantity



G(2)-QCD

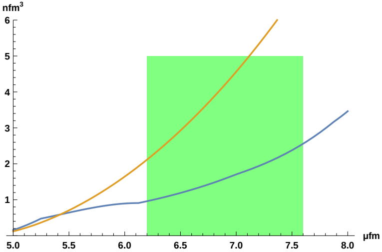


Free Fermi gas

(O. Hajizadeh, A. Maas, arXiv:1609.06979)

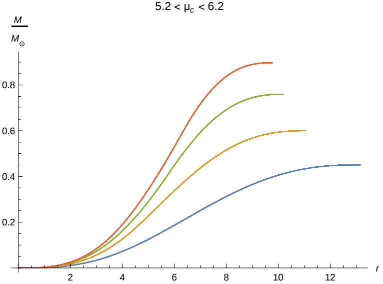
- $\mu(r)$ curves diverge further after the crossing point in free case than in the G(2)-QCD diagram.
- The softer the equation of state, the more rapid drop in $\mu(r)$ for smaller radii(where accumulated mass is still negligible). $d\mu \sim \; - n(\mu_c)\mu_c$

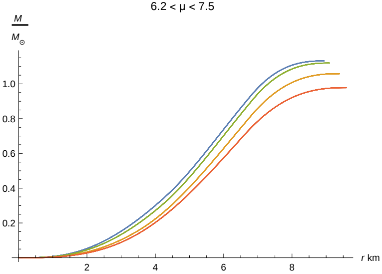
$$\frac{d\mu(r)}{dr} = -\frac{[\mu(r)]\,[M(r)+4\pi r^3 p(r)]}{r\,[r-2M(r)]}$$



Orange: Free neutron gas, blue: G(2)-QCD nuclear matter

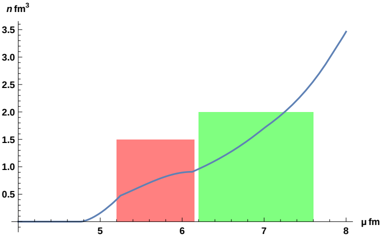
Central chemical potential, less effective at higher values : Phase transition?





High density core ($6.2 < \mu_c < 7.5$), contributes less than the lower region in the accumulated mass.

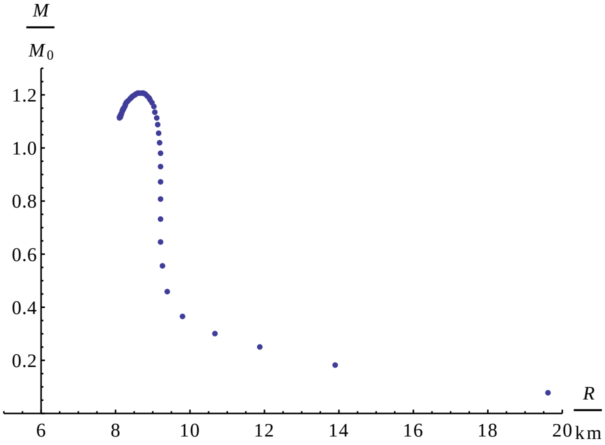
High density: Transition to a softer region?



If $\frac{d^2 n_1(\mu)}{d\mu^2} > \frac{d^2 n_2(\mu)}{d\mu^2}$ the rapidity of drop in $\mu(r)$ is enhanced at higher value of centeral chemical potential for $n_1(\mu)$. Therefore the higher μ_c gets the less it contribute in the accumulated mass, comparing to the stiffer e.o.s.

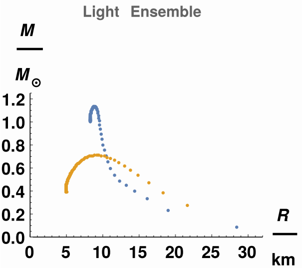
Systematic variation of the equation of state

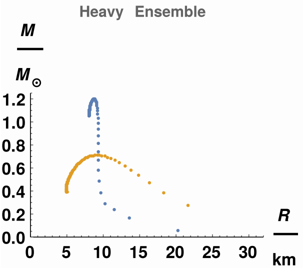
- We can get slightly higher mass by reducing the region of the free Fermi gas approximation.



Systematic variation of the equation of state

- Variation of theory parameter: M-R curve for different quark's current mass gives qualitatively same result
  light ensemble: $m_\pi = 1.254 fm$, heavy ensemble: $m_\pi = 1.653 fm$.





Conclusion

- Equation of state derived from G(2)-QCD simulation on the lattice gives the qualitative expected behavior for M-R curves of neutron stars, and maximum value within the allowed region.
- Different phases in the e.o.s are indicated by the mass-radius curve behavior for stars with different centeral chemical potential.
- The variation of the parameters of the theory at a preliminary level doesn't seem to affect the result significantly.

Outlook

- Inclusion of Goldstones: Stable composite star?
- Study of the bulk properties to estimate viability of ideal fluid approximation and to see if there is color superconductivity in the core of the neutron star
- Improving the lattice data:
  - Precision of e.o.s
  - Estimation of lattice artifacts
- Further spectroscopy to see if hybrids are heavier than $G(2)$ nucleon, so that neutronic matter exists like neutron stars.