Observable Spectrum in theories with a Brout-Englert-Higgs effect

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Gauge Symmetry 24
Graz
Austria

NAWI Graz
Natural Sciences

FWF Österreichischer Wissenschaftsfonds

What is this talk about?

- Brout-Englert-Higgs Physics
  - The issue of gauge invariance

Review: 1712.04721 Update: 2305.01960
Philosophical aspects: 2110.00616

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- Fröhlich-Morchio-Strocchi mechanism
- Reformulations and effective theories
- Impact beyond the Standard Model
  - Qualitative changes
  - Changed phenomenology

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The issue
-
Clashing formal theory with effective phenomenology

A toy model

A toy model: Higgs sector of the SM

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- Essentially the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W^{\mu\nu}_{a}$$

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- Ws W^{a}_{μ} 

- Coupling g and some numbers f^{abc}

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$$W^a_{\mu\nu}=\partial_\mu W^a_\nu-\partial_\nu W^a_\mu+gf^a_{bc}W^b_\mu W^c_\nu$$

$$D^{ij}_\mu=\delta^{ij}\partial_\mu-igW^a_\mu t^{ij}_a$$

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A toy model: Higgs sector of the SM

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$$L = -\frac{1}{4} W^{a}_{\mu\nu} W_{a}^{\mu\nu} + \left(D^{ij}_{\mu} h^{j} \right)^{+} D^{\mu}_{ik} h_{k} + \lambda \left(h^{a} h_{a}^{+} - v^{2} \right)^{2}$$

$$W^{a}_{\mu\nu} = \partial_{\mu} W^{a}_{\nu} - \partial_{\nu} W^{a}_{\mu} + g f^{a}_{bc} W^{b}_{\mu} W^{c}_{\nu}$$

$$D^{ij}_{\mu} = \delta^{ij} \partial_{\mu} - i g W^{a}_{\mu} t^{ij}_{a}$$

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- Higgs h_i h
- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}
- Parameters selected for a BEH effect

A toy model: Symmetries

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- Local SU(2) gauge symmetry

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-g\,f_{bc}^{a}W_{\mu}^{c})\phi^{b} \qquad h_{i}\rightarrow h_{i}+g\,t_{a}^{ij}\phi^{a}h_{j}$$

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- Global SU(2) custodial (flavor) symmetry
    - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $\qquad$ $h\rightarrow h\Omega$

Textbook approach

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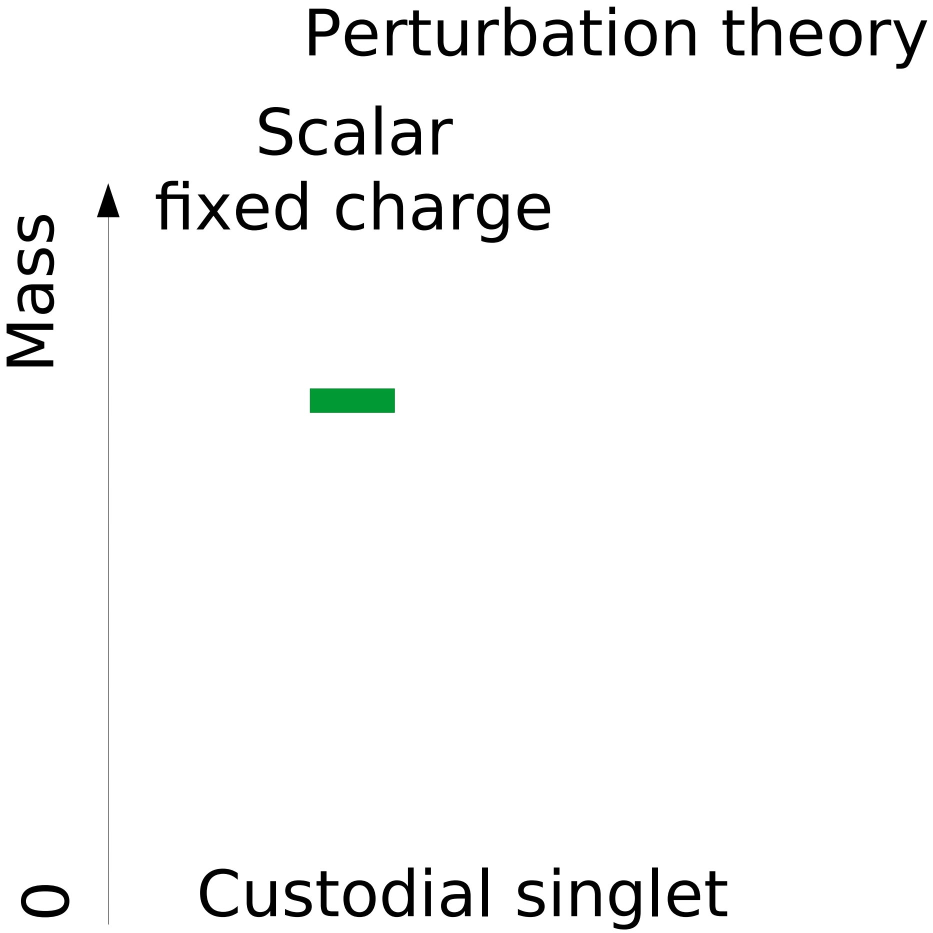
- Minimize the classical action
- Choose a suitable gauge and obtain ‘spontaneous gauge symmetry breaking’: SU(2) → 1
- Get masses and degeneracies at tree-level
- Perform perturbation theory

Physical spectrum

Perturbation theory

Physical spectrum



Perturbation theory

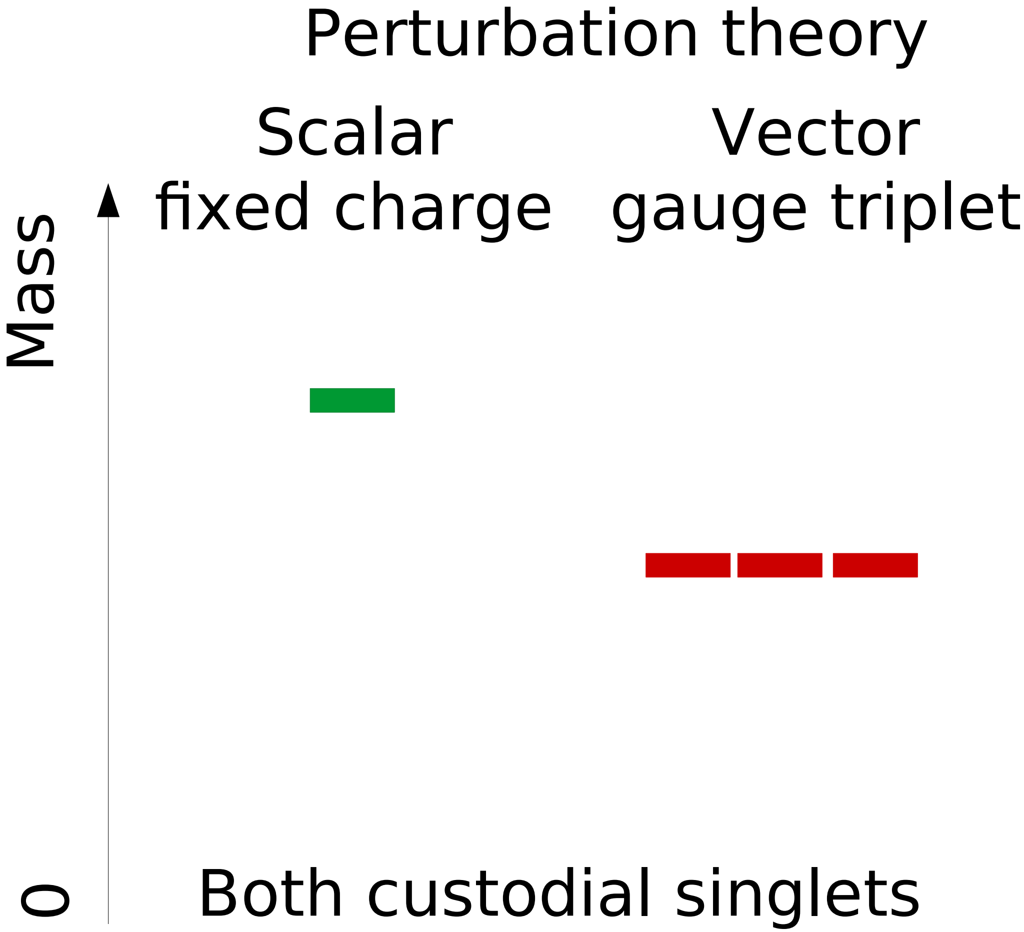
Scalar
fixed charge

Mass

0

Custodial singlet

Physical spectrum



Perturbation theory

Scalar
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Vector
gauge triplet

Both custodial singlets

The origin of the problem

[Fröhlich et al.'80,
Banks et al.'79]

- Elementary fields are gauge-dependent

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- Physics has to be expressed in terms of manifestly gauge-invariant quantities
  - And this includes non-perturbative aspects...
  - ...even at weak coupling [Gribov’78,Singer’78,Fujikawa’82]

Physical states

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- Need physical, gauge-invariant particles

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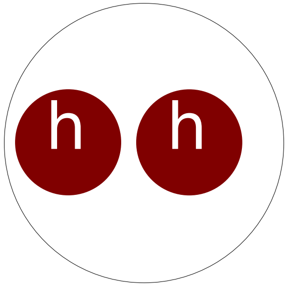

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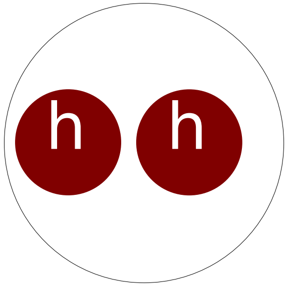

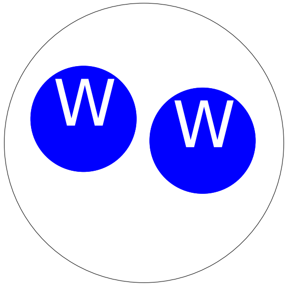

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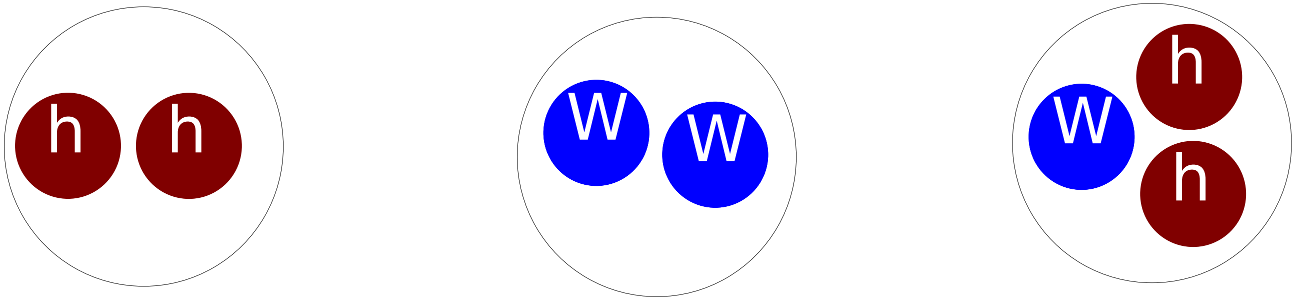

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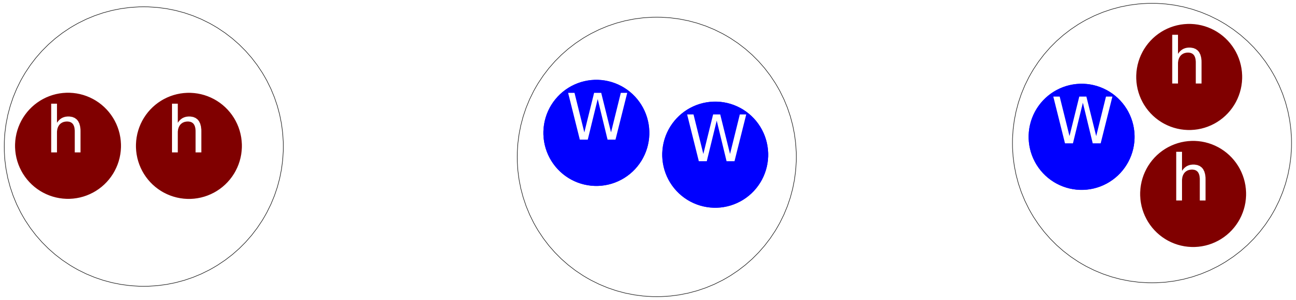

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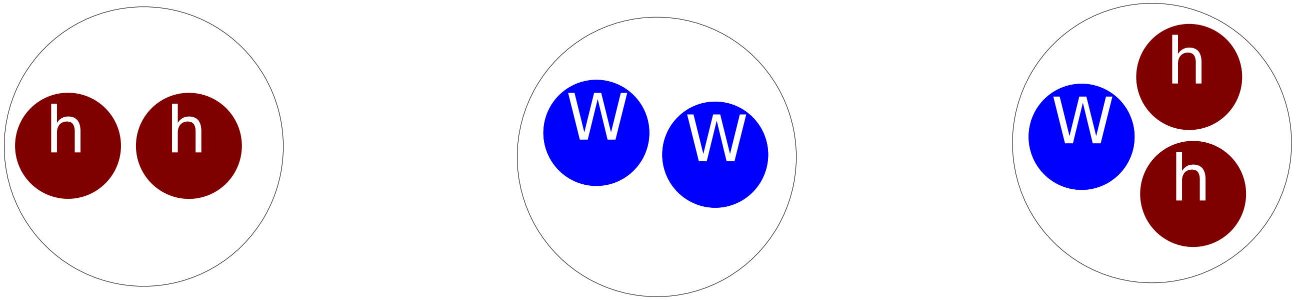

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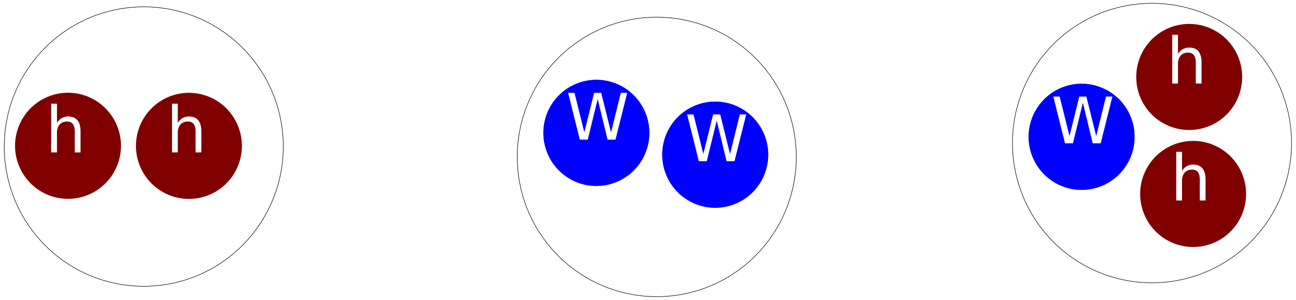


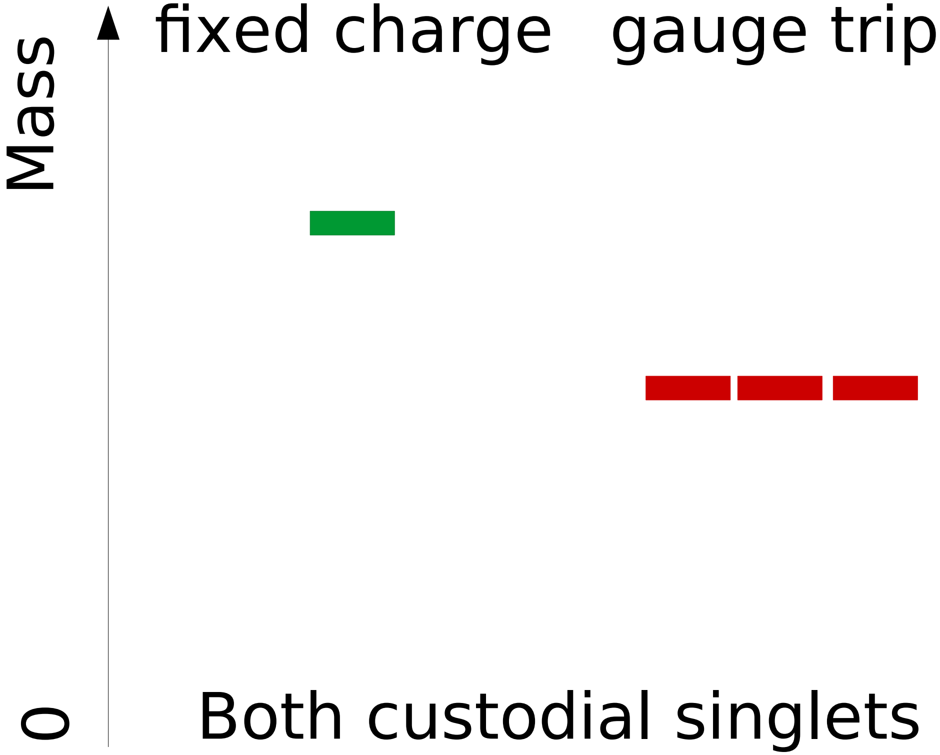
- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)
- Can this matter?

Physical spectrum

Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

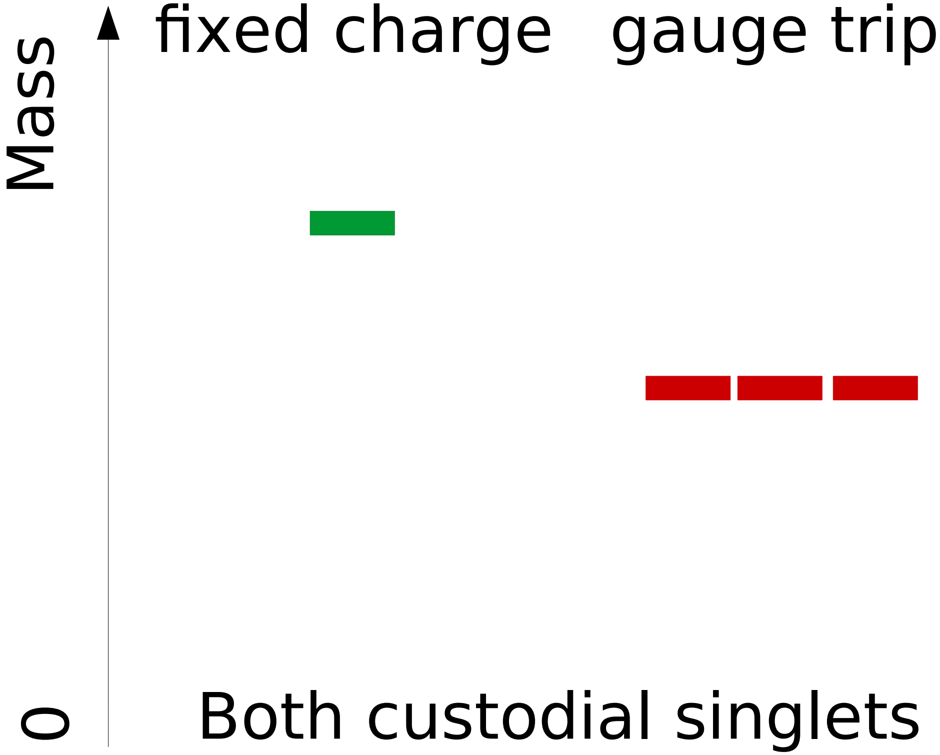


Both custodial singlets

Remember: Experiment tells that somehow the left is correct!

Physical spectrum

Perturbation theory



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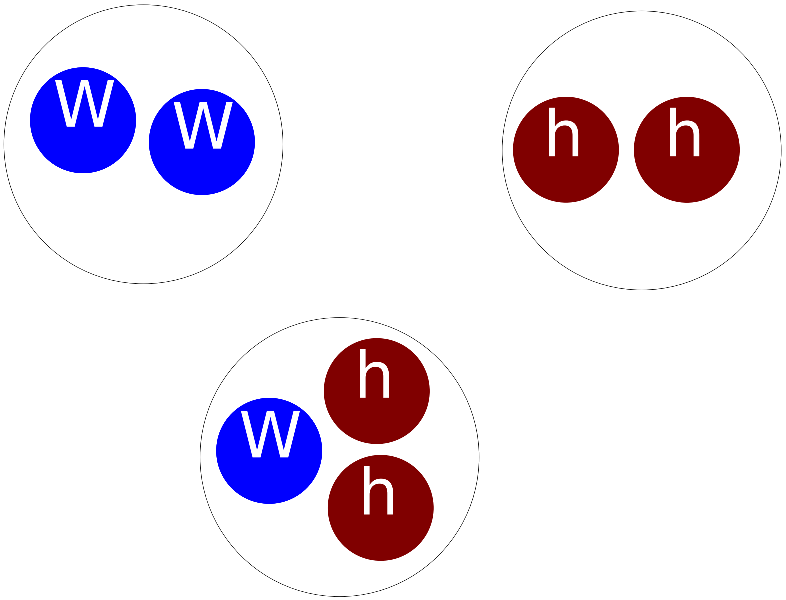

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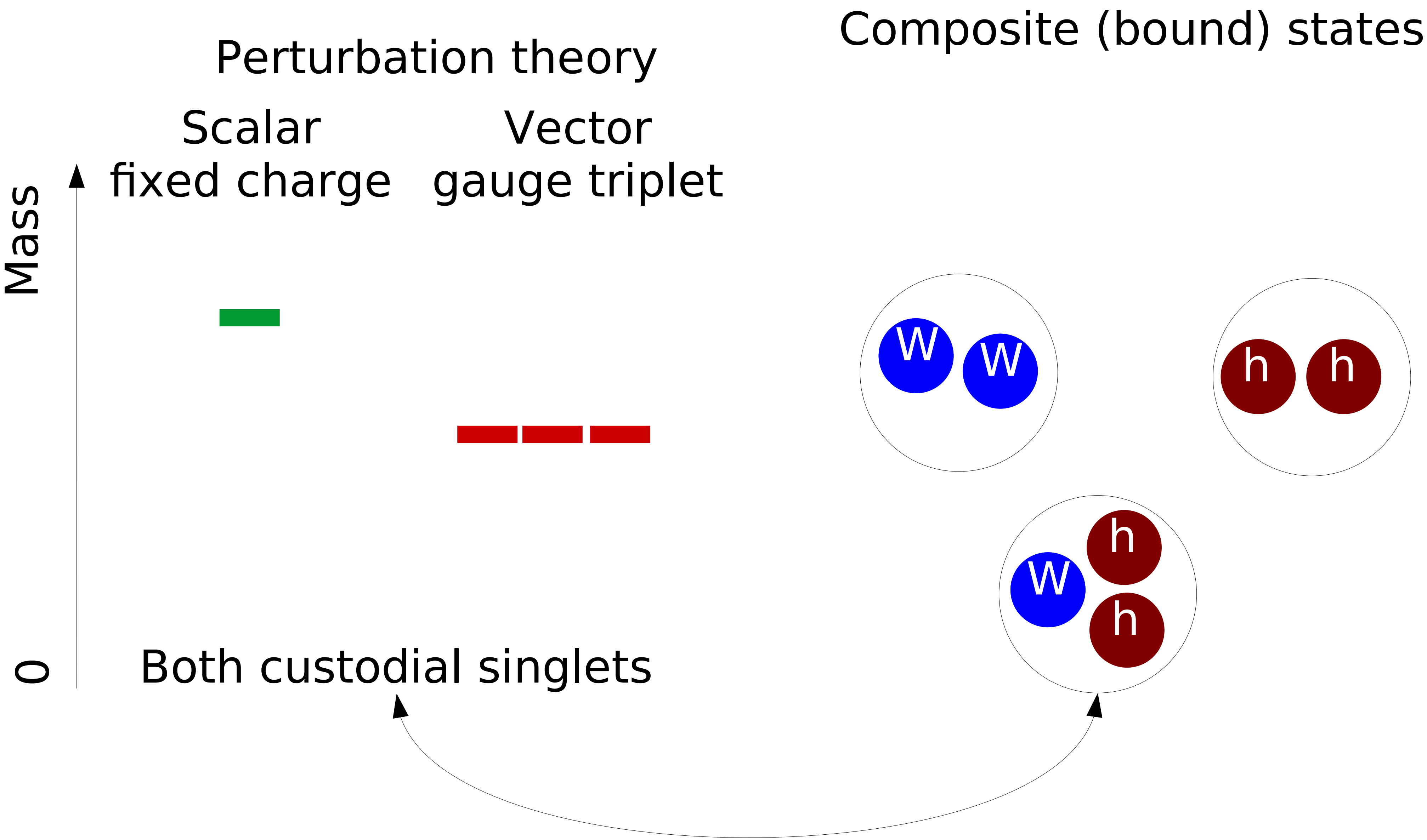
Physical spectrum



Perturbation theory

Composite (bound) states

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Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

Physical particles

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
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- J^{PC} and custodial charge only quantum numbers

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    - Standard lattice spectroscopy problem
    - Standard methods
      - Smearing, variational analysis, systematic error analysis etc.
    - Very large statistics ($>10^5$ configurations)

Physical spectrum

[Maas'12, Maas & Mufti'14]

Perturbation theory

Gauge-invariant

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Vector
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Scalar
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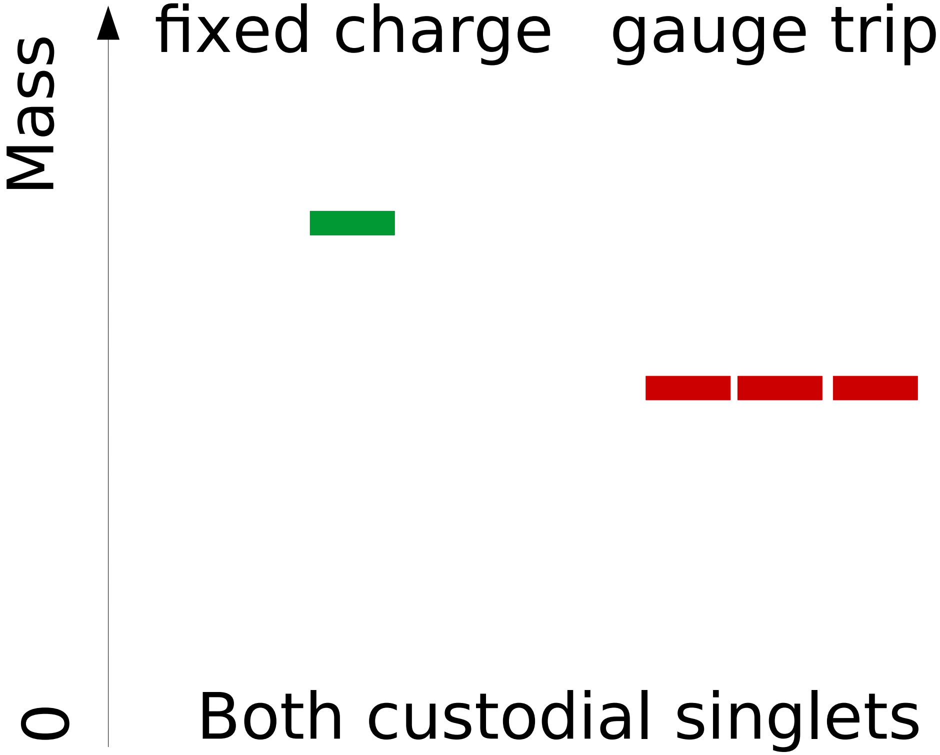

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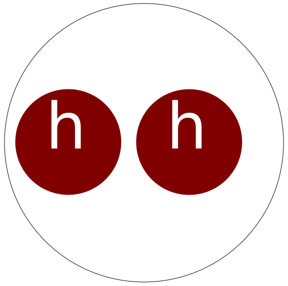

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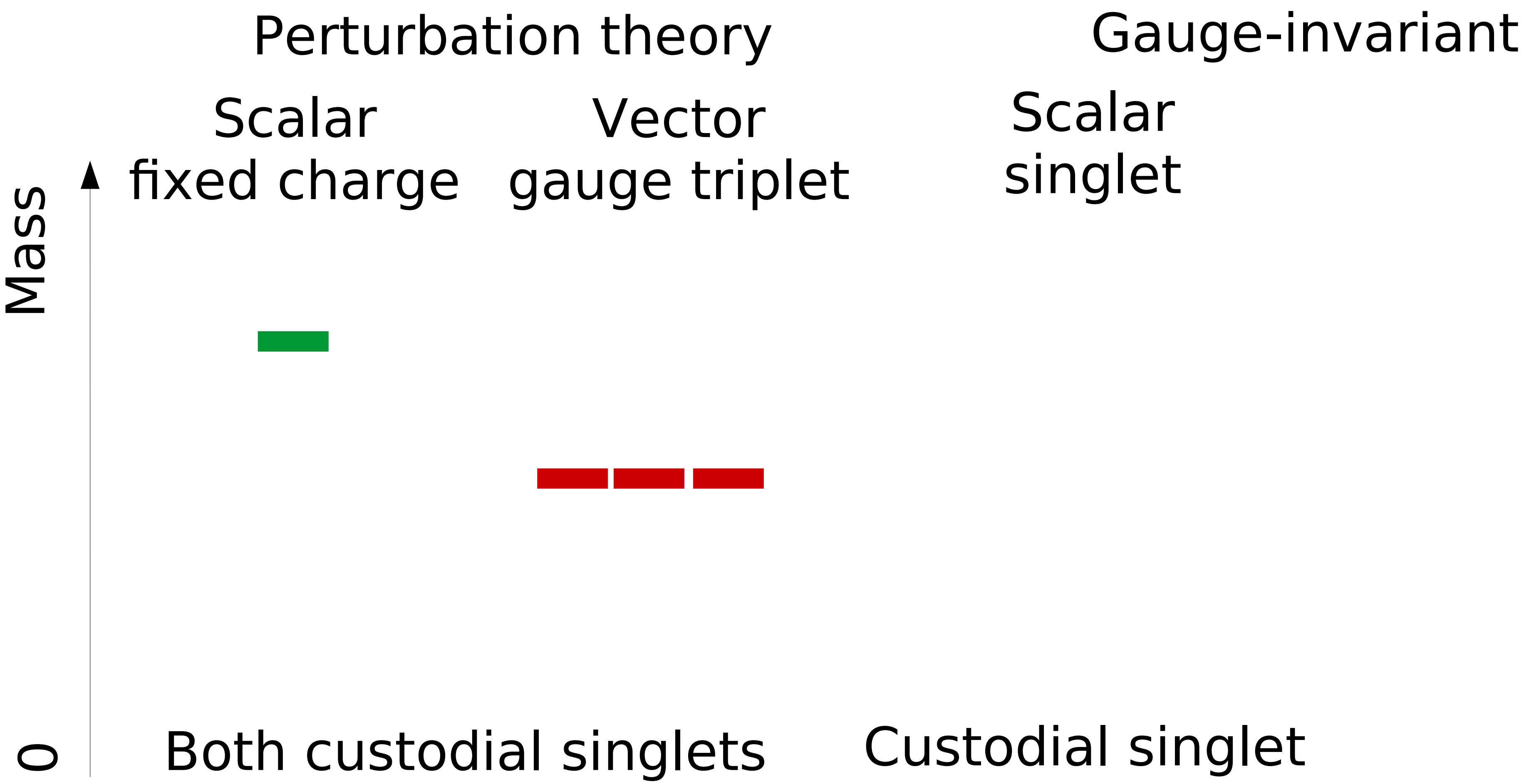

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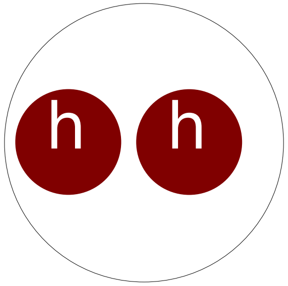

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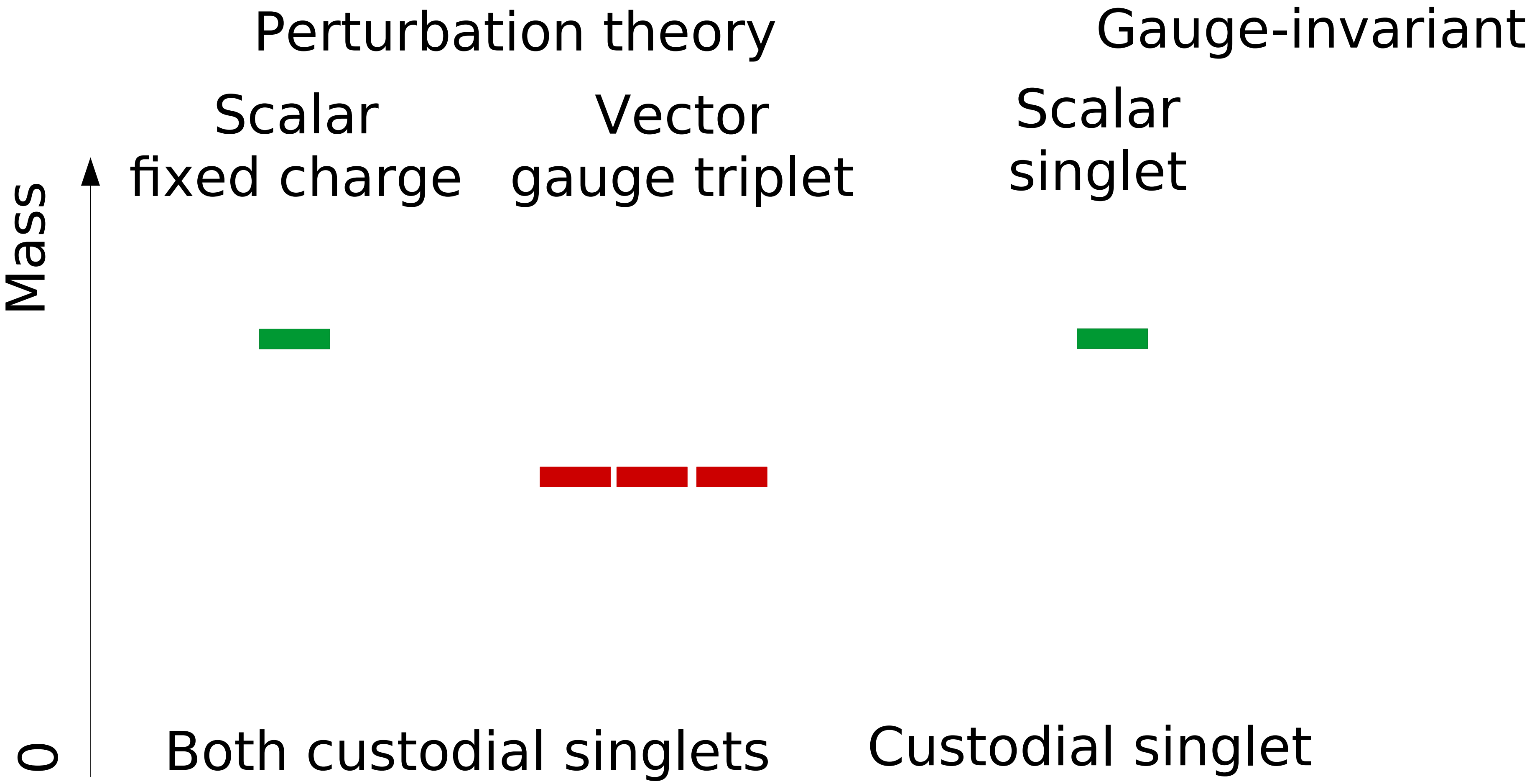

Perturbation theory

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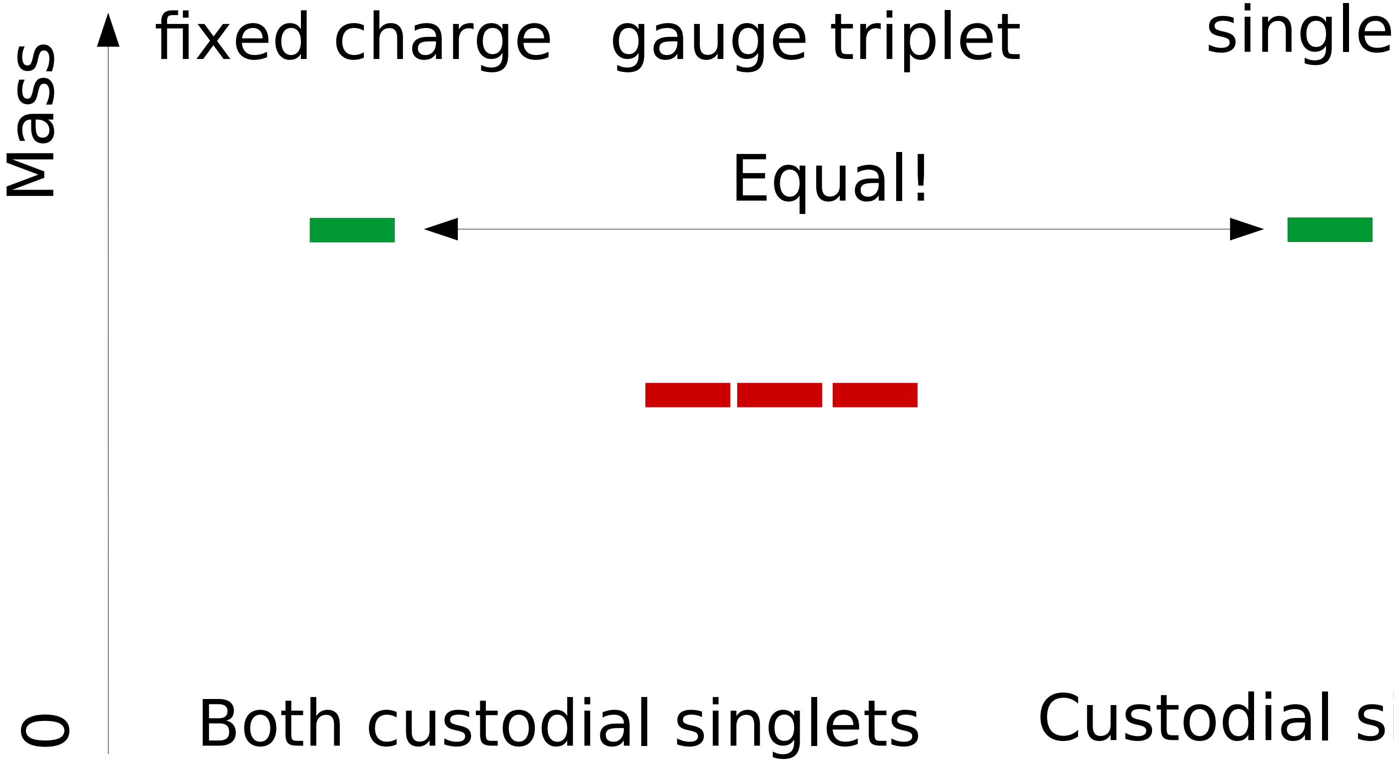

Equal!

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Custodial singlet

Physical spectrum

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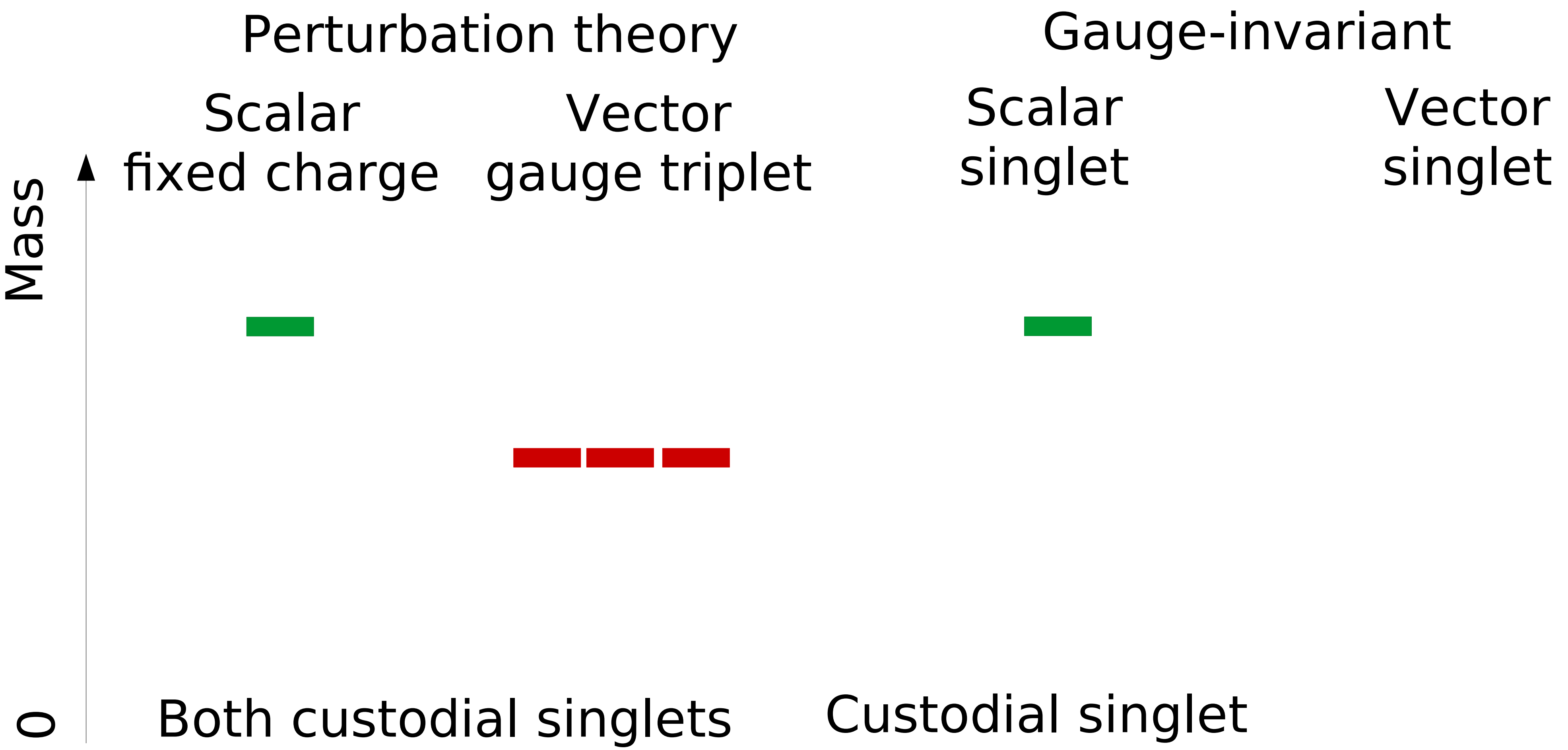

Perturbation theory

Scalar
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Gauge-invariant

Scalar
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Vector
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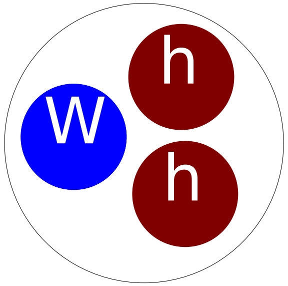
Mass

0

Both custodial singlets

Custodial singlet

$$tr\, t^a \frac{h^+}{\sqrt{h^+ h}} D_\mu \frac{h}{\sqrt{h^+ h}}$$



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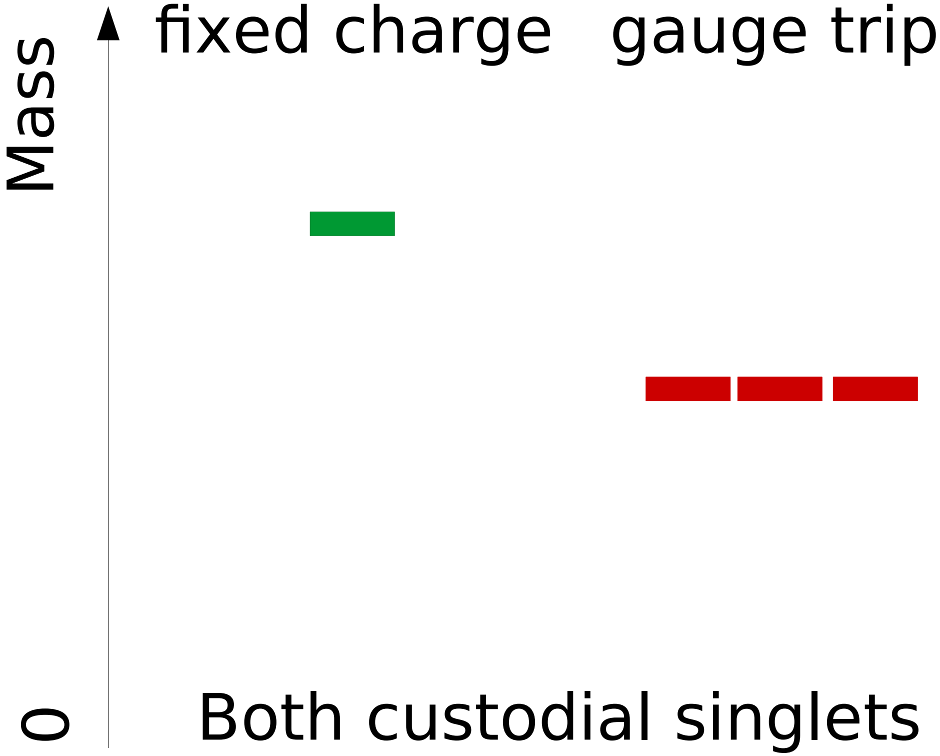

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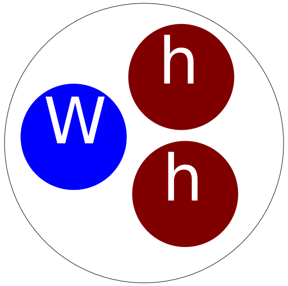

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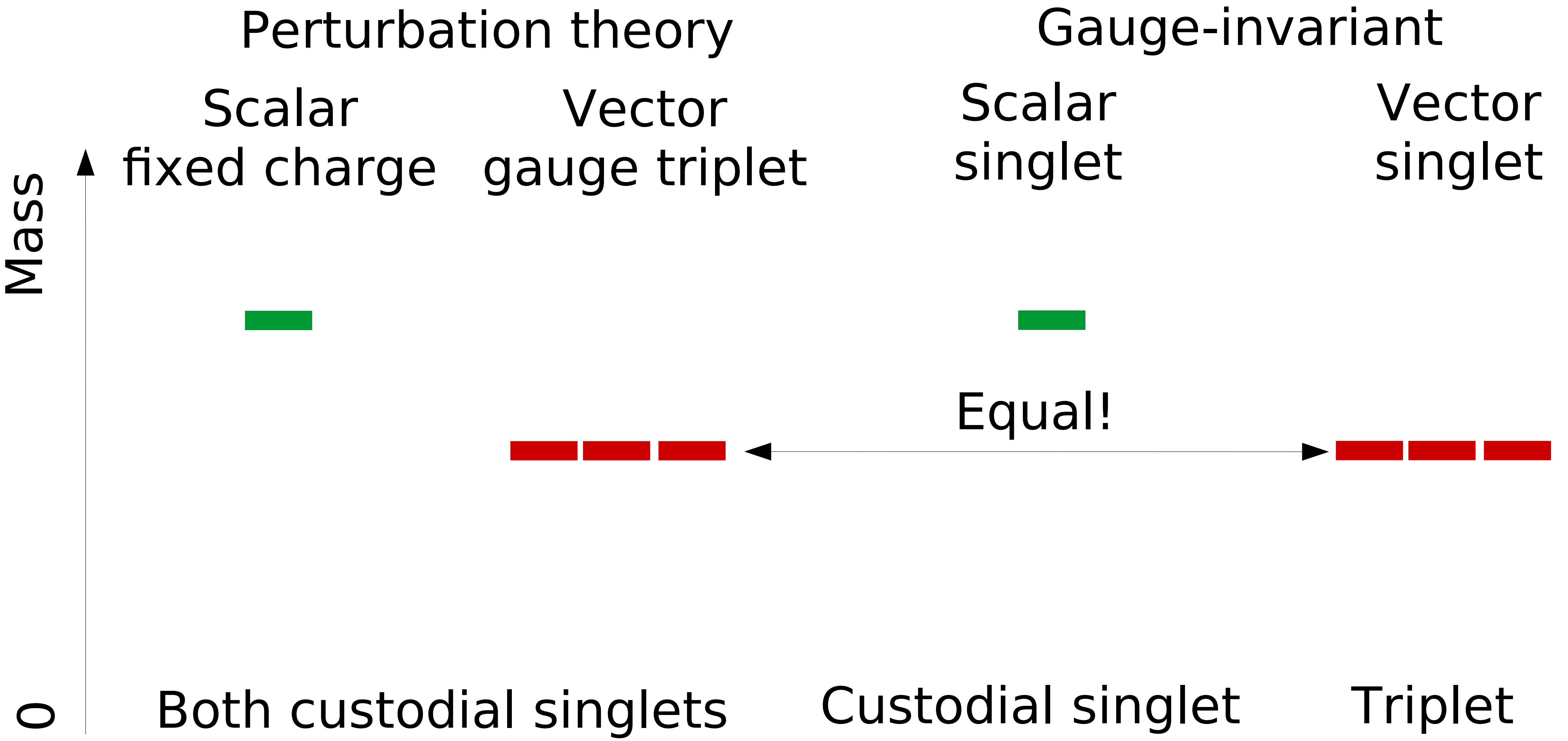

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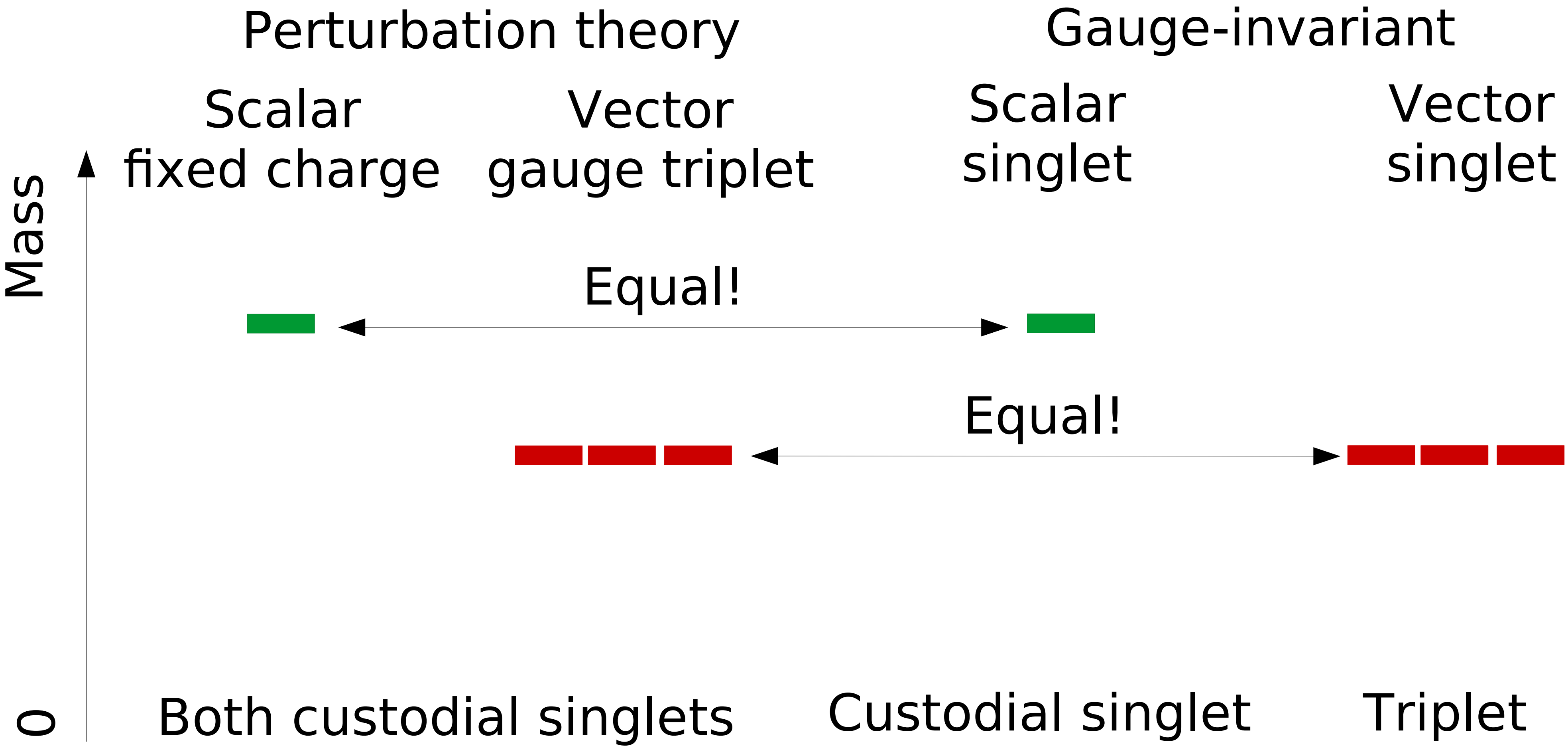
[Maas'12, Maas & Mufti'14]



Perturbation theory

Gauge-invariant

Scalar
fixed charge

Vector
gauge triplet

Scalar
singlet

Vector
singlet

Mass

0

Equal!

Equal!

Both custodial singlets

Custodial singlet

Triplet

Why?

A microscopic origin
-
Fröhlich-Morchio-Strocchi mechanism

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
Maas, Sondenheimer & Törek'17
Maas & Sondenheimer ‘20]

- J^{PC} and custodial charge only quantum numbers
  - Different from perturbation theory
    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods?

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- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas’12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

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Higgs field

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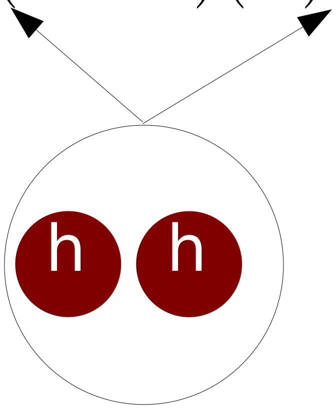

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Trivial two-particle state

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What about this?
→ Duijfe van Egmond’s talk

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What about the vector?

[Fröhlich et al.’80,’81
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Matrix from group structure

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Exactly one gauge boson for every physical state

Phenomenological Implications
-
Can we measure this?

Bound states as extended objects

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- Two possibilities to measure extension

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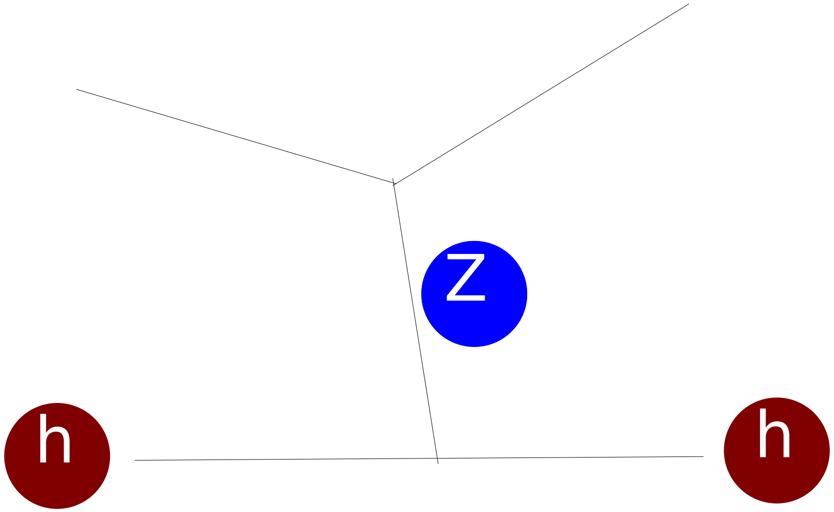
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process

[Maas, Raubitzek, Törek'18]



Bound states as extended objects

- Two possibilities to measure extension
  - Form factor
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  - Elastic scattering
    - Standard vector boson scattering process at low energies
    - Use this one

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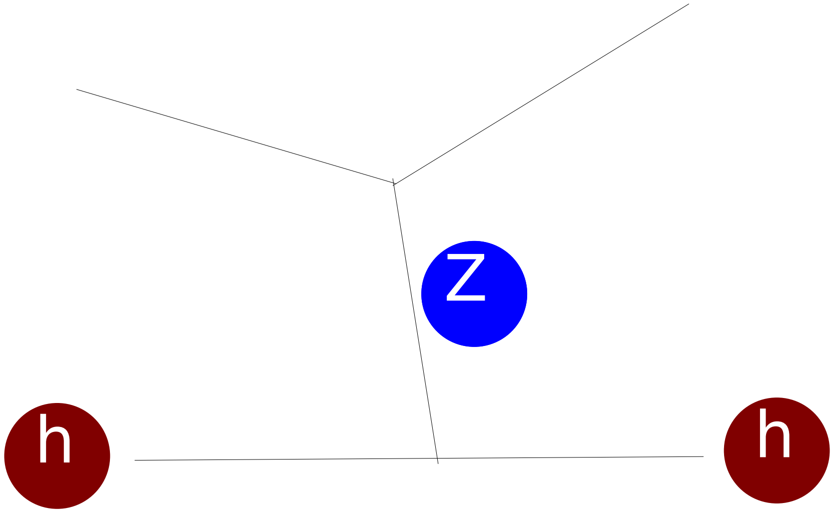

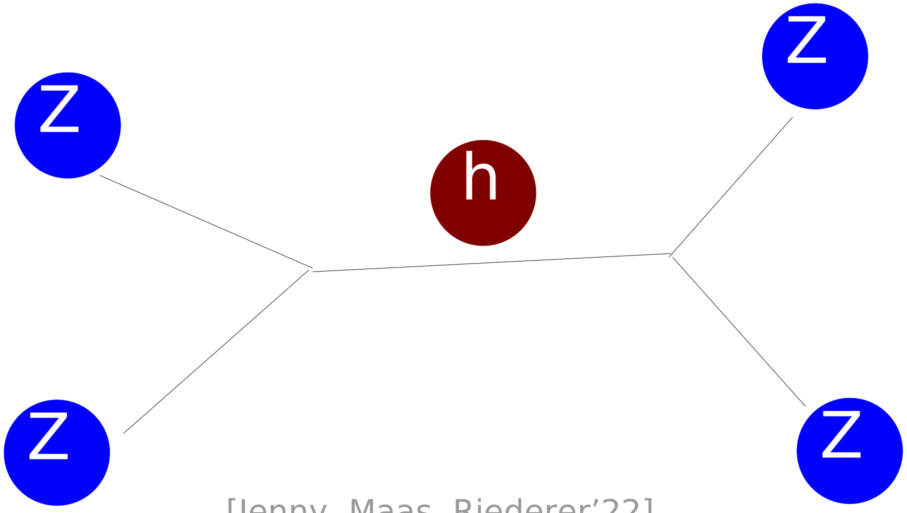

[Jenny, Maas, Riederer'22]

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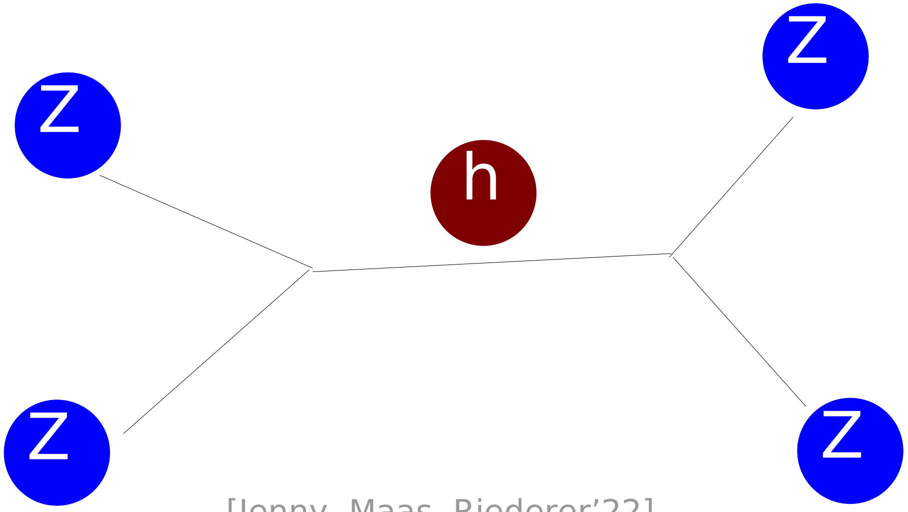

[Jenny, Maas, Riederer'22]

Radius from elastic scattering in VBS

- Elastic region: $160/180\,GeV \leqslant \sqrt{s} \leqslant 250\,GeV$
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Cross section

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Matrix element

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Legendre polynom

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Scattering length~"size"

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Scattering length~"size"

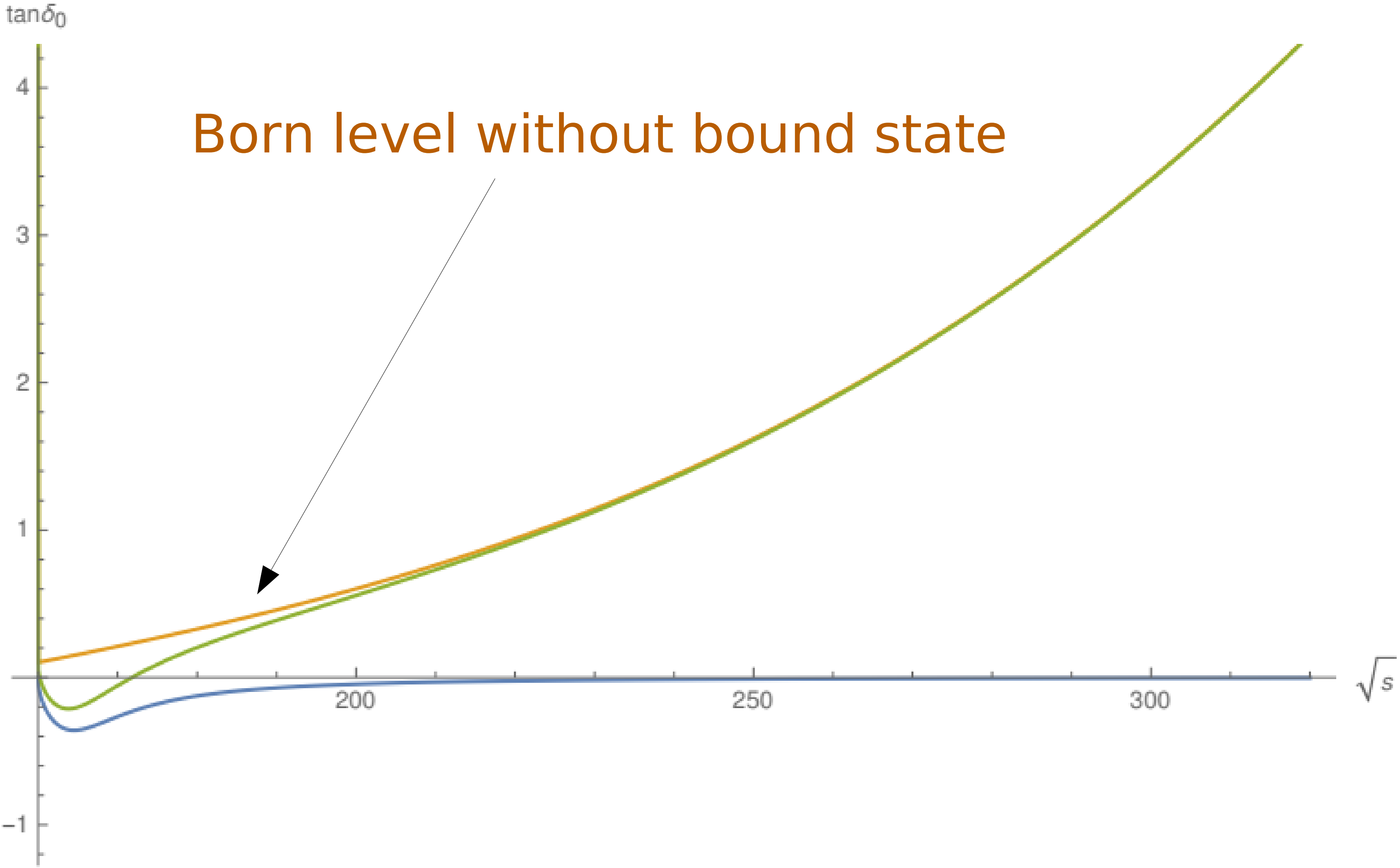
Phase shift

→ Lattice Lüscher analysis

Impact of a finite size of the Higgs

- Consider the Higgs: $J=0$

Impact of a finite size of the Higgs



Born level without bound state

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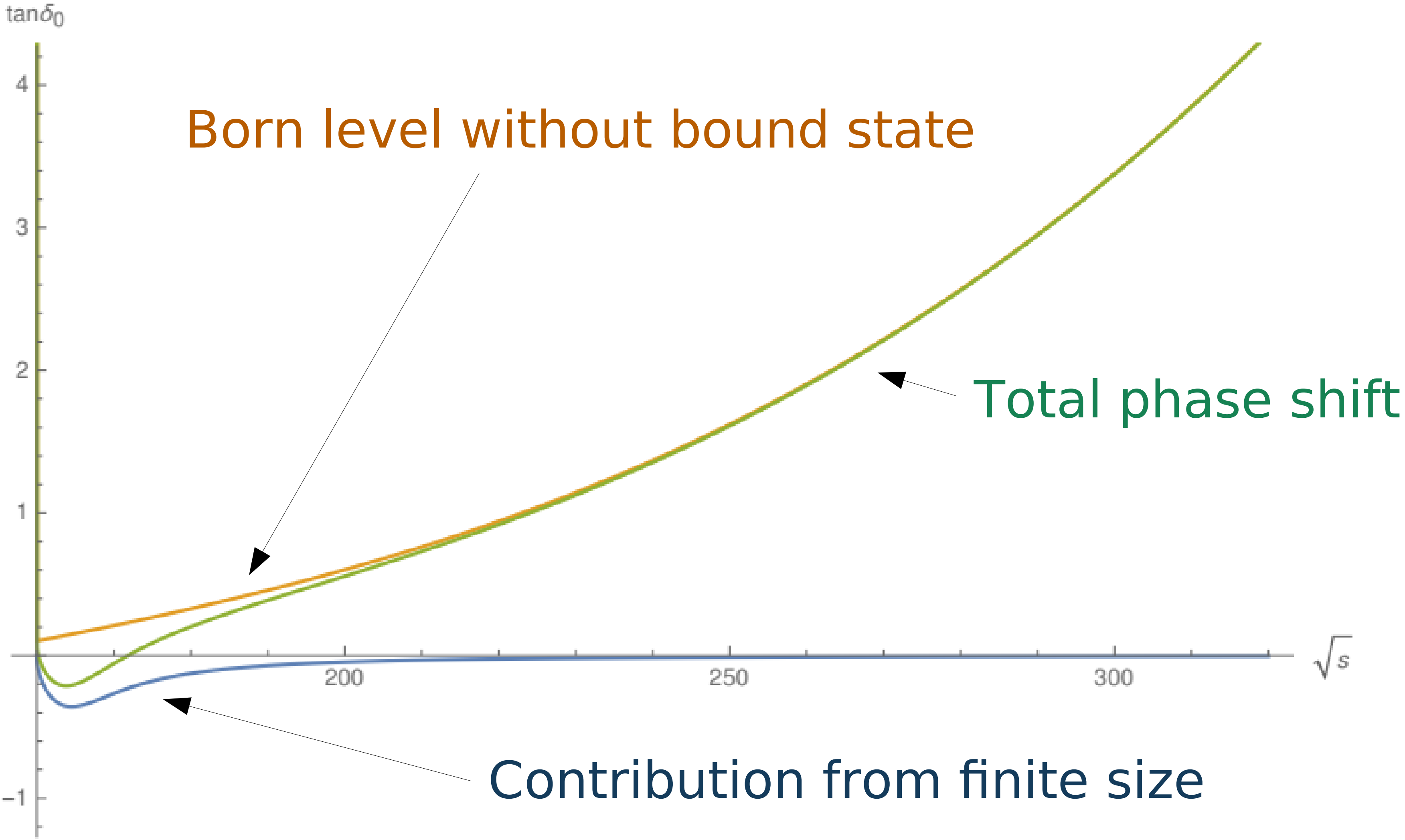


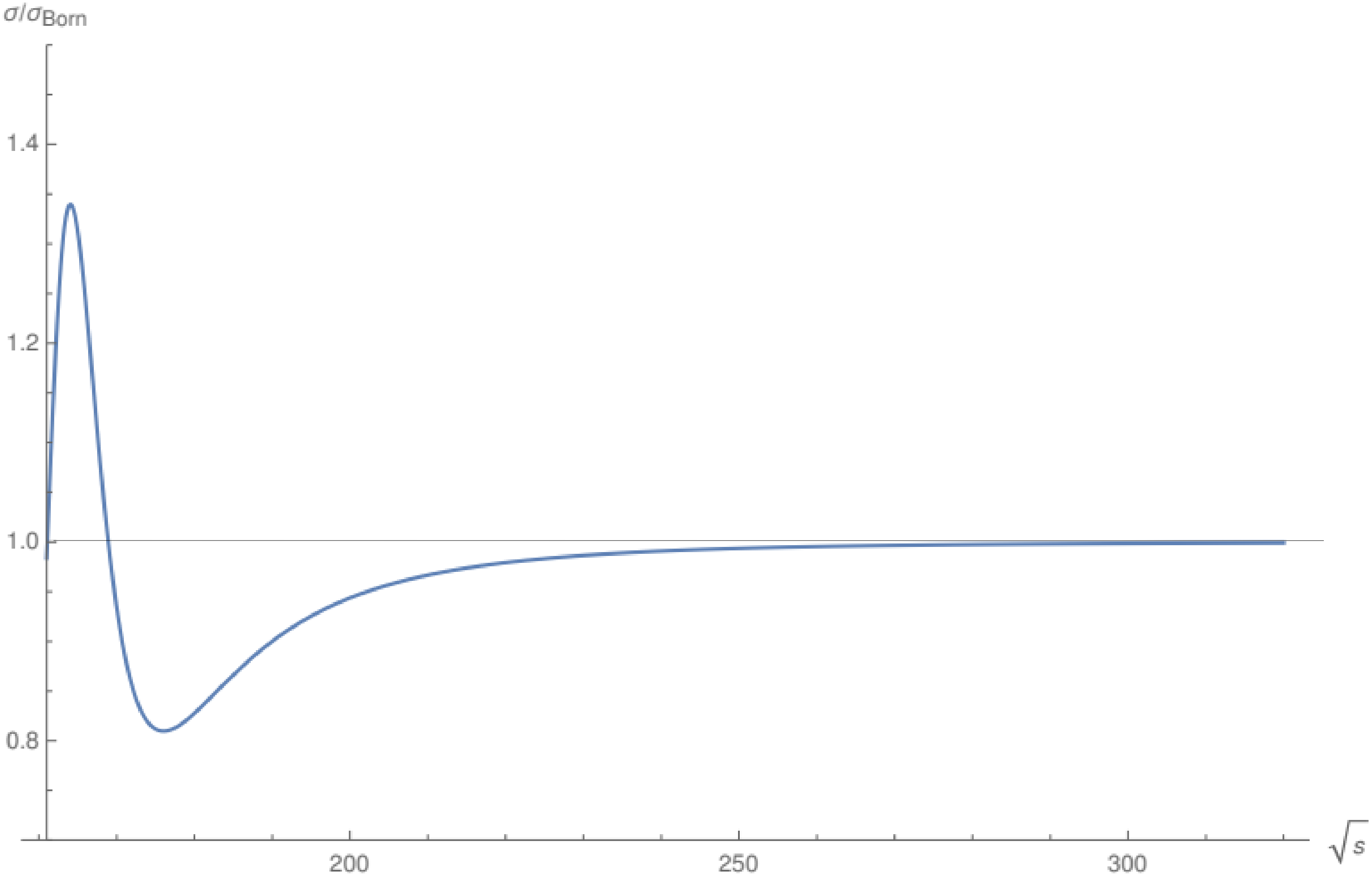
Born level without bound state

Total phase shift

Contribution from finite size

- Consider the Higgs: *J=0*
- Mock-up effect
    - Scattering length 1/(40 GeV)

Impact of a finite size of the Higgs



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Impact on the radius of the Higgs

- Reduced SM: Only W/Z and the Higgs
    - Parameters slightly different
        - Higgs 145 GeV and weak coupling larger

[Jenny, Maas, Riederer’22]

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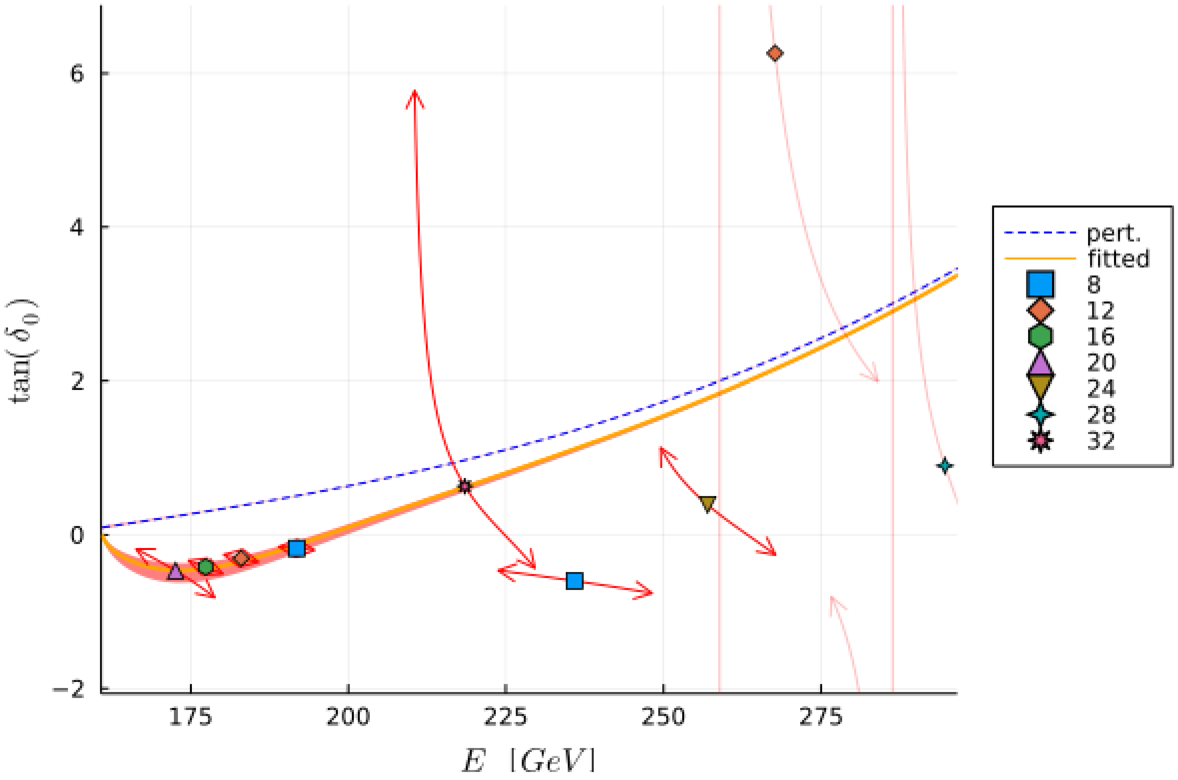

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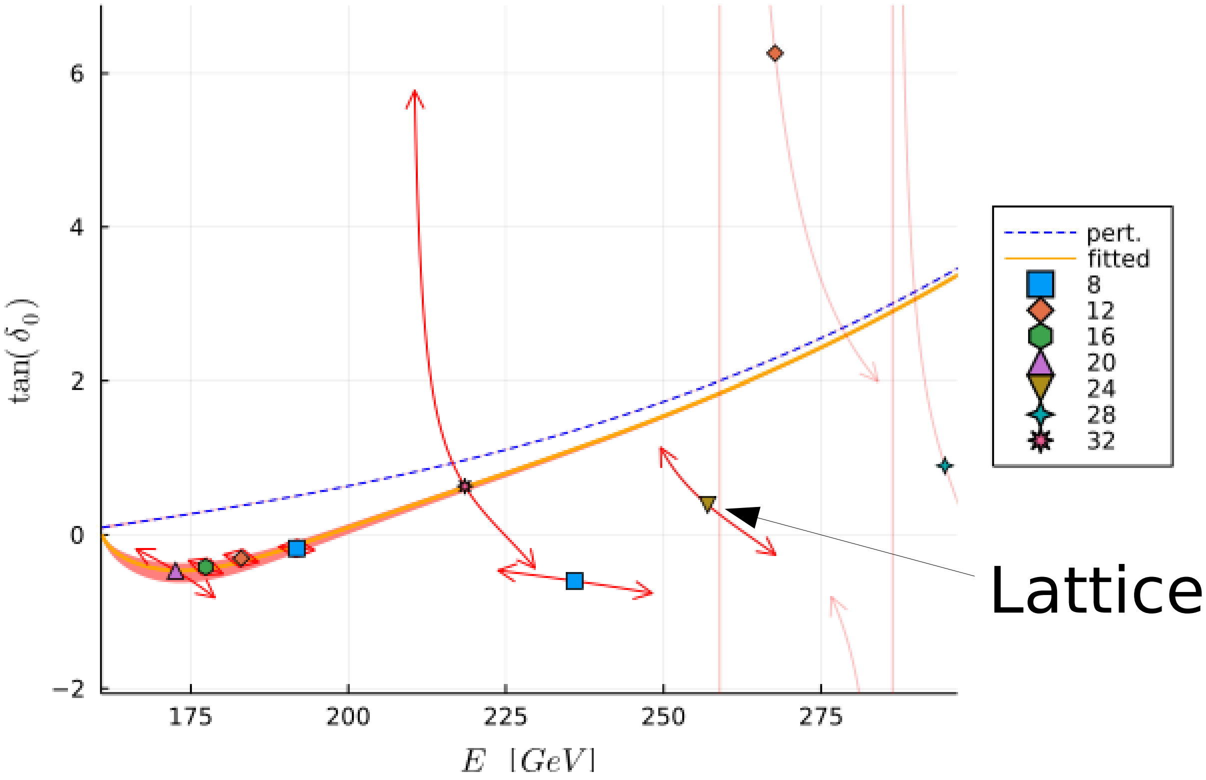

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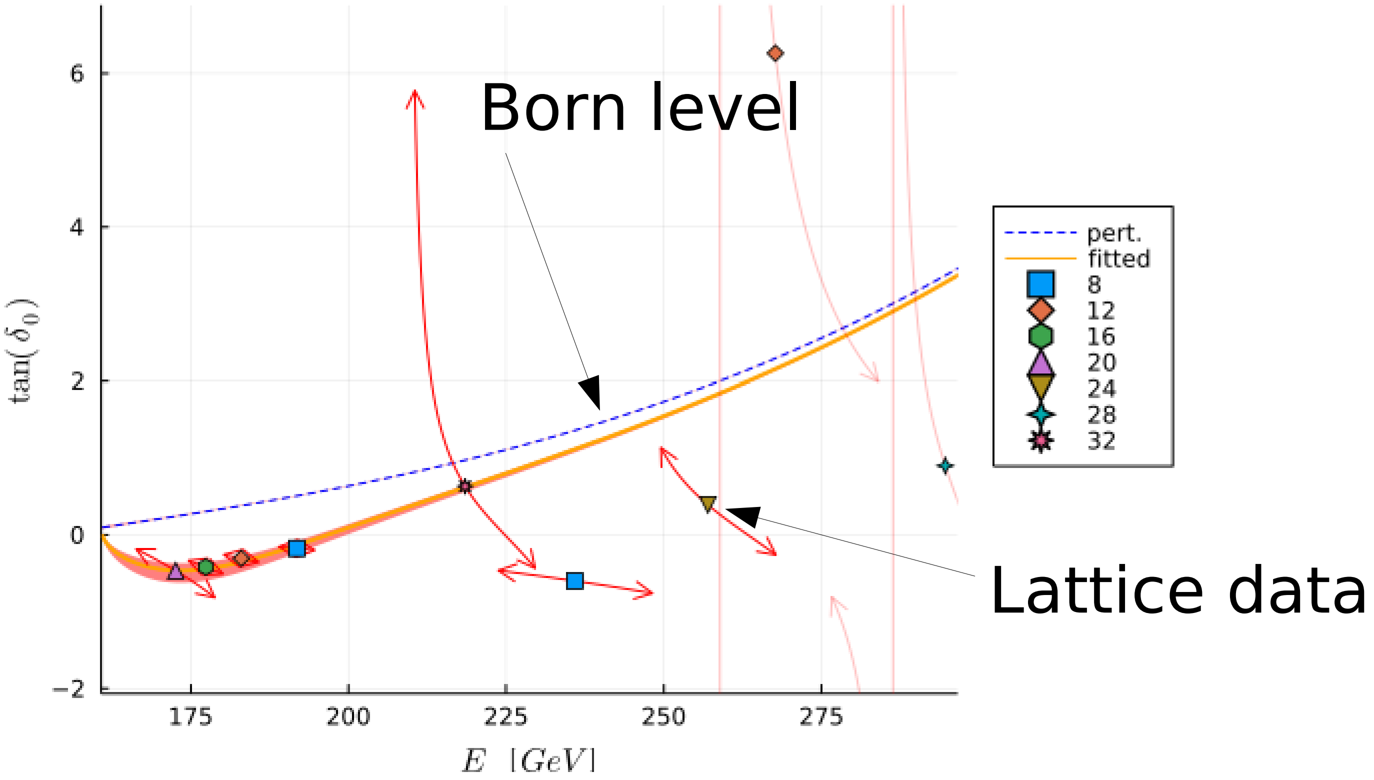

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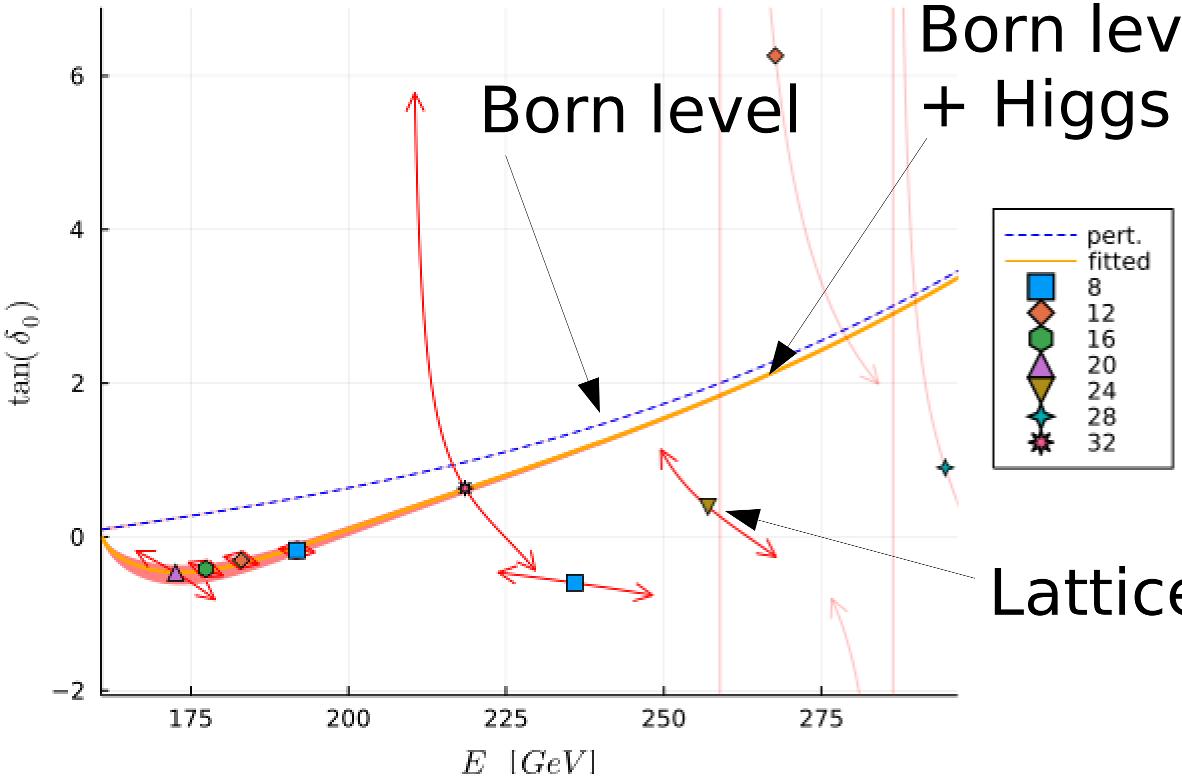

Born level

Born level FMS

+ Higgs size of (40 Gev)$^{-1}$

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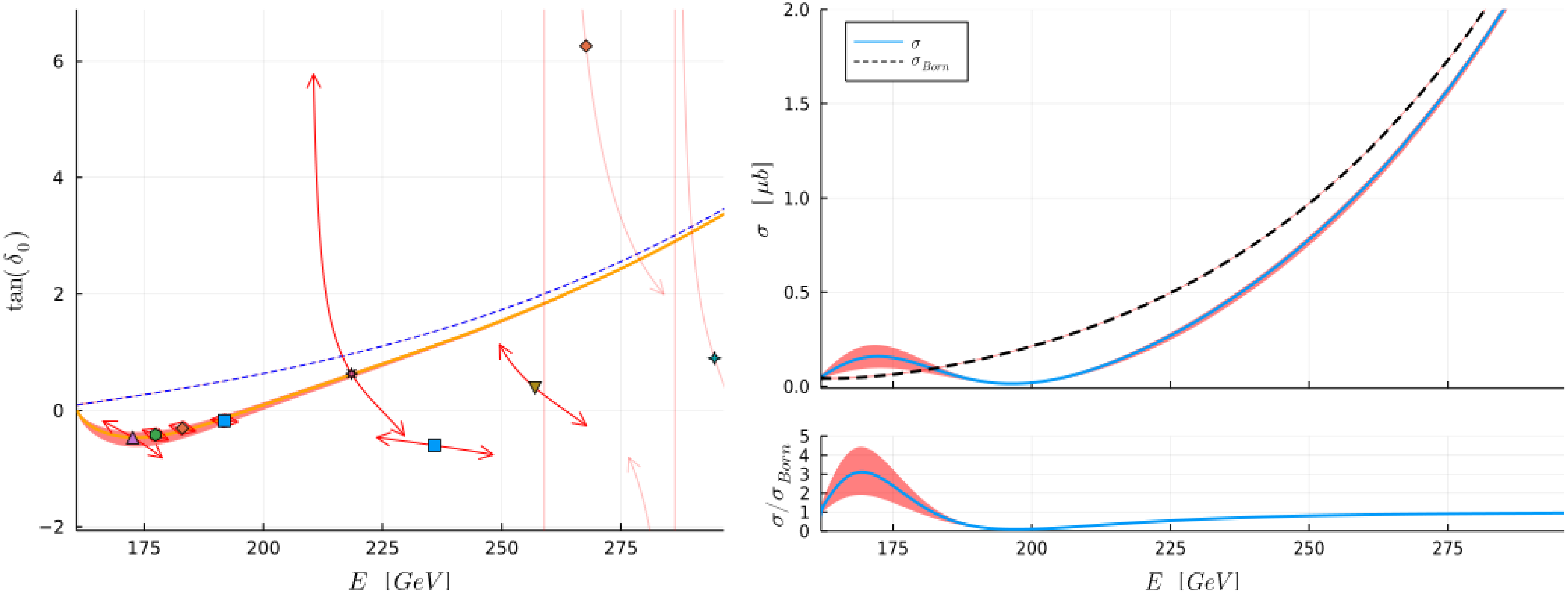

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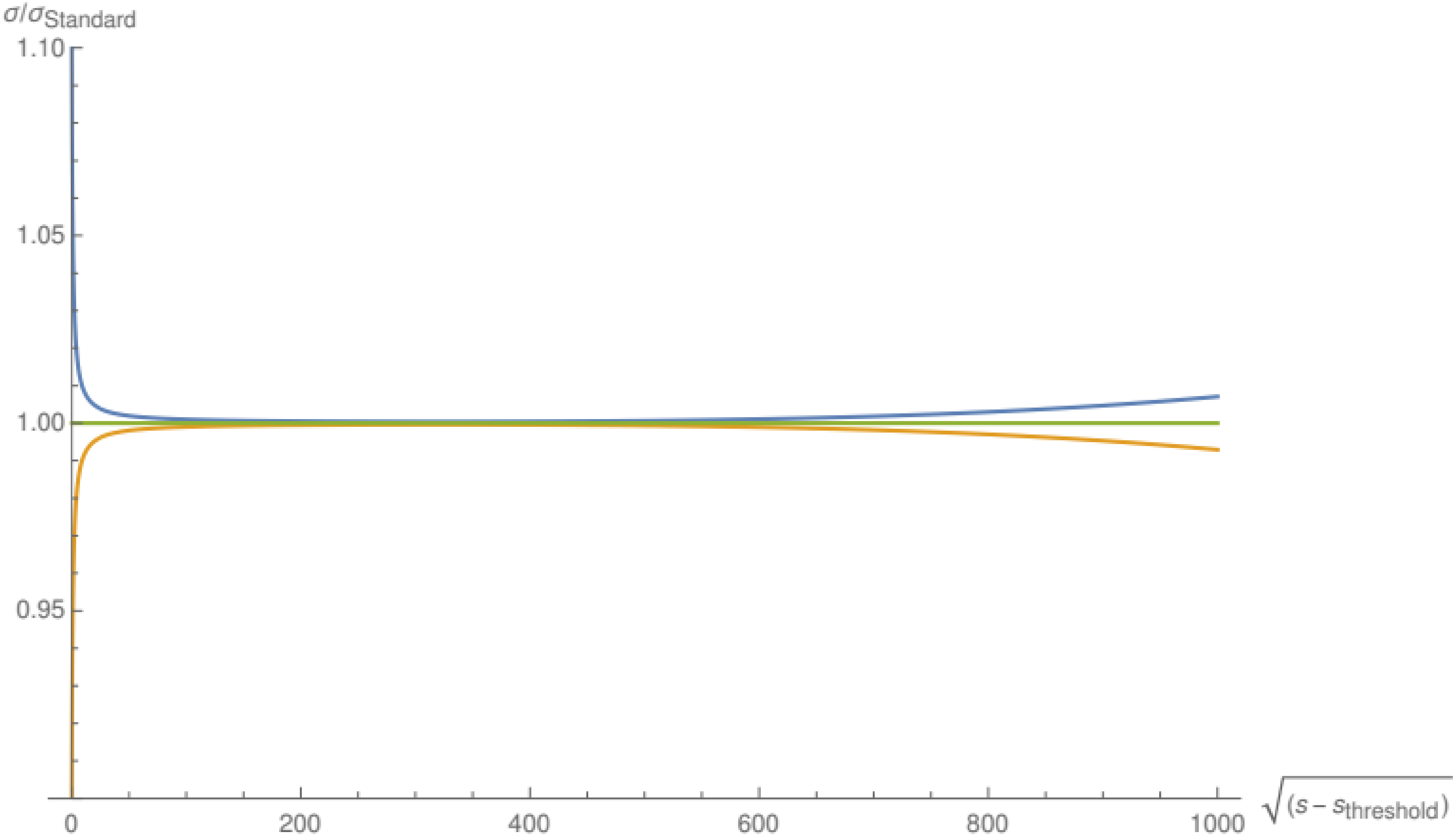
Generic behavior

Has been done for several observables

Generic behavior: DIS-like



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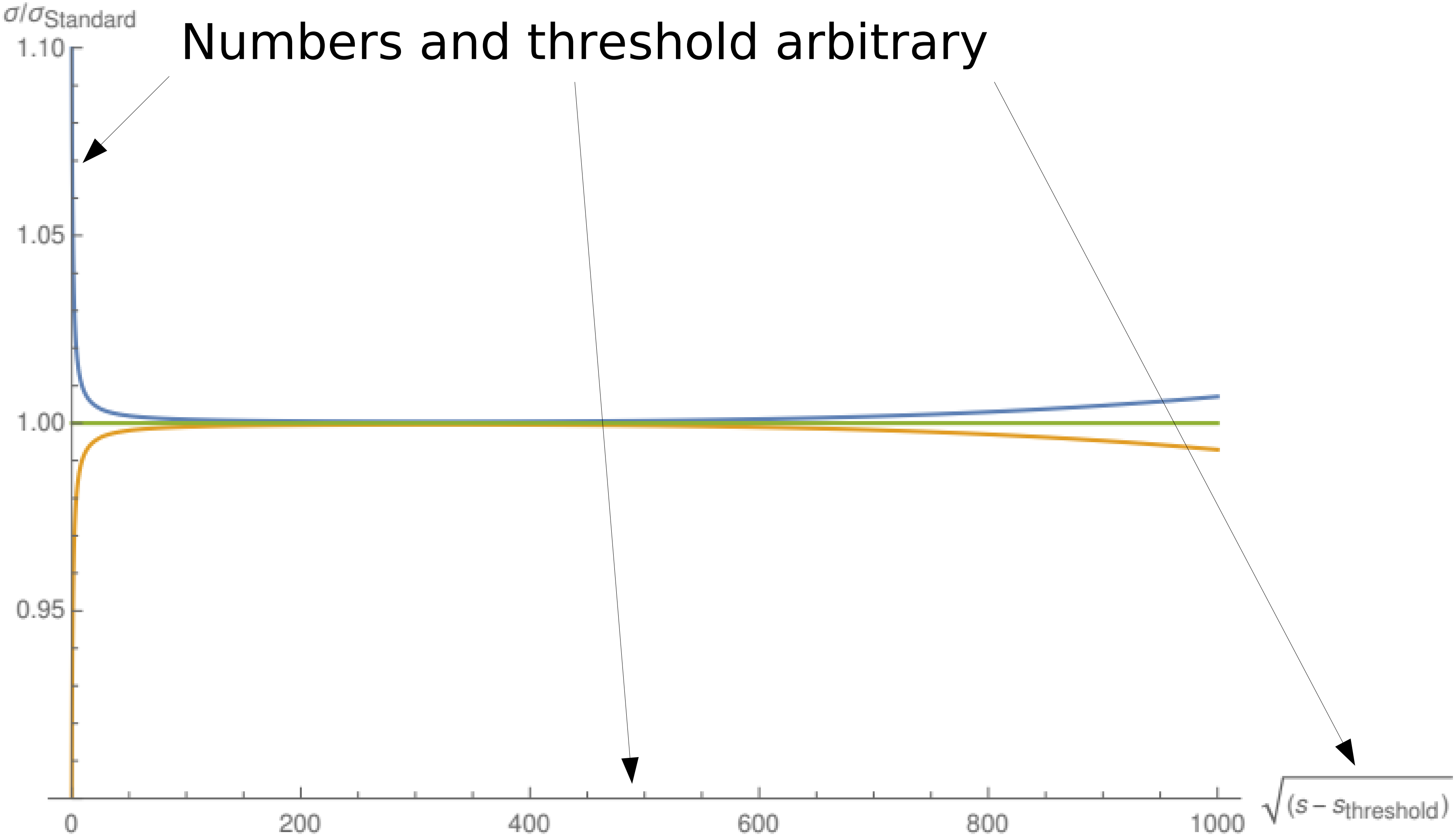

Numbers and threshold arbitrary

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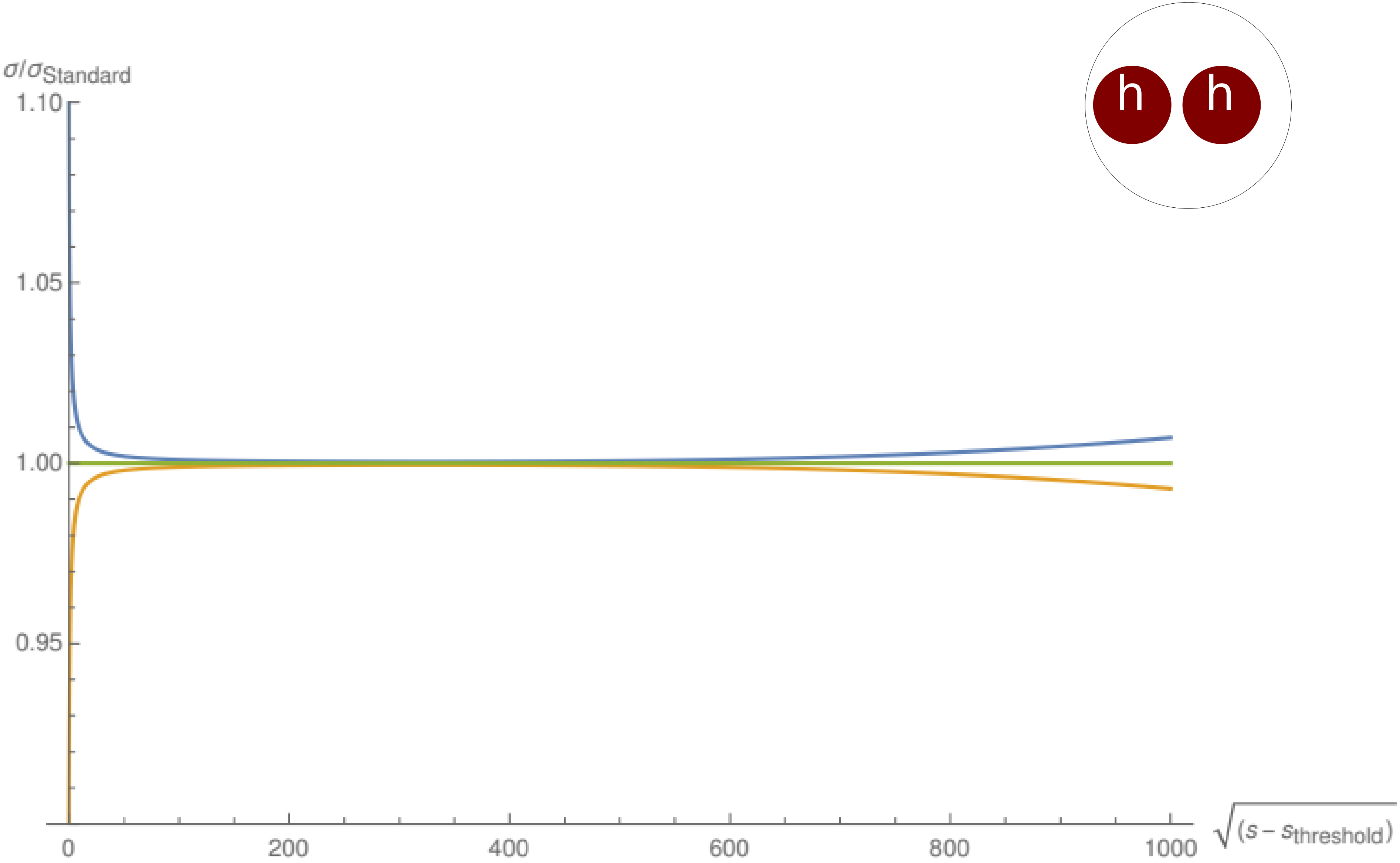

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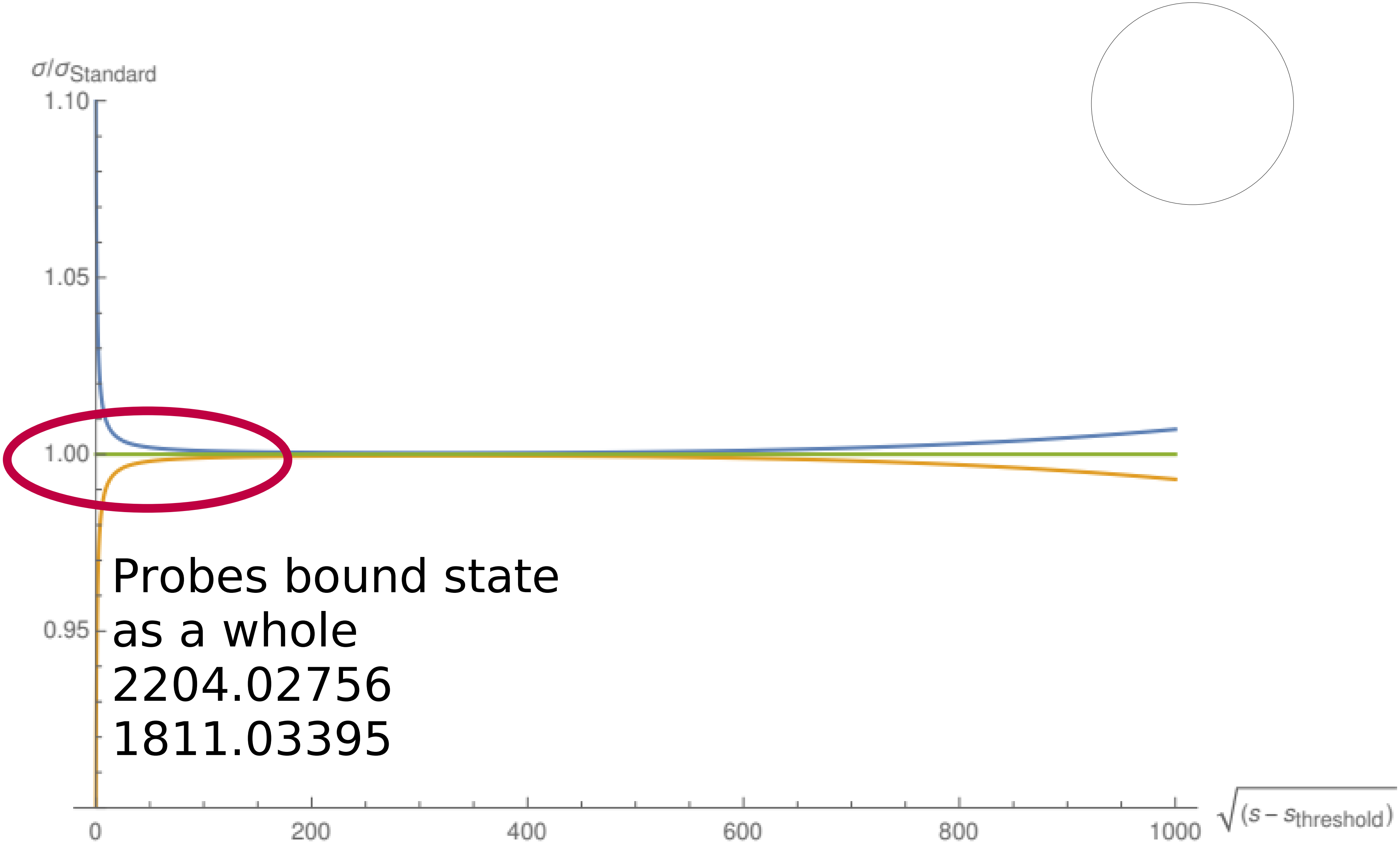
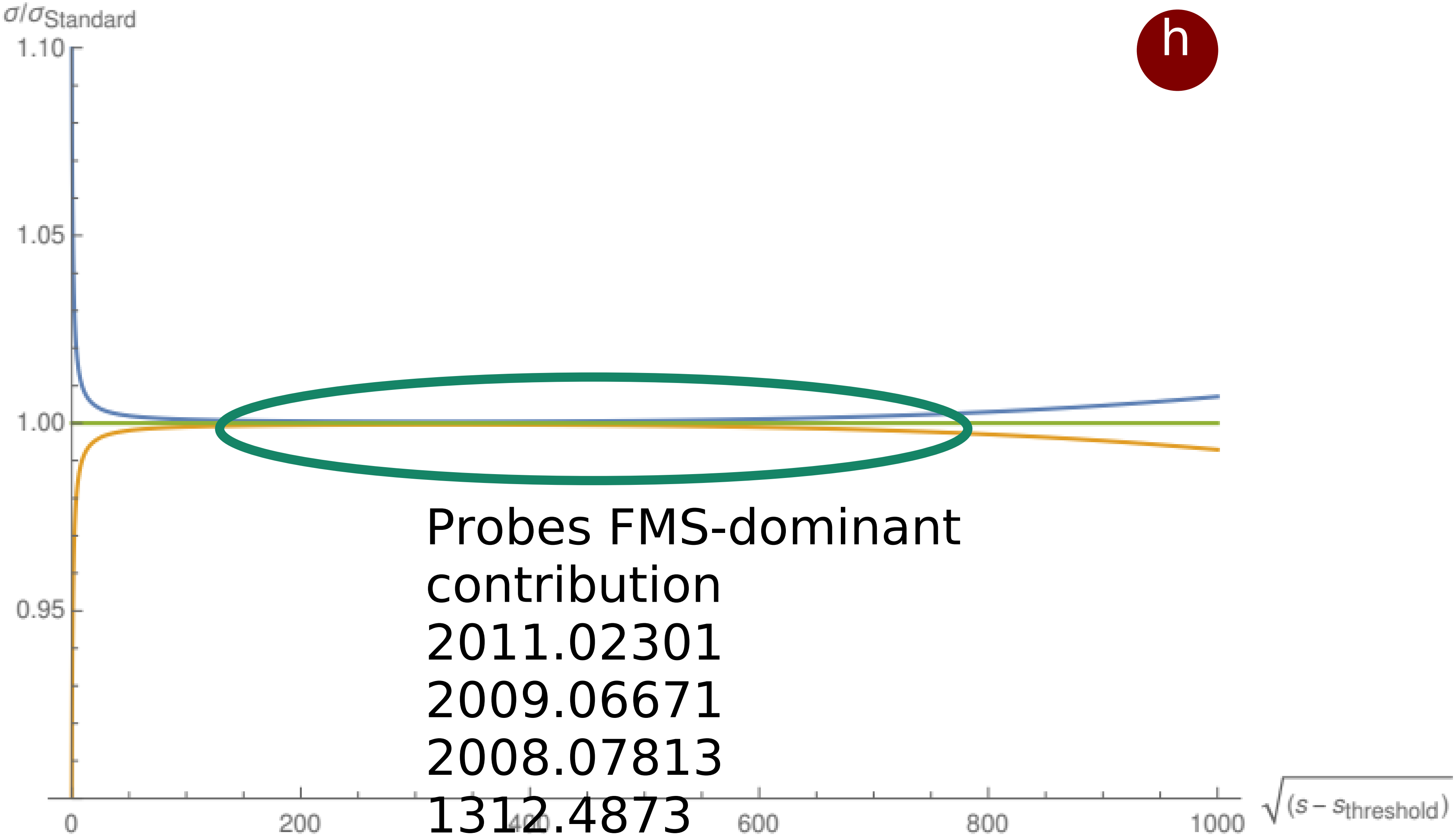


Probes bound state
as a whole
2204.02756
1811.03395

Has been done for several observables

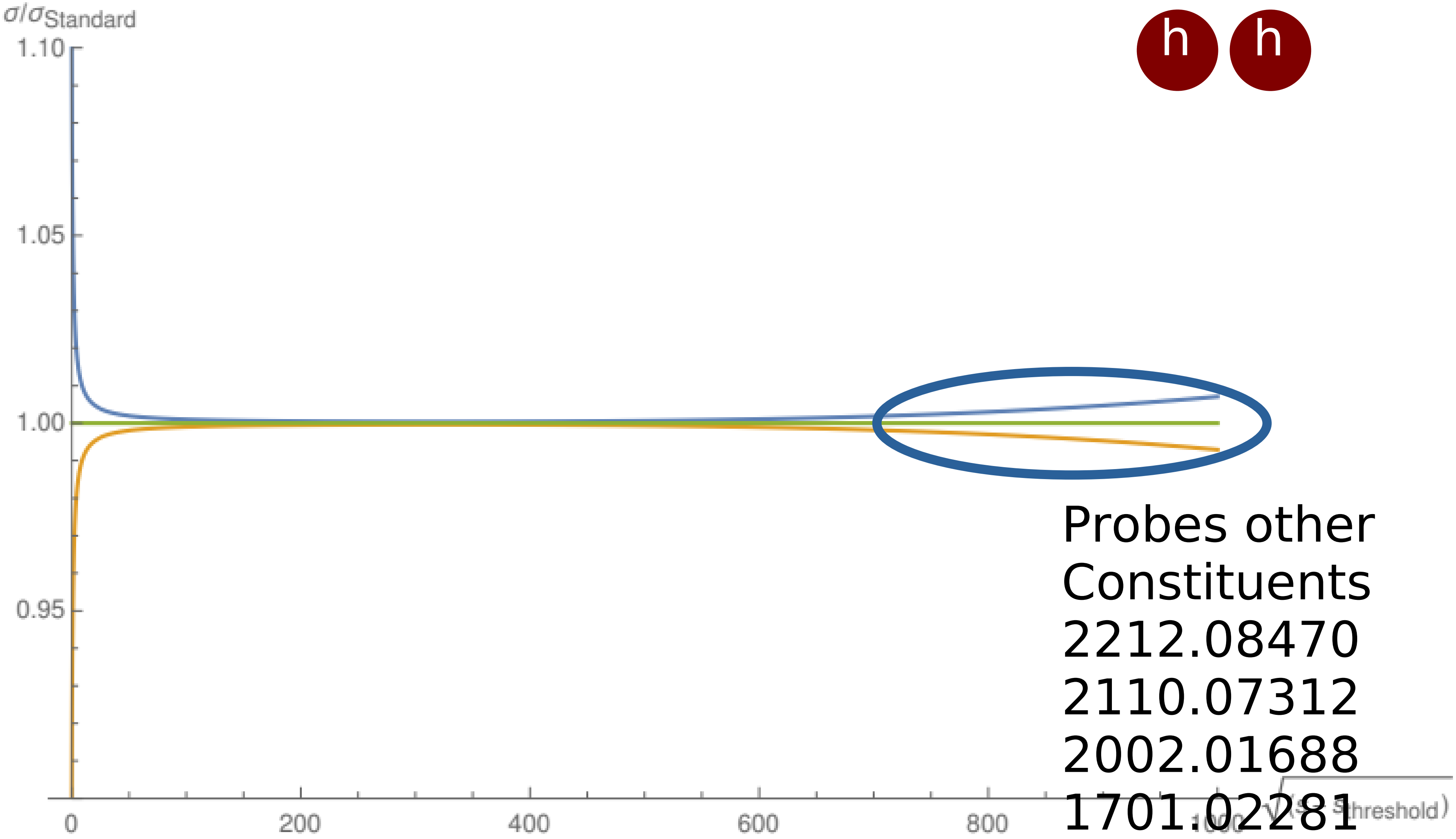
Generic behavior: DIS-like



Probes FMS-dominant contribution
2011.02301
2009.06671
2008.07813
1312.4873

Has been done for several observables

Generic behavior: DIS-like



Probes other Constituents
2212.08470
2110.07312
2002.01688
1701.02281

Has been done for several observables

Elimination
-
What are the true degrees of freedom?

Decovariantization

$$L = -\frac{1}{4} W^{a}_{\mu\nu} W^{\mu\nu}_{a} + \left(D^{ij}_{\mu} h^{j} \right)^{+} D^{\mu}_{ik} h_{k} + \lambda \left(h^{a} h_{a}^{+} - v^{2} \right)^{2}$$

$$W^{a}_{\mu\nu} = \partial_{\mu} W^{a}_{\nu} - \partial_{\nu} W^{a}_{\mu} + g f^{a}_{bc} W^{b}_{\mu} W^{c}_{\nu}$$

$$D^{ij}_{\mu} = \delta^{ij} \partial_{\mu} - i g W^{a}_{\mu} t^{ij}_{a}$$

Decovariantization

- Perform a variable transformation

$$h=\sqrt{\phi^{+}\ \phi} \quad Z^a=tr\ \tau^a \phi^{+}\ D\phi$$

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Non-trivial tree-level structure defects or large λ

Well-defined theory, can be simulated on the lattice

[Jersak et al.'85, Evertz et al.'86]

The structure of the particles

- Elementary particles appear point-like
    - But equivalent theories: At the quantum level extended objects!
    - Others: Looks like having a substructure
        - Peaks at 1/2, 1/3, 1/4,...

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- Properties of particles do not uniquely determine if elementary or composites of a gauge theory

Generalizing

[Berghofer et al.’21]

- Rewriting of a gauge theory as an ungauged theory

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    - Possible for QED
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    - (Quantum) gravity as dynamical triangulation → Talk by Renate Loll [Regge ’61, Ambjorn et al. ’12]
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  - (Quantum) gravity as dynamical triangulation → Talk by Renate Loll [Regge '61, Ambjorn et al. '12]
  - Generalization: Dressing field method → Talk by Jordan Francois
- Conversely: Many (all?) ungauged theories can be written explicitly as a gauge theory
  - (Generalized) Kretschmann hypothesis: Always possible [Kretschmann '17, Einstein '18, Kibble '67, Pitts '09, Francois '18]

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
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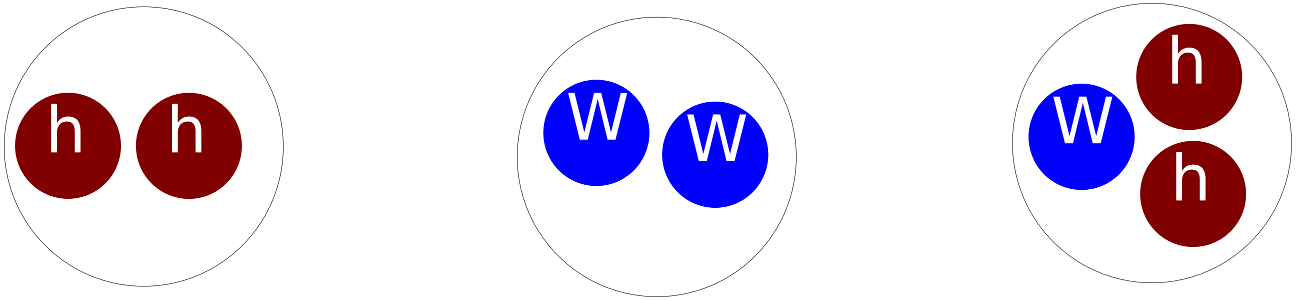
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



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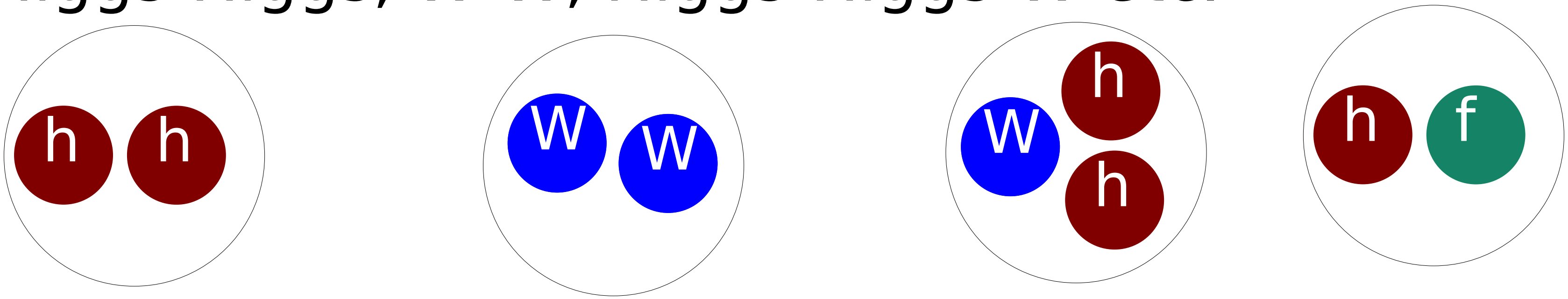

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[Fröhlich et al.'80,
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  - Global SU(3) generation
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Flavor on the lattice

[Afferrante,Maas,Sondenheimer,Törek'20]

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  - Compressed mass scales
  - One generation
  - Degenerate leptons and neutrinos
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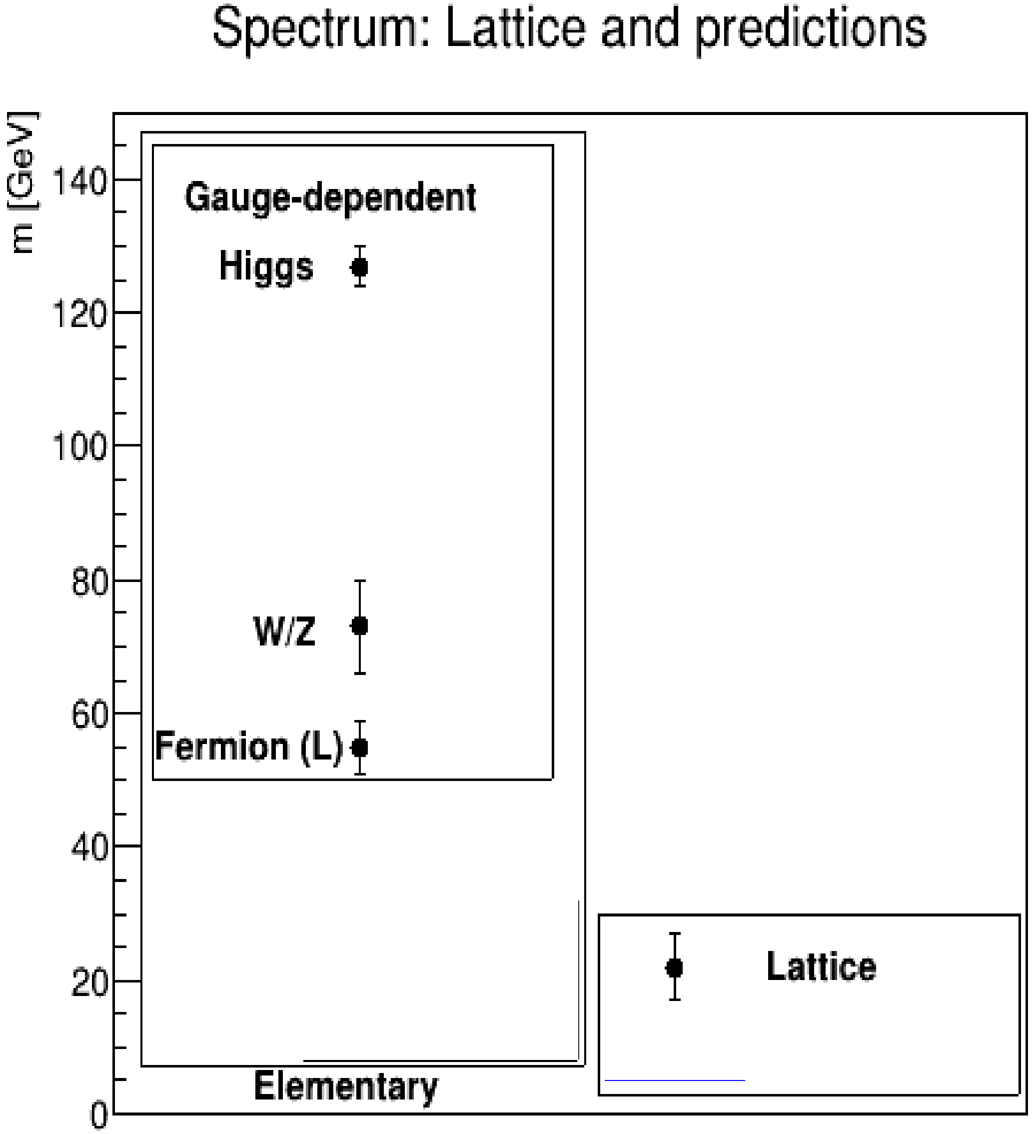

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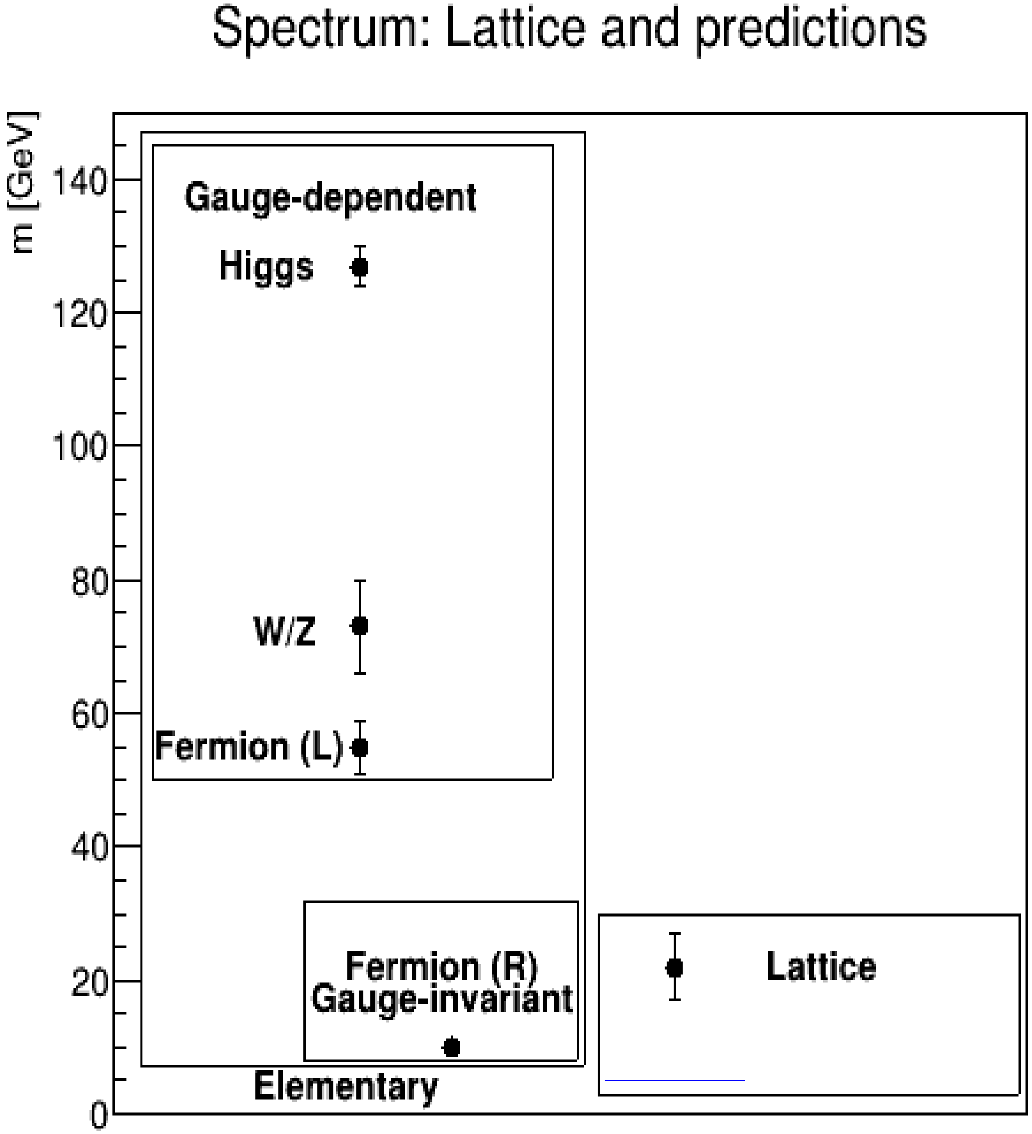

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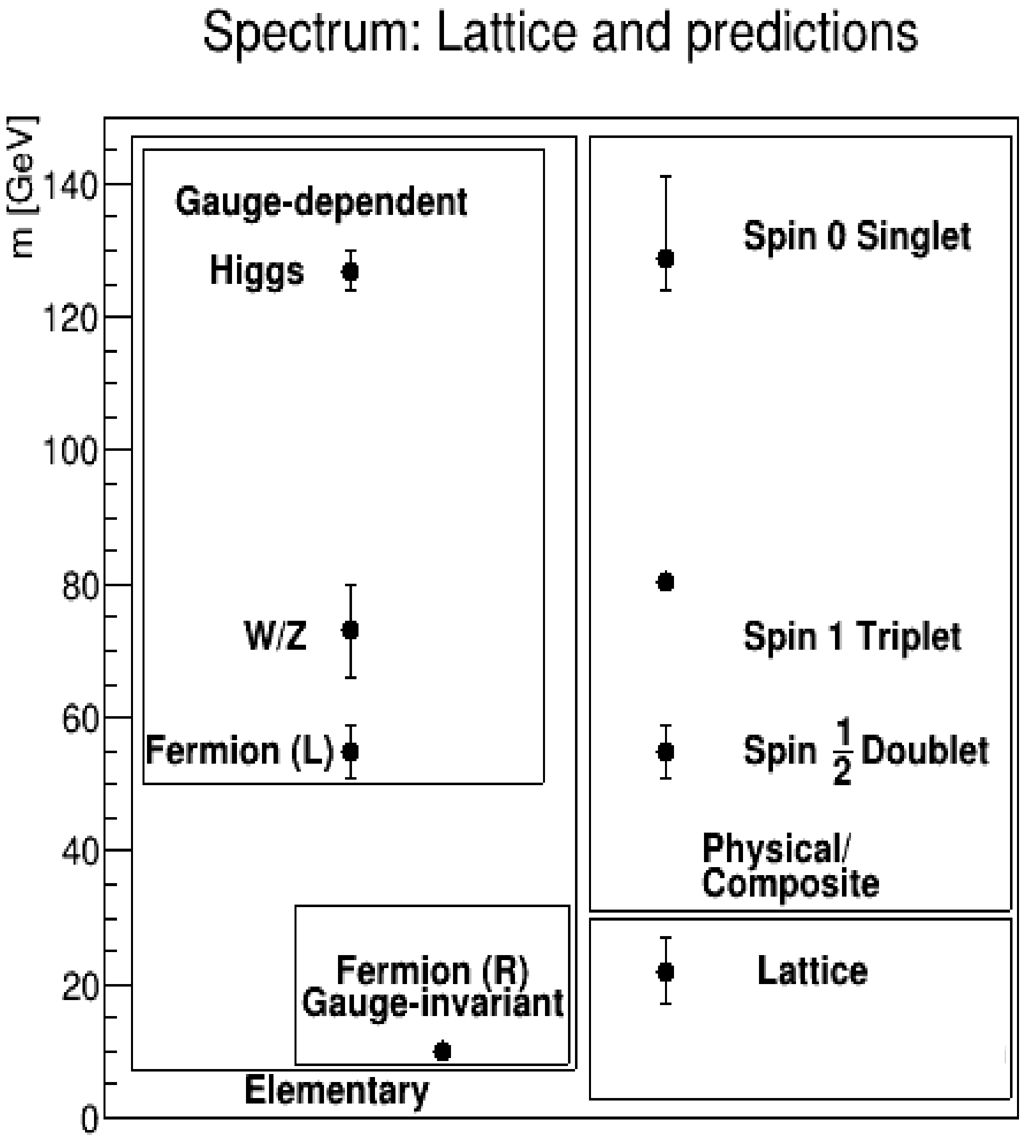

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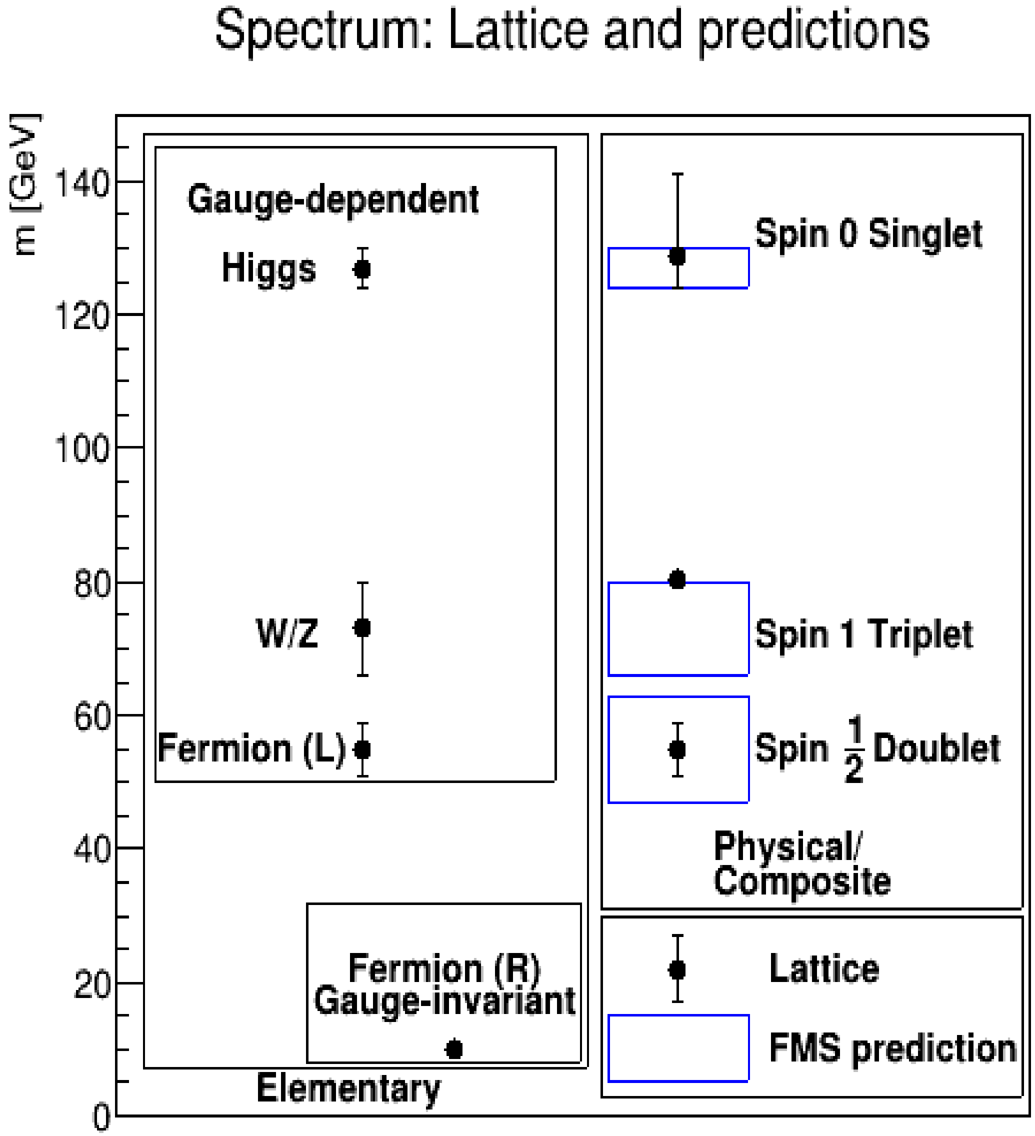

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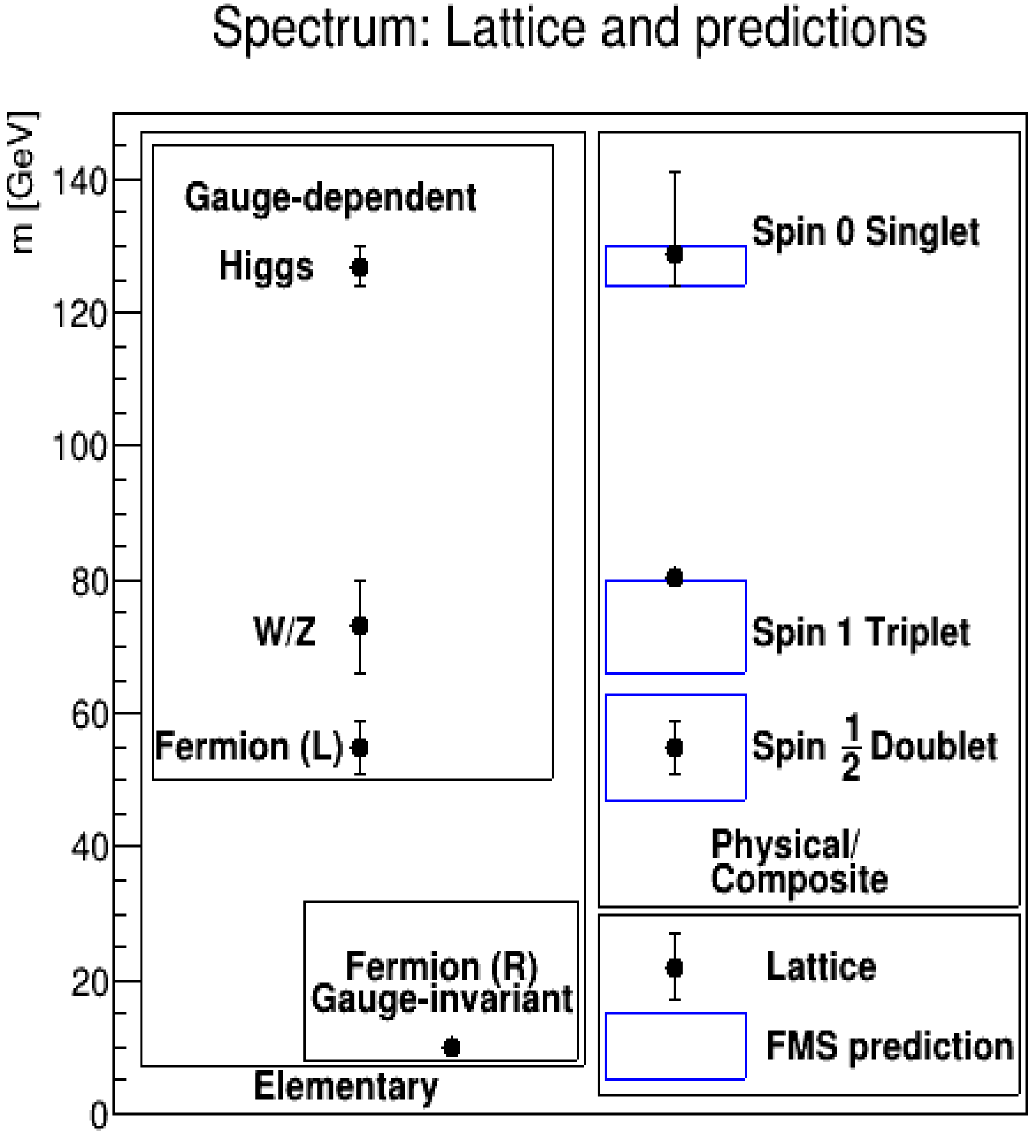
- Supports FMS prediction – grant for unquenching ‘24-’28



Interesting option

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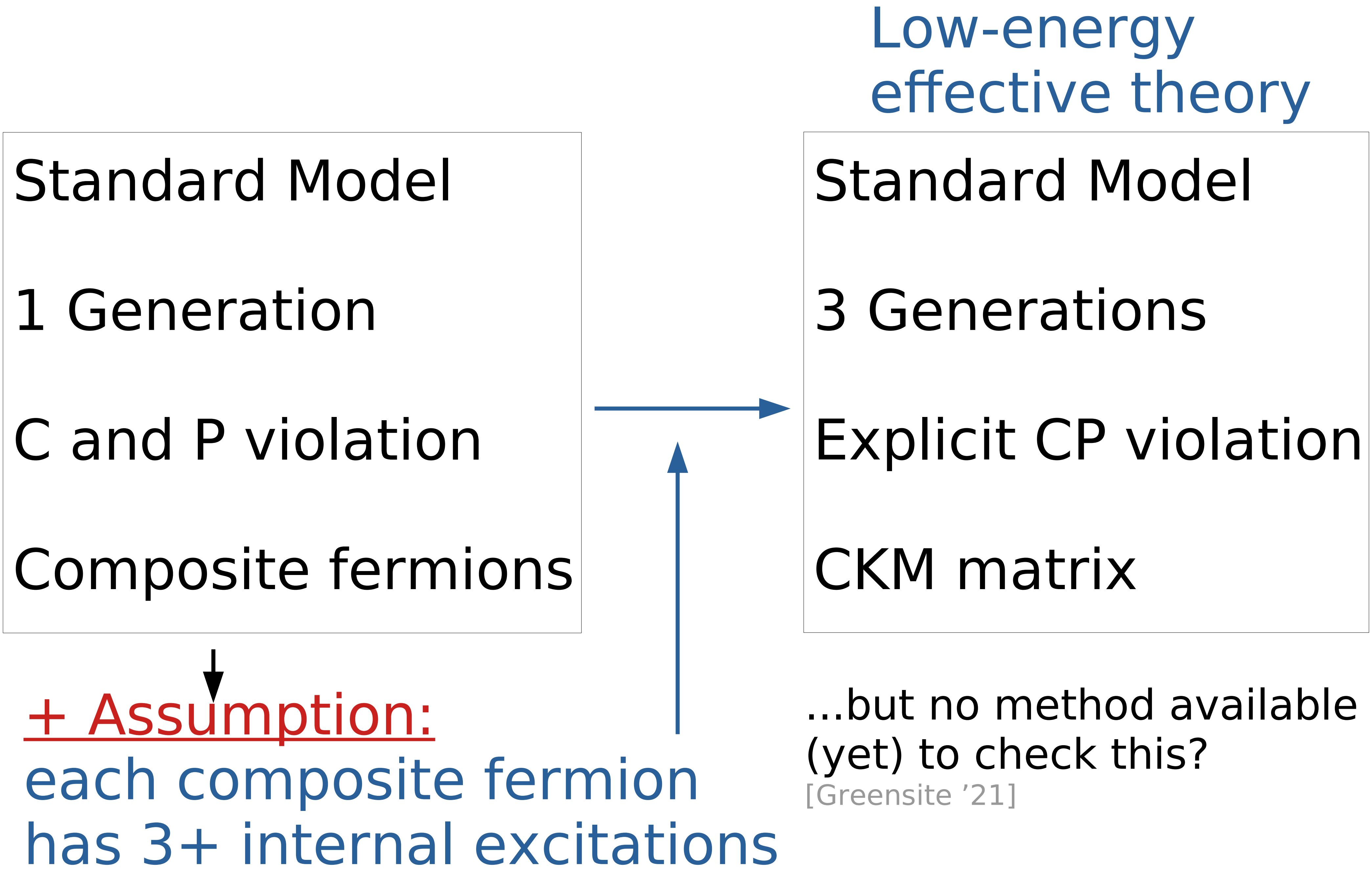

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Low-energy effective theory

Standard Model

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Explicit CP violation

CKM matrix

...but no method available (yet) to check this?

[Greensite '21]

New physics
-
Qualitative changes

Beyond the standard model

[Maas'15
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  - Generally qualitative differences
    - Most dramatic consequences: GUTs

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  - Gauging of the SU(2)xU(1) subgroup of the O(4) Higgs global symmetry
- Why do all gauge couplings become similarly strong at about the same energy of $\sim 10^{15}$ GeV?

Grand-unified theories

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    - Standard model low-energy effective theory
- Standard scenario of double breaking
    - Two Brout-Englert-Higgs effect
    - One breaks at 10^{15} GeV
    - The other at the electroweak scale
    - Requires at least one more Higgs
        - Other particle content scenario-dependent

Consequences and status

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- Many (simple) scenarios ruled out…
- …but many more alive (w or w/o SUSY)

A toy model for unification

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- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W_{a}^{\mu\nu}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

- Ws W^{a}_{μ} W

- Coupling g and some numbers f^{abc}

A toy model for unification

- Consider an SU(3) with a single fundamental scalar
- Looks very similar to the standard model Higgs

$$L = -\frac{1}{4} W^{a}_{\mu\nu} W^{\mu\nu}_{a} + \left(D^{ij}_{\mu} h^{j} \right)^{+} D^{\mu}_{ik} h_{k}$$

$$W^{a}_{\mu\nu} = \partial_{\mu} W^{a}_{\nu} - \partial_{\nu} W^{a}_{\mu} + g f^{a}_{bc} W^{b}_{\mu} W^{c}_{\nu}$$

$$D^{ij}_{\mu} = \delta^{ij} \partial_{\mu} - i g W^{a}_{\mu} t^{ij}_{a}$$

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- Ws W^a_μ W
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- Parameters selected for a BEH effect

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- Local SU(3) gauge symmetry

$$W^{a}_{\mu}\rightarrow W^{a}_{\mu}+(\delta^{a}_{b}\partial_{\mu}-g\,f^{a}_{bc}W^{c}_{\mu})\phi^{b} \qquad h_{i}\rightarrow h_{i}+g\,t^{ij}_{a}\phi^{a}h_{j}$$

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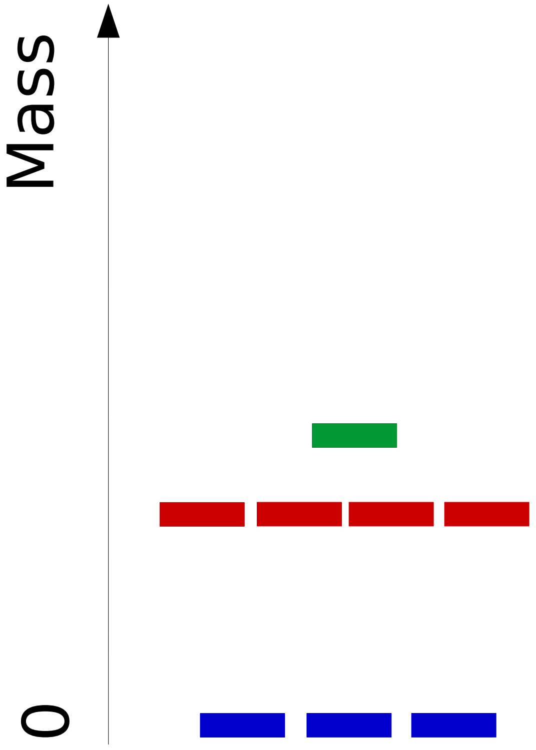
- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $h\rightarrow\exp(ia)h$

Spectrum

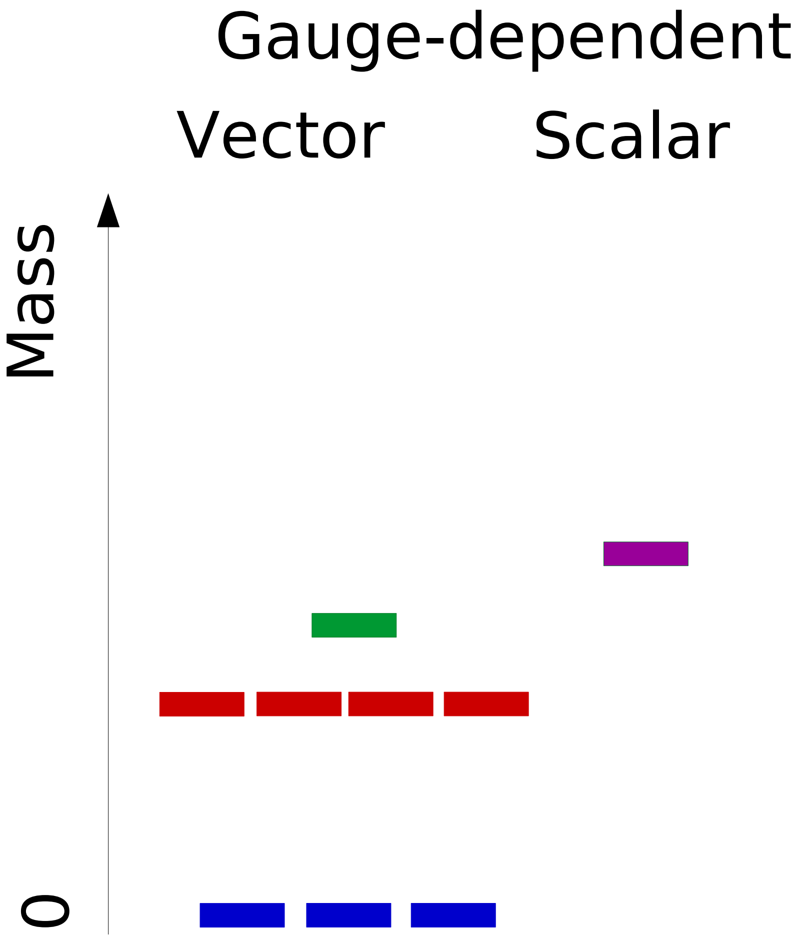
Gauge-dependent

Vector



‘SU(3)→ SU(2)’

Spectrum



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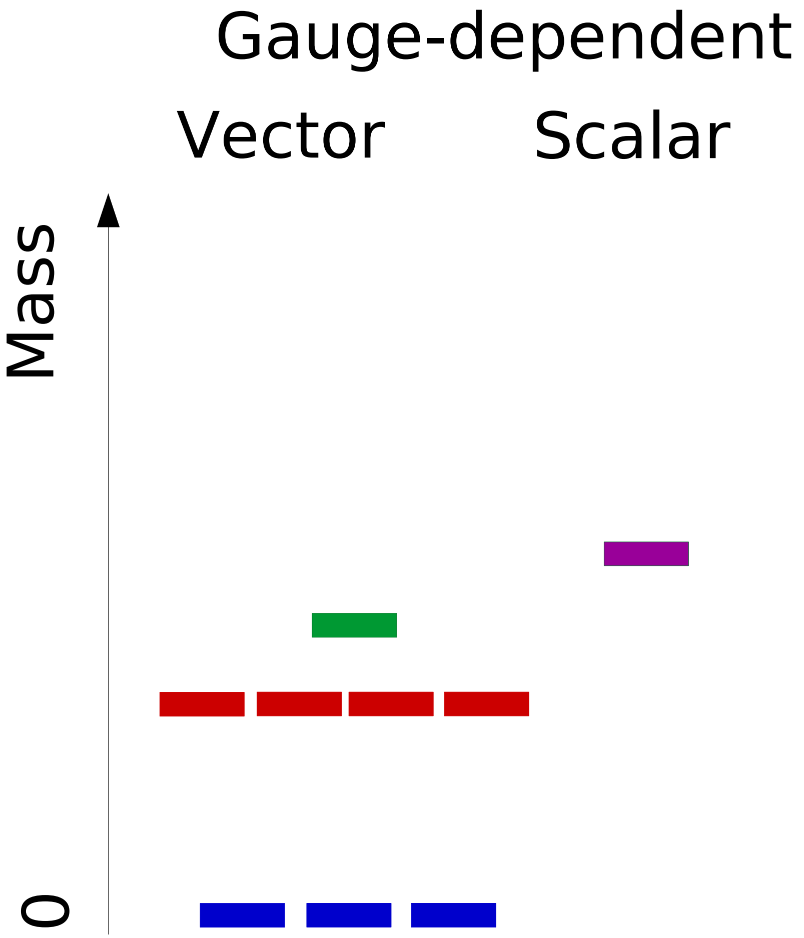


Gauge-dependent

Vector Scalar

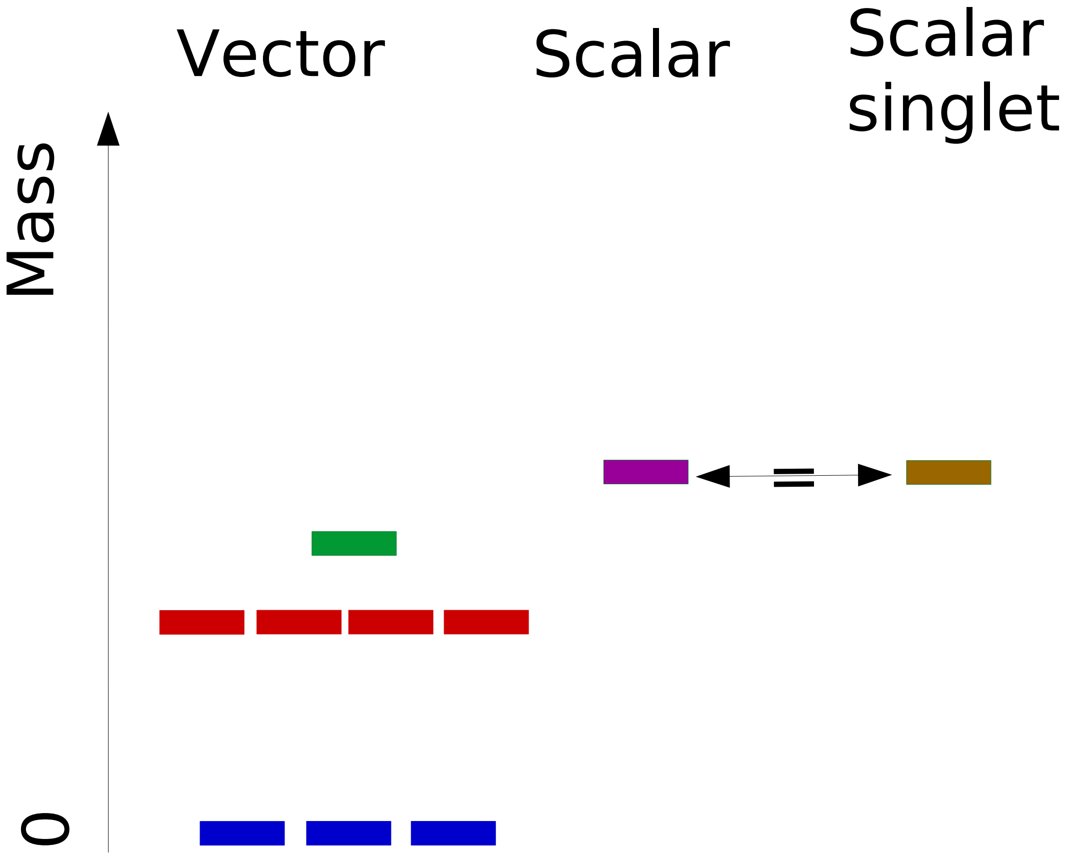
Confirmed in gauge-fixed lattice calculations [Maas et al.’16]

Spectrum

[Maas & Törek’16,'18
Maas, Sondenheimer & Törek'17]

Gauge-dependent

Gauge-invariant



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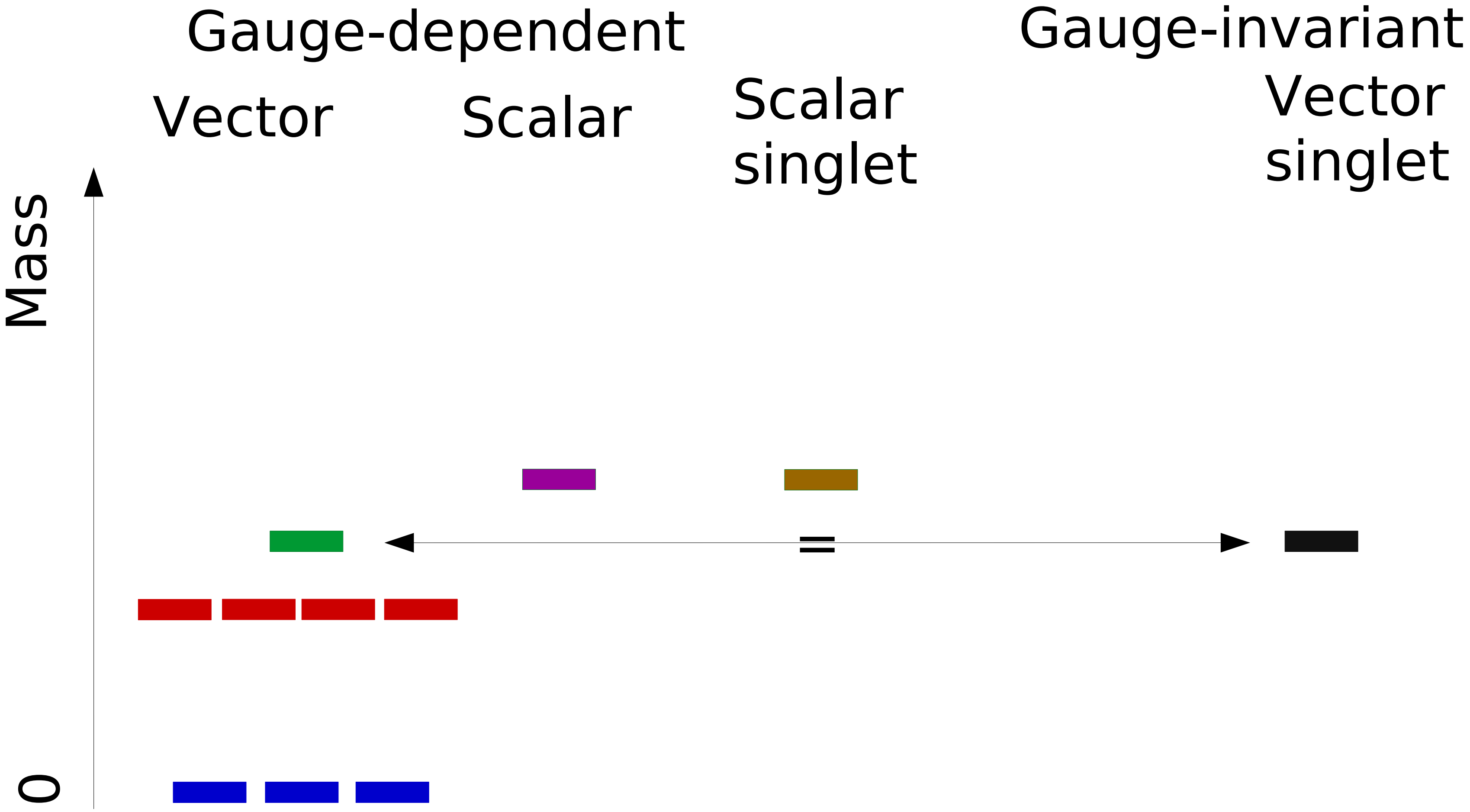

What about the vector?

[Maas & Törek’16]

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1) Formulate gauge-invariant operator

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Matrix from group structure

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Matrix from group structure

c^{ab} projects out only one field

Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

A different kind of states

- Group theory forced same gauge multiplets and custodial multiples for SU(2)
  - Because Higgs is bifundamental
  - Remainder is bound state/resonance or not

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- Now: States without elementary analouge
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue – U(1) acts as baryon number
  - Lightest must exist and be absolutely stable

Possible new states

- Quantum numbers are $J^{PC}{}_{\text{Custodial}}$

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- What is the lightest state?
  - Prediction with constituent model

Spectrum

[Maas & Török’16,'18
Maas, Sondenheimer & Török'17]

Gauge-dependent

Gauge-invariant

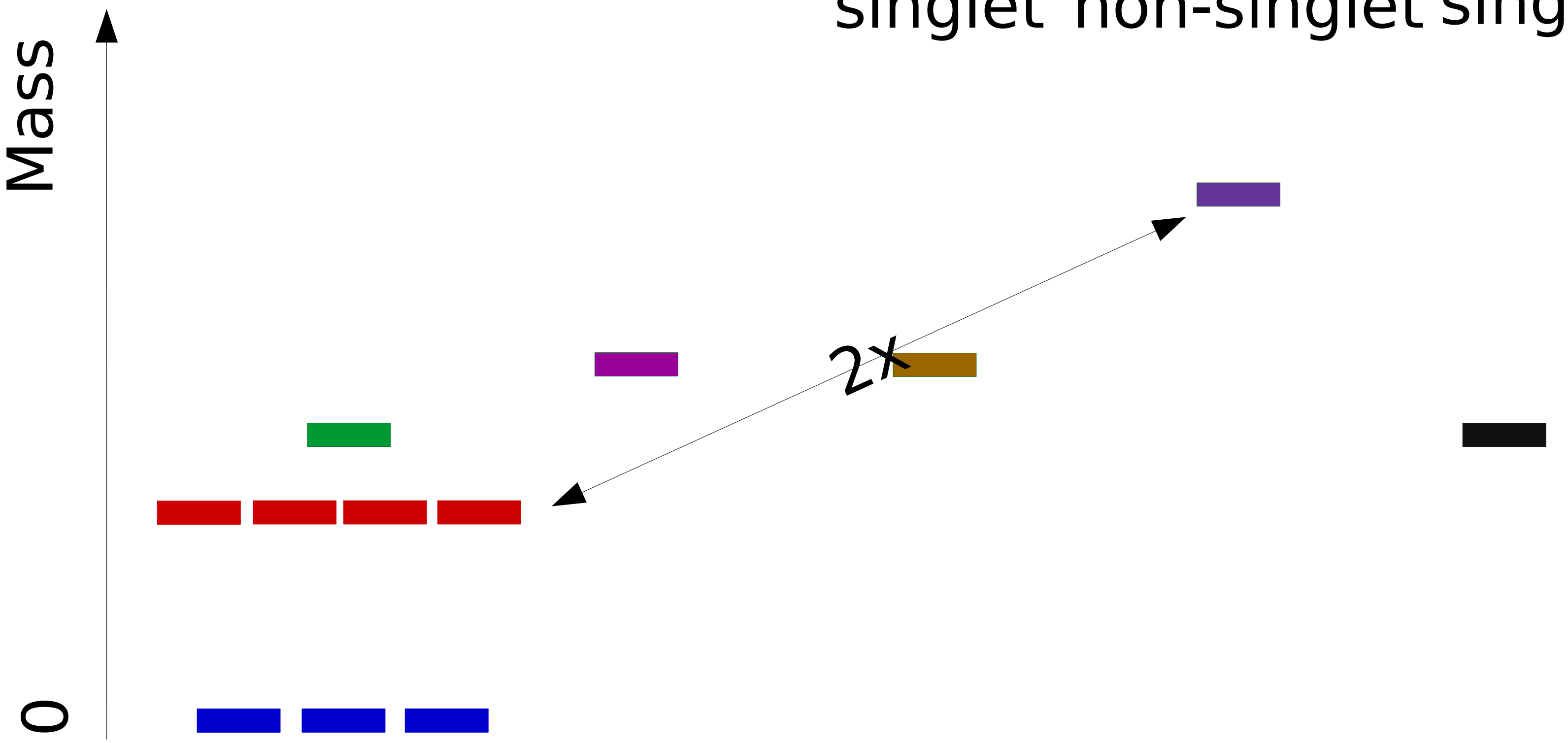
Vector

Scalar

Scalar
singlet

Scalar
non-singlet

Vector
singlet



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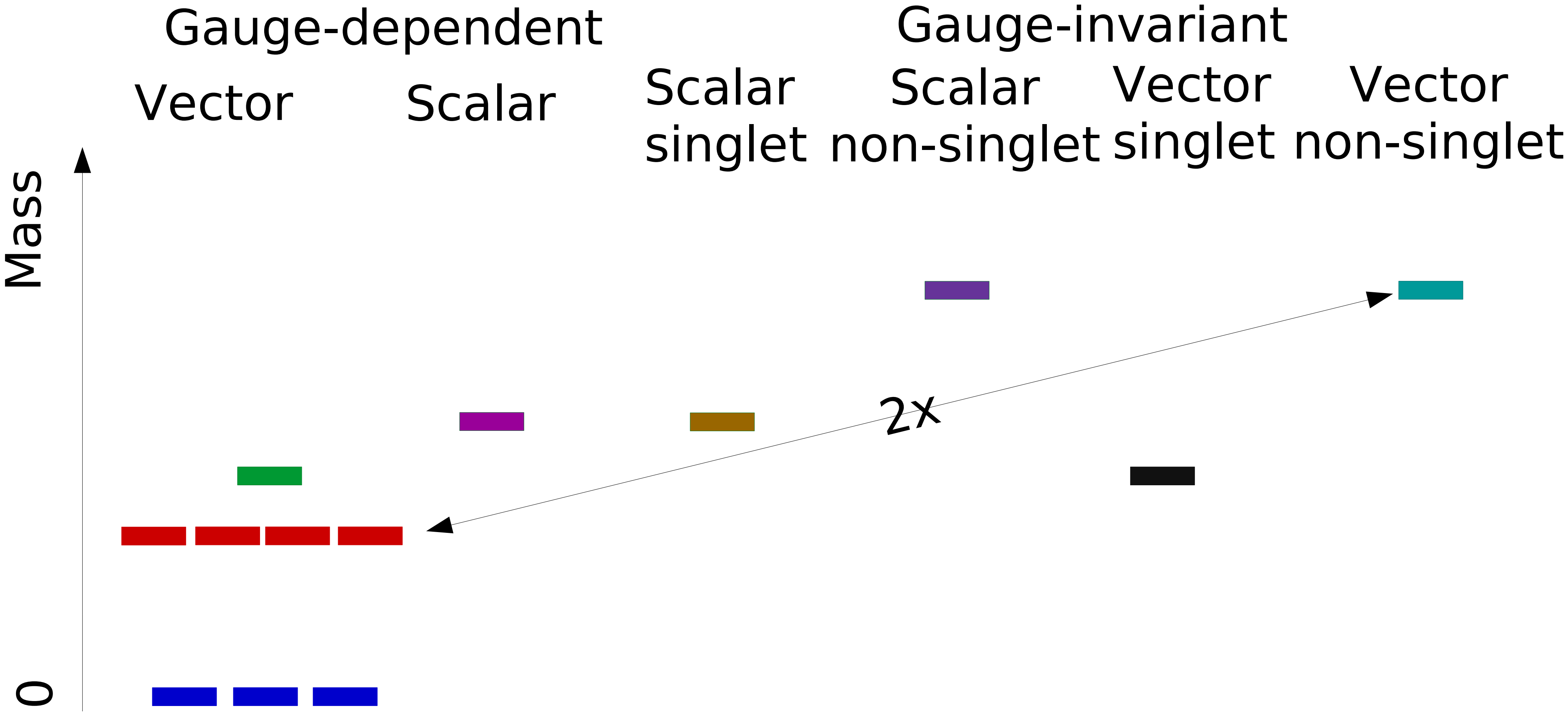

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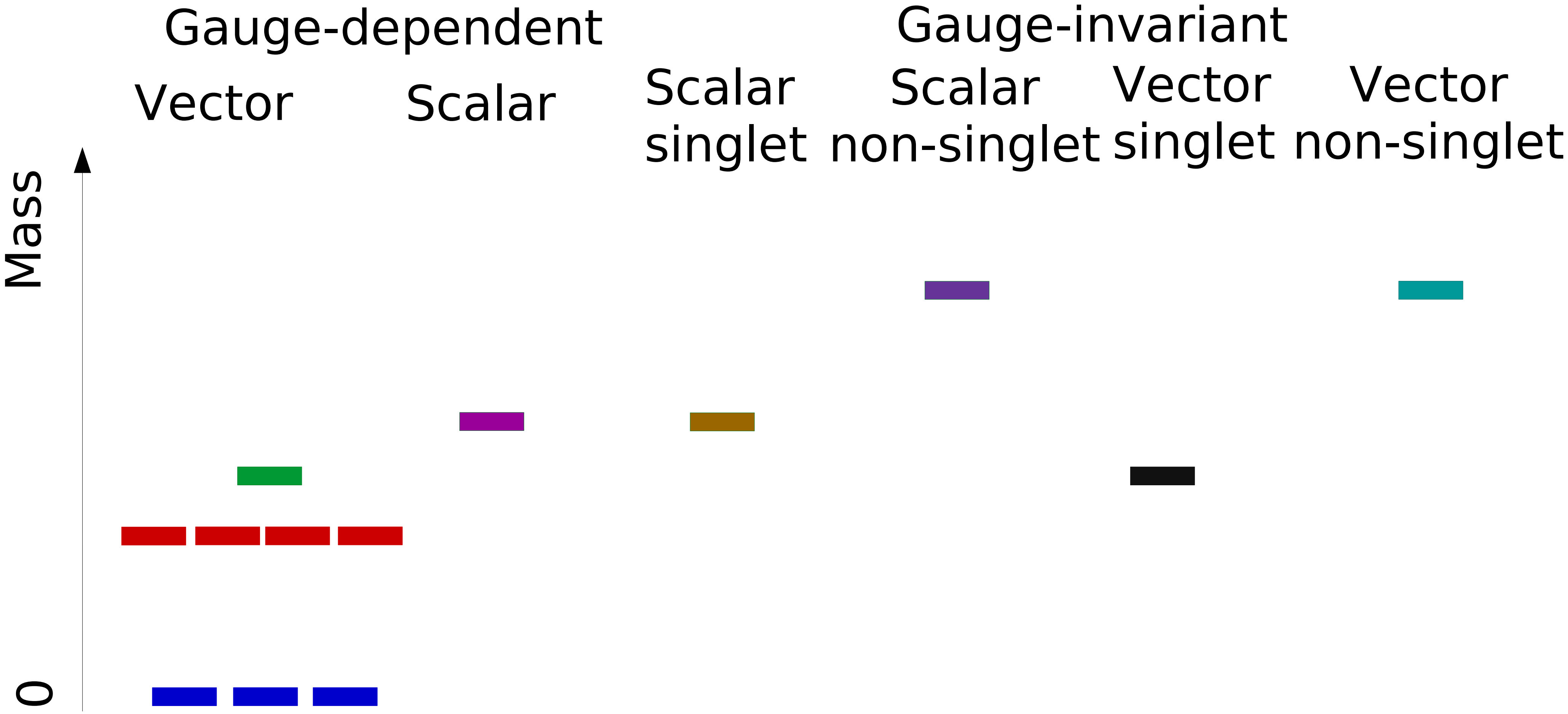

Gauge-dependent

Gauge-invariant

Vector

Scalar

Scalar singlet

Scalar non-singlet

Vector singlet

Vector non-singlet

- Qualitatively different spectrum
- No mass gap!

Possible states

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- What is the lightest state?
  - Prediction with constituent model
  - Lattice calulations

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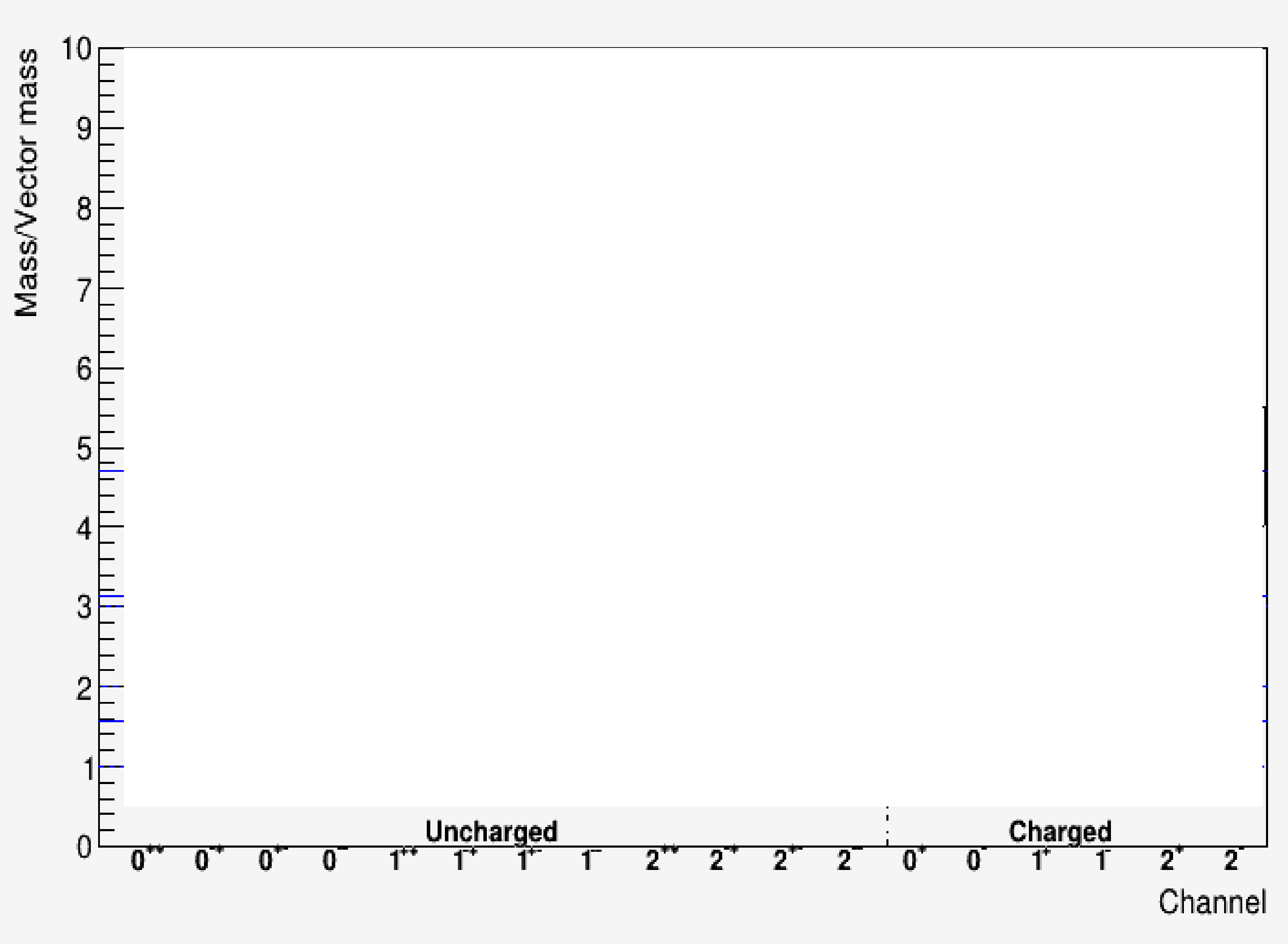
  - Simpelst non-trivial state operator: $0^{++}{}_1$
  - $\epsilon_{abc}\phi^a D_\mu \phi^b D_\nu D^\nu D^\mu \phi^c$
- What is the lightest state?
  - Prediction with constituent model
  - Lattice calulations
  - All channels: J<3
  - Aim: Ground state for each channel
    - Characterization through scattering states

Typical spectrum

PRELIMINARY

[Dobson et al.’22]



Typical spectrum

PRELIMINARY

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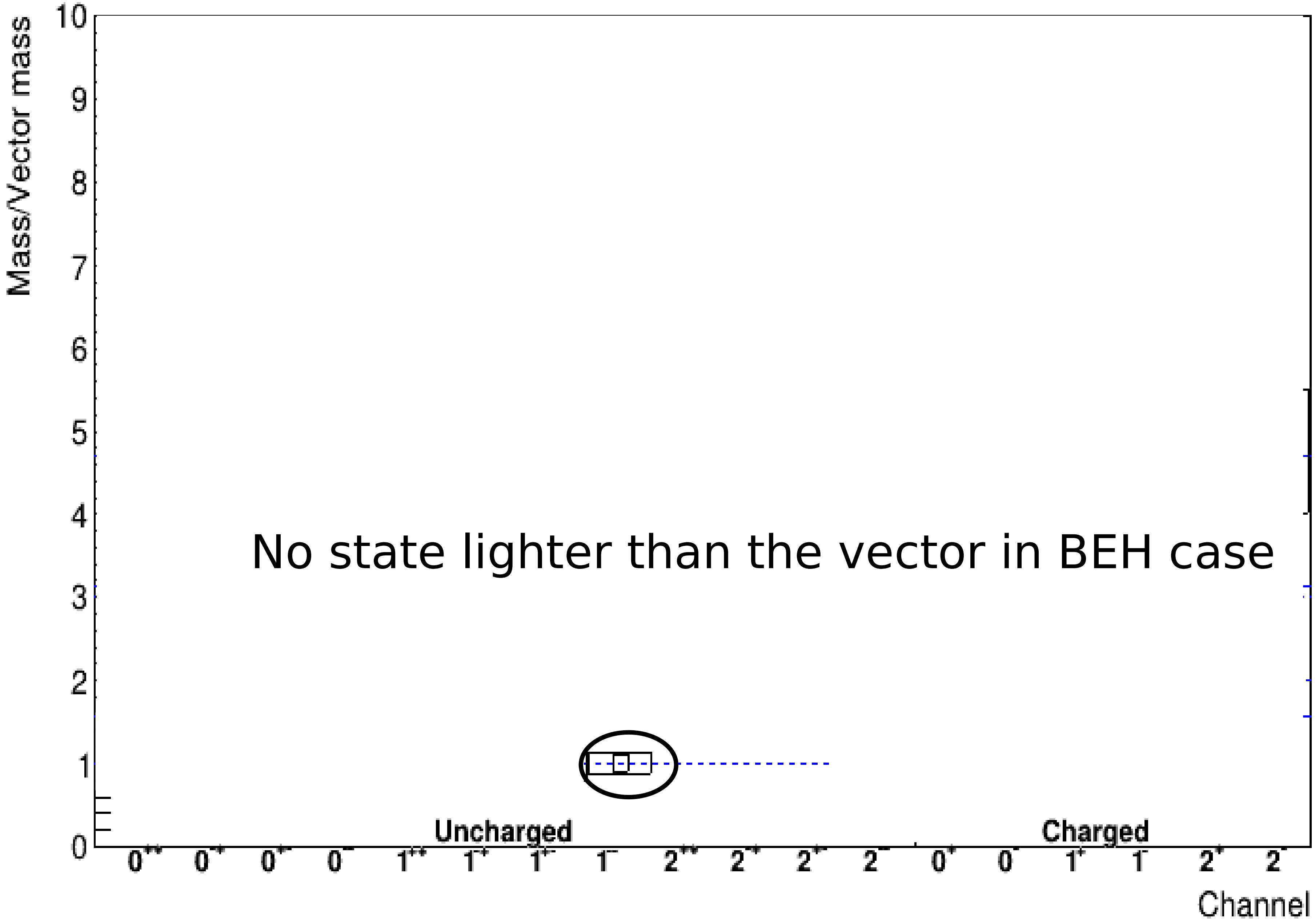
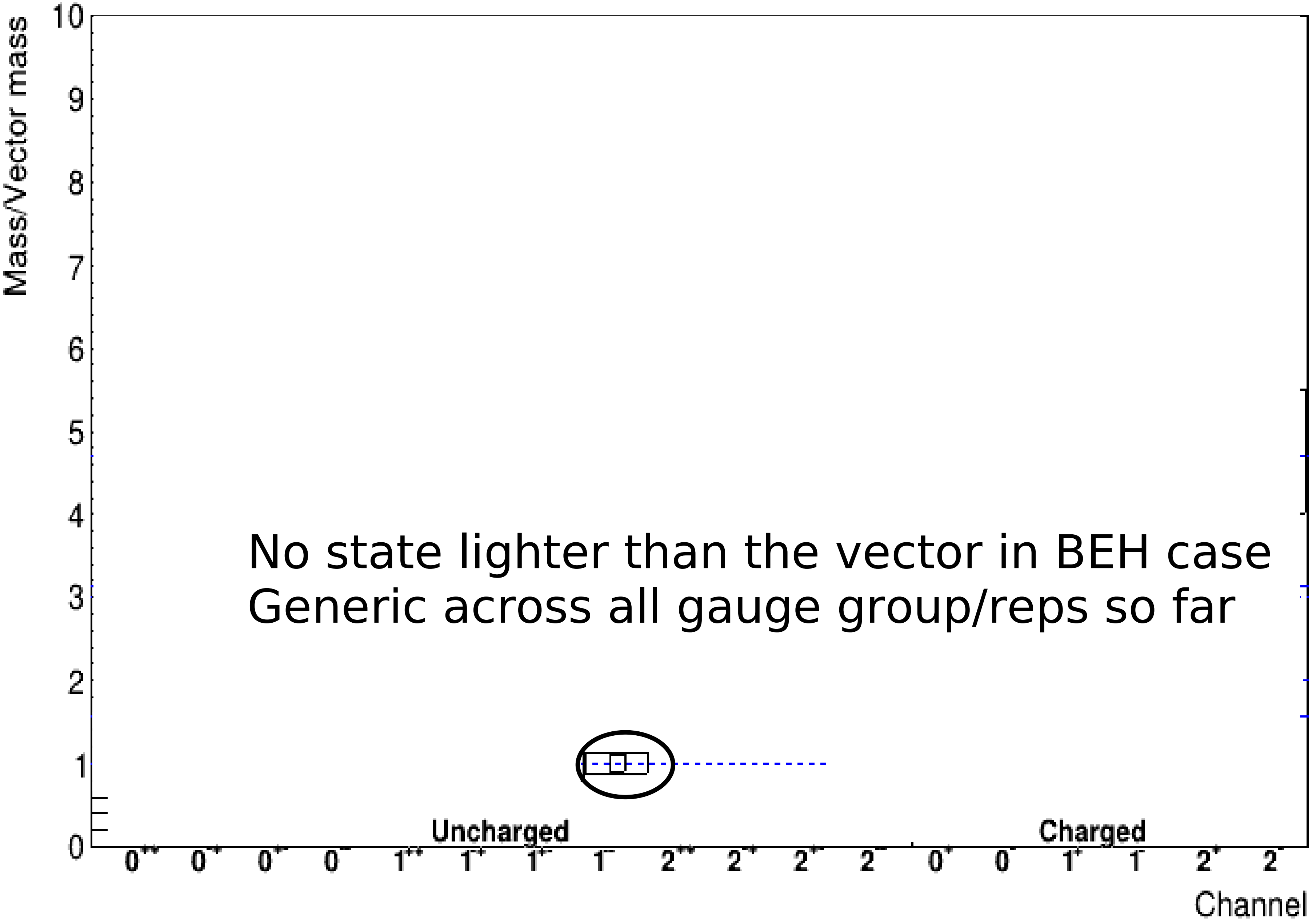


No state lighter than the vector in BEH case

Typical spectrum

PRELIMINARY

[Dobson et al.’22]



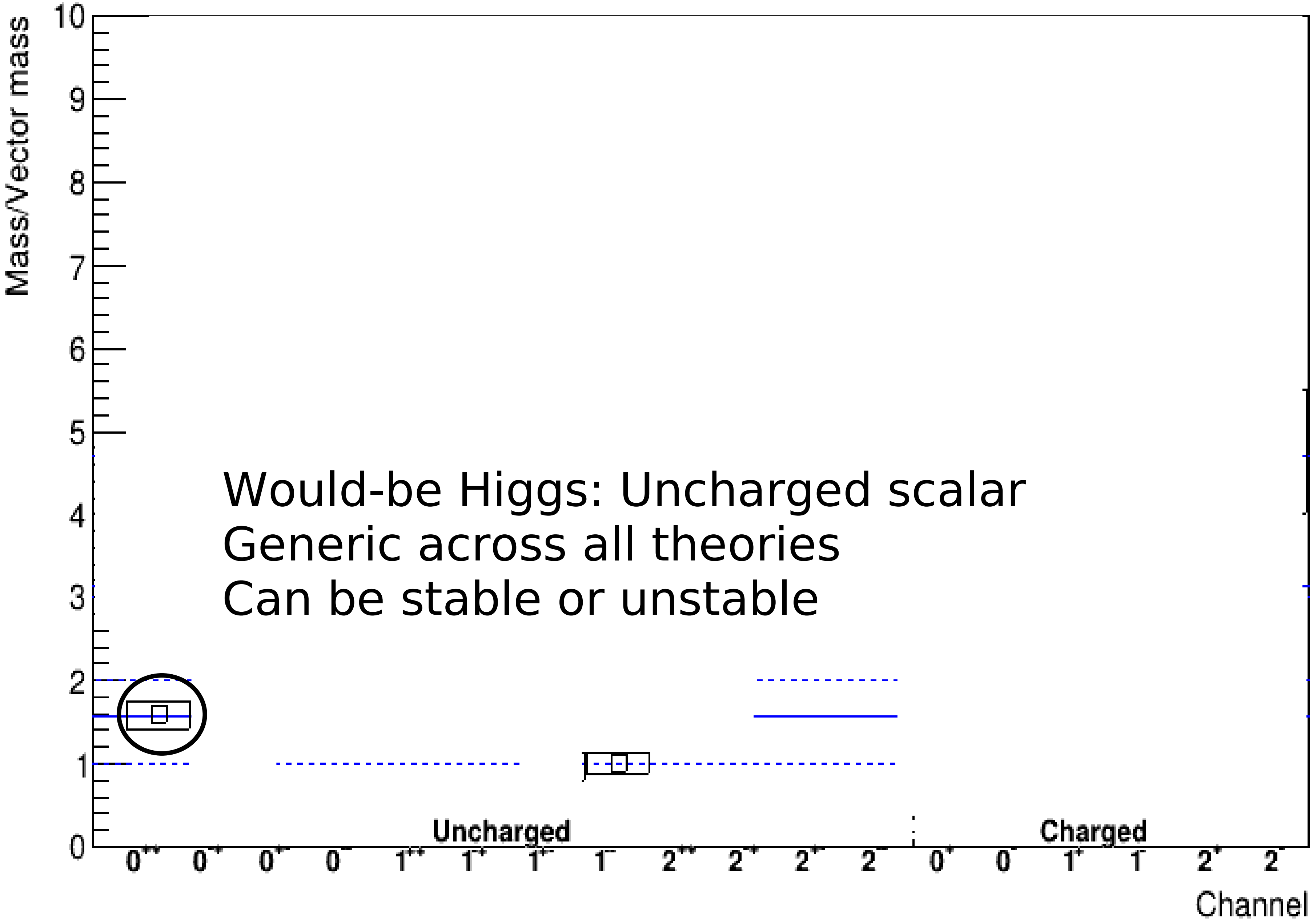
No state lighter than the vector in BEH case

Generic across all gauge group/reps so far

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

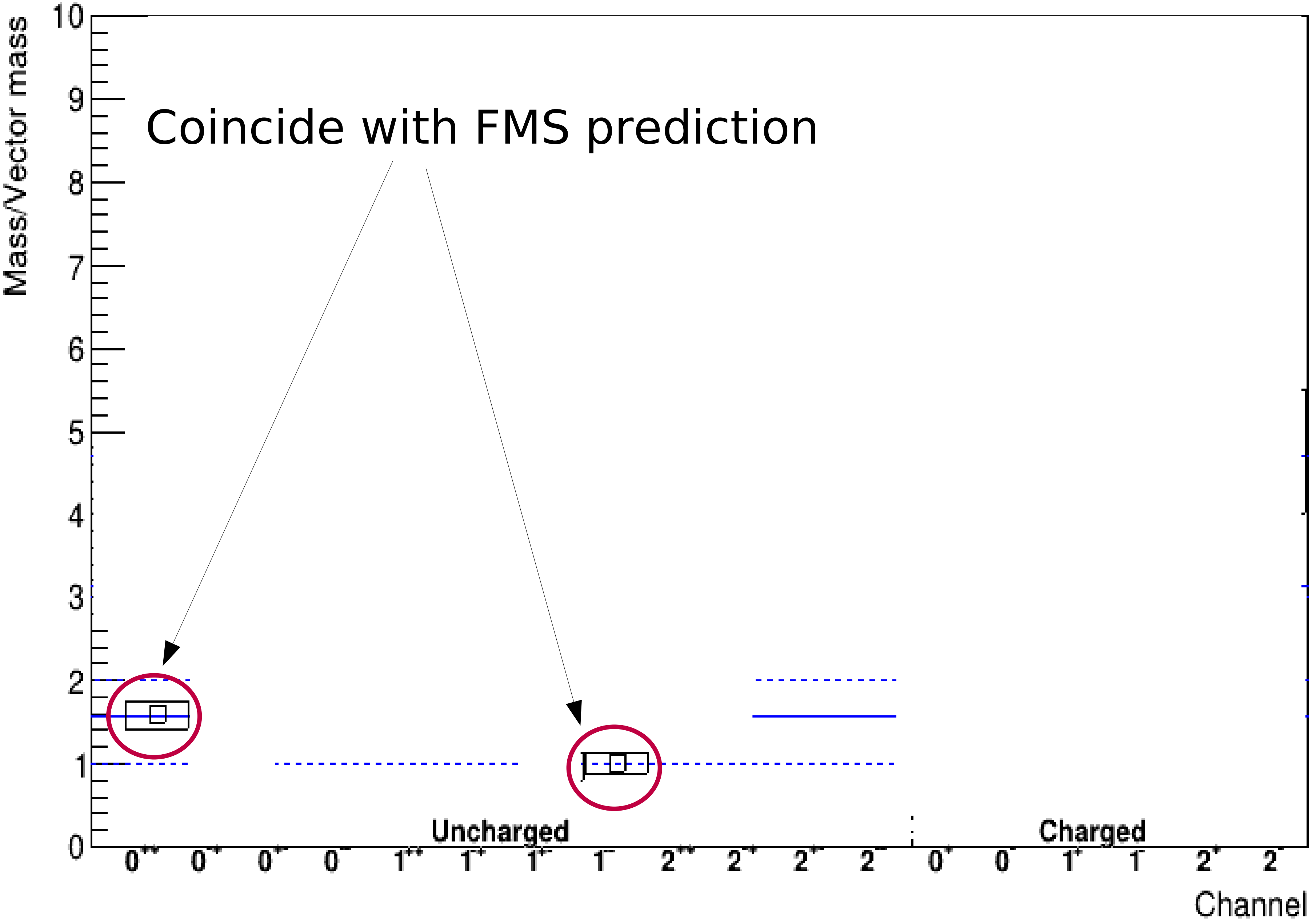


Would-be Higgs: Uncharged scalar
Generic across all theories
Can be stable or unstable

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22
Maas et al.’16’18]



Coincide with FMS prediction

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22
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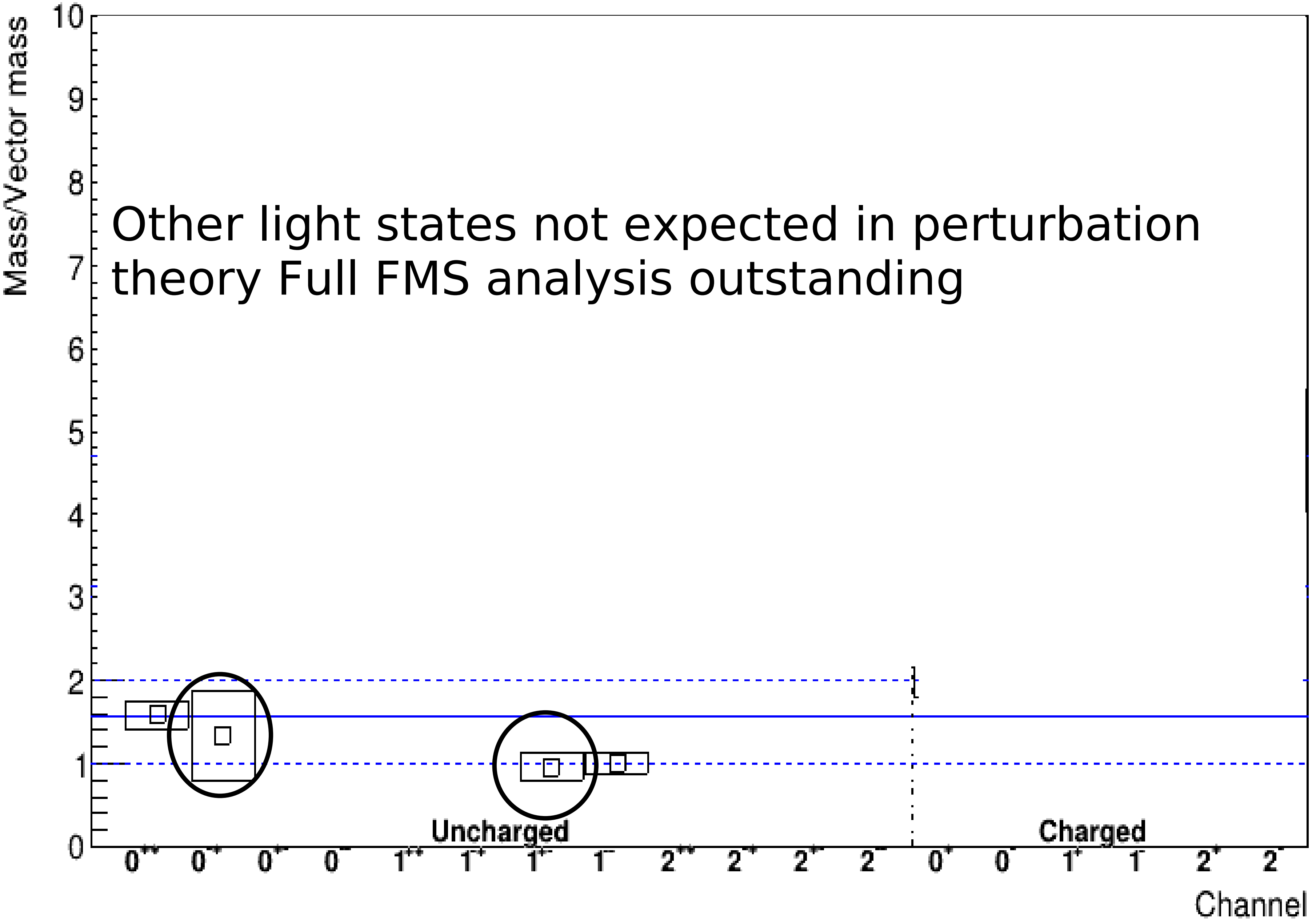

Other light states not expected in perturbation theory Full FMS analysis outstanding

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
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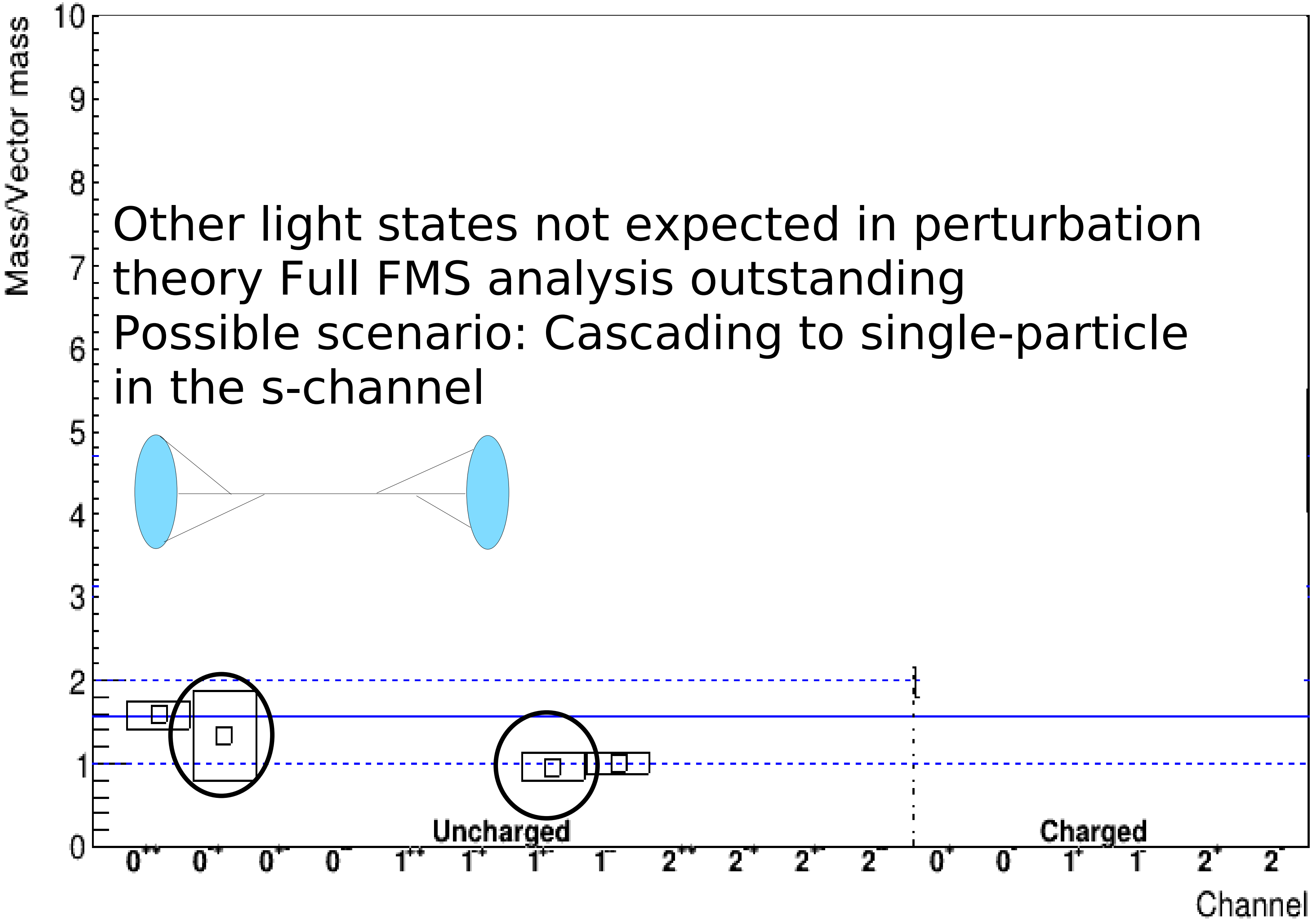

Other light states not expected in perturbation theory Full FMS analysis outstanding

Possible scenario: Cascading to single-particle in the s-channel

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22
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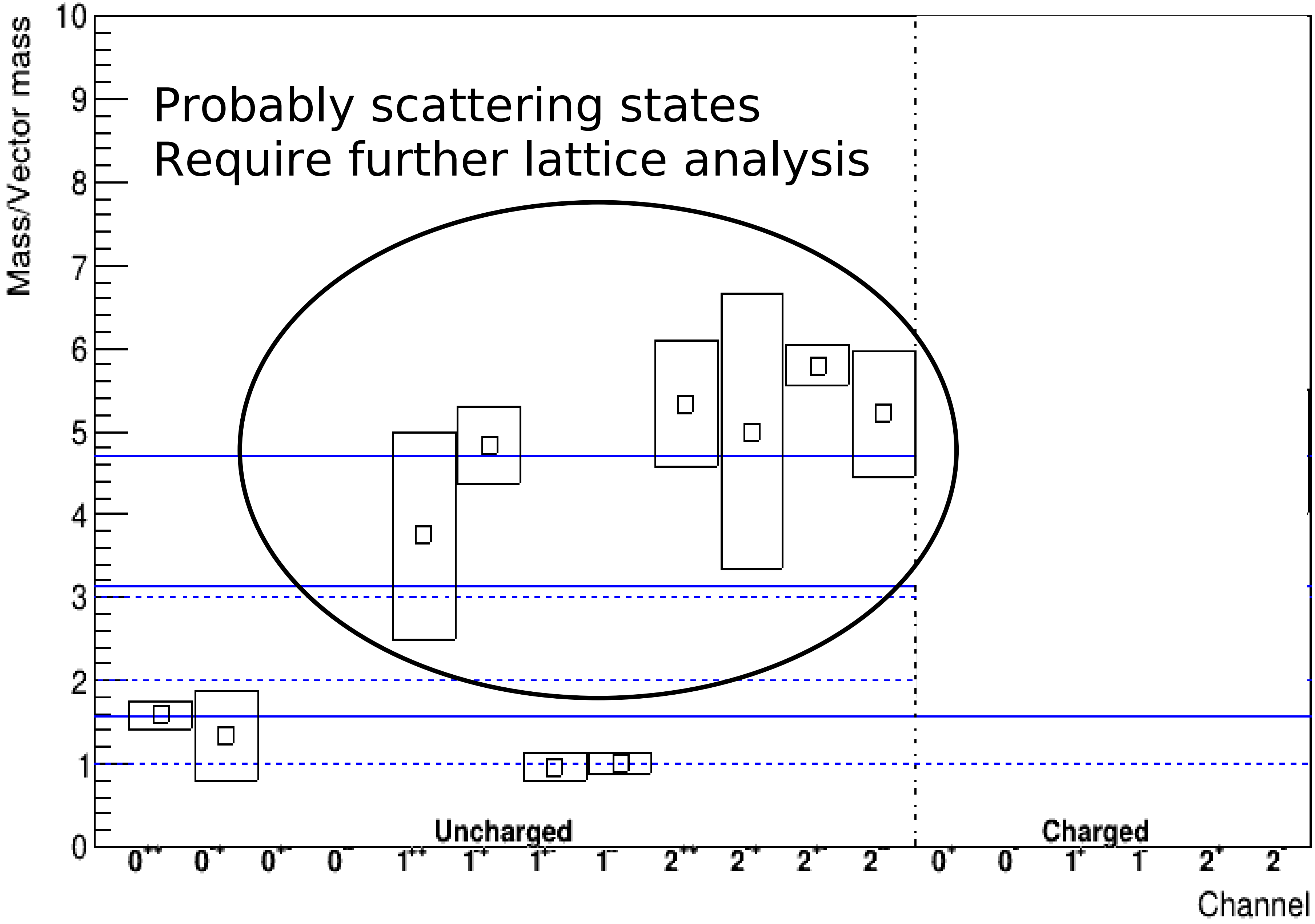
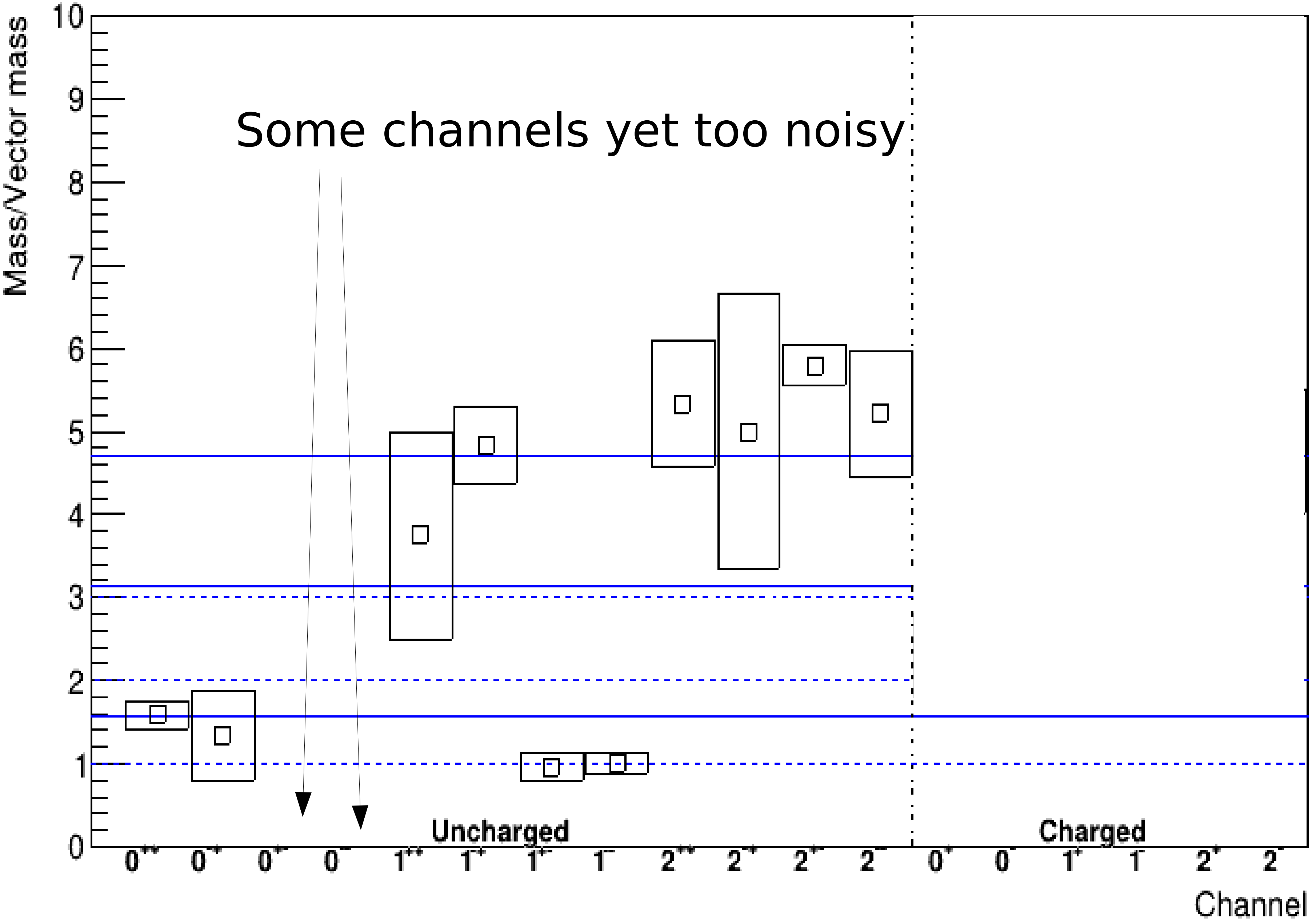


Probably scattering states
Require further lattice analysis

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22
Maas et al.’16’18]



Some channels yet too noisy

Typical spectrum

PRELIMINARY

[Dobson et al.'22
Maas '17
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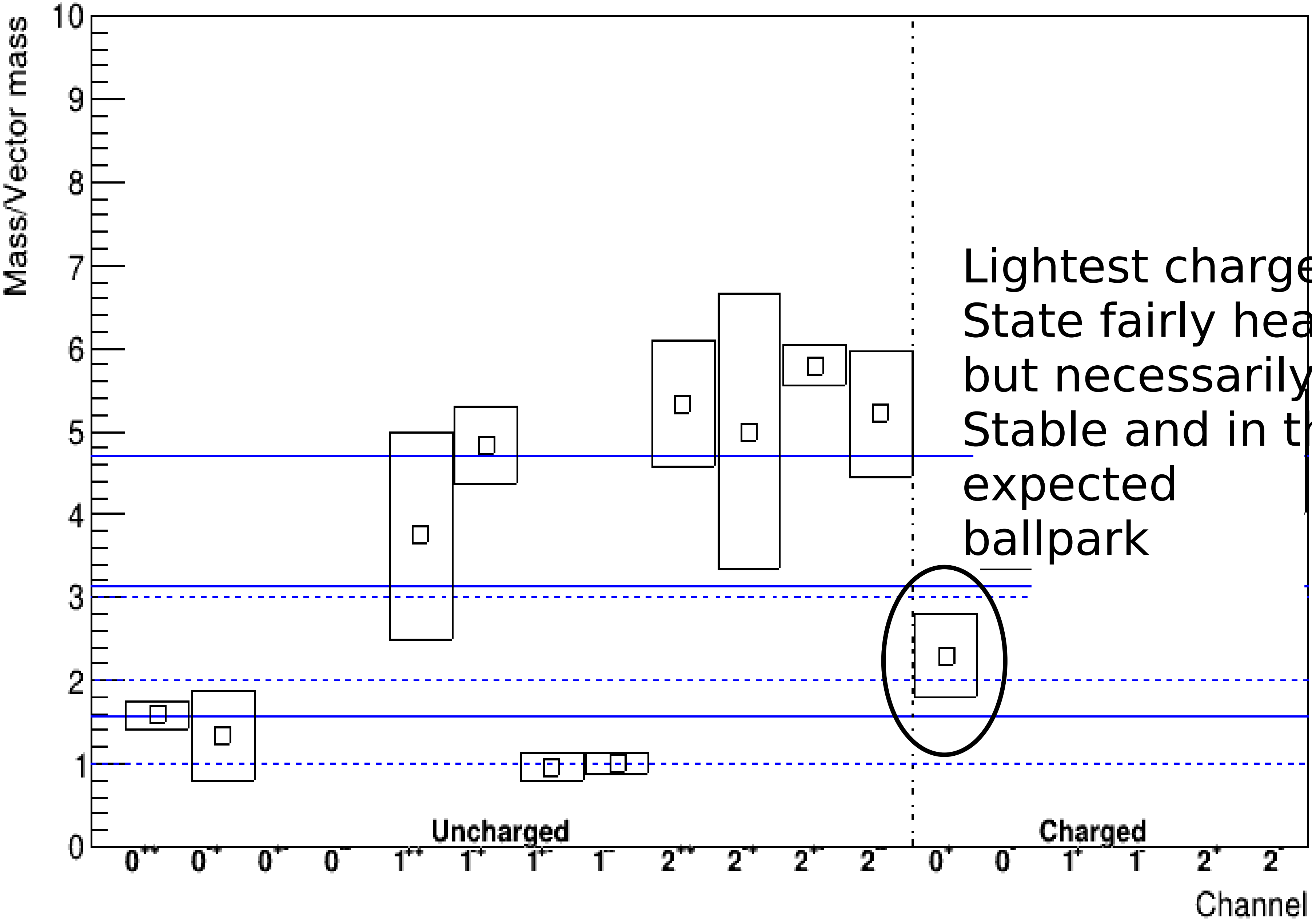

Lightest charged
State fairly heavy,
but necessarily
Stable and in the
expected
ballpark

Typical spectrum

PRELIMINARY

[Dobson et al.’22
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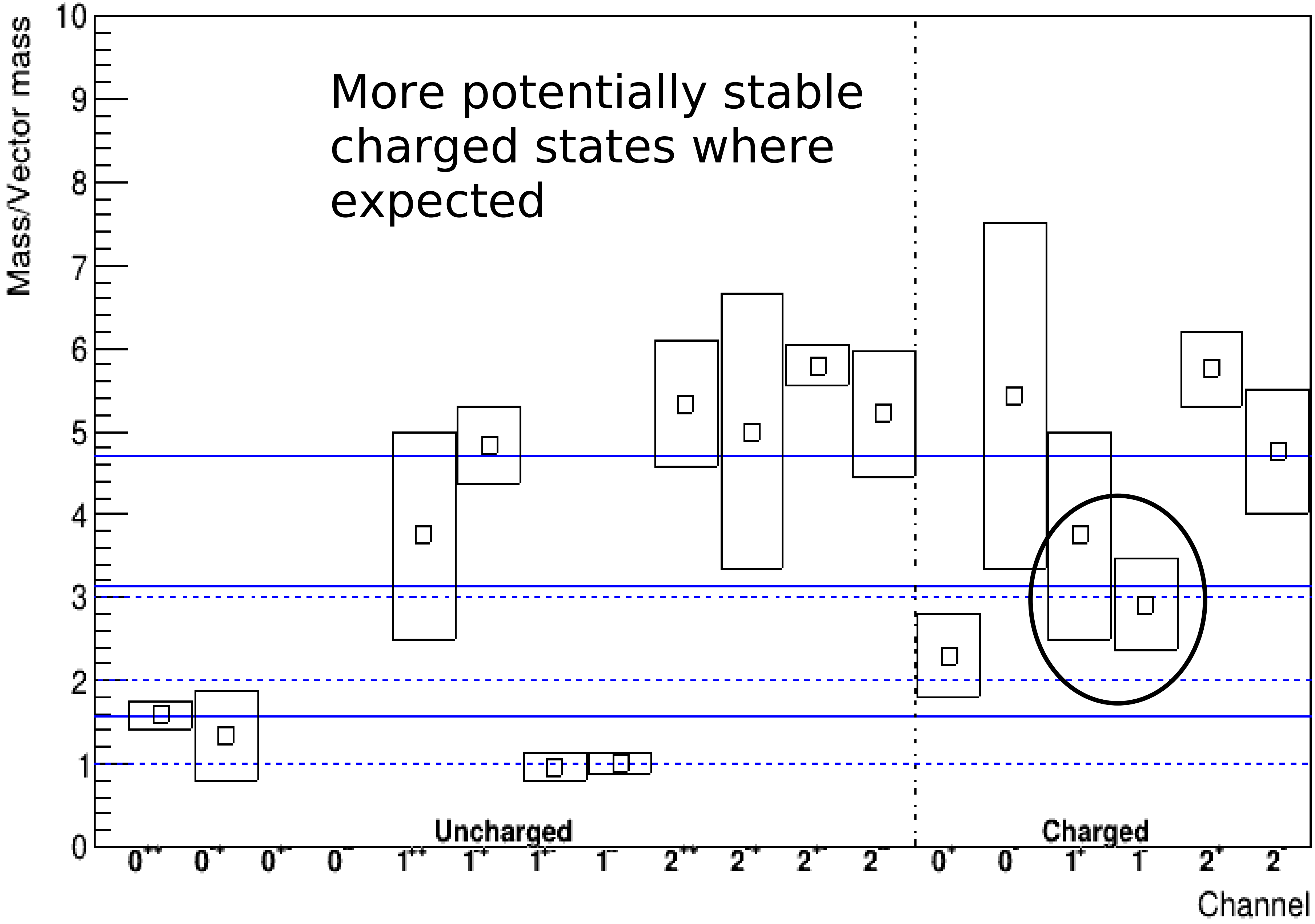
Jenny et al.’22
Maas et al.’16’18]



More potentially stable charged states where expected

Typical spectrum

PRELIMINARY

[Dobson et al.’22
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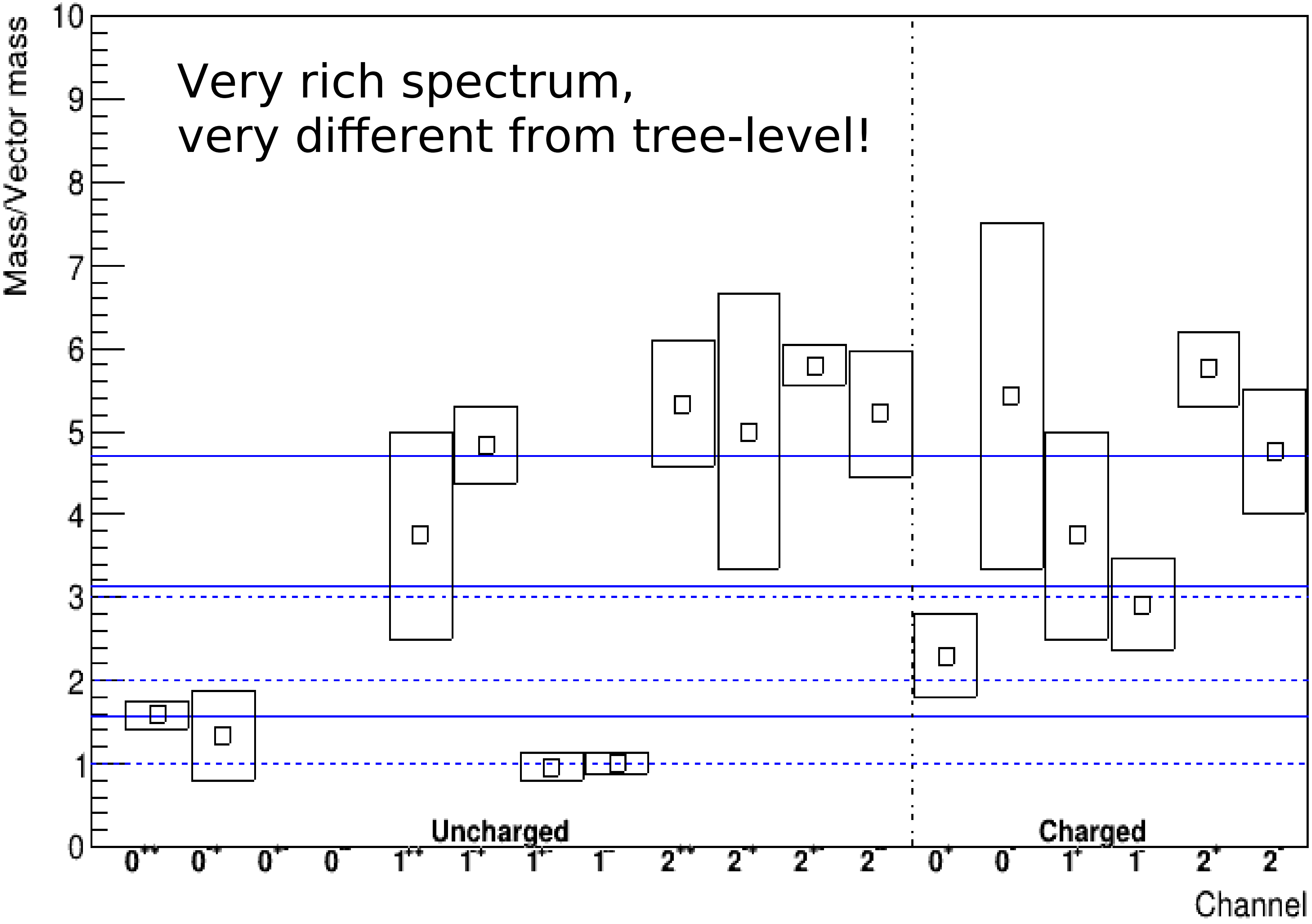

Very rich spectrum,
very different from tree-level!

Experimental consequences

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[Maas & Törek'18
Maas'17]

- Add fundamental fermions

Experimental consequences

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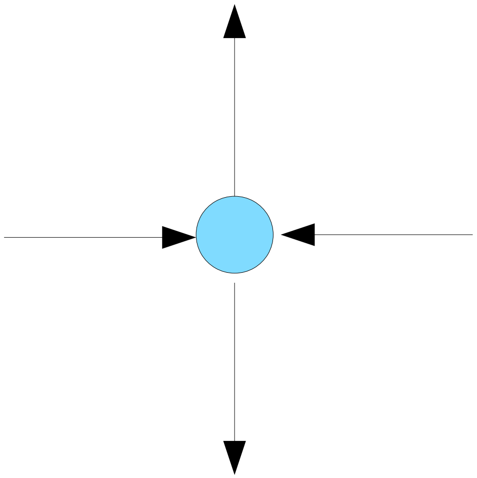

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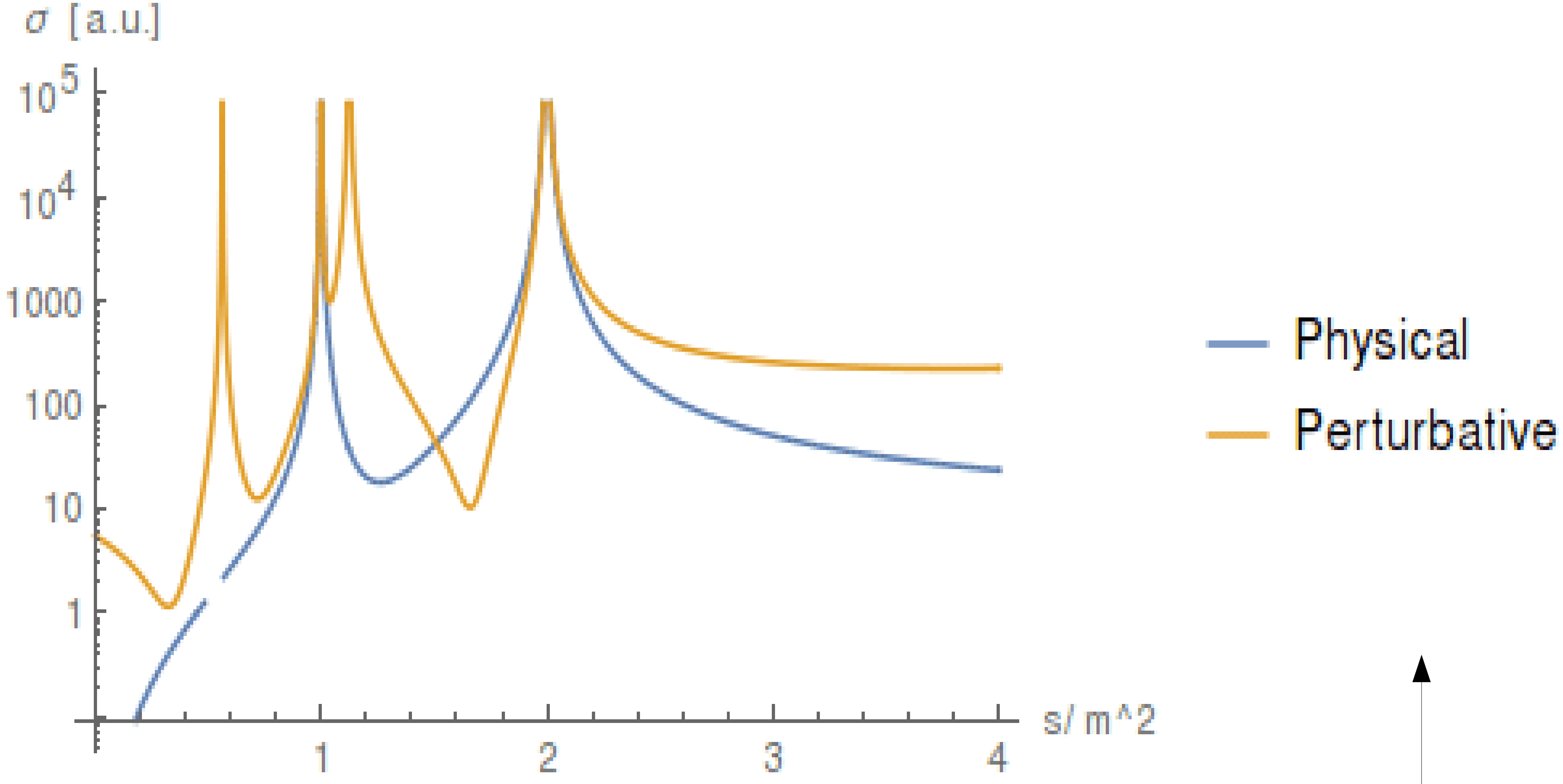

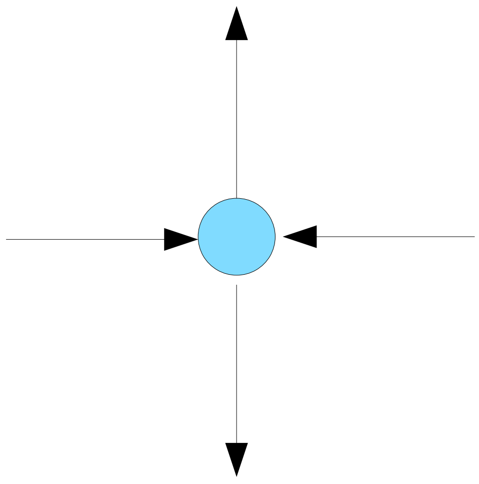

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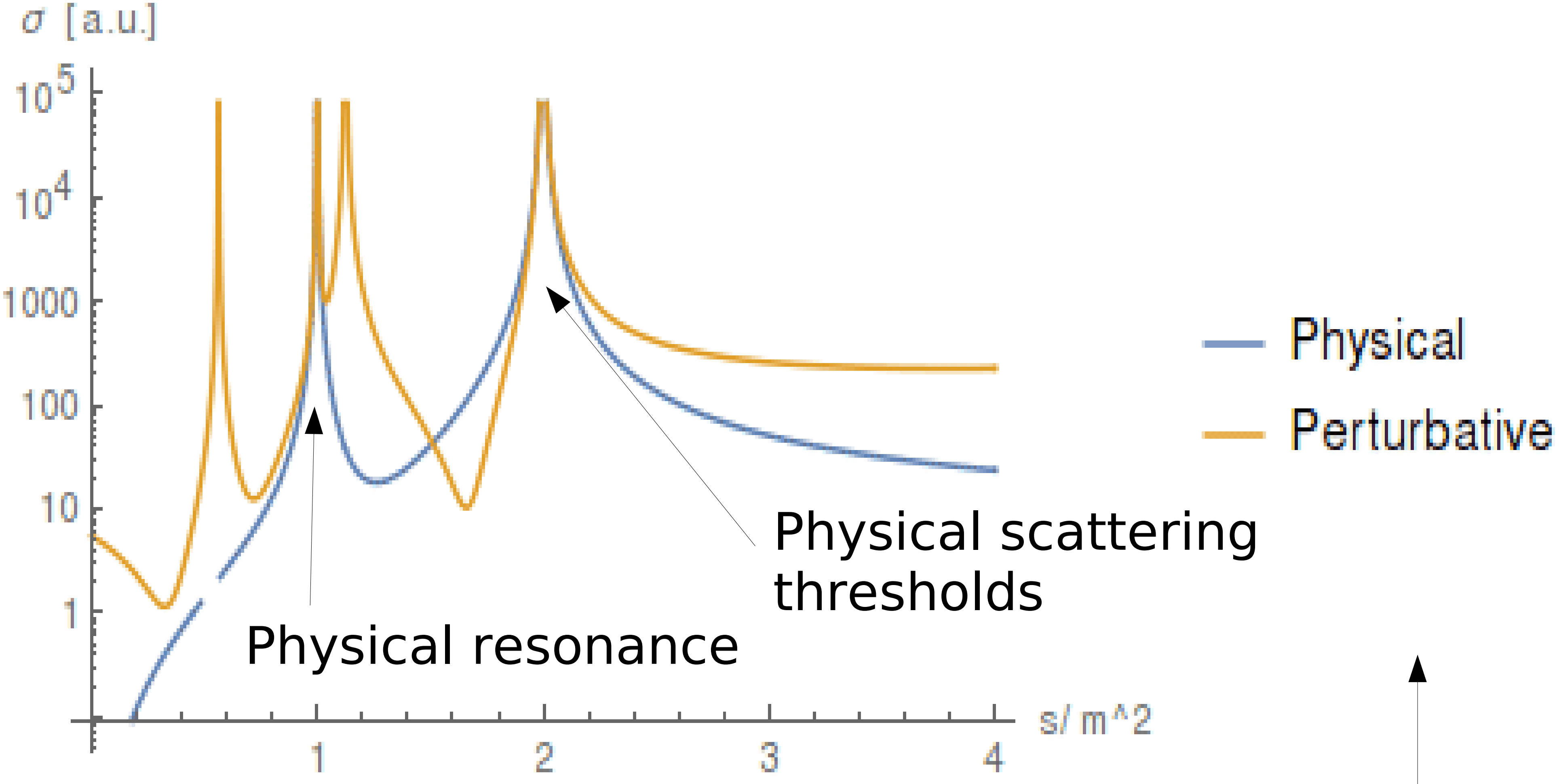
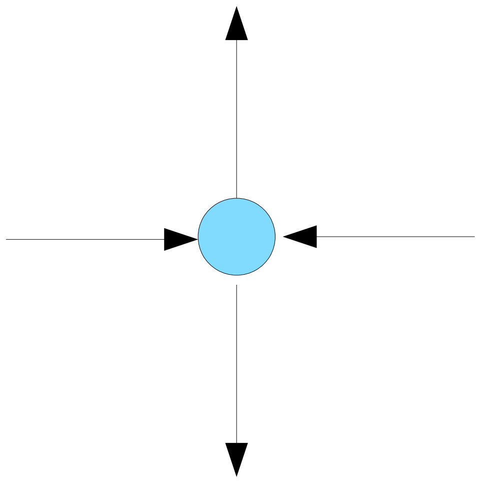


Physical scattering thresholds

Physical resonance



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Experimental consequences

[Maas & Török'18
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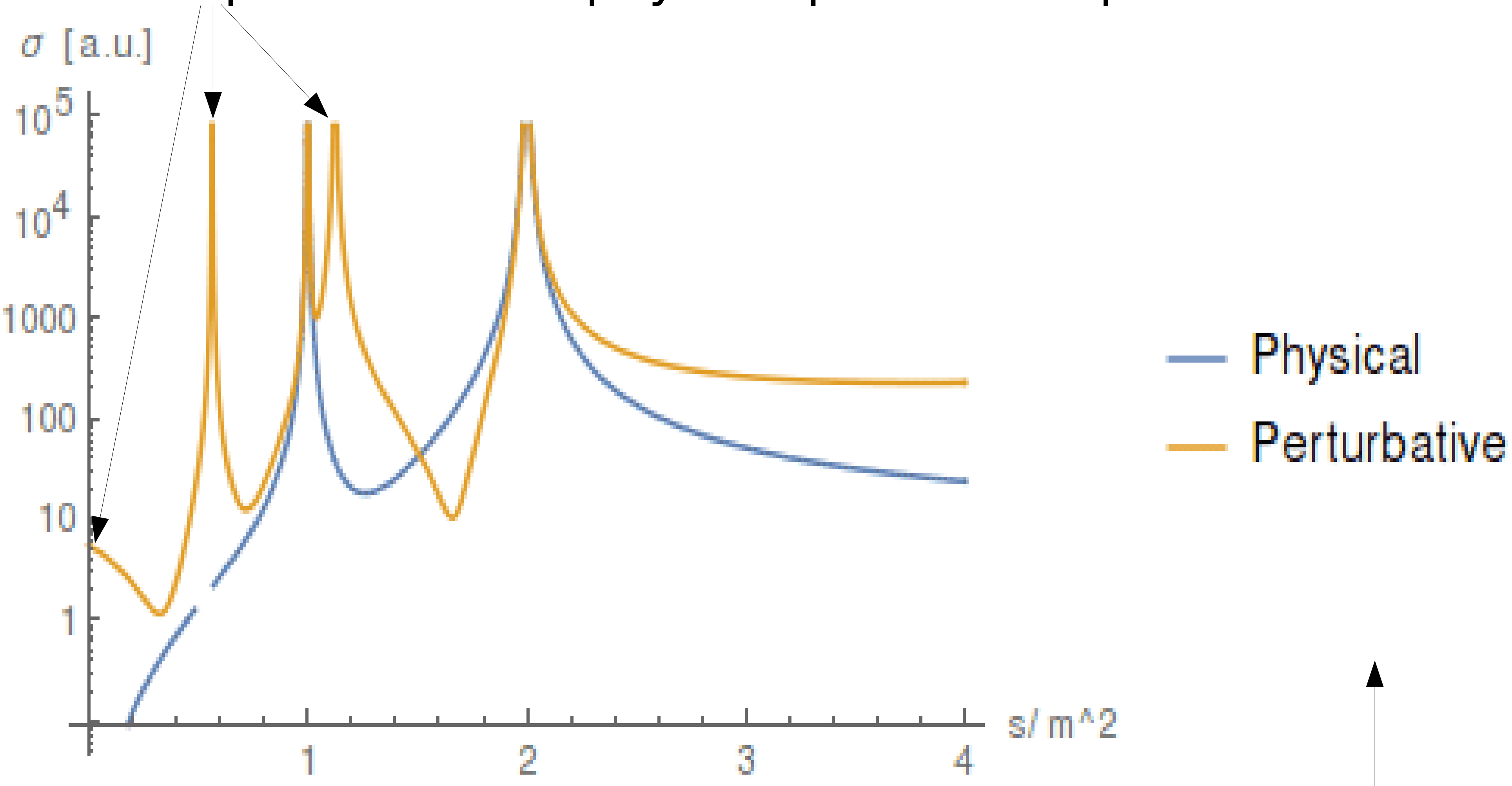
Ghost peaks from unphysical particles in perturbation theory



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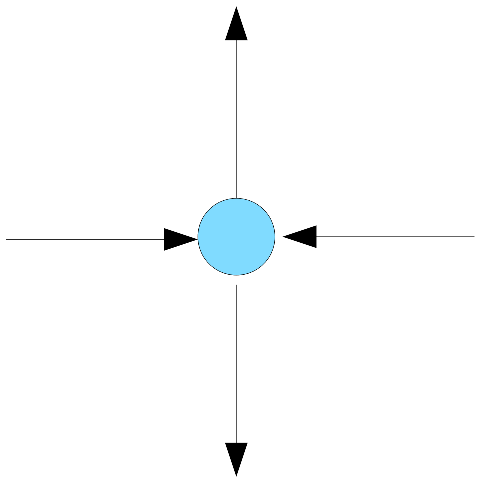

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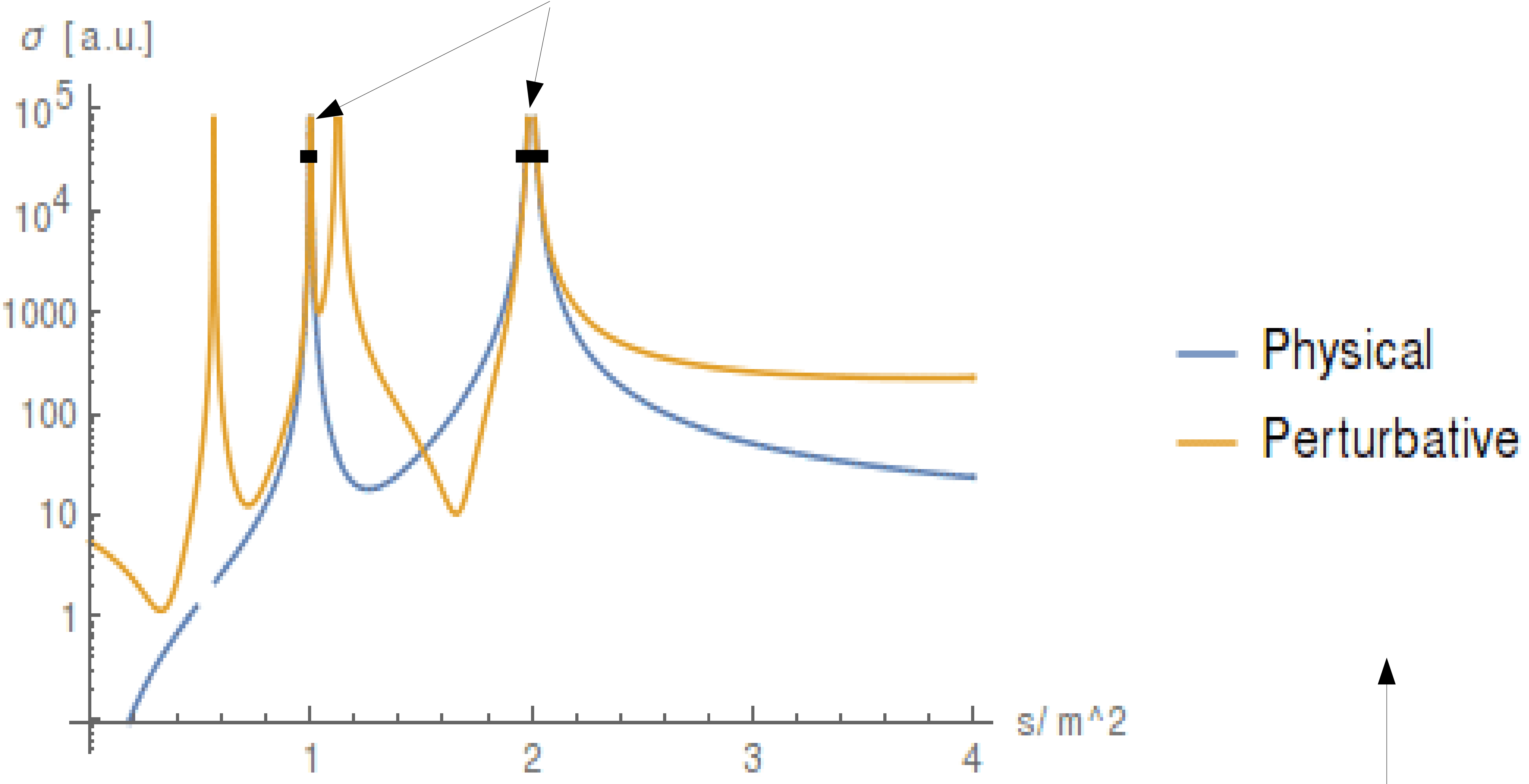
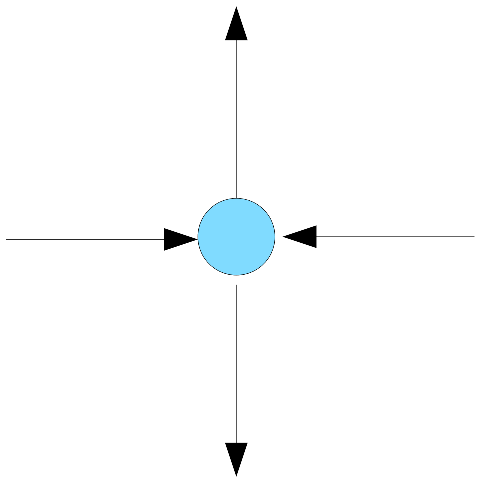


Close to true structures identical!

- Add fundamental fermions
- Bhabha scattering



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[Maas et al.'17]

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- Massless composite vector boson!
  - Made from massive objects

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- Massless composite vector boson!
  - Made from massive objects
    - Without associated symmetry
    - Not a Goldstone of dilation symmetry breaking, like SM photon

A massless composite vector

- Appears impossible with a fundamental Higgs [Maas et al.’17, Sondenheimer’20]

A massless composite vector

- Appears impossible with a fundamental Higgs [Maas et al.’17, Sondenheimer’20]
- Possible with an adjoint Higgs

Simpelst case

- SU(2) with an adjoint Higgs
    - Breaks by BEH to a U(1)

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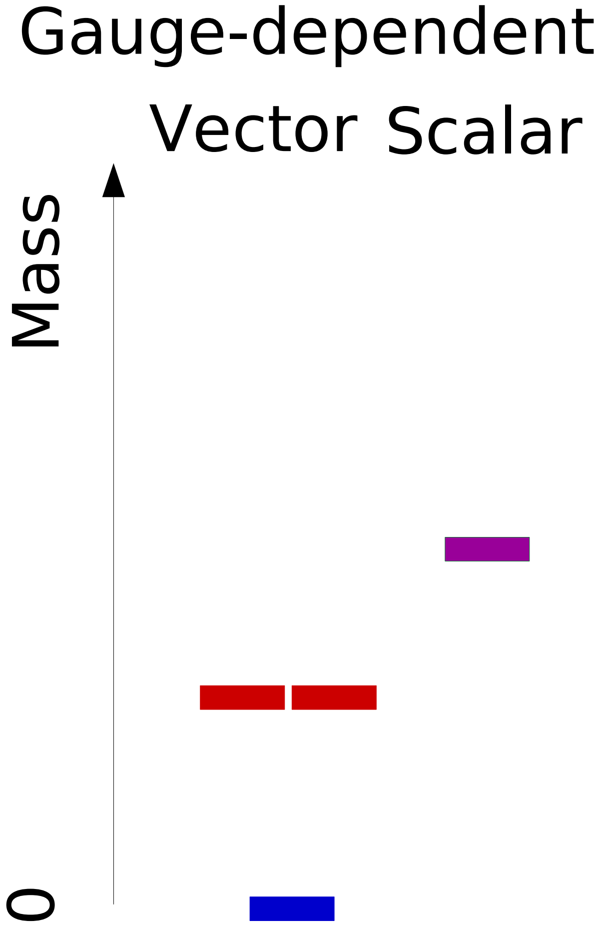
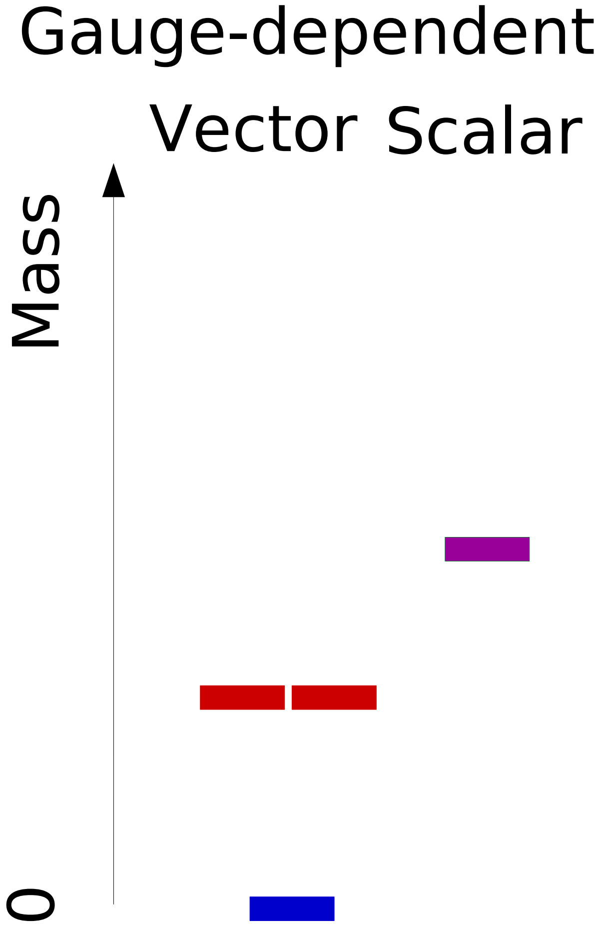
  - Breaks by BEH to a U(1)
  - Massless gauge boson
  - Two massive gauge boson
  - One massive scalar



Simpelst case



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  - Two massive gauge boson
  - One massive scalar
- Should happen at arbitrary weak coupling
  - Check by calculating running coupling
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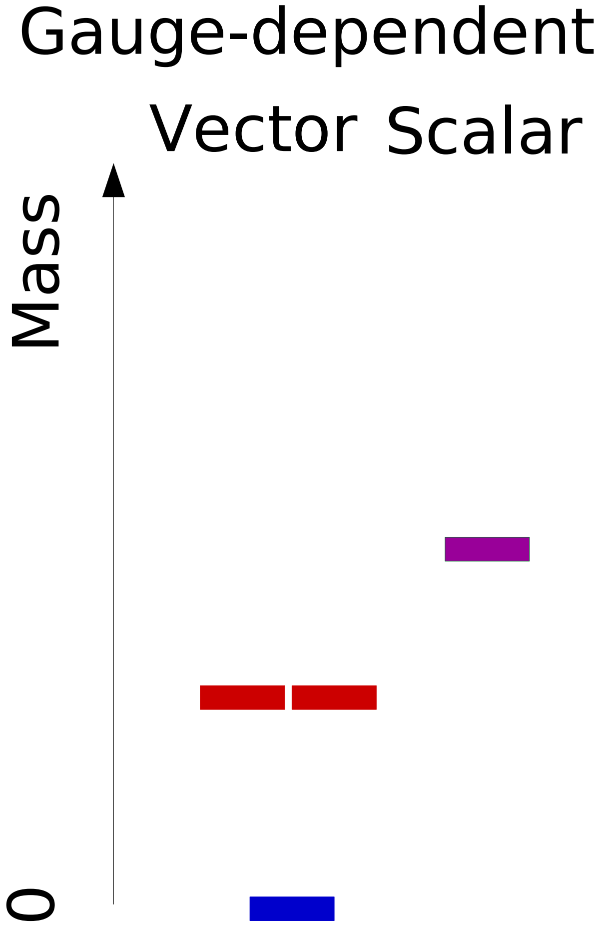
Simpelst case



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- Massless state: Autocorrelation bad!

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- Appears impossible with a fundamental Higgs [Maas et al.'17, Sondenheimer'20]
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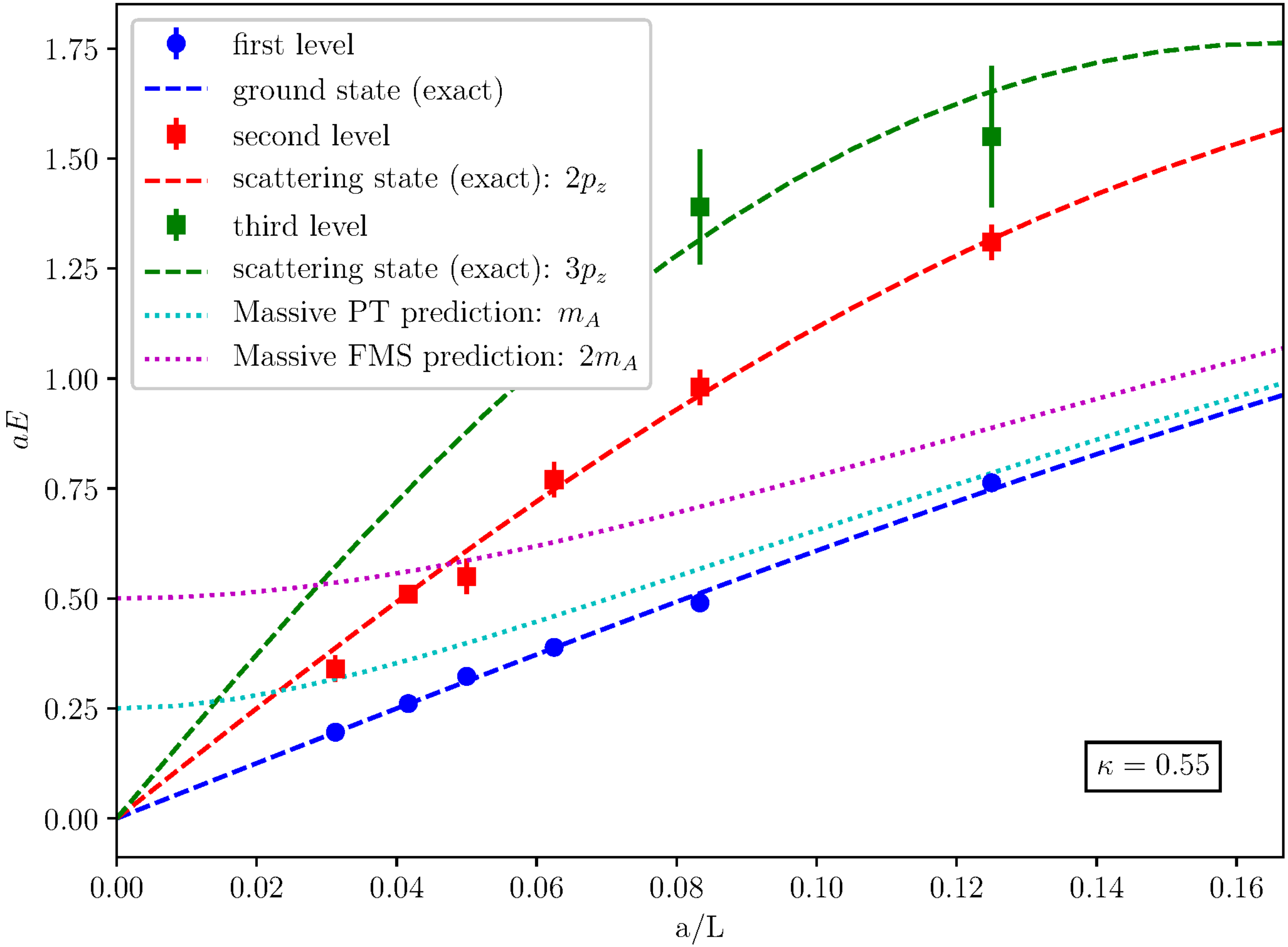
Result

[Afferrante et al.'20]



No massive states seen yet – but no suitable methods available

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Review: 1712.04721 Update: 2305.01960
Philosophical aspects: 2110.00616

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