

Geon Propagator in Causal Dynamical Triangulations

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21. September 2026 , ÖPG-CMD Joint Meeting 2026

Supervisors: *Axel Maas, Simon Plätzer*

Dark matter from pure gravity

Geons as candidates for dark matter

- Self-bound state of gravitational waves / gravitons

[Wheeler, Phys. Rev. **97** (1955)]

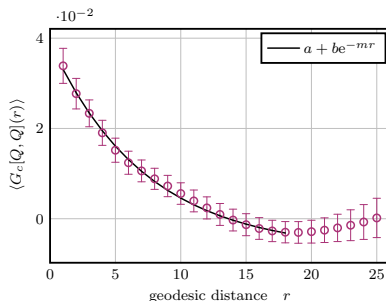
- Hints for a massive state from curvature-curvature correlation functions $G_c[Q, Q](r)$ in lattice quantum gravity

[Maas, Plätzer, and Pressler, Phys. Lett. B **879** (2026), [2504.11047]]

Geons may form (primordial) black holes

- Next generation gravitational wave detectors allow to probe black holes of broad mass range
- Detection of quantum signatures in gravitational wave signals (equation of state for astrophysical objects formed from geons?)

[Bagui et al., Living Rev. Rel. **28** (2025), [2310.19857]]



Taking a conservative approach to quantum gravity

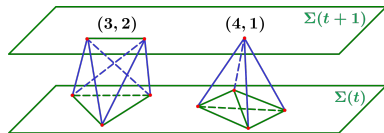
Functional integral quantization of pure gravity

$$Z(G, \Lambda) = \int \mathcal{D}[g] e^{iS^{\text{EH}}[g_{\mu\nu}]}, \quad S^{\text{EH}}[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\Lambda)$$

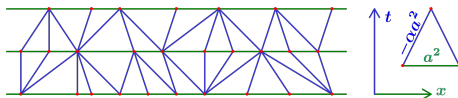
- Approximation of a manifold with piecewise flat building blocks – lattice regulator with UV cut-off $a > 0$

[Regge, Nuovo Cim. **19** (1961)]

$$Z_{a,N}^{\text{CDT}} = \sum_T \frac{1}{C_T} e^{iS^{\text{Regge}}[T]}$$



- Summation only over foliated spacetimes with fixed spatial topology (spherical)
- Connection with the continuum theory via scaling limit



[Ambjørn et al., Contemp. Phys. **47** (2006), [hep-th/0509010]] [Ambjørn et al., Phys. Rept. **519** (2012), [1203.3591]] [Loll, Class. Quant. Grav. **37** (2020), [1905.08669]]

Monte Carlo simulation and phase structure

Numerical simulations of CDT quantum gravity

- Change to Euclidean time (Wick rotation) $t \rightarrow i\tau$ allows to use Monte Carlo techniques

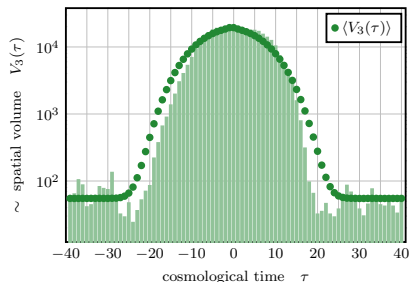
$$(-, +, +, +) \rightarrow (+, +, +, +), \quad e^{iS} \rightarrow e^{-S_E} \quad \sim \quad \text{Boltzmann weights}$$

- Causal information remains although space and time appear with equal sign

Phase structure of CDT

- Determined by Newton constant κ and asymmetry parameter α
- de Sitter phase: spatial volume profile of classical Euclidean de Sitter space
- Allows for connection with classical general relativity and cosmology

[Ambjørn et al., Phys. Rev. Lett. **100** (2008), [0712.2485]] [Ambjørn et al., Phys. Rev. D **78** (2008), [0807.4481]]



Curvature on simplicial manifolds

Description of a geon via an invariant scalar operator

- Usage of Riemann curvature tensor only on smooth (pseudo) Riemannian manifolds

Quantum Ricci curvature

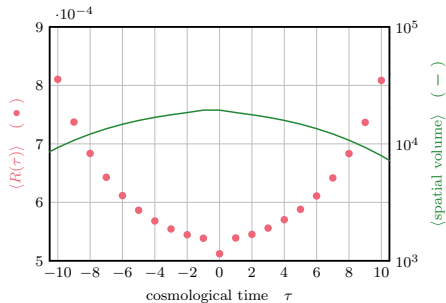
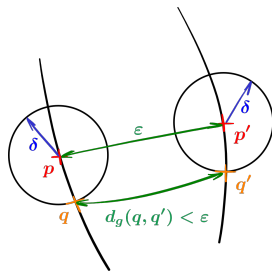
[Klitgaard et al., Phys. Rev. D **97** (2018), [1712.08847]]

- Measure geodesic distance $d_g(q, q')$ between point pairs (q, q') on the $(d-1)$ -spheres with centres p, p'
- Scalar quantity can be obtained from limit $\varepsilon \rightarrow 0$

$$Q = \frac{1}{[N_0(S_p^\delta)]^2} \sum_{q, q' \in S_p^\delta} \frac{d(q, q')}{\delta}$$

$$= c_q \{1 - c_2 R \delta^2 + \mathcal{O}(\delta^3)\}$$

- Extract curvature scalar R from a quadratic fit of Q



Correlation between the values of an observable R at a geodesic distance r apart

- Formulation in a diffeomorphism invariant way
- Not possible to track events across different configurations

$$\begin{aligned} G_g[R, R](r) &= \int d^4x \sqrt{g(x)} \int d^4y \sqrt{g(y)} R(x) R(y) \delta(d_g(x, y) - r) \\ &\rightarrow \sum_{p, p' \in T} R(p) R(p') \delta_{d(p, p'), r} \\ \langle G[R, R](r) \rangle &= \int D[g] e^{-S_E[g_{\mu\nu}]} G_g[R, R](r) \end{aligned}$$

- Normalization of the correlator with $\langle G[1, 1](r) \rangle$ – also quantum observable
- Replace $R \rightarrow \Delta R = R - \langle \bar{R} \rangle$ to obtain connected correlator

$$\langle \tilde{G}_c[R, R](r) \rangle \equiv \frac{\langle G_c[R, R](r) \rangle}{\langle G[1, 1](r) \rangle} = \langle \tilde{G}[R, R](r) \rangle - 2 \langle \tilde{G}[R, 1](r) \rangle \langle \bar{R} \rangle + \langle \bar{R} \rangle^2$$

[Ambjørn et al., JHEP **02** (1999), [hep-lat/9812015]] [van der Duin et al., Eur. Phys. J. C **84** (2024), [2404.17556]]

Constructing a causal observable

Determination of causally connected points

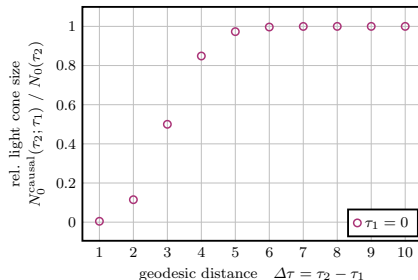
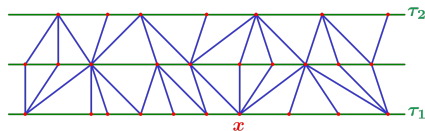
- Sum over all points y at τ_2

$$N_{0,x(\tau_1)}^{\text{causal}}(\tau_2) \leq N_0(\tau_2)$$

within the light cone of $x(\tau_1)$ and over
all points x at τ_1

Correlator of curvature scalars with a
geodesic separation of $\Delta\tau$

$$\sum_{\substack{x(\tau_1), y(\tau_2) \\ \text{caus. conn.}}} \Delta R(x(\tau_1)) \Delta R(y(\tau_2))$$



$$\begin{aligned} D_{RR}(\tau_1, \tau_2) = & \langle \tilde{G}[R, R] \rangle - \langle \tilde{G}[R, 1] \rangle \langle \bar{R}(\tau_2) \rangle - \\ & - \langle \bar{R}(\tau_1) \rangle \langle \tilde{G}[1, R] \rangle + \langle \bar{R}(\tau_1) \rangle \langle \bar{R}(\tau_2) \rangle \end{aligned}$$

Constructing a causal observable

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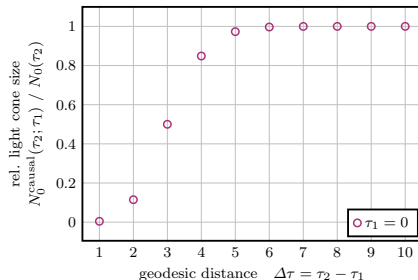
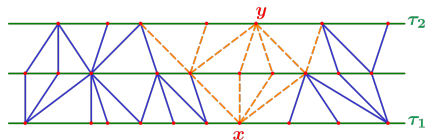
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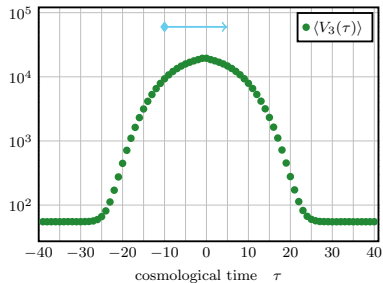
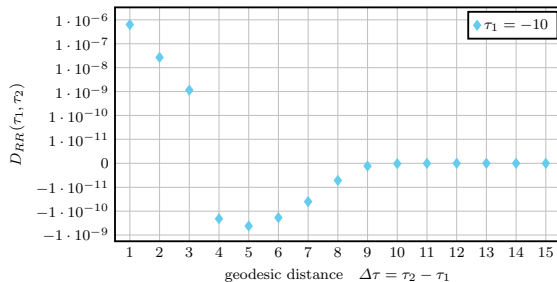
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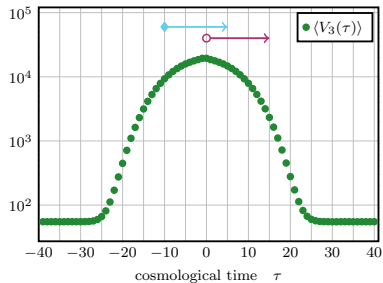
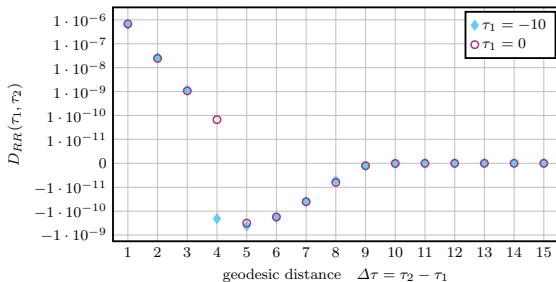


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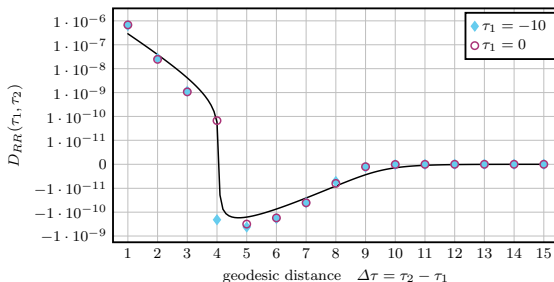
Geon propagator



Geon propagator



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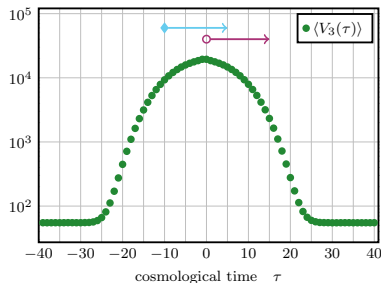
Exponential fit of the propagator to extract mass spectrum

$$\begin{aligned} D_{RR}(\Delta\tau) &= c_1 \cdot e^{-m_1 \cdot \Delta\tau} - c_2 \cdot e^{-m_2 \cdot \Delta\tau} \\ &= c_1 \cdot e^{-m_1 \cdot \Delta\tau} (1 - \tilde{c}_2 \cdot e^{\Delta m \cdot \Delta\tau}) \end{aligned}$$

[Ambjørn et al., Phys. Rev. D **78** (2008),
[0807.4481]]

$$m_1 = 1.8 \pm 0.6, \quad m_2 = 1.3 \pm 0.5$$

$$\rightarrow m_i = \mathcal{O}(M_{\text{Planck}})$$

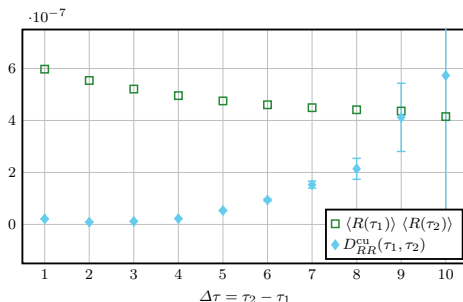


Two-point function for causally unconnected points

- Summation only over points outside the light cone

$$G_c[R(\tau_1), R(\tau_2)] = \sum_{\substack{x(\tau_1), y(\tau_2) \\ \text{caus. unconn.}}} \Delta R(x(\tau_1)) \Delta R(y(\tau_2))$$

- Long distance behaviour dominated by mean curvature – hint for cluster decomposition



Curvature correlator for causally connected points

- Particle like behaviour of the propagator
- Mass parameters independent of cosmological time
- Geon mass $m_i \approx \mathcal{O}(M_{\text{Planck}})$ – detection possible when clumping to more massive objects?

Future work and possible modifications

- Reduction of systematic errors – construct propagator only from dual lattice
- Extract gauge field (e.g. $g_{\mu\nu}$) from triangulation and build graviton propagator
- Build correlators from other scalar operators
- Extension to higher order correlation functions

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Causal structure and Wick rotation

Summation (integration) of curved Euclidean $(+, +, +, +)$ spaces and of Lorentzian $(-, +, +, +)$ spaces leads to inequivalent results

- Global timeslicing of universes in the summation (introduction of a cosmological time)
- Fix spatial topology – no branching points are allowed

Wick rotation of curved spacetimes

- Not possible in general Lorentzian manifolds, only due to global hyperbolic nature of CDT spacetimes
- Euclidean spaces obtained from Wick rotation form only subset of general curved Euclidean spaces

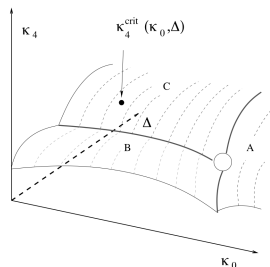
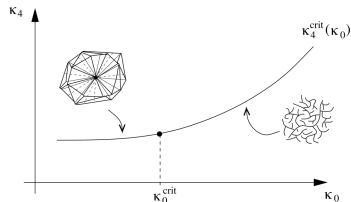
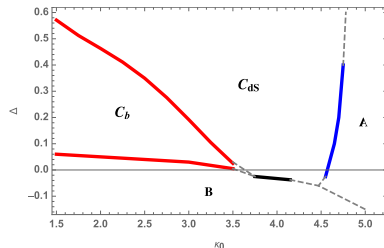
Phase diagram of (C)DT

Phases of Euclidean DT

- Crumpled and branched-polymer phase have both not a good classical limit (Hausdorff dimension ∞ and 2)

Phases of Lorentzian DT

- Phases A and B are similar to Euclidean version
- Extended, 4D universes emerge from phase C – connection with classical GR



[Ambjørn et al., (2010), [1004.0352]] [Ambjørn et al., Universe **7** (2021), [2103.15610]]

Dual lattice and fractional time

Timelike links on the regular lattice

- Connection of timeslices formed from spatial simplices
- Only one step required to connect a vertex at time t to a vertex at time $t + 1$

Timelike links on the dual lattice

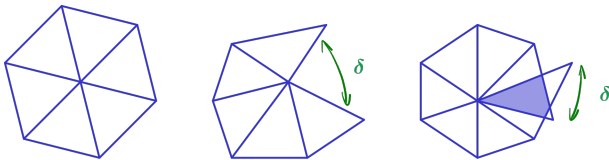
- Not possible to connect a (m, n) -simplex at dual timeslice t to a (m, n) -simplex at dual timeslice $t + 1$ with only one step
- Shortest possible timelike connecting between simplices of equal type
 $(4, 1) \rightarrow (3, 2) \rightarrow (2, 3) \rightarrow (1, 4) \rightarrow (4, 1)$
- Divide dual timeslice into four slices of fractional time $(4, 1) \rightarrow t, (3, 2) \rightarrow t + \frac{1}{4}, (2, 3) \rightarrow t + \frac{1}{2}, (1, 4) \rightarrow t + \frac{3}{4}$

[Klitgaard et al., Eur. Phys. J. C **80** (2020), [2006.06263]]

Curvature in Regge calculus

Measure curvature from deficit angle

- Flat space: regular pattern of six triangles meet at one vertex
- Positively (negatively) curved surface: less (more) than six triangles meet at one vertex
- Problem: divergent in infinite-volume limit



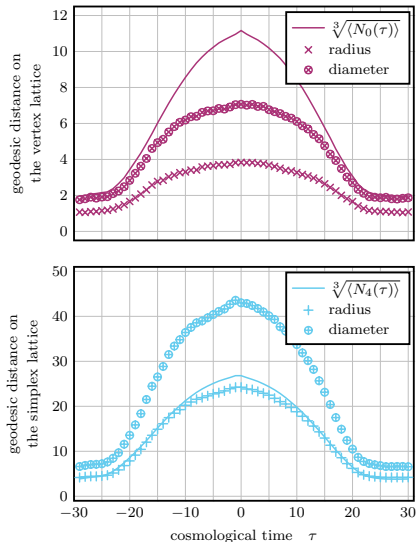
Linear extension of the timeslices

Measures for the linear extension

- Spatial volume increases approximately with 3rd power of linear size
- Eccentricity of a vertex v – largest geodesic distance between v and any other vertex v' of a connected graph
- largest eccentricity = diameter, smallest eccentricity = radius

Physical size of the piecewise flat building blocks $a \approx 2\ell_{\text{Planck}} \rightarrow$ linear size of the CDT universes $\approx 20\ell_{\text{Planck}}$

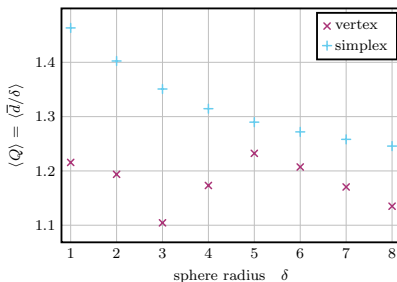
[Ambjørn et al., Phys. Rev. D **78** (2008),
[0807.4481]]



Quantum Ricci curvature on the vertex and simplex (dual) lattice

Measuring curvature on the vertex (regular) lattice

- Anisotropic shape of sphere in spatial and temporal direction: most points of δ -sphere reside on timeslices $\tau \pm \delta$ for large δ
- Occurrence of vertices where hundreds of links meet, leads to small extension in terms of the vertex content $\rightarrow$ finite size effects already for small δ



Curvature on the simplex (dual) lattice

- Larger extension of the configurations in terms of the simplex content (exactly 5 neighbours for each simplex)
- Better behaviour of the average sphere distance as a function of δ – allows for an extraction of the Ricci curvature

[Klitgaard et al., Eur. Phys. J. C **80** (2020), [2006.06263]]

Geon propagator – quantum Ricci curvature

