Possible Substructure of an Electron

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Abstract

The standard model is one of the most successful theories in theoretical particle physics. Although the introduction of the Higgs field solved the problem of the electroweak interaction, it also introduced new problems. One of them is the fact of some observables of the theory not being gauge-invariant. This master thesis presents one possible solution to this issue. The main idea is to use the Fröhlich-Morchio-Strocchi mechanism as a basis. Essentially, it states that a bound state of the Higgs boson is gauge-invariant. In this thesis we determine the bound state of a Higgs boson and an electron. To identify the possible structure of this state, we use the parton model, a well known method within the QCD. This thesis determines cross sections of the bound states, since these can be more easily compared to data gained from experiments.

Kurzfassung

Das Standard Modell ist eine der erfolgreichsten Theorien in der theoretischen Teilchenphysik. Die Einführung des Higgsfeldes löste zwar das Problem mit den Massen in der elektroschwachen Wechselwirkung, jedoch traten hier wiederum neue Probleme auf. Eines davon ist, dass einige Observablen der Theorie nicht eichinvariant sind. Eine mögliche Lösung für dieses Problem wird in dieser Masterarbeit angeführt. Die Idee ist den Fröhlich-Morchio-Strocchi-Mechanismus als Grundlage zu verwenden. Dieser sagt aus, dass ein Bindungs-zustand des Higgs eichinvariant ist. In dieser Arbeit bestimmen wir den Bindungszustand eines Higgsbosons und eines Elektrons. Um die mögliche Struktur dieses Zustandes zu bestimmen, verwenden wir das in der QCD wohlbekannte Partonmodell. In dieser Arbeit werden die Wirkungsquerschnitte der Bindungszustände bestimmt, da diese mit den experimentellen Daten leichter vergleichbar sind.

Contents

1 Introduction

In physics, we usually gain most insights by way of observations and experiments. In particle physics, these experiments comprise various collisions of two particles at high speeds. There are many accelerators which are designed for doing exactly that. The most famous of them is the LHC in Geneve, which is currently the largest and most powerful accelerator in the world. The LHC speeds two protons up and scatters them. Although the technical realisation of this is by no means easy, the suiting theoretical description is even more difficult. Since protons consist of quarks and gluons, causing a collision of two such protons leads to a complex variety of particles.

There were other experiments, which worked with an electron and its antiparticle, the positron. These are more complicated on a technical level, but the scattering process is elementary. These new demands on the experiments also demand new accelarators. For that, two new accelerators are being planned and discussed, which promise to be even more accurate. This means they should reach higher energies than the present experiments.

However there are some indications in theoretical physics that these scattering processes are not as simple as is widely believed. For more than the last 30 years, physicists have been thinking that the structure of the Higgs boson is more complex than a simple elementary particle. Furthermore also the electron can no longer be seen as a simple particle anymore.

The idea is that the electron we can "see" is not an elementary particle anymore, instead it is a bound state of the elementary electron and the Higgs boson [8]. It is very hard to imagine this picture, especially as the Higgs has a much higher mass than the electron, which we measured. How is this possible?

In nuclear physics we already observed such an effect, for example that an atom or nuclei is lighter than the masses of its constituents. This effect is called mass defect. In the case of the bound state of an electron and a Higgs boson, this effect has to be very strong. Hence we can theoretically formulate such a bound state, so that it looks like an electron, which will be shown in this thesis later. But how can we measure such a state?

The interpretation is that by colliding two of the bound states at high energies only a collision between two components of the bound states are affected. As the electron and the positron are much lighter than the Higgs boson, they are faster and so the collision between them is much more likely. For this reason, we want to calculate the cross section and find a possible structure for such a bound state in this master thesis.

In the first section, we want to emphasize the nessecity of the bound state in the standard model. In the second section, a possible method to describe a

structure of a bound state is explained. This is the parton model, which was introduced by Feynman and Björken for the hadrons in the QCD [1]. The next two sections contain the calculations for the bound state of an electron and Higgs boson. To be able to find a reasonable characterization of this state, we need something to compare to. Here we choose the results of the perturbative theory, as this theory fits the most with the experiment.

Considering the experiment, we only have enough data from the scattering of an electron and positron into a muon and antimuon. For that reason, we fit our calculations for this process. We are also interested in other cases, for example a top and an antitop quark as a final state of the scattering and their cross sections.

The results for the top case can not be compared with experiments at this time, but hopefully the planned experiments will be sensitive enough to measure explicitly the scattering of two Higgs bosons in the case of a bound state.

2 Electroweak Interaction

As the scientific field of electroweak interaction is very broad, this section will only focus on the aspects relevant to this master thesis. This section will therefore introduce the basic theoretical concepts which this thesis is based on, namely the Glashow-Weinberg-Salam theory, the Higgs mechanism and the Fröhlich-Morchio-Strocchi mechanism. For an overall description of electroweak interaction in general, please see [1], [2].

The notation and the information for section two and three are taken from [1].

2.1 Glashow-Weinberg-Salam Theory

The Glashow-Weinberg-Salam Theory is based on an interaction which contains the weak and the electromagnetic sector. For this the gauge group is $SU(2)_W \times U(1)_Y$. This new group has a quantum number, the so-called weak isospin and the hypercharge.

In this theory, it is important to differentiate left-handed and right-handed particles, as the weak interaction takes a different effect on each of them. So there are different representations for these particles: the right-handed ones are singlets and the left-handed ones are doublets. The W boson only interacts with left-handed fermions and the Z boson acts differently on each of them, depending if they are right- or left-handed. In this theory the chiral and parity symmetry are violated. The W boson additionally violates the flavor and charge symmetry, but this is not relevant to this thesis (for more information see [1], [2]).

Another very important fact is that the Brout-Englert-Higgs mechanism is needed. This can be seen from these properties: First of all it is needed to break down the group $SU(2)_W \times U(1)_Y$ to the electroweak gauge group $U(1)$, for that we get the relation

$$eQ = e(I_w^3 + \frac{y}{2}) \tag{1}$$

Second, some of the gauge bosons have masses. Also, since they act differently on the left-handed and right-handed fermions, we cannot simply write a mass term for the fermions in the Lagrangian.

For these reasons we need a new mechanism which generates the masses, the afore-mentioned BEH mechanism. In order to give the particles their mass, the BEH mechanism spontaneously breaks the gauge symmetry of the electroweak interaction.

2.2 Higgs Sector

Experiments showed that the following properties should be fullfilled

- It should generate the masses for the quarks and leptons.
- It must generate the masses of the Z and W bosons.
- It must satisfy the relation $m_W = m_Z cos\theta_W$

The simplest field, which fulfills these, is a two doublet field. As the Higgs field ϕ is a real four component vector, we have a $O(4)$ symmetry, which can be split into $SU(2) \times SU(2)$. The $SU(2)_R$ is the global symmetry and is also called the custodial symmetry, which remains unbroken. The custodial symmetry is important as its breaking by the QED is responsible for the mass of the Z boson not being the same as the mass of the W boson [22].

The BEH mechanism only acts on the gauged sector so only the $SU(2)_L$ is affected. The $SU(2)_L$ will be broken spontaneously, which is often called hiding the symmetry. This breaking or hiding is done by choosing a vacuum expectation value for the Higgs field [1].

Since the gauge symmetry U(1) is manifested here, the photon must be massless and the mixing of operators of the group $SU(2)_W$ and $U(1)_Y$ guarantees that even if there is no gauge symmetry manifested, every expectation, similar to a manifested electromagnetic symmetry, is met in the end.

Unfortunately, this discussion can only be used when working in a gauge, which prefers one direction, for example the unitary gauge. Excluding the gauge invariant observables, for example the cross section, different approaches have to be taken in any other gauge.

2.3 Fröhlich-Morchio-Strocchi Mechanism

One promising approach to finding a gauge invariant description is the Fröhlich-Morchio-Strocchi mechanism [8], [9]. As already mentioned, the electroweak symmetry breaking is only hiding the symmetry by the gauge choice. The actual symmetry can never be broken [21]. This means, the standard method of the standard model, the perturbation theory, is only possible in suitable gauges [13]. Every gauge that singles out a direction will lead to the same result as above, for example the 't Hooft gauge ($\xi = 1$) which will be used in the further paper.

The Higgs, $W^{\pm}$ and Z^0 bosons are carrying the weak charge, a non-abelian charge, which means it is gauge variant and therefore not observable. Therefore, the question remains, what did we measure?

The Fröhlich-Morchio-Strocchi mechanism provides a possible answer. The main idea is to define gauge invariant operators. For a non-abelian theory, as in this case these are bound states which can be classified by global quantum numbers, for example the spin or the custodial structure.

In the standard model, only the left-handed leptons and neutrinos carry the weak charge, as a result they are not gauge invariant. The theory of Fröhlich, Morchio and Strocchi observes the gauge invariant composite operators

$$O_{1/2}(x) = \phi_i(x)\psi_i(x) \tag{2}$$

with ϕ being the Higgs field and ψ_i standing for the left-handed fermion. As the Higgs field is a scalar, the bound state still has the spin 1/2. Now the expansion of the correlation function looks like this:

$$\langle(\phi_i(y)\psi_i(y))^\dagger\phi_j(x)\psi_j(x)\rangle \sim v^2\langle\psi_i(y)^\dagger\psi_i(x)\rangle + \mathcal{O}(\frac{\eta}{v}) \tag{3}$$

This function can now be developed in perturbation theory. By doing so, one gets a propagator and a scattering continuum, which is unimportant here. With the propagator we can calculate the poles, which tell us the mass of the bound state. The mass of the bound state is the same as for the fermion [8], [9].

Additionally, it should be mentioned that the flavor symmetry does not exist in this case, but the custodial symmetry preserves the multiplet structure of our theory.

So a bound state with the Higgs has the same quantum numbers as the elementary fermion and is therefore gauge invariant.

3 Parton Model

The parton model has been developed for the quantum chromodynamic by Feynman and Bjorken. This model describes a way to calculate the cross section for a hadron scattering. The hadron consists of constituents, which are called the partons. These partons are quarks as well as gluons. If we now take a look at the scattering, only one of these partons scatters. So in order to get the whole cross section, one has to sum over all cross sections of the partons.

We want the total cross section for the hadron scattering, for that reason we have to factorise our process. In this case here, we look at the constituents and then sum over all possible processes. There are also other methods for factorisation which are described in [2], [14], [15].

The notation of this section and the information are taken from the book "An introduction to Quantum Field Theory" [1].

3.1 An Example

Take for example a scattering process with one proton and one electron. The electron strikes a quark out of the proton, which sets the quark free from the proton. The rest of the hadron transforms into a jet of hadrons, which is collinear to the stricken quark.

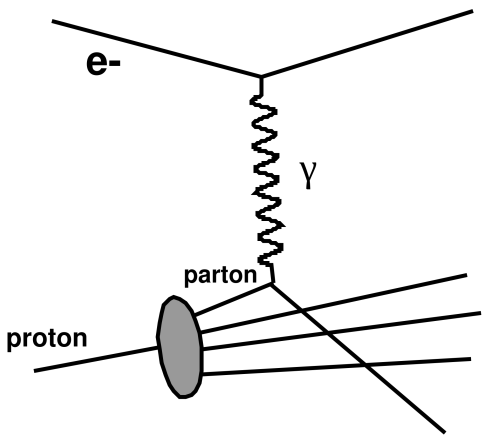


Figure 1: Scattering process of an electron and a proton [18]

As you can see in figure 1, all this can be described by the cross section of the electron and, for example, one of the quarks of the proton.

This parton only has a part of the momentum of the proton. As a result the momentum of the parton p_1 is x*P, where P is the momentum of the proton and x is the longitudinal fraction of the momentum. We can now take the cross section of an electron-muon scattering since only the coupling constants change. Additionally, the mass can be neglected here as we only look at high energy. As a consequence we have the differential cross section

$$\frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha^2 Q_i^2}{\hat{s}^2}\left(\frac{\hat{s}^2 + (\hat{s}+\hat{t})^2}{\hat{t}^2}\right) \tag{4}$$

where Q_i is the electric charge of the quark. In order to get the cross section, we have to express the Mandelstam variables in terms of measurable quantities, in other words, in quantities of the proton.

The momentum transfer q of this collision can be measured. For that we only need the final momentum and the energy of the electron. As the momentum transfer q^μ is a spacelike vector, we can define the quantity Q

$$Q^2 \equiv -q^2 \tag{5}$$

For that the variable $\hat{t}$ is simply $-Q^2$ [1]. As has already been mentioned, the momentum of the parton can be given by the longitudinal fraction and the momentum of the proton. There also is a function $f_i(x)$ for each parton of the type i, which gives the probability of striking this parton in the proton. These functions are called parton distribution functions and they will be discussed in more detail later on.

For a better understanding, let us take a look at an example in figure 1 [1]. We have a collision of an electron and a proton, where actually only one quark or gluon takes part. For the scattering of an electron and a quark, we can take the cross section for the electron-muon scattering as this case is related to our example, because we can also take a look at the massless limit. So for our cross section, we get

$$\frac{d\sigma}{d\hat{t}}(e^- q \to e^- q) = \frac{2\pi\alpha^2 Q_f^2}{\hat{s}^2}[\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}] \tag{6}$$

The variables $\hat{s}, \hat{t}$ and $\hat{u}$ are the so-called Mandelstam variables for the center of mass system of the scattering process at proton level. These variables must fulfill the condition

$$u + t + s = \sum_i m_i = 0 \tag{7}$$

Consequently we can write $u = -(t+s)$, so that the cross section only depends on the variables $\hat{s}$ and $\hat{t}$. As already mentioned, these variables

are described at the parton center of mass system and therefore must be rewritten in a form, which depends on the observales of the hadronic system.

As the momentum of the exchange particle q is a space-like vector and can be measured by the energy and the final momentum of the electron, the quantity of Q stays invariant. Then the variable $\hat{t}$ is $-Q^2$. For the variable $\hat{s}$, we look at the definition of this variable.

$$\hat{s} = (p+k)^2 = 2pk = 2xPk = xs \tag{8}$$

it follows that the total cross section for this scattering process looks like this:

$$\frac{d^2\sigma}{dxdQ^2} = \sum_i f_i(x) Q_i^2 \frac{2\pi\alpha^2}{Q^4}[1 + (1 - \frac{Q^2}{xs})^2] \tag{9}$$

The function $f_i(x)$ depends on the structure of the state. Finding them is no easy task.

3.2 Parton Distribution Function

Take for instance our example from above. For the quarks the parton distribution functions as shown in picture 2, they have a great impact by small values for x.

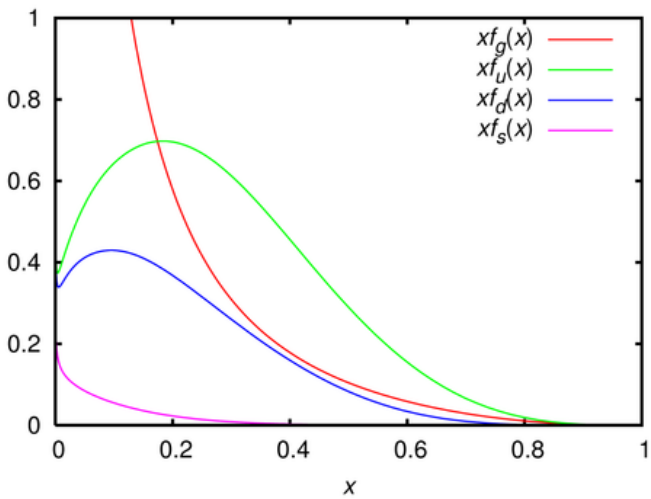


Figure 2: Parton Distribution Functions for quarks and gluons [19]

As already mentioned, the parton distribution functions tell us the probability of finding the constituent of the type i with the longitudinal fraction x. These functions depend on the structure of the bound state and therefore

can not be calculated. As for the QCD, these functions were extracted from the experiment. The parton distribution functions have to be normalized. This leads us to the sum rules.

$$\int_0^1 dxx \sum_i f_i(x) = 1 \tag{10}$$

The sum over all momenta of the partons must be the total momentum of the hadron. This must also be true for the charge of the partons.

$$\int_0^1 dx \sum_i f_i(x) = 1 \tag{11}$$

The parton distribution functions are not easy to compute as there is no description of how to do it. The functions from figure 2 are extracted from data of the deep inelastic scattering. For more details see [1].

3.3 Analogy to Fermion-Higgs Bound State

The idea here is to take the concept of the Fröhlich-Morchio-Strocchi mechanism and look at bound states for elementary particles. As already shown in section 1, a bound state of an electron and a Higgs "looks" like an electron. We want to describe the structure for this state in this master thesis. In order to do so we need a method to specify a possible structure, for this reason we adopt the parton model of the quantum chromodynamic. But contrary to the example above for the parton distribution functions, we choose the form for our functions.

So one schematic scattering process for our case looks like the figure 3

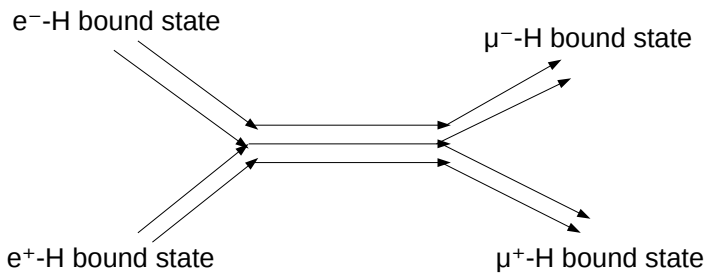


Figure 3: A schematic scattering process for the bound state

We assumed here that the final states are either bound states of a Higgs and a muon or the electron has the most energy and interacts softly. This is called fragmentation.

In the following subsection, the cross sections will be calculated and possible parton distribution functions will be discussed.

4 Cross Section

In this chapter, the elementary cross sections at tree level for the partons will be calculated and discussed. Later on, the transformation from the center of mass system of the parton to the one of the bound state will be shown. In the last part, the Breit-Wigner propagator will be explained since it is a necessity in the next chapter.

Before that, the notation should be defined.

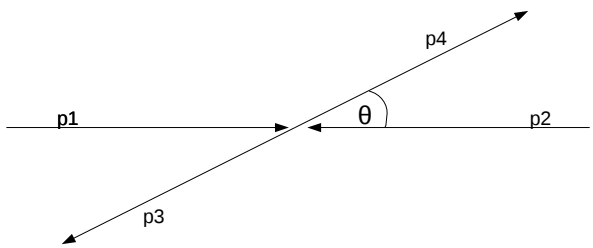


Figure 4: A schematic process with all parameters

As can be seen in figure 4, the incoming particles have the momenta p1 and p2, whereas the outgoing momenta are notated with p3 and p4. The scattering angle θ is defined between the momenta p1 and p3 and we notate the momentum of the exchange particle to be k. Also, the four component column spinors are defined by u and v and the row one by $\bar{u}$ and $\bar{v}$. The spinor u stands for the particle and v for the anti-particle. These spinors further have indices $s1, s2, s3$ and $s4$, which denote the spin state. Also we denote the $sin\theta_W$ with s and the $cos\theta_W$ with c, and θ_W stands for the Weinberg angle.

The notation of this section and the information are taken from [2].

4.1 Electron Cross Section

We are interested in the differential cross section of the scattering process of an electron and a positron into a muon and an antimuon. For the differential cross section in dependence of the energy and the rapidity, which is defined as [1]

$$\eta = -ln\left(tan\left(\frac{\theta}{2}\right)\right) \tag{12}$$

We need the invariant matrix element M, which is the sum over all possible Feynman diagrams. In this case, we have four possible scatterings with different exchange bosons.

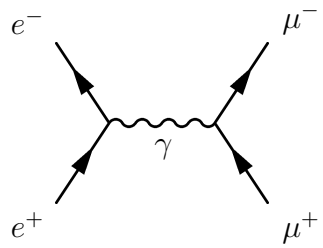


Figure 5: Feynman diagram with the photon γ

One of them has the photon as the exchange boson, as seen in the Feynman diagram in figure 5. This process is the simplest. The matrix element S_γ looks like

$$M_\gamma = \bar{v}(p2)^{s2}(-ie\gamma^\mu)u(p1)^{s1}D_{\mu\nu}\bar{u}(p3)^{s3}(-ie\gamma^\nu)v(p4)^{s4}$$

with ξ being the gauge parameter and the propagator $D_{\mu\nu}$

$$D_{\mu\nu} = \frac{-ig_{\mu\nu}}{k^2} + \frac{i(1-\xi)k_\mu k_\nu}{k^4}$$

As the photon has no transversal momentum, as a result the term $k_\mu k_\nu$ always produces zero.

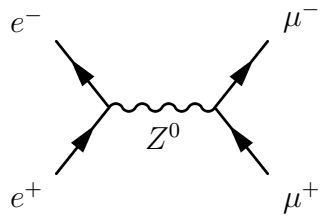


Figure 6: Feynman diagram with the Z boson

Furthermore there is a diagram with the Z boson as you can see in figure 6, with the matrix element M_Z

$$M_Z = \bar{v}(p2)^{s2}C^\mu u(p1)^{s1}D_{\mu\nu}\bar{u}(p3)^{s3}C^\nu v(p4)^{s4}$$

with the coupling C^μ

$$C^\mu = \left(ie\gamma^\mu\left(\frac{(I^3_{W,f} - s^2Q_f)(1-\gamma_5)}{2sc} - \frac{sQ_f(1-\gamma_5)}{2c}\right)\right)$$

where f stands for the type of fermion, which interacts with the boson, and also the propagator $D_{\mu\nu}$

$$D_{\mu\nu} = \frac{-ig_{\mu\nu}}{k^2 - M_Z^2} + \frac{i(1-\xi)k_\mu k_\nu}{(k^2 - M_Z^2)(k^2 - \xi M_Z^2)}$$

Here, the term $k_\mu k_\nu$ does not disappear, therefore this term is gauge variant. This is not fatal as the matrix element M has to be invariant. So there has to be another term, which will cancel the gauge dependant out of the sum.

There are two Feynman diagrams with a Higgs as exchange particle.

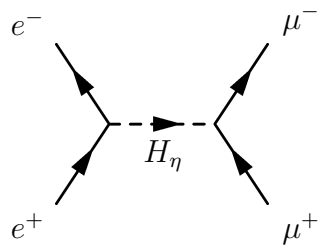


Figure 7: Feynman diagram with Higgs boson H_η

On the one hand the real Higgs H_η in figure 7 with the matrix element M_η

$$M_\eta = c\bar{v}(p2)^{s2}u(p1)^{s1}D_{\mu\nu}\bar{u}(p3)^{s3}v(p4)^{s4}$$

with the constant c

$$c = \left(-ie\frac{1}{2s}^2 \frac{M_e}{M_W}\frac{M_\mu}{M_W}\right) \tag{13}$$

and the propagator

$$D_{\mu\nu} = \frac{i}{k^2 - M_\eta^2} \tag{14}$$

This term is gauge invariant in perturbative theory.

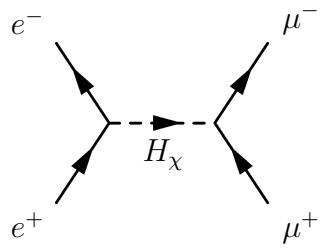


Figure 8: Feynman diagram with Higgs boson H_χ

On the other hand, we have a diagram with the imaginary Higgs particle H_χ, as can be seen in figure 8

$$M_\chi = c\bar{v}(p2)^{s2}(\gamma_5)u(p1)^{s1}D_{\mu\nu}\bar{u}(p3)^{s3}(\gamma_5)v(p4)^{s4}$$

with the constant

$$c = \left(-e\frac{I^3_{W,e}}{2s}\frac{M_e}{M_W}\frac{I^3_{W,\mu}}{2s}\frac{M_\mu}{M_W}\right) \tag{15}$$

and the propagator

$$D_{\mu\nu} = \frac{i}{k^2 - \xi M_\eta^2} \tag{16}$$

Comparable to the term for the Z boson, this is gauge dependent. The sum over this term is gauge invariant. The calculation for the term $|M|^2$ was done with the program FORM [11]. Therefore, it will not be elaborated in detail. It should be mentioned that the cross section is calculated with the mandelstam variable. Here the variable s is the energy of the center of mass system.

Now we take a look at how the cross section behaves differently when applying various values for the variables it depends on, like the energy $\sqrt{s}$ or the rapidity η. The cross section seen in figure 9 drops with one over s, but

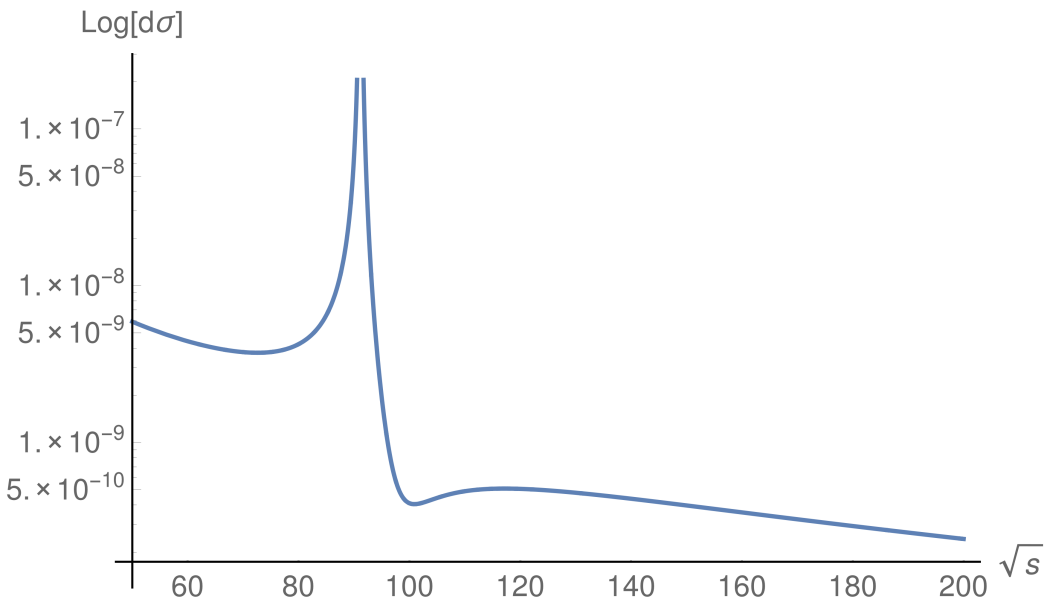


Figure 9: The cross section over the energy $\sqrt{s}$ with $\eta = 0$

near the mass of the Z boson and the Higgs boson the cross section goes to

infinity. This behavior is a result of the propagator, which has the form

$$\frac{1}{s - M^2} \tag{17}$$

Then at an energy equal to the mass of the boson there is a singularity. This singularity is not integrable, thus there will be a problem for the case of the parton as we have to integrate over the longitudinal fraction x. This problem and the solution for that will be discussed later in this section.

If the energy is constant and the rapidity for the energy $\sqrt{s}$ 40GeV is variated, we get a cross section of the following form

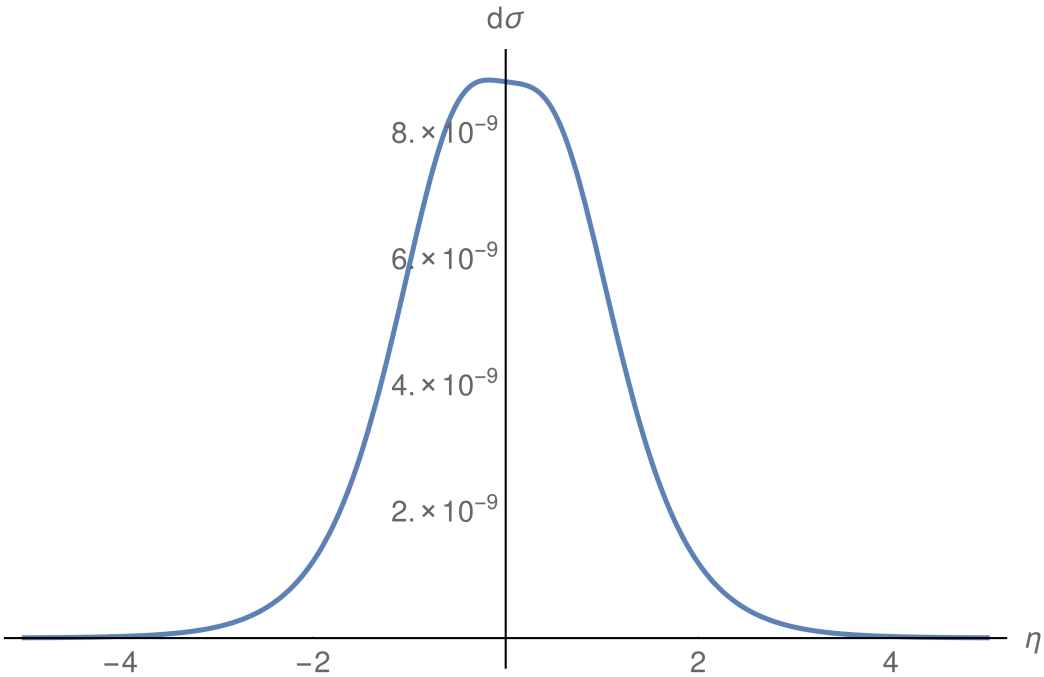


Figure 10: The cross section over the rapidity η

For a better discussion of the form of this cross section in figure 10, the cross sections for each possible process are also shown. In order to get these cross sections we choose the 't Hooft gauge ($\xi = 1$).

As a result of that, we can see that the two Higgs boson processes seen in figure 12 are very small due to the small coupling of the electron to the Higgs boson. The coupling goes for both cases with the term M_e/M_W, which is roughly 10^{-6}.

Then the dominant term in the cross section is the photon process, seen in figure 11. Near the Z boson mass the Z boson process is dominating. Looking closer at the case with the Z boson, there is a slant in the cross

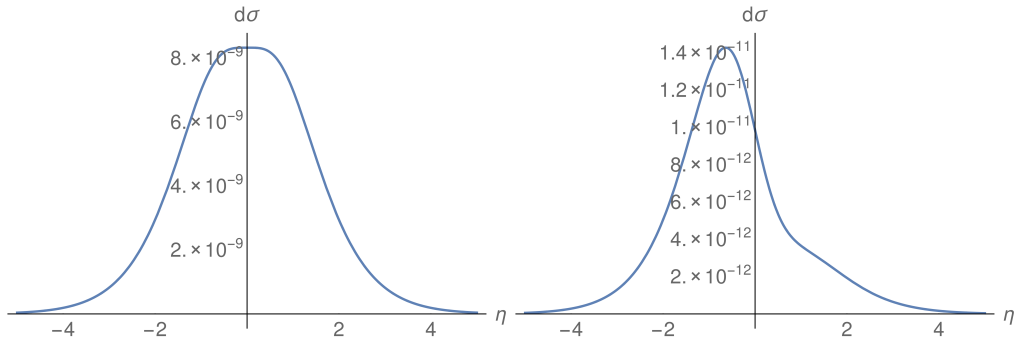


Figure 11: The cross section of the first two possible scattering processes

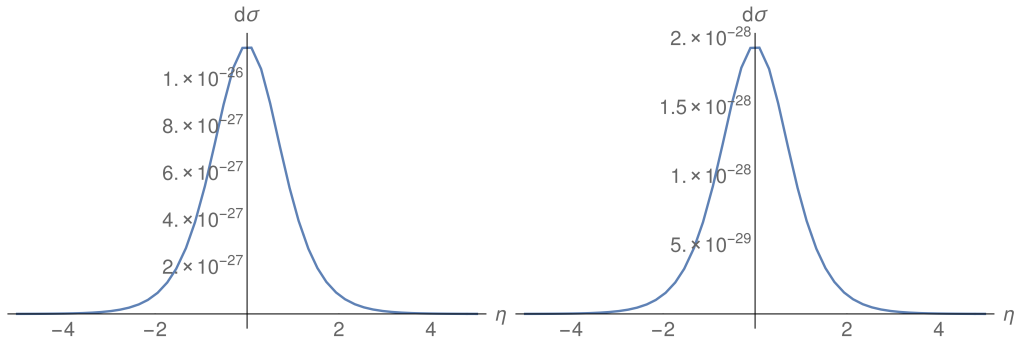


Figure 12: The cross section of the last two possible scattering processes

section. On that account there is a prefered scattering angle. In the cross section for the Z boson, there is a $cos\theta$ term, which is charge asymmetric and is typical for the weak interaction [2].

4.1.1 Higgs Boson Cross Section

As for the scattering process of the two Higgs bosons as inital states, the invariant matrix element M is easier than the case above. There is only one process since two Higgs can only scatter into the elementary Higgs boson H_η so that the final states would be muons shown in figure 13.

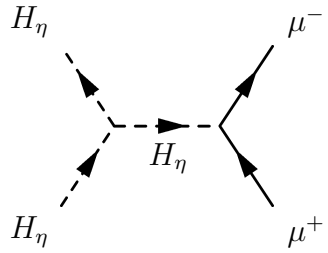


Figure 13: Feynman diagram for the Higgs boson cross section

The external line for the Higgs boson is one. The form of the matrix element is

$$M = -\frac{3}{2s}\frac{M_\eta^2}{M_W}\frac{i}{k^2 - M_\eta^2}\bar{u}(p3)^{s3}\left(-\frac{1}{2s}\frac{M_\mu}{M_W}\right)v(p4)^{s4}$$

As we only look at on-shell particles, the cross section for the Higgs is not zero over an energy $\sqrt{s}$ 250GeV and additionally it drops with 1 over s shown in figure 14.

To compare the two cross sections as in figure 15, lets take a look at the cross sections for the electron and the Higgs boson case at the energy $\sqrt{s}$ equals 251 GeV.

The cross section of the two Higgs boson scatterings is very small compared to the one of the electron and positron, when the final states are muons. If the final states are changed to top and anti-top, the case looks different.

Here, the Higgs cross section is dominant, as you can see in figure 16. The coupling of the Higgs to the top is very strong, roughly 1600 times stronger than in the case of the muons.

The electron cross section also has processes with the Higgs boson, but on the other hand it also couples at the electron, as has already been shown. Thus, the Higgs parts are supressed in the cross section of the electron and positron scattering.

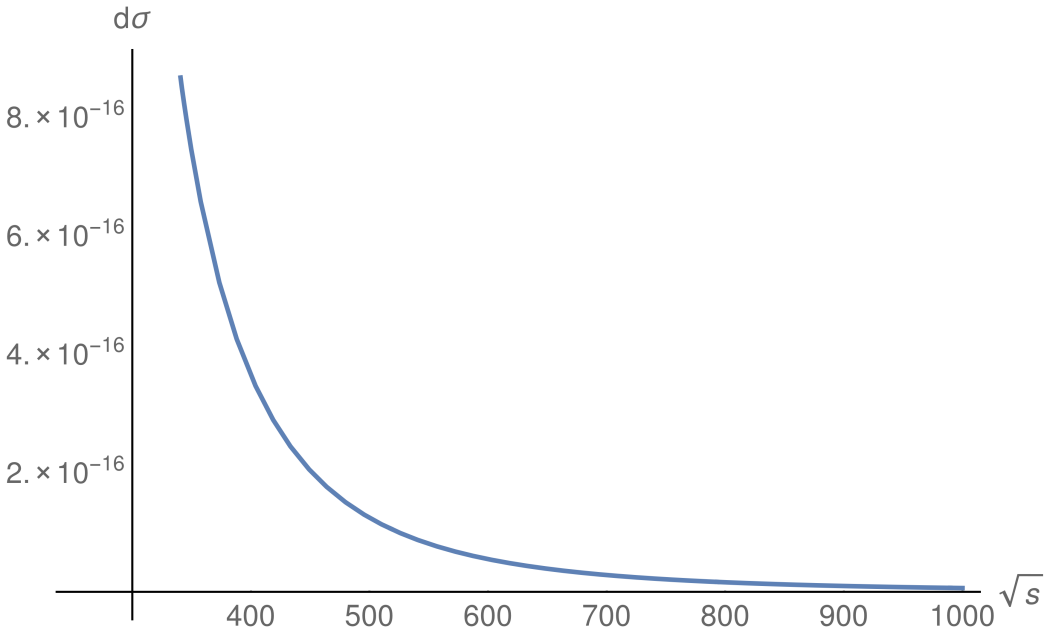


Figure 14: The cross section for the Higgs boson over the energy $\sqrt{s}$

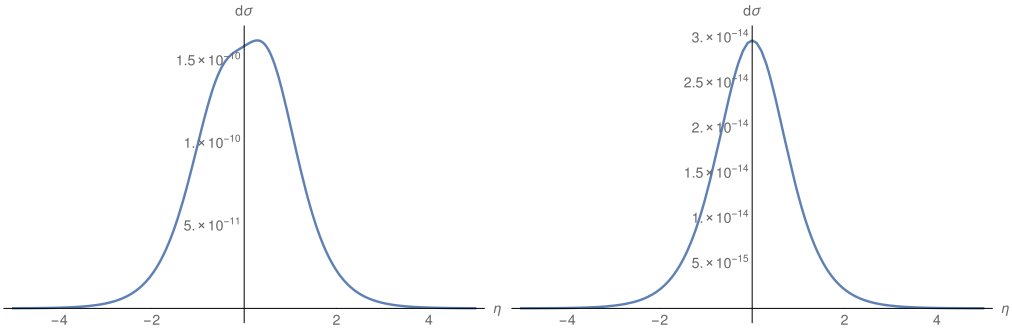


Figure 15: The first one is for the electron and the second one for the Higgs. Both cross sections have muons as final states

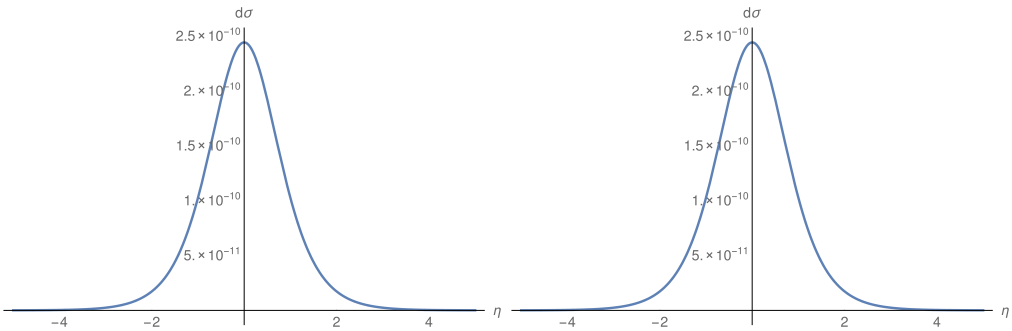


Figure 16: The first one is for the electron and the second one for the Higgs. Both cross sections have top and anti-top as final states

4.2 Transformation to Bound State System

For the cross section as defined in section 3 [1],

$$\frac{d\sigma}{dsd\eta} = \int_0^1 dx \sum_i f_i(x) \left(\frac{d\sigma}{dsd\eta}\right)_i \tag{18}$$

the elementary cross section $\left(\frac{d\sigma}{dsd\eta}\right)_i$ has to be rewritten so that the parameters are defined in the properties of the bound state. This has already been shown in section three, but for a simple case with only one parton and massless particles. For that reason, the transformation for our case will be shown here again [1].

Here we have two bound states, which collide, so there are two partons. One from the first bound state with the longitudinal fraction x and the other parton from the second bound state with the longitudinal fraction y. The

two inital bound states are labeled with the indices 1 and 2 and the two partons with A and B. The variables with a ' denote the quantities in our parton system and those without a ' in the bound state system. The two momenta of the partons can be written as $p_A = x * p_1$ and $p_B = y * p_2$.

First we want to transform the mandelstam variable s'

$$\begin{aligned} s' &= (p_A + p_B)^2 = p_A^2 + p_B^2 + 2p_A * p_B \\ &= m_A^2 + m_B^2 + 2xE_1yE_2 - 2\sqrt{(xE_1)^2 - m_A^2}\sqrt{(yE_2)^2 - m_B^2}cos\theta_{12} \end{aligned} \tag{19}$$

In the center of mass system of the bound state, the two energies E_1 and E_2 are both $\frac{1}{2}E_{cm}$ and the angle $\theta_{12} = \pi$. Since we only look at scatterings of two similar particles, the masses m_A and m_B are the same, so s' has the form

$$s' = 2m_A^2 + 2xy\left(\frac{E_{cm}}{2}\right)^2 + 2\sqrt{\left(x\frac{E_{cm}}{2}\right)^2 - m_A^2}\sqrt{\left(y\frac{E_{cm}}{2}\right)^2 - m_A^2} \tag{20}$$

The next Mandelstam variable is t'. Due to the fact that t' only depends on the energy of the system, which is here the variable s' and the rapidity η_{AB} and we already transformed s', we only need to look at the transformation for the rapidity. The rapidity transforms by a shift, so we only have to calculate the radipity of, for example, the virtual photon η_{12} and add it to the rapidity η_{AB}, which is defined as [1]

$$q^0 = M * cosh\eta_{12} \tag{21}$$

where $M^2 = q^2$ and q is the momentum of the photon.

$$q = xp_1 + yp_2 \tag{22}$$

As a result, the rapidity η_{12} is

$$cosh\eta_{12} = \frac{q^0}{M} = \frac{(x+y)\frac{E_{cm}}{2}}{\sqrt{s'}} \tag{23}$$

We know that a sign of this radipity should depend on the longitudinal fractions x and y. For example, when x is larger than y, the momentum of the parton A is stronger and therefore the rapidity eta_{12} should shift the virtual photon closer to the parton B. But the equation (23) does not reflect this behavior, so we multiply this equation with a sign[x-y] to get this behavior.

The last Mandelstam variable u' is calculated with

$$u' = 2m_A + 2m_E - s' - t' \tag{24}$$

with m_E being the mass of the final state.

With these transformations, the cross sections are finished and can be used to calculate the total cross section of the bound state scattering in the parton model.

4.3 Breit-Wigner Propagator

After rewriting the cross section in the bound state system, we can see that the propagators now have the form

$$\frac{1}{(xys - M^2)} \tag{25}$$

Since we want to calculate the total cross section of bound states scattering, we have to integrate over the longitudinal fraction x, as shown in section 3. For this integration, the propagator has a non integrable singularity. This is the behavior of an unstable particle [1]. In order to be able to calculate the cross section, we need to expanded the propagator by an imaginary part. Hence we get

$$\frac{1}{xys - M^2 + iM\Gamma} \tag{26}$$

This is the so-called Breit-Wigner propagator [16] and Γ represents the width of the boson. With this expansion, the cross section is now integrable.

To show the difference between these two propagators, we plotted the rate between them.

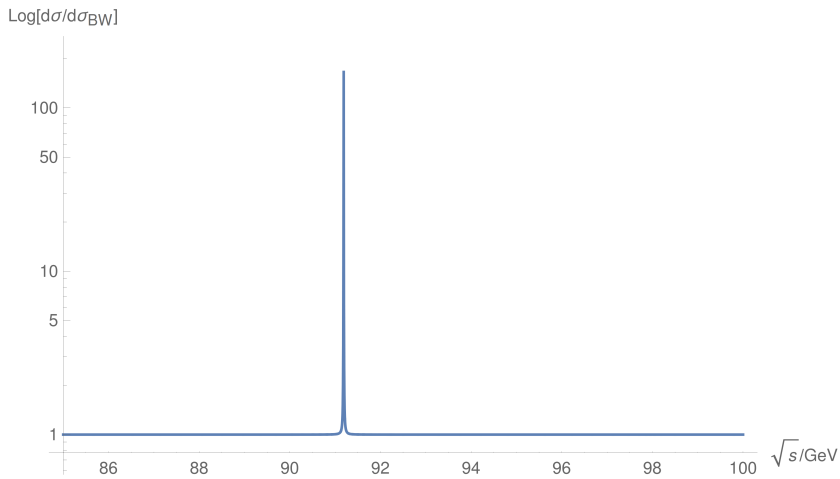


Figure 17: Ratio between the two cross sections

As seen in figure 17, the imaginary part of the propagator only makes a big difference near the boson mass. In addition, the cross section does not go to infinity at these energies anymore.

So we expand our propagators with the particular width Γ_Z=2.4952GeV and Γ_{H^0}=4MeV [10].

5 Parton Distribution Functions

First of all, we define the sum rules for the bound state. One of the sum rules represents the conservation of charge. Accordingly, the sum rule looks as follows:

$$\int_0^1 dx f_e[x] = 1 \tag{27}$$

where $f_e[x]$ is the parton distribution function for the electron since only the electron carries a charge. The second sum rule expresses the conservation of the momentum and therefore is

$$\int_0^1 dx x[f_e[x] + f_H[x] = 1 \tag{28}$$

with $f_H[x]$ being the parton distribution function of the Higgs boson. What we seek to find is a description which fits the experimental data. As mentioned before, the perturbation theory depicts this data perfectly. We will compare the cross section of the bound state with the sum over the cross section of the elementary ones.

To do so, we first choose a form for our functions. Starting with the form shown in figure 18.

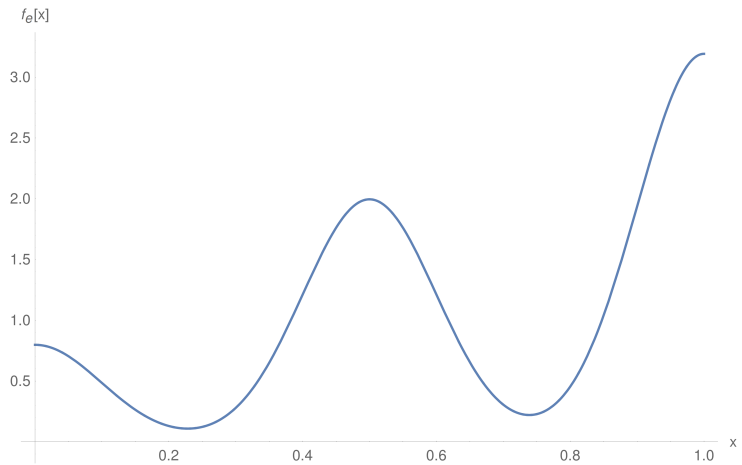


Figure 18: A possible form of the parton distribution function of an electron

The parton distribution function has three peaks. The first one is situated at zero, which means we always have a bound state. The next one lies by half of the energy. Therefore, the two partons are treated equally. The last one is at x=1, which means all the energy is in this parton. So we have a

parton distribution function of the form

$$f[x] = \frac{1}{\sqrt{2\pi w^2}}(ae^{-\frac{x^2}{2w^2}} + be^{-\frac{(x-0.5)^2}{2w^2}} + ce^{-\frac{(x-1)^2}{2w^2}}) \tag{29}$$

where w is the width of the gauss curve and a, b and c are the parameters which will be variated. For the Higgs boson the parameters are a=d, b=f and c=g.

We start with the parton distribution function for the electron and try to find a form which satisfies the sum rule and provides the same cross section. As we look at on-shell particles, the parton distribution function also has to depend on the energy, so that under the energy of the Higgs mass only the electron has a non-zero function. This function has to be a δ-function by one so that the same result as for the pertubative cross section can be achieved.

As the cross section for the Higgs boson is so small compared to the one for the electron, we only paid attention that the cross section for the parton case with the electron fits the one for the elementary electron. In the second subsection, where the parton distribution function for the Higgs is determined, we look at the ratio of the total case.

5.1 PDF for Electron and Positron

First of all we just want to look at the peak at zero, so we set the parameters b and c to zero. To satisfy the sum rule (27) the parameter a has to be 2. Now we take a look at the rate of the cross section of the parton model and the cross section of the pertubative theory.

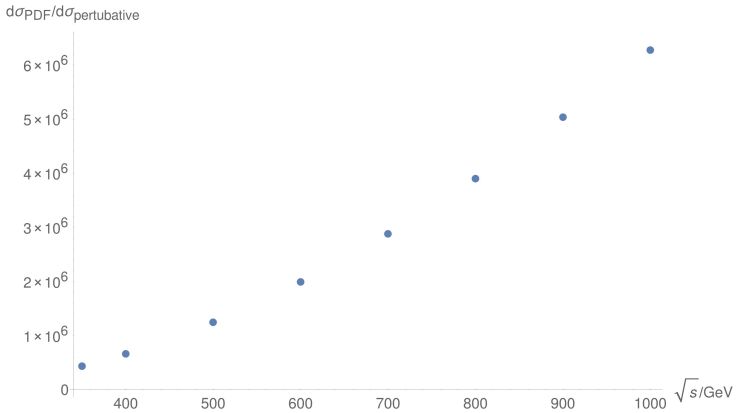


Figure 19: The ratio of the two cross sections with a=2

From figure 19 we can see that the cross section for the parton case is too large. So the next idea is to choose a smaller case. By doing so the sum rule (27) is not satisfied, but a possible way to fix that is to add one of the other peaks. But first we wanted to look at what happens with the cross section for the value a=0.01.

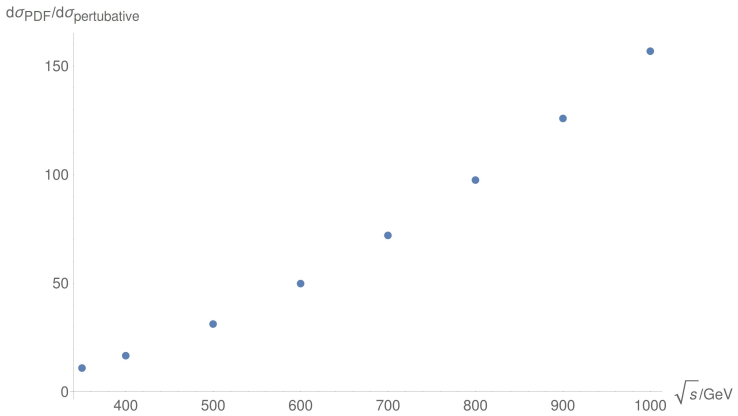


Figure 20: The ratio of parton model and perturbative theory with a=0.01

We can see in figure 20 that there is no longer such a considerable difference. Even for such a small parameter, the cross section with this parton distribution function is too large to fit. From this we can learn that there cannot be a peak at zero. Now we try the same thing for the peak at 0.5.

The parameter b must be 1 in order to get the correct result for the sum rule (27) and therefore the ratio looks like this:

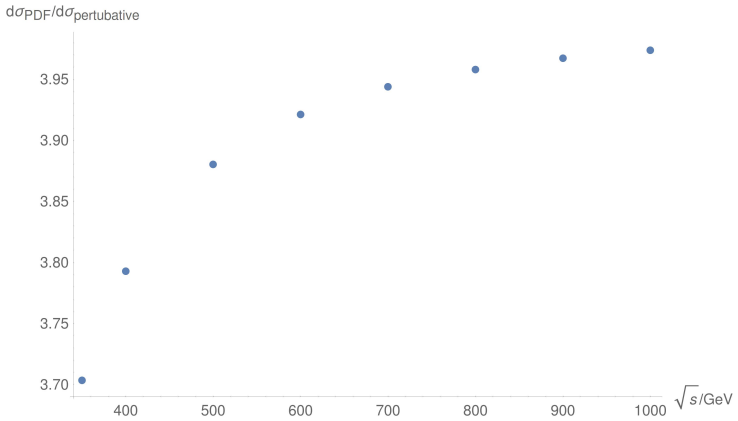


Figure 21: The ratio of parton model and perturbative theory with b=1

In figure 21 the ratio is already much smaller, but it is the same as for the case with the peak by zero, the cross section is too big.

To solve this problem, we also try to choose a very small parameter and ignore the sum rule (27) for the time being. When we look at figure 22, we

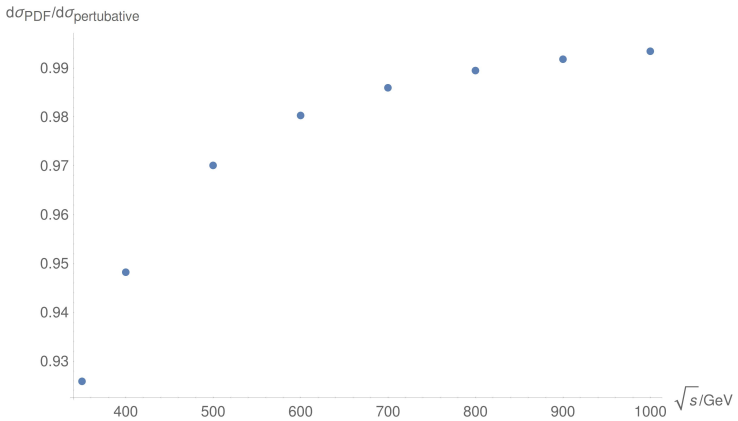


Figure 22: The ratio of parton model and perturbative theory with b=0.5

see that the highest possible parameter for b is 0.5.

Now we look at the last peak, where the parameter too has to be 2 to fullfil the sum rule (27). For this choosen parton distribution function the ratio of the two cross section is

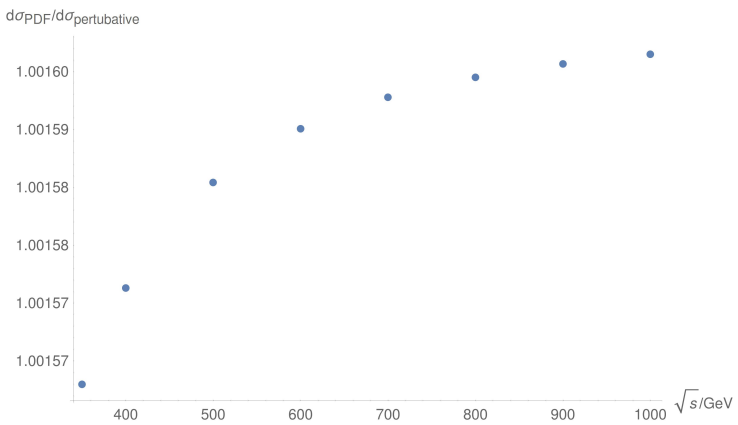


Figure 23: The ratio of parton model and perturbative theory with c=2

With this function, shown in figure 23 we get a ratio which fits until the third decimale place, which accords to the inaccuracy of the experiment [].

So the peak at x is 1 and at x is 0.5 produces a fitting cross section. We now try to find a parton distribution where both peaks exist and the sum rule is achieved. First we choose the peak at a half to be the strong part. For the parameters b=0.99 and c=0.01, we get the ratio

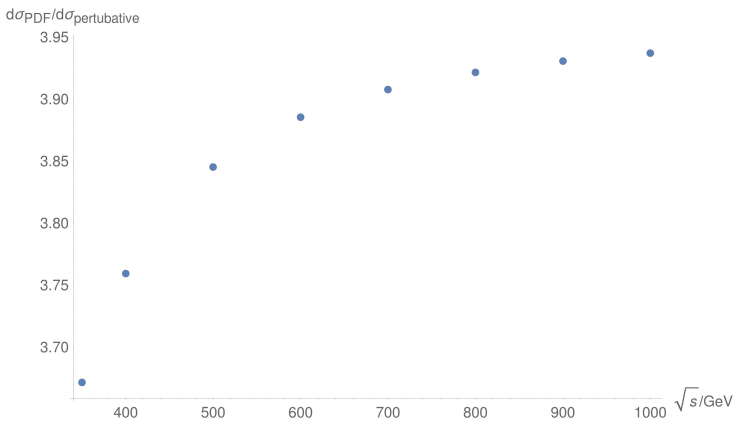


Figure 24: The ratio of parton model and perturbative theory with b=0.99 and c=0.01

As this ratio (figure 24) also is too large we reduced the parameter b and increased c. For example, when b is 0.2 and c is 1.6, we get

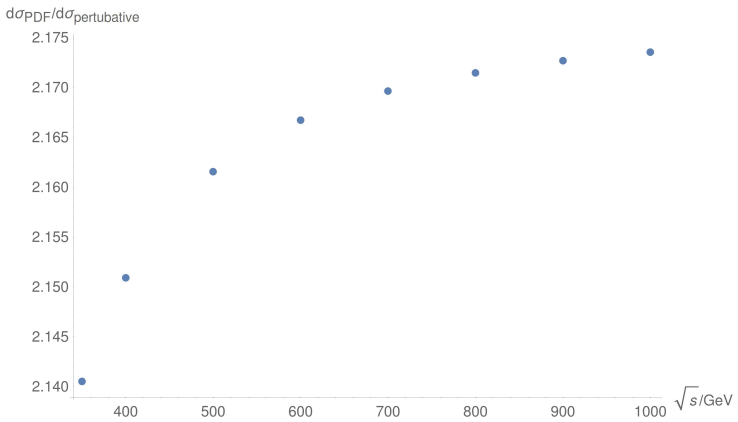


Figure 25: The ratio of parton model and perturbative theory with b=0.2 and c=1.6

This ratio in figure 25 is still too large, so we try b=0.01 and c=1.98 Also here in figure 26 the ratio is not fitting. So it is only possible to have one peak in this parton distribution function. Since the parton distribution function with only one peak at x is a 0.5 fits either the sum rule or the ratio, the only possible structure is the one with the peak at x is 1.

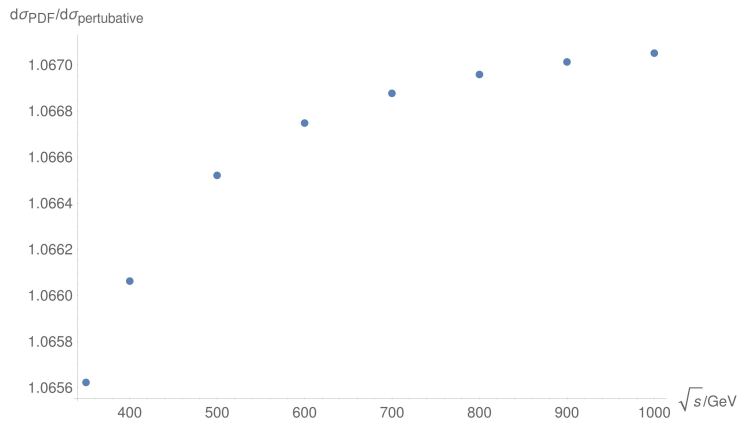


Figure 26: The ratio of parton model and pertubative theory with b=0.01 and c=1.98

5.2 PDF for Higgs Boson

With the parton distribution function for the electron found, we can now search for one for the Higgs. These two functions have to satisfy the second sum rule (28). Here we applied the same process as for the electron above. We start with the peak at x=1 and set the other parameters f and g zero. For the structure with a peak at zero, we found out that the parameter d can be set up to 4. Otherwise the sum rule (28) is larger than our error margin of one percent.

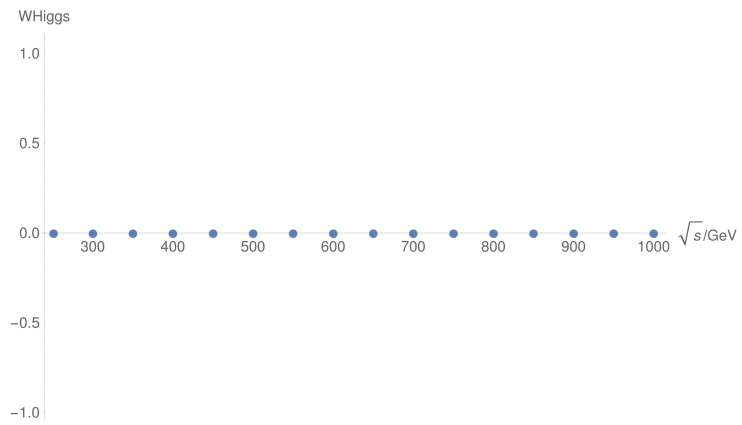


Figure 27: The cross section for the Higgs with only a peak at zero

With this structure in figure 27 the total ratio looks the same as in the last section. When looking at the cross section for the Higgs boson, we see that it is almost zero for all energies.

When we look at the peak at a half, the parameter has to be smaller than 0.02 to satisfy the sum rule (28).

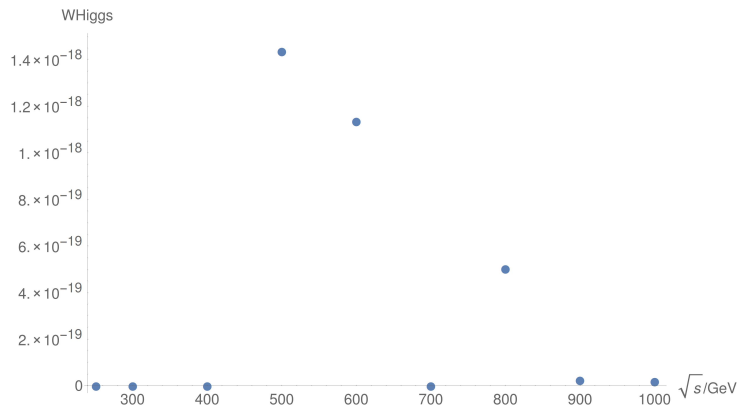


Figure 28: The cross section for the Higgs with only a peak at a half

The cross section for the parton case shown in figure 28 of the Higgs boson is almost zero too for a energy $\sqrt{s}$ smaller than 500 GeV. For energies beyond 500GeV, the cross section of the parton case is about two magnitudes smaller than the pertubative one. As an example, the cross section for the energy $\sqrt{s} = 300GeV$ and a radipity of zero is $1.5 * 10^{-48295}$. For the energy $\sqrt{s} = 1000GeV$ and a radipity of zero it is $1.8 * 10^{-20}$. Also, at a energy of about 700GeV, the cross section also falls to a small value, which differs more than the other values.

For the last peak we can observe almost the same behavior as for the case with x is a half.

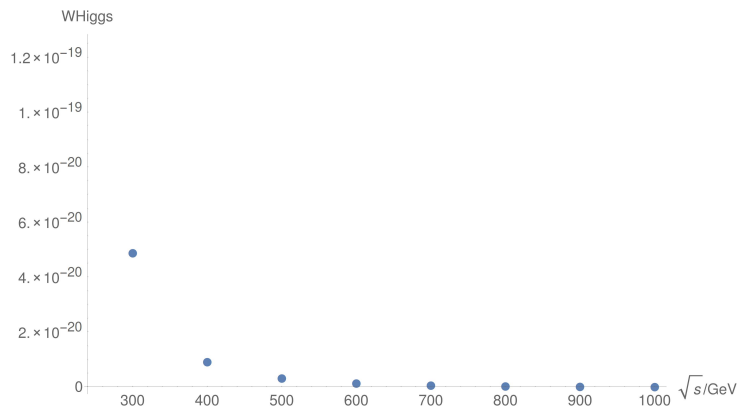


Figure 29: The cross section for the Higgs with only a peak at one

Here the cross section in figure 29 is about four magnitudes smaller than

the pertubative one for an energy higher than 250GeV, but it has a rational valued cross section at every single point. In this case, the cross section has the behavior we seek, which means it falls with the energy.

So we see that for the ratio of the two total cross sections, the structure of the parton distribution function of the Higgs boson is not very important. Yet we can now say that we can put most of the structure of the Higgs boson into the peak at zero, which means for only a small amount of the total energy of this bound state. Also, we choose to have a peak at 1 to have a non-zero cross section for all energies.

5.3 Other Final States

As a result, our parton distribution function for the Higgs and the electron has the following structure

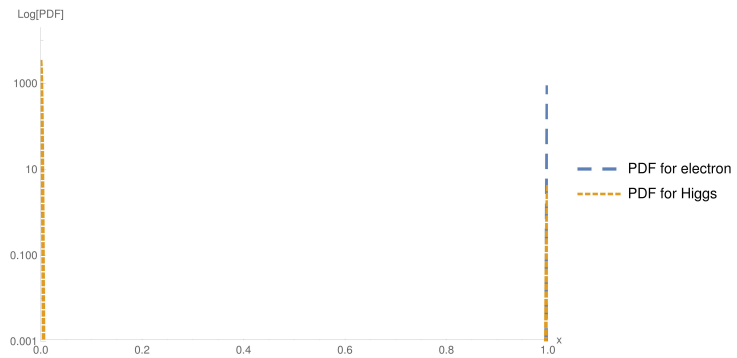


Figure 30: The parton distribution functions for the electron and the Higgs boson

With these two structures in figure 30 the results for the cross section in the parton case fits the one of the perturbative theory very well, as shown in figure 31.

We did the same calculation with these parton distribution functions for the case with other final states, like the bottom quark and the top quark. Figure 32 shows that the parton distribution function also matches the case with the bottom quark with an exception for the energy $\sqrt{s}$ is 251GeV, which is near the threshold of the Higgs boson mass.

As for the top quark case the behavior is totally different. The ratio is very small. Aside from that it is also rising. This is due to the fact that the Higgs boson scattering here is the dominant process and the cross section with the partons is much smaller than the pertubative one.

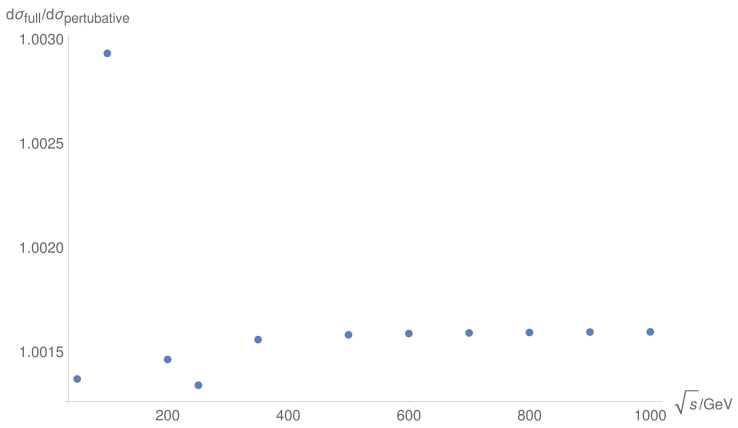


Figure 31: The cross section for the final parton distribution functions

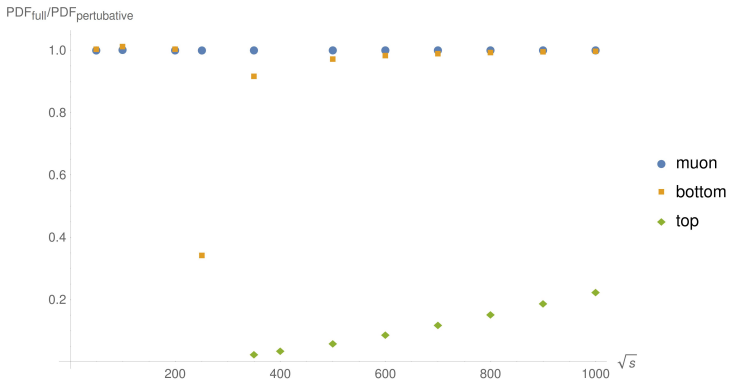


Figure 32: The ratio of parton model and perturbative theory for different final states

6 Summary and Outlook

Let us summarize what was done in this master thesis. We started with the Fröhlich-Morchio-Strocchi mechanism, which shows that taking bound states makes the theory gauge invariant. So we asked the question how such a bound state would have to look like. In this thesis we only looked at the possible bound state of a Higgs boson and an electron. In order to get a result easy to compare with the experiment, we calculated the cross sections. We also need to factorize it, so they apply to a collision of two bound states. This split them into possible individual scatterings of all the constituents. To obtain the total cross section we further had to define the parton distribution functions. Therefore we needed to choose a form and change them until they fit the data gained from the experiment of the electron-positron scattering into two muons.

Taking the previous section into account, we now know the parton distribution funtions, which provide us with more in-depth information regarding the structure of the bound state. There is a high probability of the Higgs boson being in the center, since it has only a small portion of the total energy. The electron on the other hand carries a large amount of this total energy. Therefore it would move around the Higgs boson, a pattern reminding us of the atom model.

From the cross sections for other final states, we can see another interesting possibility. If we were able to measure the process of an "effective" electron (the bound state) and an "effective" positron scattering into a top quark, we would be able to verify if this is indeed a reasonable model. As has already been mentioned in the introduction, such an experiment is already being planned, but its realisation will most likely have to wait for the next 20 years.

Needless to say, there are many ways to further improve this calculation as well as the model. A higher order could be considered, or the same calculations could be done at the lattice. There is also the possibility to study other bound states and their respective properties. From our current point of view, off-shell particles would be the most obvious option for further research.

In conclusion, we can see from this thesis that having bound states instead of our elementary particles is a possible theory for the standard model. The future will tell how successful it will be.

Appendix

The Program

```
(*general parameter*)
mZ = 91.18765; (*Z boson mass*)
mW = 80.385; (*W boson mass*)
mH = 125; (*Higgs boson mass*)
me = 0.00051; (*electron mass*)
aa = 100; (*Precision for the calculation*)
sin = Sin[28.74]; (*sinus of the Weinberg angle*)
cos = Cos[28.74]; (*cos of the Weinberg angle*)
alpha = −1/137;
γ1 = 2.5; (*width for the Z boson*)
γ2 = 10^ − 3; (*width for the Higgs boson*)
YE = −1;
qE = −1;
IwE = qE − YE/2;

(*parameter for the muon case*)
mm = 0.105;
qT = qE;
IwT = IwE;

(*parameter for the top case*)
qT = 2/3;
YT = 1/3;
```

```
IwT = qT - YT/2;

mm = 173.34;

(*parameter for the bottom case*)

qT = -1/3;

YT = 1/3;

IwT = qT - YT/2;

mm = 4.15;

(*Matrixelement for the electron scattering in parton model*)

MM[s1_, n_, x_, y_]:=64me^2mm^2qE^2qT^2/(2me^2+s1xy/2+2Sqrt[(-me^2+s1x^2/4)(-me^2+s1y^2/4)])^2+
16me^2qE^2qT^2/(2me^2+s1xy/2+2Sqrt[(-me^2+s1x^2/4)(-me^2+s1y^2/4)])+
16mm^2qE^2qT^2/(2me^2+s1xy/2+2Sqrt[(-me^2+s1x^2/4)(-me^2+s1y^2/4)])-
(8IwE^6IwT^6me^2mm^2 (2me^2 + 2mm^2 - s1x^2))/
(Cos^2 mW^2 Sin^4 (mZ^4 - 2mZ^2 (2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)]) +
(2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)])^2 + γ1^2)) +
(4IwE^6IwT^6me^2mm^2 (4me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)])
(2me^2 + 2mm^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)]))/
(mW^4 Sin^4 (mZ^4 - 2mZ^2 (2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)]) +
(2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)])^2 + γ1^2)) +
(me^2mm^2 (me^2 - 2 (2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)]))
(mm^2 - 2 (2me^2 + s1xy/2 + 2Sqrt[(-me^2 + s1x^2/4) (-me^2 + s1y^2/4)])))/
```

$$\left(16\mathrm{mW}^4\sin^4\left(\mathrm{mH}^4-2\mathrm{mH}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.\right.$$
$$\left.\left.\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 2^2\right)\right)+$$
$$\left(8\mathrm{me}^2\mathrm{mm}^2\left(\mathrm{IwE}^3-2\mathrm{qE}\sin^2\right)\left(-\mathrm{IwT}^3+2\mathrm{qT}\sin^2\right)\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\left(\left(-2\mathrm{me}^2+\mathrm{mH}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)\right.$$
$$\left.\left(-2\mathrm{me}^2+\mathrm{mZ}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\gamma 1\gamma 2\right)$$
$$\mathrm{Tanh}\left[n+\mathrm{ArcCosh}\left[\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]\right.$$
$$\left.\mathrm{Sign}[x-y]\right])/$$
$$\left(\cos^2\mathrm{mW}^2\sin^4\left(\left(-2\mathrm{me}^2+\mathrm{mZ}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 1^2\right)\right.$$
$$\left.\left(\left(-2\mathrm{me}^2+\mathrm{mH}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 2^2\right)\right)-$$
$$\left(32\mathrm{me}^2\mathrm{mm}^2\mathrm{qEqT}\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\left(-2\mathrm{me}^2+\mathrm{mH}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)$$
$$\mathrm{Tanh}\left[n+\mathrm{ArcCosh}\left[\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]\right.$$
$$\left.\mathrm{Sign}[x-y]\right])/$$
$$\left(\mathrm{mW}^2\sin^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)\right.$$
$$\left(\mathrm{mH}^4-2\mathrm{mH}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.$$
$$\left.\left.\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 2^2\right)\right)+$$
$$(8\mathrm{qE}^2\mathrm{qT}^2$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$

$$\mathrm{Tanh}\left[n+\mathrm{ArcCosh}\left[\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]\right.$$
$$\left.\mathrm{Sign}[x-y]]\right)^2)/$$
$$\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+$$
$$(8\mathrm{qE}^2\mathrm{qT}^2$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}\left[n+\mathrm{ArcCosh}\left[\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]\right.$$
$$\left.\mathrm{Sign}[x-y]]\right)^2)/$$
$$\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+$$
$$\left(8\mathrm{qEqT}\left(-2\mathrm{me}^2+\mathrm{mZ}^2-\frac{\mathrm{s1}xy}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)\right.$$
$$(-\mathrm{qE}\sin^2\left(\mathrm{IwT}^3-2\mathrm{qT}\sin^2\right)$$
$$\left(2\mathrm{mm}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.$$
$$2\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2+$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2)+$$

$$\mathrm{IwE}^3$$
$$\Bigg(\mathrm{IwT}^3\left(\mathrm{mm}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\Bigg(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\Bigg]$$
$$\mathrm{Sign}[x-y]])^2)-$$
$$\mathrm{qT}\sin^2\left(2\mathrm{mm}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.$$
$$2\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\Bigg(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\Bigg]$$
$$\mathrm{Sign}[x-y]])^2+$$
$$\Bigg(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\Bigg]$$
$$\mathrm{Sign}[x-y]])^2))))/$$
$$\Bigg(\cos^2\sin^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)$$
$$\Bigg(\mathrm{mZ}^4-2\mathrm{mZ}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 1^2\Bigg)\Bigg)+$$
$$(4$$

$$
\begin{aligned}
&(\mathrm{IwE}^6 \\
&\Bigg(\mathrm{IwT}^6 \Bigg(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)} \\
&\mathrm{Tanh}[n + \mathrm{ArcCosh}[\\
&\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right] \\
&\mathrm{Sign}[x - y]])^2 - \\
&2\mathrm{IwT}^3\mathrm{qT}\sin^2\Bigg(\mathrm{mm}^2\left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) + \\
&\Bigg(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)} \\
&\mathrm{Tanh}[n + \mathrm{ArcCosh}[\\
&\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right] \\
&\mathrm{Sign}[x - y]])^2) + \\
&\mathrm{qT}^2\sin^4\Bigg(2\mathrm{mm}^2\left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) + \\
&\Bigg(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} - 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)} \\
&\mathrm{Tanh}[n + \mathrm{ArcCosh}[\\
&\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right] \\
&\mathrm{Sign}[x - y]])^2 + \\
&\Bigg(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)} \\
&\mathrm{Tanh}[n + \mathrm{ArcCosh}[\\
&\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right] \\
&\mathrm{Sign}[x - y]])^2)) - \\
&2\mathrm{IwE}^3\mathrm{qE}\sin^2 \\
&\Bigg(\mathrm{IwT}^6\Bigg(\mathrm{me}^2\left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) +
\end{aligned}
$$

$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2)-$$
$$2\mathrm{IwT}^3\mathrm{qT}\sin^2\left(\mathrm{mm}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.$$
$$\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2)+$$
$$\mathrm{qT}^2\sin^4\left(2\mathrm{mm}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.$$
$$2\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2+$$
$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x-y]])^2))+$$

$$\mathrm{qE}^2 \sin^4$$
$$\left(\mathrm{IwT}^6 \left(2\mathrm{me}^2 \left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) + \right.\right.$$
$$\left(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} - 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n + \mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x - y]])^2 +$$
$$\left(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n + \mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x - y]])^2) -$$
$$2\mathrm{IwT}^3\mathrm{qT} \sin^2 \left(2\mathrm{mm}^2 \left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) + \right.$$
$$2\mathrm{me}^2 \left(2\mathrm{me}^2 + 4\mathrm{mm}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) +$$
$$\left(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} - 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n + \mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x - y]])^2 +$$
$$\left(\mathrm{me}^2 + \mathrm{mm}^2 - \frac{\mathrm{s1}x^2}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2 + \frac{\mathrm{s1}x^2}{4}\right)}\right.$$
$$\mathrm{Tanh}[n + \mathrm{ArcCosh}[$$
$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$
$$\mathrm{Sign}[x - y]])^2) +$$
$$2\mathrm{qT}^2 \sin^4 \left(2\mathrm{mm}^2 \left(2\mathrm{me}^2 + \frac{\mathrm{s1}xy}{2} + 2\sqrt{\left(-\mathrm{me}^2 + \frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2 + \frac{\mathrm{s1}y^2}{4}\right)}\right) + \right.$$

$$2\mathrm{me}^2\left(2\mathrm{me}^2+4\mathrm{mm}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+$$

$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}-2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$

$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$

$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$

$$\mathrm{Sign}[x-y]])^2+$$

$$\left(\mathrm{me}^2+\mathrm{mm}^2-\frac{\mathrm{s1}x^2}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{mm}^2+\frac{\mathrm{s1}x^2}{4}\right)}\right.$$

$$\mathrm{Tanh}[n+\mathrm{ArcCosh}[$$

$$\left.\frac{\sqrt{\mathrm{s1}}(x+y)}{2\sqrt{2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}}}\right]$$

$$\mathrm{Sign}[x-y]])^2))))/$$

$$\left(\cos^4\sin^4\left(\mathrm{mZ}^4-2\mathrm{mZ}^2\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)+\right.\right.$$

$$\left.\left.\left(2\mathrm{me}^2+\frac{\mathrm{s1}xy}{2}+2\sqrt{\left(-\mathrm{me}^2+\frac{\mathrm{s1}x^2}{4}\right)\left(-\mathrm{me}^2+\frac{\mathrm{s1}y^2}{4}\right)}\right)^2+\gamma 1^2\right)\right);$$

(*Cross section for the electron*)

WQs[s_, n_, x_, y_]:=

SetPrecision[

((1−

Tanh[

$n+\mathrm{Sign}[x-y]*$

ArcCosh[

$(x+y)$Sqrt[s]/(2Sqrt[$2*x*y*s/4+2$me^2+

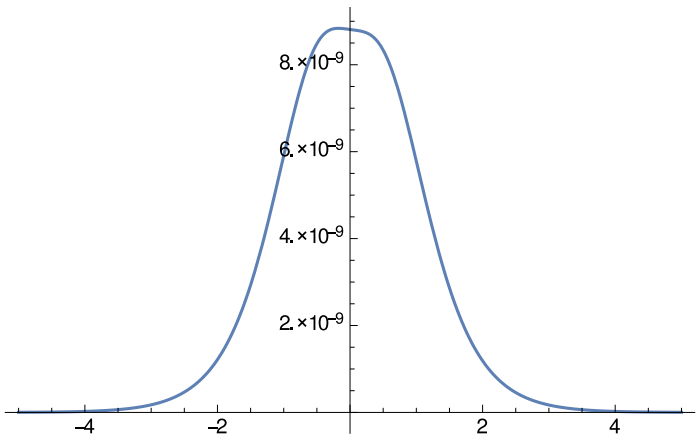
2Sqrt[($x^{2}s/4-$me^2)($y^{2}s/4-$me^2)]])]]^2))

MM[$s,n,x,y]*1/16*$

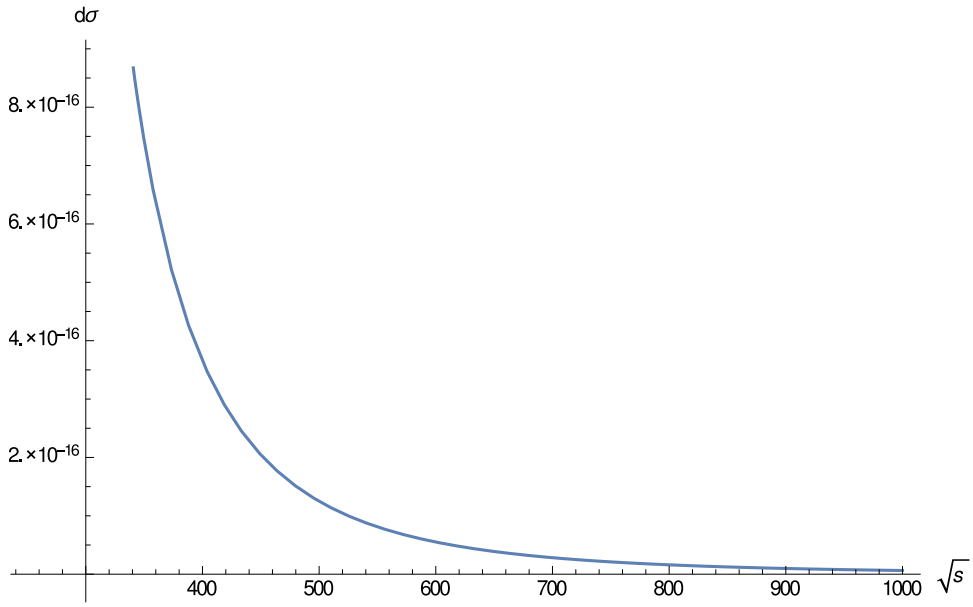
$1/(2*x*y*s/4+2$me^2 + 2Sqrt[($x^{2}s/4-$me^2)($y^{2}s/4-$me^2)])*

```
e^2*
Sqrt[
(2*x*y*s/4 + 2me^2 + 2Sqrt[(x^2s/4 - me^2)(y^2s/4 - me^2)]-
4*mm^2)/
(2*x*y*s/4 + 2me^2 + 2Sqrt[(x^2s/4 - me^2)(y^2s/4 - me^2)]-
4*me^2)] * HeavisideTheta[x^2s - 4*mm^2]*
HeavisideTheta[y^2s - 4*mm^2], aa];

(*Matrixelement for the Higgs boson scattering*)
MH[s_, n_, x_, y_]:=
SetPrecision[-(9 mH^4 (mm^4 - 2mm^2 (2mH^2 + sxy/2 + 2 Sqrt[(-mH^2 + sx^2/4)(-mH^2 + sy^2/4)]))) / (16mW^4 Sin^4(-mH^2 - sxy/2 - 2 Sqrt[(-mH^2 + sx^2/4)(-mH^2 + sy^2/4)])^2),
aa];

(*Cross section for the Higgs boson*)
WQS2[s_, n_, x_, y_]:=
SetPrecision[
((1-
Tanh[
n + Sign[x - y]ArcCosh[((x + y) * Sqrt[s])/
(2Sqrt[2*x*y*s/4 + 2mH^2+
2Sqrt[(x^2s/4 - mH^2)(y^2s/4 - mH^2)]])]]^2))*
MH[s, n, x, y] * 1/16*
1/(2*x*y*s/4 + 2mH^2 + 2Sqrt[(x^2s/4 - mH^2)(y^2s/4 - mH^2)])*
e^2*
Sqrt[
```

```
(2 * x * y * s/4 + 2mH^2 + 2Sqrt[(x^2s/4 − mH^2)(y^2s/4 − mH^2)]−
4 * mm^2)/
(2 * x * y * s/4 + 2mH^2 + 2Sqrt[(x^2s/4 − mH^2)(y^2s/4 − mH^2)]−
4 * mH^2)] * HeavisideTheta[x^2s − 4 * mH^2]*
HeavisideTheta[y^2s − 4 * mH^2], aa];

(*Testofthecrosssection, whenxandyareone,
thecrosssectionshouldbethesameasforthepertubativetheory*)

m = 40^2;
Plot[{WQs[m, n, 1, 1]}, {n, −5, 5}, PlotRange → All]
```




```
Plot [WQS2[s^2, 0, 1, 1], {s, 251, 1000}, AxesLabel → {"√s", "dσ"},
ImageSize → 500]
```



(*whenγ1andγ2aresetzero, thelimitshouldgotoinfinity*)

Limit[WQs[q, 0, 1, 1], $q \to$ mZ^2]

0.000370022

(*Cross section for the electron in parton system*)
WEle[s_, n_]:=NIntegrate $\left[\text{fe}[x, s] * \text{fe}[y, s] * \text{WQs}[s, n, x, y], \left\{x, \frac{2\text{mm}}{\sqrt{s}}, 1\right\},\right.$
$\left.\left\{y, \frac{2\text{mm}}{\sqrt{s}}, 1\right\}, \text{WorkingPrecision} \to 70, \text{PrecisionGoal} \to 10, \text{MaxRecursion} \to 10,\right.$
AccuracyGoal $\to$ 15, Method $\to$ "GaussKronrodRule"];

(*Cross section for the Higgs boson in parton system*)
WHiggs[s_, n_]:=NIntegrate $\left[\text{fh}[x, s]\text{fh}[y, s]\text{WQS2}[s, n, x, y], \left\{x, \frac{2\text{mH}}{\sqrt{s}}, 1\right\},\right.$
$\left.\left\{y, \frac{2\text{mH}}{\sqrt{s}}, 1\right\}, \text{WorkingPrecision} \to 70, \text{PrecisionGoal} \to 10, \text{MaxRecursion} \to 10,\right.$
AccuracyGoal $\to$ 10, Method $\to$ "GaussKronrodRule"]

(*parameter for the PDF*)

$a = 0$;

```
b = 0;
c = 2;
d = 0;
f = 0;
g = 0.01;
tw = 0.001;
tr = 0.001;
w = 0.001;

(*PDF for the electron*)
fe[x_, s_]:=
SetPrecision[
HeavisideTheta[-s + 4 * mH^2]*
(2/Sqrt[(2 * Pi * w^2)] * (E^(-(x - 1)^2/(2 * w^2))))+
HeavisideTheta[s - 4 * mH^2]
(1/Sqrt[(2 * Pi * tw^2)]*
(a * E^(-(x)^2/(2 * tw^2)) + b * E^(-(x - 0.5)^2/(2 * tw^2)))+
c/Sqrt[(2 * Pi * tw^2)] * E^(-(x - 1)^2/(2 * tw^2))), aa];

(*PDF for the Higgs bososn*)
fh[x_, s_]:=
SetPrecision[HeavisideTheta[s - 4 * mH^2]*
(1/Sqrt[(2 * Pi * tr^2)]*
(d * E^(-x^2/(2 * tr^2)) + f * E^(-(x - 0.5)^2/(2 * tr^2))+
g * E^(-(x - 1)^2/(2 * tr^2)))), aa];
```

```
(*Test energy and rapidity*)
s = 700^2;
n = 0;

Ladung = Integrate[fe[x, s], {x, 0, 1}]
```

0.99999999999999990551

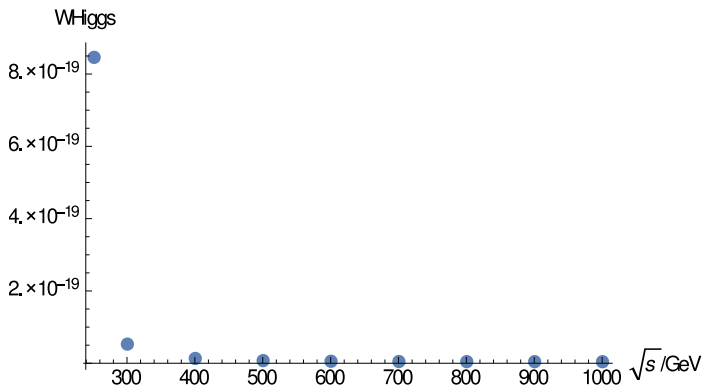
```
Impuls = Integrate[x * (fe[x, s] + fh[x, s]), {x, 0, 1}]
```

1.00419812601639302552

```
ListLogPlot[{El, T}, AxesLabel → {x, "Log[PDF]"},
PlotStyle → {{Dashing[Large], Thickness[0.009]},
{Dashing[Small], Thickness[0.007]}}, ImageSize → 500,
PlotLegends → {"PDF for electron", "PDF for Higgs"},
PlotRange → {{0, 1}, {10^ − 3, 2 * 10^4}}, Joined → True]


(*Plotfortheratioofthecrosssectionformuonandbottomas
finalstates*)
R = {{50, ((WEle[50^2, n])/WQs[50^2, n, 1, 1])},
{100, ((WEle[100^2, n])/WQs[100^2, n, 1, 1])},
{200, ((WEle[200^2, n])/WQs[200^2, n, 1, 1])},
{251, (WHiggs[251^2, n] + WEle[251^2, n])/
(WQs[251^2, n, 1, 1] + WQS2[251^2, n, 1, 1])},
{350, (WHiggs[350^2, n] + WEle[350^2, n])/
(WQs[350^2, n, 1, 1] + WQS2[350^2, n, 1, 1])},
```

```
{500, (WHiggs[500^2, n] + WEle[500^2, n])/
(WQs[500^2, n, 1, 1] + WQS2[500^2, n, 1, 1])},
{600, (WHiggs[600^2, n] + WEle[600^2, n])/
(WQs[600^2, n, 1, 1] + WQS2[600^2, n, 1, 1])},
{700, (WHiggs[700^2, n] + WEle[700^2, n])/
(WQs[700^2, n, 1, 1] + WQS2[700^2, n, 1, 1])},
{800, (WHiggs[800^2, n] + WEle[800^2, n])/
(WQs[800^2, n, 1, 1] + WQS2[800^2, n, 1, 1])},
{900, (WHiggs[900^2, n] + WEle[900^2, n])/
(WQs[900^2, n, 1, 1] + WQS2[900^2, n, 1, 1])},
{1000, (WHiggs[1000^2, n] + WEle[1000^2, n])/
(WQs[1000^2, n, 1, 1] + WQS2[1000^2, n, 1, 1])}};

(*Plot for the ratio of the cross section for top as final states*)
R1 =
{{350, (WHiggs[350^2, n] + WEle[350^2, n])/
(WQs[350^2, n, 1, 1] + WQS2[350^2, n, 1, 1])},
{400, (WHiggs[400^2, n] + WEle[400^2, n])/
(WQs[400^2, n, 1, 1] + WQS2[400^2, n, 1, 1])},
{500, (WHiggs[500^2, n] + WEle[500^2, n])/
(WQs[500^2, n, 1, 1] + WQS2[500^2, n, 1, 1])},
{600, (WHiggs[600^2, n] + WEle[600^2, n])/
(WQs[600^2, n, 1, 1] + WQS2[600^2, n, 1, 1])},
{700, (WHiggs[700^2, n] + WEle[700^2, n])/
(WQs[700^2, n, 1, 1] + WQS2[700^2, n, 1, 1])},
```

```
{800, (WHiggs[800^2, n] + WEle[800^2, n])/
(WQs[800^2, n, 1, 1] + WQS2[800^2, n, 1, 1])},
{900, (WHiggs[900^2, n] + WEle[900^2, n])/
(WQs[900^2, n, 1, 1] + WQS2[900^2, n, 1, 1])},
{1000, (WHiggs[1000^2, n] + WEle[1000^2, n])/
(WQs[1000^2, n, 1, 1] + WQS2[1000^2, n, 1, 1])}};

ListPlot [{R}, AxesLabel → {"√s", "dσ_full/dσ_pertubative"} , ImageSize → 500,
PlotLegends → {"for top"}, PlotRange → All]


(*Plot only for the parton Higgs boson cross section*)
H = {{251, WHiggs[251^2, n]}, {300, WHiggs[300^2, n]}, {400, WHiggs[400^2, n]},
{500, WHiggs[500^2, n]}, {600, WHiggs[600^2, n]}, {700, WHiggs[700^2, n]},
{800, WHiggs[800^2, n]}, {900, WHiggs[900^2, n]}, {1000, WHiggs[1000^2, n]}};

ListPlot [H, AxesLabel → {"√s/GeV", "WHiggs"} , PlotMarkers → {Automatic, Small},
PlotRange → All]
```



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