Dark Matter on the Lattice

SIMPs in an Sp(4) dark sector

FAKT Workshop 2024
Particle Physics Retreat

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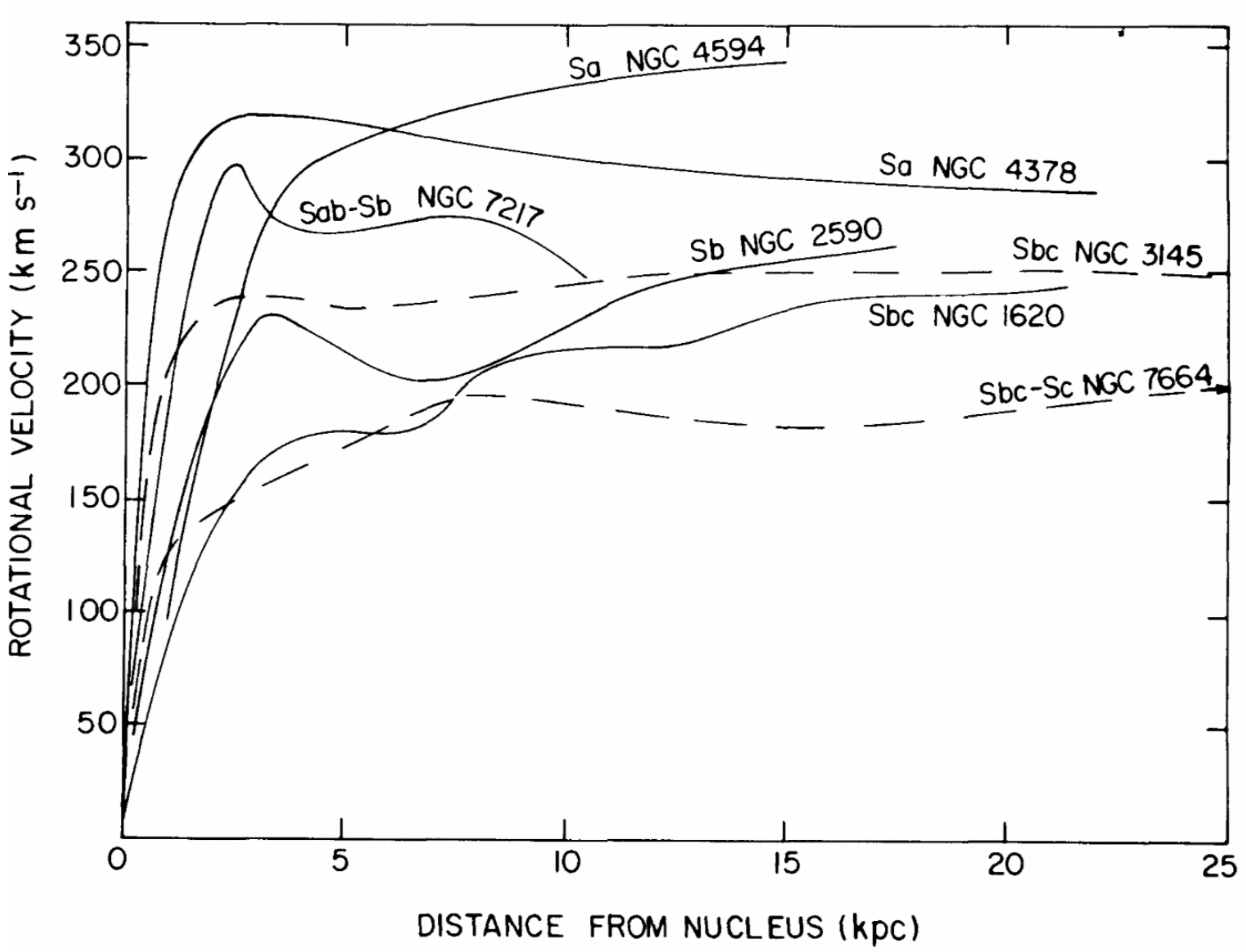
With Axel Maas und Fabian Zierler

Dark Matter

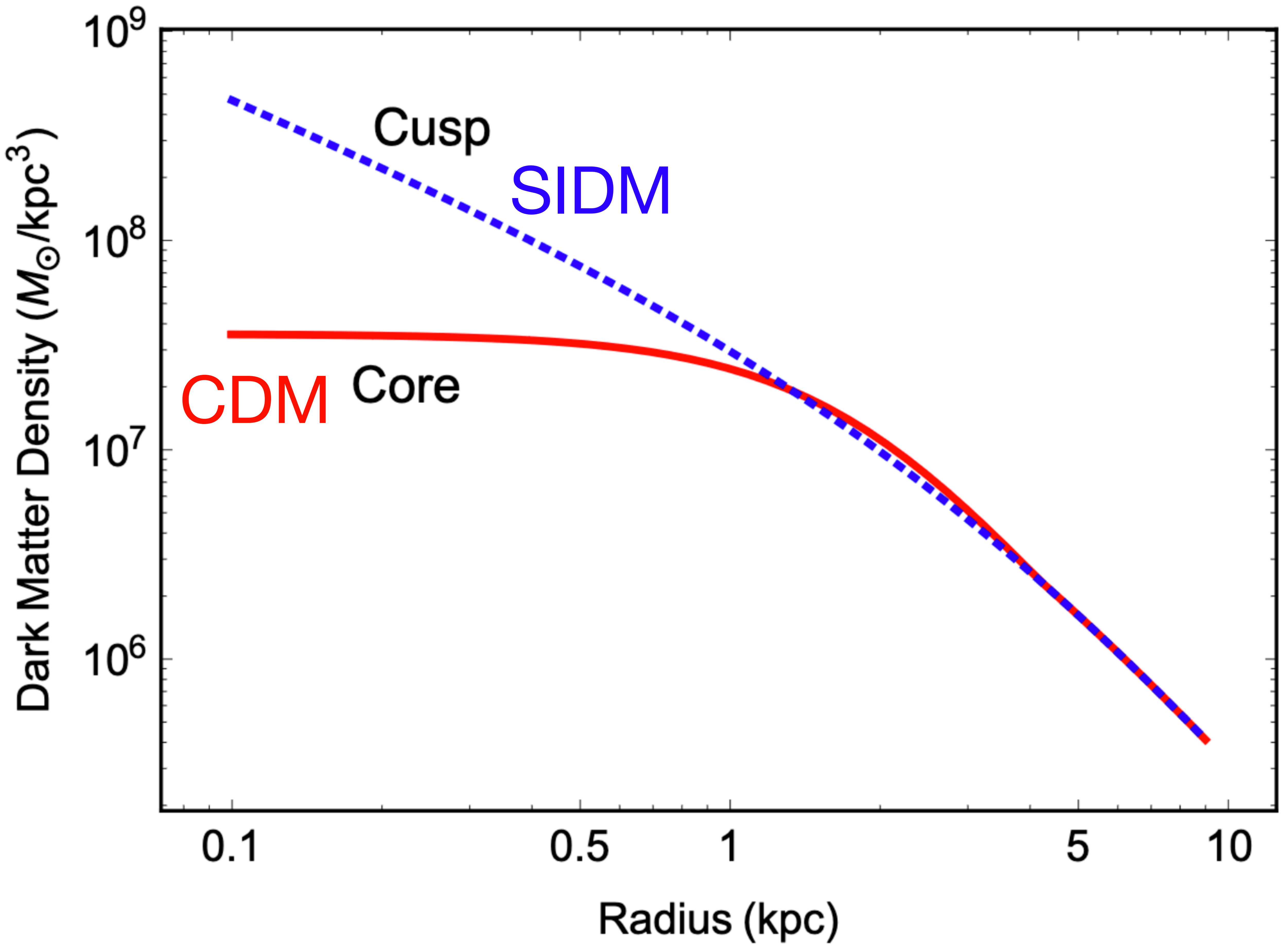
- Collection of phenomena beyond the standard model
    - Rotation curves, structure formation, etc.
- Possible explanations:
    - Modified gravity
    - Non observable form of matter
    - Particle beyond the SM



Rubin et al.: Ap.J.L. 225 (1978)

Dark Matter - Self-interaction

- Observations are in conflict with cold dark matter (CDM) models
  - "cusp vs. core", "too big to fail", etc.
- Possible solution:
  - Self-interacting dark matter (SIDM)
- Constraint by "bullet cluster"

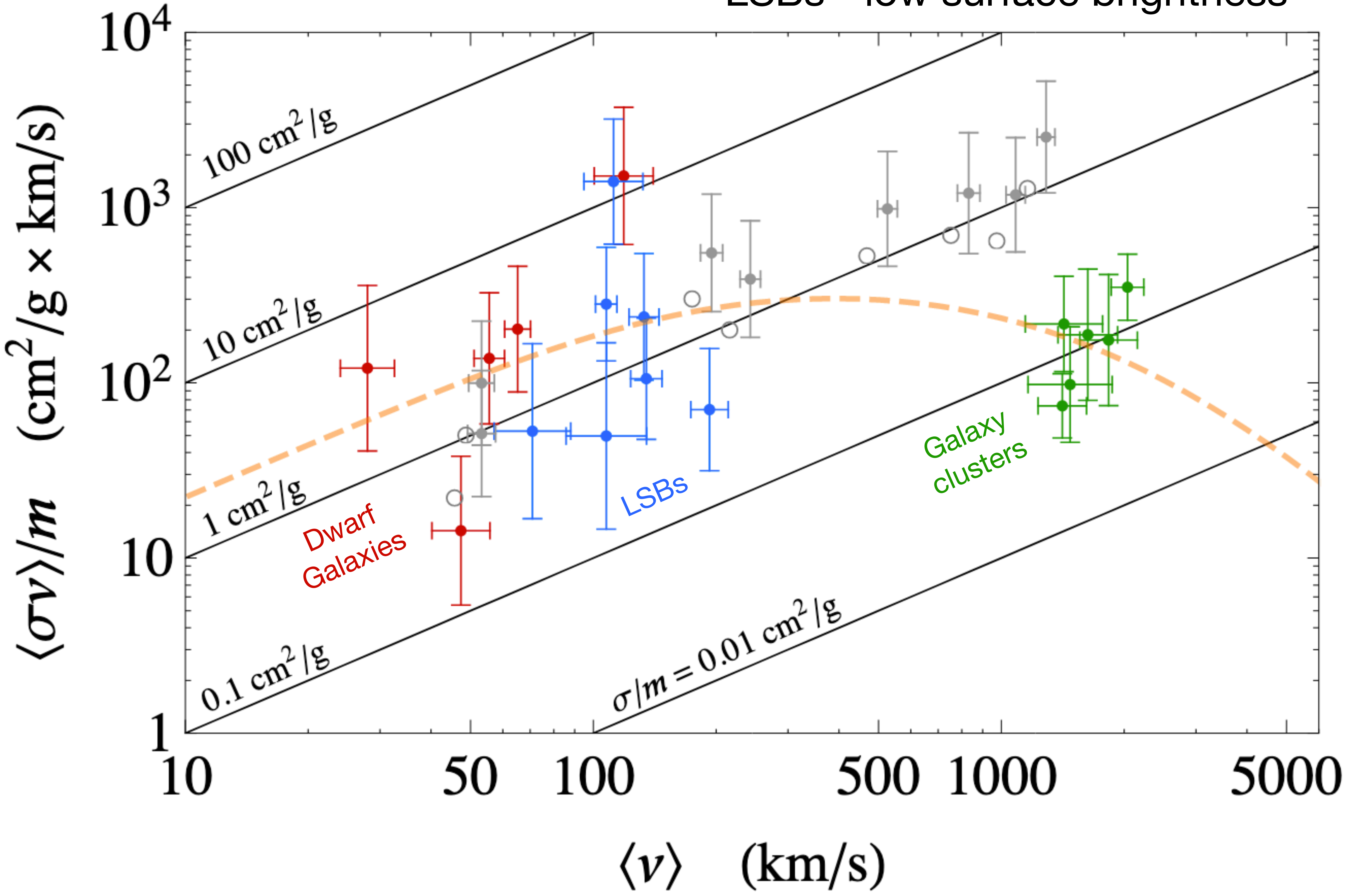


Tulin, Yu: arXiv:1705.02358 (2017)

Dark Matter - Velocity-dependence

- "Dark halos as particle colliders"
- Cross-section from shape of halos
- Results prefer velocity-dependent cross section
- This work:
  - Blueprint: How to compare lattice results to this

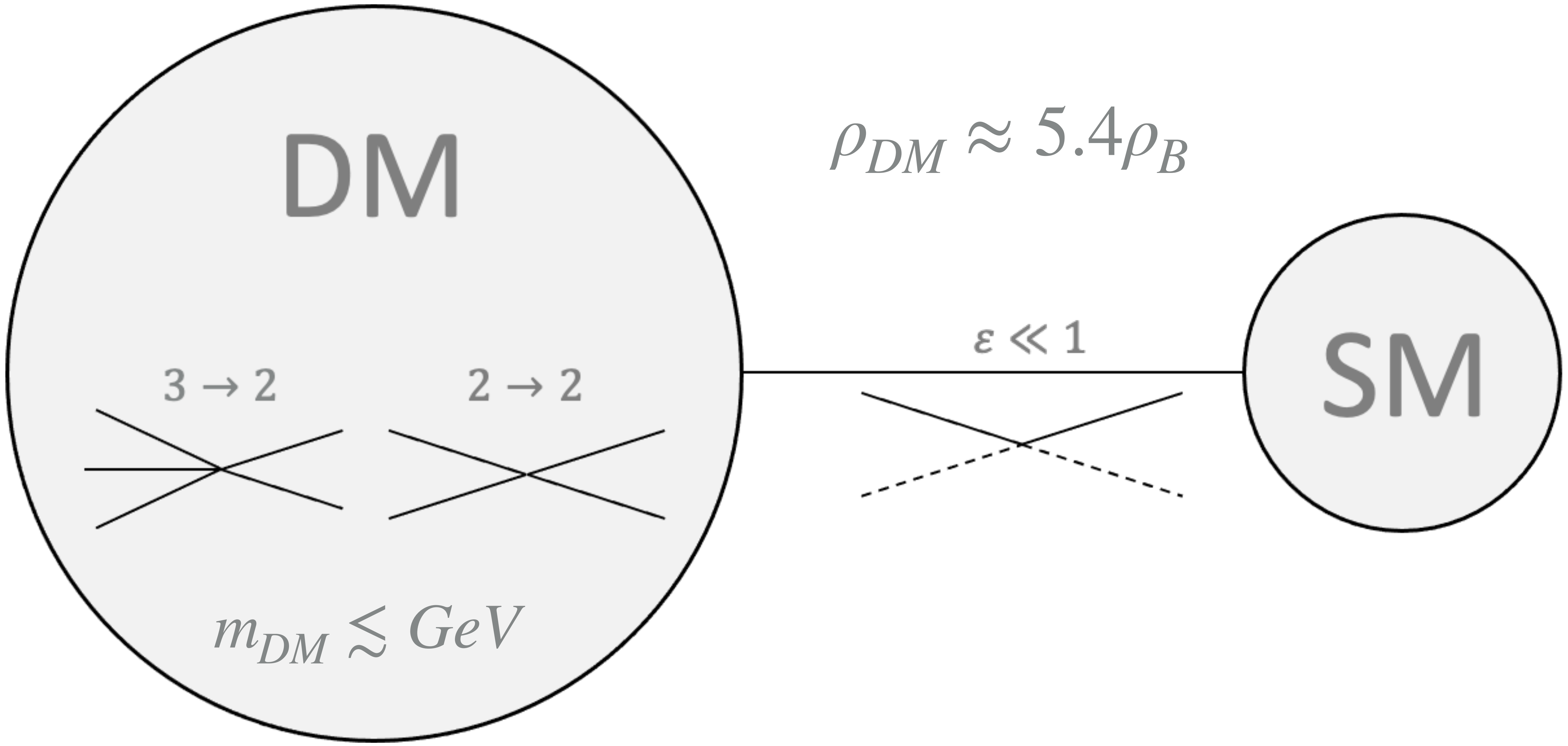
LSBs - low surface brightness



Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Dark Matter - SIMP

- One possible realization of SIDM
    - DM as a thermal relic of the early universe via freeze-out
- Number lowering process in the dark sector
    - Heat up of DM
- Heat flow from DM to SM via coupling
    - Mediator enables direct detection

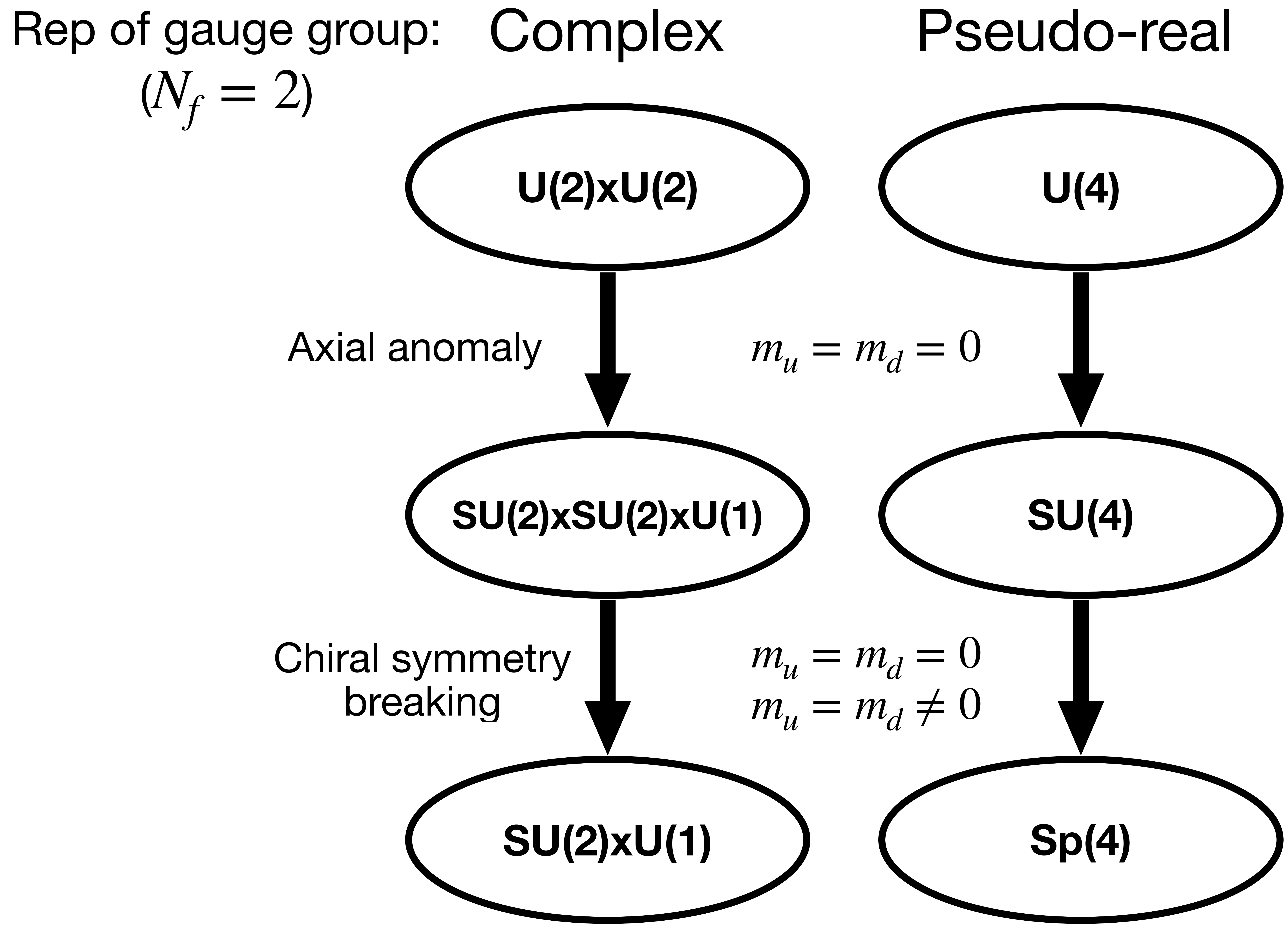


Hochberg et al.: Phys. Rev. Lett. 113 (2014)

Minimal realisation

- $N_f = 2$ fundamental fermions in pseudo-real representation of gauge group
- Enlarged flavour symmetry
- Result: 5 pNGBs

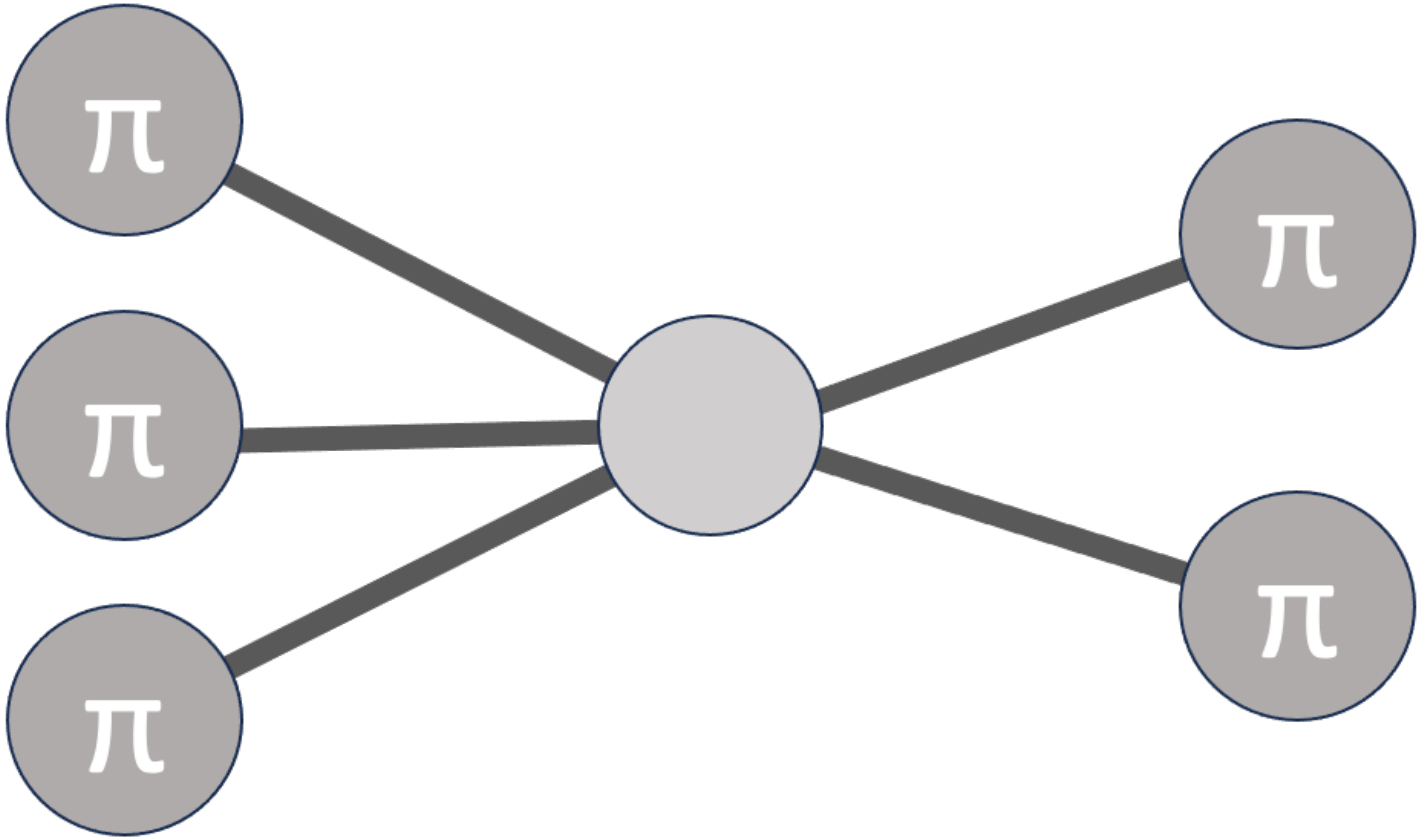
➡ $3 \rightarrow 2$ process possible



Kulkarni et al.: SciPost Phys. 14 (2023)

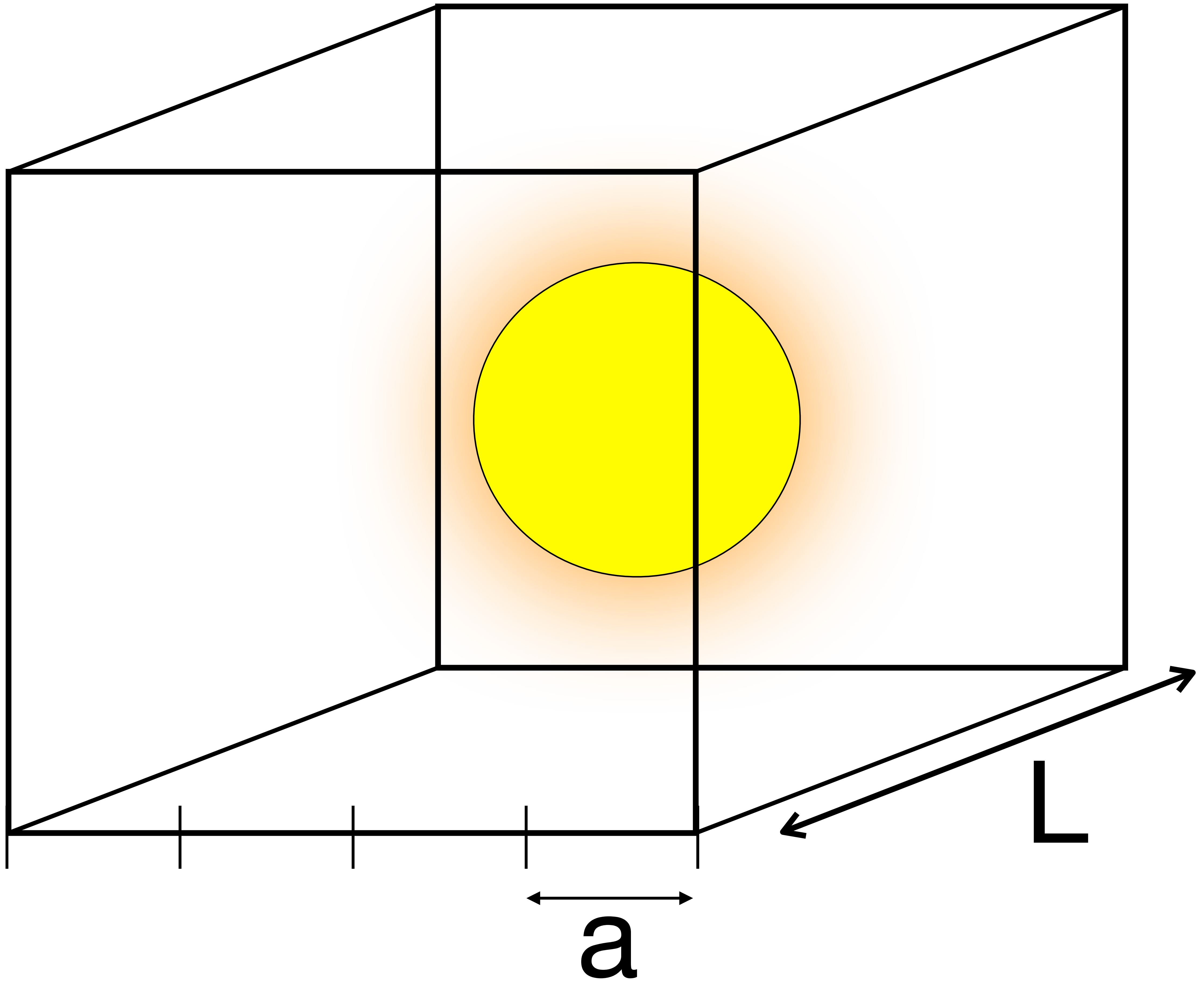
Sp(4) gauge with $N_f = 2$

- "Zoo" of dark particles:
  - 5 "dark" Pions
  - 10 "dark" Rhos
  - and more
- Even number of colours:
  - No fermionic bound states



Lattice

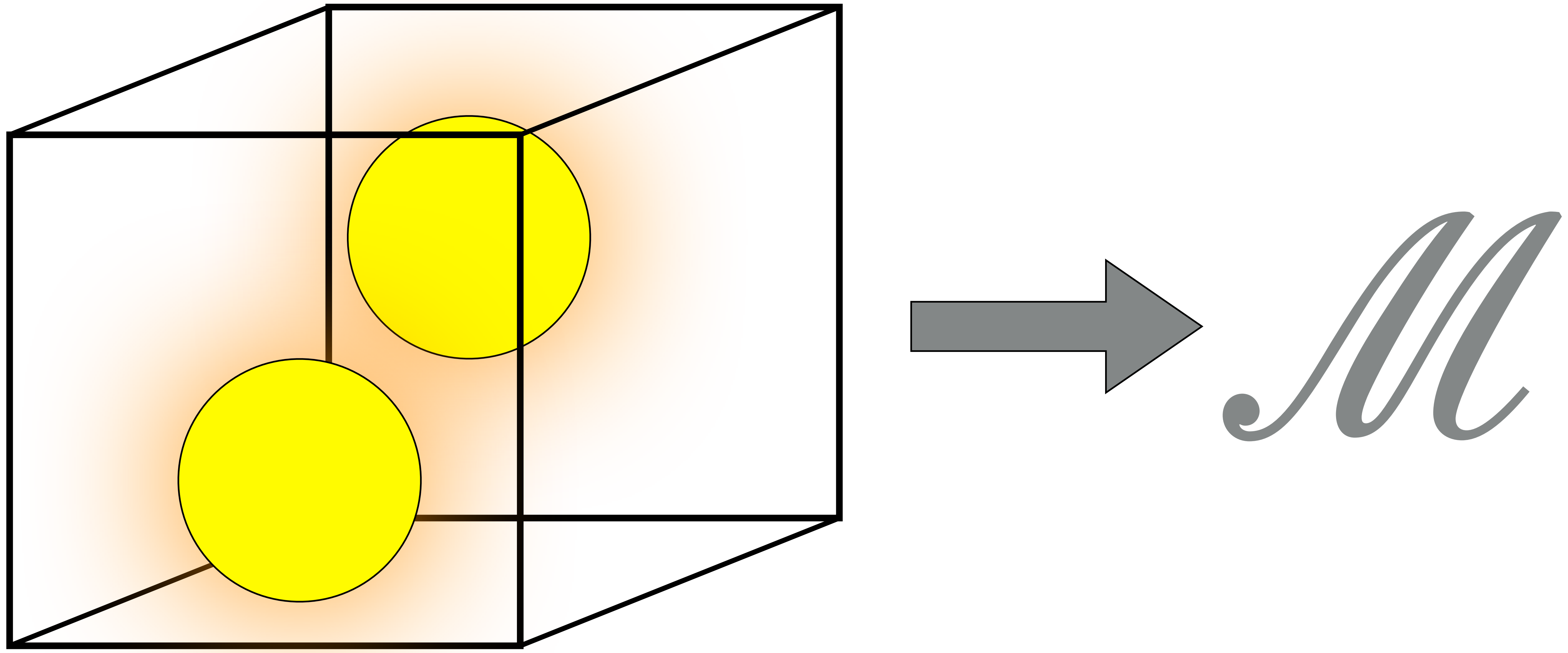
- Sample gauge configurations in a discretized space-time
- Challenges:
  - IR and UV cutoff because of a and L
  - Discretization artifacts
  - Finite volume effects



Lattice - Scattering

- Particles enclosed in a box
  - Energy levels are shifted in finite volume due to scattering effects
- Energy shift $\leftrightarrow$ scattering properties
- $\tan(\delta) = \dfrac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{\vec{0}}(1, q^2)}$

"Lüscher Zeta function"



Lüscher et al.: Commun. Math. Phys. 104/105 (1986)

Comparison to halo data

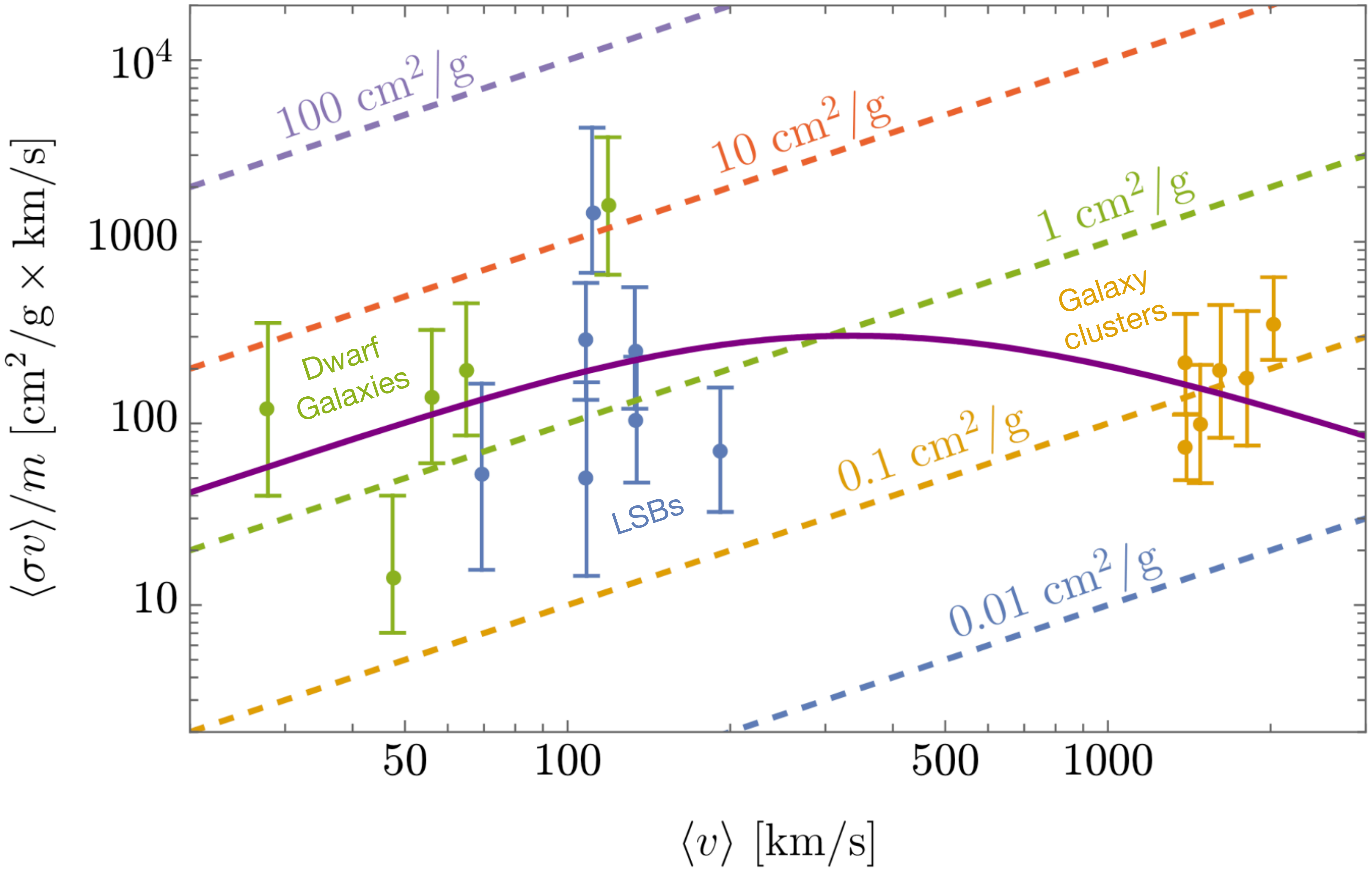
- Effective range-expansion (s-wave)
- $P\cot(\delta) = -\frac{1}{a} + \frac{P^2}{2r_e} + \mathcal{O}(P^4)$
- $\frac{< \sigma v >}{m} = \int_0^\infty v\,\sigma f_{MB}(v)\,dv$

➡$a = 22.2$ fm

➡$r_e = -2.59 \times 10^{-3}$ fm

➡$m_{DM} = 16.7 GeV$

LSBs - low surface brightness



Kondo et al: J. High Energ. Phys. (2022)

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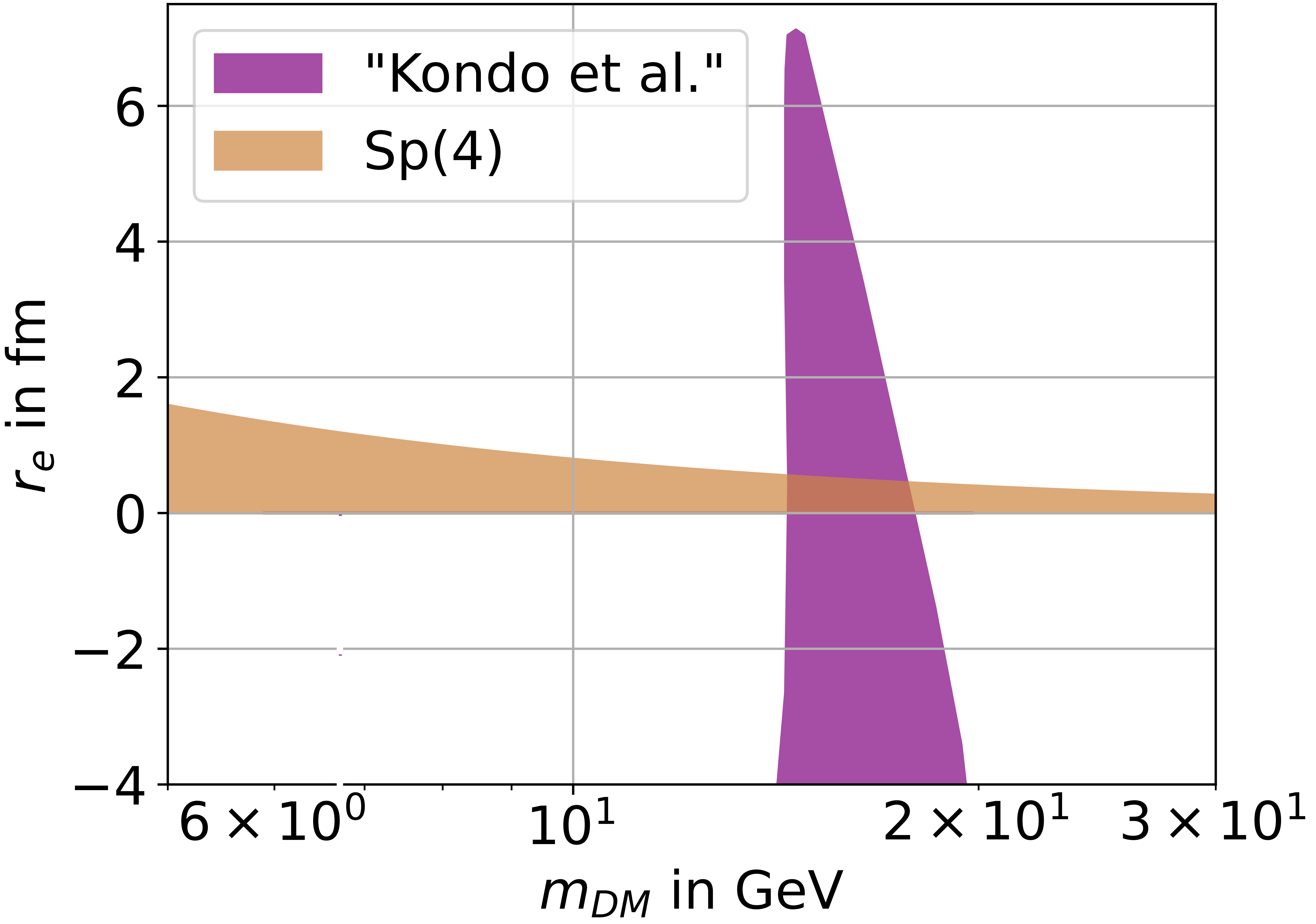

Kondo et al: J. High Energ. Phys. (2022)

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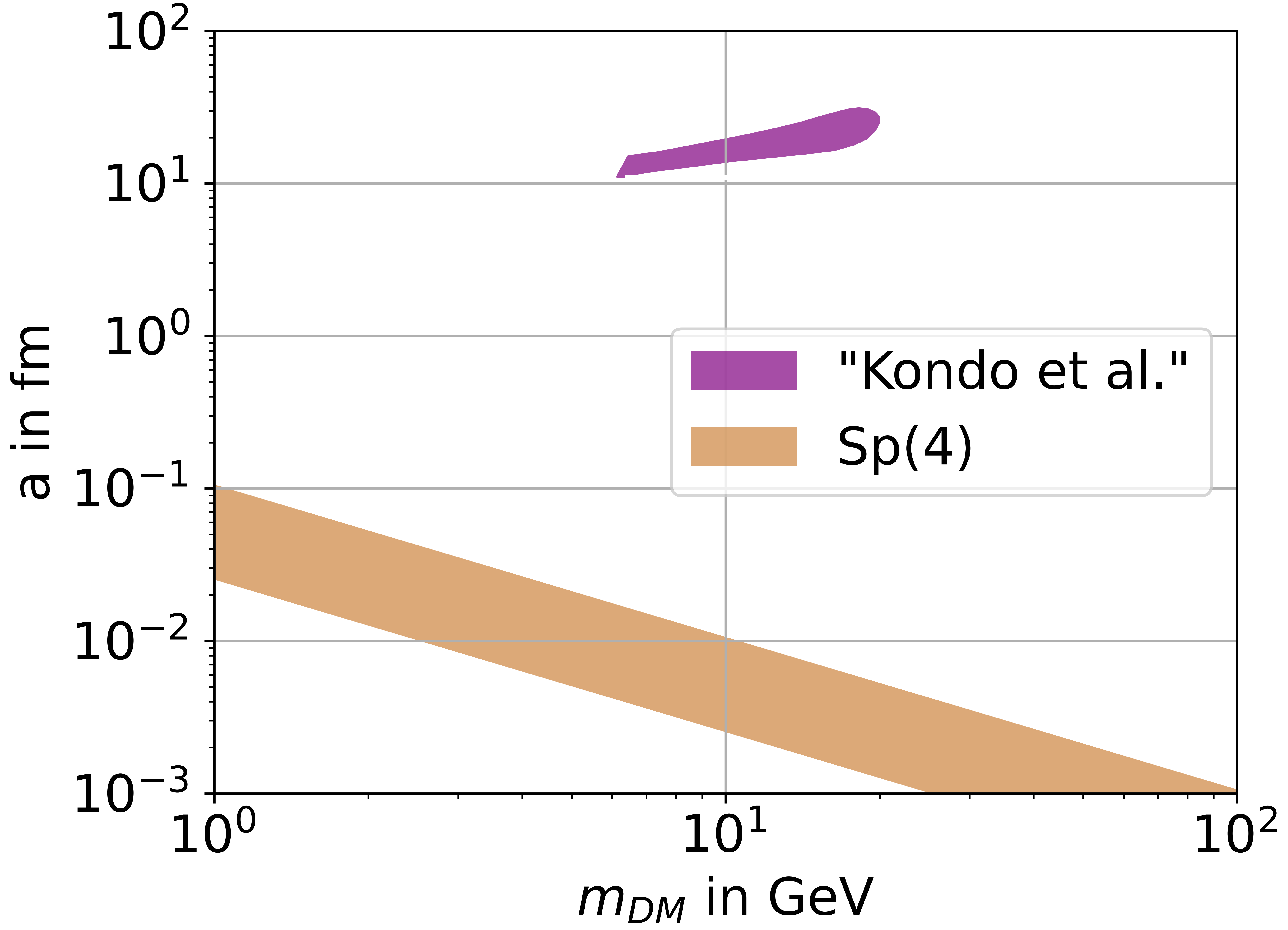
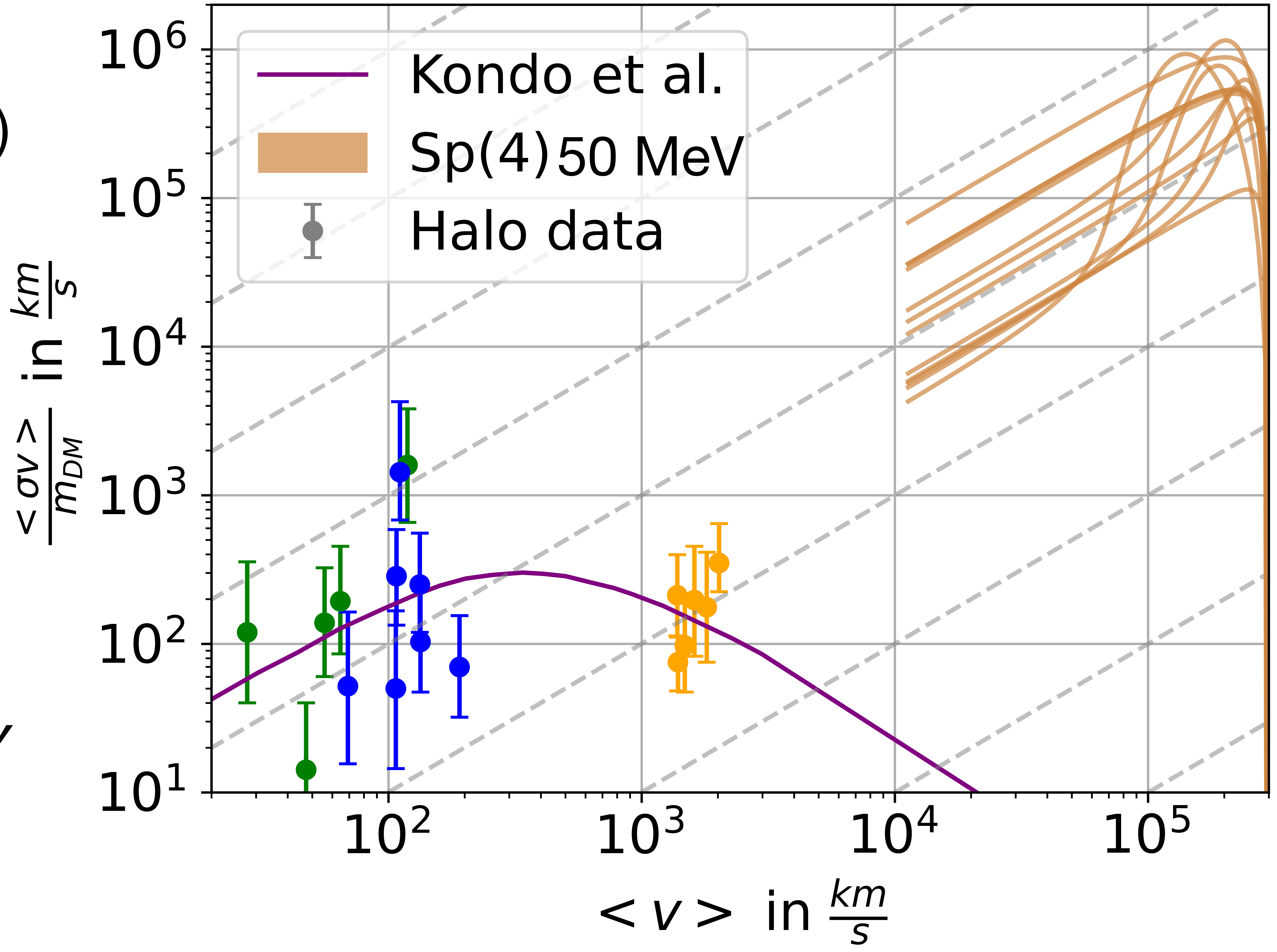


Kondo et al: J. High Energ. Phys. (2022)

Comparison to halo data

- Effective range-expansion (s-wave)
- $P\cot(\delta) = -\frac{1}{a} + \frac{P^2}{2r_e} + \mathcal{O}(P^4)$
- Parameters do not agree
- Relativistic speeds:
  - $\frac{<\sigma v>}{m} = \int_1^\infty v(\gamma)\,\sigma f_{MJ}(\gamma)\,d\gamma$
- $m_{DM} \approx 50MeV$



Kondo et al: J. High Energ. Phys. (2022)

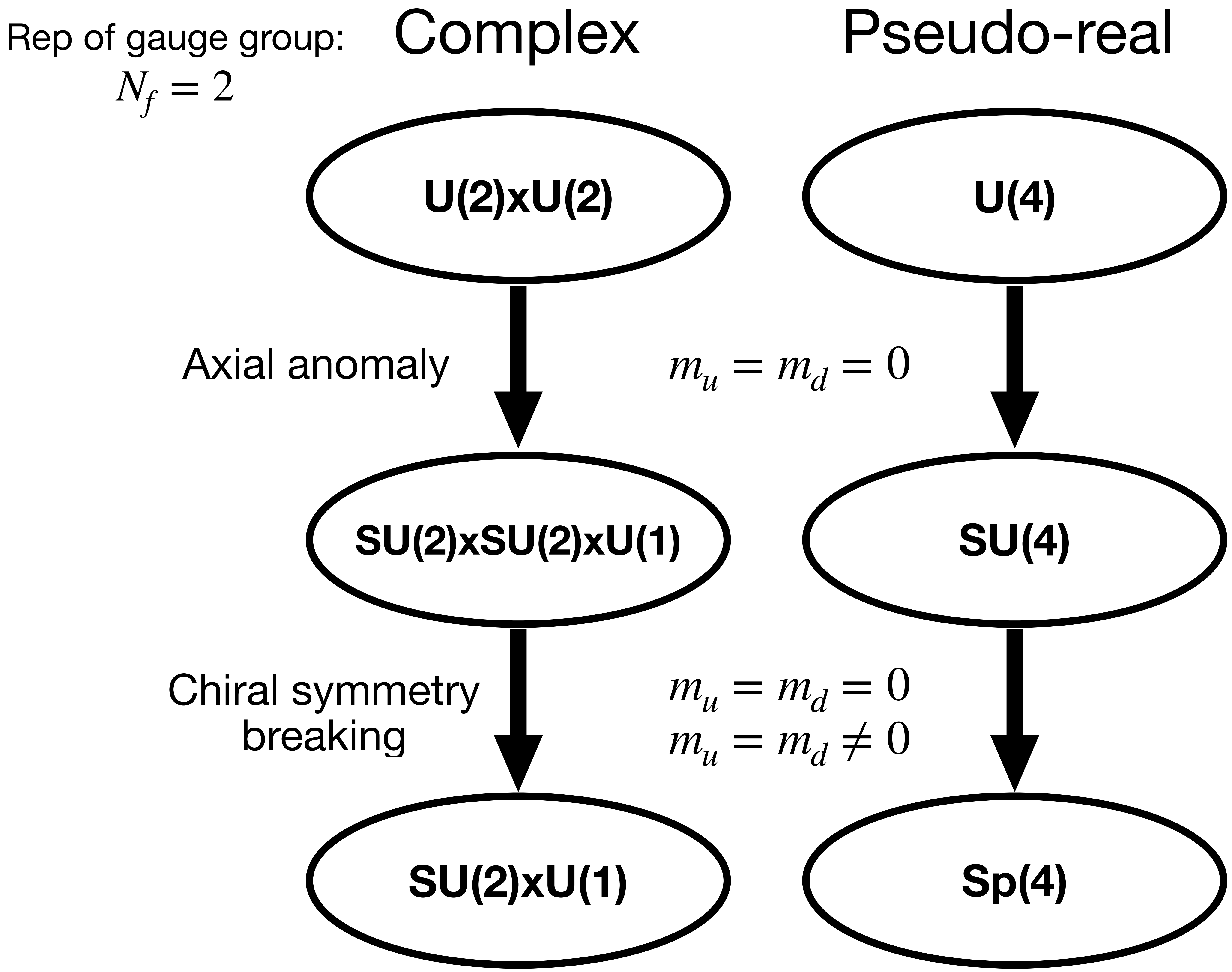
Outlook

- Lattice technicalities
- Provide low energy constants for "dark" χ-pT
- Full $\pi\pi\pi \to \pi\pi$ scattering cross section from the lattice

Thank you!

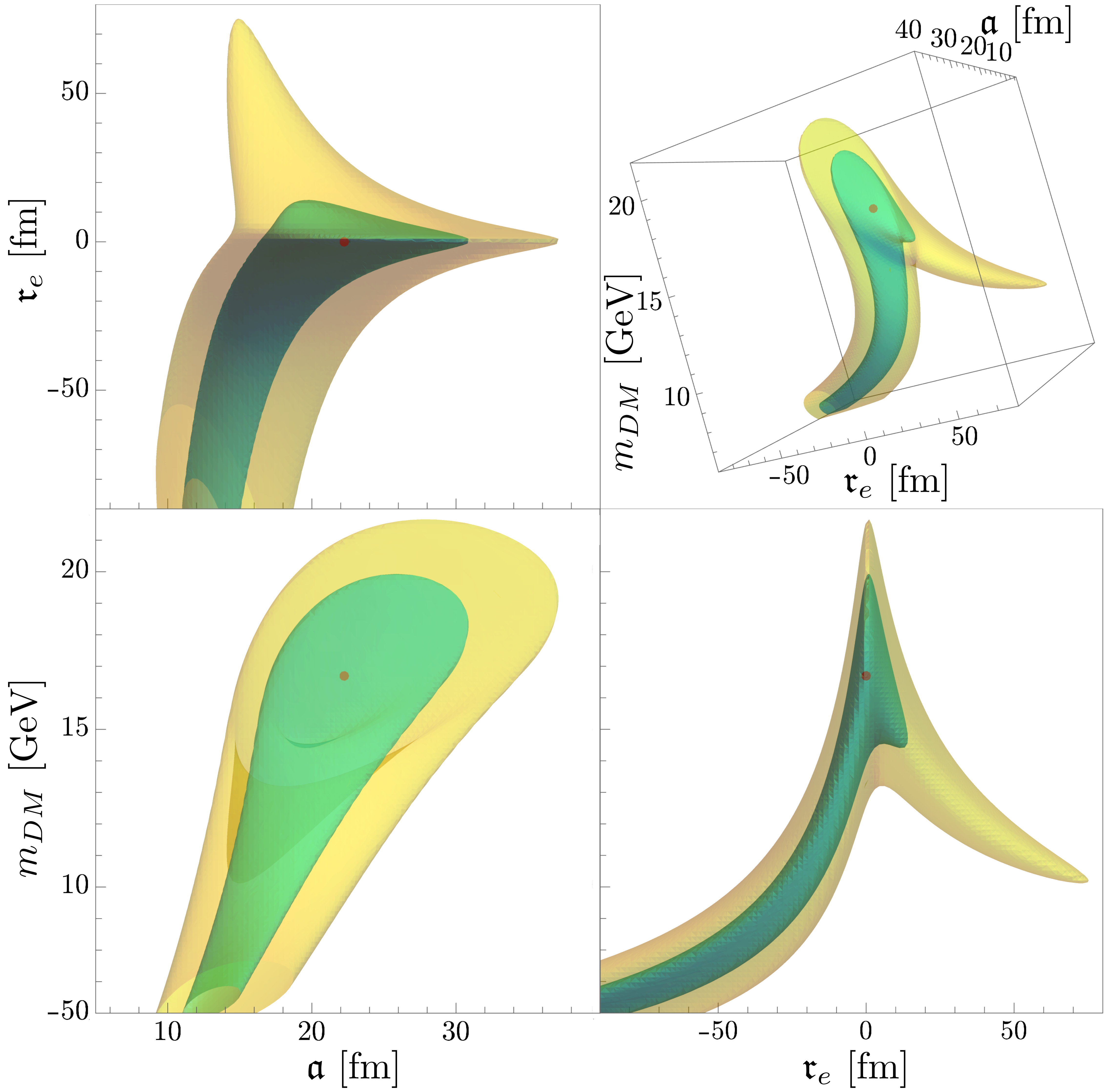
$Sp(4)_c$ vs. $Sp(4)_f$ - clarification

- Symplectic groups always have a even dimension - Sp(2N)
- Flavour symmetry:
  - Needed for symmetry breaking pattern
- Gauge symmetry: Needed for the pseudo-real representation
  - Also SU(2) or Sp(6) for example possible



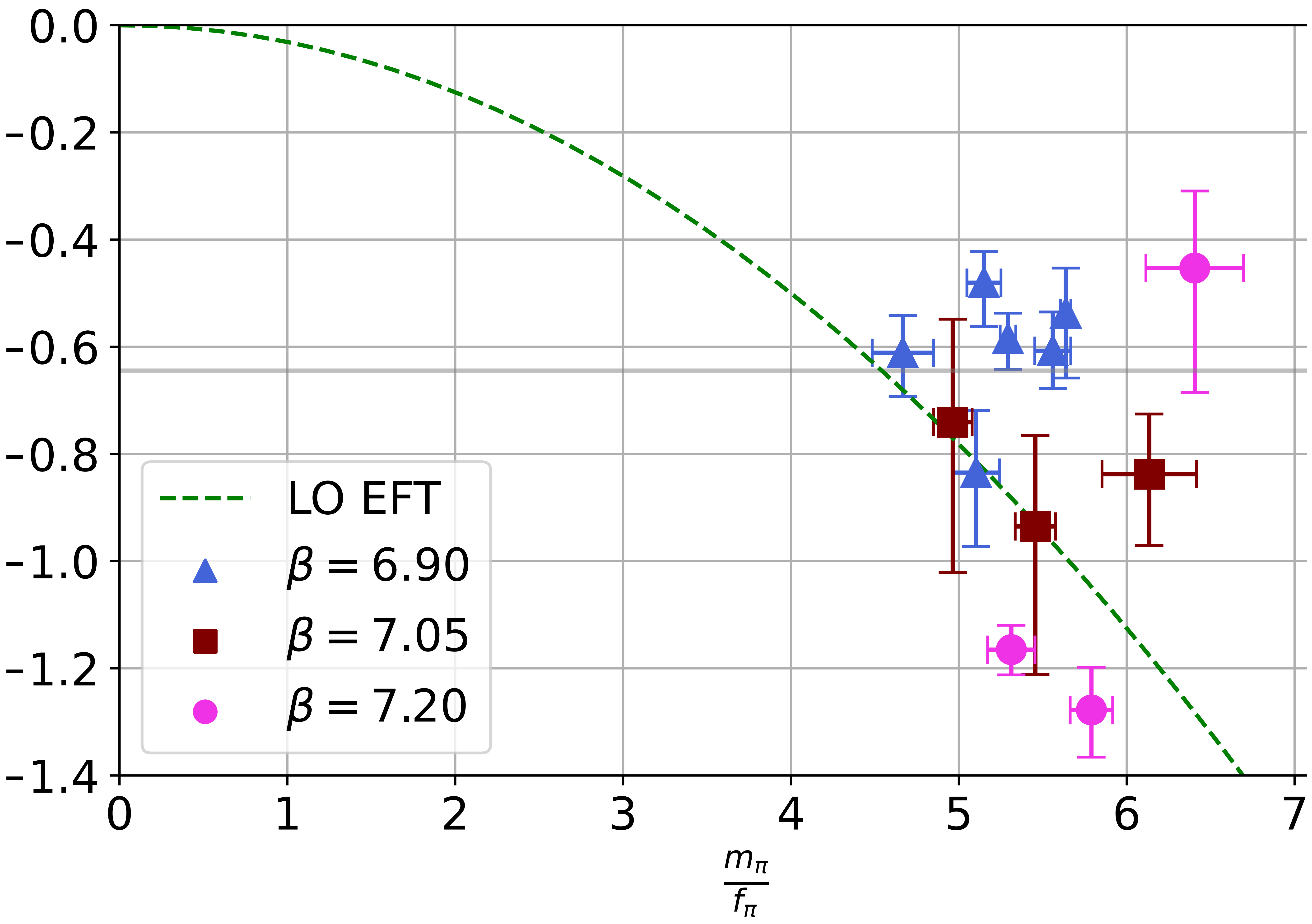
Comparison to halo data

- Effective range-expansion (s-wave)
- $$k\cot(\delta) = -\frac{1}{a} + \frac{k^2}{2r_e} + \mathcal{O}(k^4)$$
- Best fit:
  - $a = 22.2$ fm
  - $r_e = -2.59 \times 10^{-3}$ fm
  - $m_{DM} = 16.7 GeV$



Comparison chiral perturbation theory

- χpT prediction:
  - $$a_0 m_\pi = -\frac{1}{32}\left(\frac{m_\pi}{f_\pi}\right)^2$$
- Pion mass on edge or beyond validity

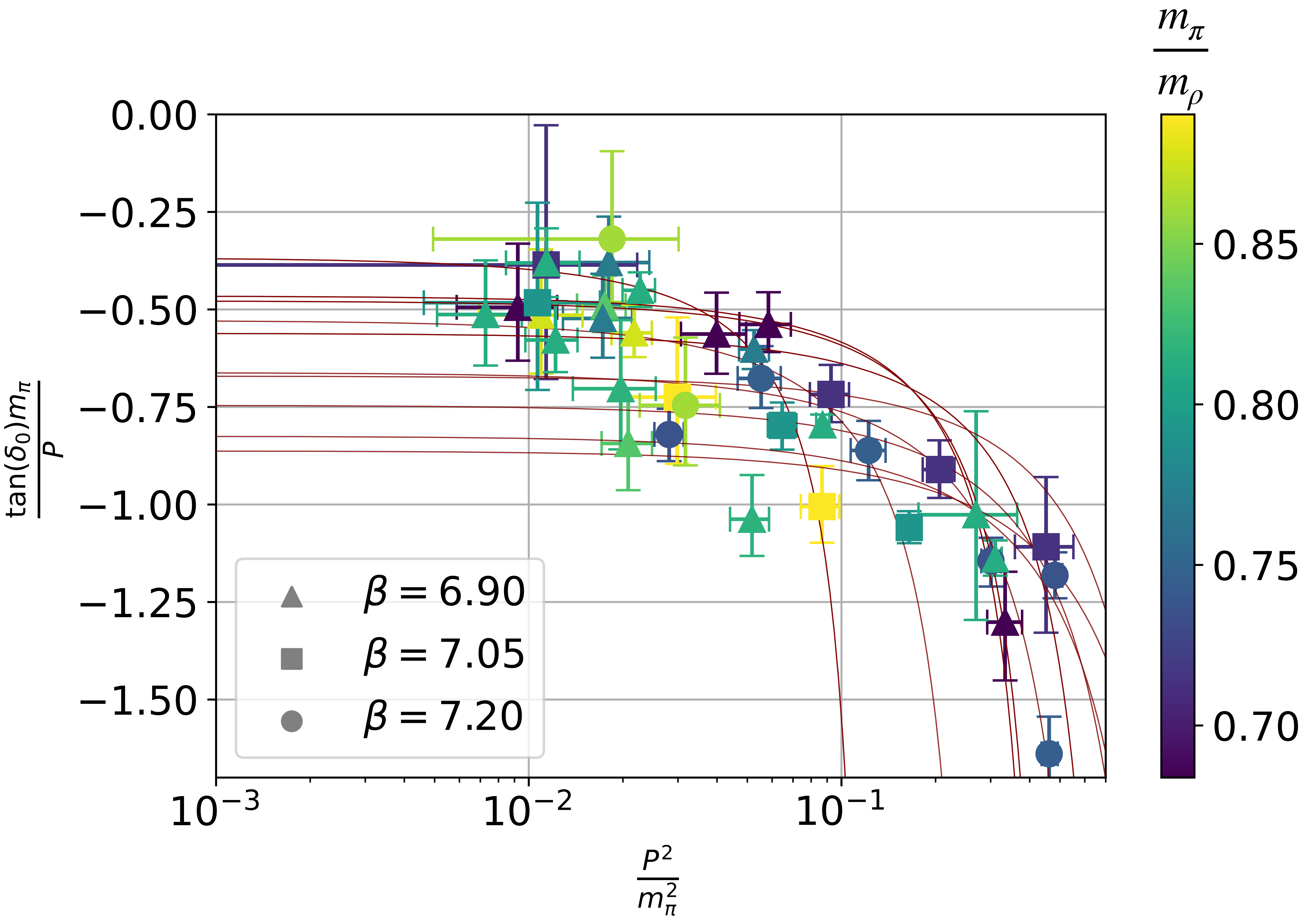


Bijens et al.: JHEP (03:028, 2011)

Comparison to astrophysical constraints

- Scattering length:
- $a_0 m_\pi = -0.65^{+0.2}_{-0.3}$
- $\frac{\sigma}{m} < 0.19 \frac{cm^2}{g}$
- Fixes the lattice constant

➡ $m_{DM} > 115 MeV$



Eckert et al.: A&A 666, A41 (2022)

5 dark Pions

$Sp(4)_f$

- Pions form a 5-plet of the flavour symmetry
  - $\pi^+, \pi^0, \pi^-, \Pi_{ud}, \Pi_{\bar{u}\bar{d}}$
- What are the possible scattering channels?
- Tensor products of the corresponding representations
- 3 Isospin channels in $\pi\pi$:
- I=0 (1-dim), I=1 (10-dim), I=2 (14-dim)

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$
$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$
$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$\pi\pi \rightarrow \pi\pi$ (I=0,1,2)
$\pi\pi \rightarrow \rho$ (I=1)
$\pi\pi \rightarrow \pi\pi\pi$ (I=1)
$\pi\pi \rightarrow \pi\pi\rho$ (I=0,1,2)
etc.

Phenomenology of scattering channels

$$Sp(4)_f$$

- I=2 (14-dim):
  - (Probably) contributes most to $\pi\pi$-scattering
  - 14 out of 25 possible combinations of Pions
- Considered in this talk

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$
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$\pi\pi \rightarrow \pi\pi\pi$ (I=1)

$\pi\pi \rightarrow \pi\pi\rho$ (I=0,1,2)

etc.

Phenomenology of scattering channels

- I=0 (1-dim):
  - (Probably) no large contribution to $\pi\pi$-scattering
  - Mixing with the „singlet“
  - Numerically challenging („connected diagrams“)
- Not considered in this work

$$Sp(4)_f$$

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$\pi\pi \to \rho$ (I=1)

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$\pi\pi \to \pi\pi\rho$ (I=0,1,2)

etc.

Phenomenology of scattering channels

- I=1 (10-dim):
  - Mixing with the Rho
  - $\pi\pi\pi \to \pi\pi$
  - No contribution to $\pi\pi$-s-wave scattering
- Tackled in the future

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$
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$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

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$\pi\pi \to \pi\pi\rho$ (I=0,1,2)

etc.

Phenomenology of scattering channels

- I=2:
  - Makes up most 2 π scattering (14/25)
  - Easiest on the lattice
- I=1:
  - No s-wave scattering
  - Mixing with dark Rho
  - $\pi\pi\pi \to \pi\pi$
- I=0:
  - Mixing with the flavour singlet

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

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etc.

Lattice - Scattering

- Can be extended to the 3 particle case
- „3 particle quantization condition“
- $\det[F_3^{-1} + \mathscr{K}_3] = 0$
    - Full $2 \to 2$ information needed