Gauge invariance and Observables in Particle Physics

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What is this talk about?

- Why an invariant formulation?
  - Path integral formulation and symmetries

Review: 1712.04721 Update: 2305.01960

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- Brout-Englert-Higgs Physics

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    - Standard Model
        - Experimental signatures

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    - Qualitative changes

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  - Standard Model
    - Experimental signatures
  - Beyond the Standard Model
    - Qualitative changes
  - Quantum (super)gravity

Review: 1712.04721 Update: 2305.01960

What’s the deal?

-

Gauge symmetry

Path integral

$$Z = \int_{\Omega} D\phi \, e^{iS[\phi]}$$

Path integral

Integral over all space-time histories of the universe

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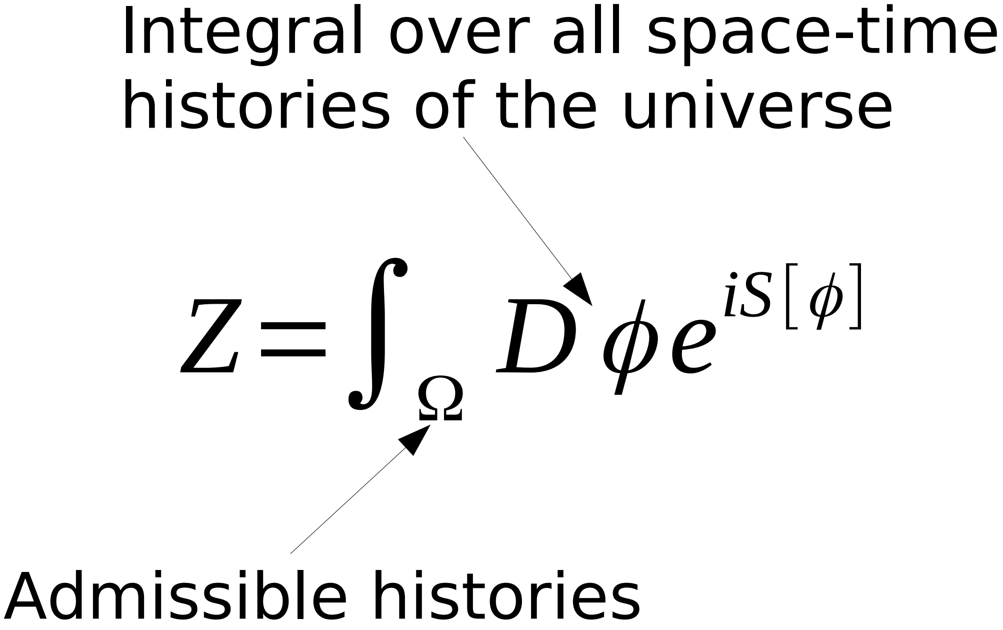
Path integral



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Admissible histories

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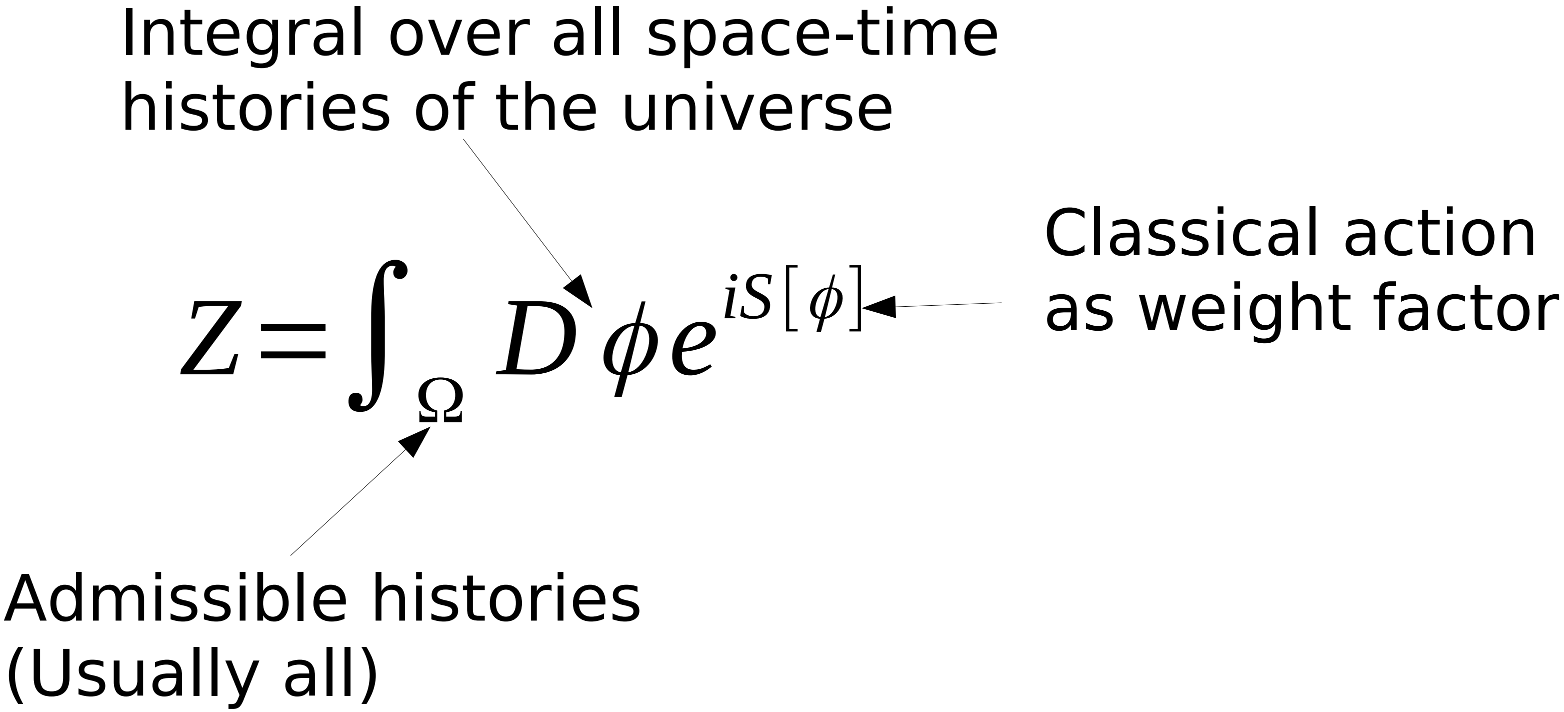

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Classical action
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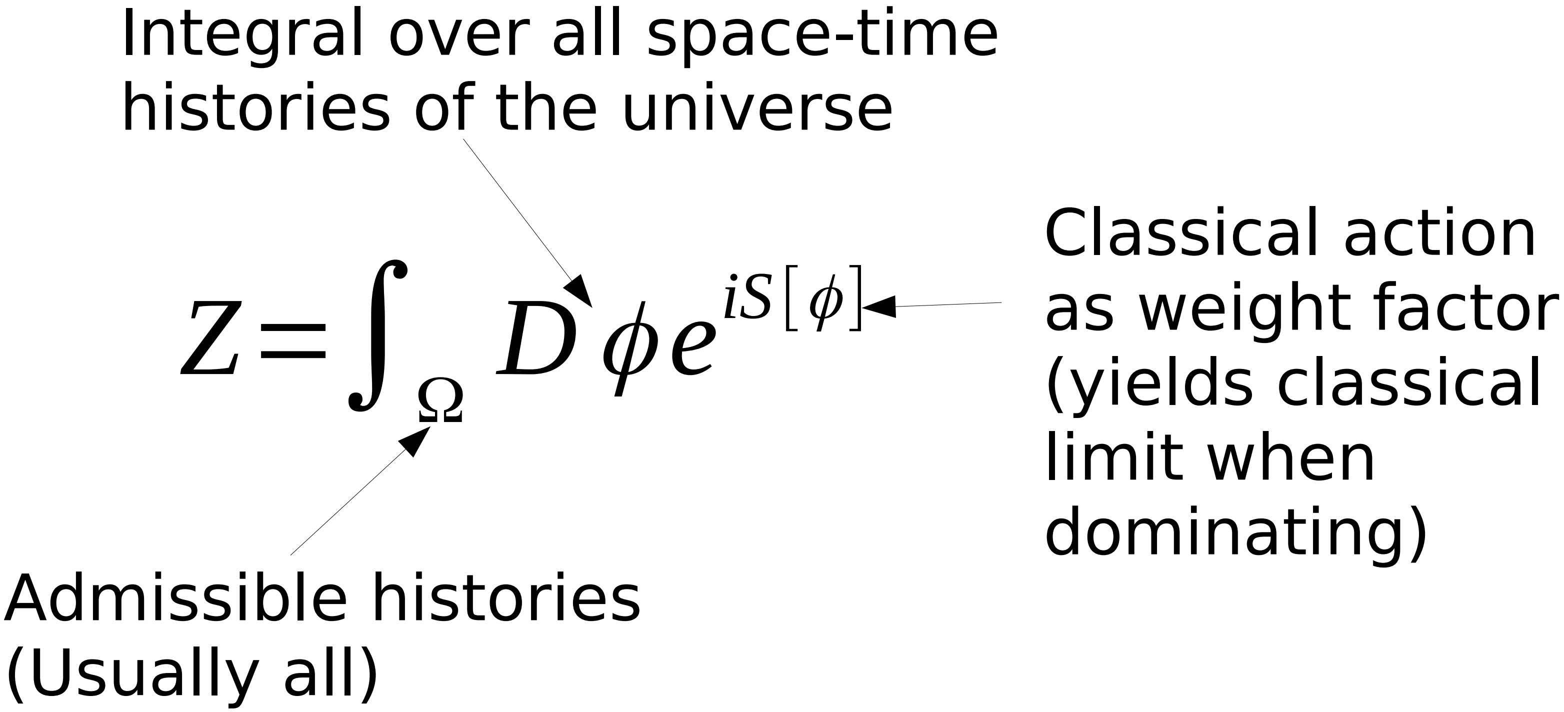

Integral over all space-time histories of the universe

$$Z=\int_{\Omega} D\phi e^{iS[\phi]}$$

Classical action as weight factor (yields classical limit when dominating)

Admissible histories (Usually all)

Path integral

$$\langle \phi(x) \ldots \phi(z) \rangle = \int_{\Omega} D\phi \, \phi(x) \ldots \phi(z) e^{iS[\phi]}$$

Expectation values are weighted averages
over space-time histories

Path integral

Dependencies on special events
is only due to external choices

$$\langle \phi(x) \ldots \phi(z) \rangle = \int_{\Omega} D\phi \, \phi(x) \ldots \phi(z) \, e^{iS[\phi]}$$

Expectation values are weighted averages
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Path integral and global symmetries

[Review: Maas'17]

$$Z=\int_{\Omega} D\phi^{a}\, e^{iS[\phi]}$$

Path integral and global symmetries

[Review: Maas'17]

Field – transforms linearly under a group $\phi^a \rightarrow G^{ab}\,\phi^b$

$$Z = \int_\Omega D\boxed{\phi^a}\, e^{iS[\phi]}$$

Path integral and global symmetries

[Review: Maas'17]

Measure is invariant
- no anomalies

$$Z=\int_{\Omega} D\phi^{a} e^{iS[\phi]}$$

Action is invariant

$$S[\phi]=S[G\phi]$$

Path integral and global symmetries

[Review: Maas’17]

$$Z=\int_{\Omega} D\phi^{a} e^{iS[\phi]}$$

Integration range
- contains all orbits $G\phi$

Path integral and global symmetries

[Review: Maas'17]

$$\langle \phi^b(x) \rangle = \int_\Omega D\phi^a e^{iS[\phi]} \phi^b(x)$$

Path integral and global symmetries

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$$\langle \phi^b(x) \rangle = \int_{\Omega} D\phi^a e^{iS[\phi]} \phi^b(x)$$

- There is no preferred point on the group orbit
  - There is no absolute orientation/frame in the internal space
  - Does not change when averaging over position
  - There is no absolute charge

Path integral and global symmetries

[Review: Maas'17]

$$\langle \phi^b(x) \rangle = \int_\Omega D\phi^a e^{iS[\phi]} \phi^b(x) = 0$$

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Path integral and global symmetries

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$$\langle \phi^b(x)\phi^c(y)\rangle = \int_\Omega D\phi^a e^{iS[\phi]}\phi^b(x)\phi^c(y)$$

- Relative charge measurement averaged over all possible starting points

Path integral and global symmetries

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- Relative charge measurement averaged over all possible starting point
  - Vanishes because no preferred absolute starting point

Path integral and global symmetries

[Review: Maas'17]

$$\langle \delta_{bc} \phi^b(x) \phi^c(y) \rangle$$

$$= \int_\Omega D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(x) \phi^c(y)$$

- Group-invariant quantity
  - Measures relative orientation
  - Created from an invariant tensor δ_{ab}

Path integral and global symmetries

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Path integral and local symmetries

[Review: Maas'17]

$$Z=\int_{\Omega} D\phi^{a} e^{iS[\phi]}$$

Path integral and local symmetries

[Review: Maas'17]

Field – transforms locally under a group $\phi^a(x) \rightarrow G^{ab}(x)\phi^b(x)$

$$Z = \int_\Omega D\phi^a e^{iS[\phi]}$$

Path integral and local symmetries

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Path integral and local symmetries

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$$= \int_\Omega D\phi^a e^{iS[\phi]} \delta_{bc} \phi^b(x) \phi^c(y) = 0$$

- No longer invariant under gauge transformations
  - Vanishes just as any other non-invariant quantity

Path integral and local symmetries

[Review: Maas'17]

$$\langle \phi^b(x) \overbrace{U^{bc}(x,y)}^{\text{Transporter}} \phi^c(y) \rangle$$

$$= \int_\Omega D\phi^a DU \, e^{iS[\phi,U]} \phi^b(x) U^{bc}(x,y) \phi^c(y)$$

- Transporter compensates gauge transformations

Path integral and local symmetries

[Review: Maas'17]

Gauge fields

$$\langle \phi^b(x) U^{bc}(x,y) \phi^c(y) \rangle$$

$$= \int_{\Omega^{\phi,U}} D\phi^a DU \, e^{iS[\phi,U]} \phi^b(x) U^{bc}(x,y) \phi^c(y)$$

- •Transporter compensates gauge transformations
  - Implemented by gauge fields

Path integral and local symmetries

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$$\neq 0$$

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Path integral and local symmetries

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Reduced integration range

$$\langle \phi^b(x)\phi^c(y)\rangle$$

$$=\int_{\boxed{\Omega_c^{\phi,U}}} D\phi^a DU e^{iS[\phi,U]}\phi^b(x)\phi^c(y)\neq 0$$

- Gauge-fixing to have non-zero results without transporters
- Reduction of integration region by gauge fixing
  - Arbitrary choice of coordinates

Path integral and local symmetries

[Review: Maas'17]

$$\langle \phi^b(x)\,\phi^c(y)\rangle$$

$$= \int_{\Omega_c^{\phi,U}} D\phi^a\, DU\, W(U,\phi)\, e^{iS[\phi,U]}\, \phi^b(x)\,\phi^c(y) \neq 0$$

Weight factor
E.g. Faddeev-Popov determinant

- Gauge-fixing to have non-zero results without transporters
- Reduction of integration region by gauge fixing
  - Arbitrary choice of coordinates
  - Weight factor to keep gauge-invariant quantities the same

Lessons

- Only invariant quantities are non-zero
    - All observables need to be invariant
    - Elementary fields are not invariant
    - True for local symmetries and global symmetries
    - Gauge fixing introduces preferred frames
        - Empirically not motivated

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- Only invariant quantities are non-zero
    - All observables need to be invariant
    - Elementary fields are not invariant
    - True for local symmetries and global symmetries
    - Gauge fixing introduces preferred frames
        - Empirically not motivated
- Are there consequences?

Brout-Englert-Higgs Physics
-
The Standard Model

A toy model

A toy model: Higgs sector of the SM

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- Consider an SU(2) with a fundamental scalar

A toy model: Higgs sector of the SM

- Consider an SU(2) with a fundamental scalar
- Essentially the standard model Higgs

$$L=-\frac{1}{4}W^{a}_{\mu\nu}W^{\mu\nu}_{a}$$

$$W^{a}_{\mu\nu}=\partial_{\mu}W^{a}_{\nu}-\partial_{\nu}W^{a}_{\mu}+gf^{a}_{bc}W^{b}_{\mu}W^{c}_{\nu}$$

- Ws W^{a}_{μ} W

- Coupling g and some numbers f^{abc}

A toy model: Higgs sector of the SM

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$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}$$

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- Higgs h_i h
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A toy model: Higgs sector of the SM

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$$L = -\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} + \left(D^{ij}_\mu h^j\right)^+ D^\mu_{ik} h_k + \lambda\left(h^a h_a^+ - v^2\right)^2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^a_{bc} W^b_\mu W^c_\nu$$

$$D^{ij}_\mu = \delta^{ij} \partial_\mu - i g W^a_\mu t^{ij}_a$$

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- Couplings g, v, λ and some numbers f^{abc} and t^{ij}_{a}
- Parameters selected for a BEH effect

A toy model: Symmetries

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- Local SU(2) gauge symmetry

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $\quad$ $h\rightarrow h\Omega$

Textbook approach

- Choose parameters to get a Brout-Englert-Higgs effect

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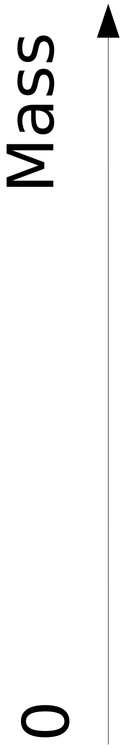
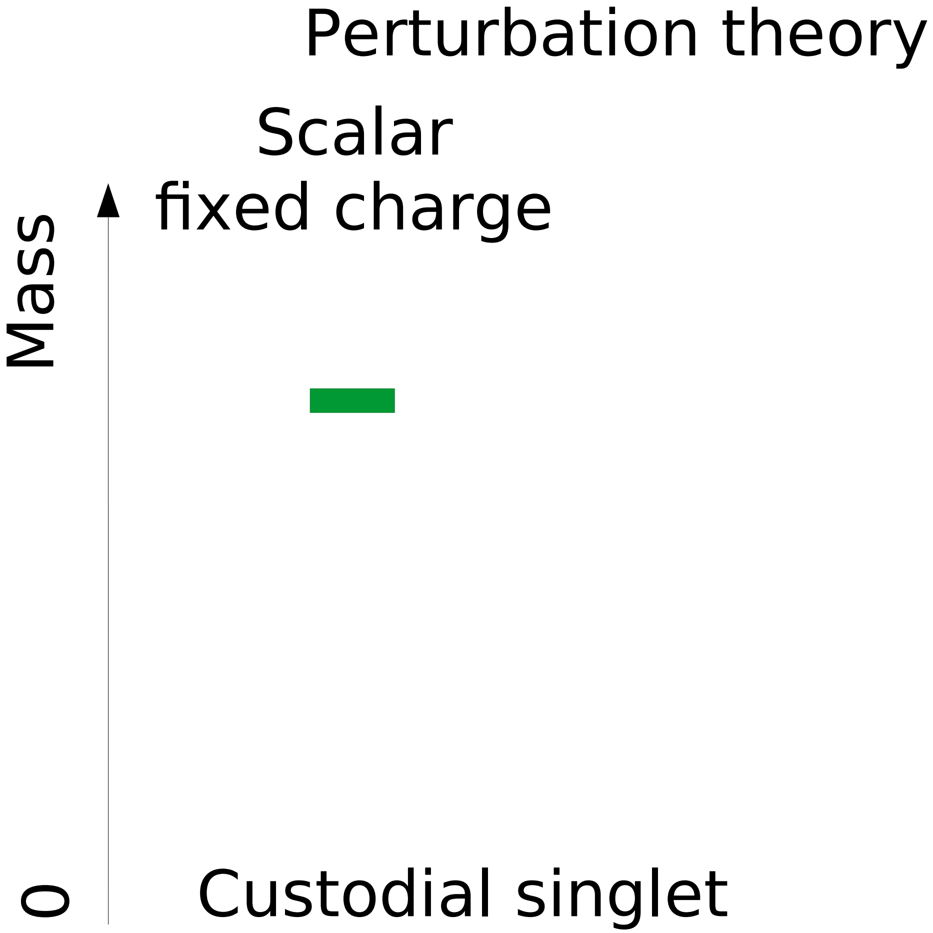
- Perform perturbation theory

Physical spectrum

Perturbation theory



Physical spectrum

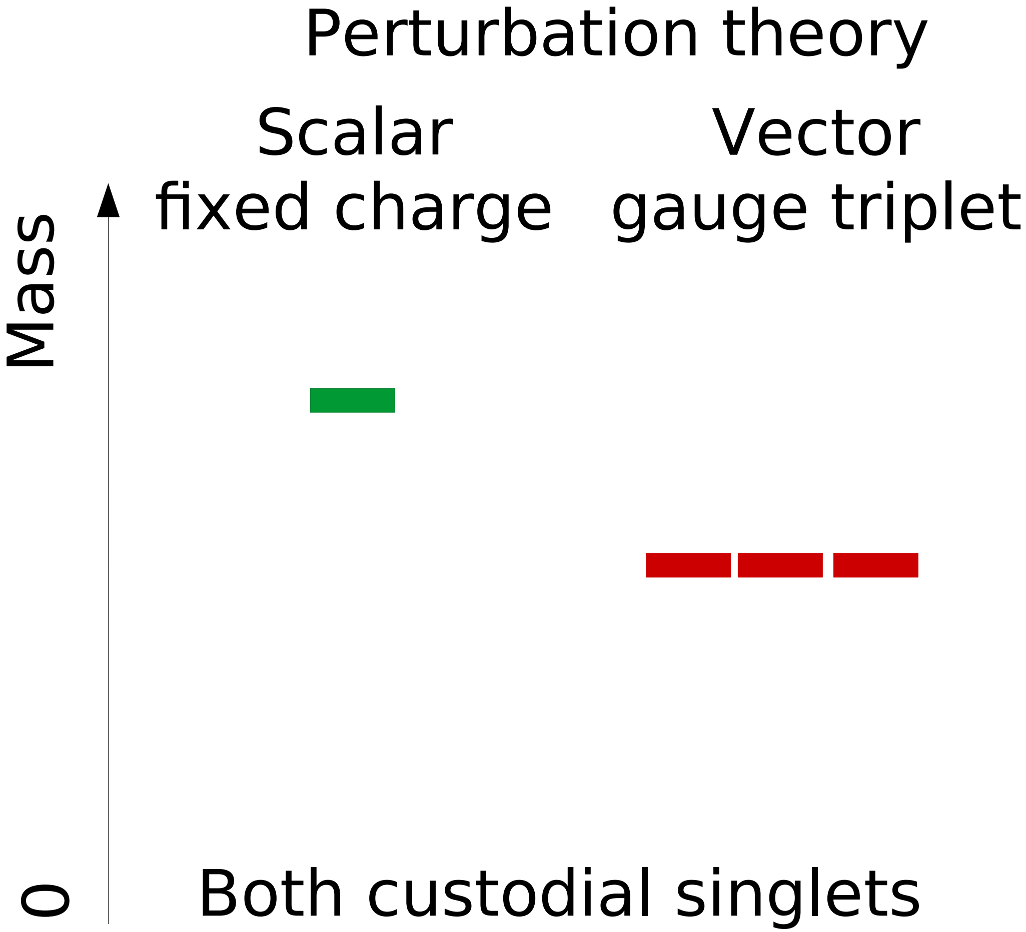


Perturbation theory

Scalar
fixed charge

Custodial singlet

Physical spectrum



Perturbation theory

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Both custodial singlets

The origin of the problem

[Fröhlich et al.'80,
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    - ...and gauge-symmetry breaking is not there

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- Physics has to be expressed in terms of manifestly gauge-invariant quantities
  - And this includes non-perturbative aspects...
  - ...even at weak coupling [Gribov’78,Singer’78,Fujikawa’82]

Physical states

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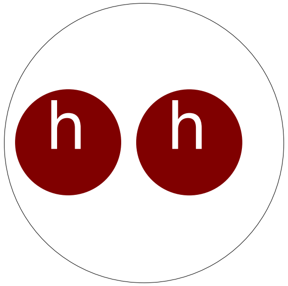

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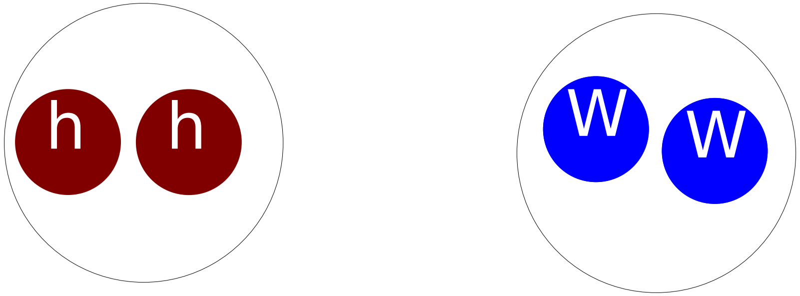

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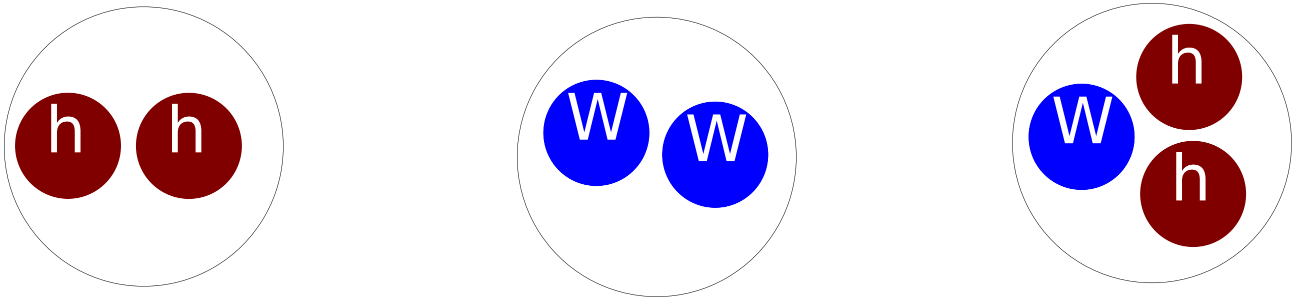

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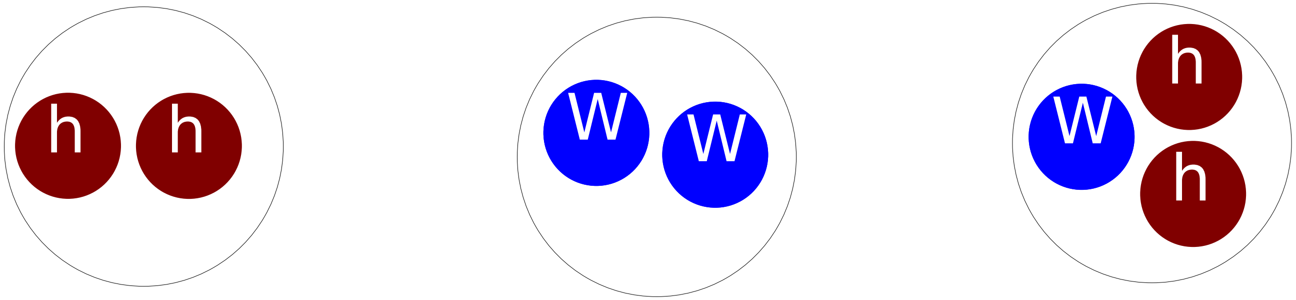

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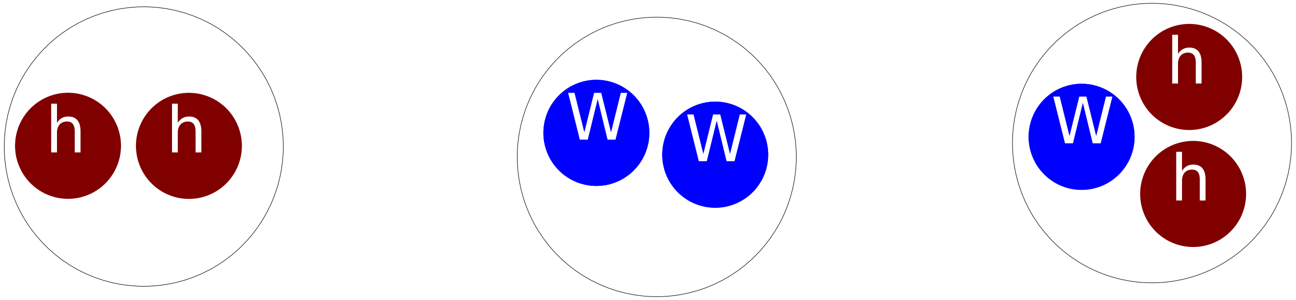

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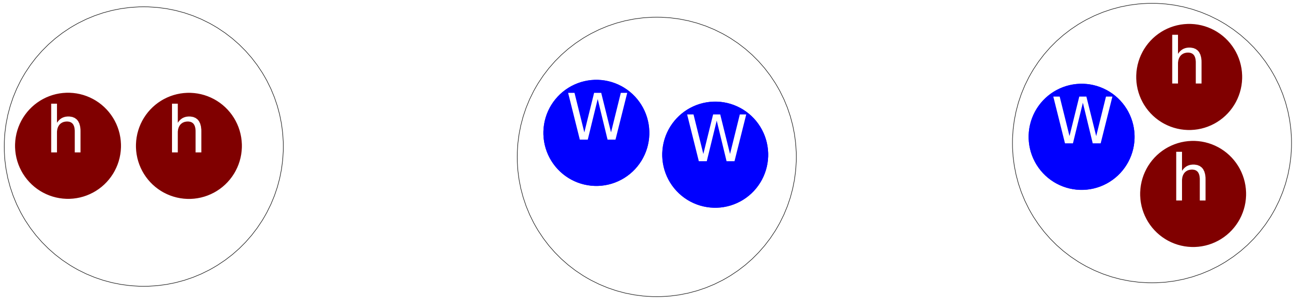

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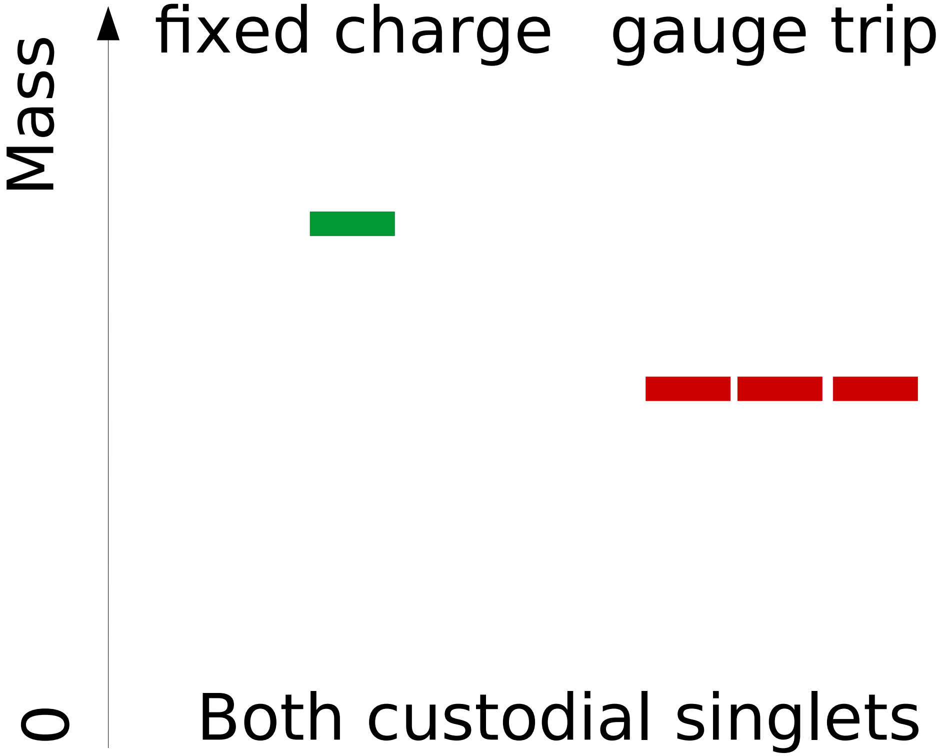
  - Think QED (hydrogen atom!)
- Can this matter?

Physical spectrum



Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Mass

0

Both custodial singlets

Remember: Experiment tells that somehow the left is correct!

Physical spectrum

Perturbation theory

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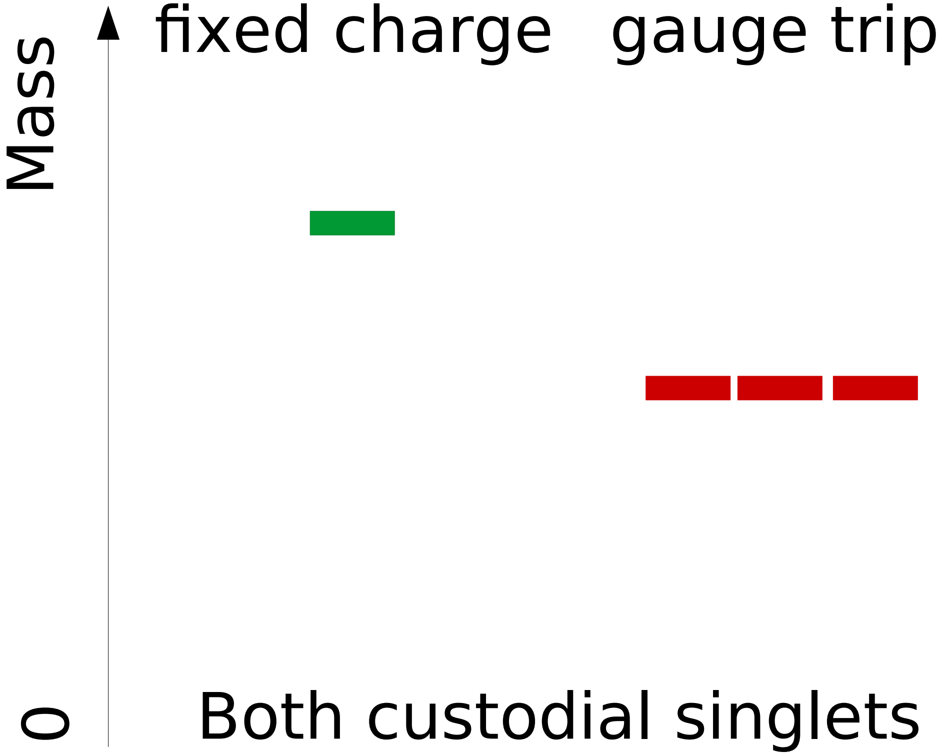
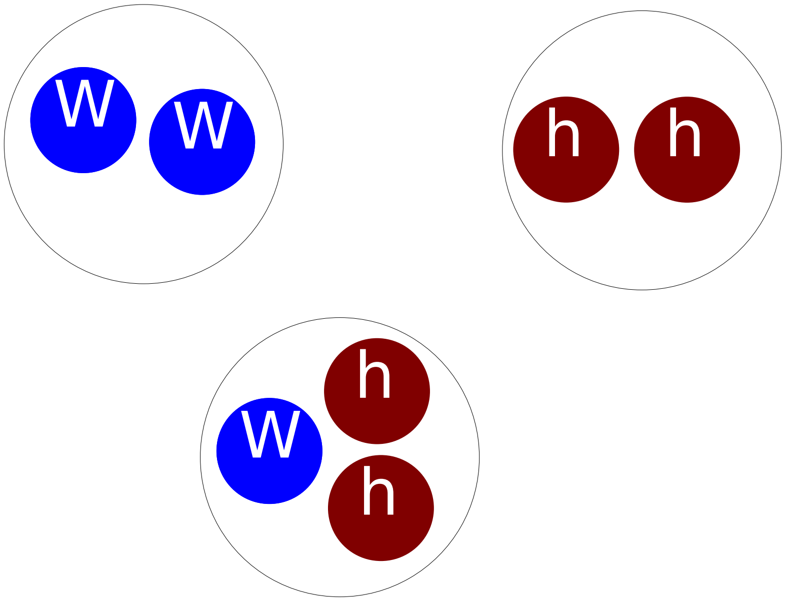
Vector
gauge triplet



Both custodial singlets

Composite (bound) states



Experiment tells that somehow the left is correct
Theory say the right is correct

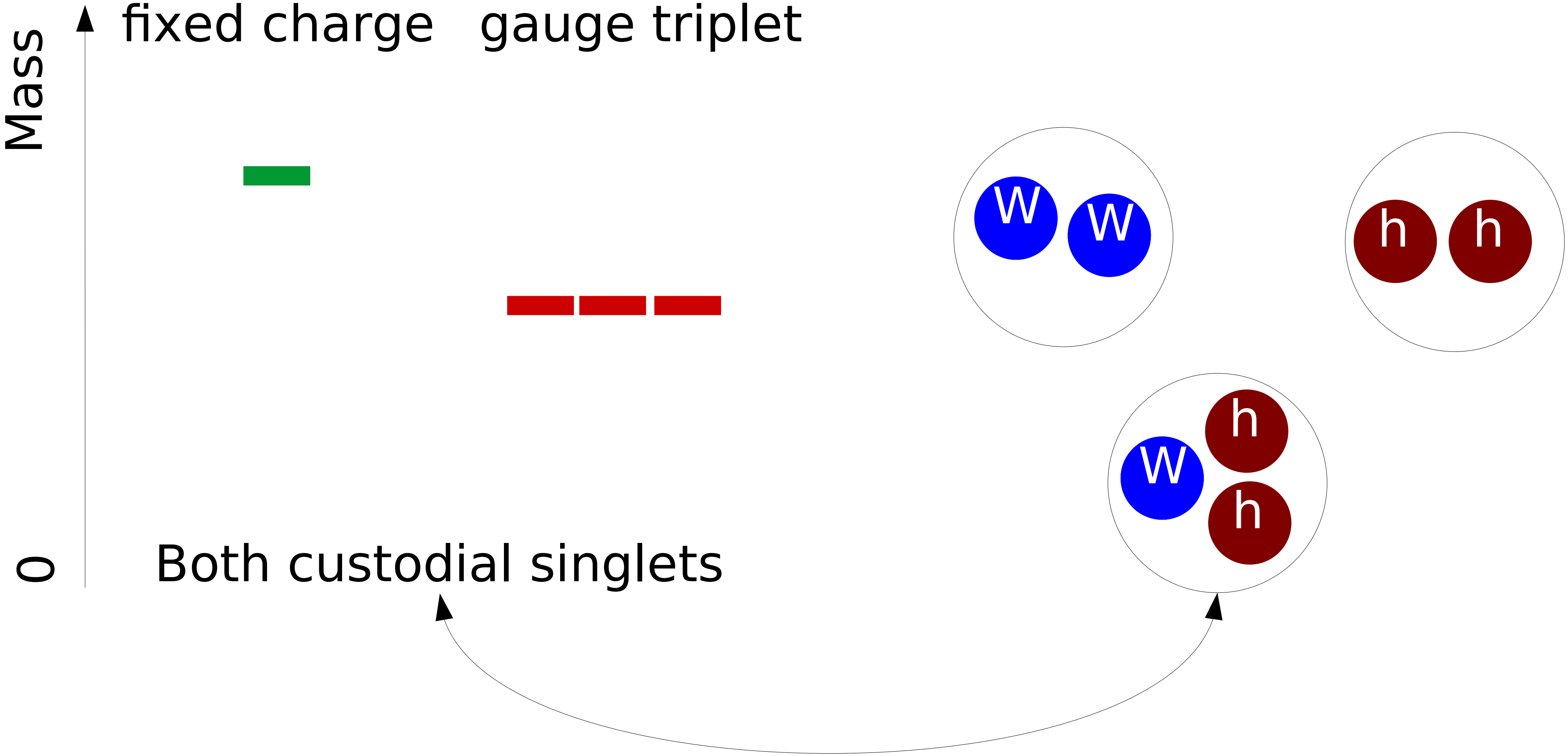
Physical spectrum

Perturbation theory

Composite (bound) states

Scalar
fixed charge

Vector
gauge triplet



Both custodial singlets

Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

Physical particles

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
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- J^{PC} and custodial charge only quantum numbers

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    - Operators limited to asymptotic, elementary, gauge-dependent states

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    - Operators limited to asymptotic, elementary, gauge-dependent states
- Formulate gauge-invariant, composite operators
  - Bound state structure – non-perturbative methods! - Lattice
    - Standard lattice spectroscopy problem
    - Standard methods
      - Smearing, variational analysis, systematic error analysis etc.
    - Very large statistics ($>10^5$ configurations)

Physical spectrum

[Maas'12, Maas & Mufti'14]

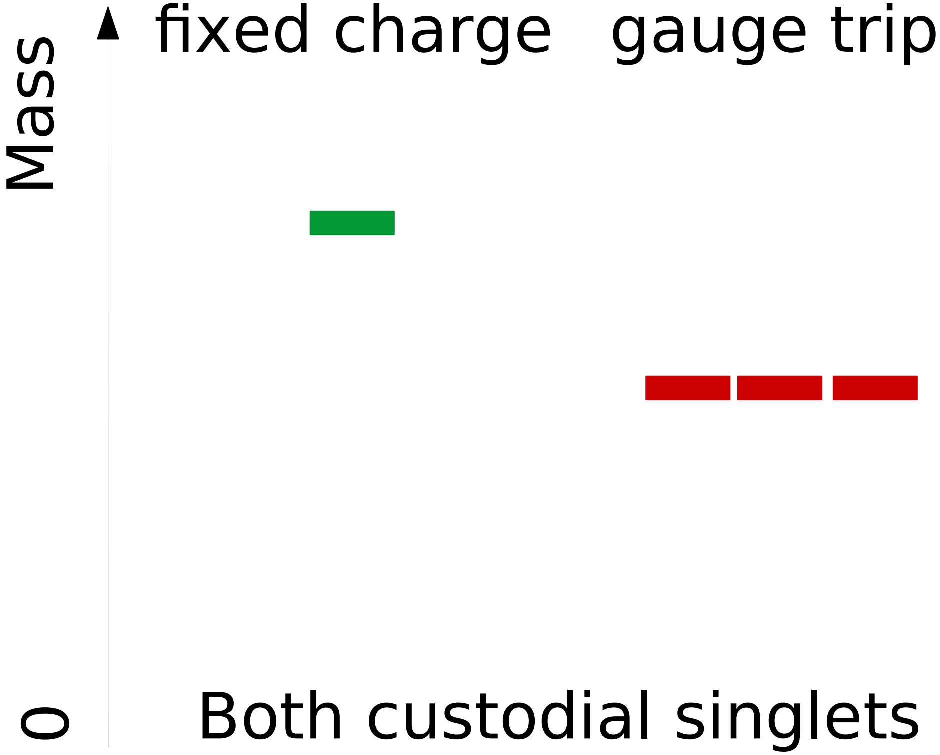
Perturbation theory

Gauge-invariant

Scalar
fixed charge

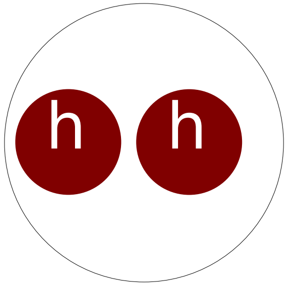
Vector
gauge triplet

Scalar
singlet



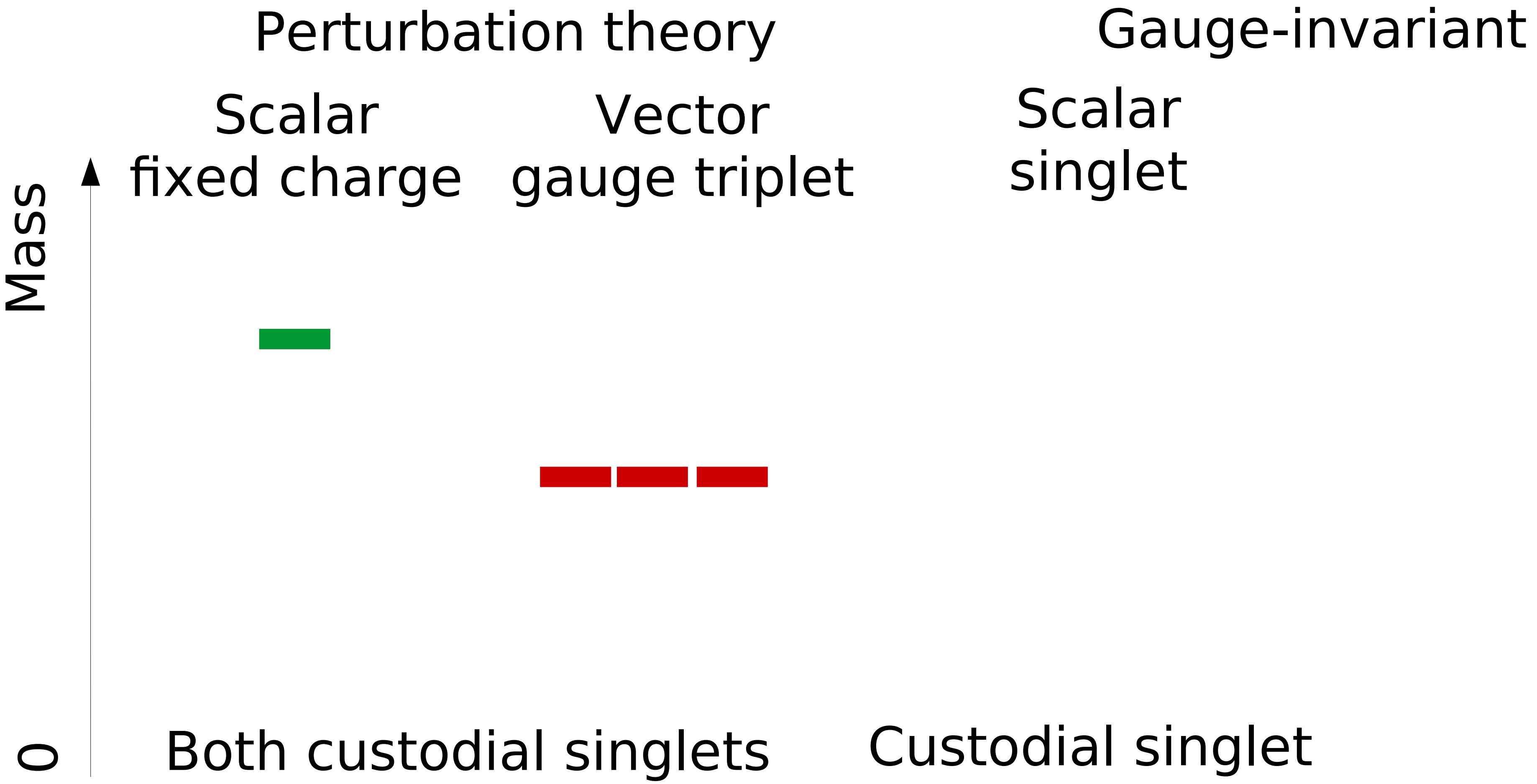
Both custodial singlets

$$h(x)^{+} h(x)$$



Physical spectrum

[Maas'12, Maas & Mufti'14]



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Custodial singlet

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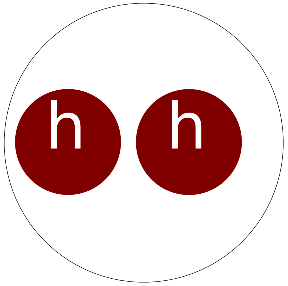

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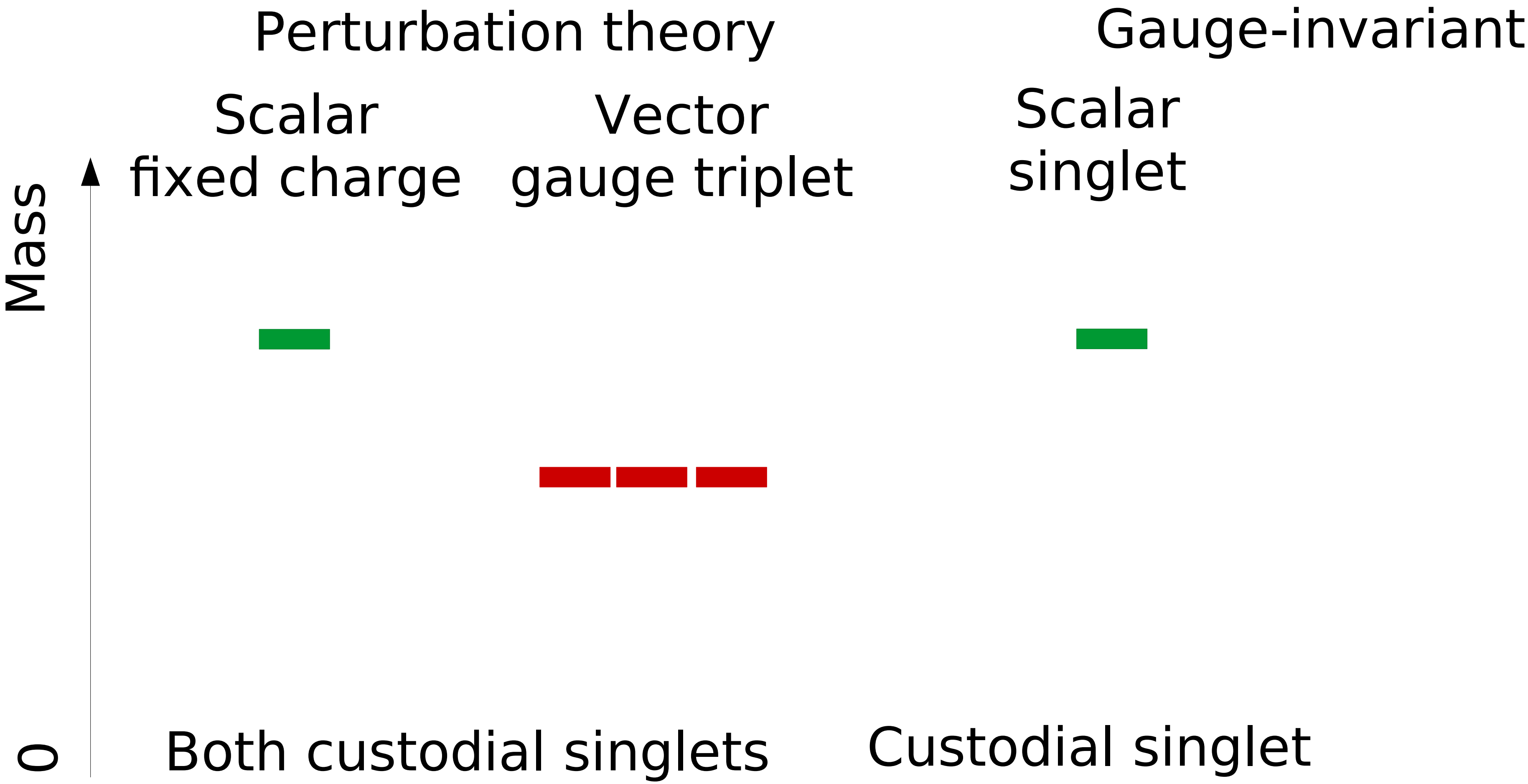

Perturbation theory

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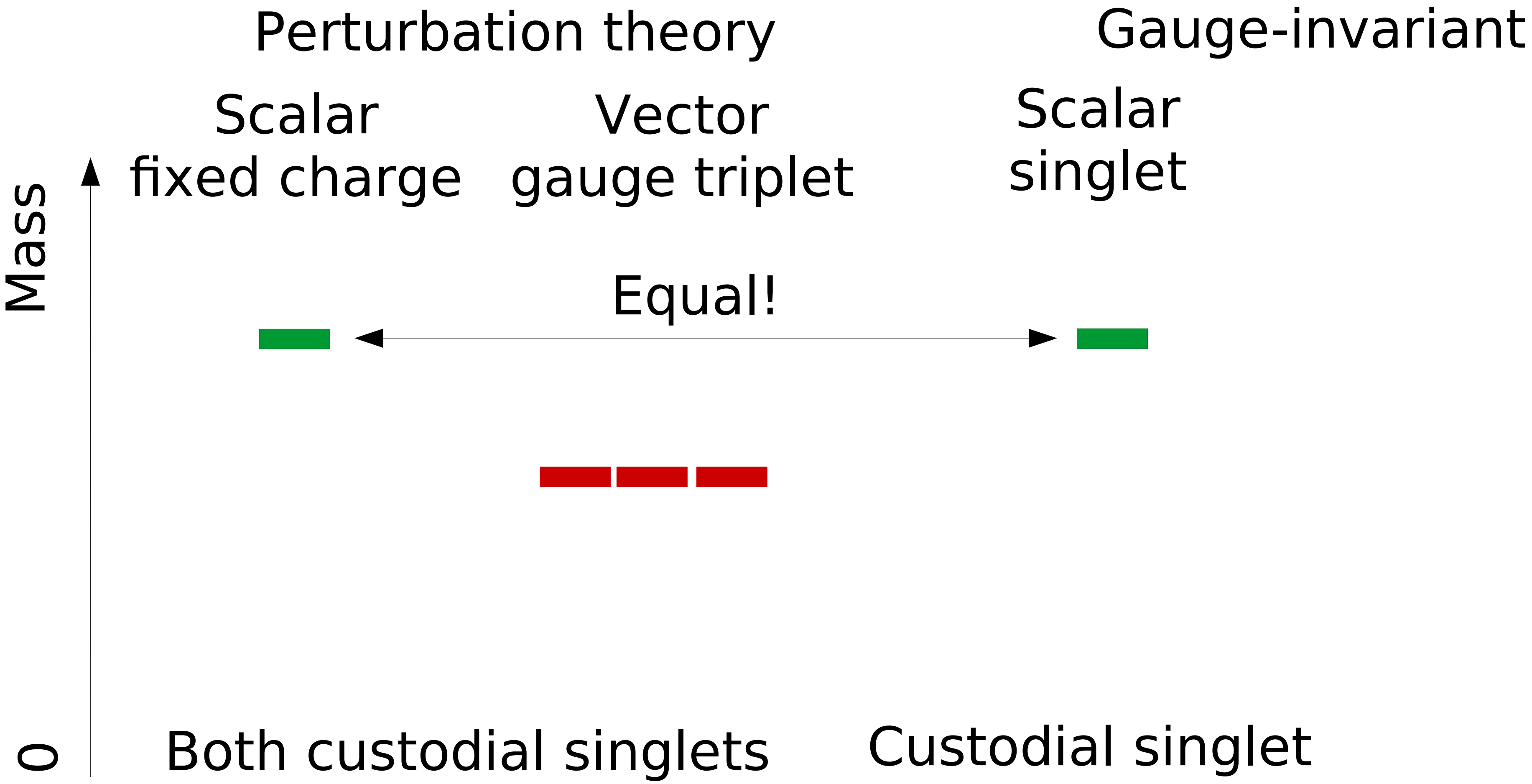


Perturbation theory

Gauge-invariant

Scalar
fixed charge

Vector
gauge triplet

Scalar
singlet

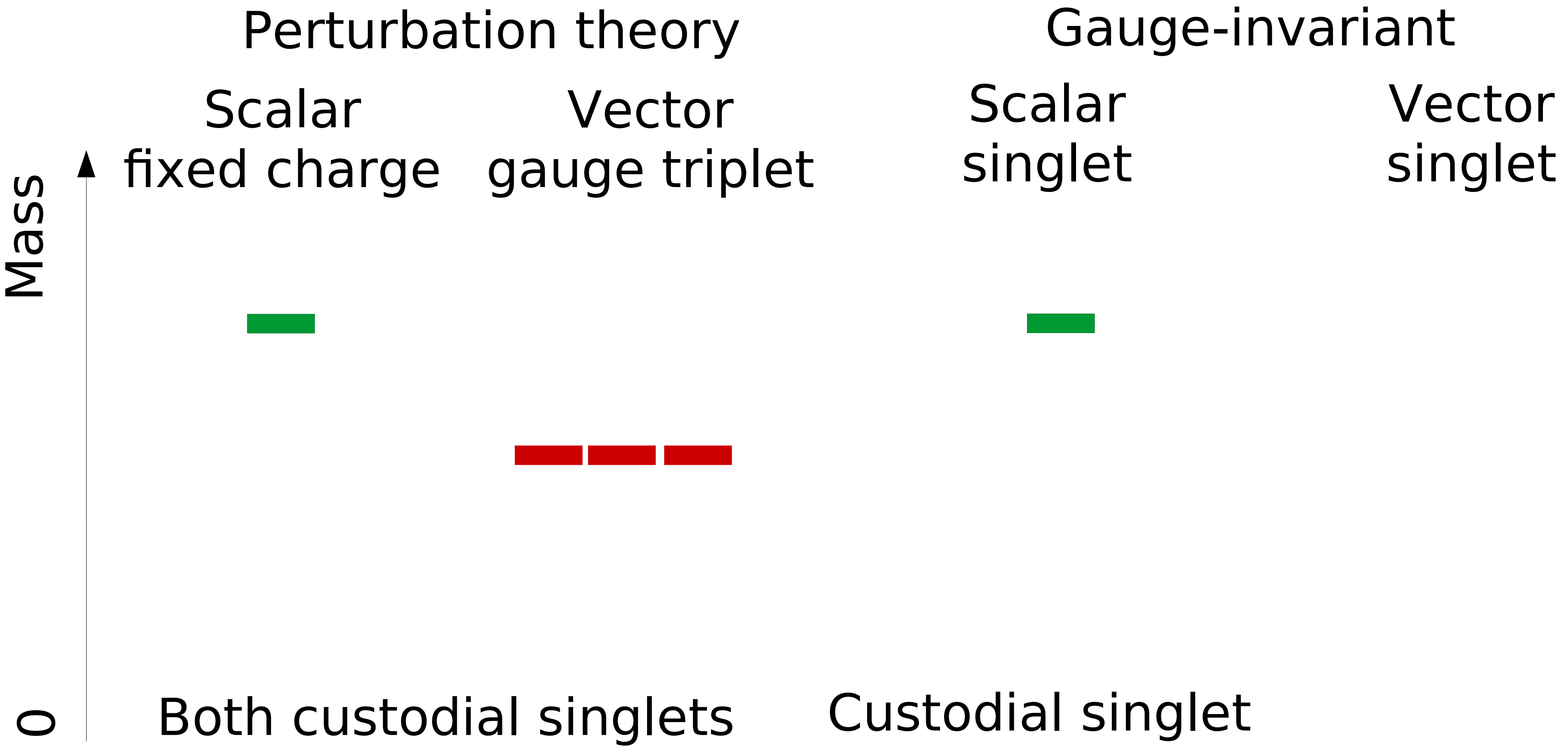
Equal!

Both custodial singlets

Custodial singlet

Physical spectrum

[Maas’12, Maas & Mufti’14]



Perturbation theory

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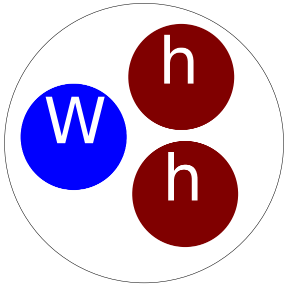
$$tr\, t^a \frac{h^+}{\sqrt{h^+ h}} D_\mu \frac{h}{\sqrt{h^+ h}}$$



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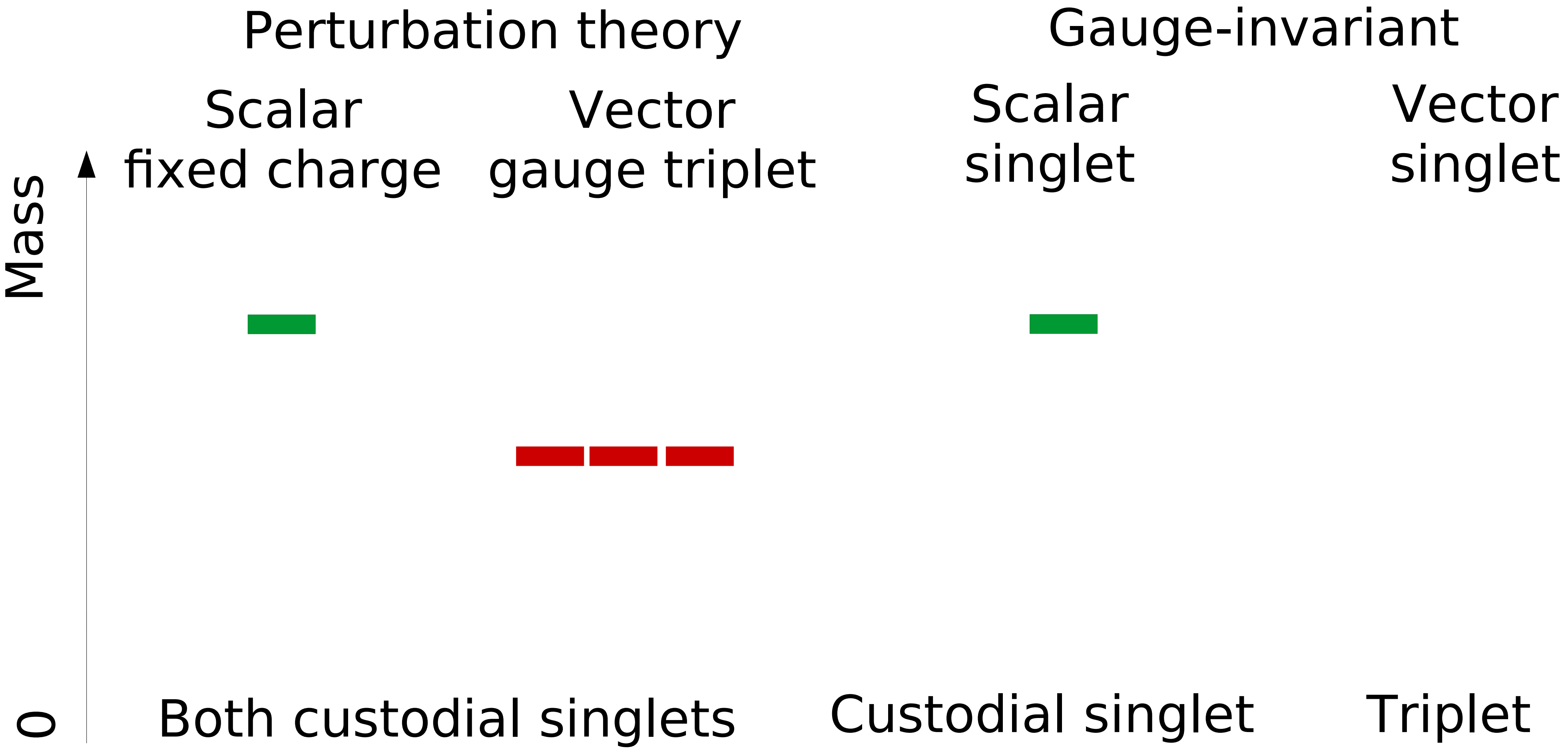

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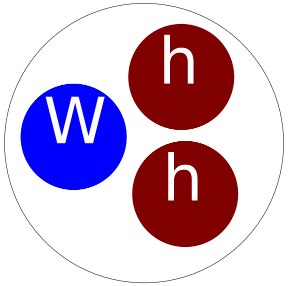

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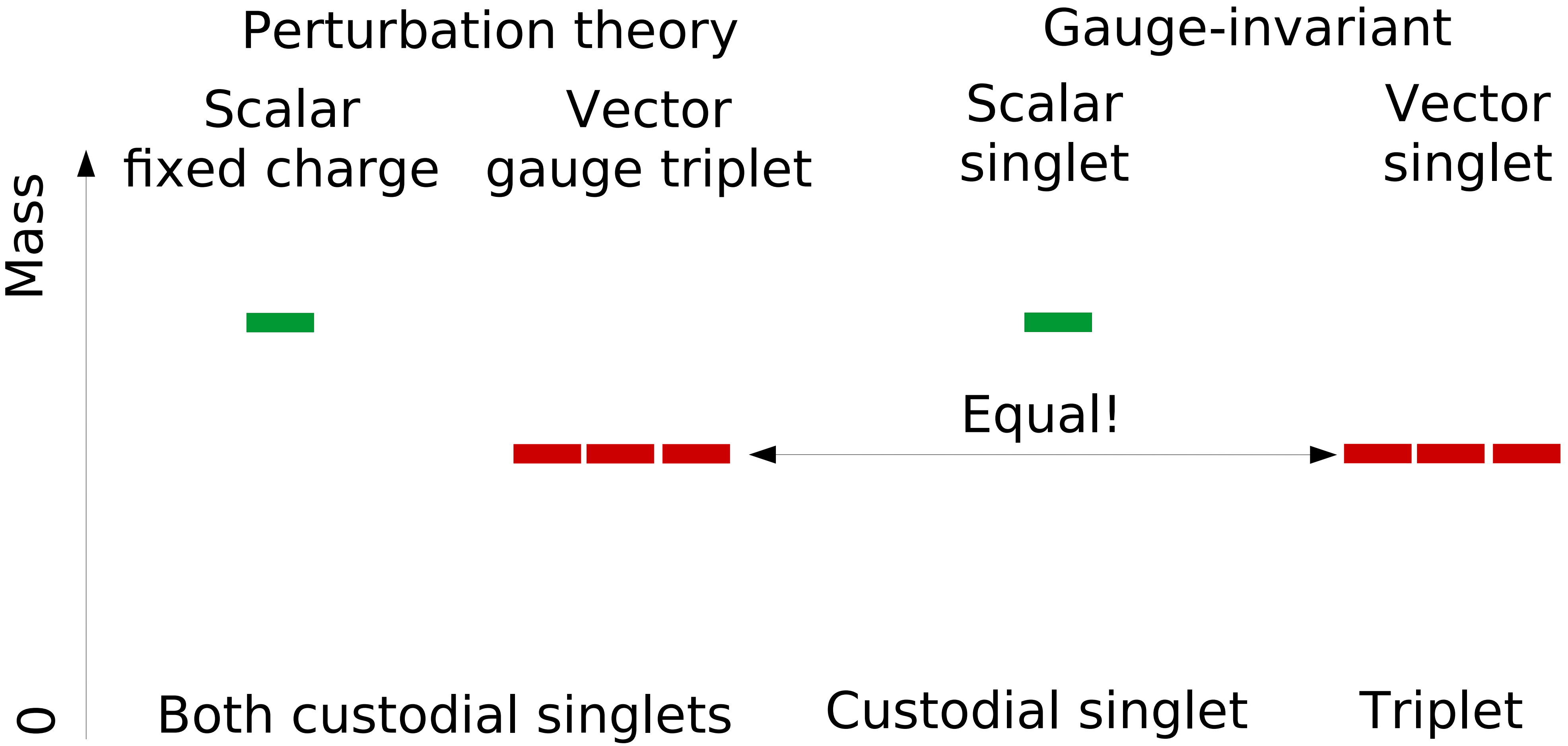

Perturbation theory

Gauge-invariant

Scalar
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Mass

0

Equal!

Both custodial singlets

Custodial singlet

Triplet

Physical spectrum

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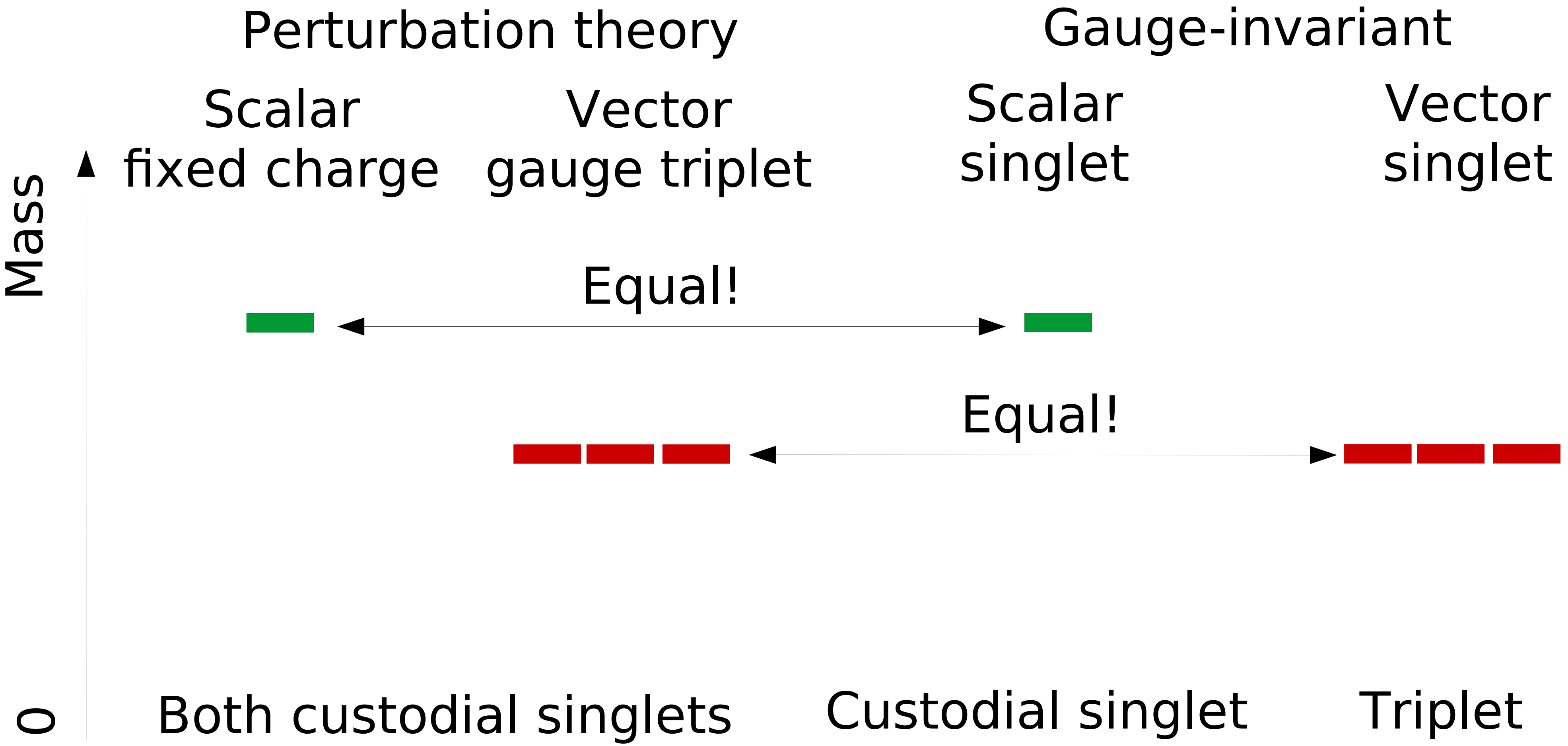

Perturbation theory

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Mass

Equal!

Equal!

0

Both custodial singlets

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Triplet

Why?

A microscopic origin
-
Fröhlich-Morchio-Strocchi mechanism

How to make predictions

[Fröhlich et al.’80,’81,
Maas & Törek'16,’18,
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  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas’12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

Augmented perturbation theory

[Fröhlich et al.'80,'81
Maas'12,'17]

1) Formulate gauge-invariant operator

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

0^+ singlet: $\left\langle \left(h^{+} h\right)\left(x\right)\left(h^{+} h\right)\left(y\right)\right\rangle$

Higgs field

Augmented perturbation theory

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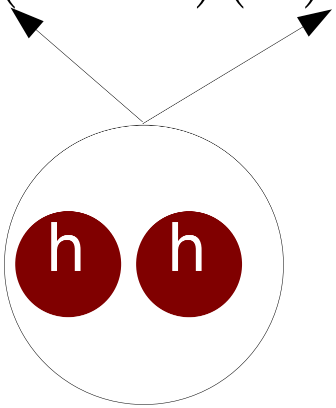

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1) Formulate gauge-invariant operator

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4) Compare poles on both sides

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Bound state mass

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Trivial two-particle state

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Standard
Perturbation
Theory

Bound
state
mass

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What about this?

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Consequences: The Higgs

[Maas'12,'17
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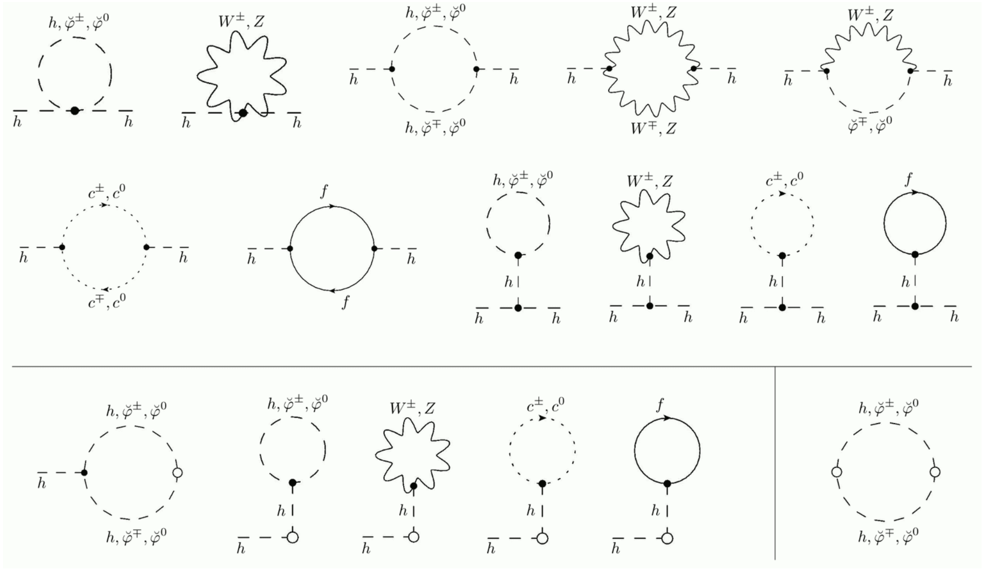

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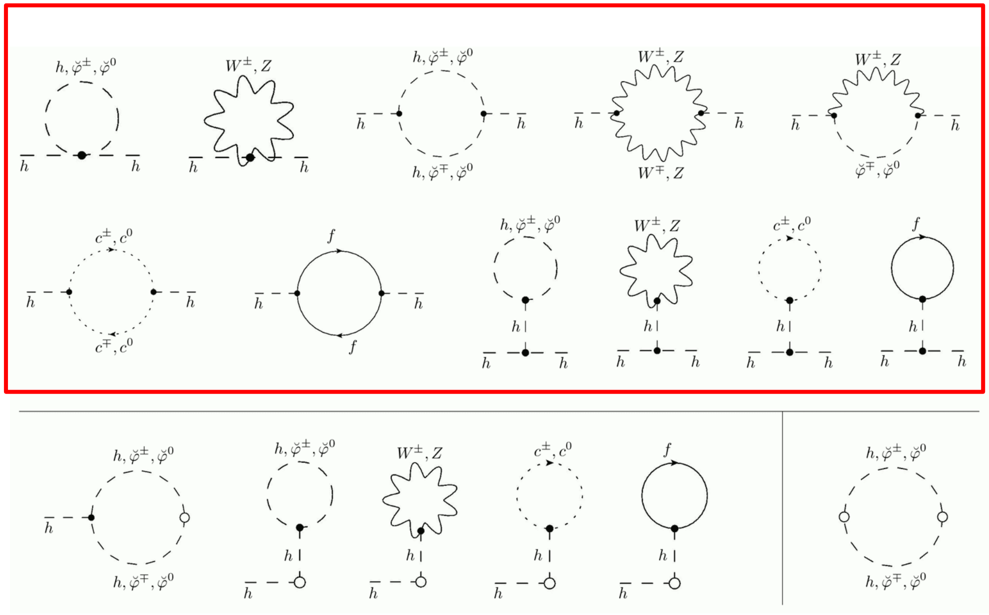

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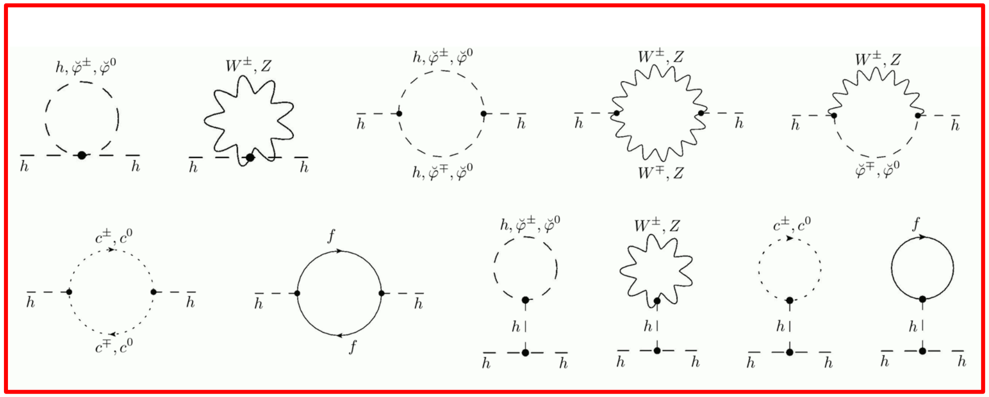

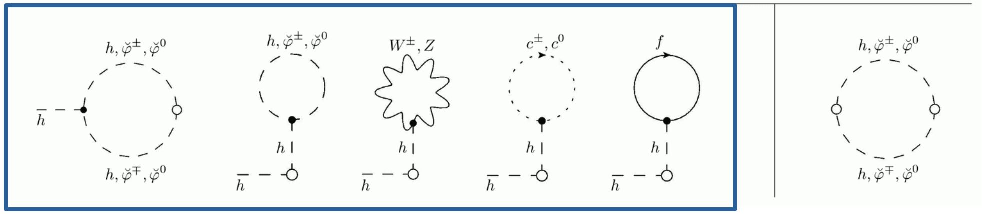

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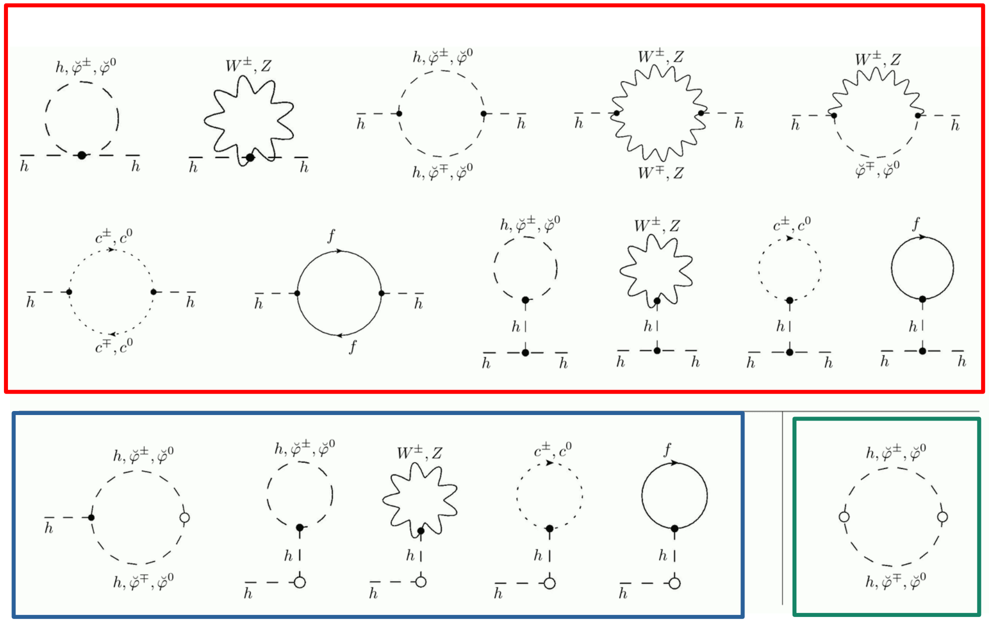

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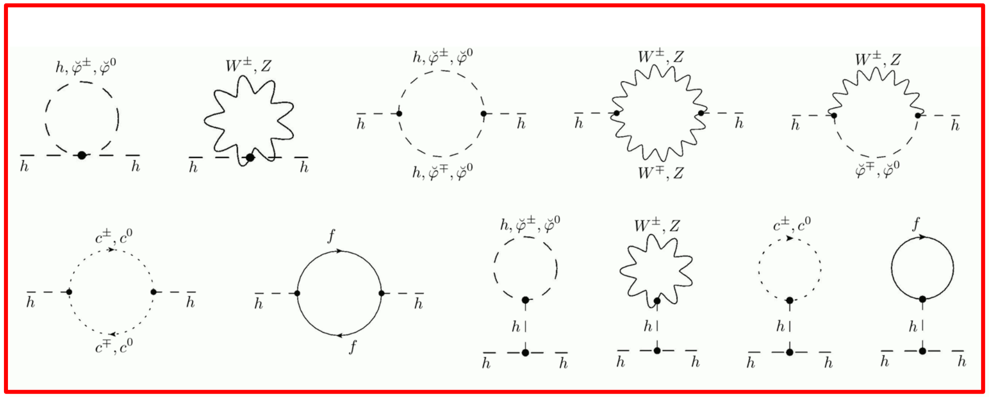

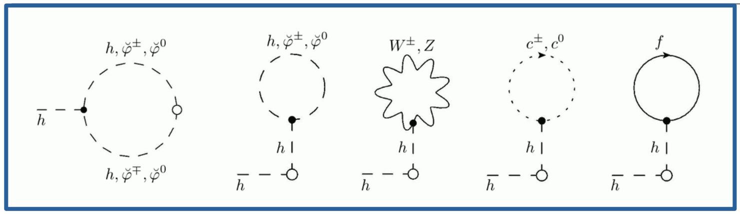

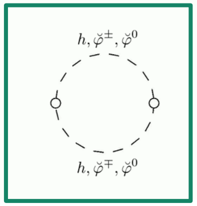

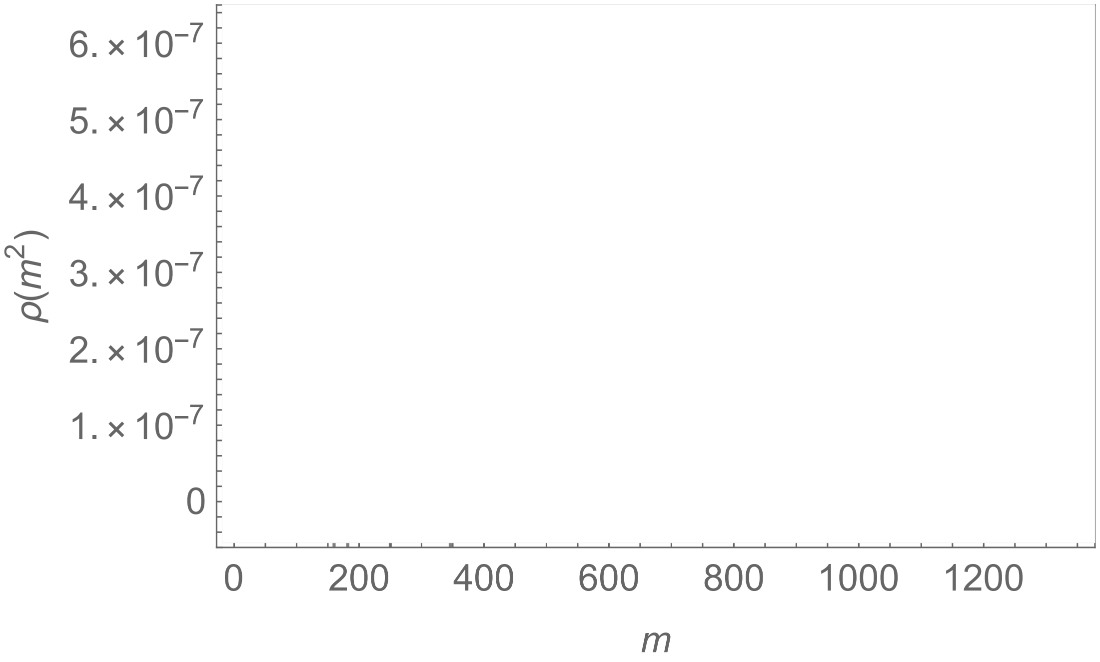

Consequences: The Higgs

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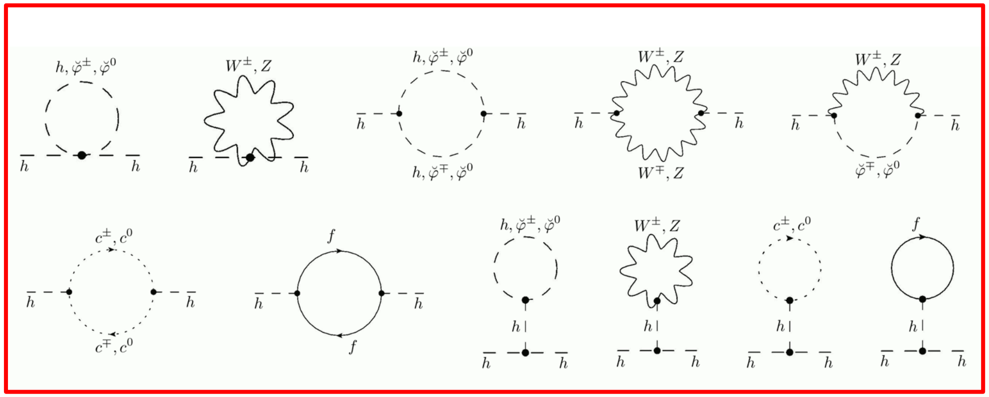

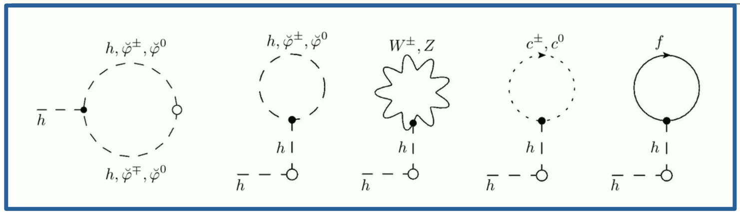

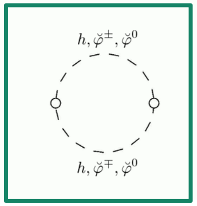

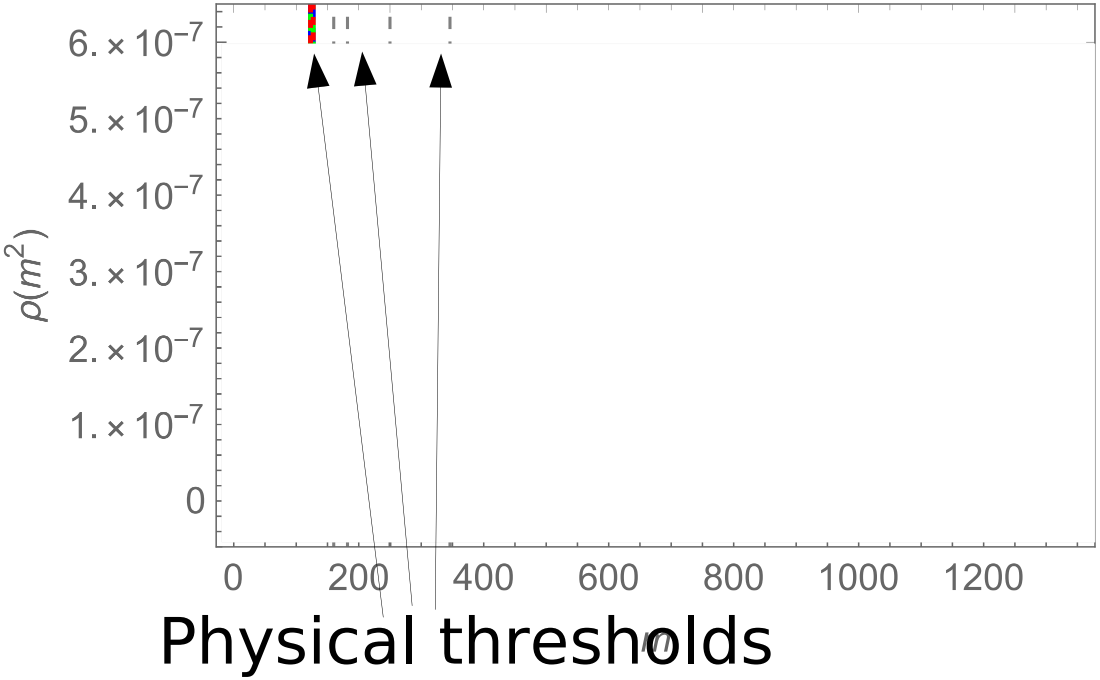

Physical thresholds

Consequences: The Higgs

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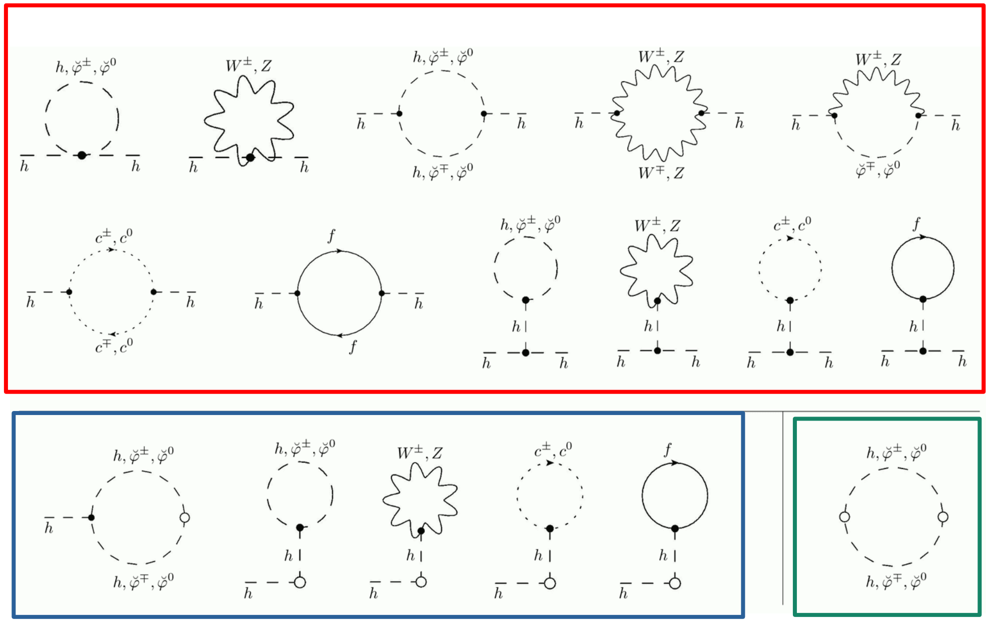
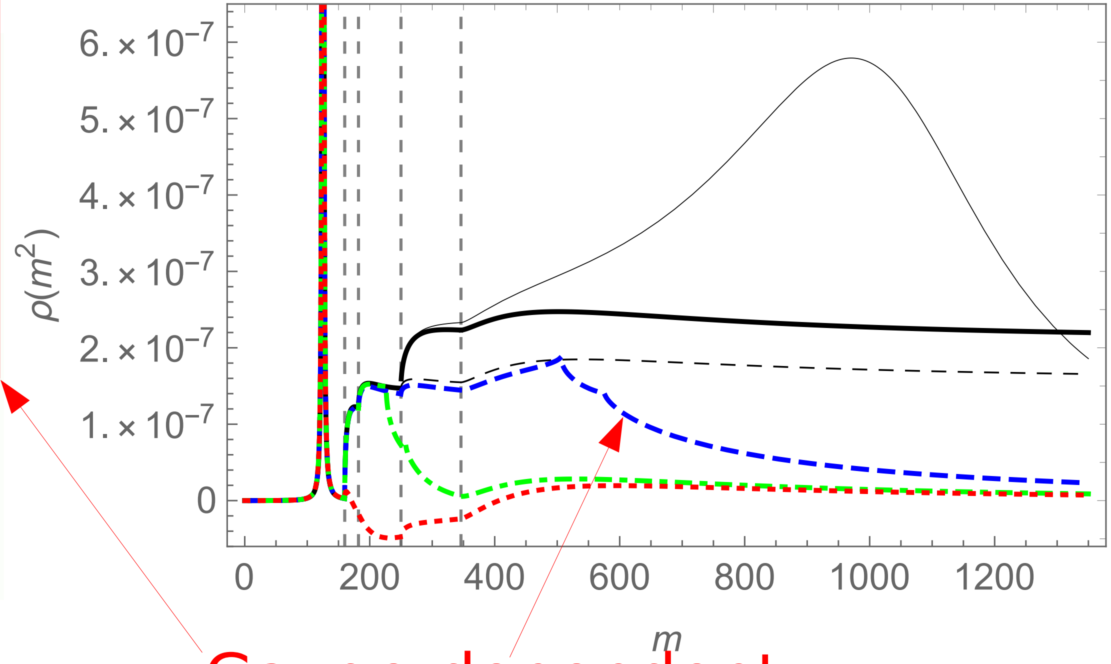
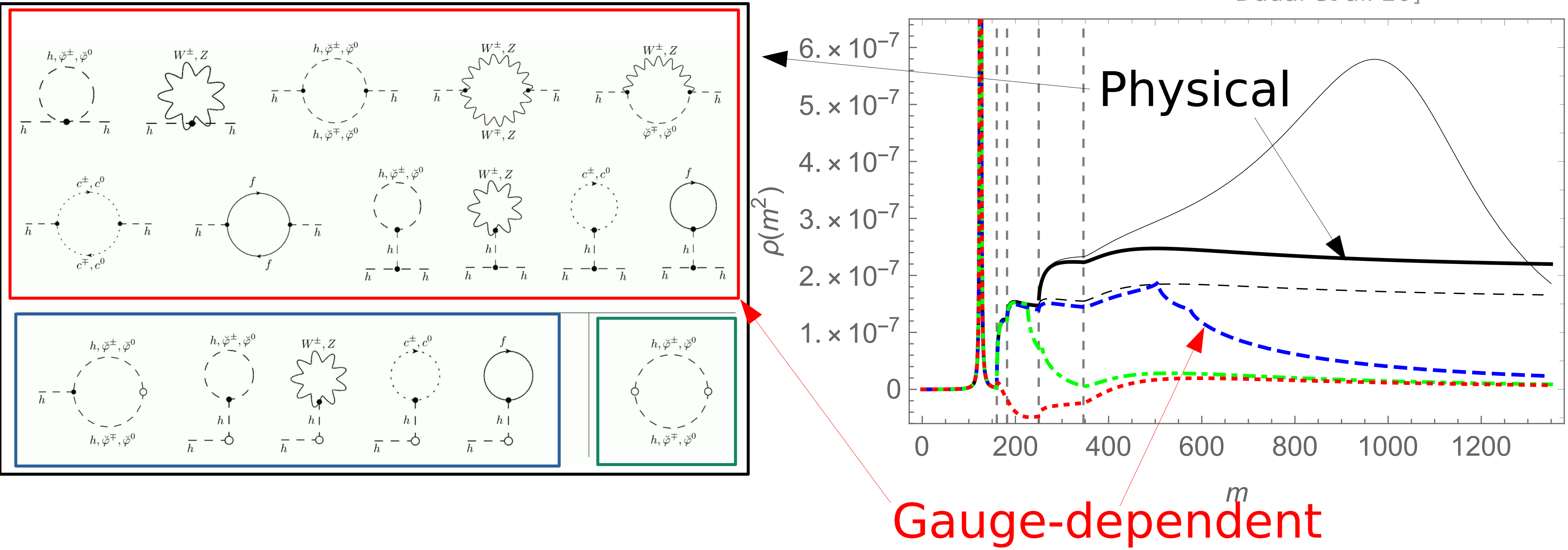




Gauge-dependent

Consequences: The Higgs

[Maas'12,'17
Maas & Sondenheimer'20
Dudal et al.'20]



Physical

Gauge-dependent

Consequences: The Higgs

[Maas'12,'17
Maas & Sondenheimer'20
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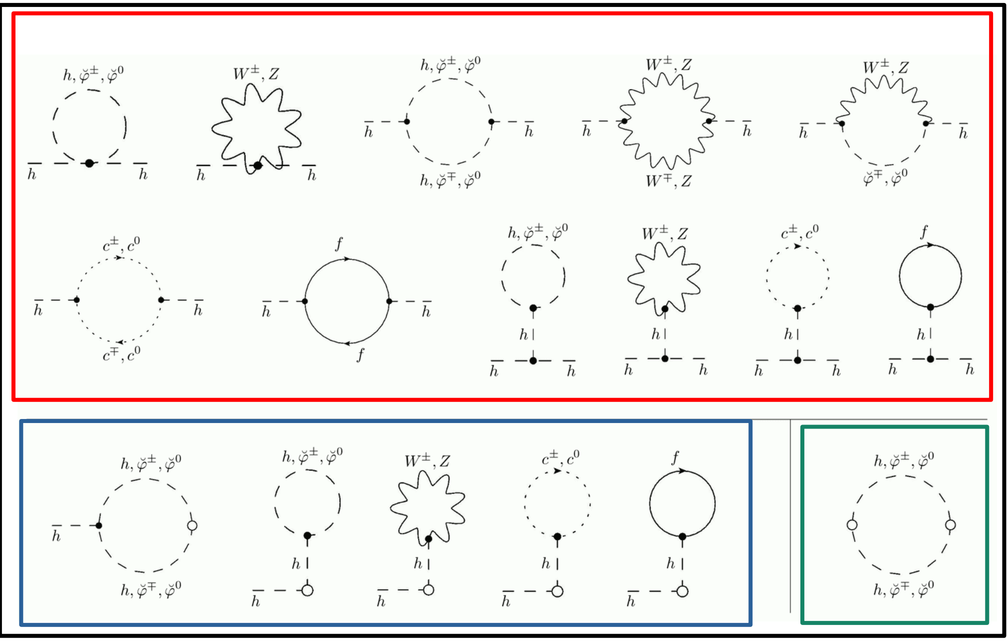

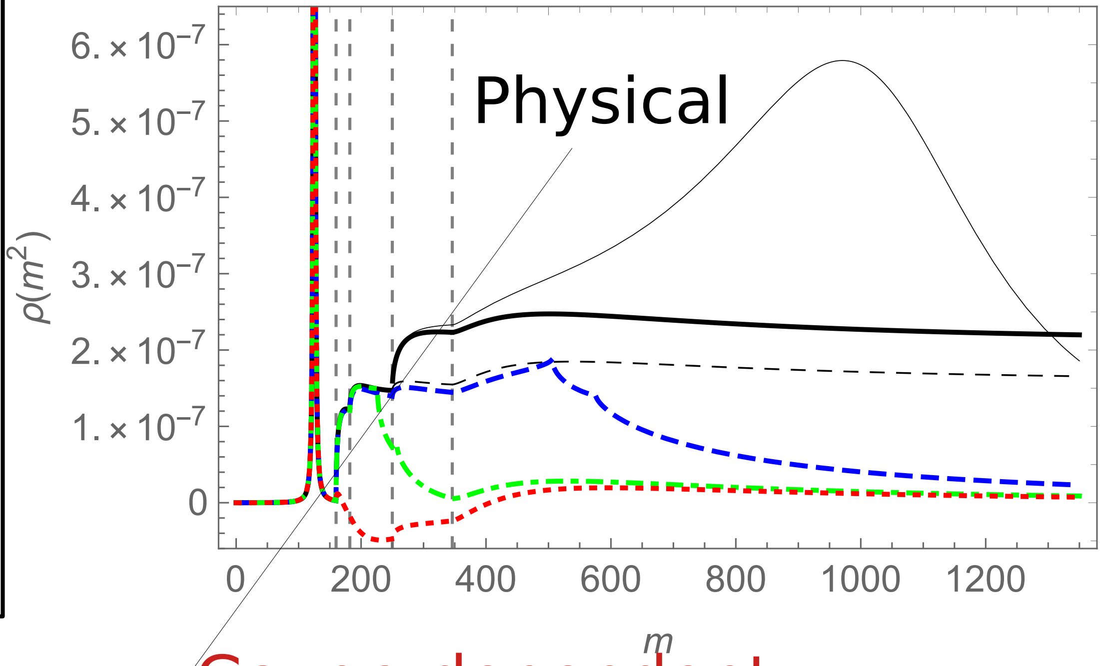

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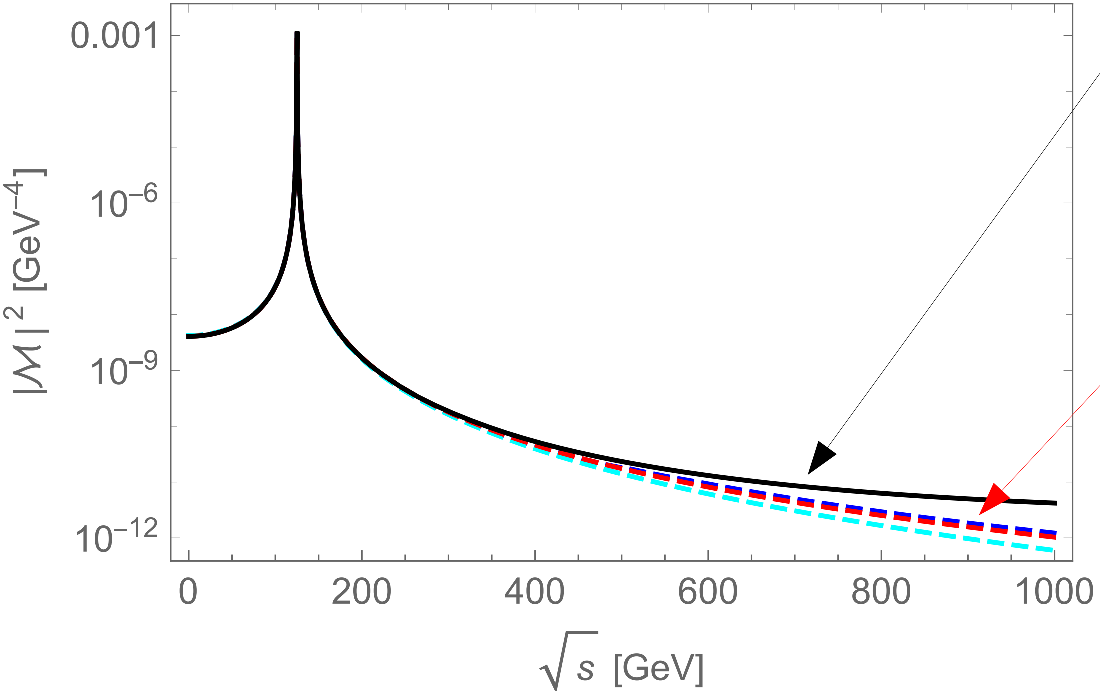

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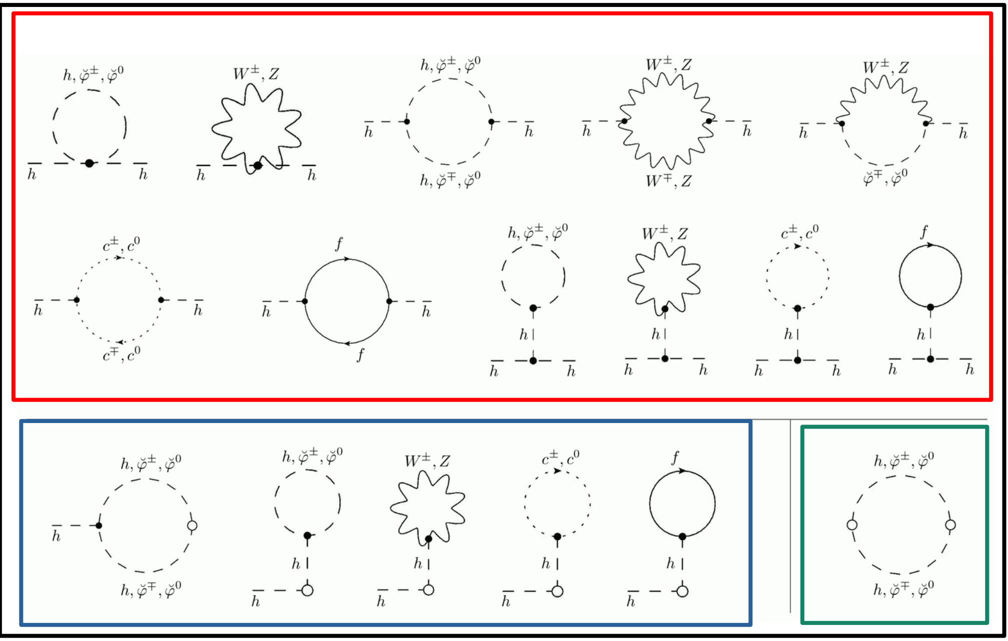

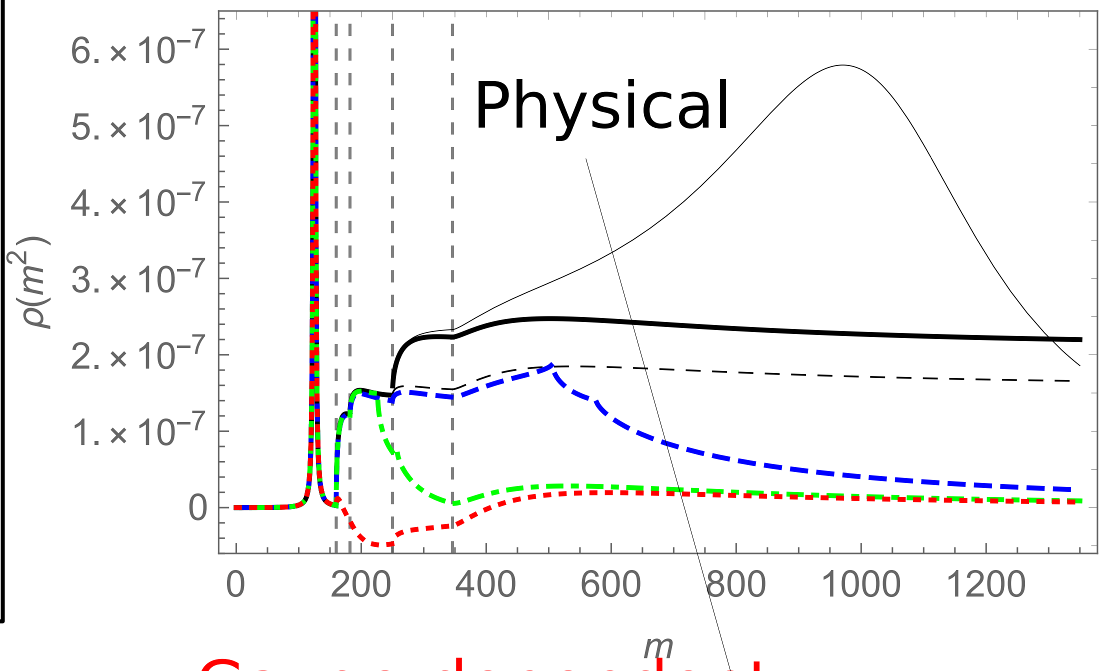

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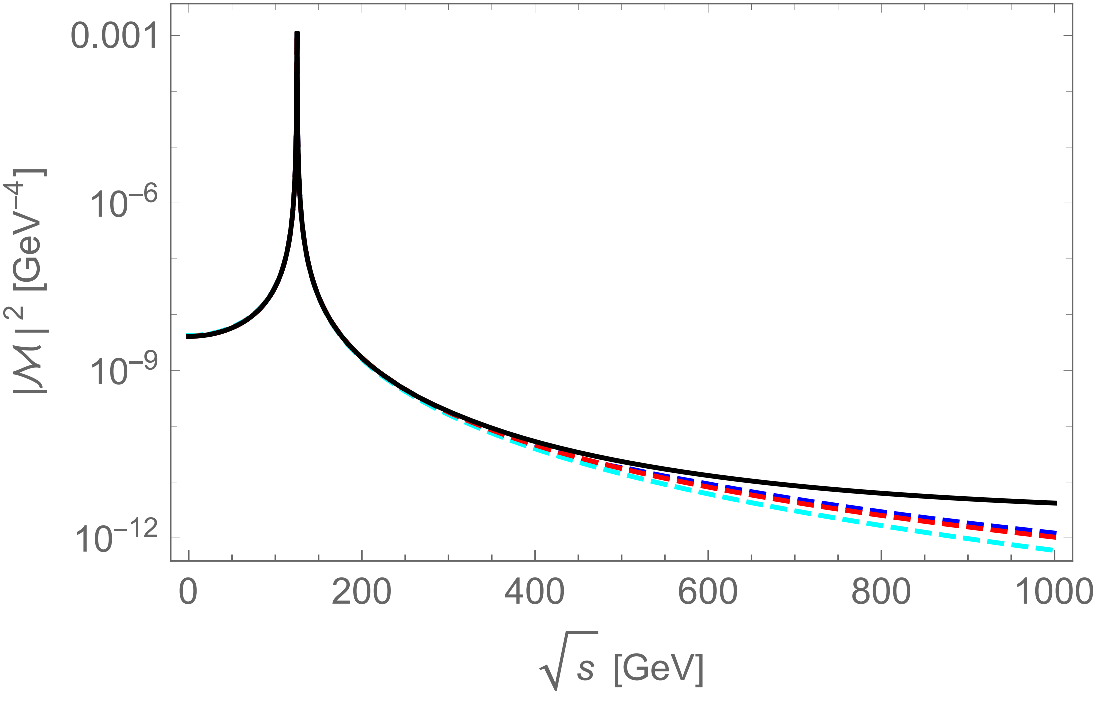

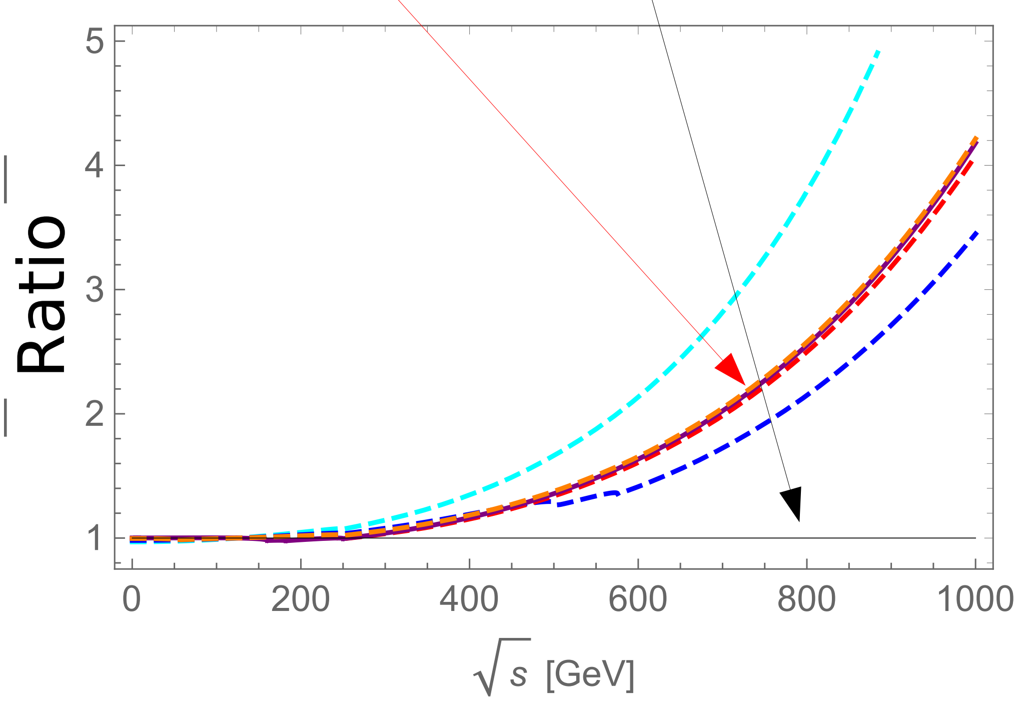

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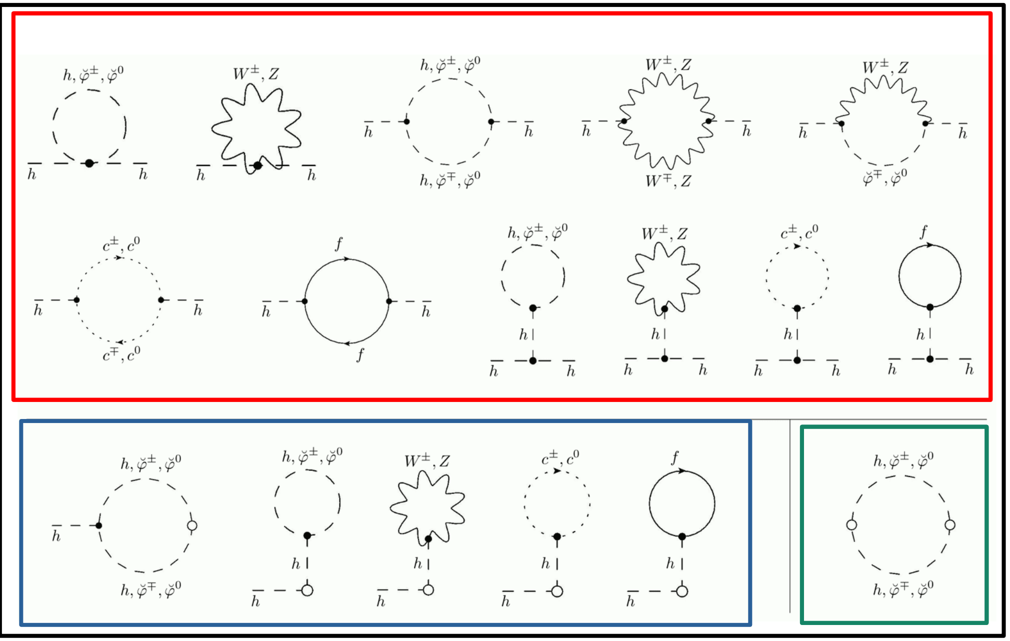

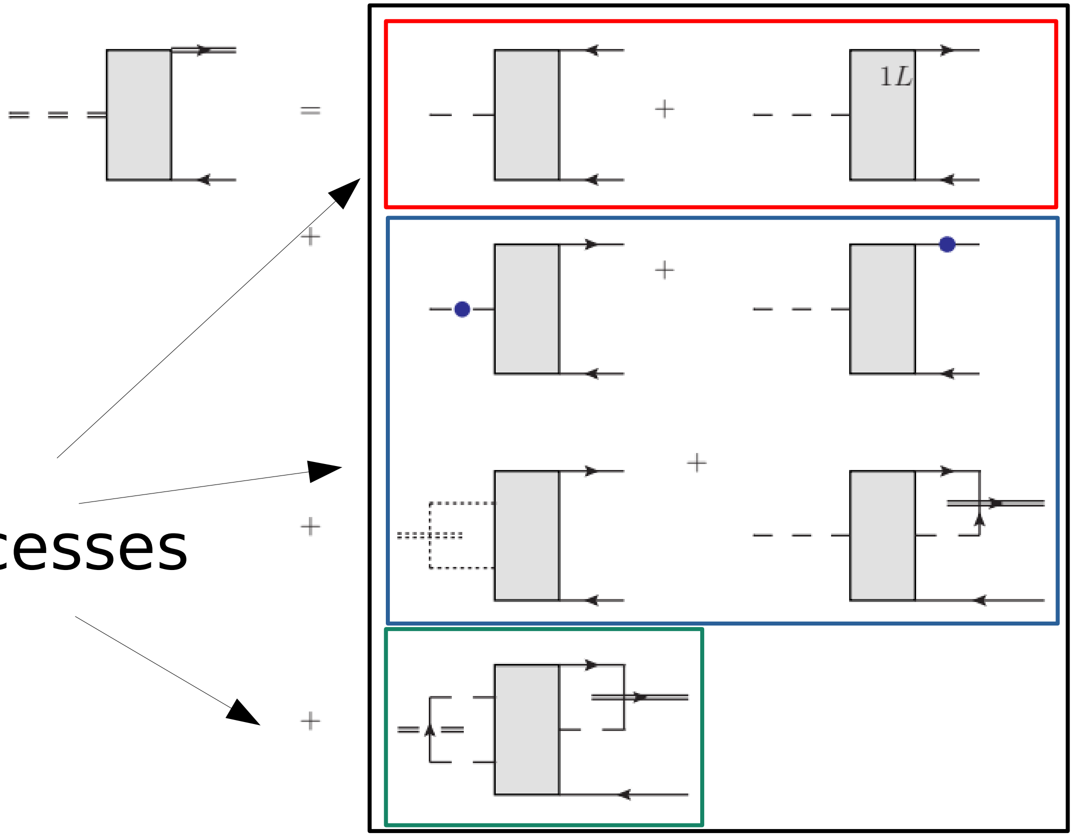

Same structure repeats itself
For decays and scattering processes

What about the vector?

[Fröhlich et al.’80,’81
Maas'12]

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Maas'12]

1) Formulate gauge-invariant operator

1^- triplet: $\left\langle\left(\tau^i h^+ D_\mu h\right)(x)\left(\tau^j h^+ D_\mu h\right)(y)\right\rangle$

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Matrix from group structure

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Matrix from group structure

c projects custodial states to gauge states

Exactly one gauge boson for every physical state

Phenomenological Implications
-
Can we measure this?

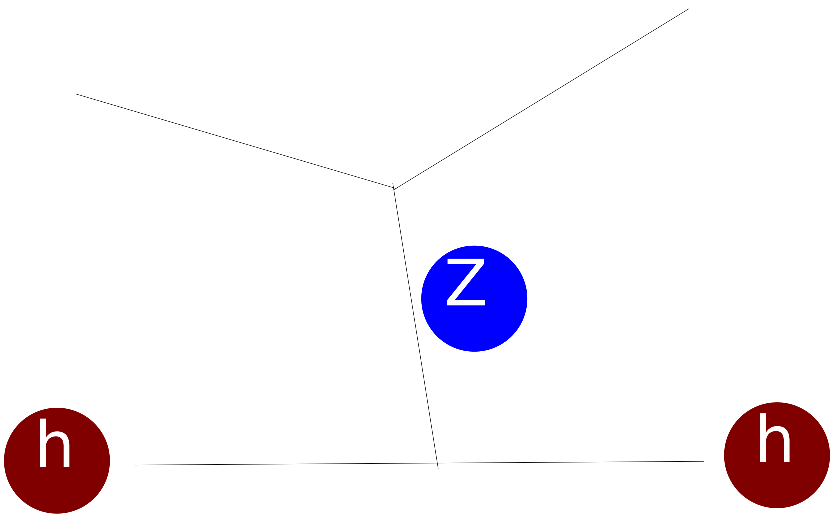
Bound states as extended objects

Bound states as extended objects

- Two possibilities to measure extension

Bound states as extended objects

- Two possibilities to measure extension
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process

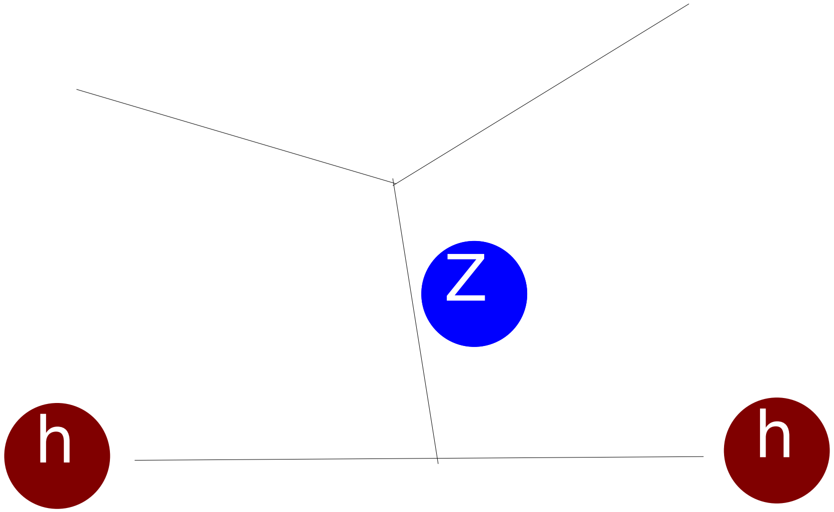


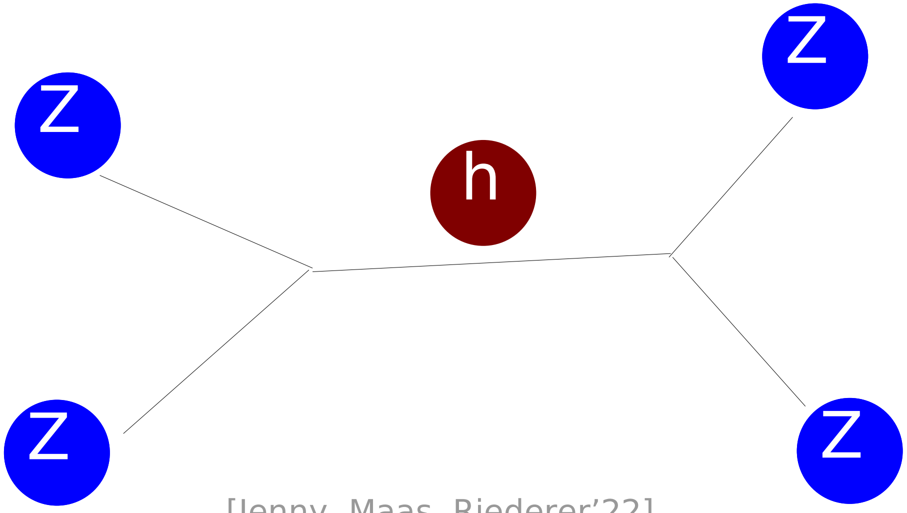
[Maas, Raubitzek, Törek’18]

Bound states as extended objects

- Two possibilities to measure extension
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process
  - Elastic scattering
    - Standard vector boson scattering process at low energies
    - Use this one

[Maas, Raubitzek, Törek’18]





[Jenny, Maas, Riederer’22]

Radius from elastic scattering in VBS

- Elastic region: $160/180\,GeV \leqslant \sqrt{s} \leqslant 250\,GeV$
  - *s* is the CMS energy in the initial/final ZZ/WW system

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Cross section

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Matrix element

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Partial wave amplitude

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Legendre polynom

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Scattering length~"size"

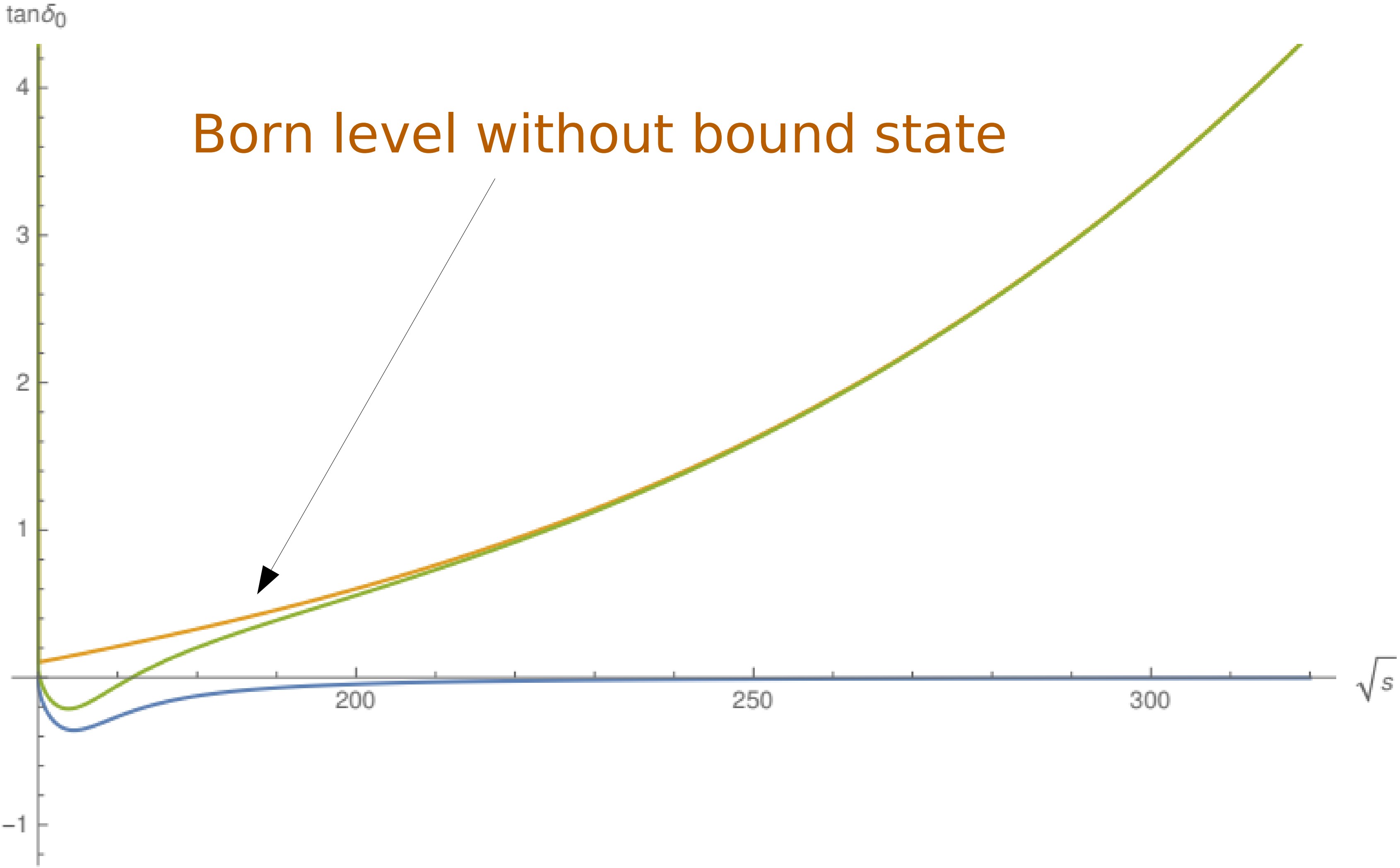
Phase shift

→ Lattice Lüscher analysis

Impact of a finite size of the Higgs

- Consider the Higgs: $J=0$

Impact of a finite size of the Higgs


Born level without bound state

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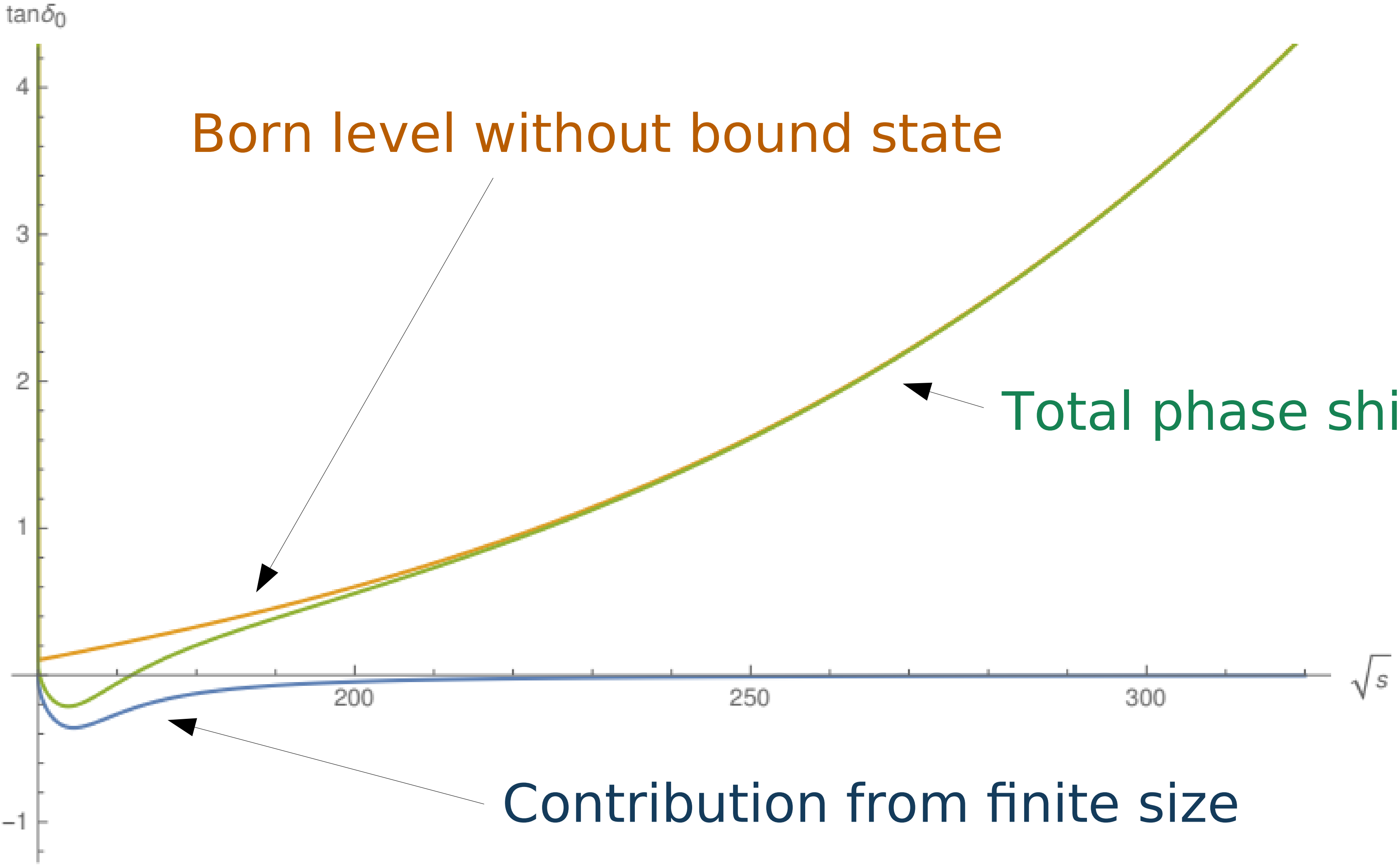


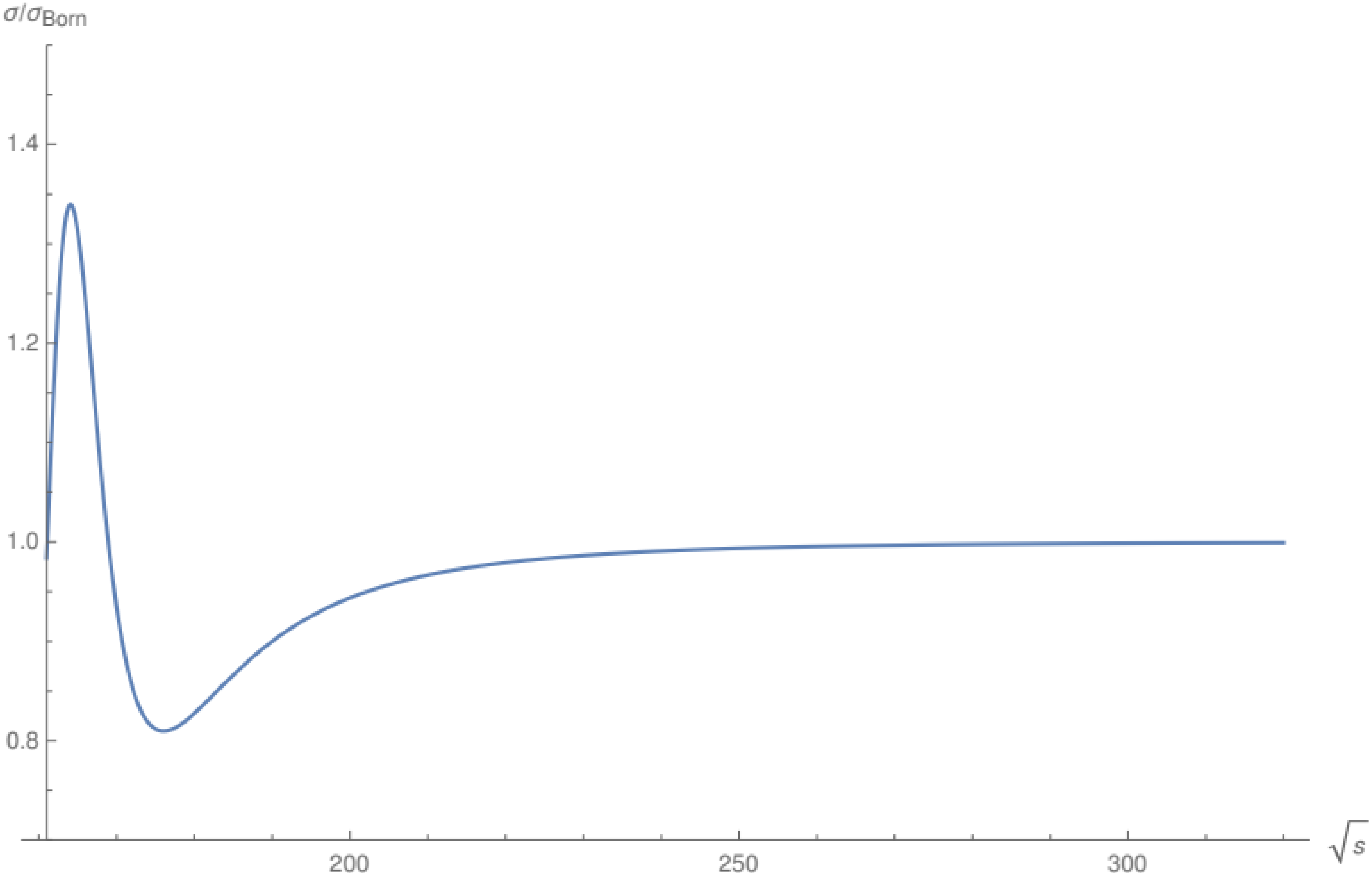
Born level without bound state

Total phase shift

Contribution from finite size

- Consider the Higgs: *J=0*
- Mock-up effect
  - Scattering length 1/(40 GeV)

Impact of a finite size of the Higgs



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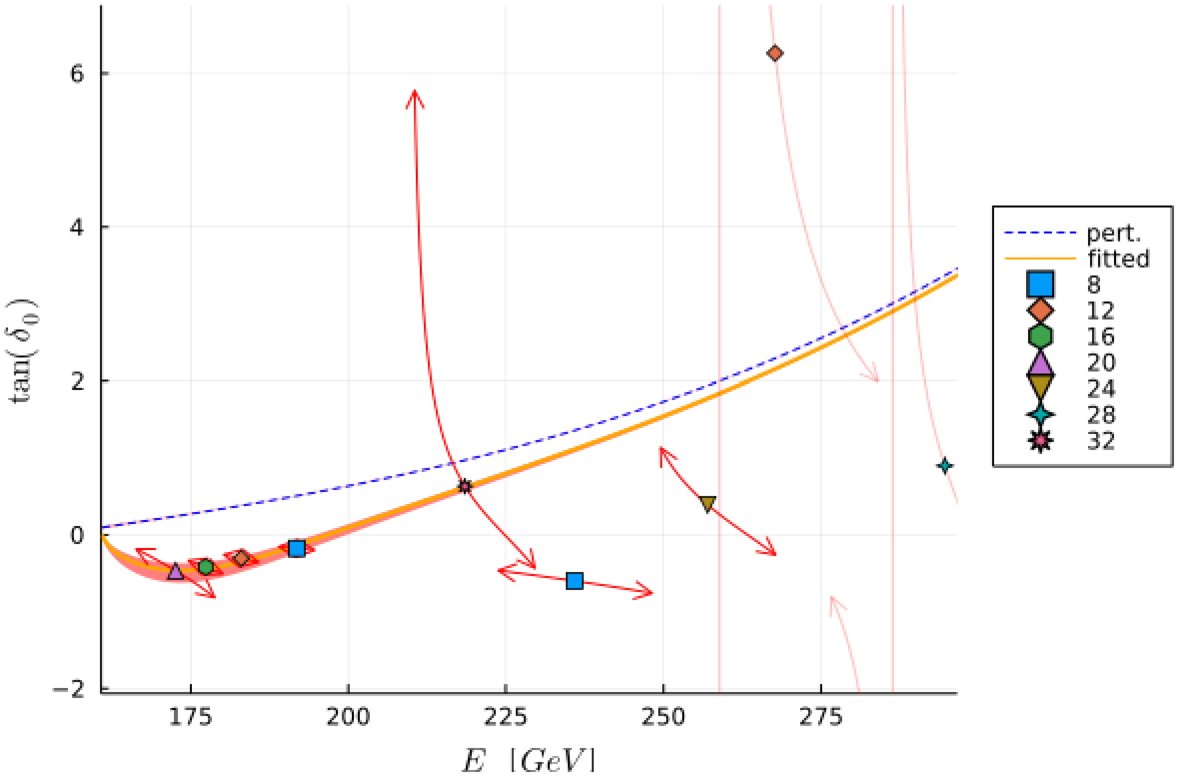

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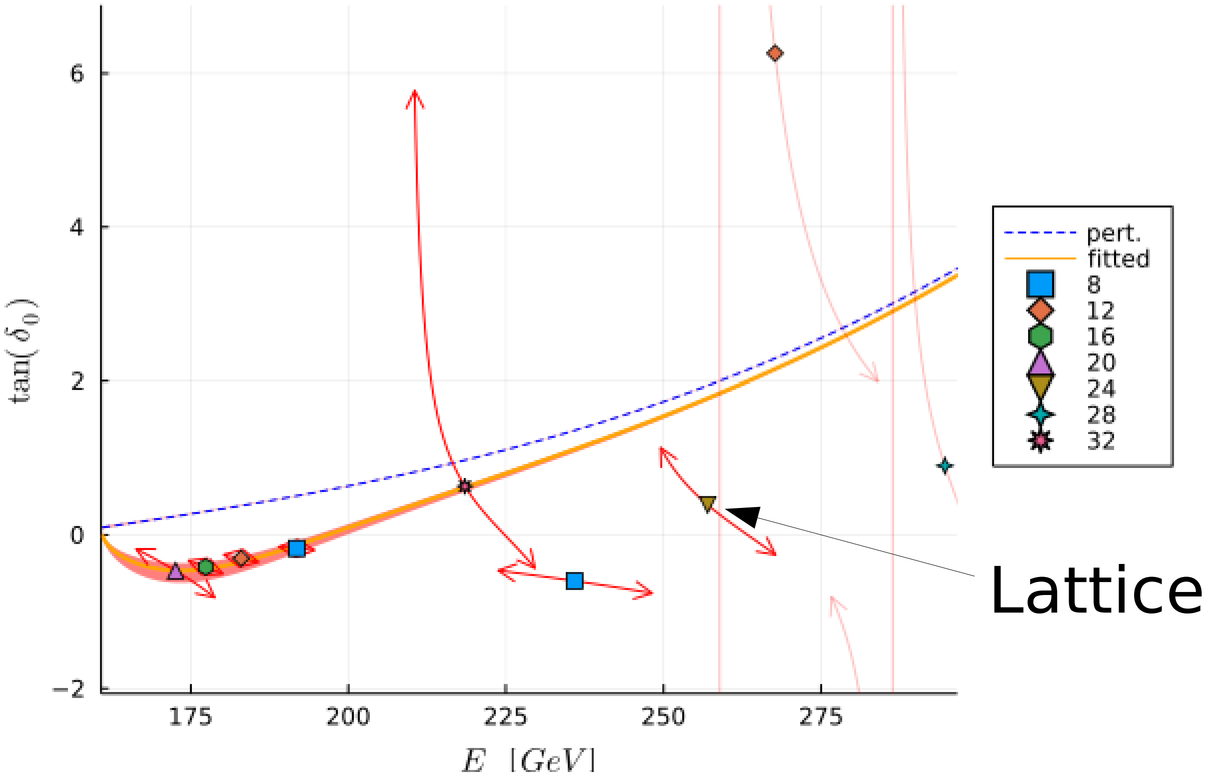

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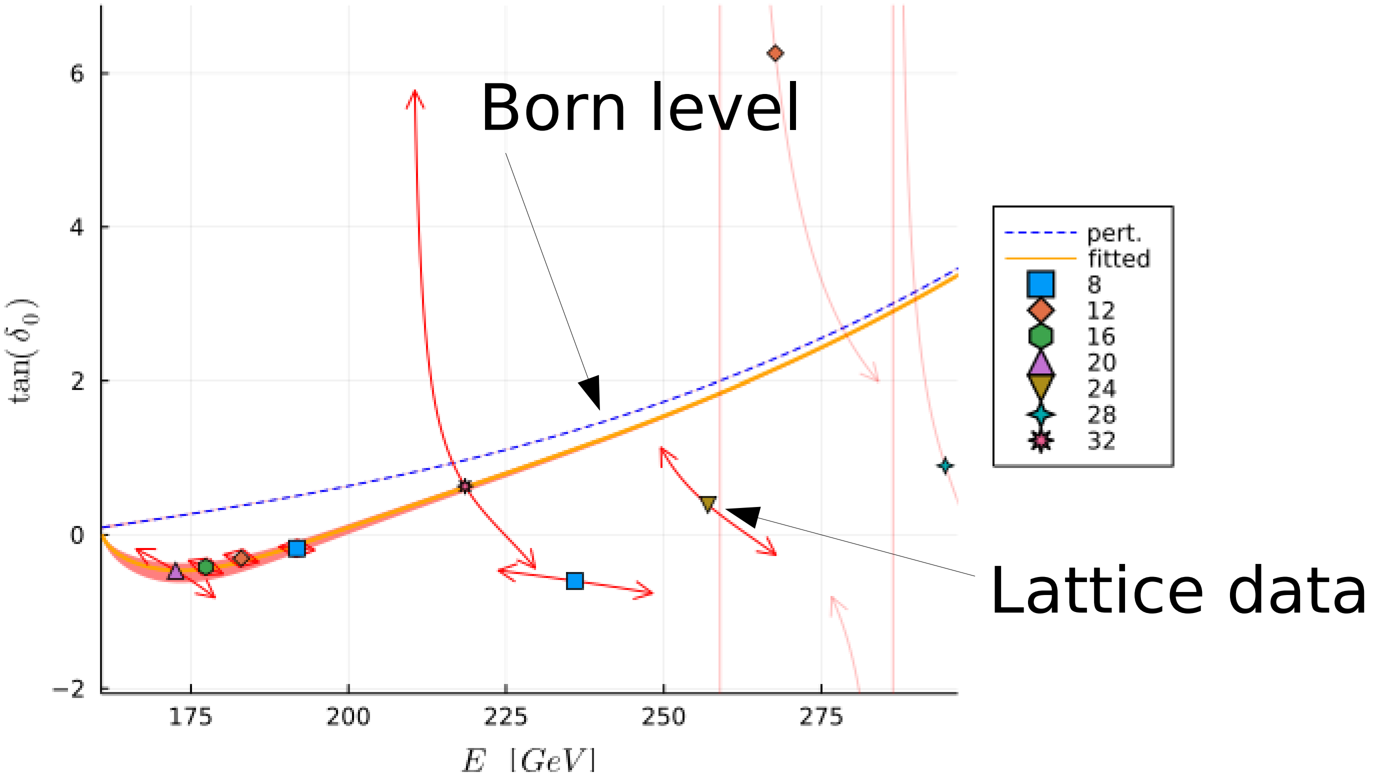

Born level

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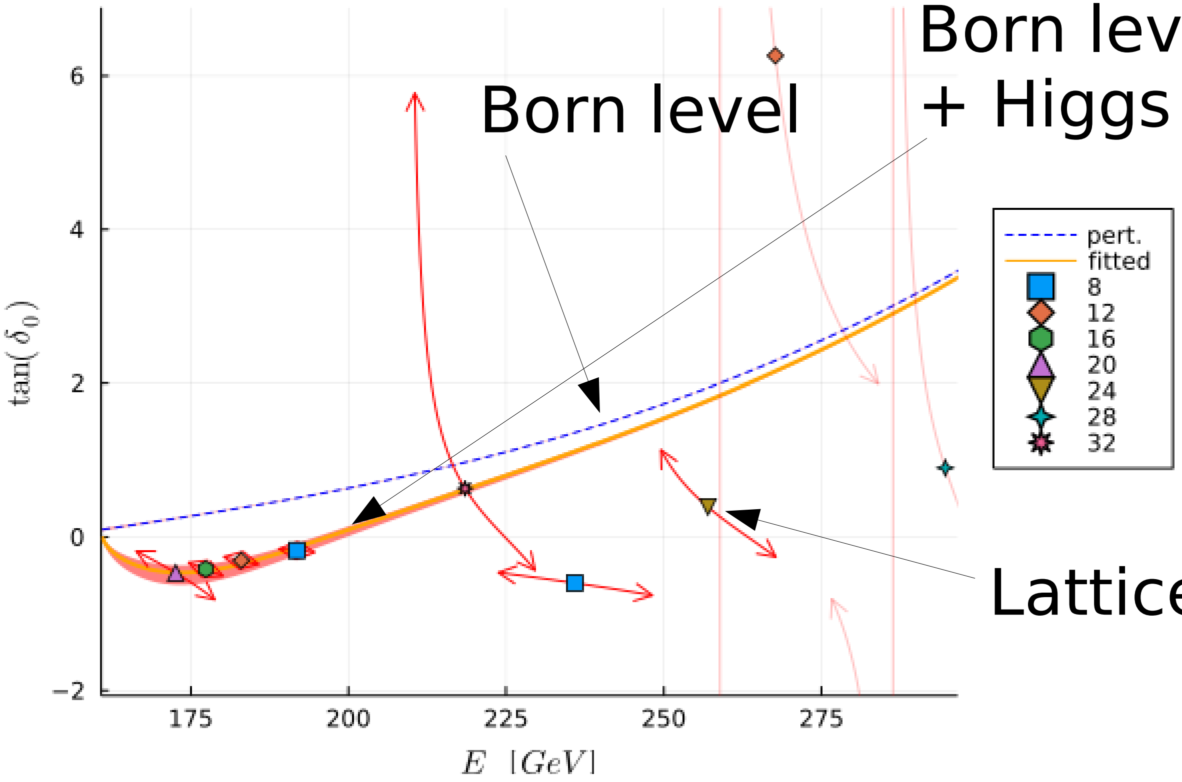

Born level

Born level FMS

+ Higgs size of (40 Gev)$^{-1}$

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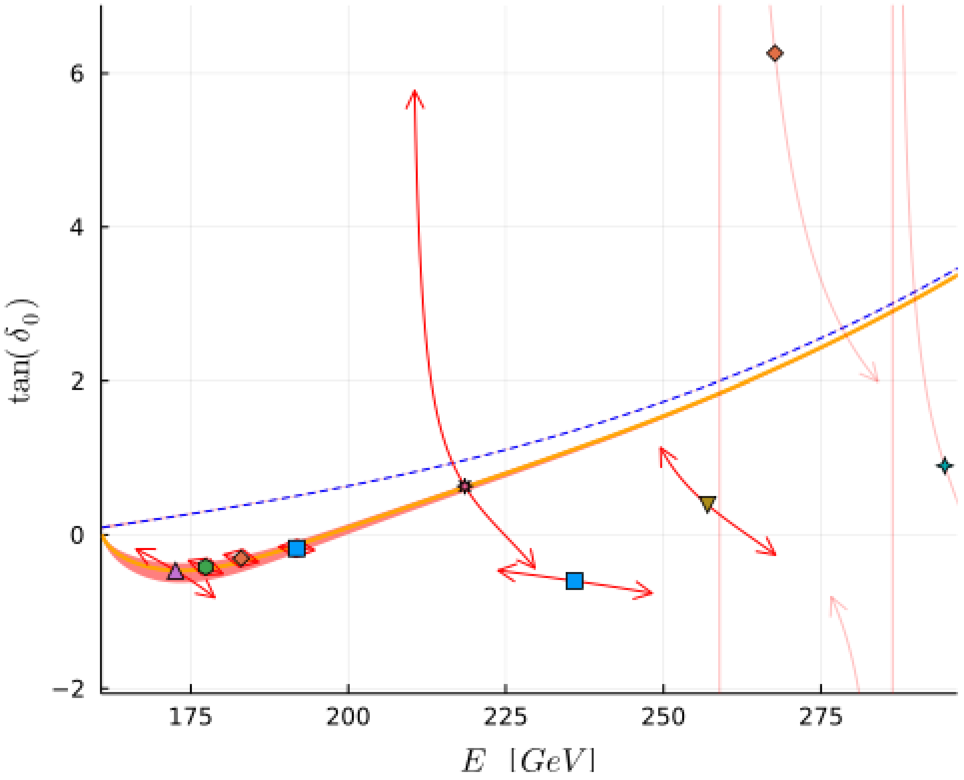

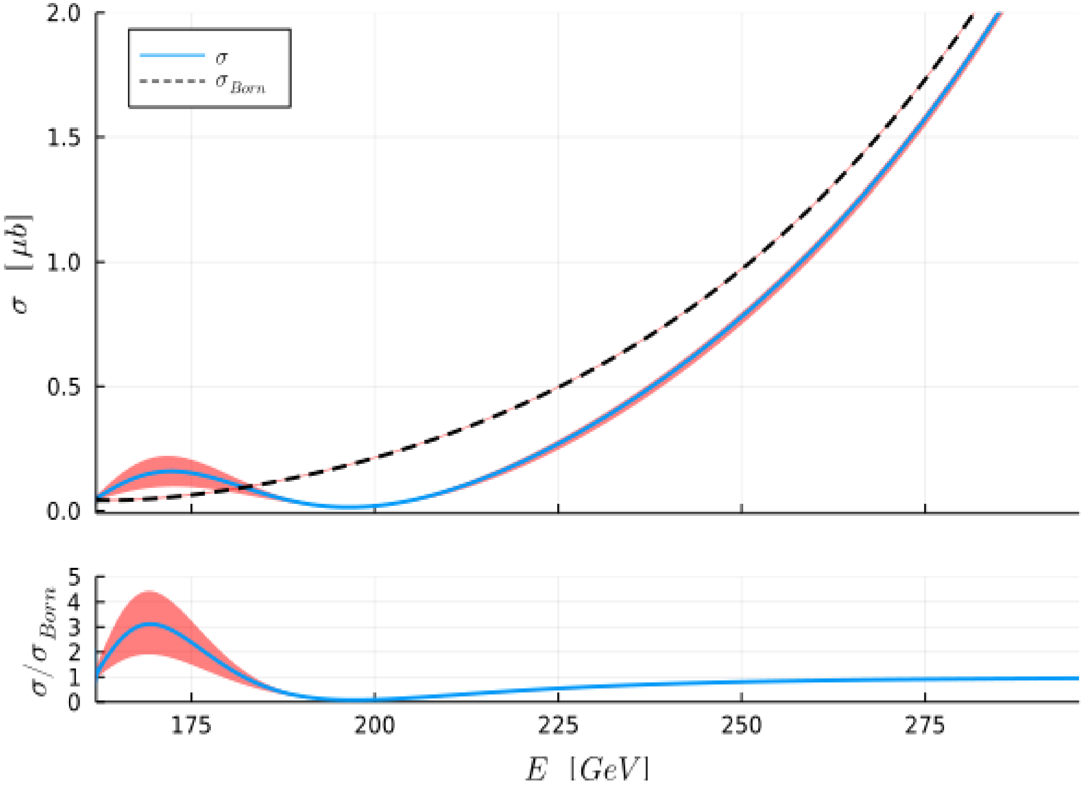

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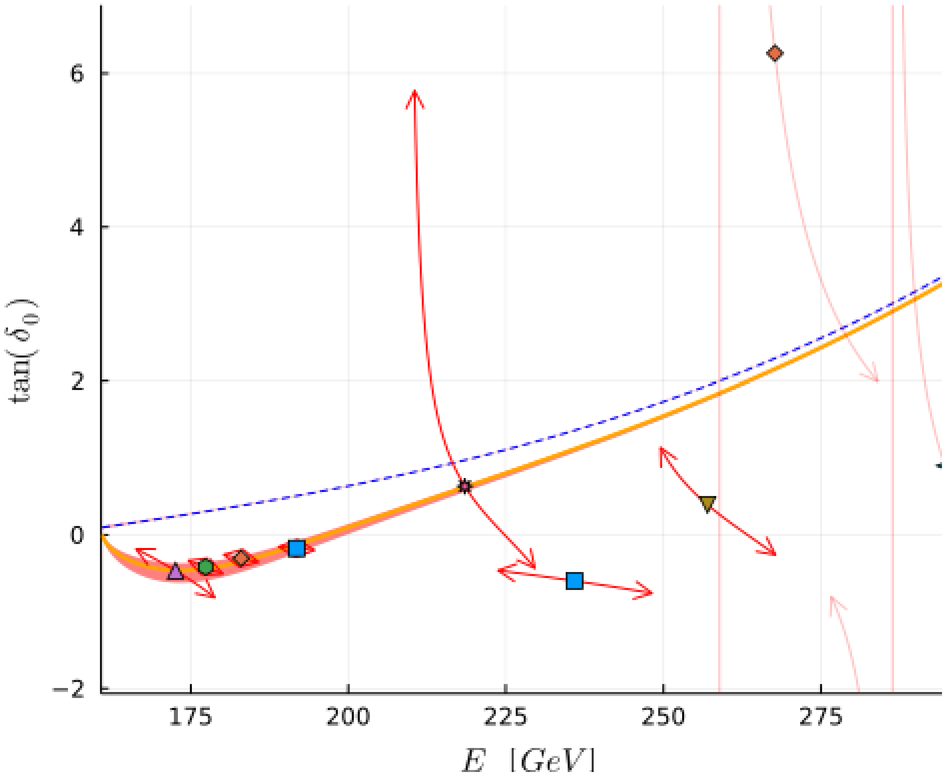
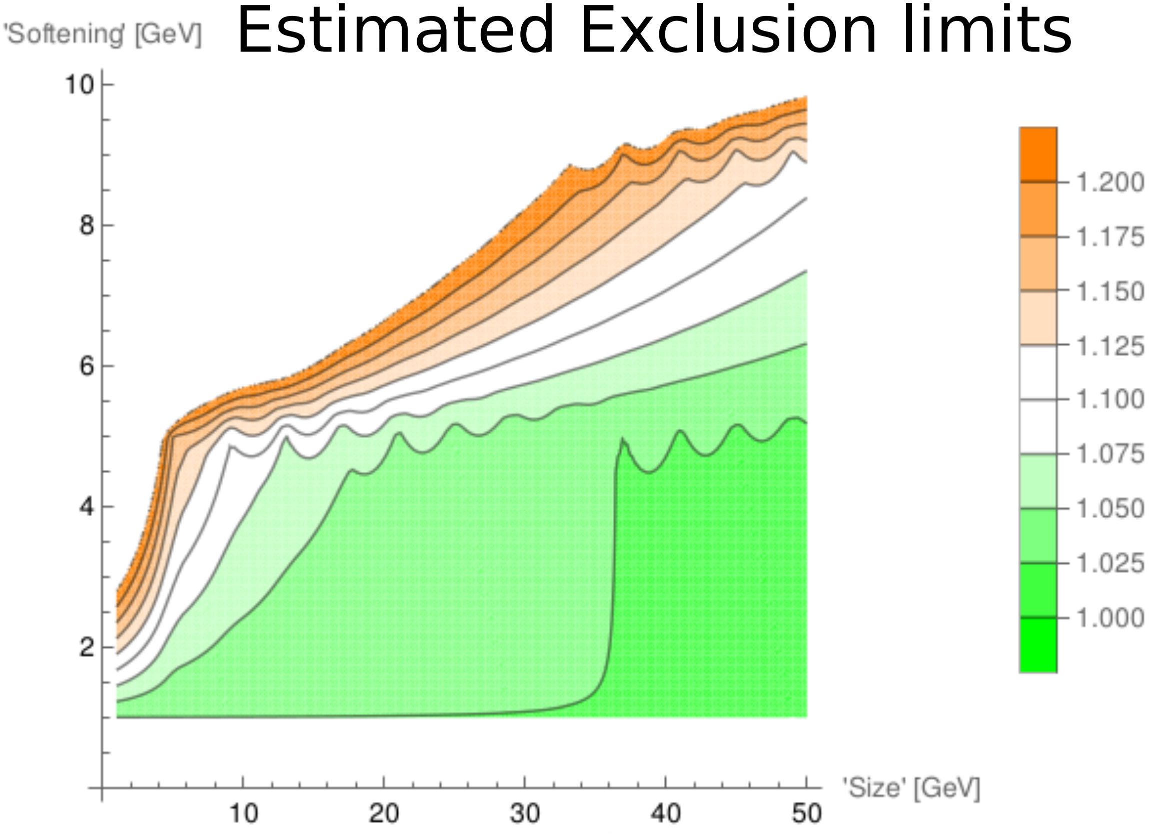


Estimated Exclusion limits



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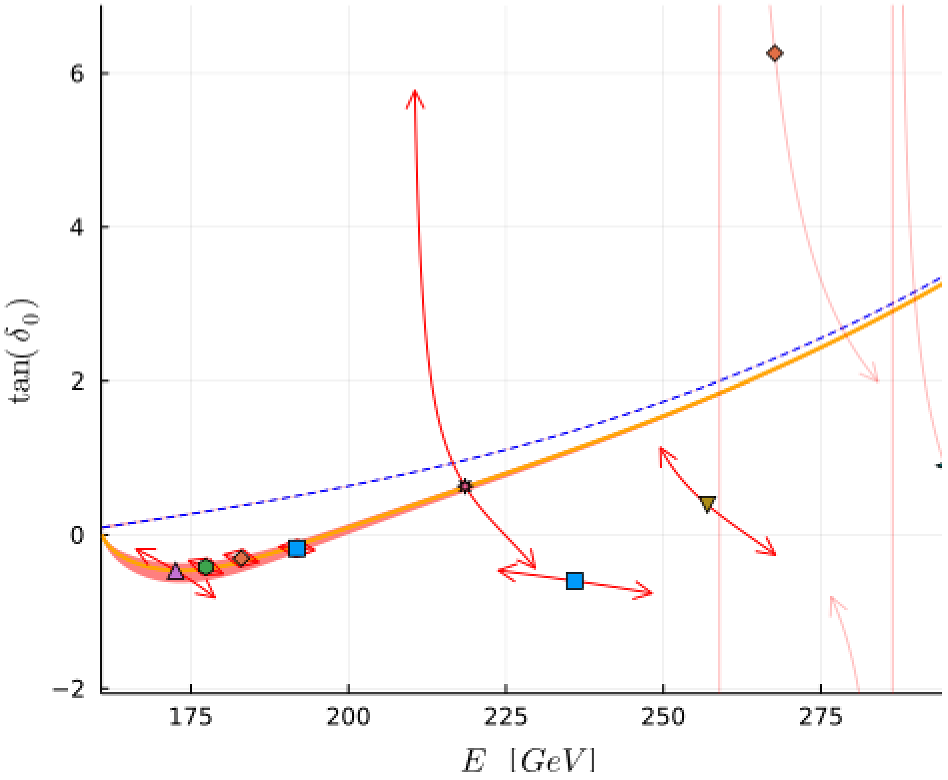

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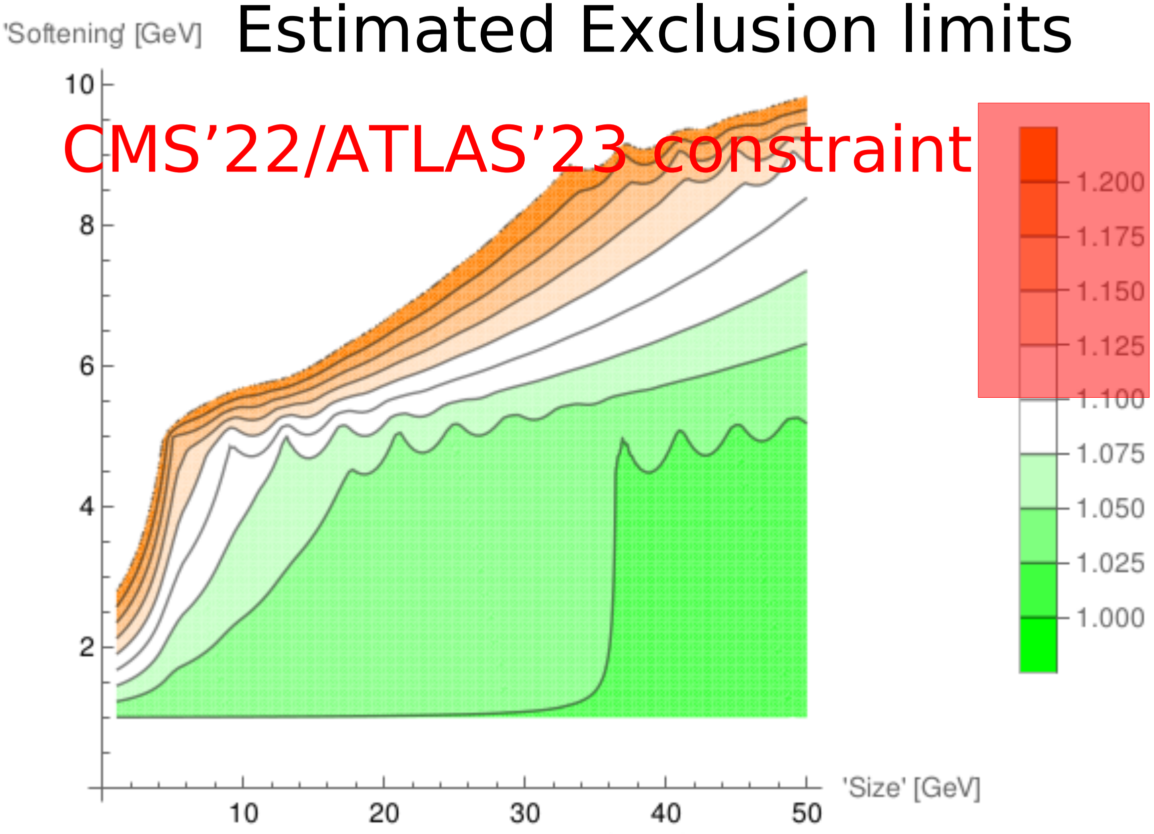

CMS’22/ATLAS’23 constraint

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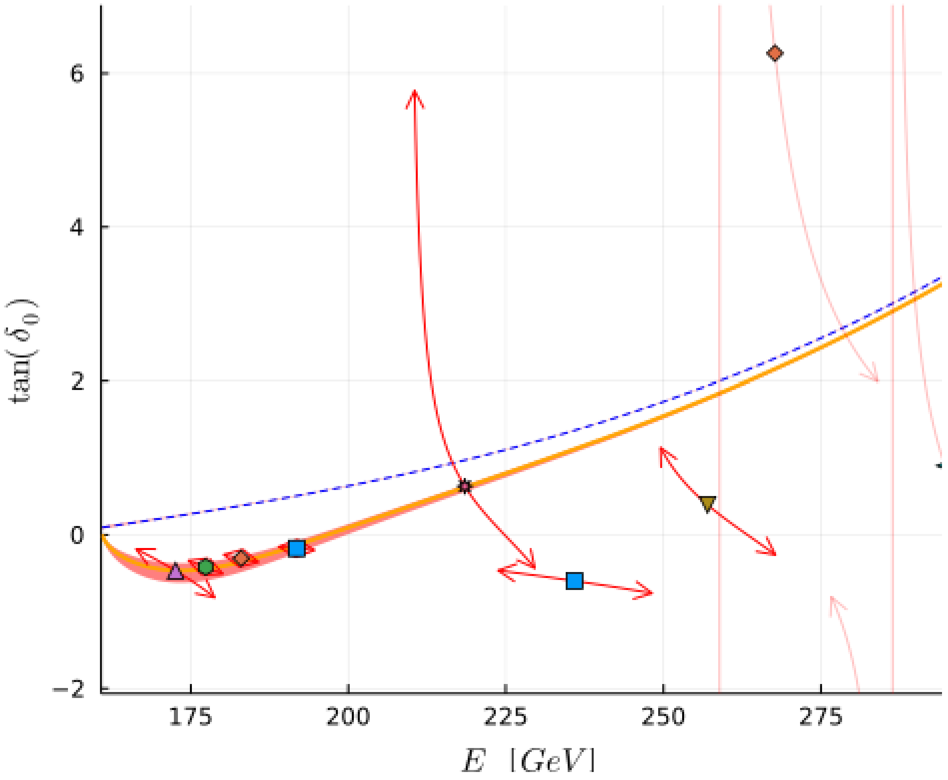

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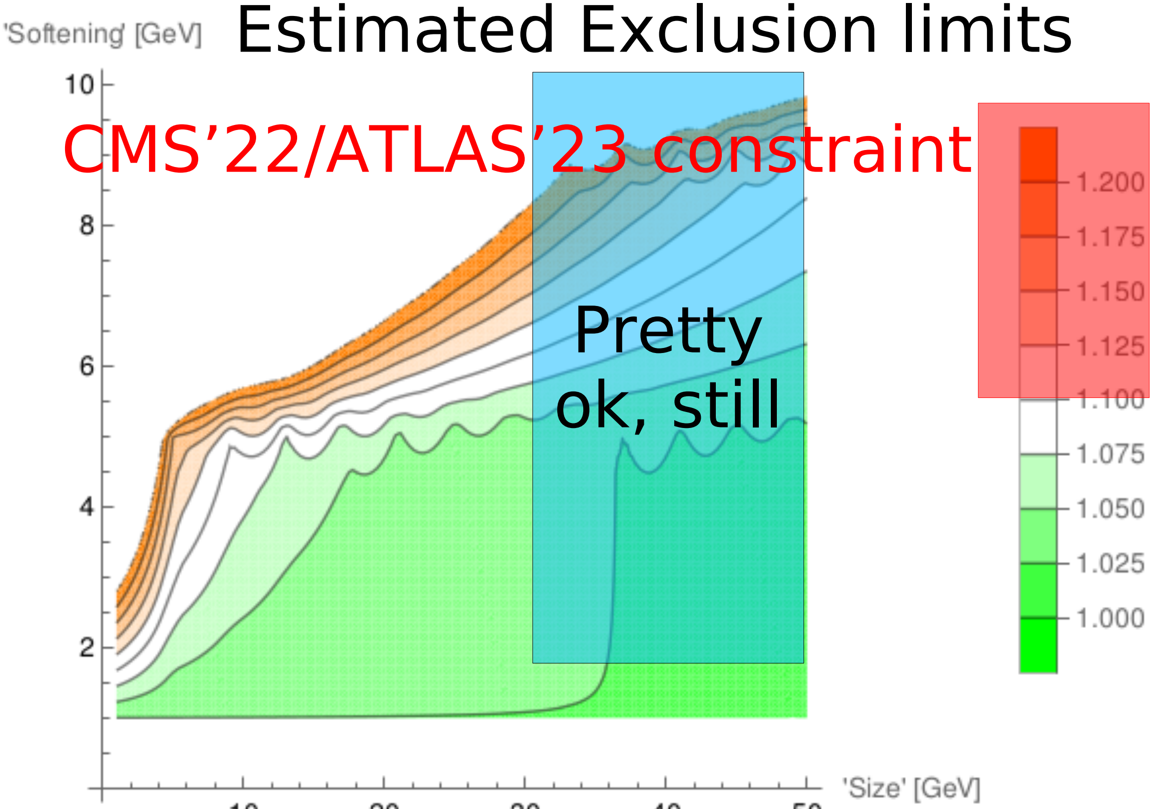

CMS’22/ATLAS’23 constraint

Pretty ok, still

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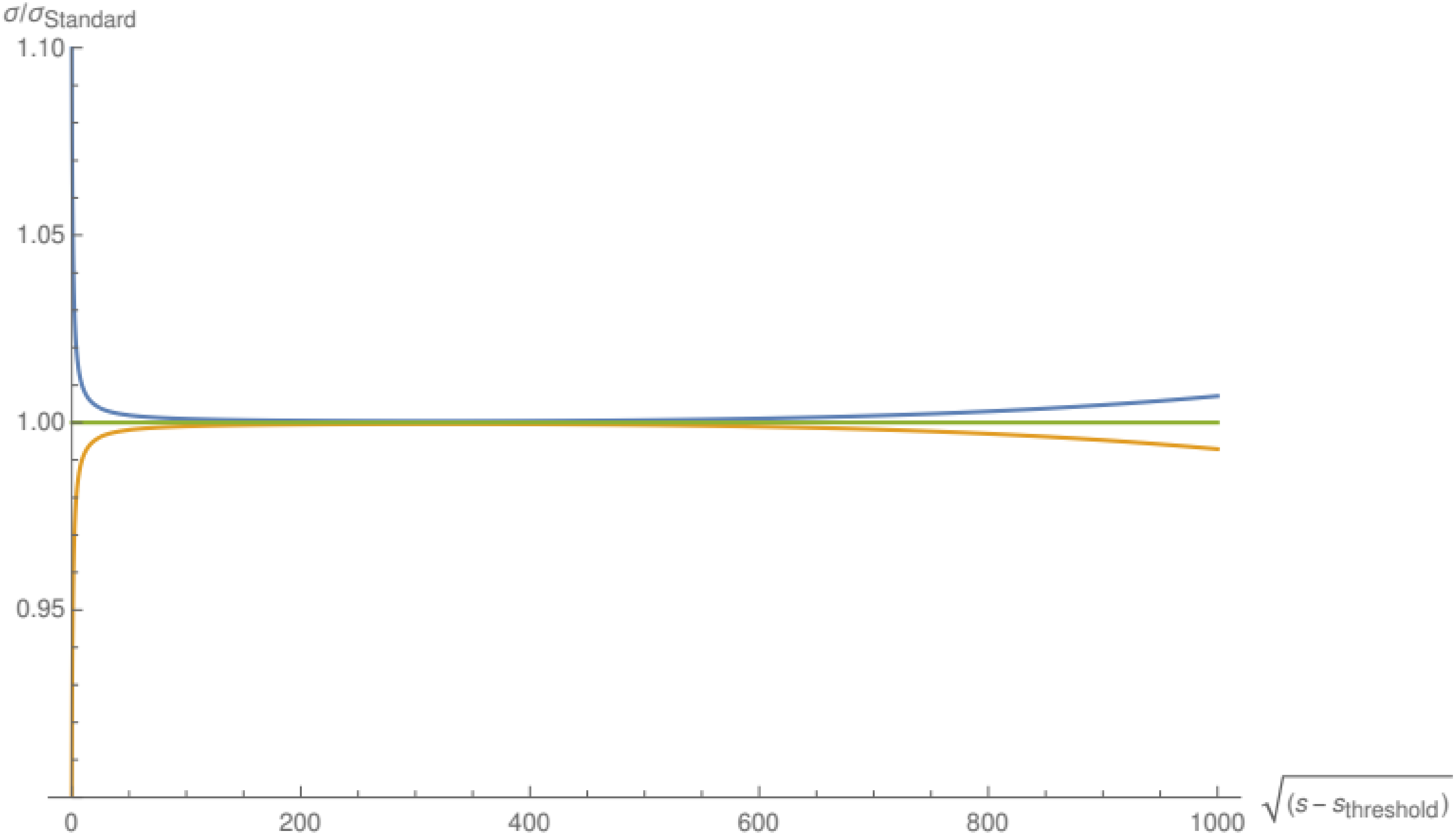
[Jenny, Maas, Riederer’22, Maas’23]

Generic behavior

Has been done for several observables

Generic behavior: DIS-like



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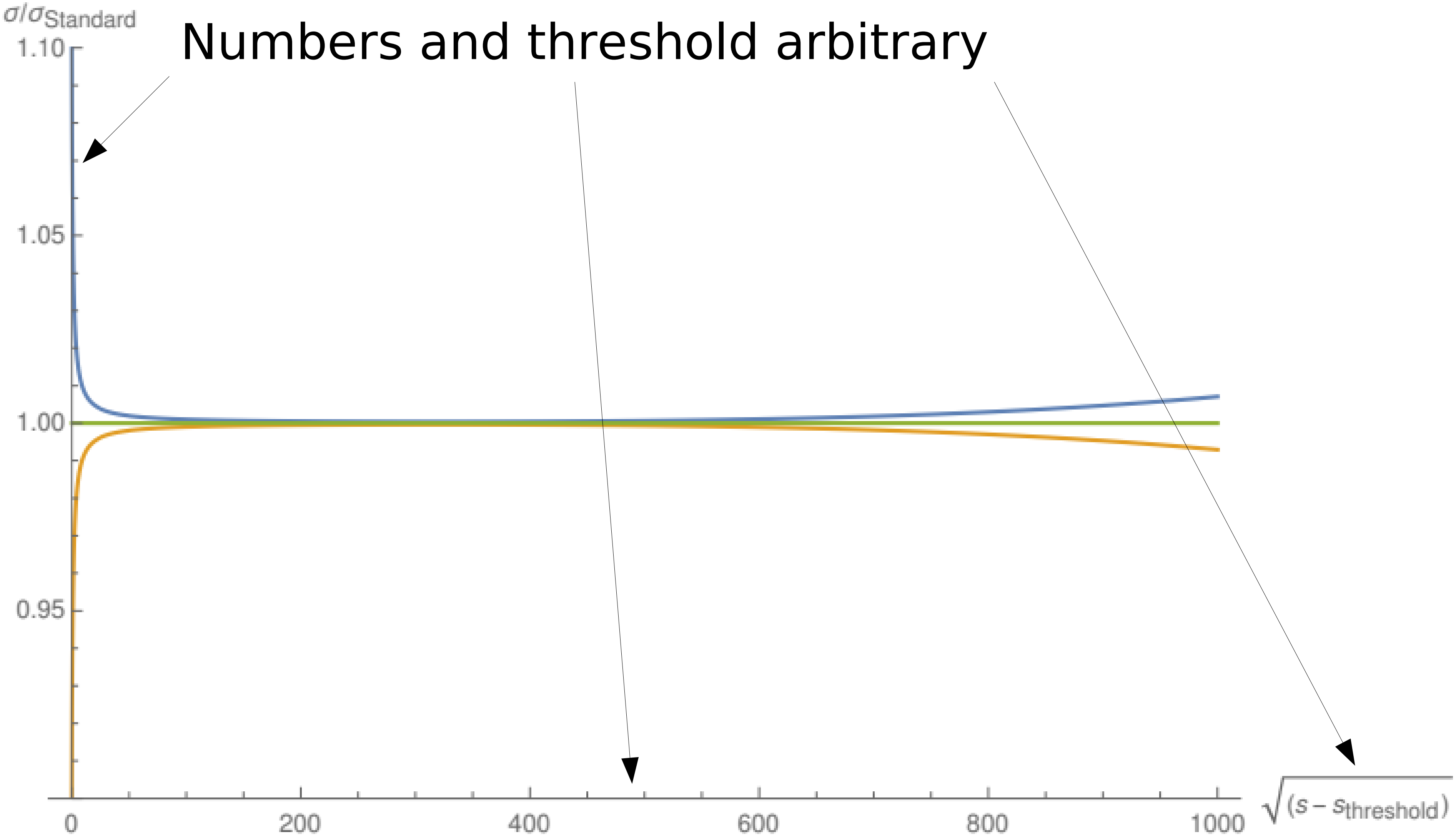

Numbers and threshold arbitrary

Has been done for several observables

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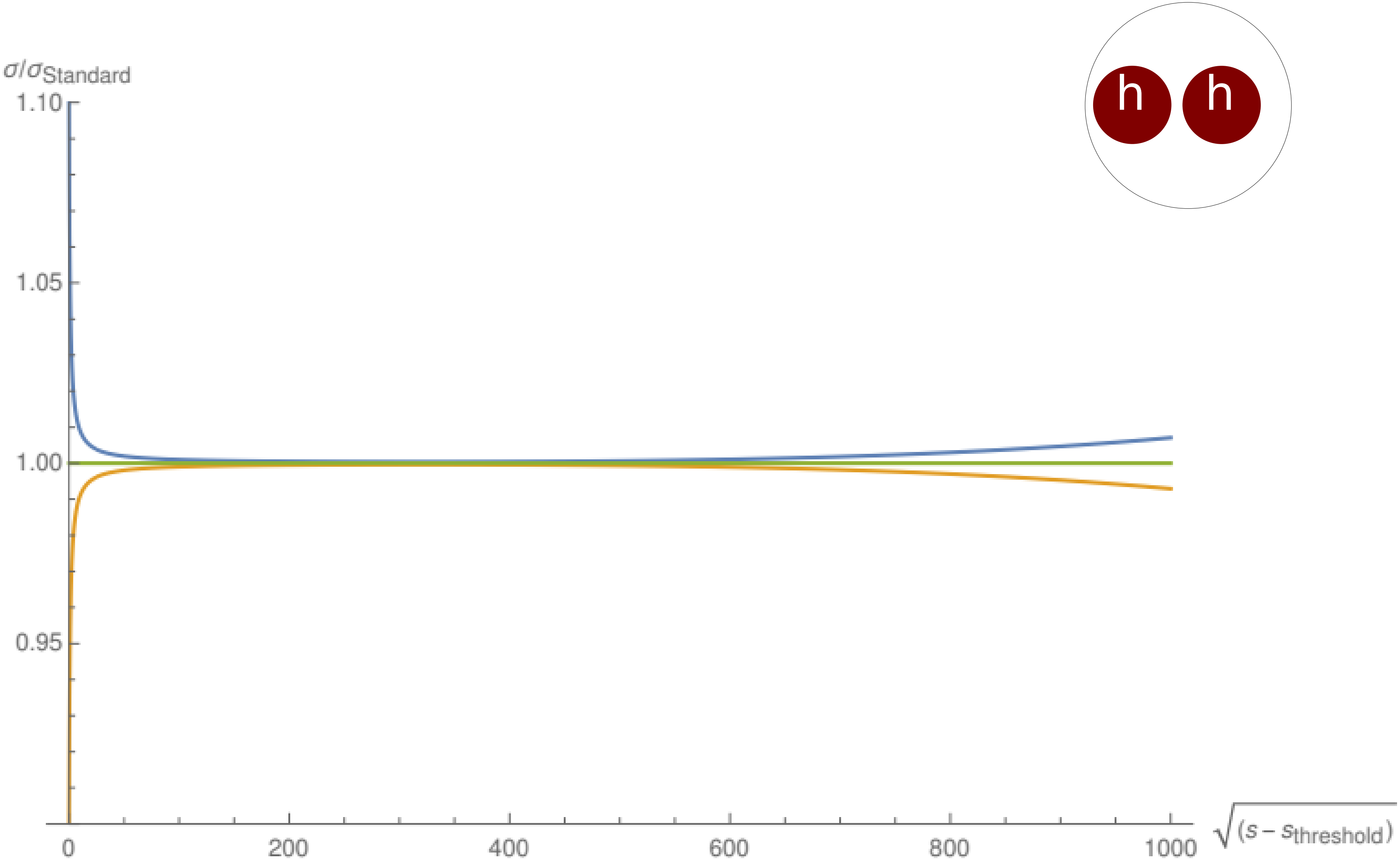

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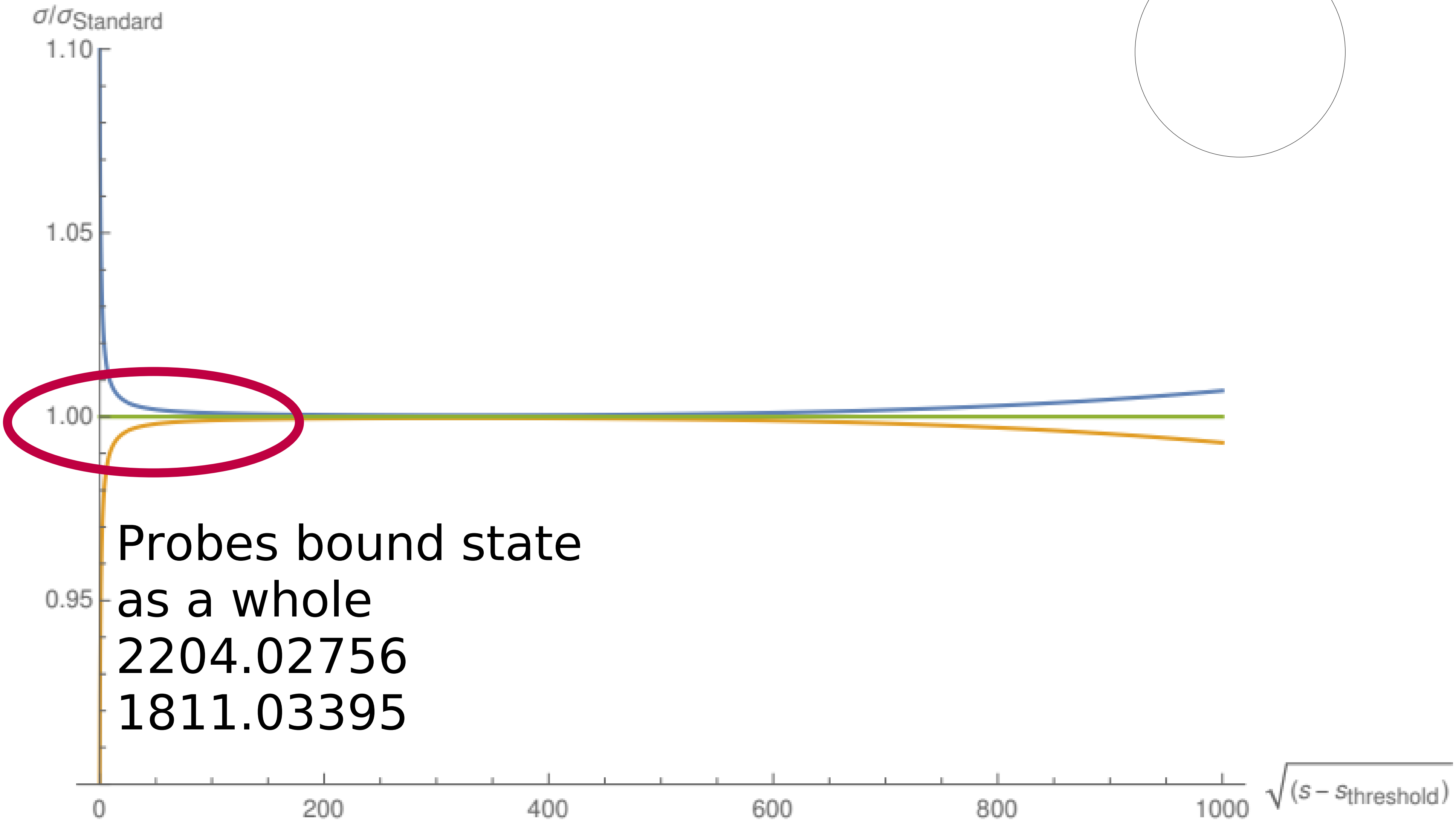
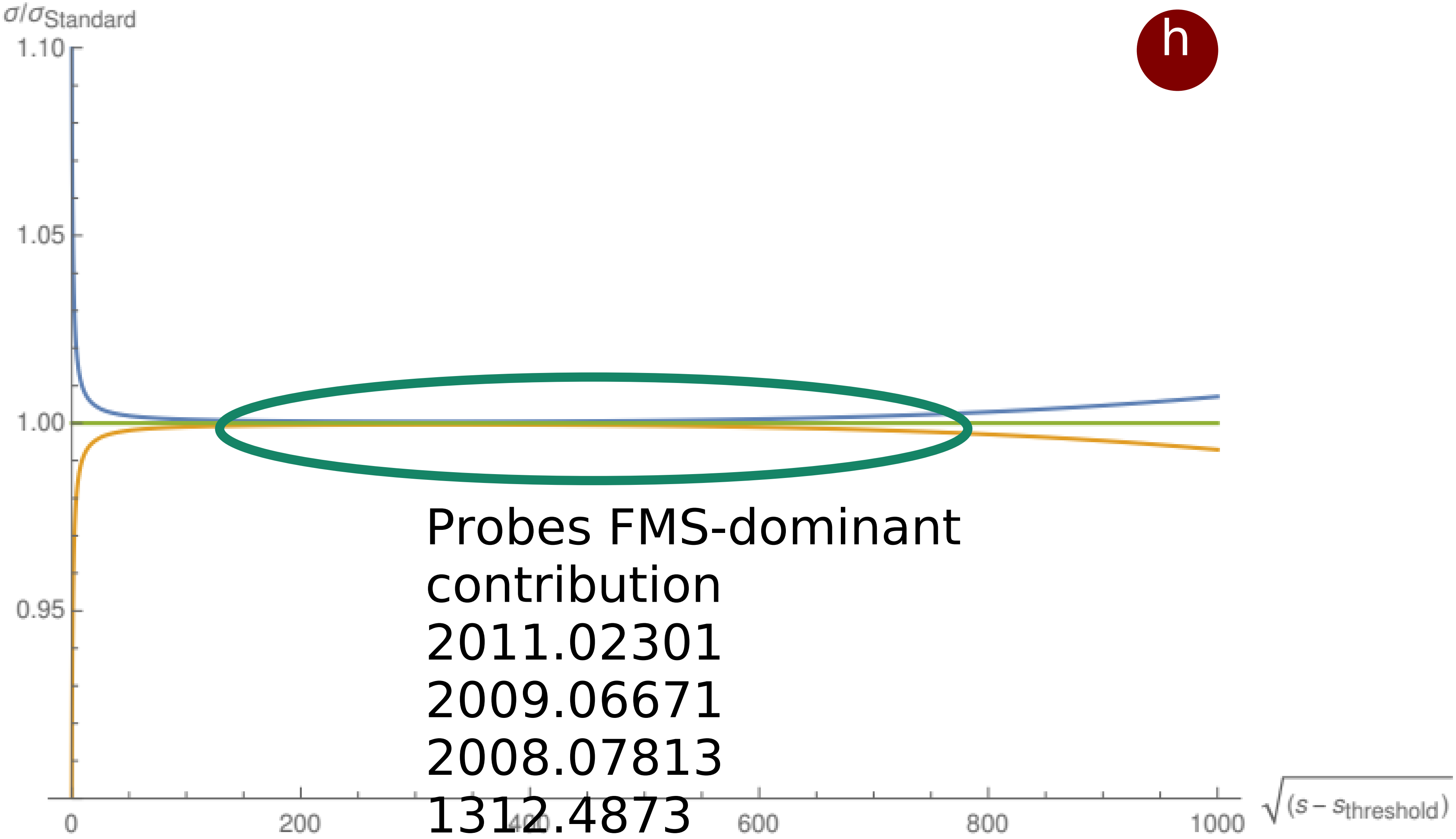


Probes bound state
as a whole
2204.02756
1811.03395

Has been done for several observables

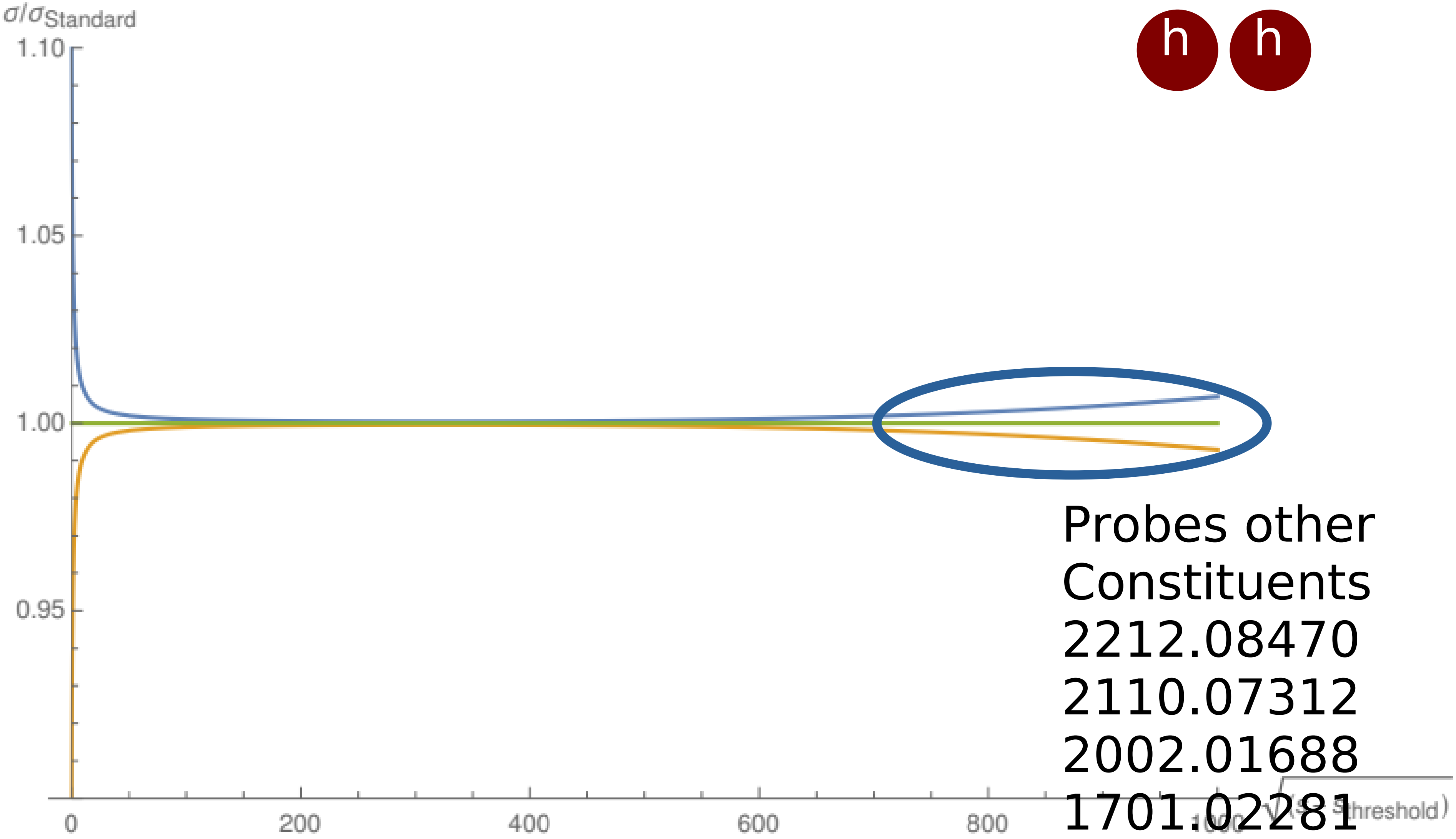
Generic behavior: DIS-like



Probes FMS-dominant contribution
2011.02301
2009.06671
2008.07813
1312.4873

Has been done for several observables

Generic behavior: DIS-like



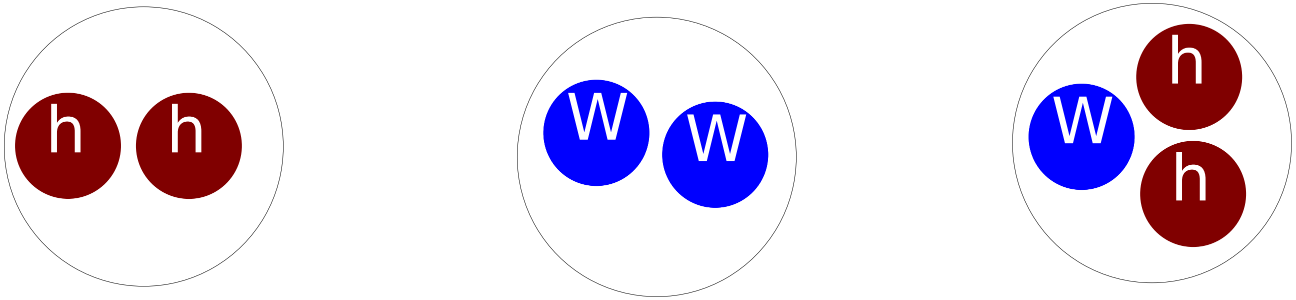
Probes other
Constituents
2212.08470
2110.07312
2002.01688
1701.02281

Has been done for several observables

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.

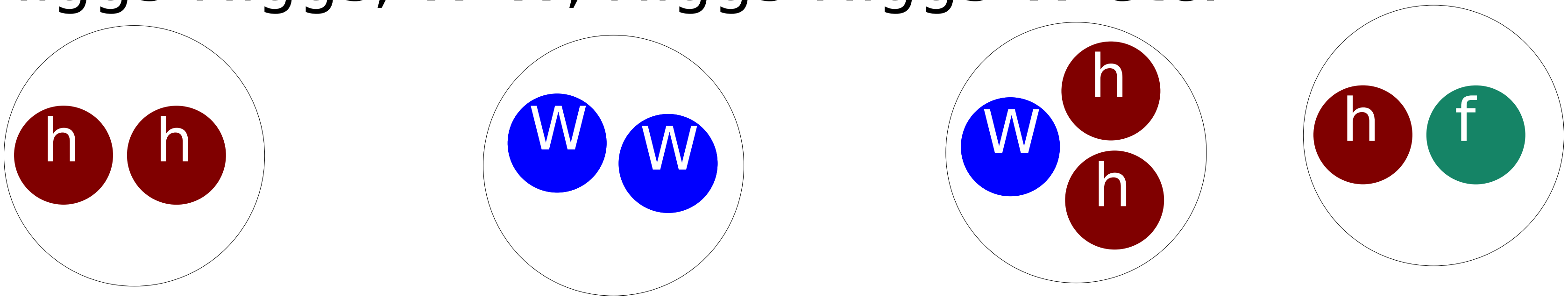


- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)
- Can this matter?

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



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[Fröhlich et al.'80,
Egger, Maas, Sondenheimer'17]

- Flavor has two components
    - Global SU(3) generation
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Flavor on the lattice

[Afferrante,Maas,Sondenheimer,Törek'20]

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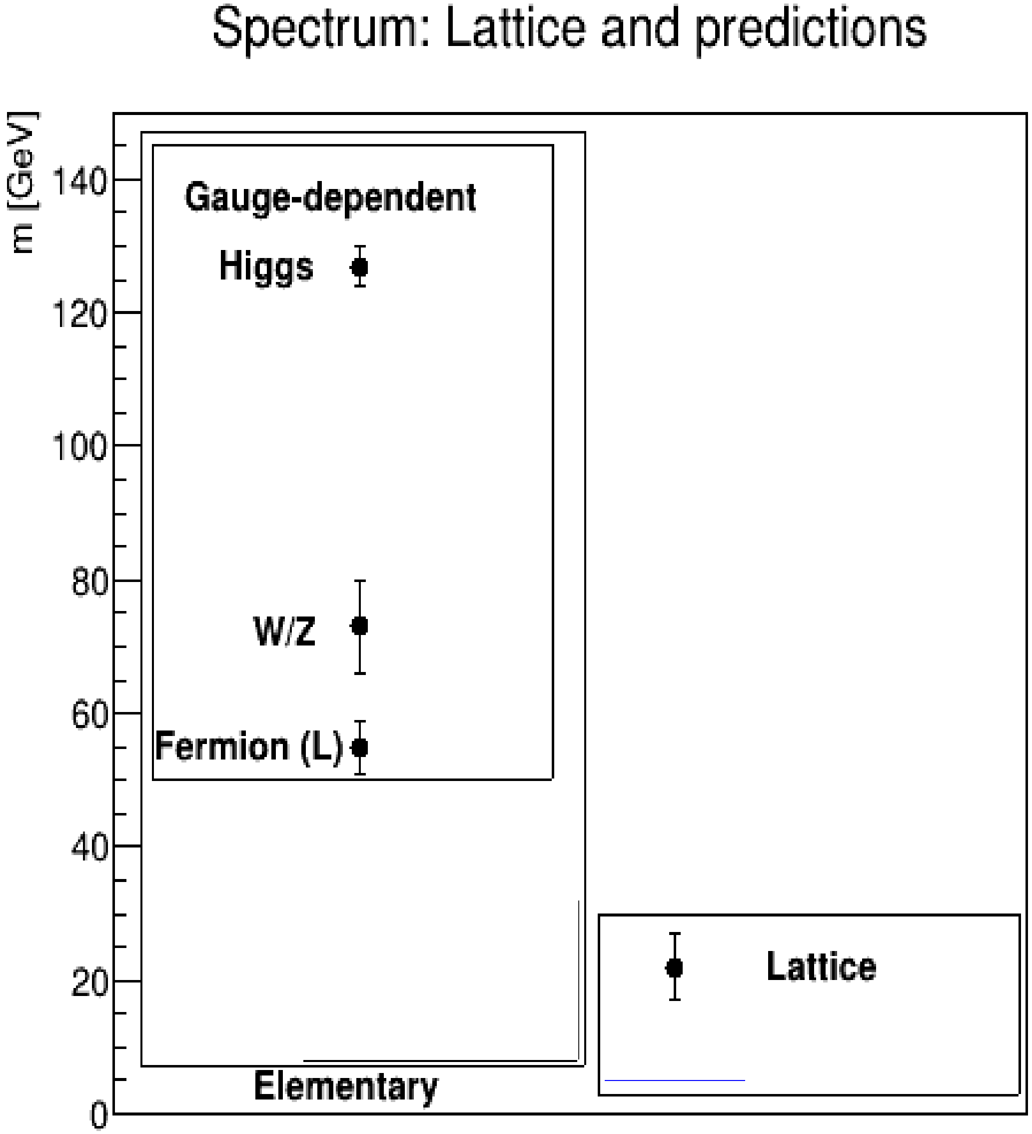

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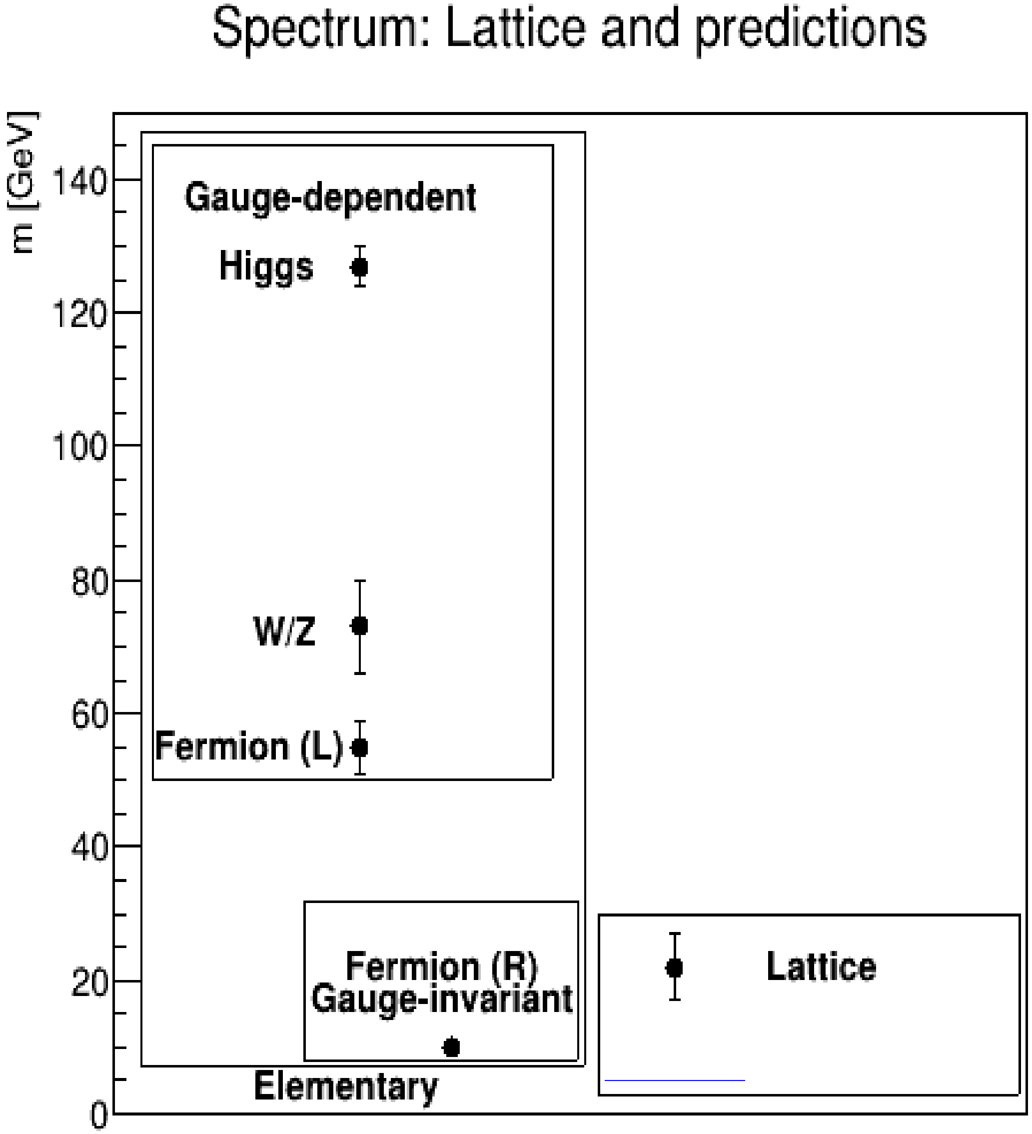

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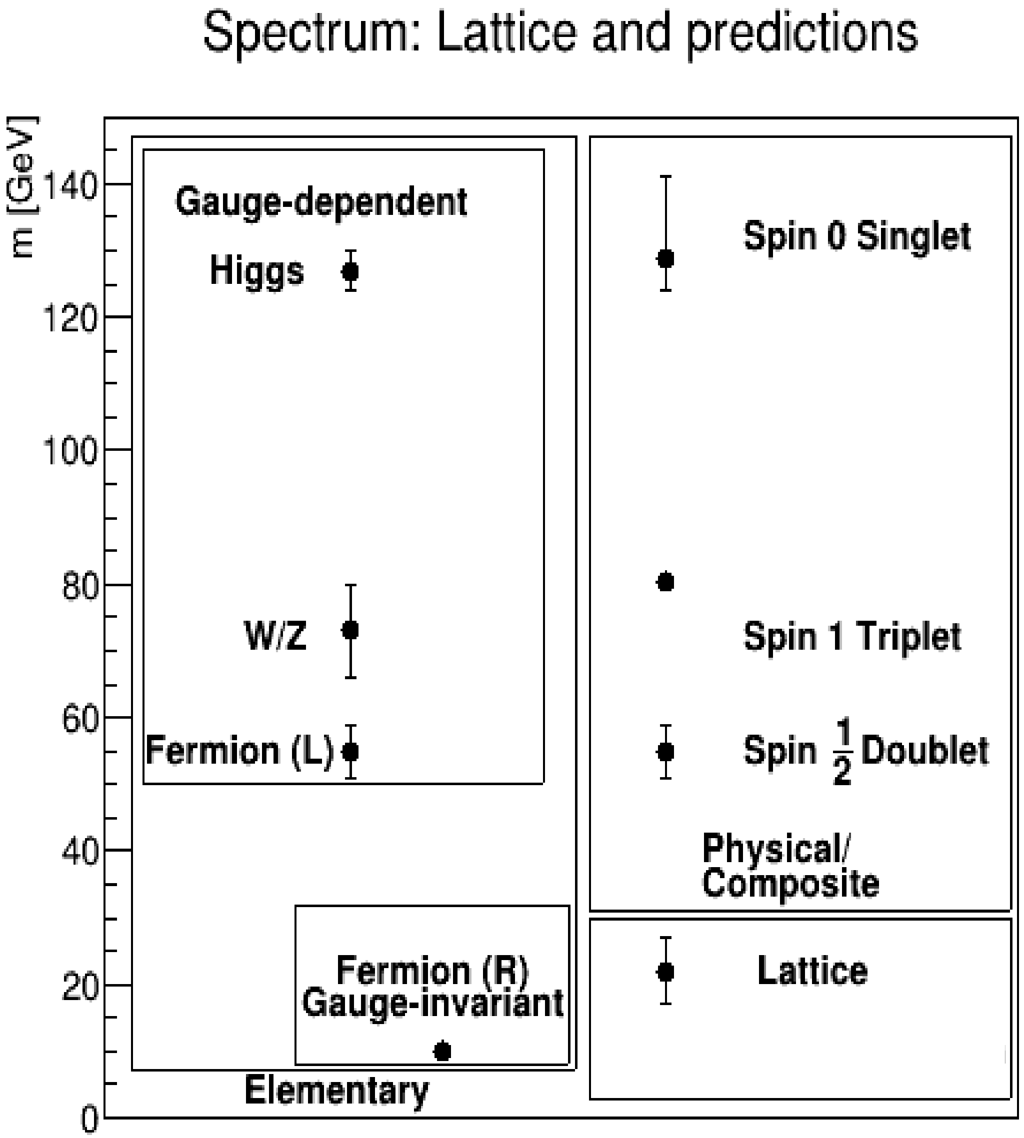

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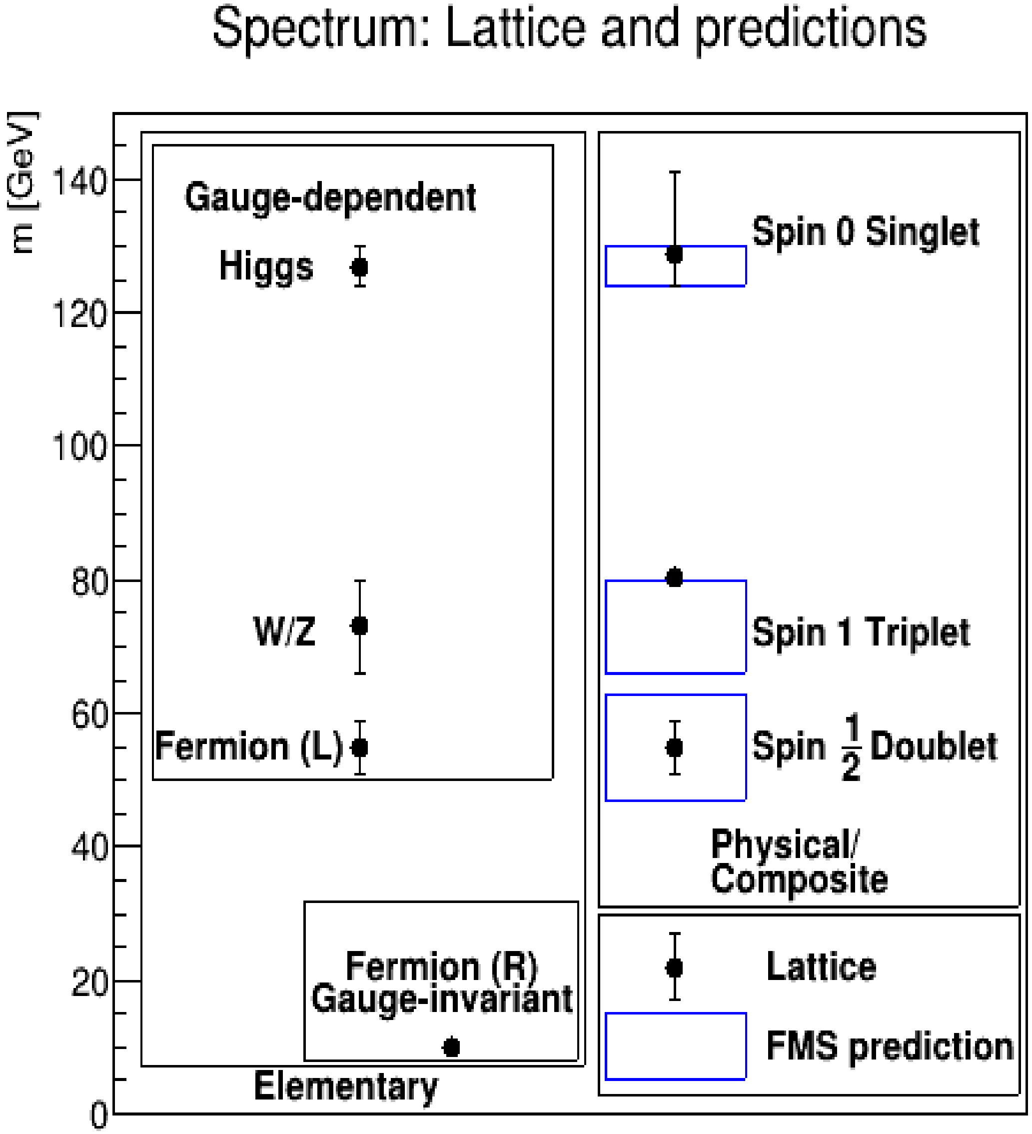

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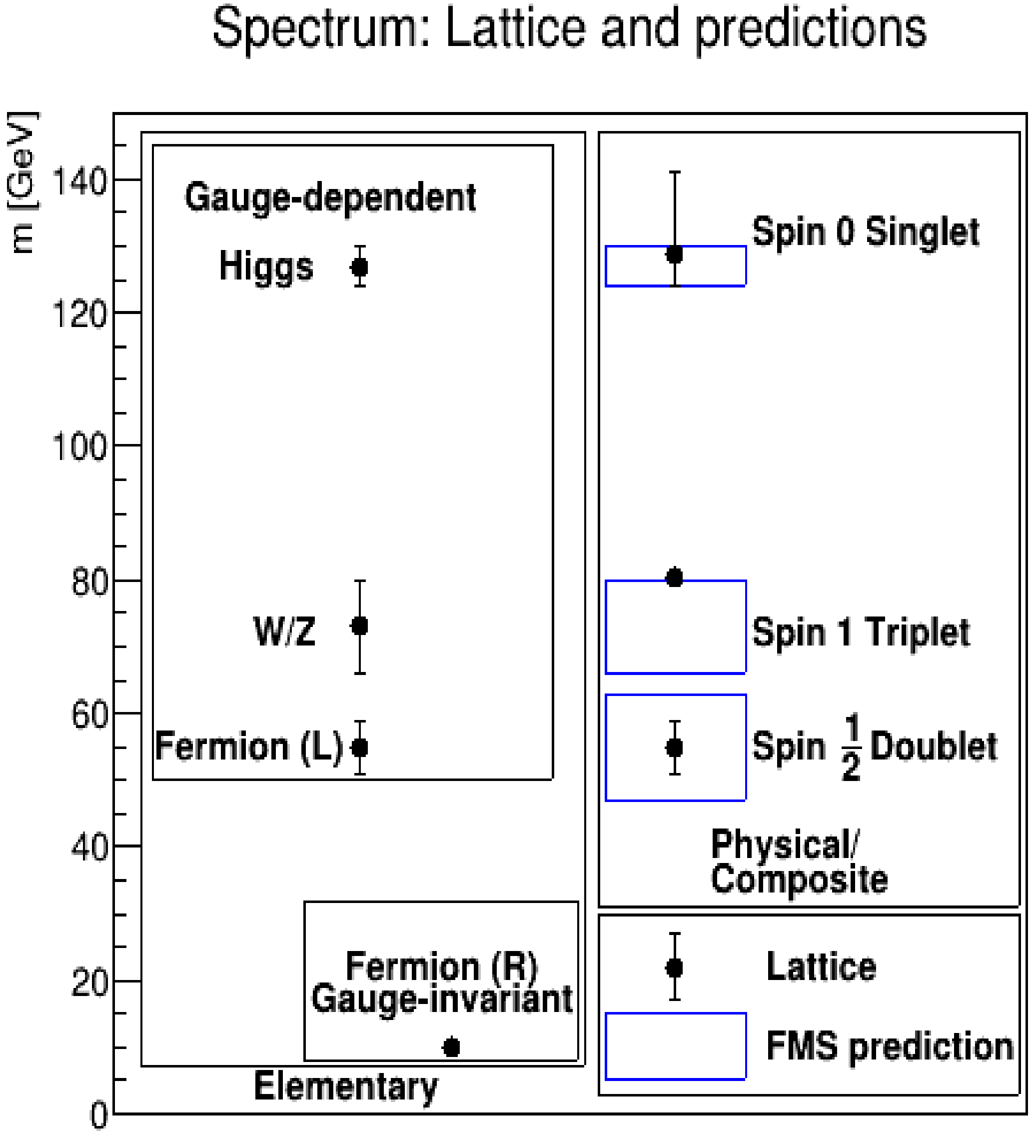

New physics
-
Qualitative changes

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Török'17]

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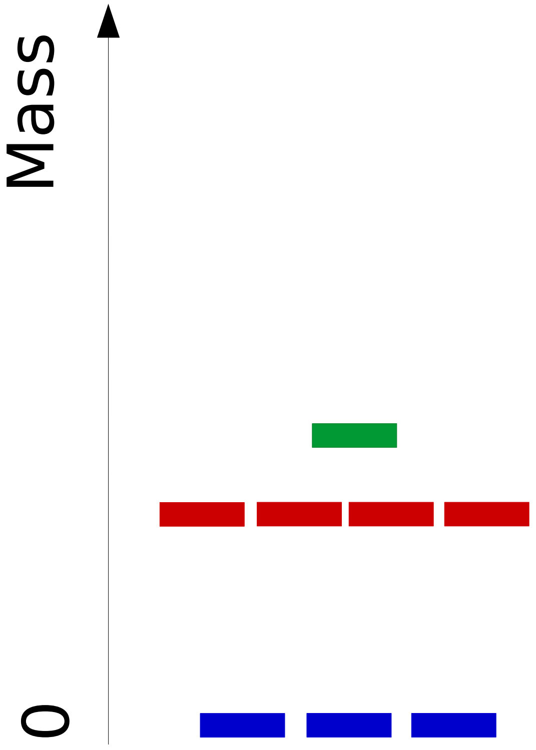
- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $\qquad h\rightarrow\exp(ia)h$

Spectrum

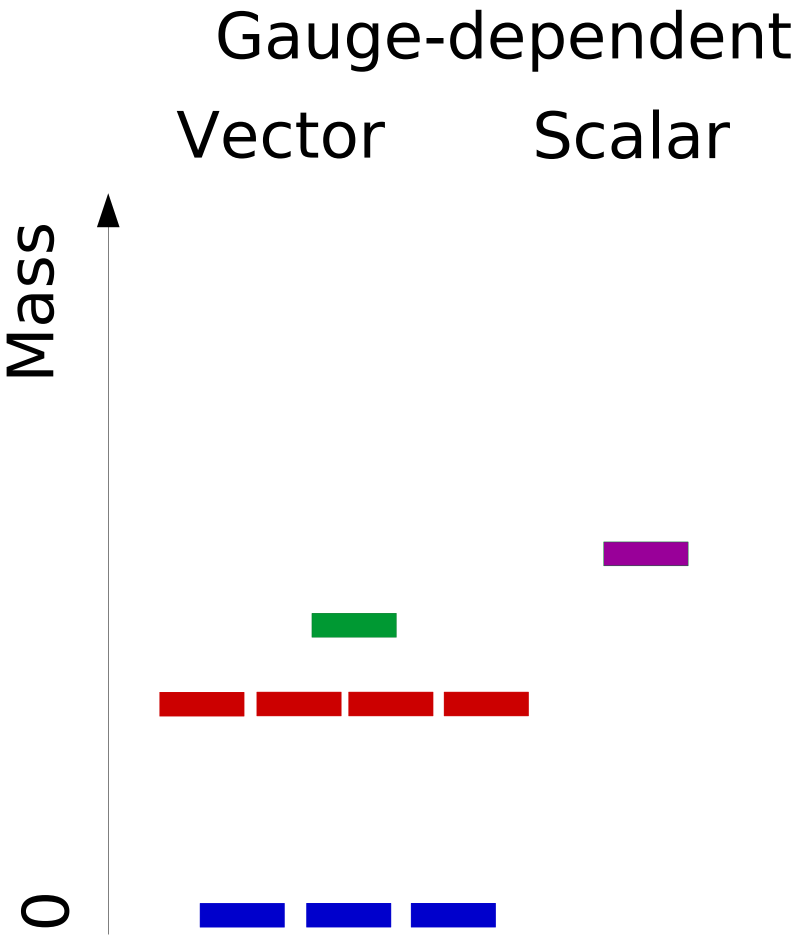
Gauge-dependent

Vector



‘SU(3)→ SU(2)’

Spectrum



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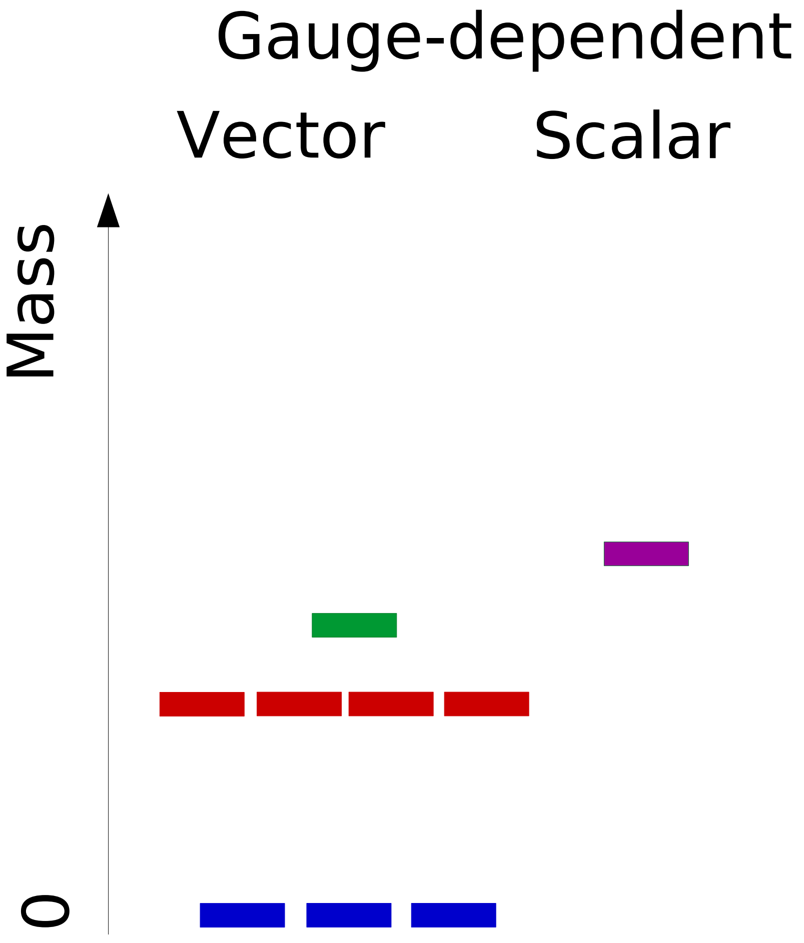

Confirmed in gauge-fixed
lattice calculations [Maas et al.'16]

Spectrum

[Maas & Törek’16,'18
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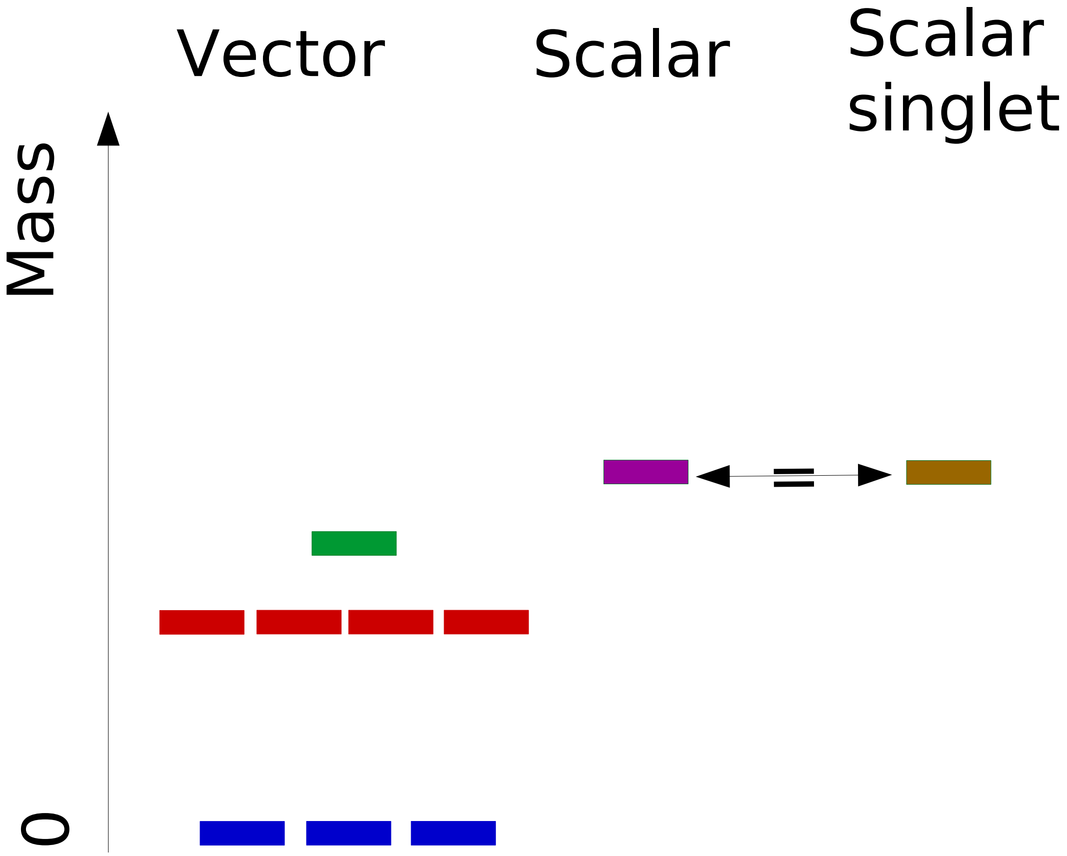
Gauge-dependent

Gauge-invariant



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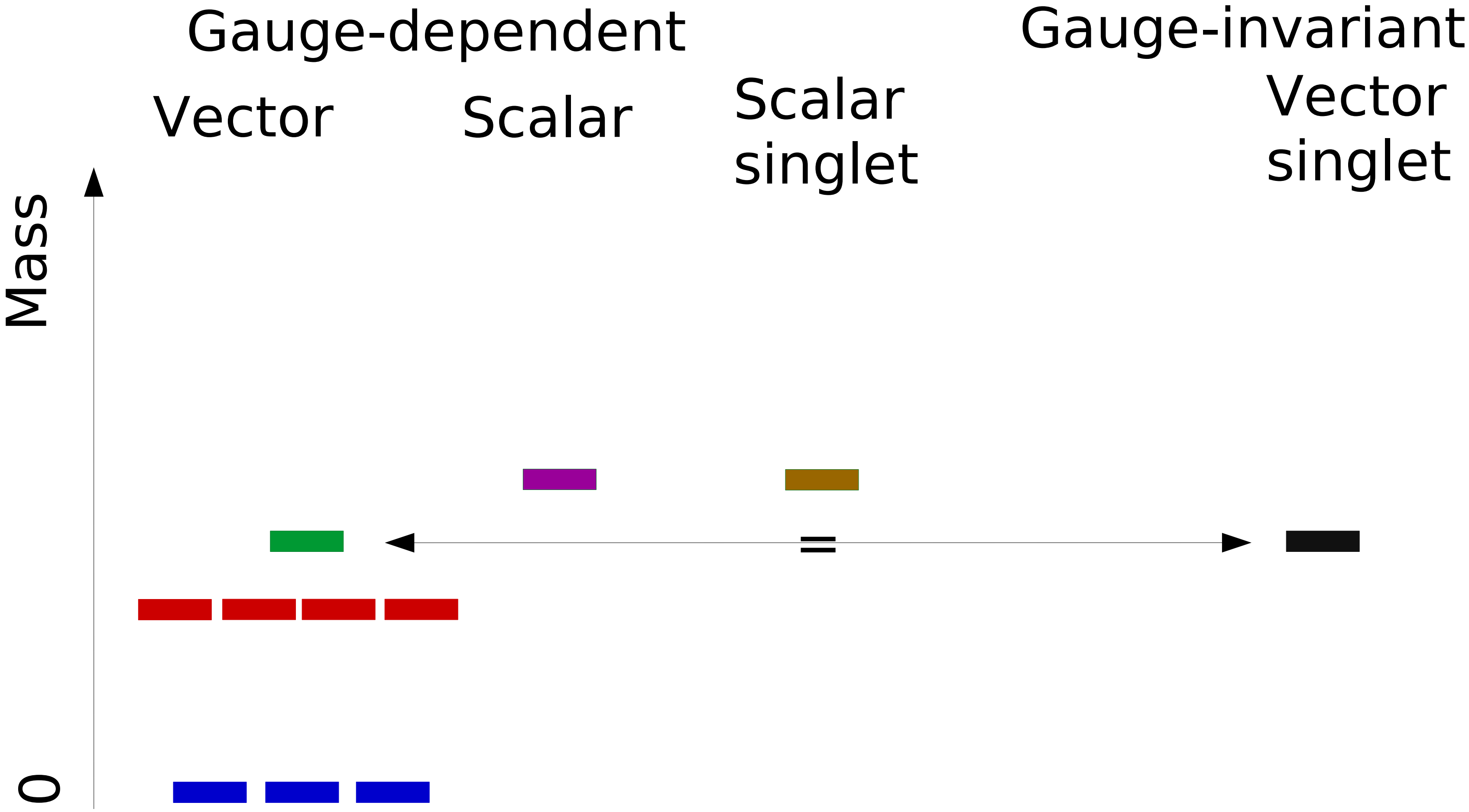

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Matrix from group structure

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Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

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- Now: States without elementary analouge
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue – U(1) acts as baryon number
  - Lightest must exist and be absolutely stable

Possible new states

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  - Prediction with constituent model

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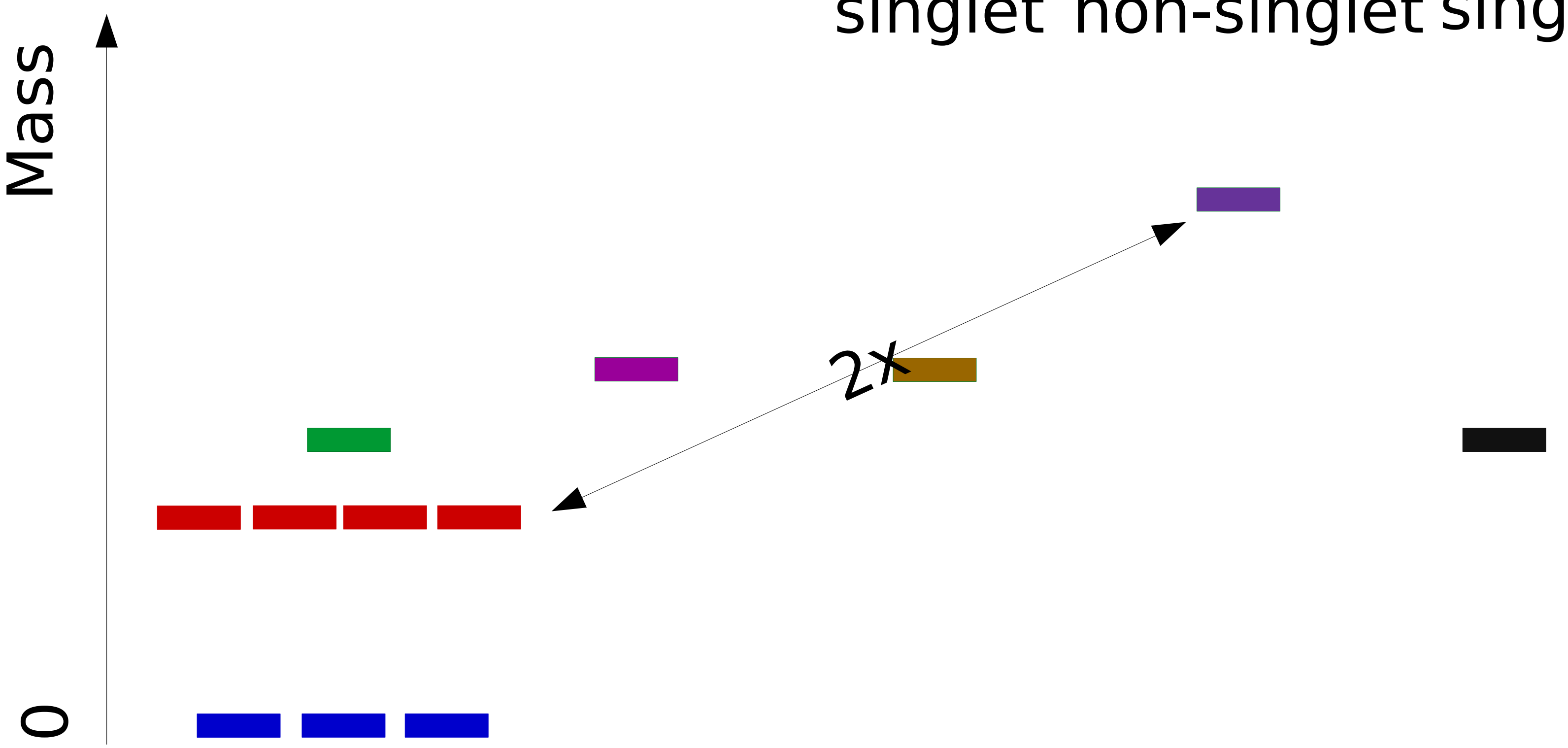

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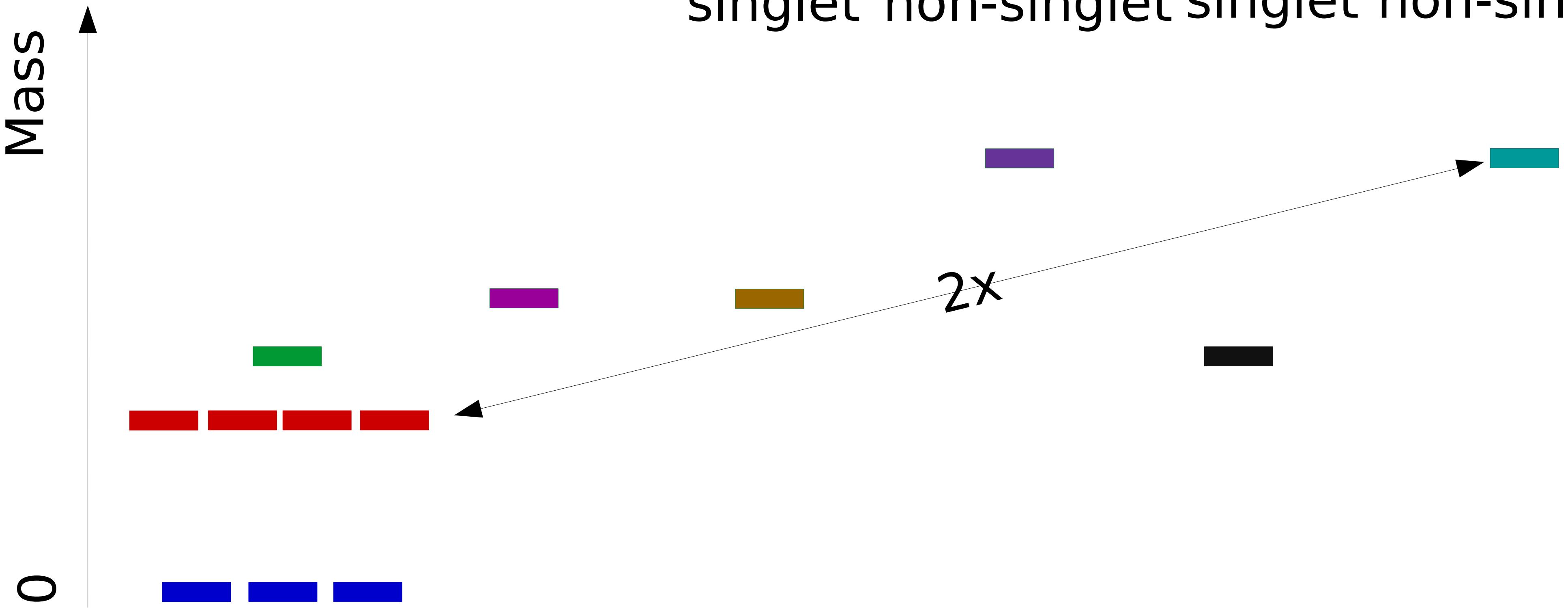

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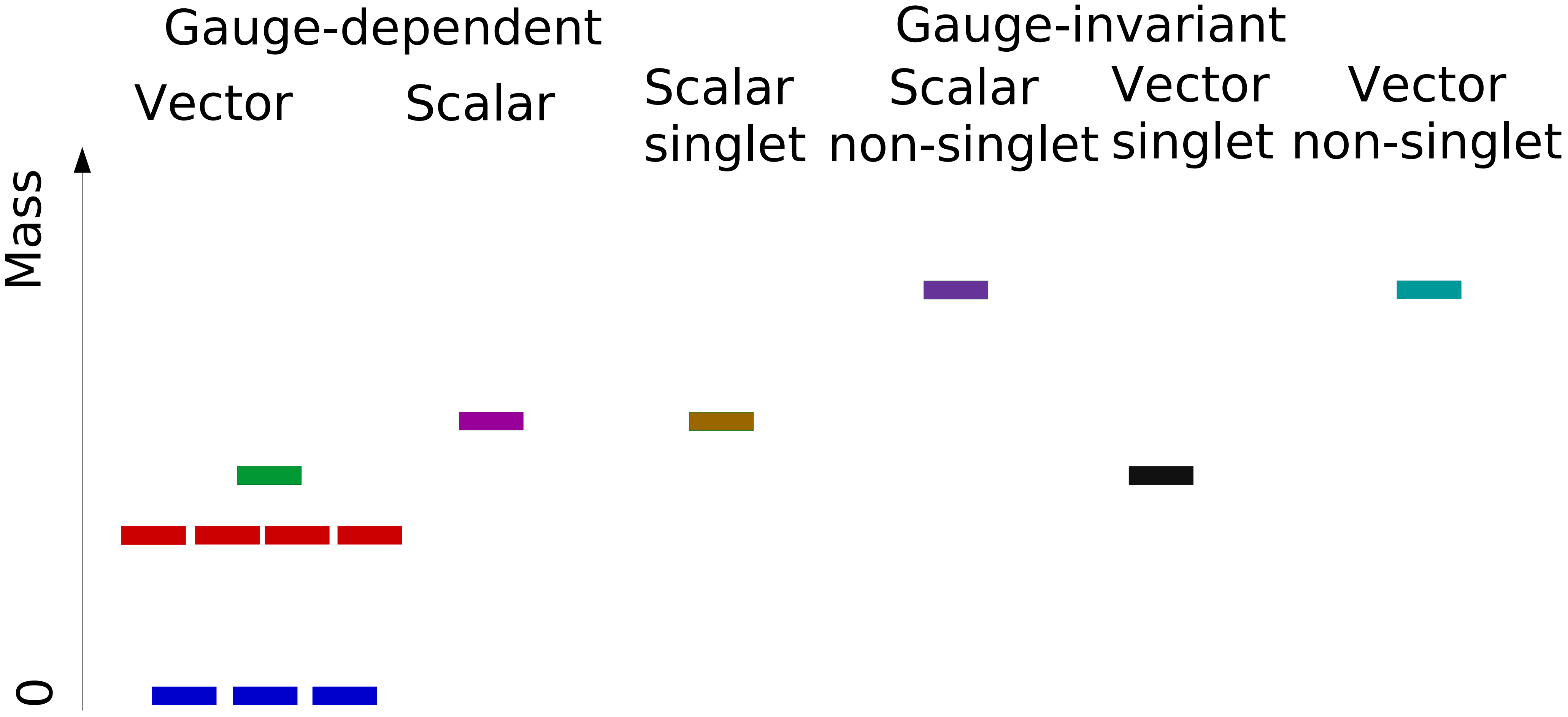

- Qualitatively different spectrum
- No mass gap!

Possible states

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  - Lattice calulations

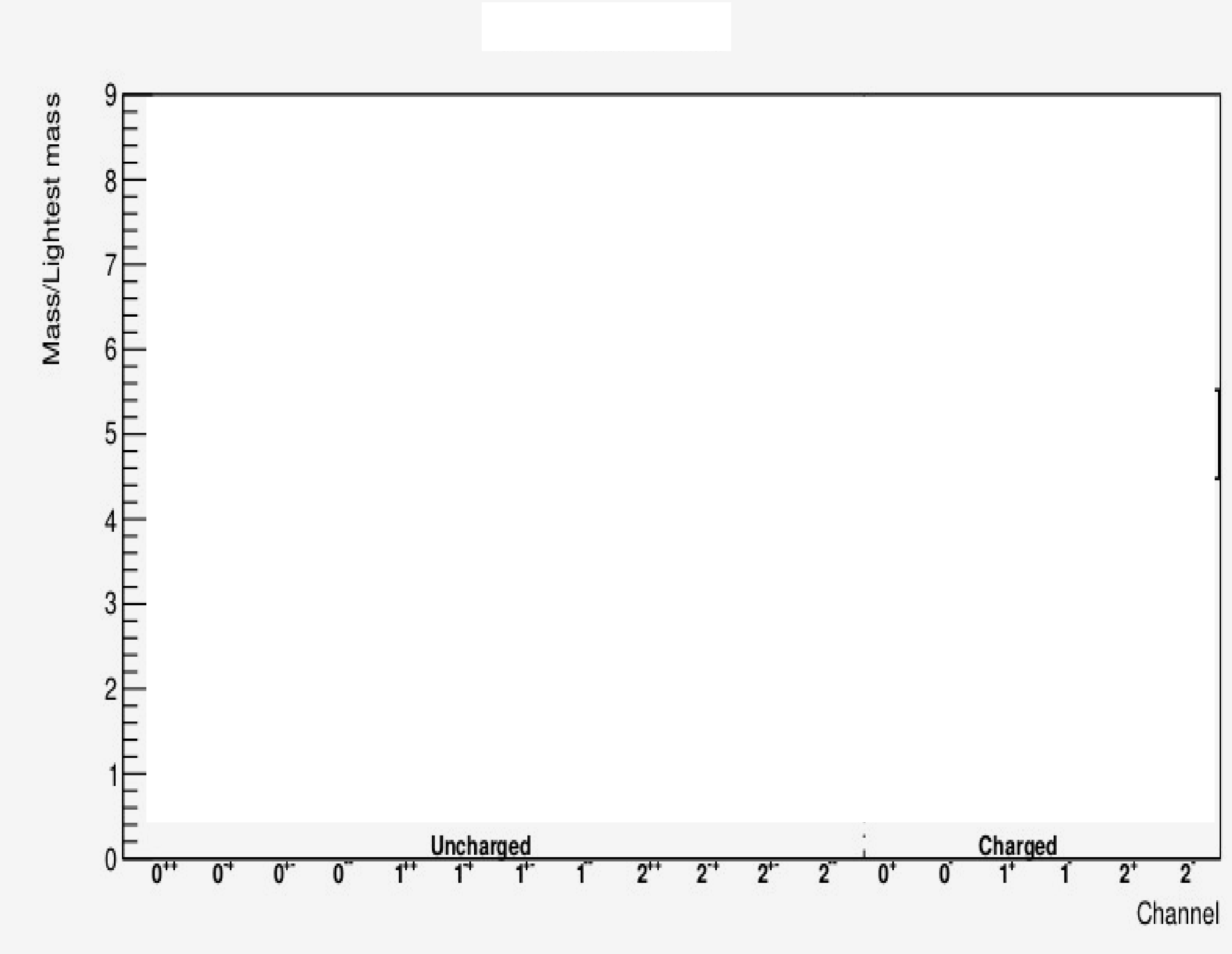
Possible states

- Quantum numbers are $J^{PC}_{\text{Custodial}}$
  - Simpelst non-trivial state operator: 0^{++}_{1}
  - $\epsilon_{abc}\phi^{a}D_{\mu}\phi^{b}D_{\nu}D^{\nu}D^{\mu}\phi^{c}$
- What is the lightest state?
  - Prediction with constituent model
  - Lattice calulations
  - All channels: J<3
  - Aim: Ground state for each channel
    - Characterization through scattering states

Typical spectrum

PRELIMINARY

[Dobson et al.’22]



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PRELIMINARY

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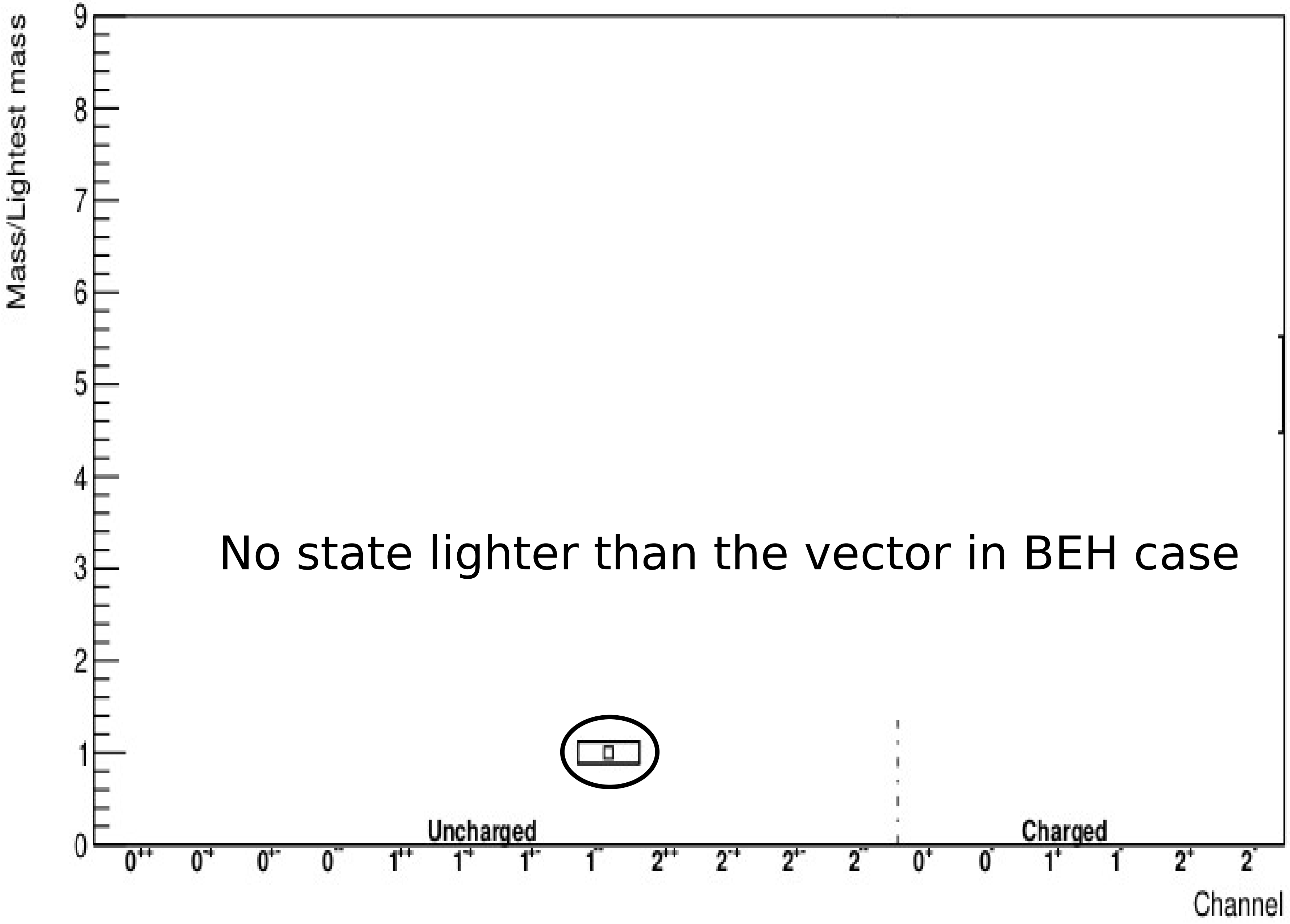
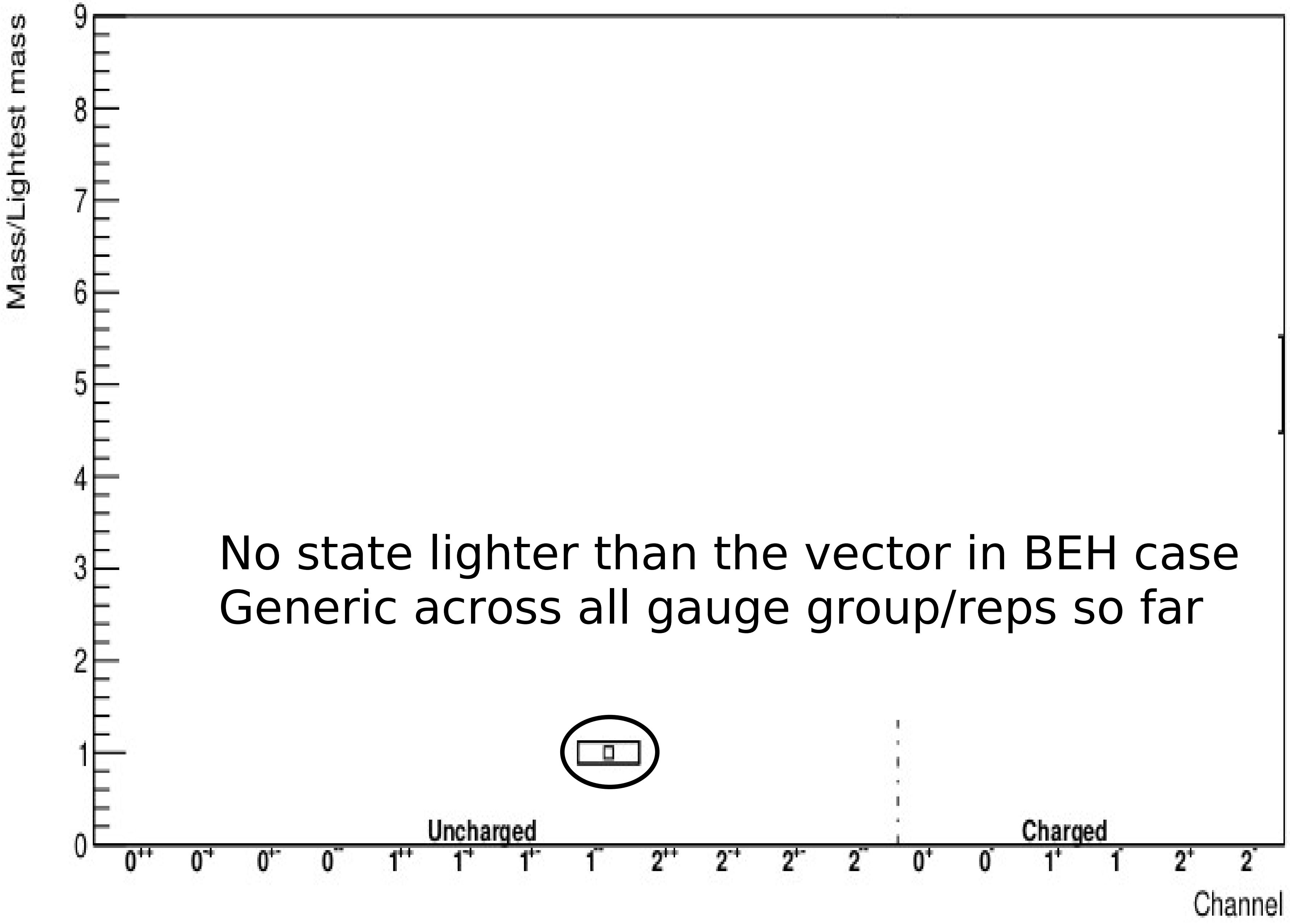


No state lighter than the vector in BEH case

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17]

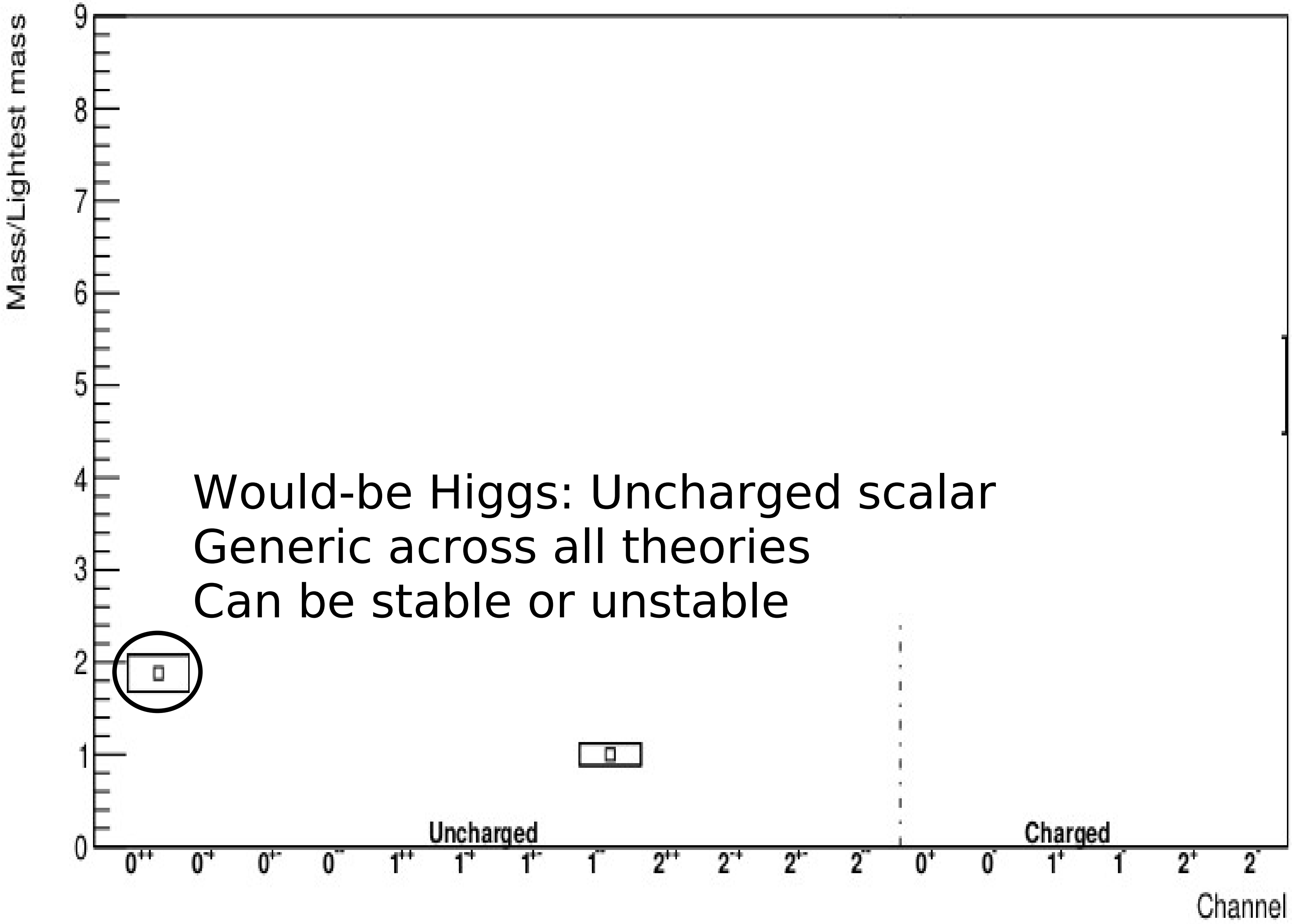


No state lighter than the vector in BEH case
Generic across all gauge group/reps so far

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

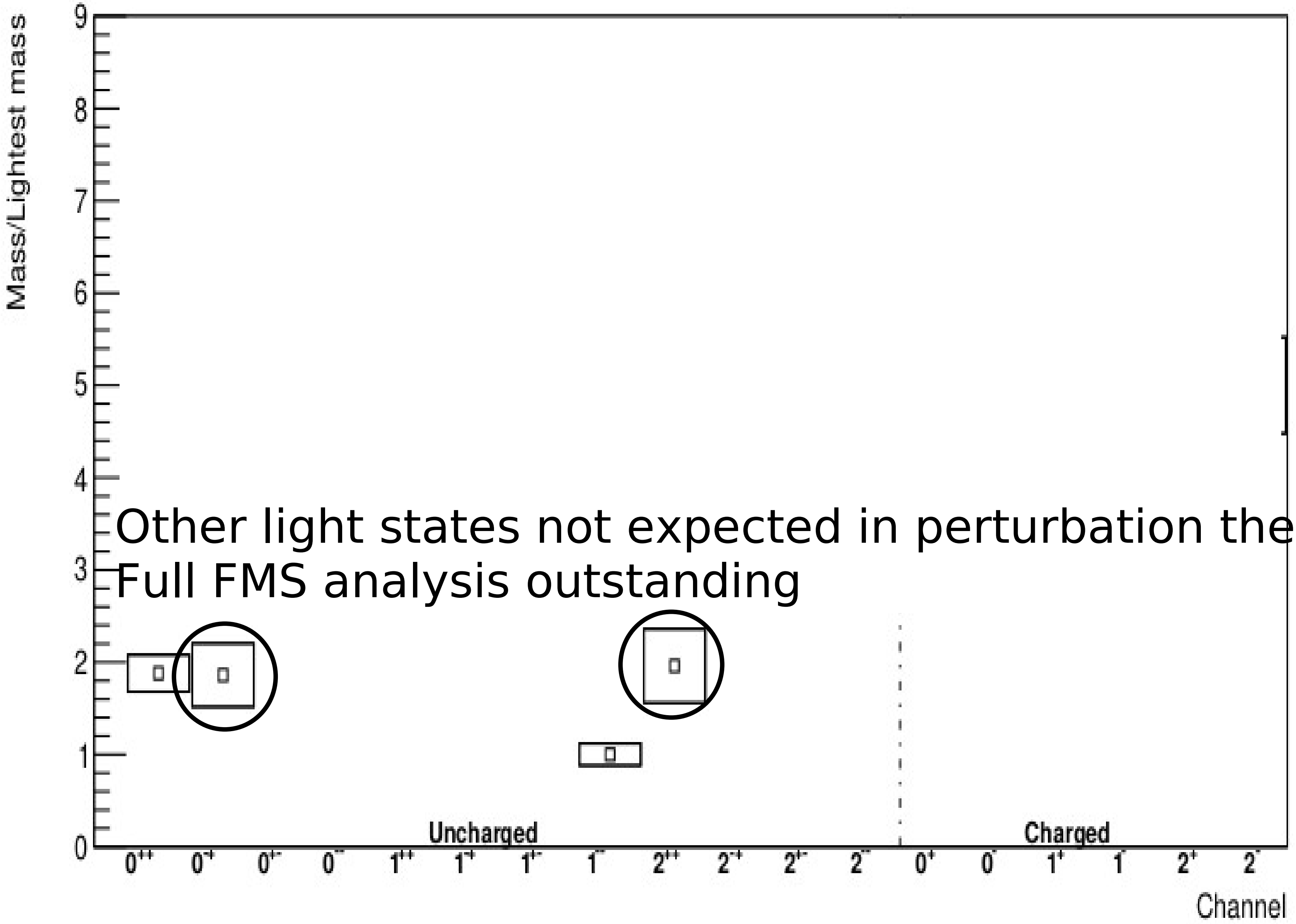


Would-be Higgs: Uncharged scalar
Generic across all theories
Can be stable or unstable

Typical spectrum

PRELIMINARY

[Dobson et al.'22
Maas '17
Jenny et al.'22]



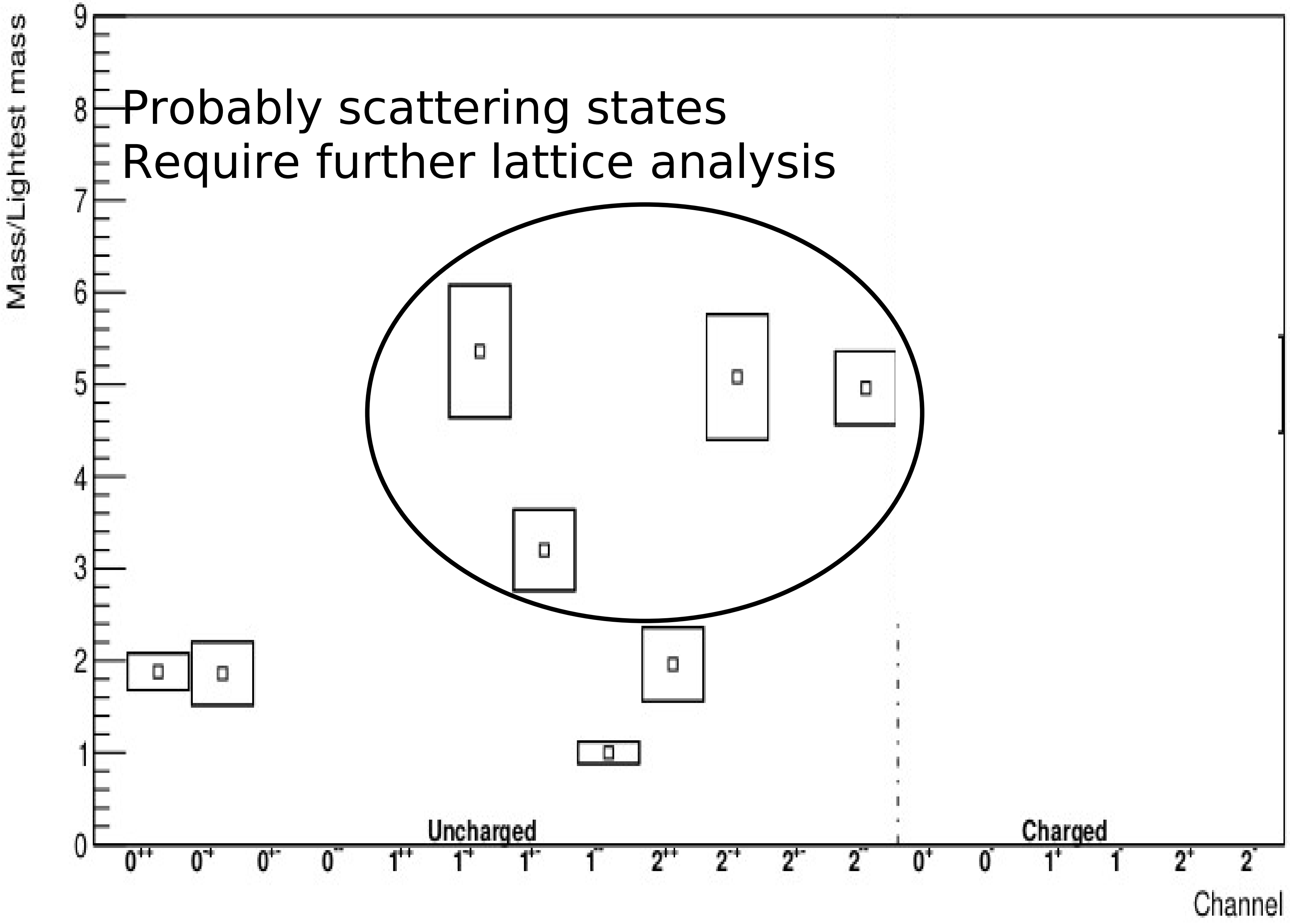
Other light states not expected in perturbation theory

Full FMS analysis outstanding

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

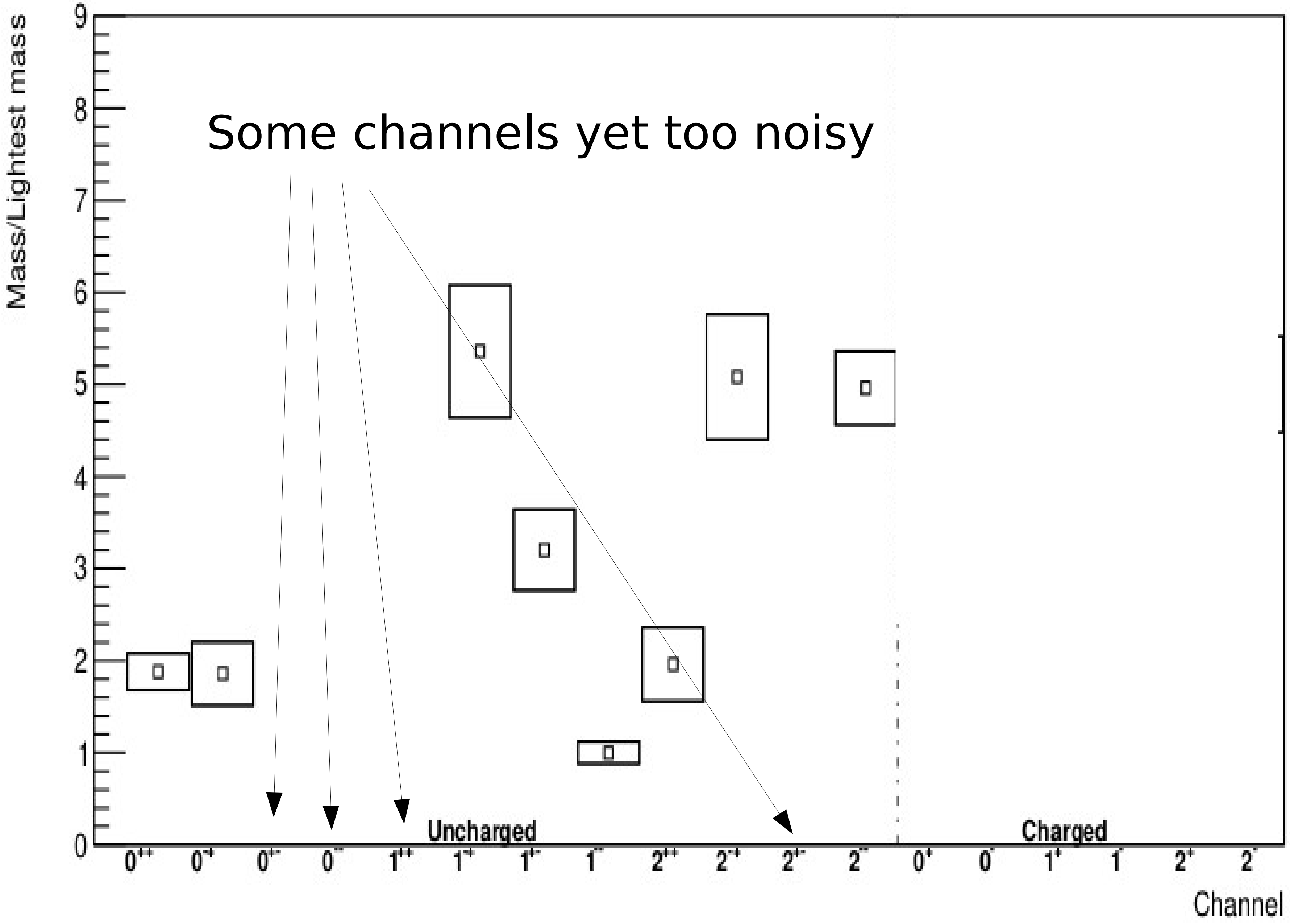


Probably scattering states
Require further lattice analysis

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

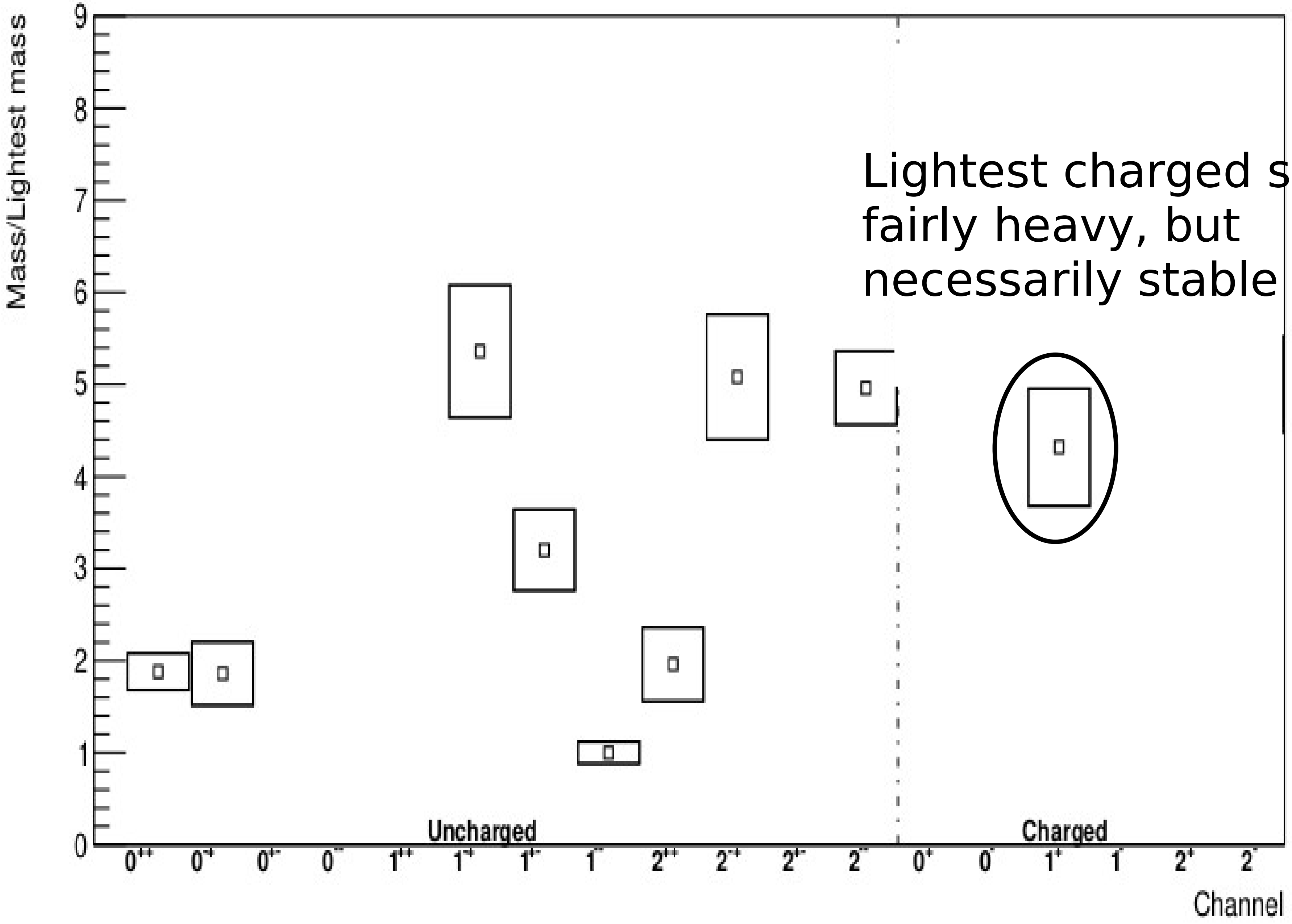


Some channels yet too noisy

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

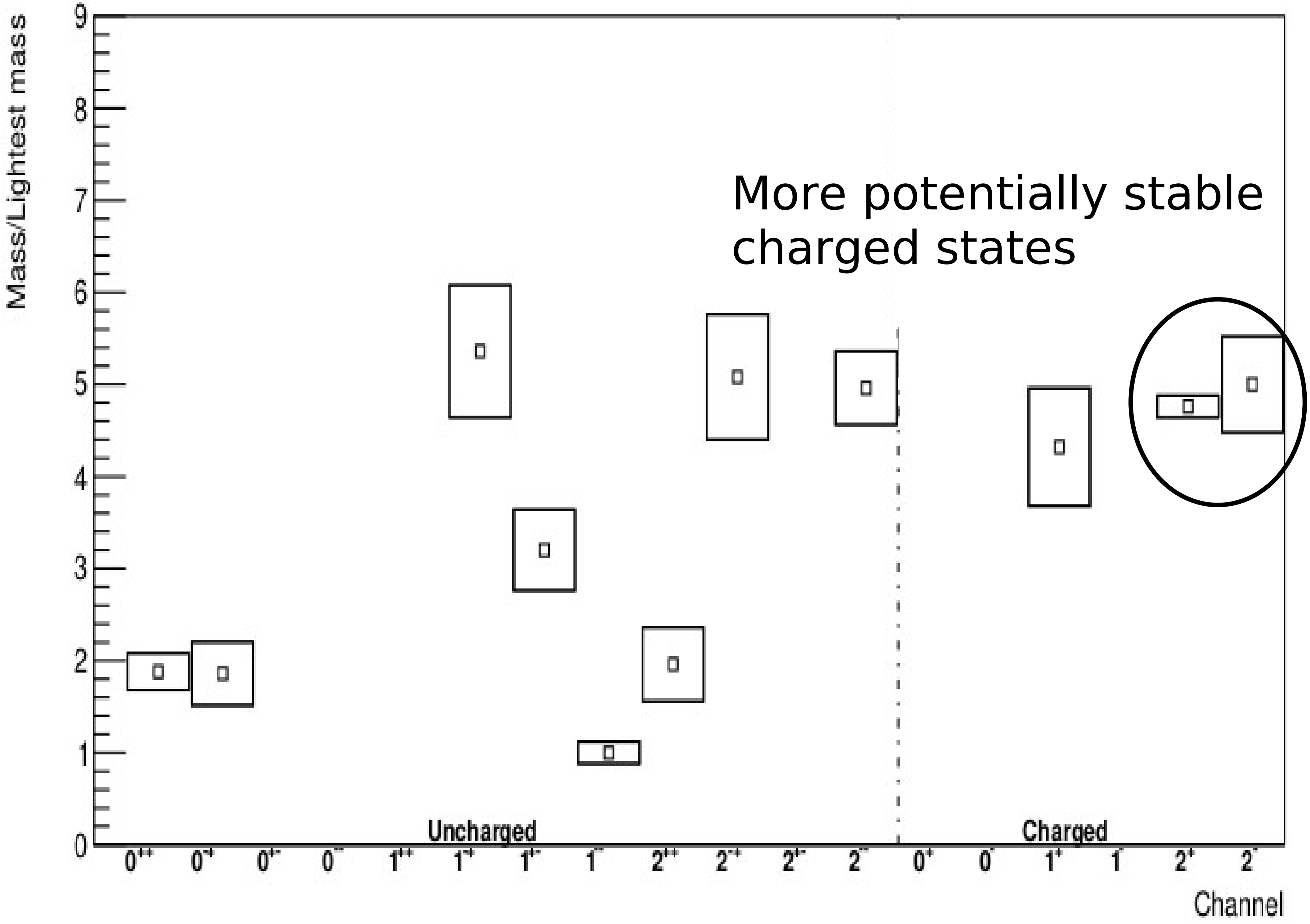


Lightest charged state fairly heavy, but necessarily stable

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

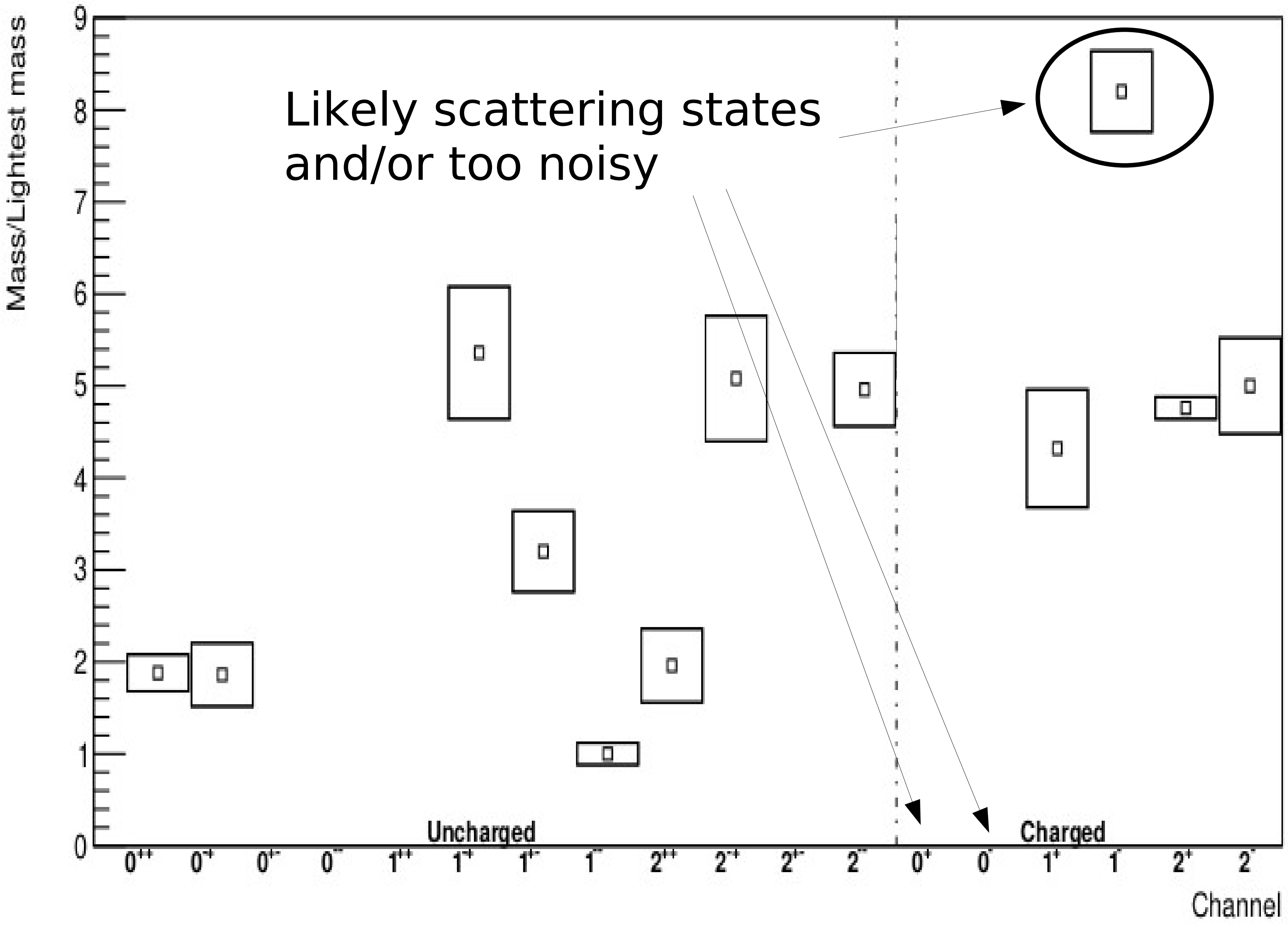


More potentially stable charged states

Typical spectrum

PRELIMINARY

[Dobson et al.'22
Maas '17
Jenny et al.'22]

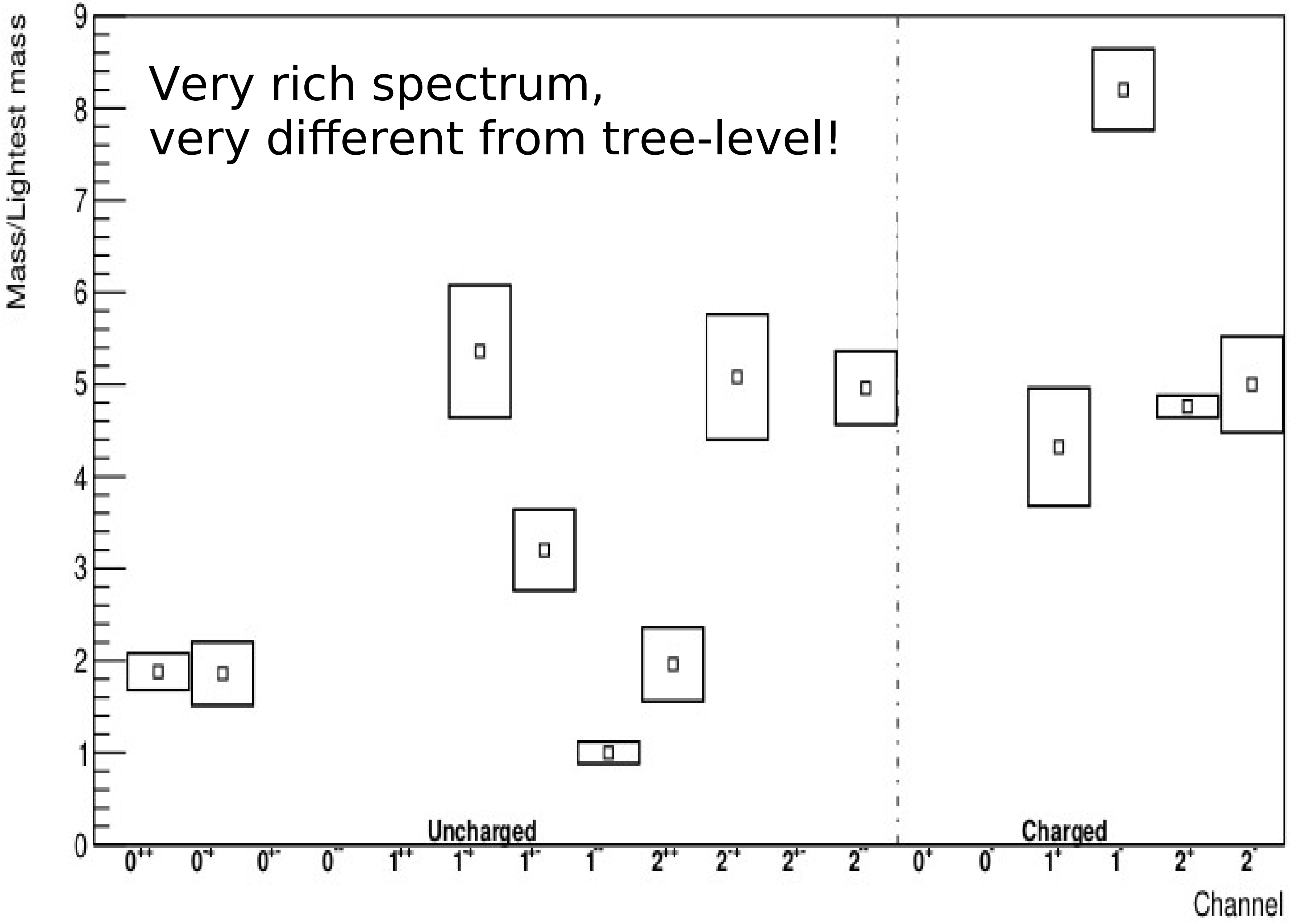


Likely scattering states and/or too noisy

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22]

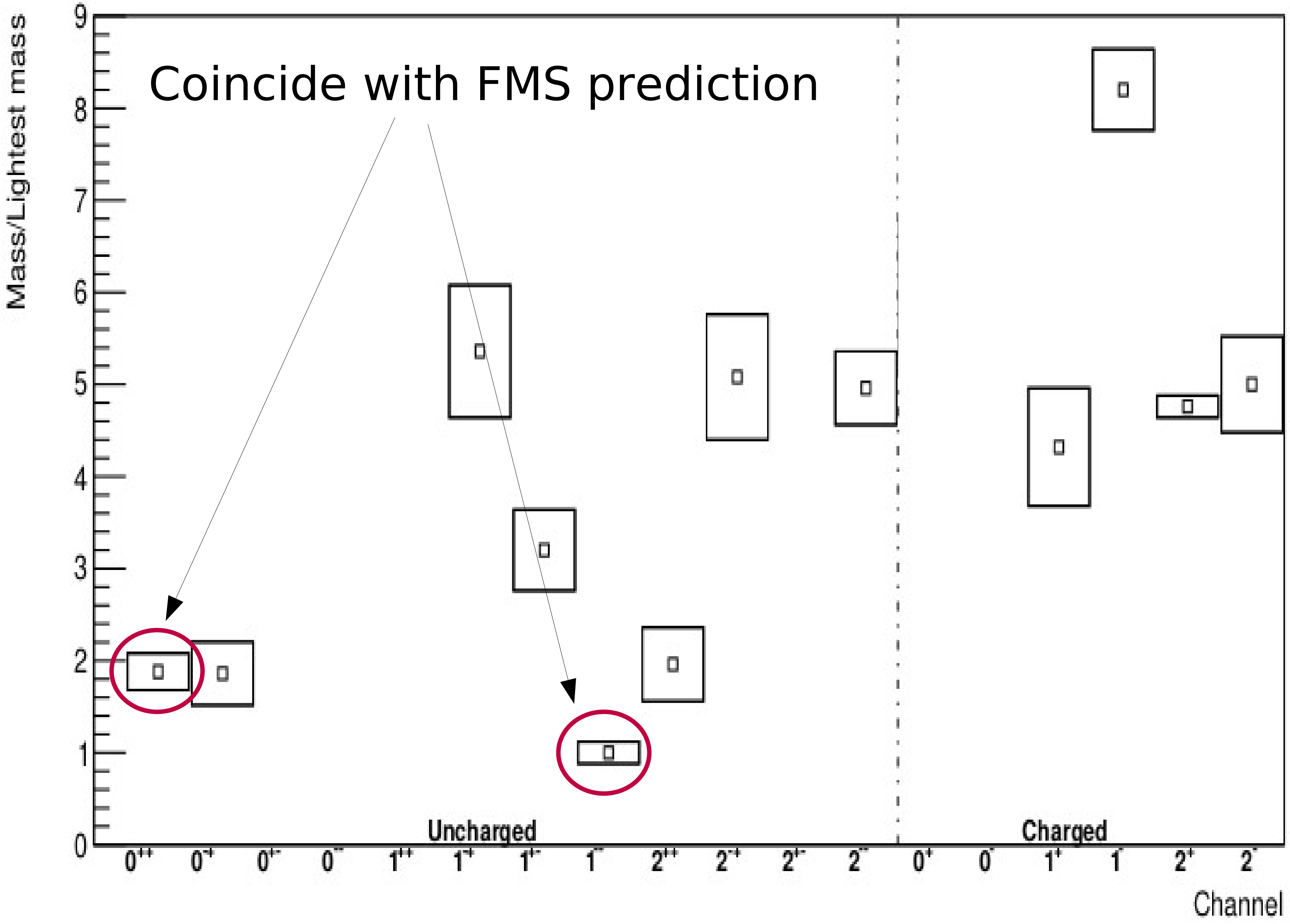


Very rich spectrum,
very different from tree-level!

Typical spectrum

PRELIMINARY

[Dobson et al.’22
Maas ‘17
Jenny et al.’22
Maas et al. ‘16, ‘18]


Coincide with FMS prediction

Experimental consequences

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[Maas & Törek'18
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- Add fundamental fermions

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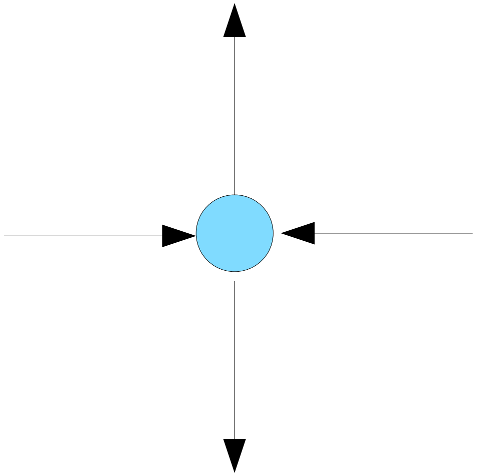

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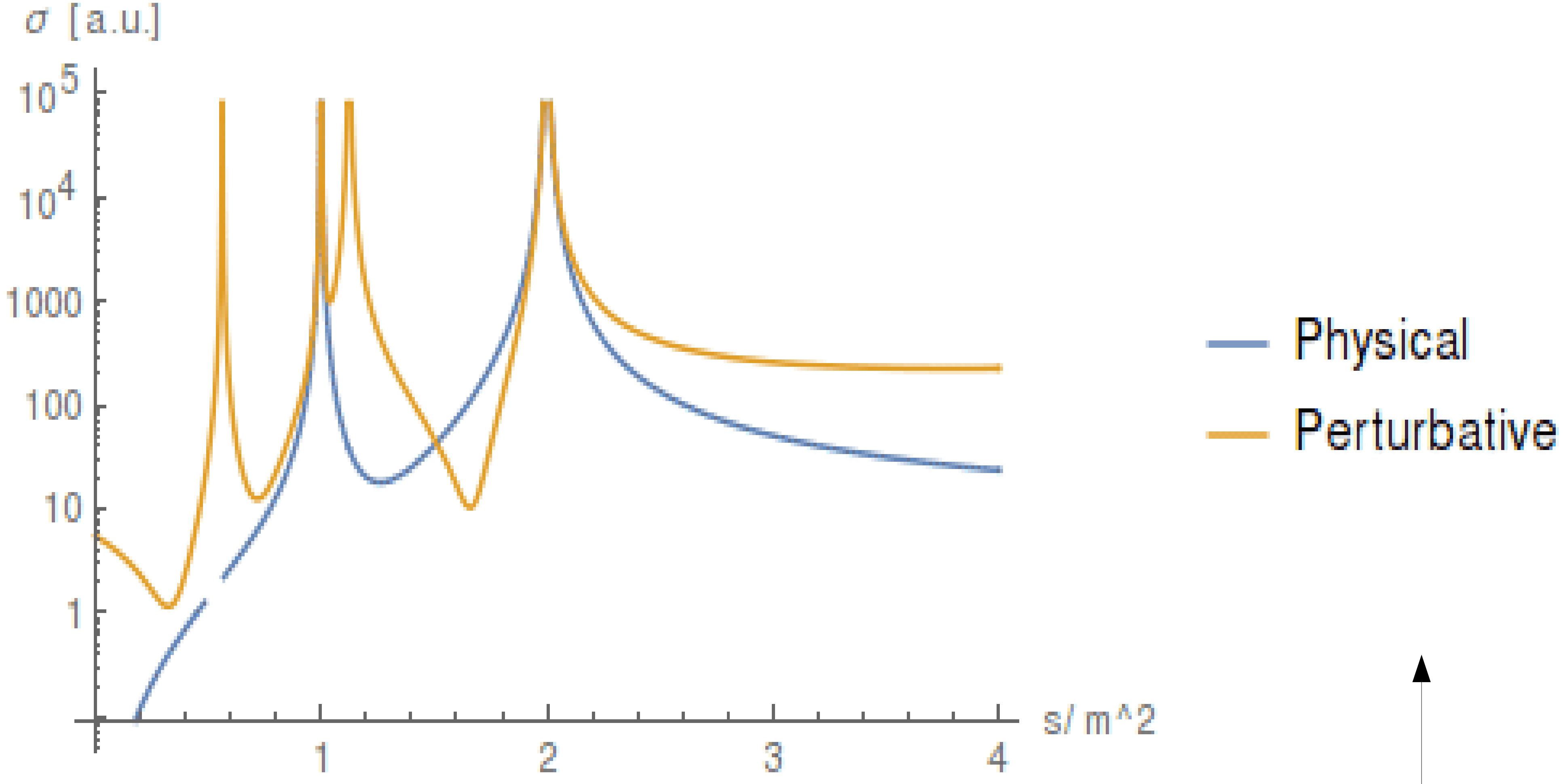

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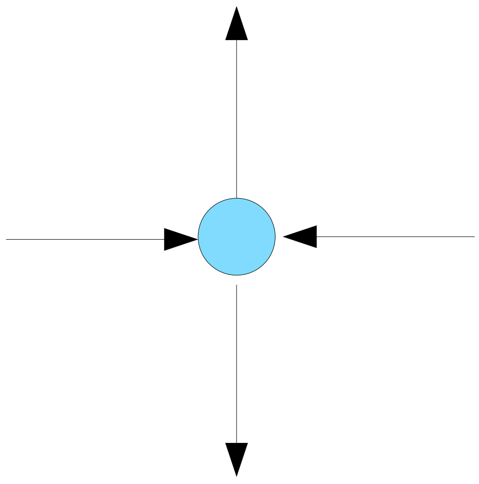

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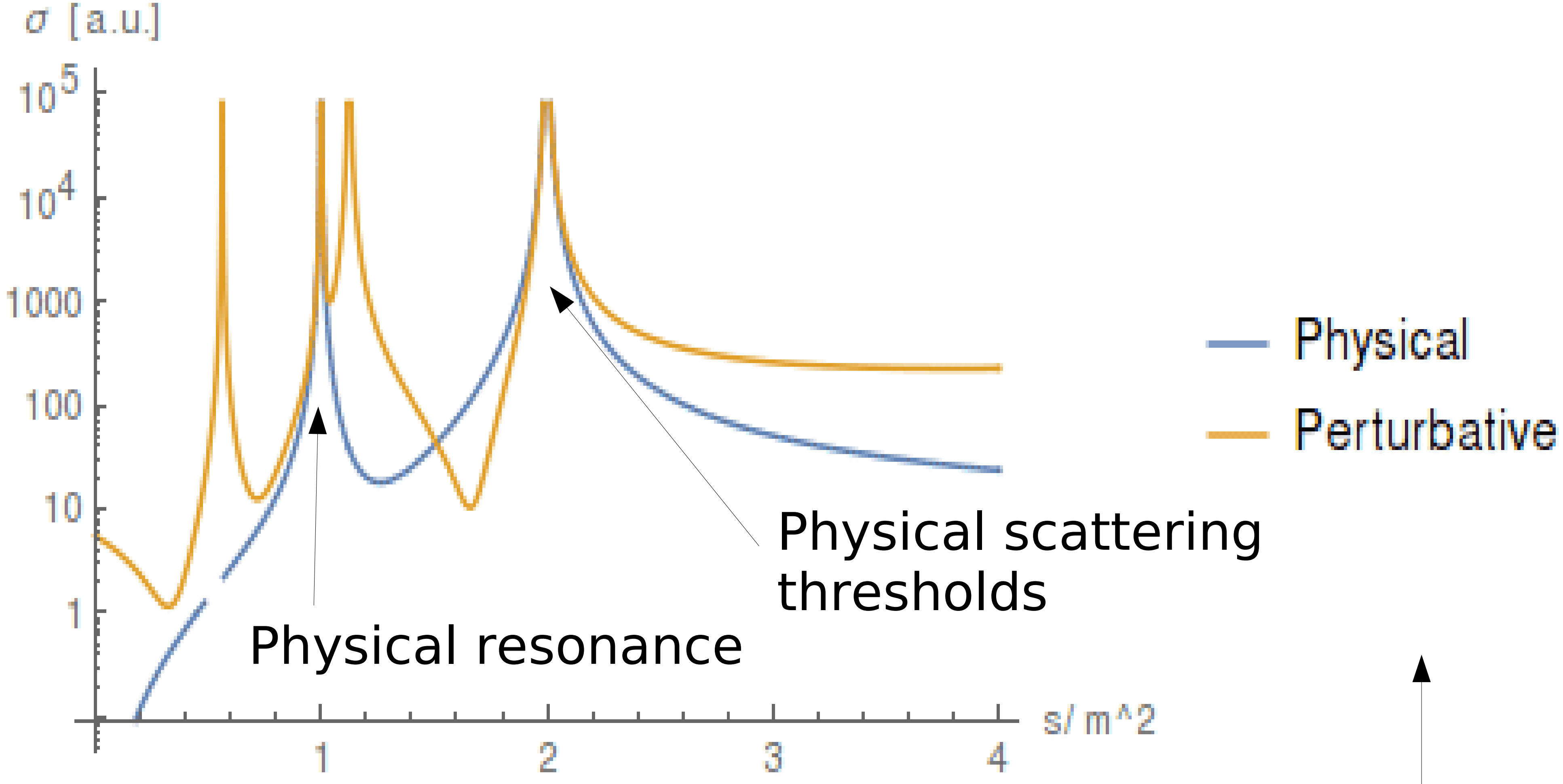
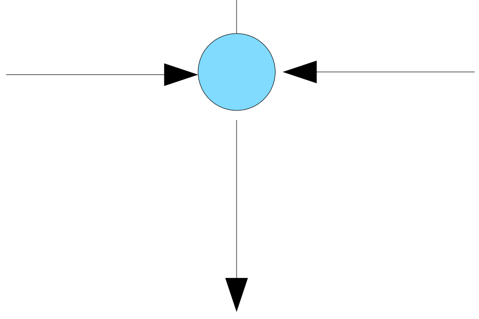


Physical scattering thresholds

Physical resonance

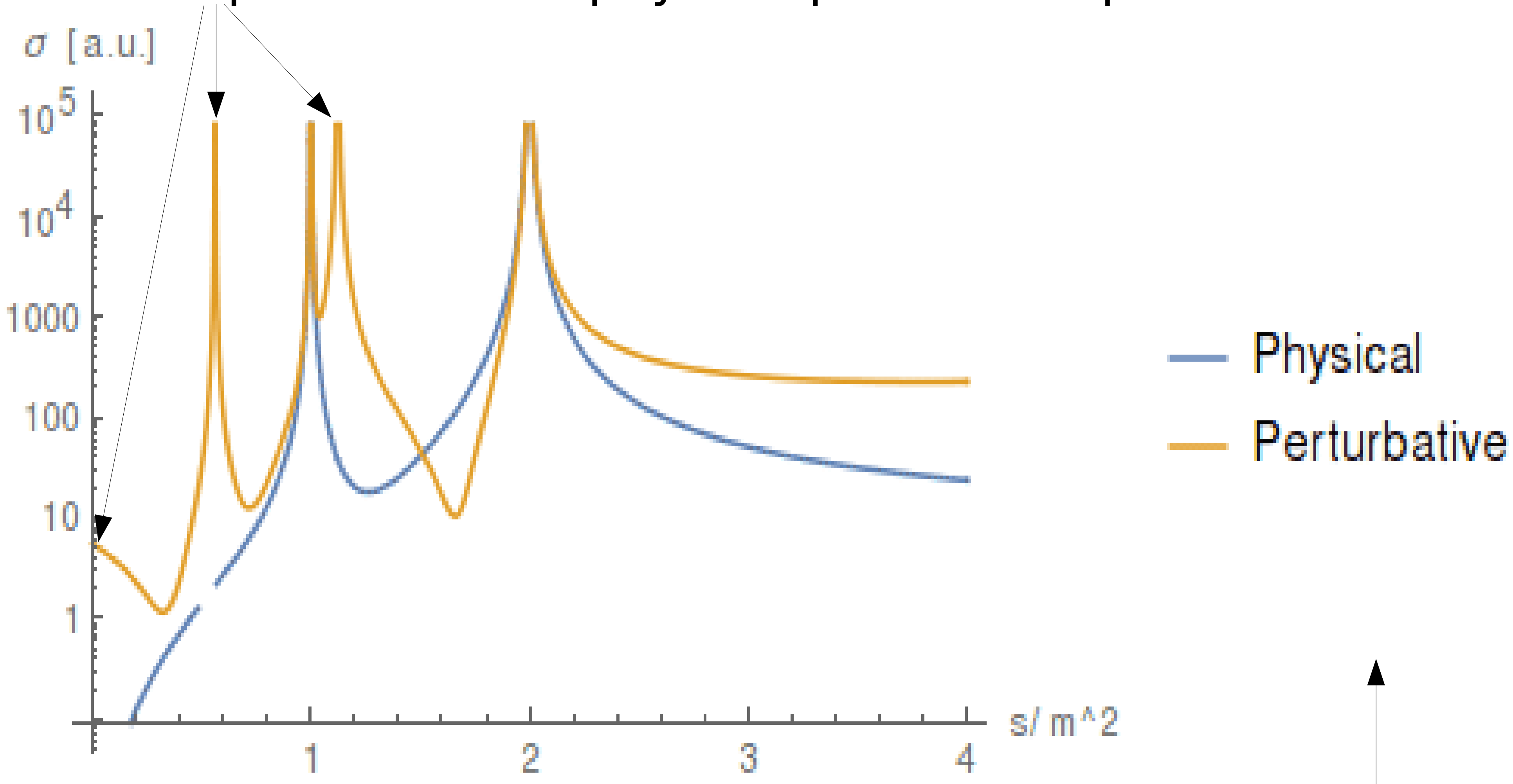
- Add fundamental fermions
- Bhabha scattering



Experimental consequences

[Maas & Török'18
Maas'17]

Ghost peaks from unphysical particles in perturbation theory



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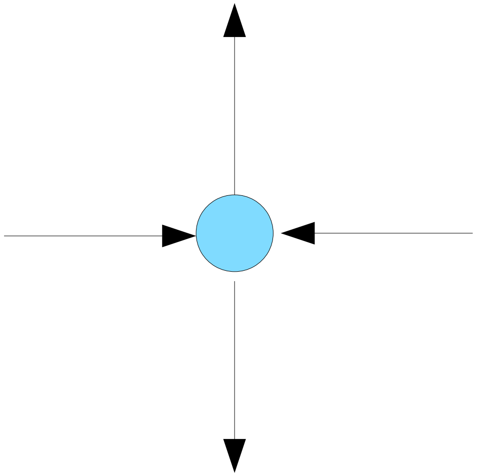

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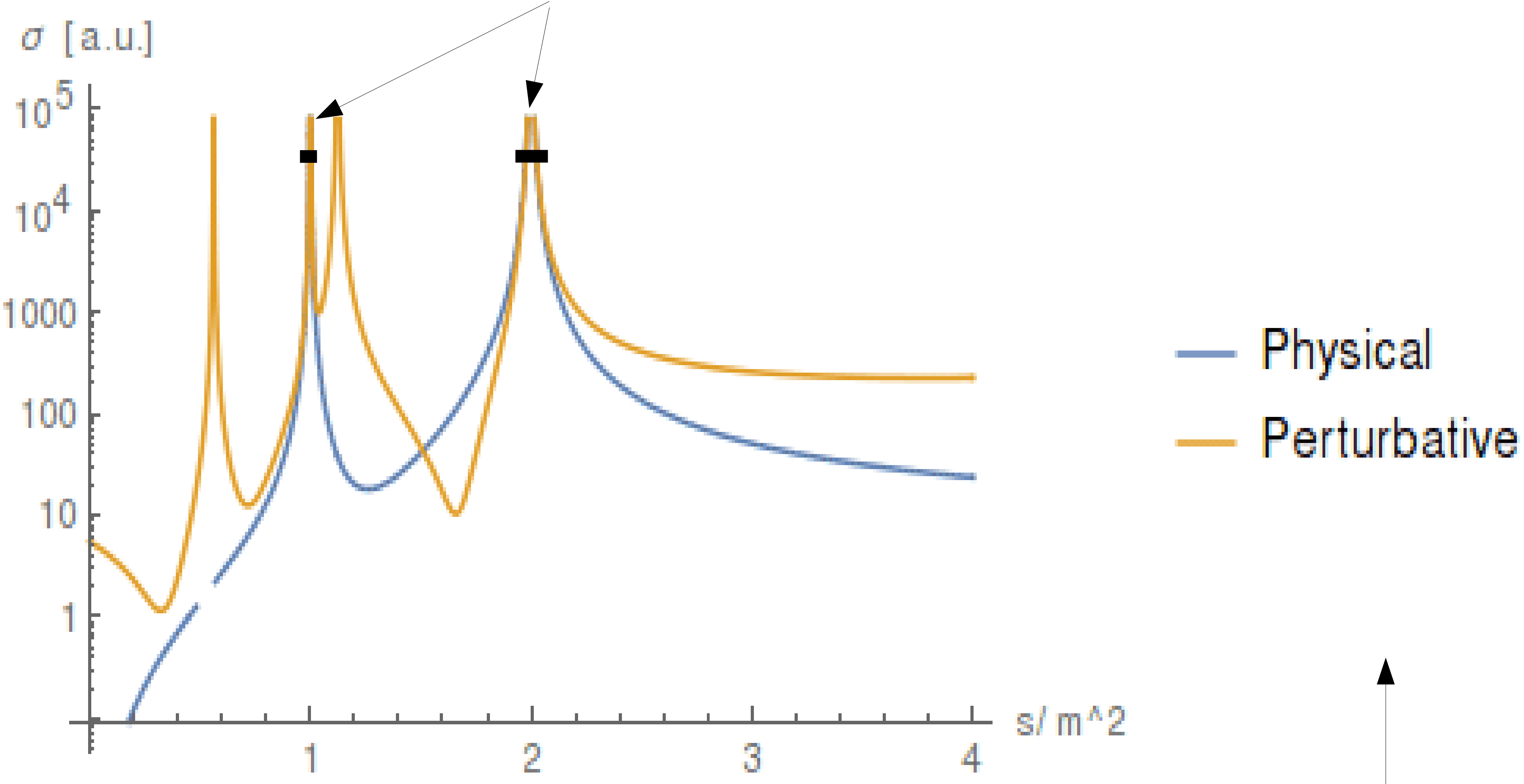
Close to true structures identical!



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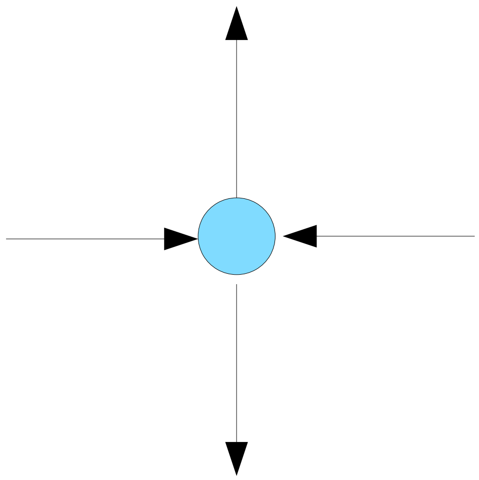

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Quantum gravity

[Maas'19,
Maas, Markl, Müller'22]

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  - Diffeomorphism and local Lorentz
- FMS mechanism applicable
  - A 'BEH effect' for gravity
  - Technically much more involved
  - First predictions agree with lattice EDT [Dai et al.'21]
  - More to come from lattice CDT

Supergravity

[Maas’23]

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  - ➔ Could explain absence of supersymmetry in experiment
- FMS mechanism as applicable as to quantum gravity

Summary

- Full invariance necessary for physical observables in path integrals

Review: 1712.04721 Update: 2305.01960

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Summary

- Full invariance necessary for physical observables in path integrals
- FMS mechanism allows estimates of quantum effects in a systematic expansion
- Gives a new perspective on particle physics and quantum gravity

Philosophy of physics perspective: 2110.00616

Review: 1712.04721 Update: 2305.01960

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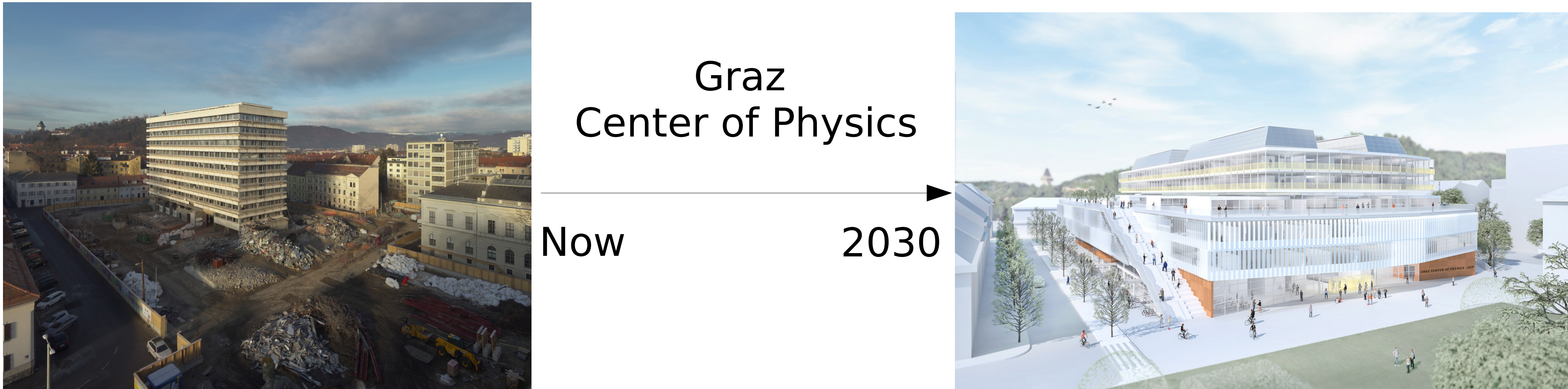

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