Gauge fixing and the Ghost DSE

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Natural Sciences

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  - Gribov-Singer ambiguity

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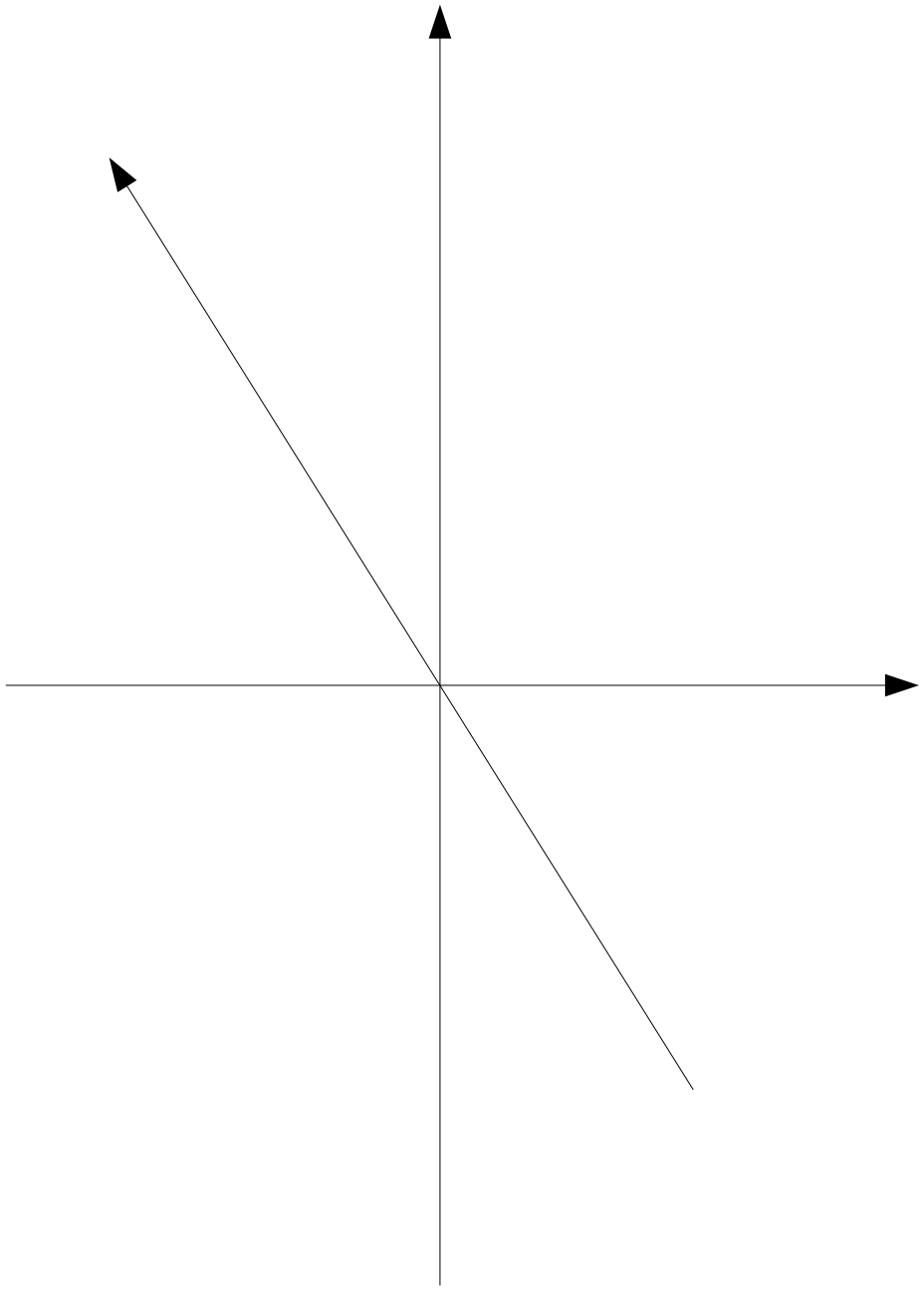

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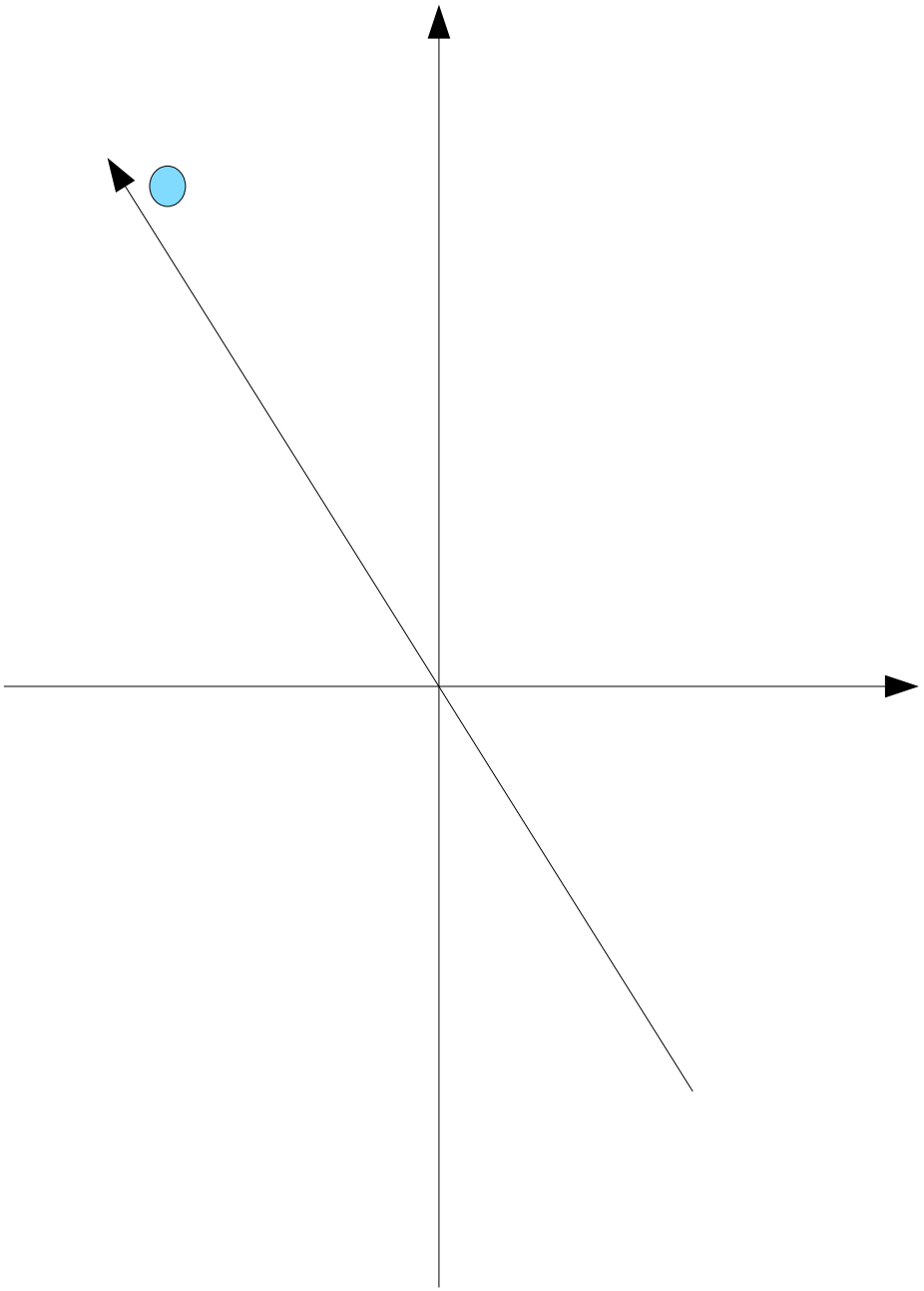

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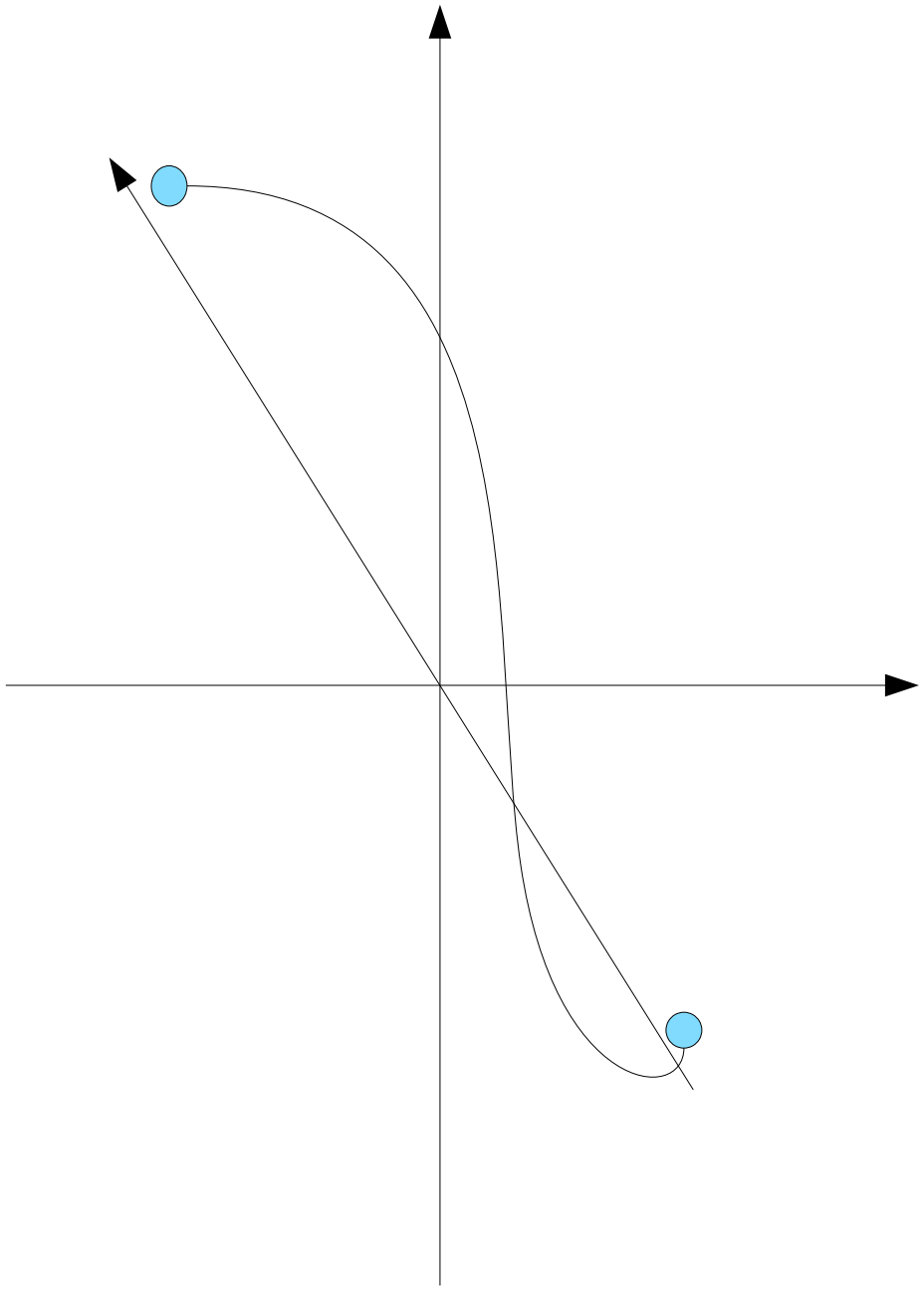

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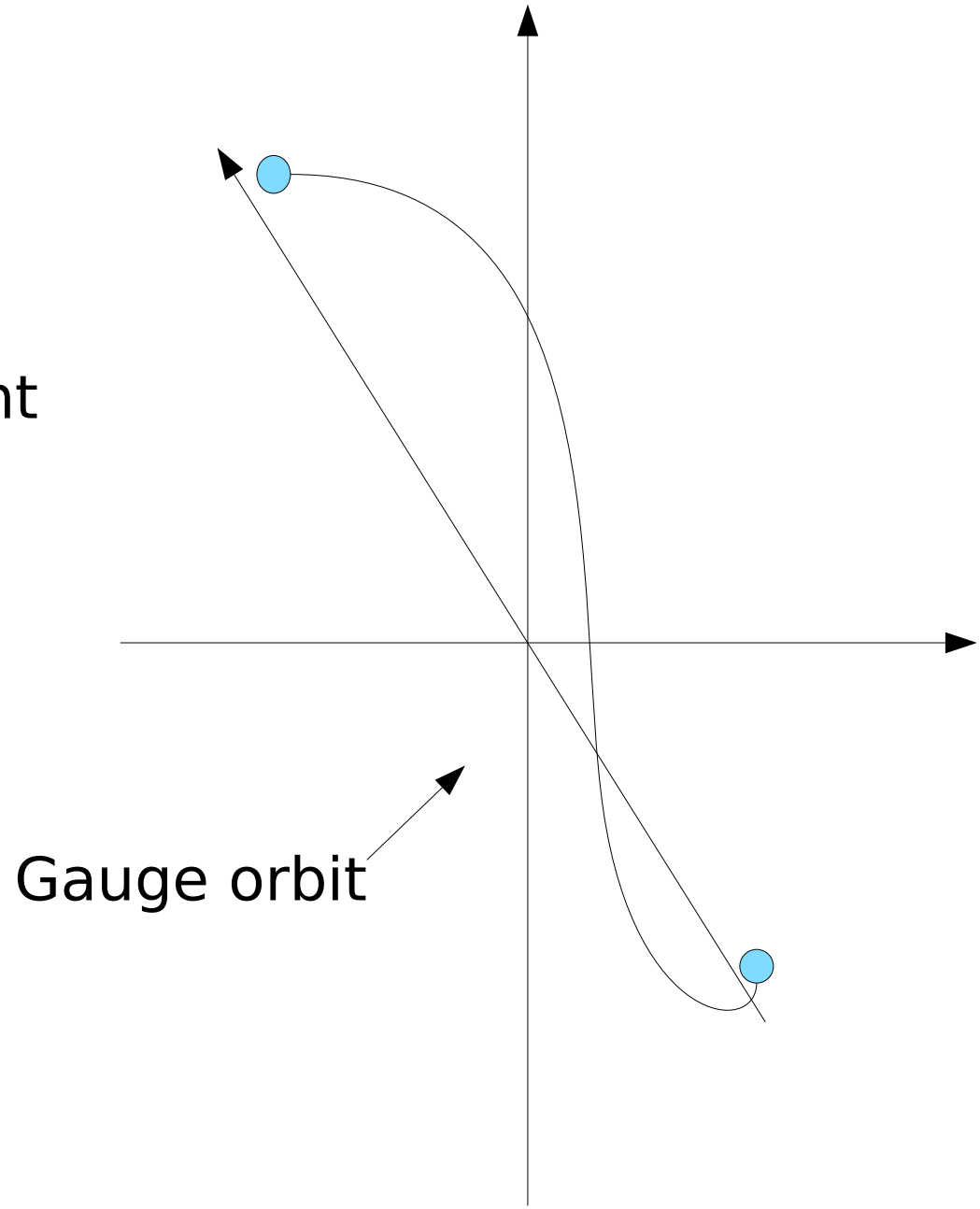

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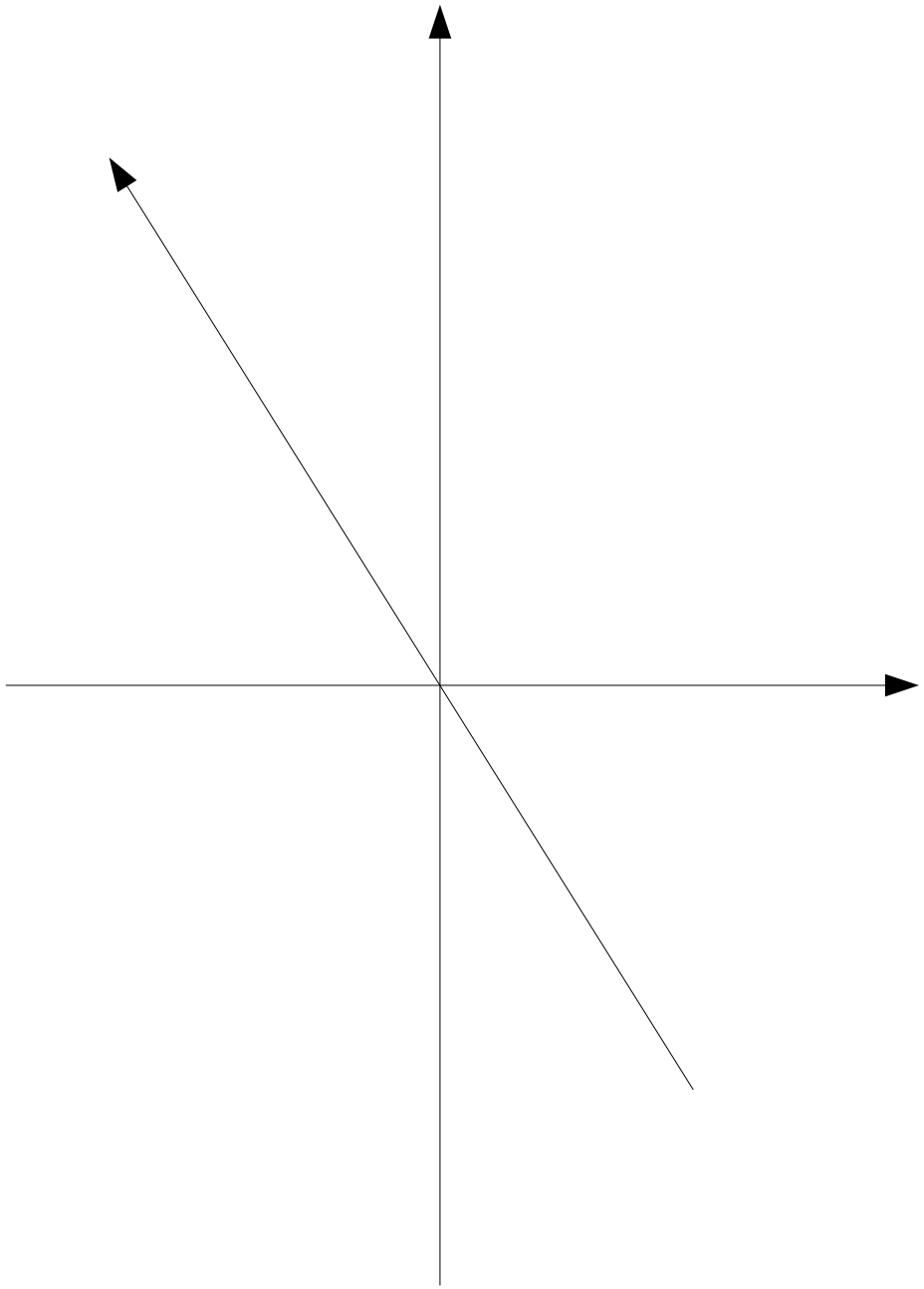

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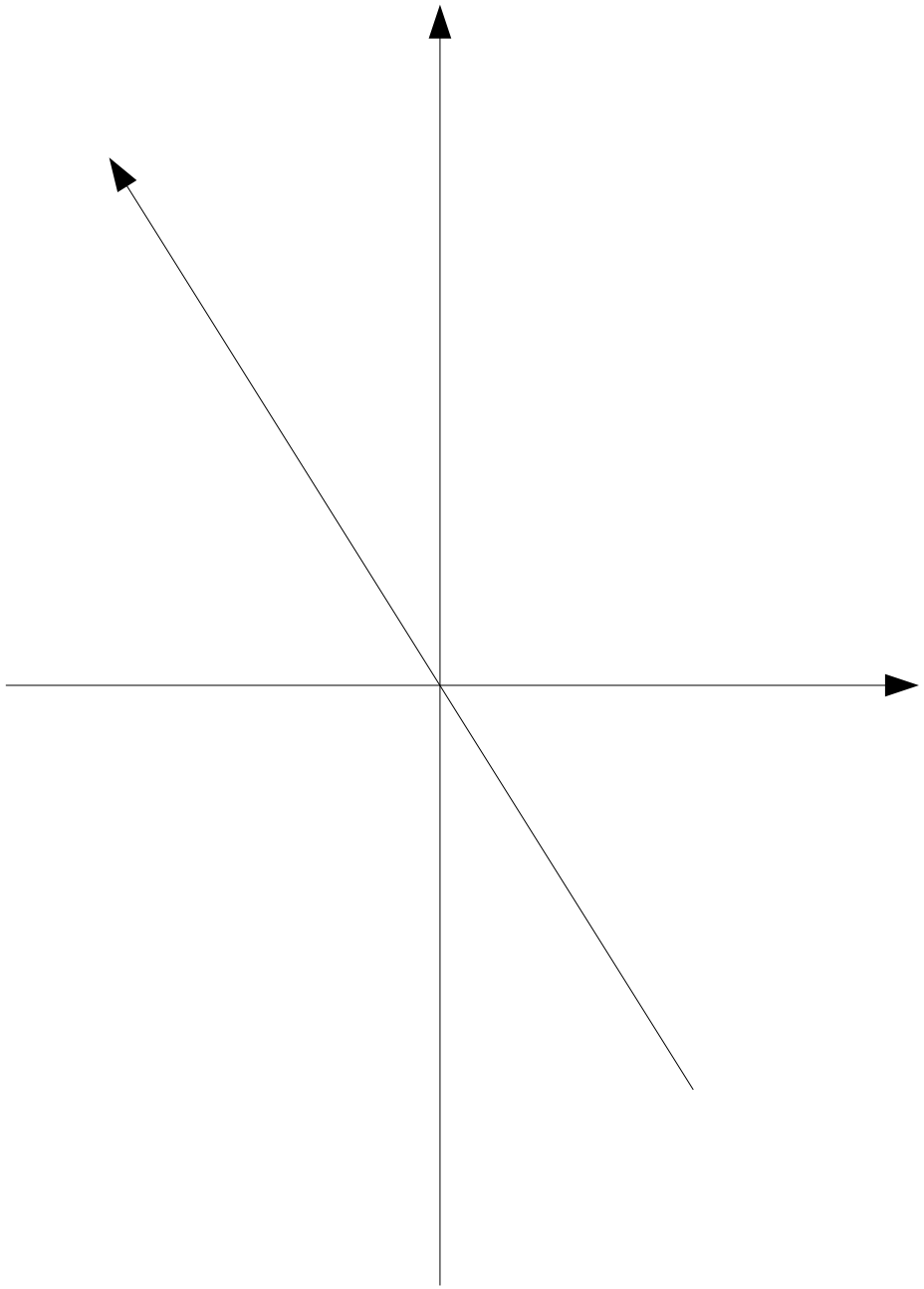

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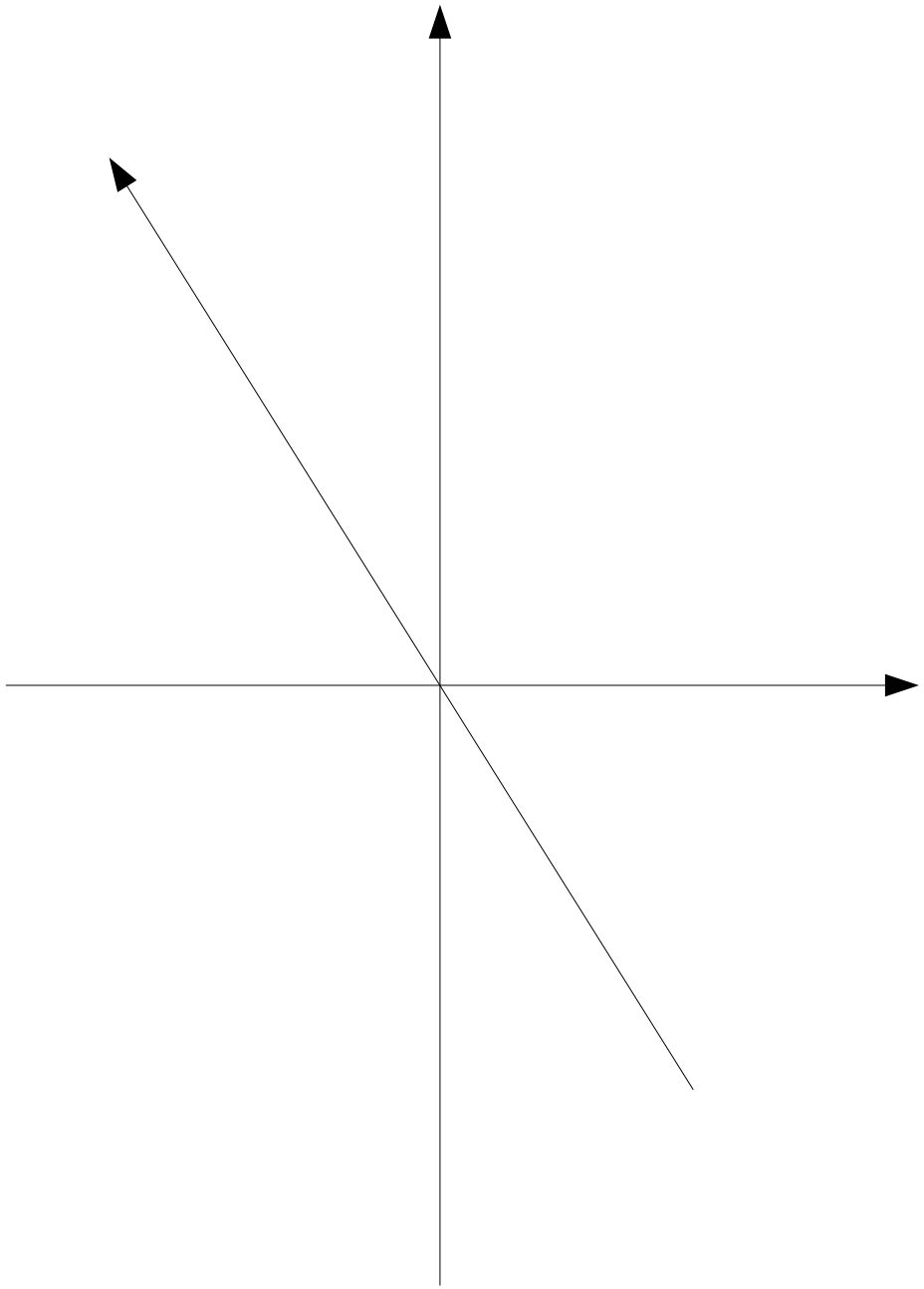

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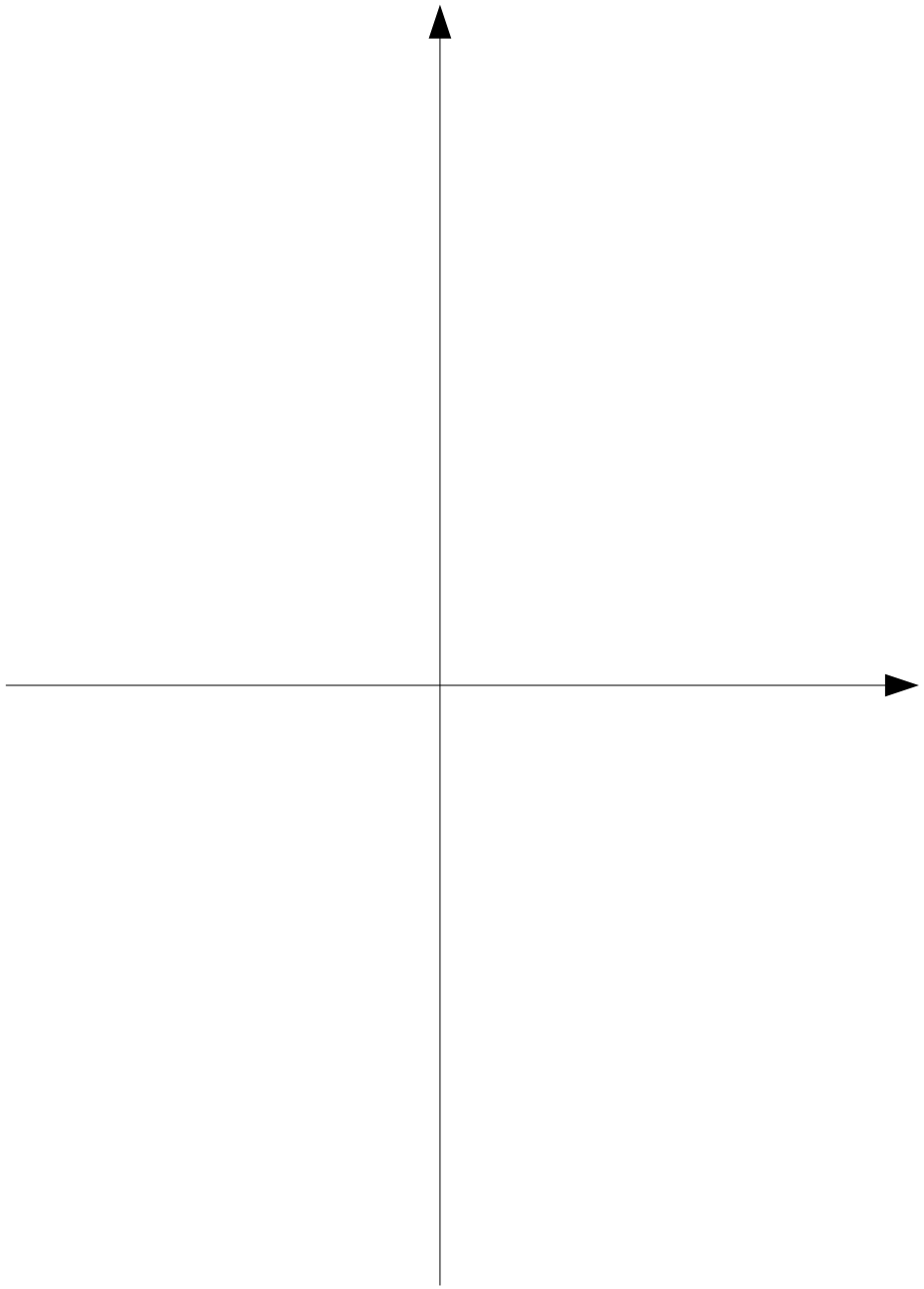
  - Reduces configuration space to a hypersurface



(Perturbative) Landau gauge

- Condition can be implemented using auxiliary fields, the ghost fields
- Lagrangian: $$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{c}^a \partial_\mu D^{ab}_\mu c^b$$

$$D^{ab}_\mu = \delta^{ab} \partial_\mu - g f^{abc} A^c_\mu$$

$$\lim_{\zeta \to 0} \int DAc\, \bar{c} \exp\left(\int dx \left(-\frac{1}{4} F^2 - \bar{c}\, \partial D c - \frac{1}{2\zeta} (\partial A)^2\right)\right)$$

- Degrees of freedom: Gluons A^a_μ Ghosts $\bar{c}^a, c^a$
  - Not physical objects
  - Pure mathematical convenience

Unique gauge-fixing [Sobreiro & Sorella, 2005]

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- Local gauge condition
  - Landau gauge: $\partial_\mu A^a_\mu = 0$
- Sufficient for perturbation theory
- Insufficient beyond perturbation theory
  - There are gauge-equivalent configurations: Gribov copies [Gribov NPB 78]
- No local gauge conditions known to select a unique gauge copy: Gribov-Singer ambiguity [Singer CMP 78]

Residual freedom

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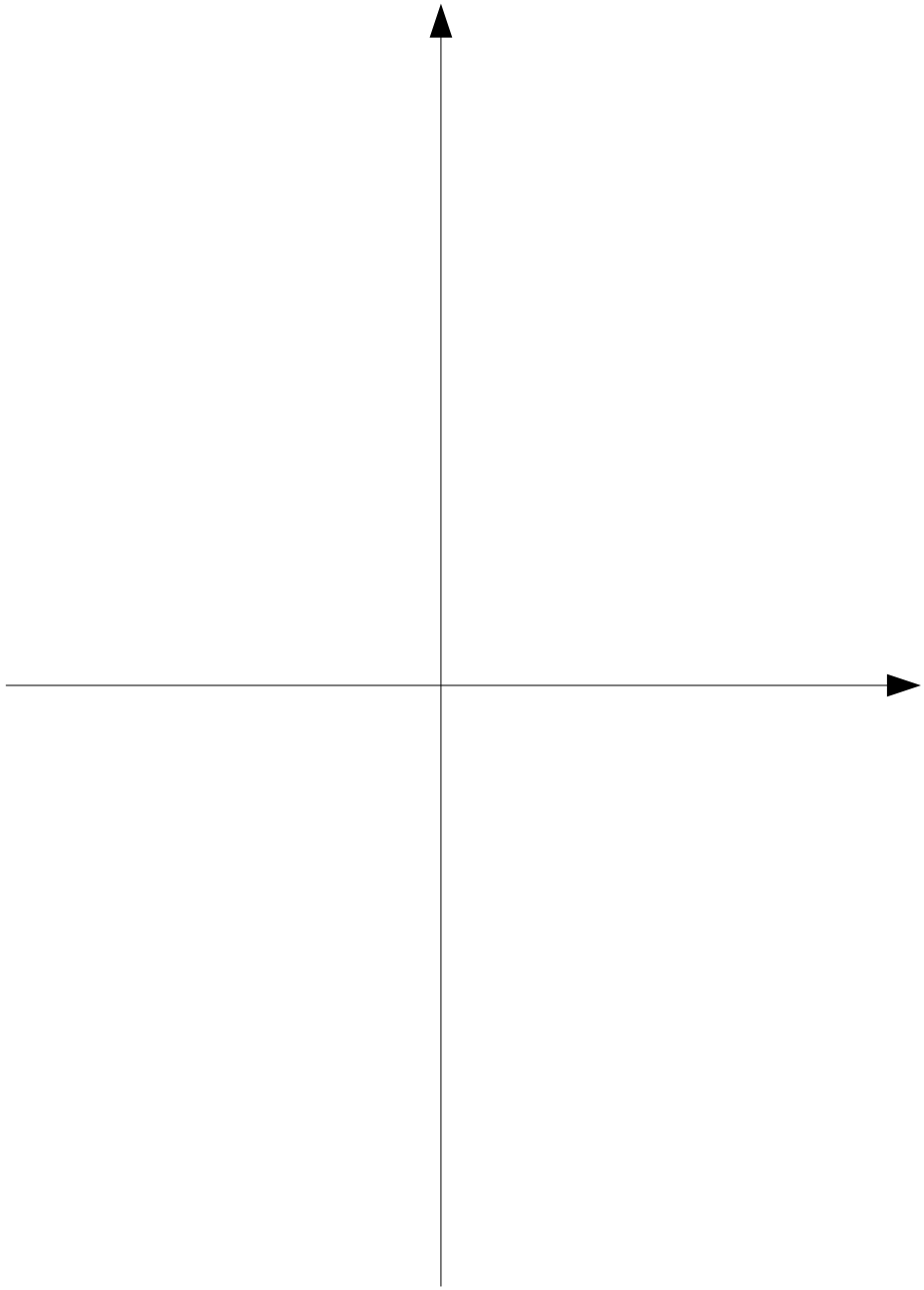
- Method: Lattice

First horizon gauges

[Gribov NPA 78, Zwanziger 93...03]



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[Gribov NPA 78, Zwanziger 93...03]

- Gribov horizon encloses all field configurations with non-negative Faddeev-Popov operator $-\partial_\mu D_\mu$
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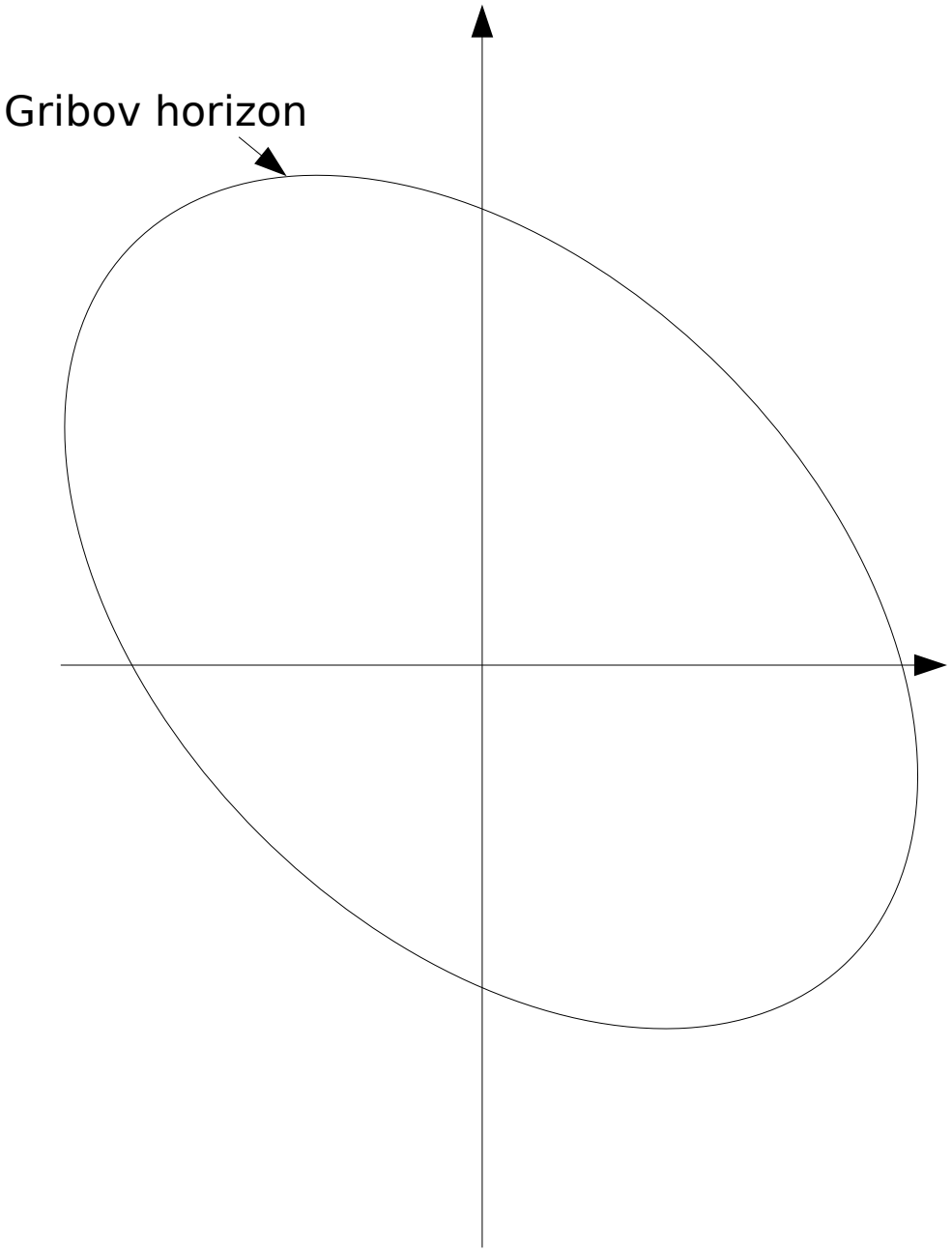

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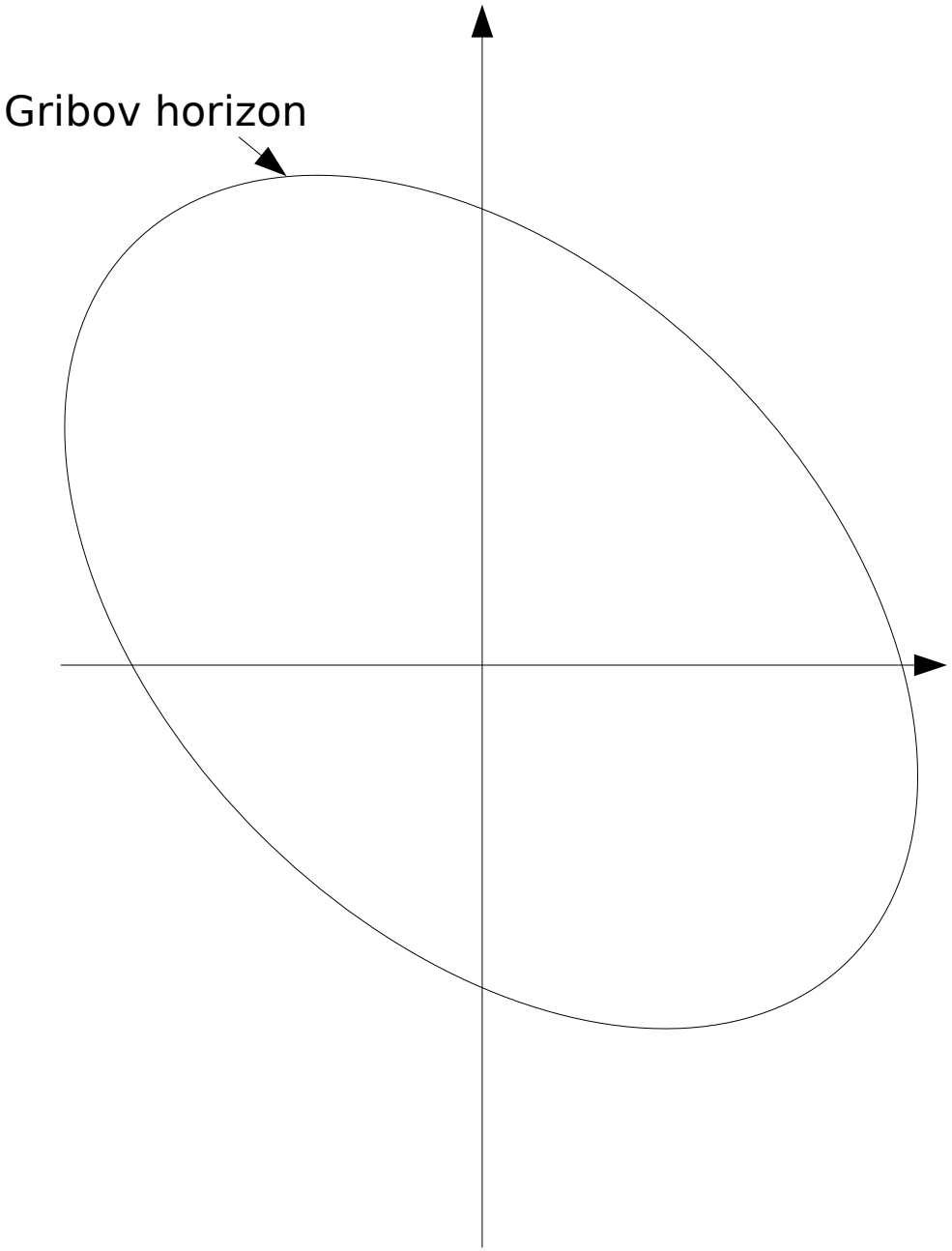

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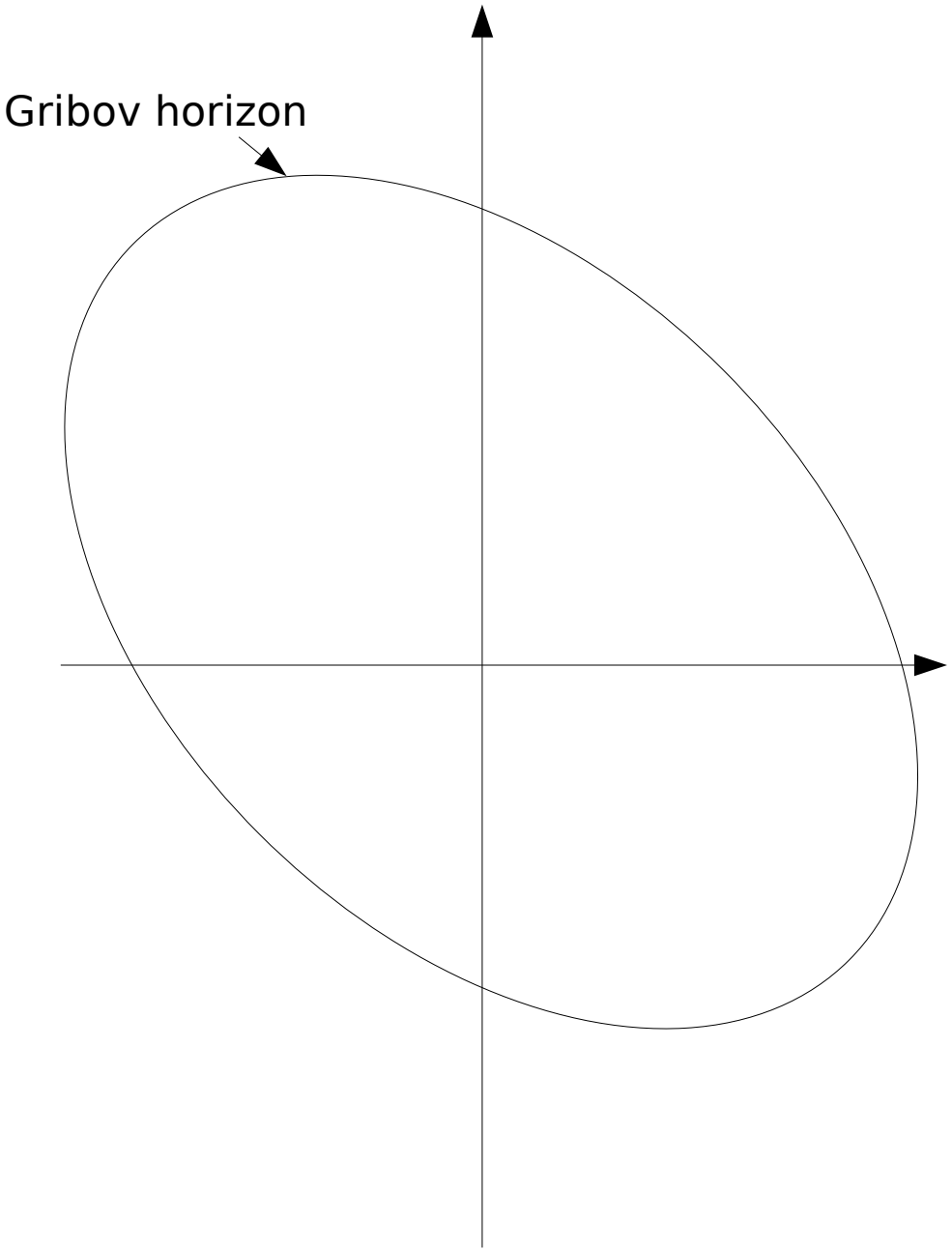

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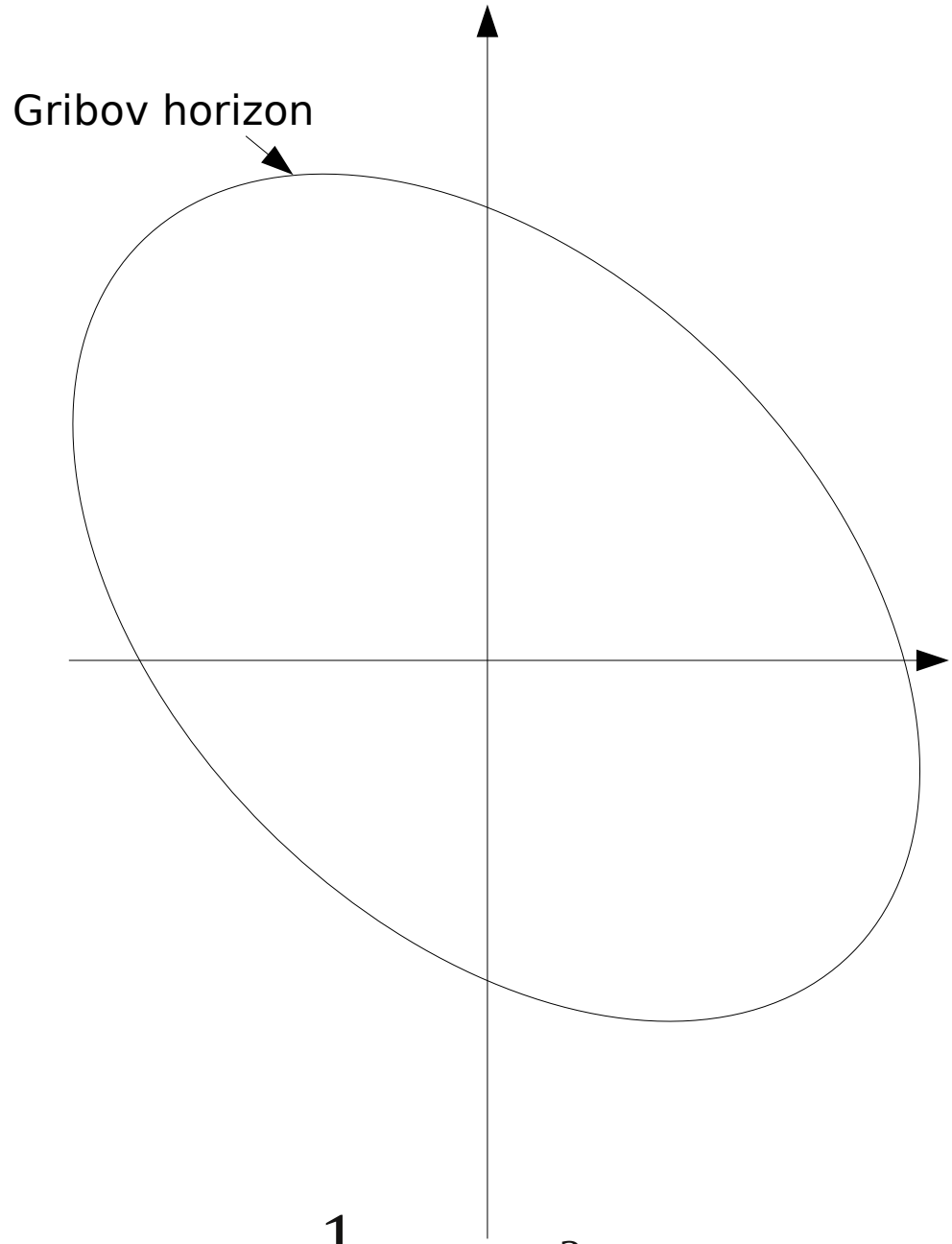
  - Literally on the lattice:



$$\lim_{\zeta\to 0}\int DAc\,\bar{c}\exp\left(\int dx\left(-\frac{1}{4}F^2-\bar{c}\,\partial D\,c-\frac{1}{2\zeta}(\partial A)^2\right)\right)\Theta(-\partial D)$$

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- Two tests

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- Derivation of functional equations assume the reconstruction theorem
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  - The quantum effective action is an analytical function of the classical fields
  - Alternatively: Free energy is an analytic function of the sources
- Is this correct?
  - Unlikely: Implies necessarily a vanishing gluon propagator at zero momentum on the lattice [Zwanziger'13]
  - At odds with lattice results [Cucchieri et al.'07, von Smekal et al'07]

Lattice results

[Maas & Zwanziger'14]

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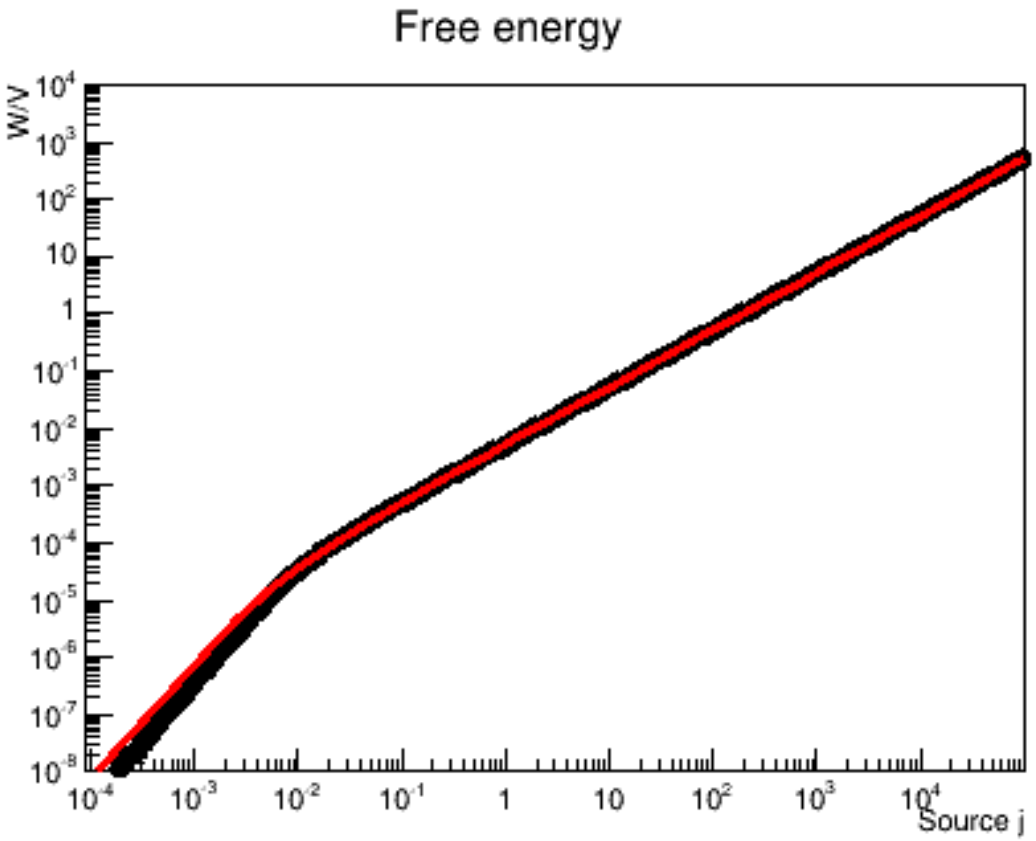

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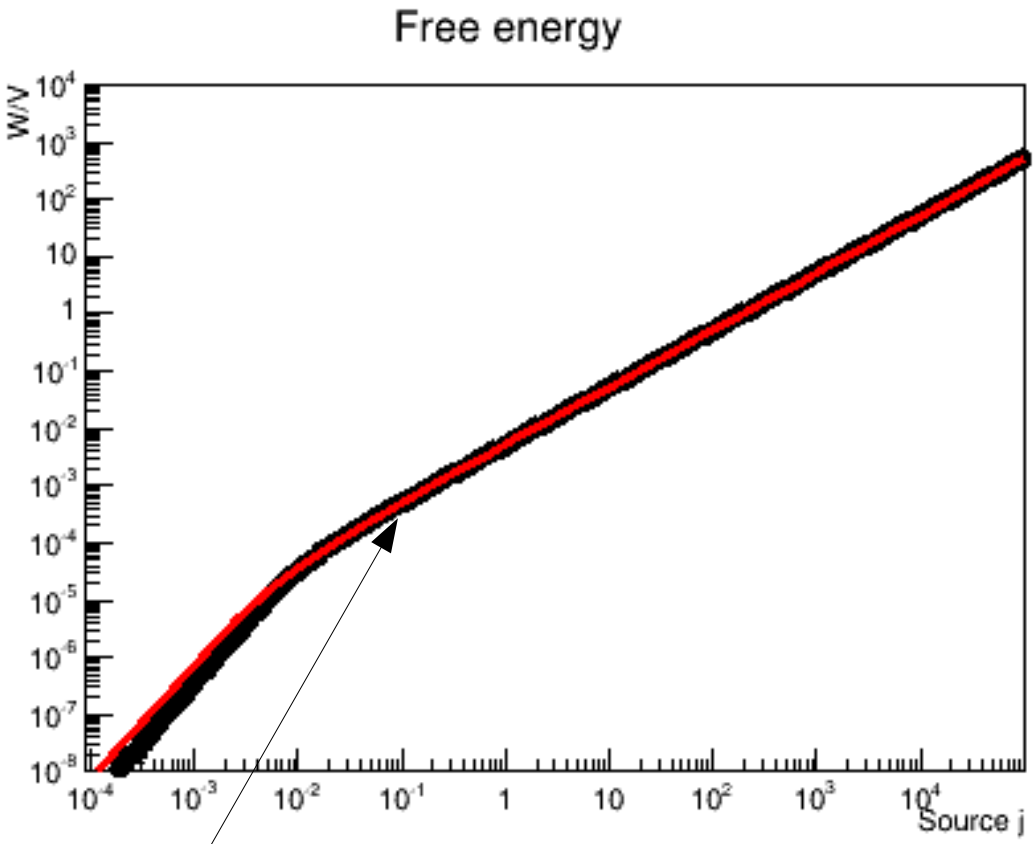

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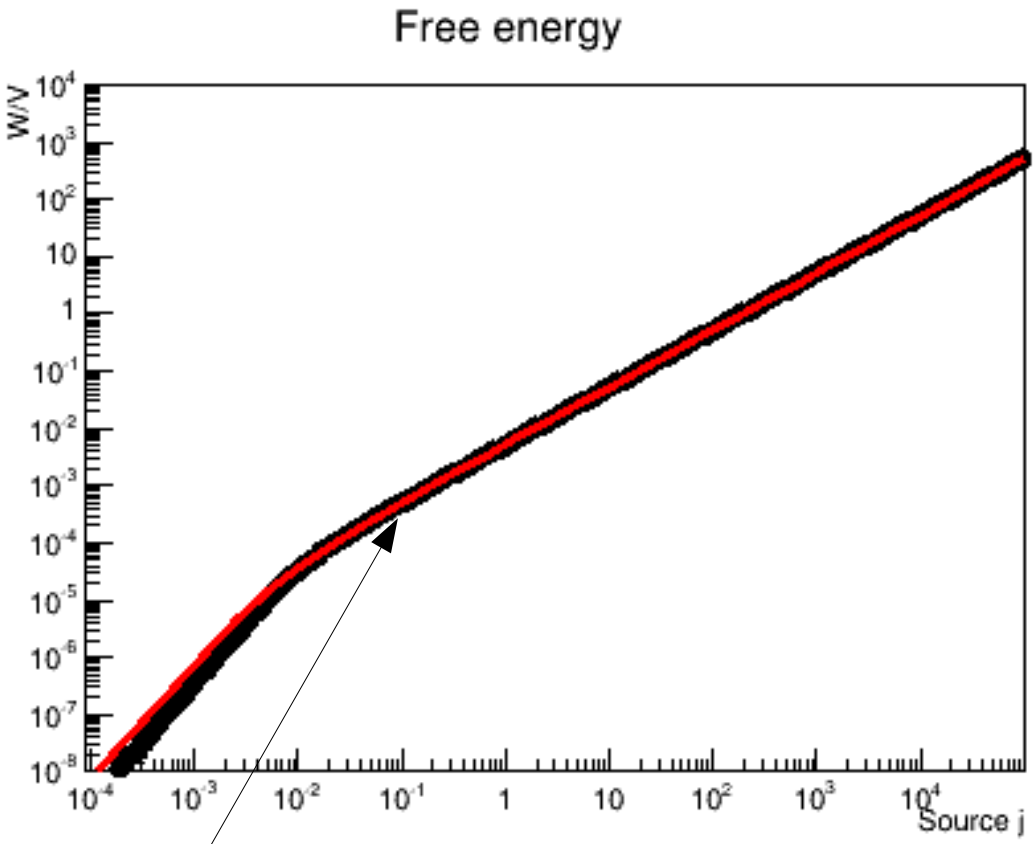

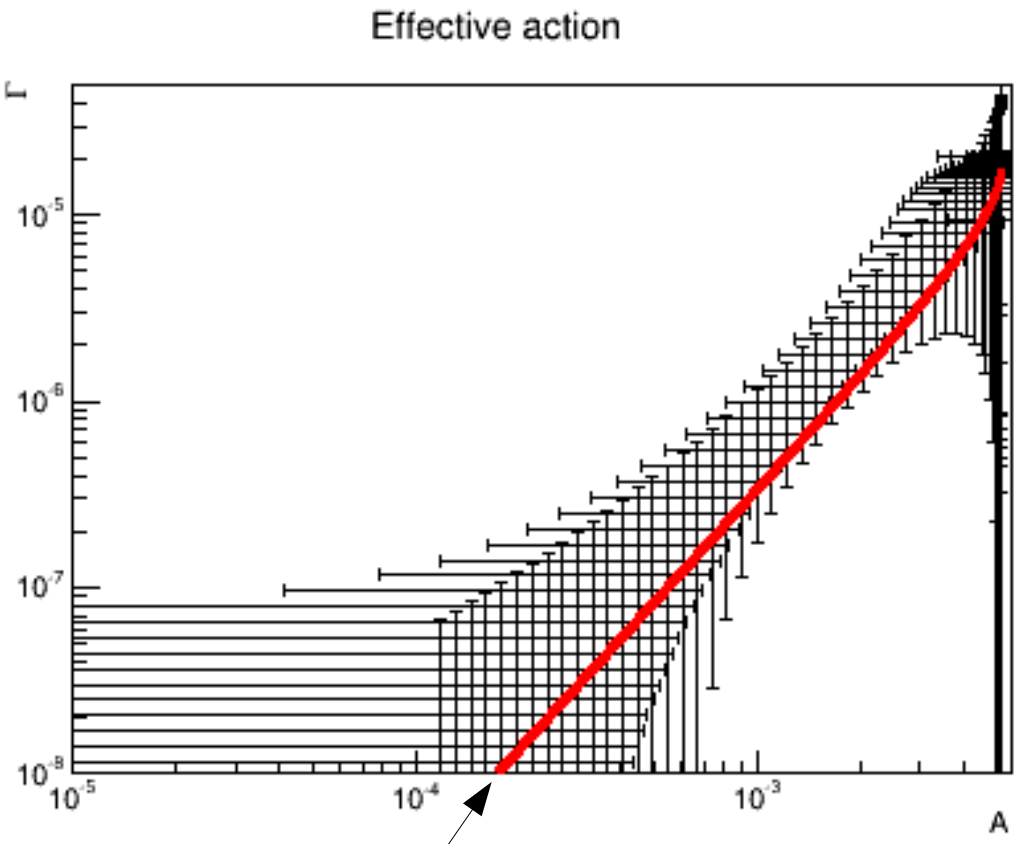

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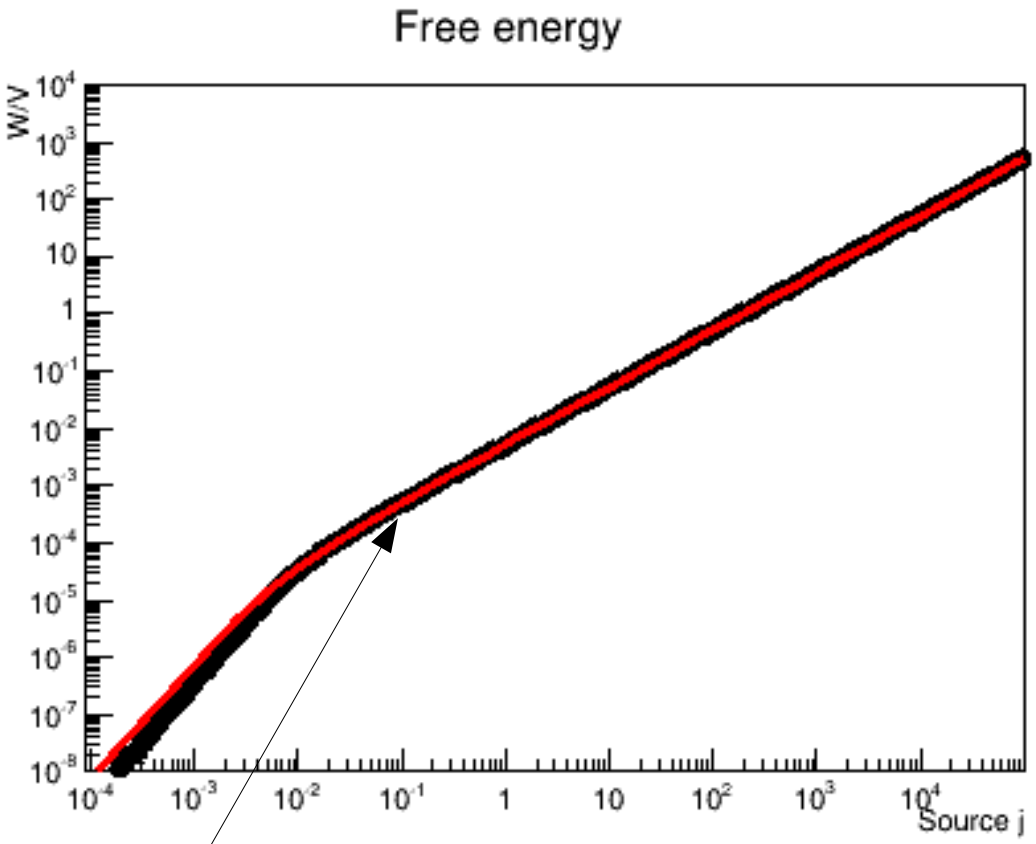

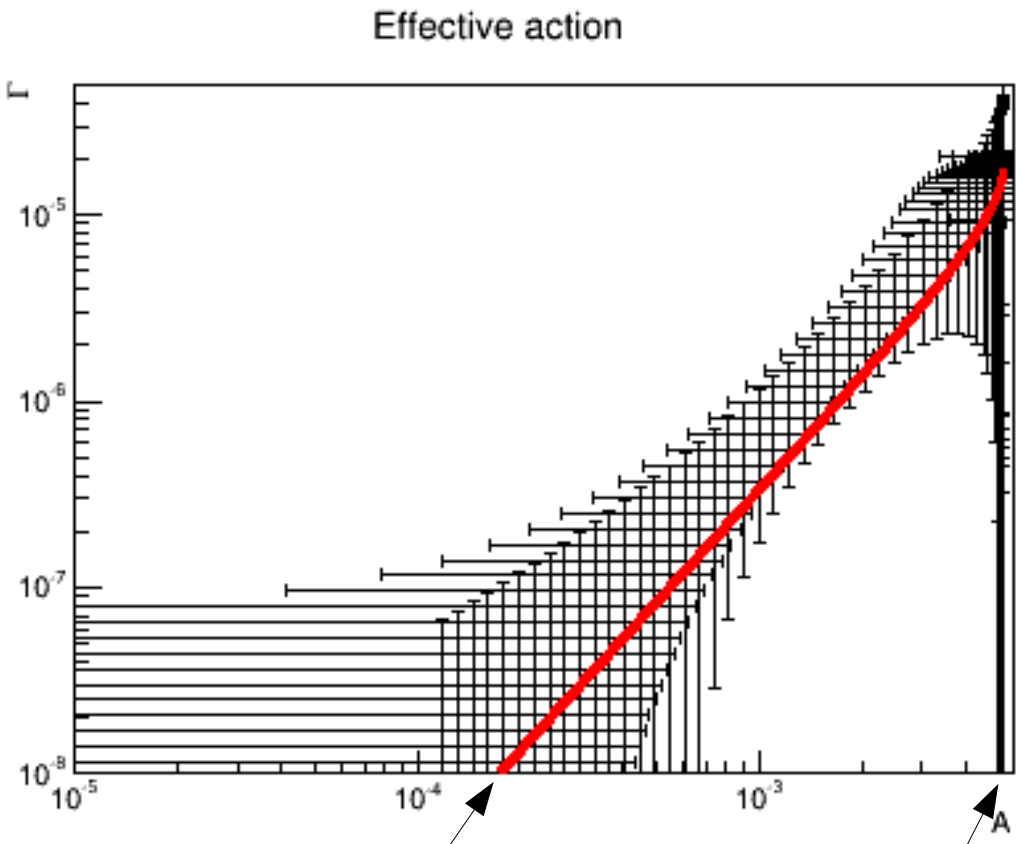

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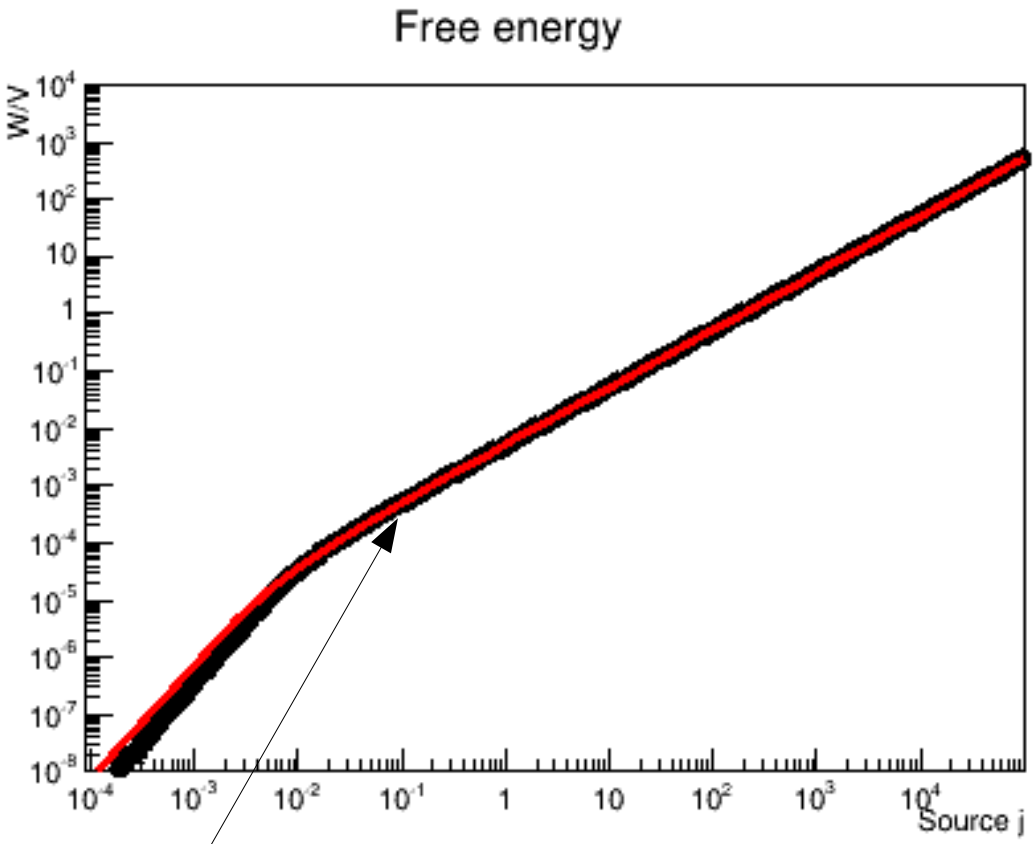

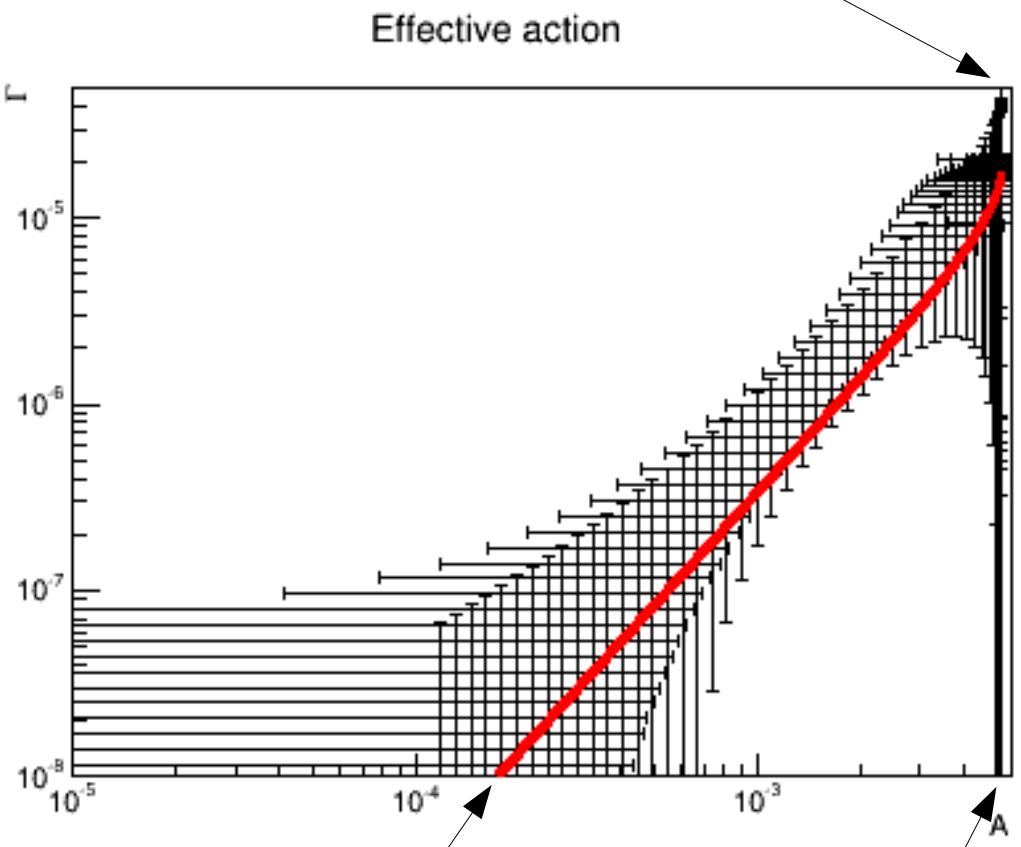

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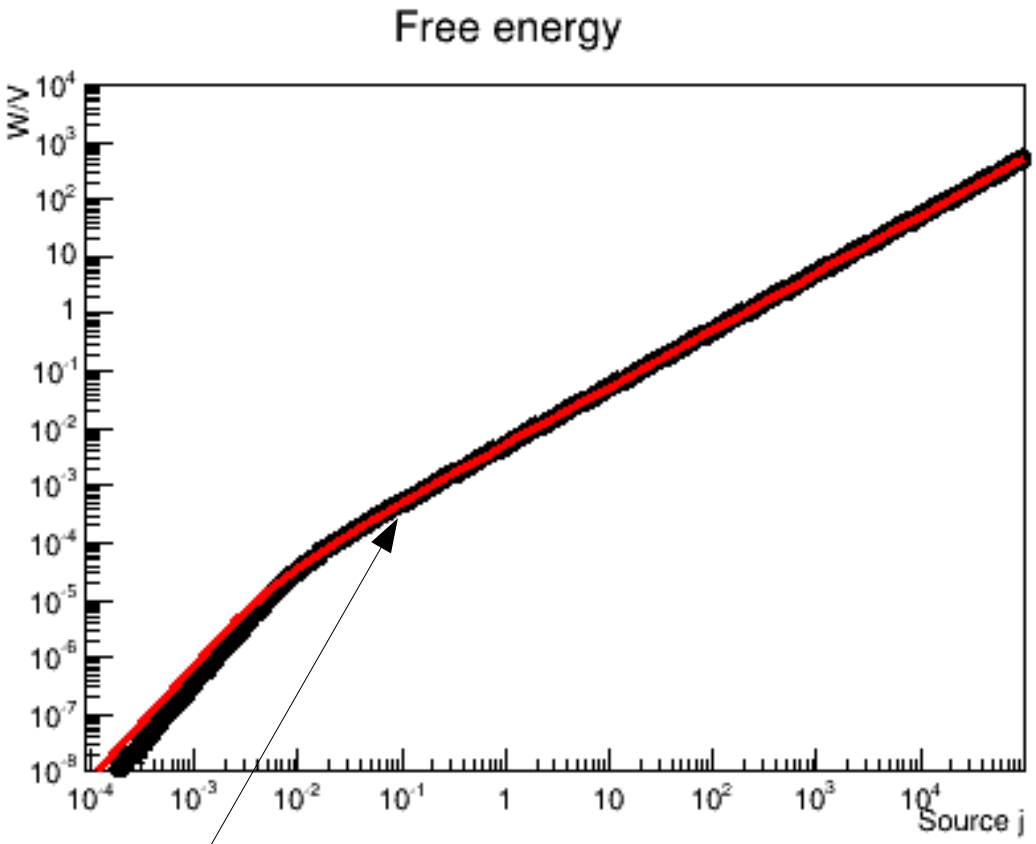

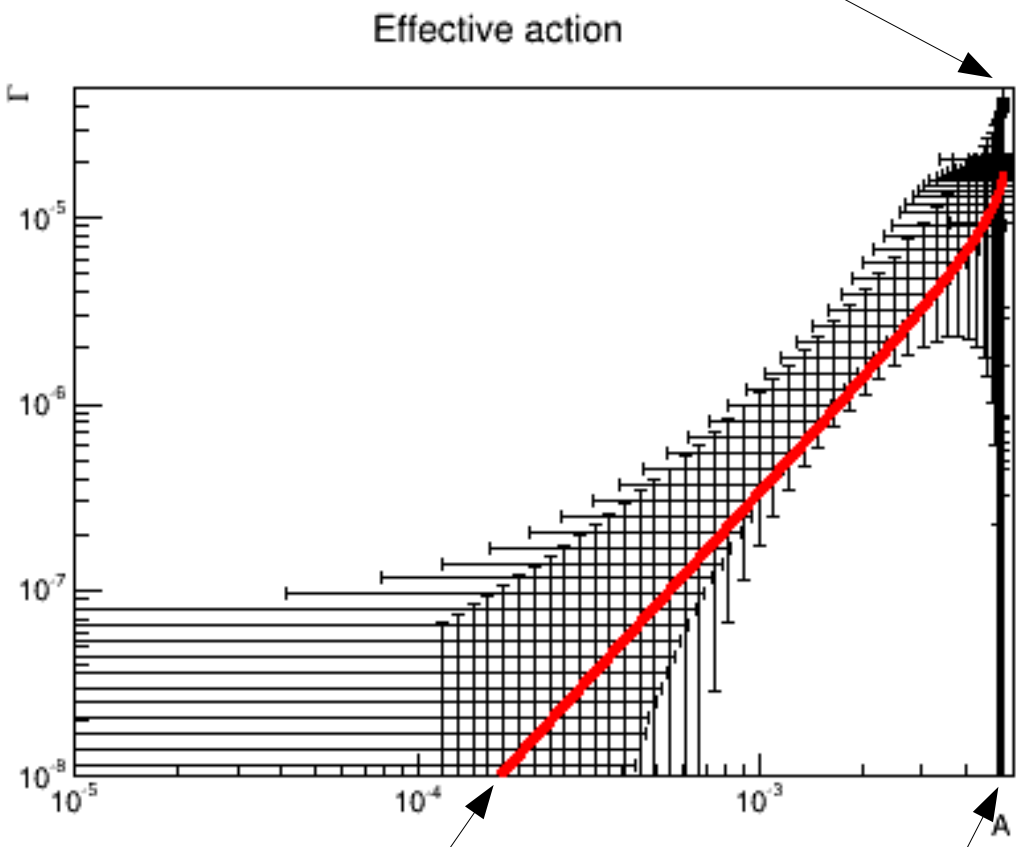

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The quantum effective action is not an analytic functional!

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- Simplest case: Ghost DSE
  - Ghost propagator, gluon propagator, ghost-gluon vertex
- Possible also for other equations
  - Link DSE shows similar results [Sternbeck et al.'15]

Determine a mis-function

[Maas'19]

Ghost DSE

$$0=\frac{1}{\left(p^2 D(p)\right)}+1-\Pi(p)$$

$$\Pi(p)=\frac{c}{p^2}\int d^d q\,\Gamma_\mu^{tl}(p,q)\,D_{\mu\nu}(p+q)\,D(p)\,\Gamma_\nu(p,q)$$

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$$f(p) = \frac{1}{\left(p^2 D^{lattice}(p)\right)} + 1 - \Pi(p)$$

$$\Pi(p) = \frac{c}{p^2} \int d^d q \, \Gamma_\mu^{tl\,lattice}(p,q) D_{\mu\nu}^{lattice}(p+q) D^{lattice}(p) \Gamma_\nu^{lattice}(p,q)$$

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Question can be recast as ratio:

$$\frac{\frac{1}{1-\Pi(p)}}{p^2 D^{lattice}(p)} \overset{?}{=} 1 \overset{?}{=} \frac{\frac{1}{1-f(p)-\Pi(p)}}{p^2 D^{lattice}(p)}$$

Lattice results

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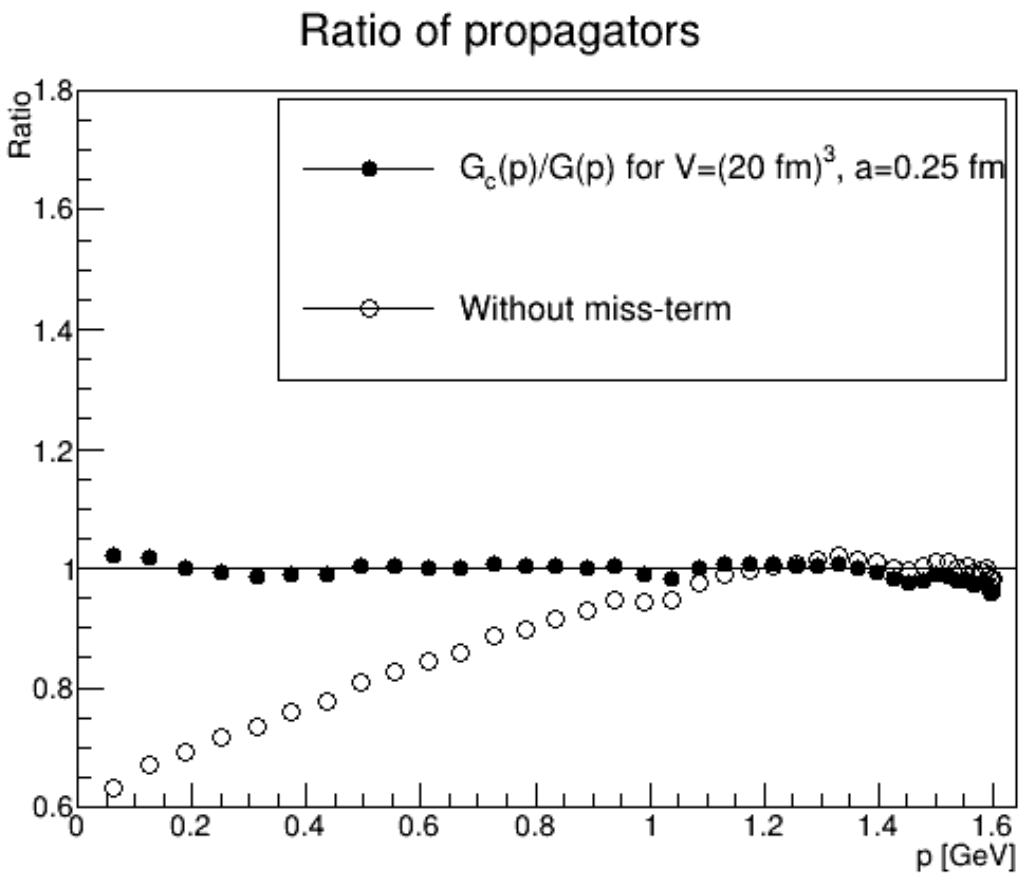
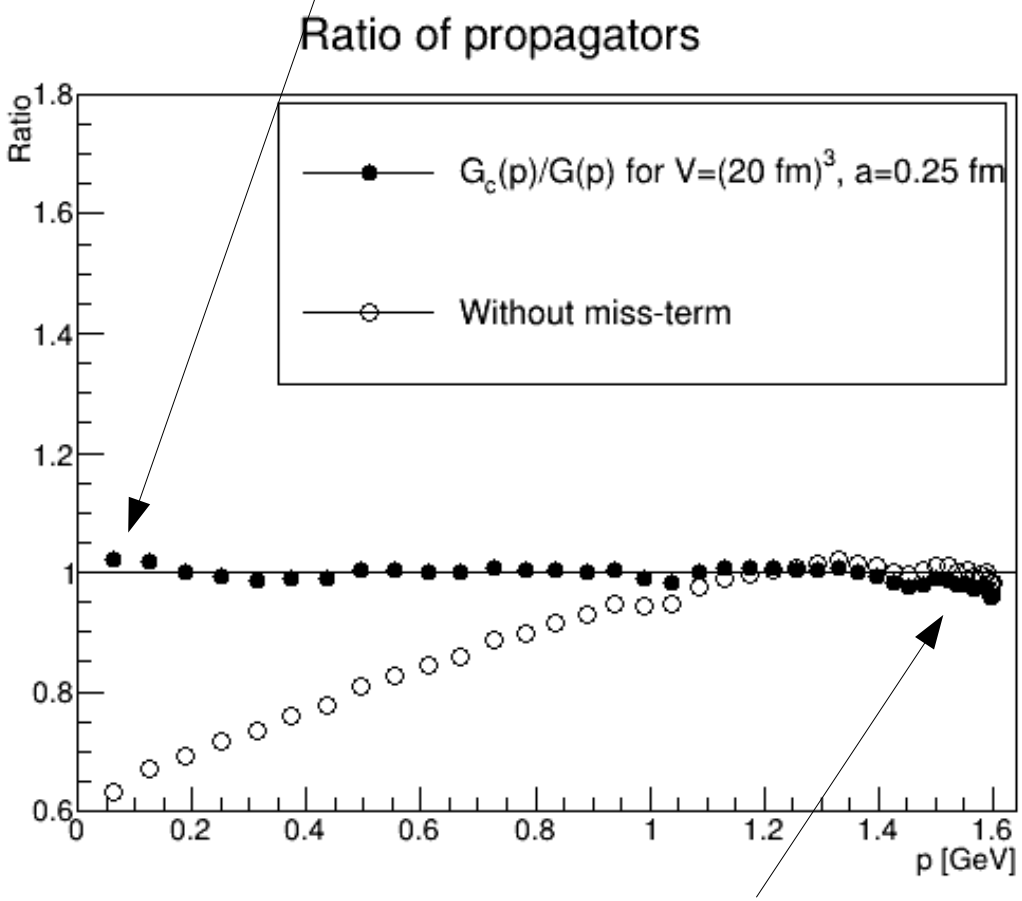


Lattice results

[Maas'19]

Substantial disagreement at low momenta



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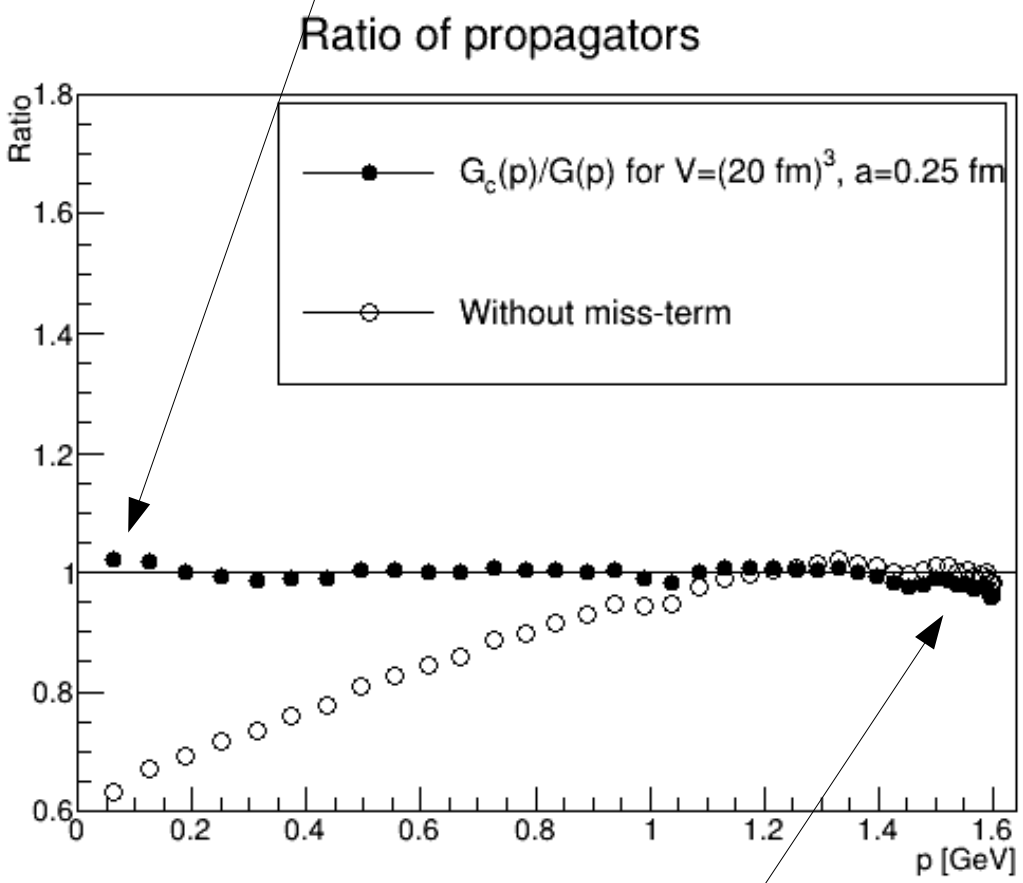

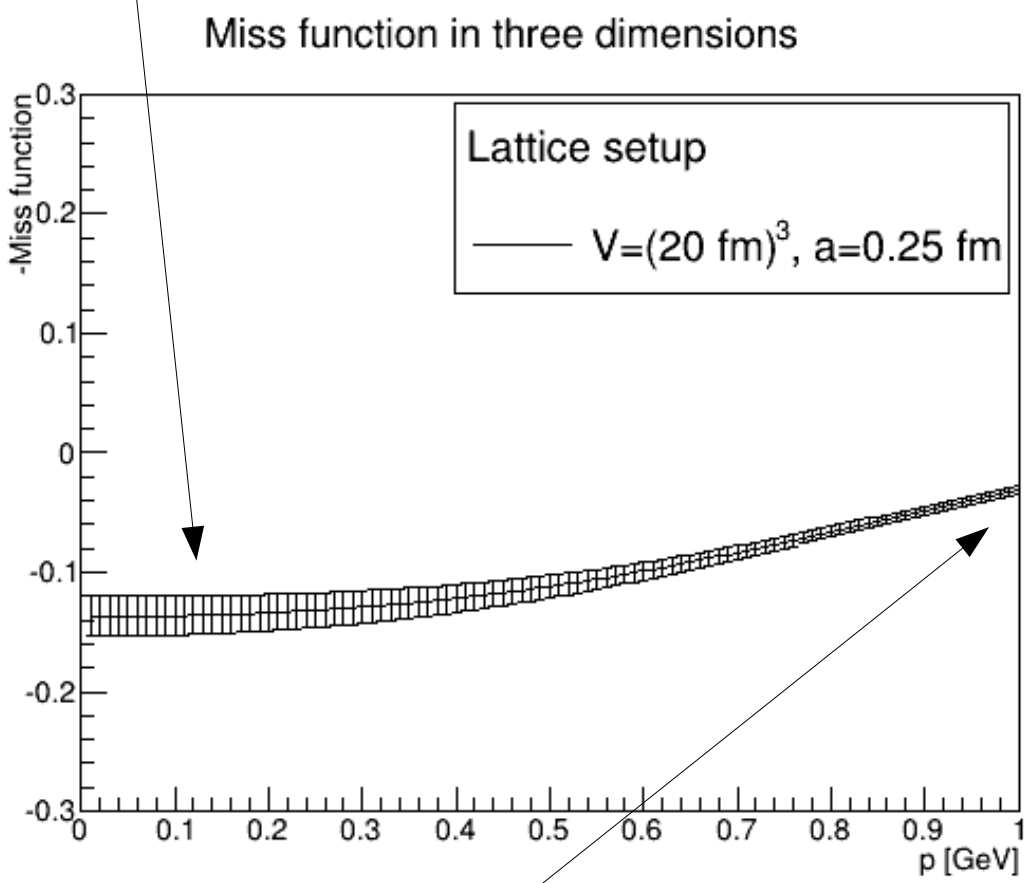

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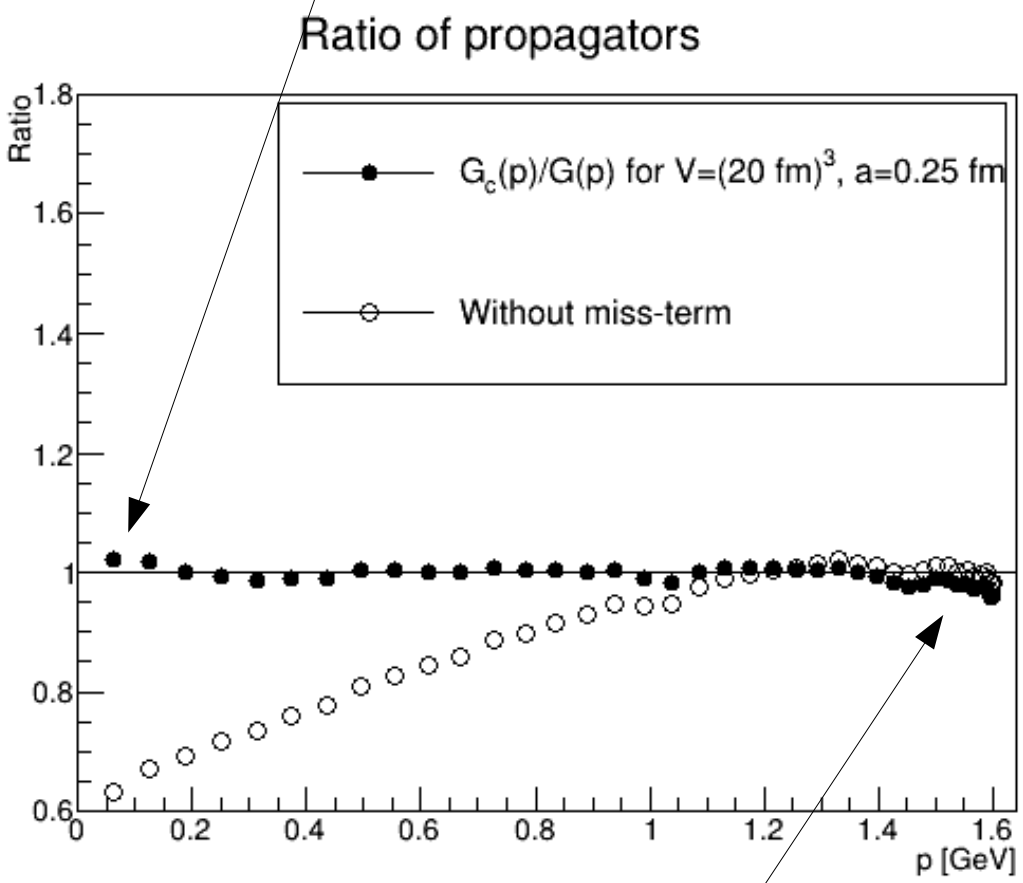

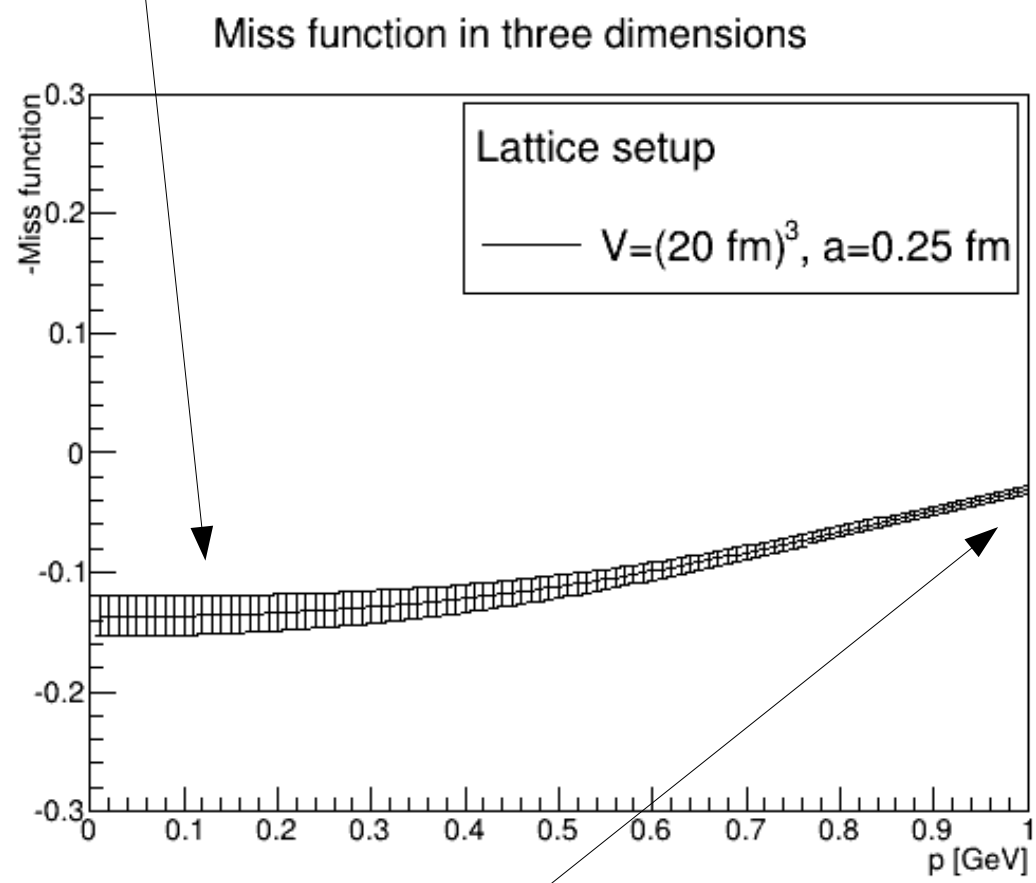

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Misfunction form:

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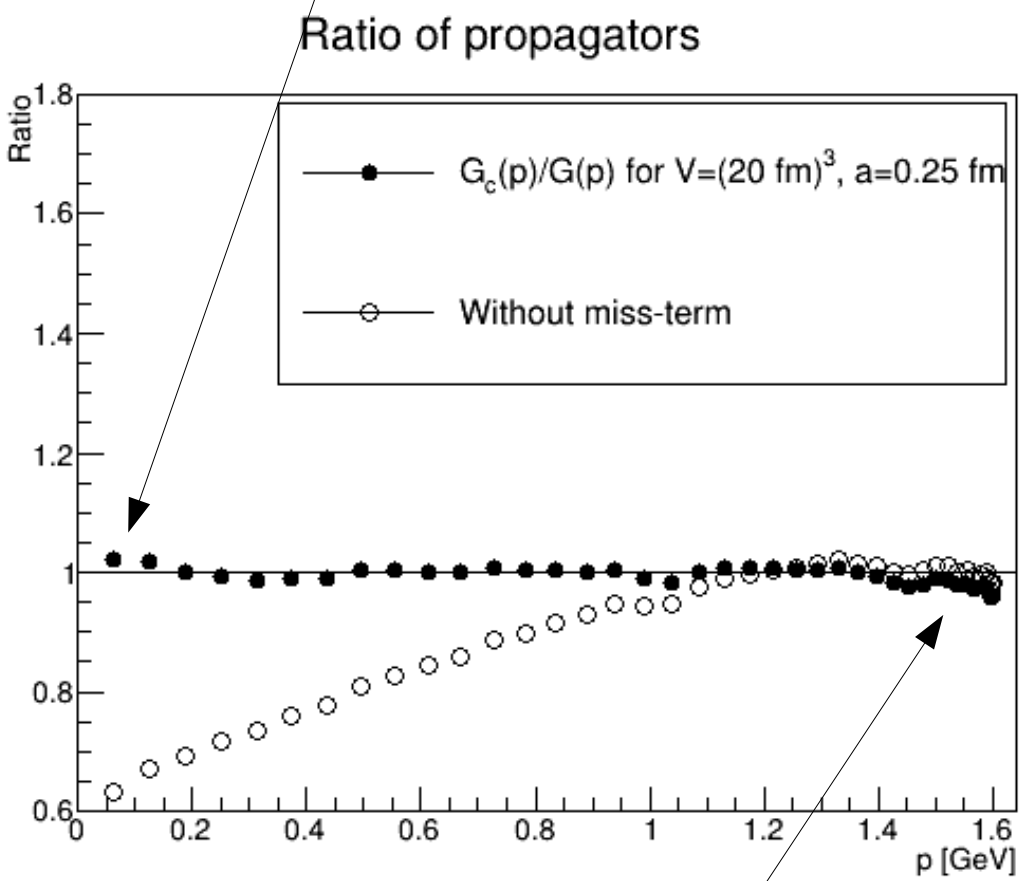

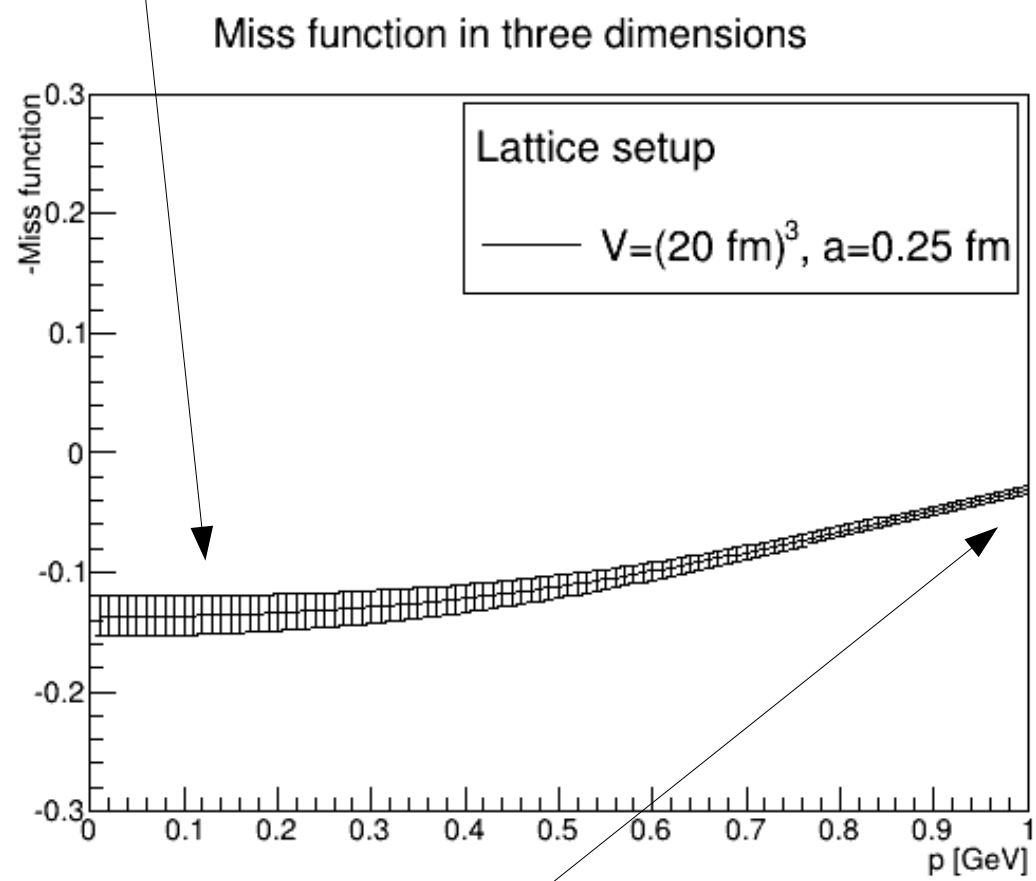

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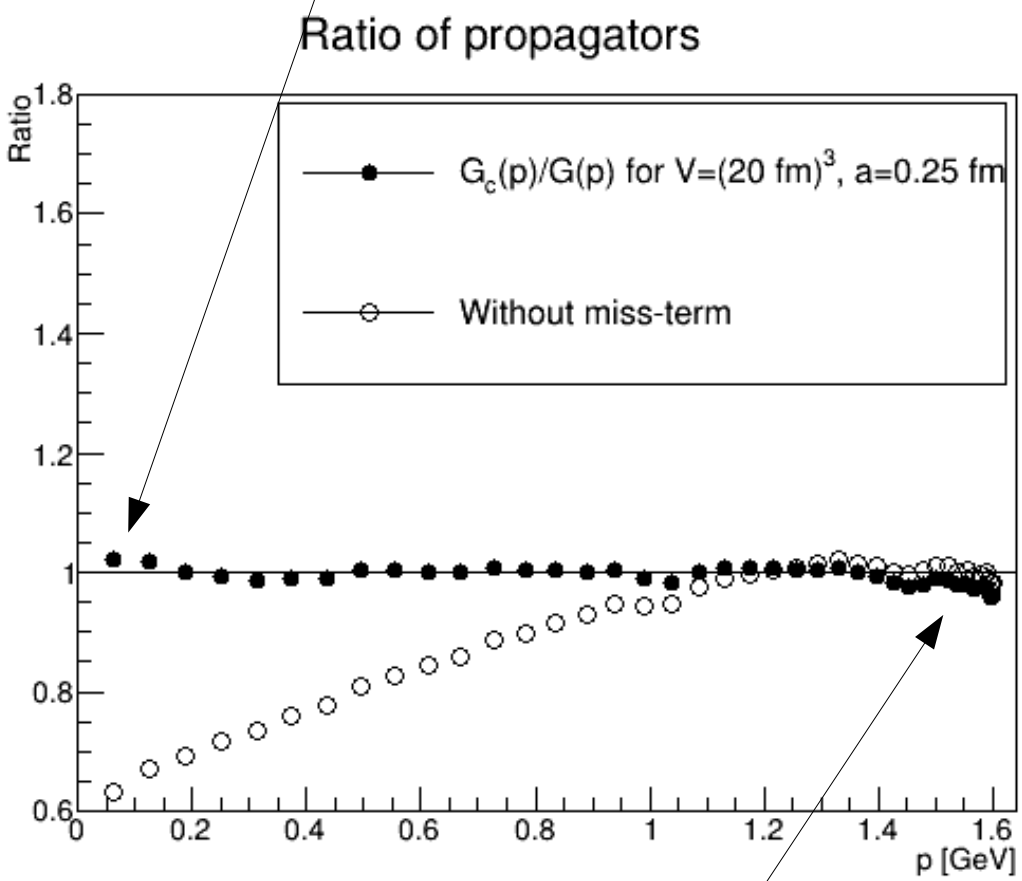
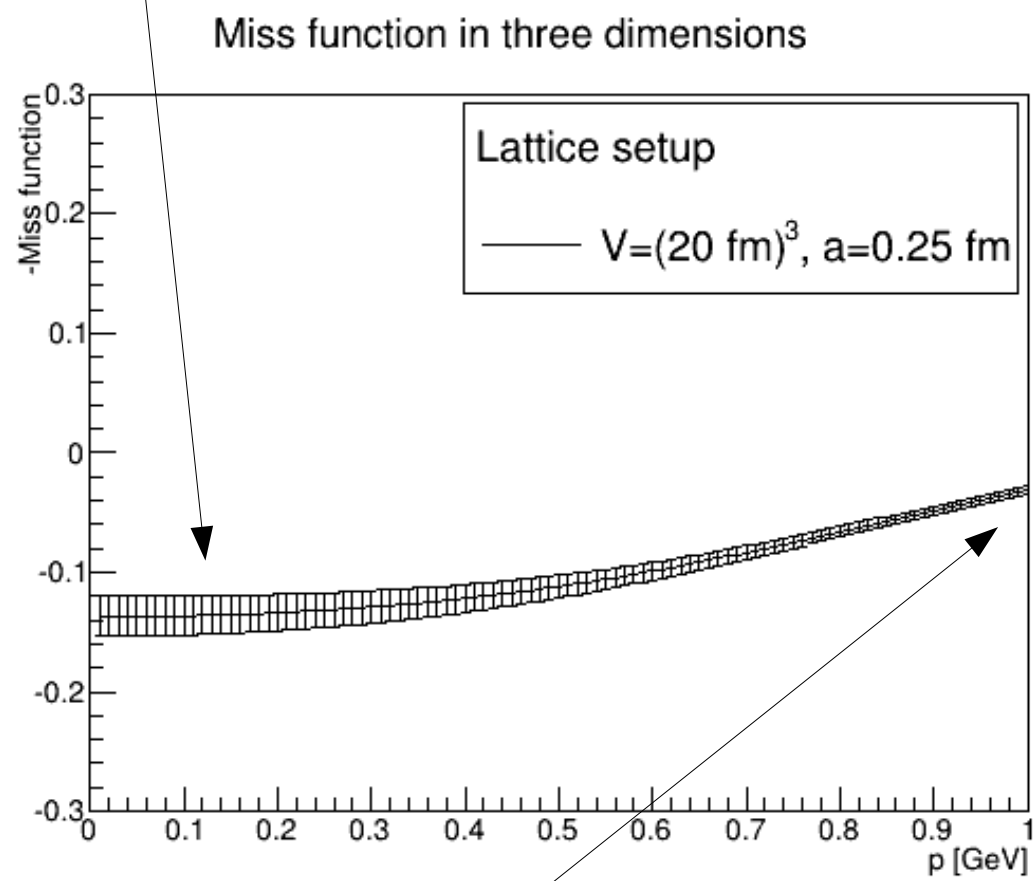
Same effect in 4 dimensions

Lattice results

[Maas'19]

Substantial disagreement at low momenta





Agreement at large momenta

Misfunction form:

$$f(p)=r\frac{1-(p/b)^e}{1+(p/c)^{2e}}$$

Same effect in 4 dimensions
...and in 2 dimensions!

[See also Maas'14]

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  - No comparability (at least in the far infrared) between lattice and continuum!
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  - But opens up more questions on results in family of Landau gauges
- The misfunction needs to be a ghost/anti-ghost-dependent term in the gauge-fixing Lagrangian!
  - Should be reconstructable/derivable somehow