

# The Spectrum Of Grand-unified theories

**Elizabeth Dobson, Axel Maas, Bernd Riederer**

22<sup>st</sup> of April 2021  
ACHT 21  
Online



**NAWI Graz**  
Natural Sciences

**FWF**

Der Wissenschaftsfonds

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- Lattice results disagree qualitatively
- Explained by manifest gauge invariance qualitatively and by the Fröhlich-Morchio-Strocchi mechanism quantitatively

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- Why?
- Unification of all gauge interactions would explain these features
- Does such a theory exist, which has as a low-energy effective theory the standard model?

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  - Add suitable Higgs particles to use a Brout-Englert-Higgs effect to break it to  $SU(3) \times U(1)$
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- Should be testable on the lattice

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- **Higgs**  $h_i$  
- Couplings  $g, v, \lambda$  and some numbers  $f^{abc}$  and  $t_a^{ij}$
- There is a global  $U(1)$  symmetry for the Higgs only

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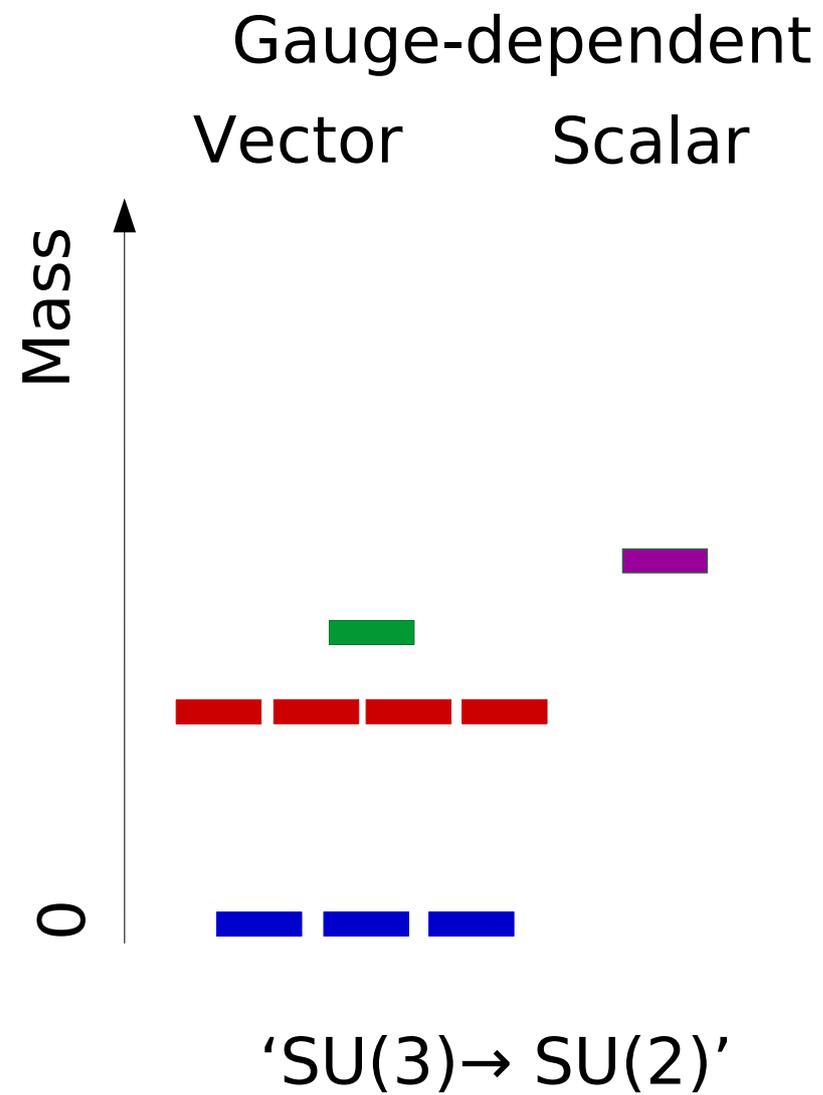
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- Get masses and degeneracies at tree-level
- Perform perturbation theory

# Spectrum



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- Physics has to be expressed in terms of manifestly gauge-invariant quantities
  - And this includes non-perturbative aspects...
  - ...even at weak coupling [Gribov'78, Singer'78, Fujikawa'82]
- Especially on the lattice: No gauge-fixing necessary

# Physical states

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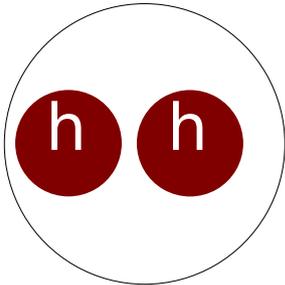
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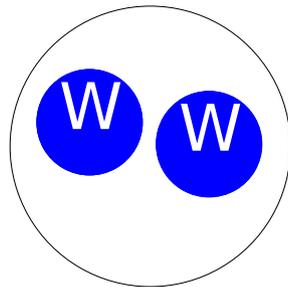
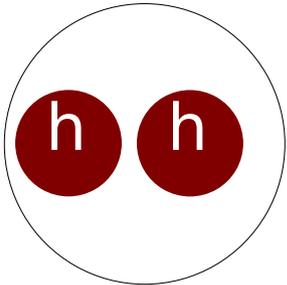
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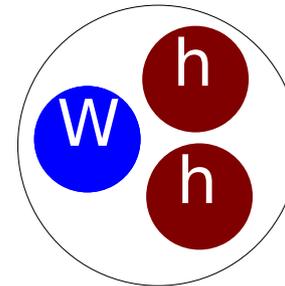
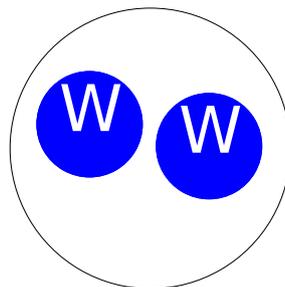
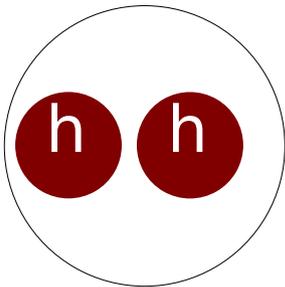
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- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



# How to make predictions

[Fröhlich et al.'80,'81,  
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- $J^{PC}$  and custodial charge only quantum numbers

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- Formulate gauge-invariant, composite operators
  - Bound state structure
- Depends on theory. Here:
  - Integer  $J$ , any  $P$ ,  $C$
  - Uncharged or charged under (Higgs)  $U(1)$

# Spectrum

Gauge-dependent

Vector

Scalar

Mass

0

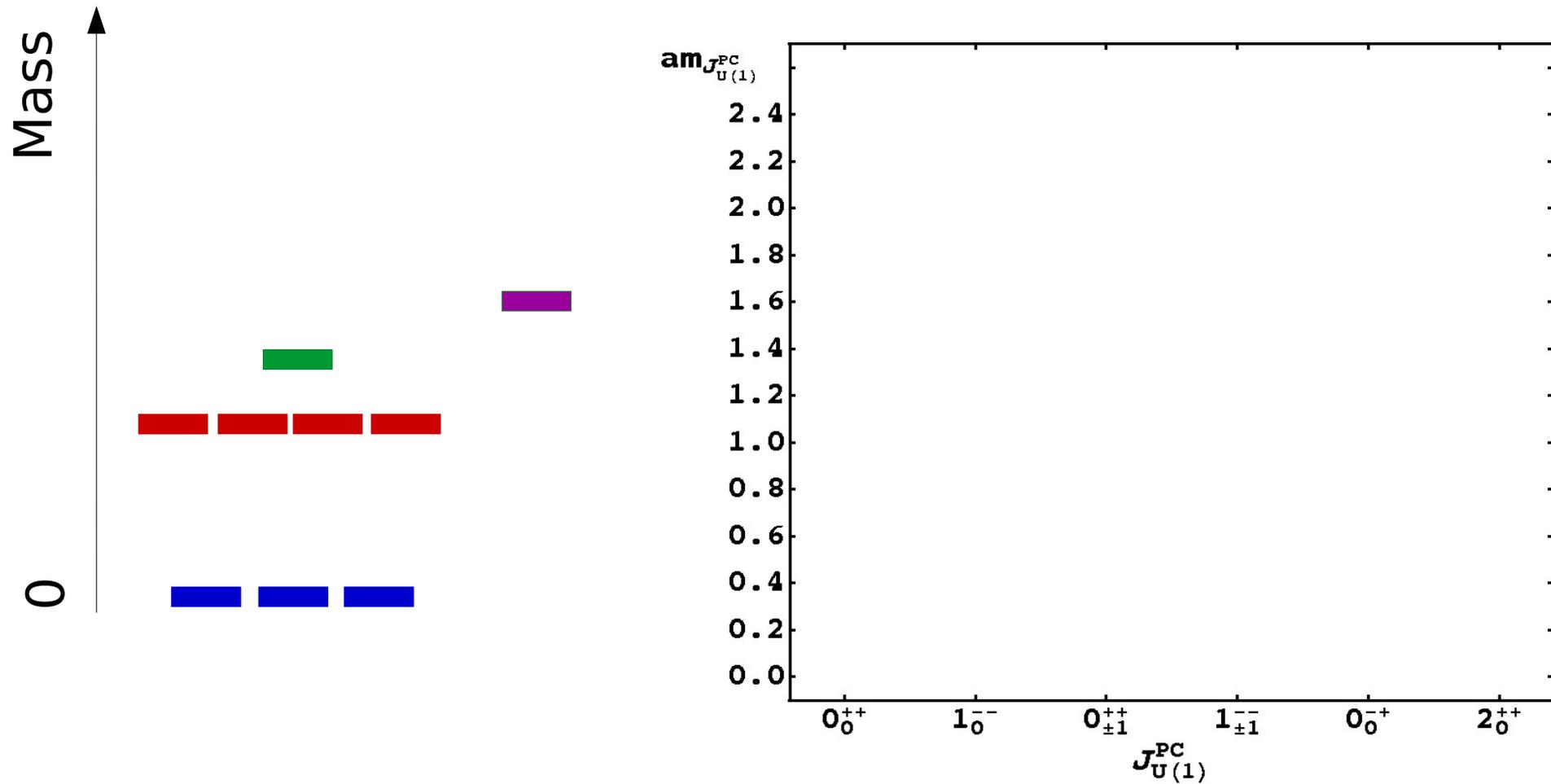


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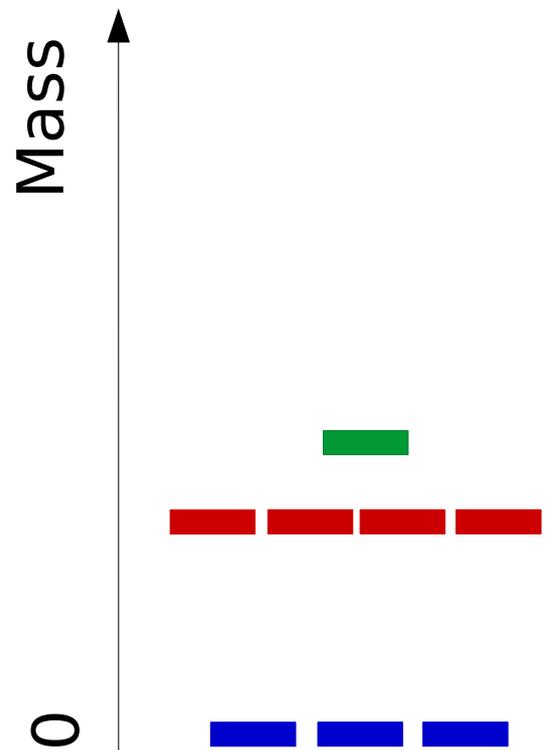
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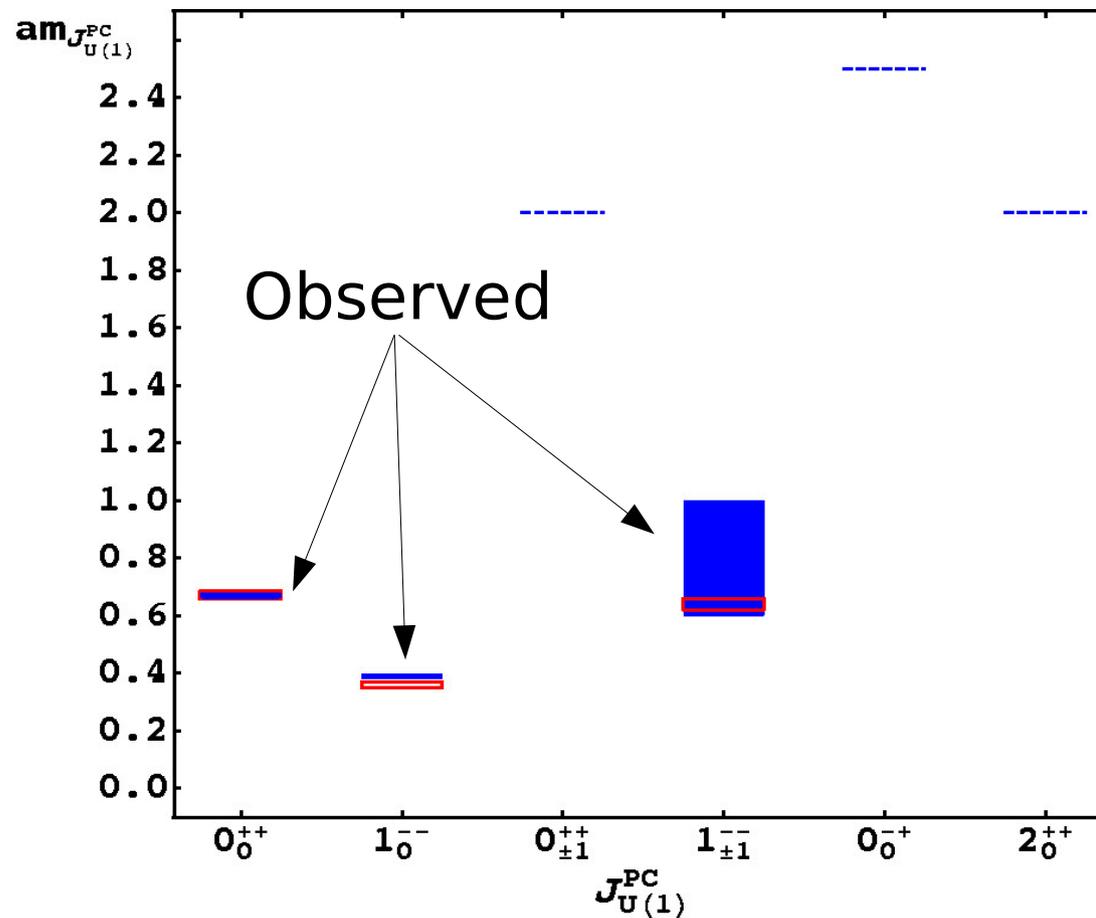
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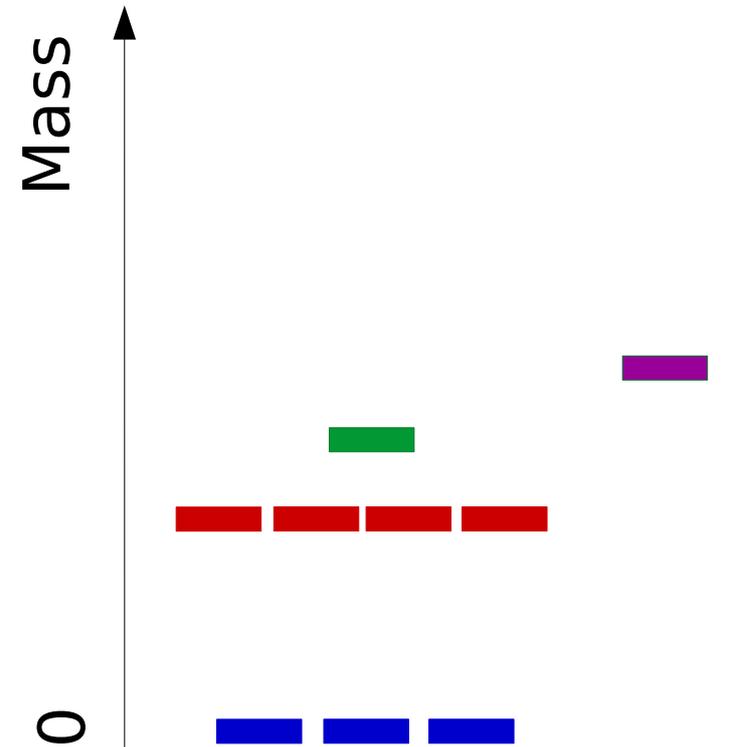
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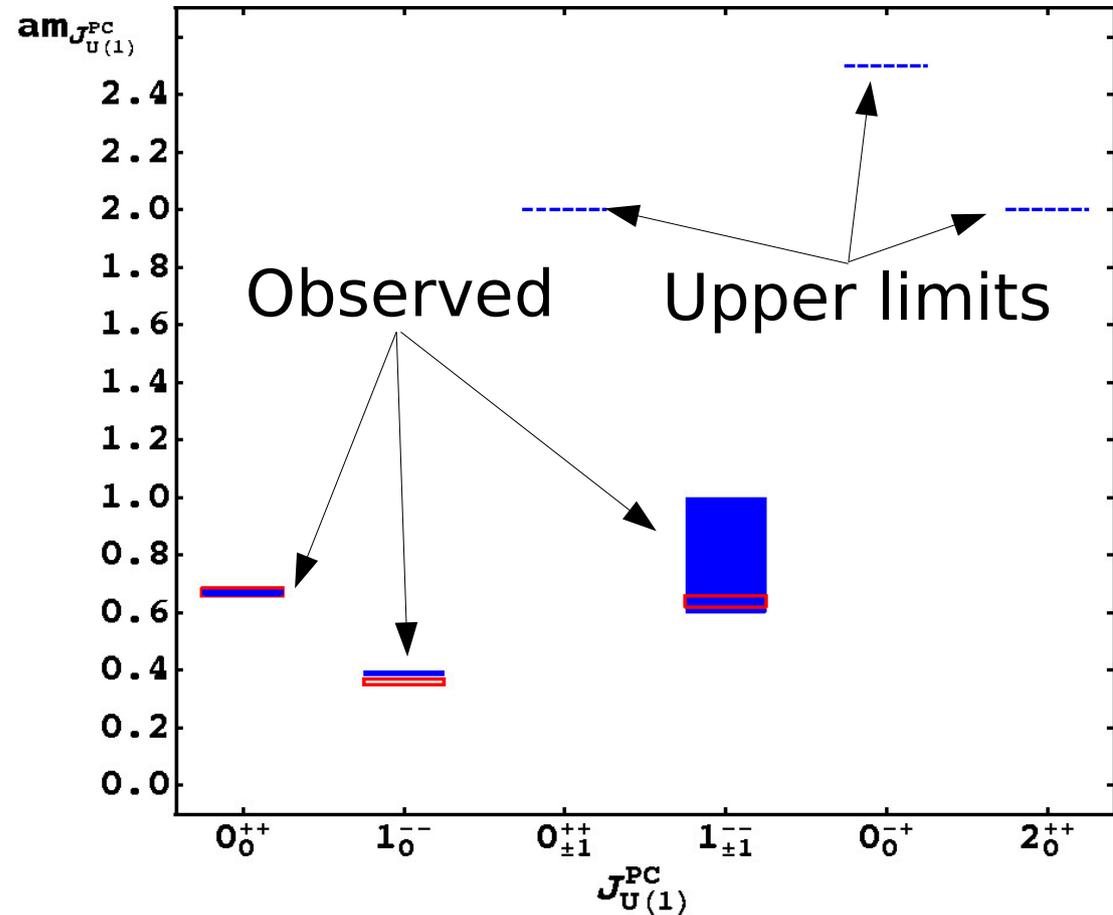
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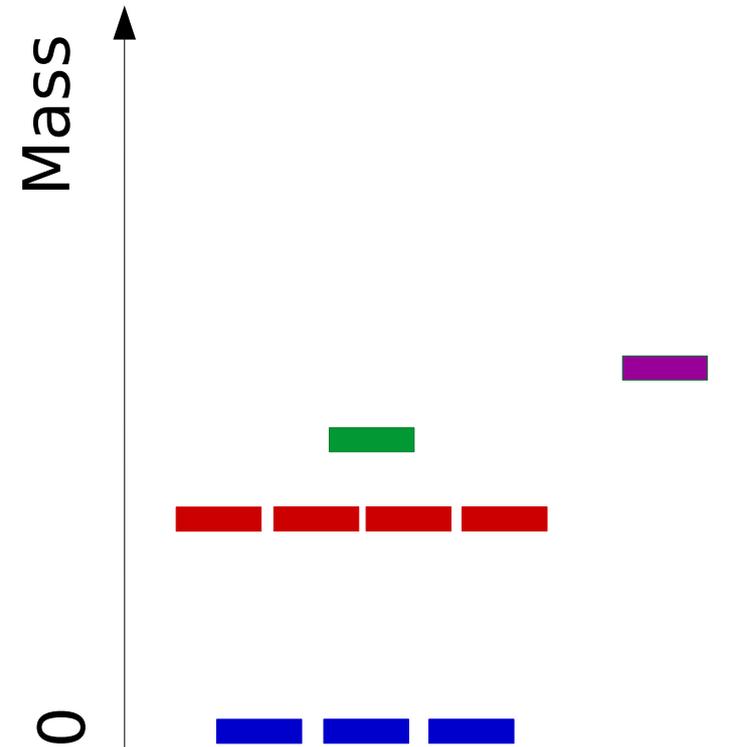


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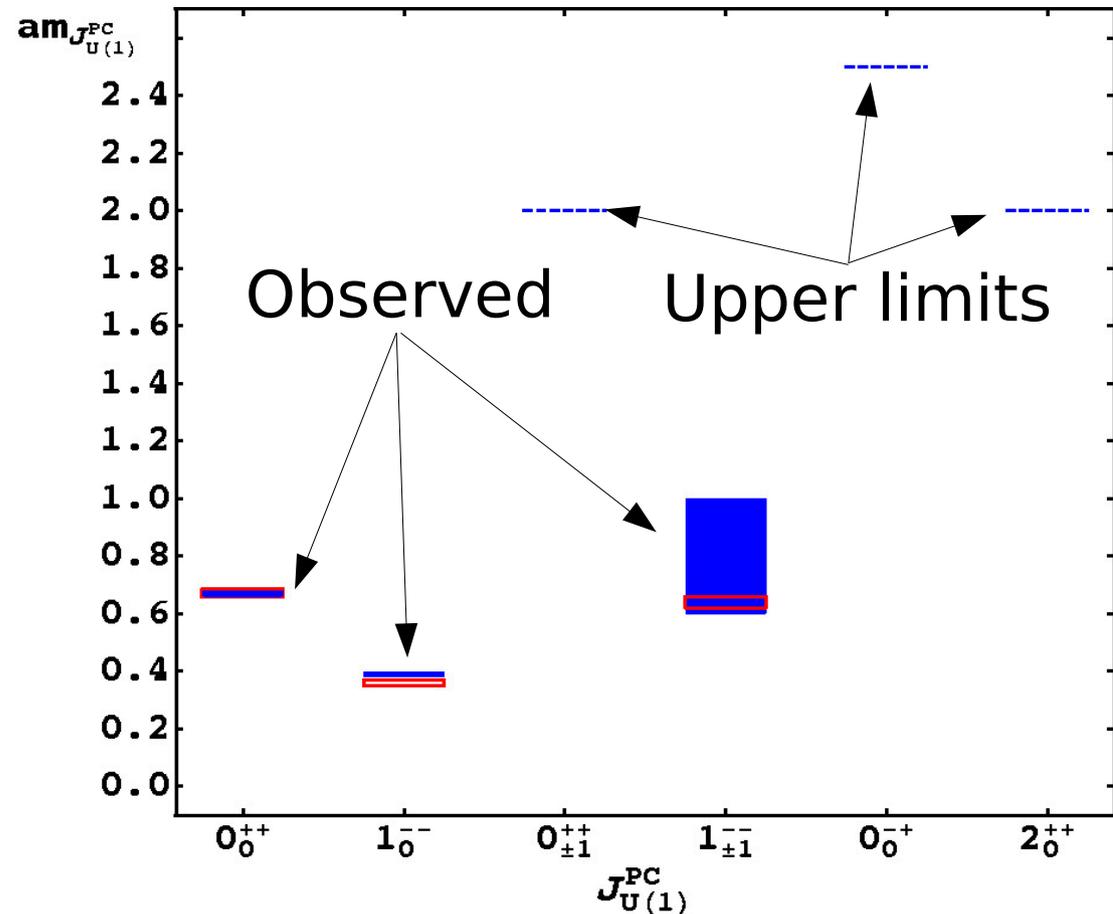
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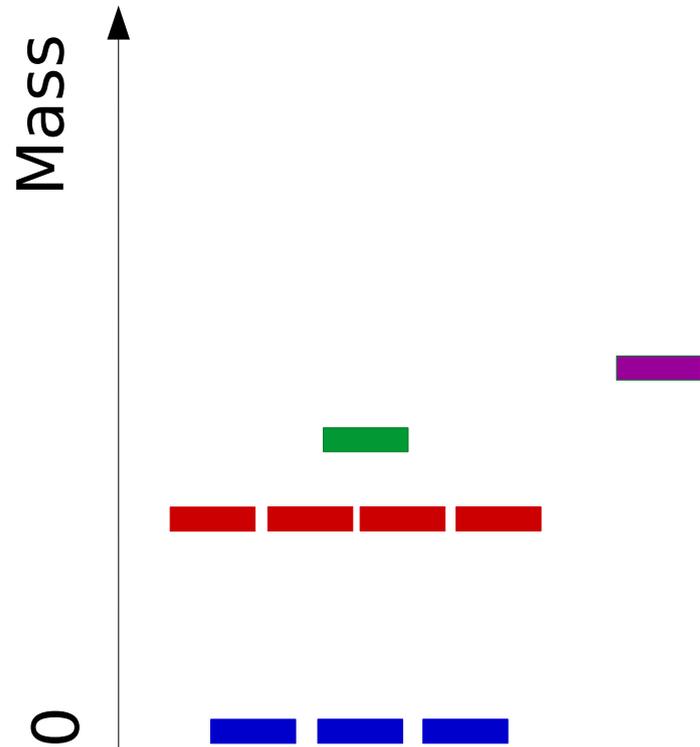


- Qualitatively different spectrum
- Gauge-dependent particles can also be calculated

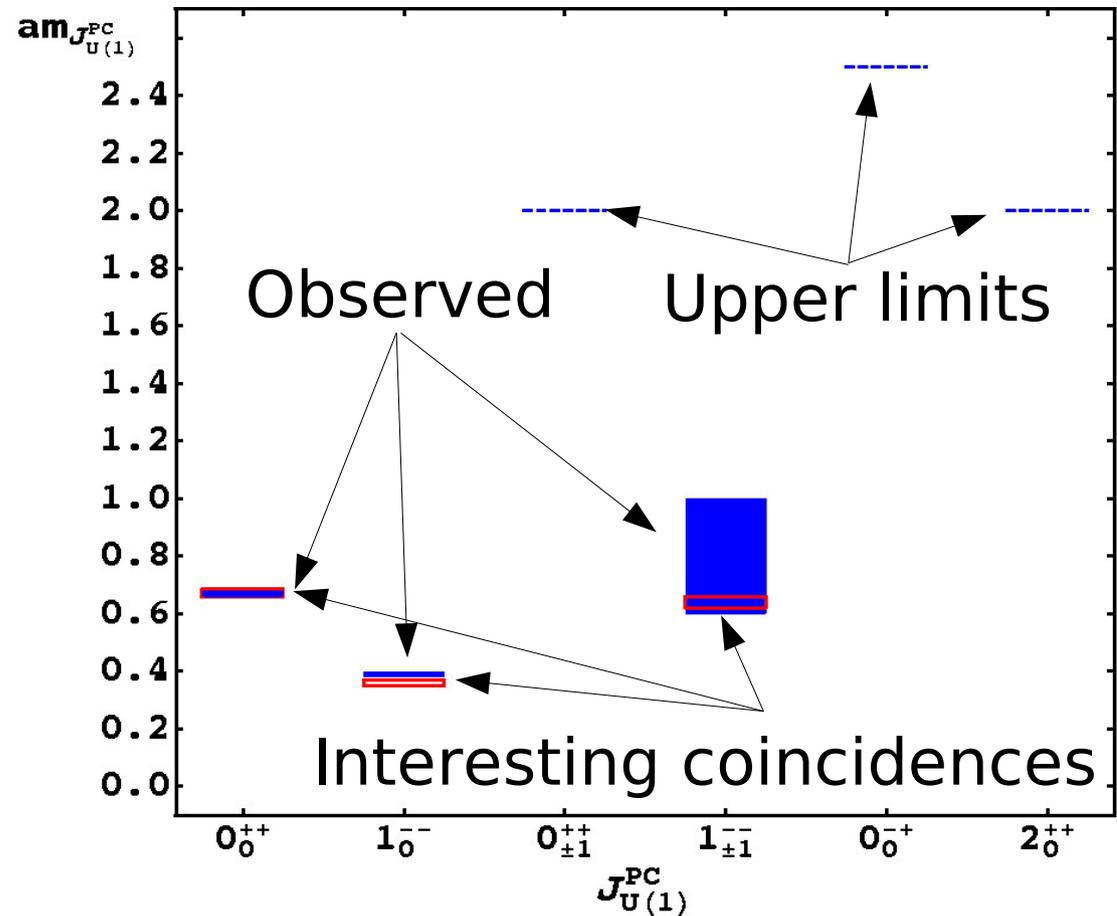
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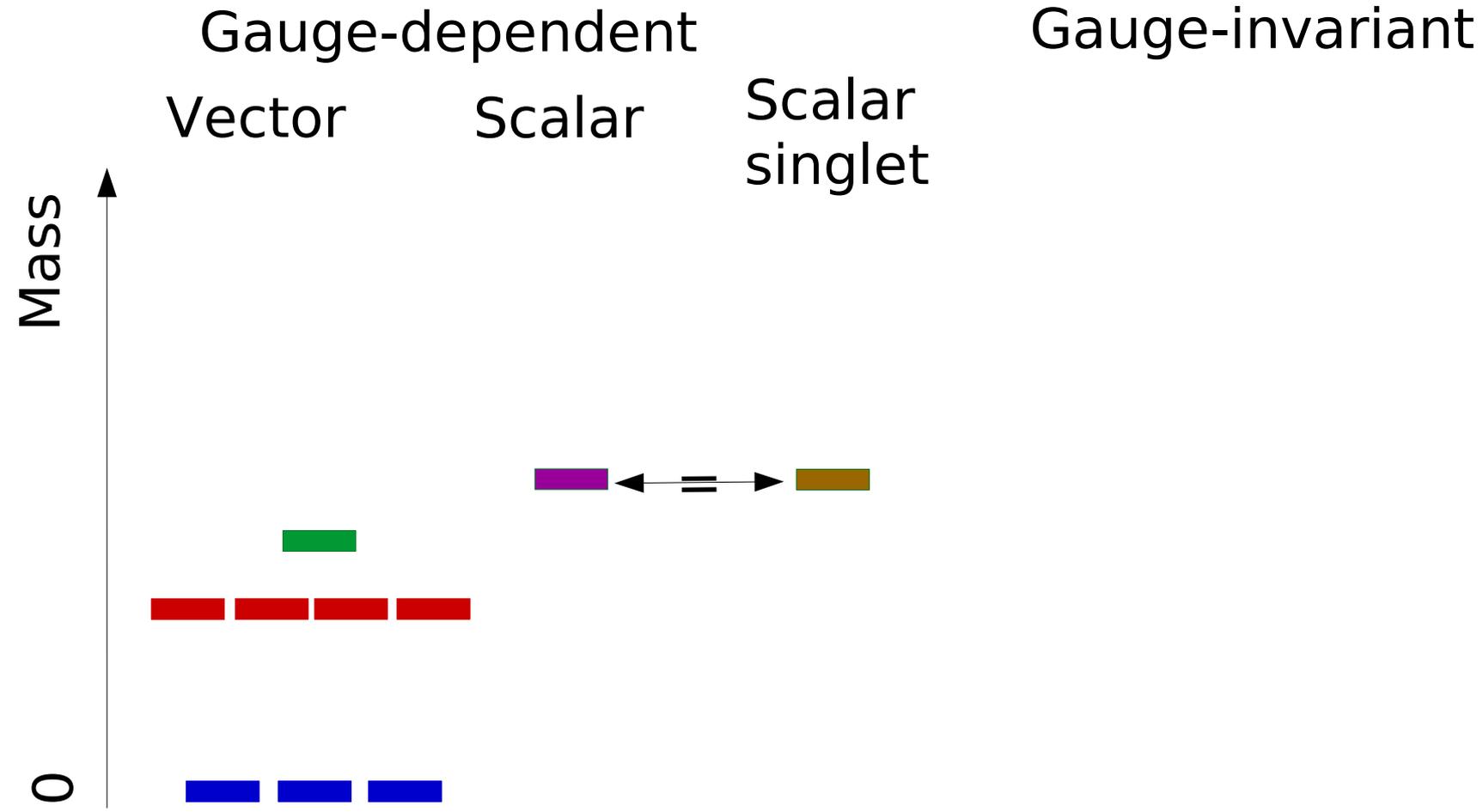
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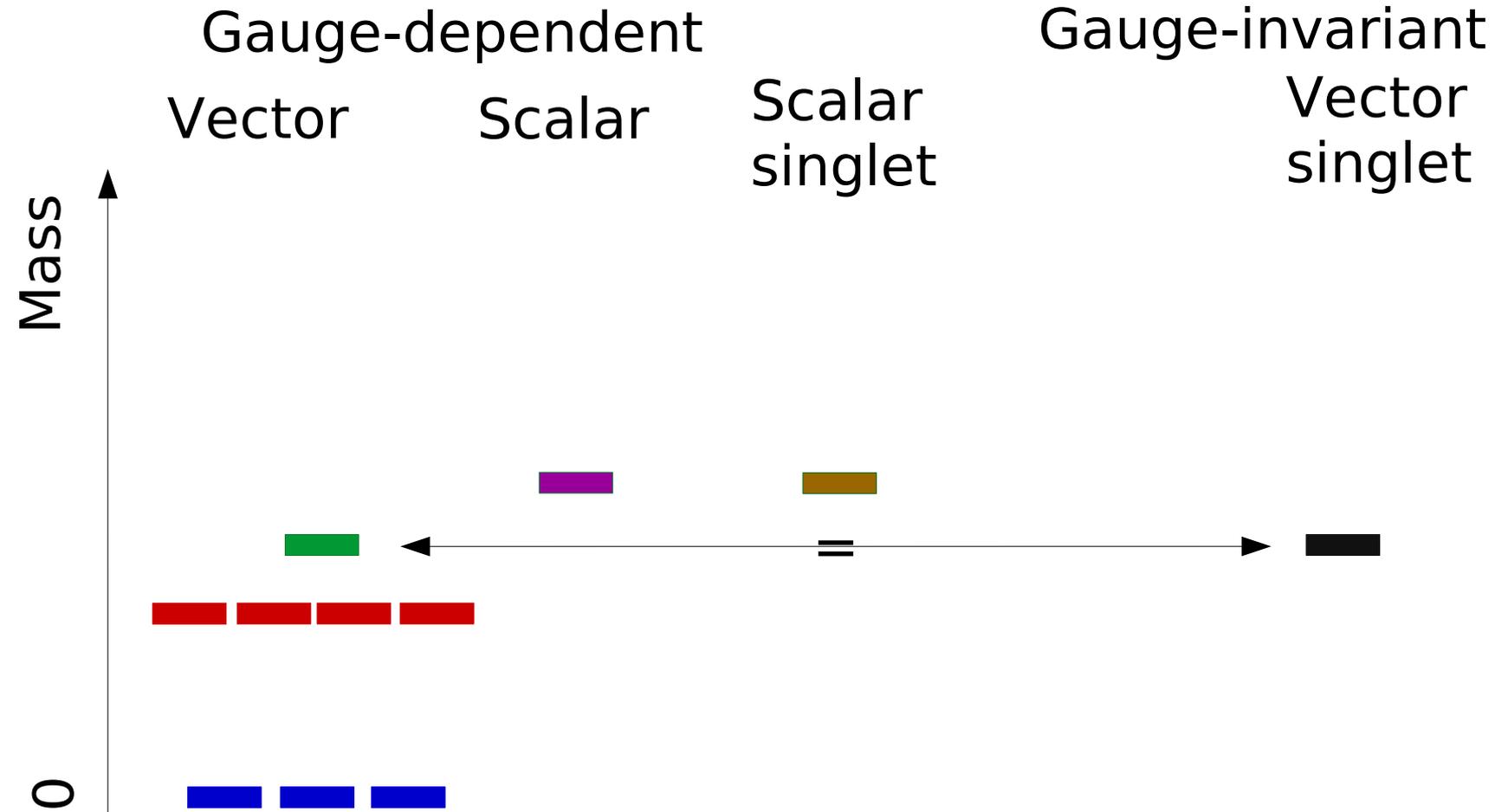
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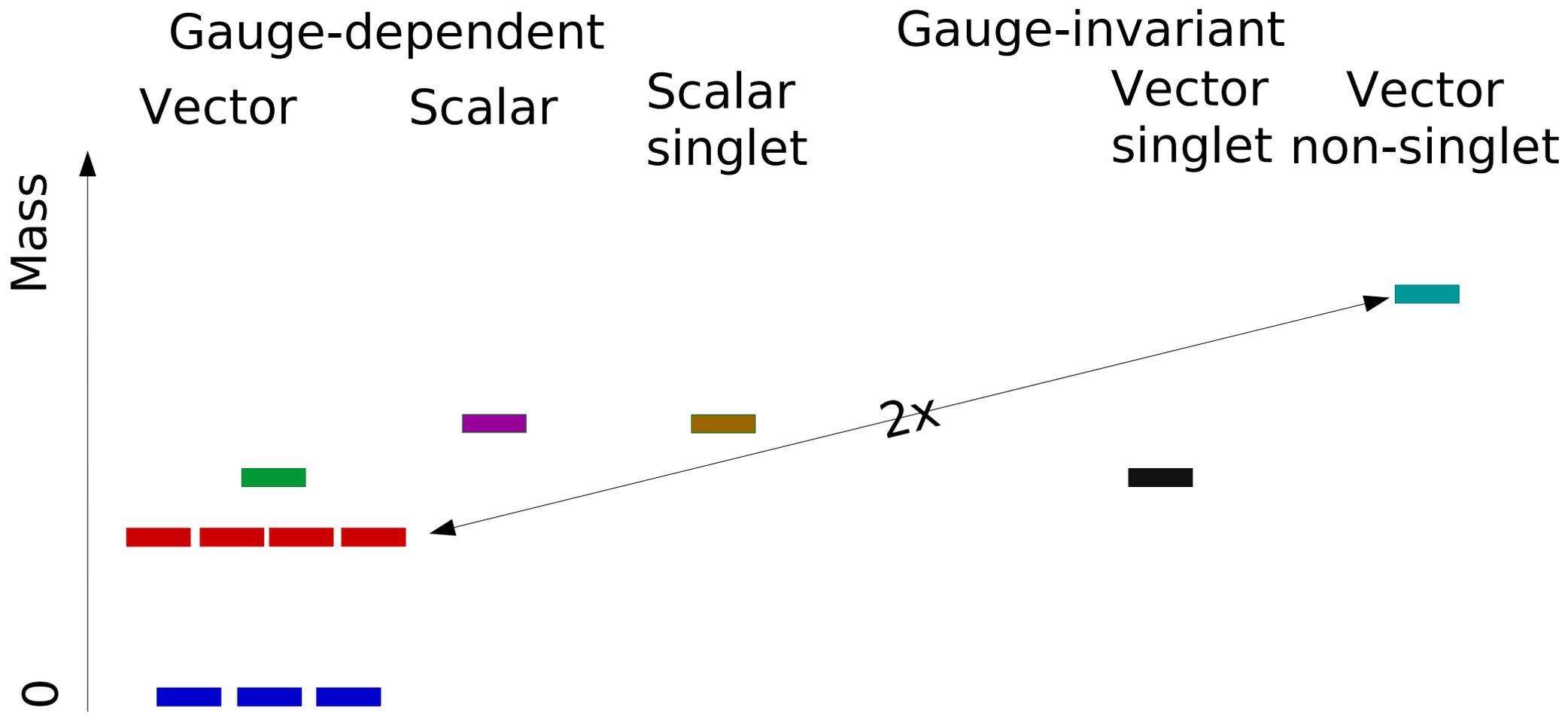
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Higgs field

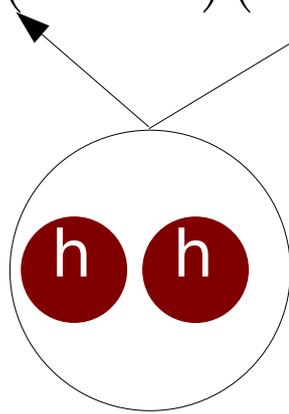


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2 x Higgs mass:  
Scattering state

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$$0^+ \text{ singlet: } \langle (h^\dagger h)(x)(h^\dagger h)(y) \rangle$$

2) Expand Higgs field around fluctuations  $h=v+\eta$

$$\begin{aligned} \langle (h^\dagger h)(x)(h^\dagger h)(y) \rangle &= v^2 \langle \eta^\dagger(x)\eta(y) \rangle \\ &+ v \langle \eta^\dagger \eta^2 + \eta^{\dagger 2} \eta \rangle + \langle \eta^{\dagger 2} \eta^2 \rangle \end{aligned}$$

Standard  
Perturbation  
Theory

3) Standard perturbation theory

Bound  
state  
mass

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4) Compare poles on both sides

# Fröhlich-Morchio-Strocchi Mechanism

[Fröhlich et al.'80,'81  
Maas'12,'17  
Maas & Sondheimer'20]

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Calculable:  $\rightarrow$  2009.06671

$$\langle (h^\dagger h)(x)(h^\dagger h)(y) \rangle = v^2 \langle \eta^\dagger(x)\eta(y) \rangle + v \langle \eta^\dagger \eta^2 + \eta^{\dagger 2} \eta \rangle + \langle \eta^{\dagger 2} \eta^2 \rangle$$

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# What about the vector?

[Maas & Törek'16]

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$1^-$  singlet

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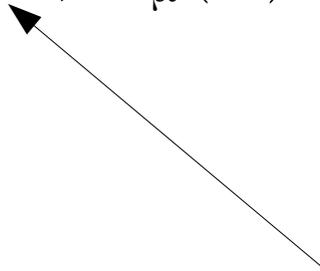
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Matrix from  
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$c^{ab}$  projects out  
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Only one state remains in the spectrum  
at mass of gauge boson 8 (heavy singlet)

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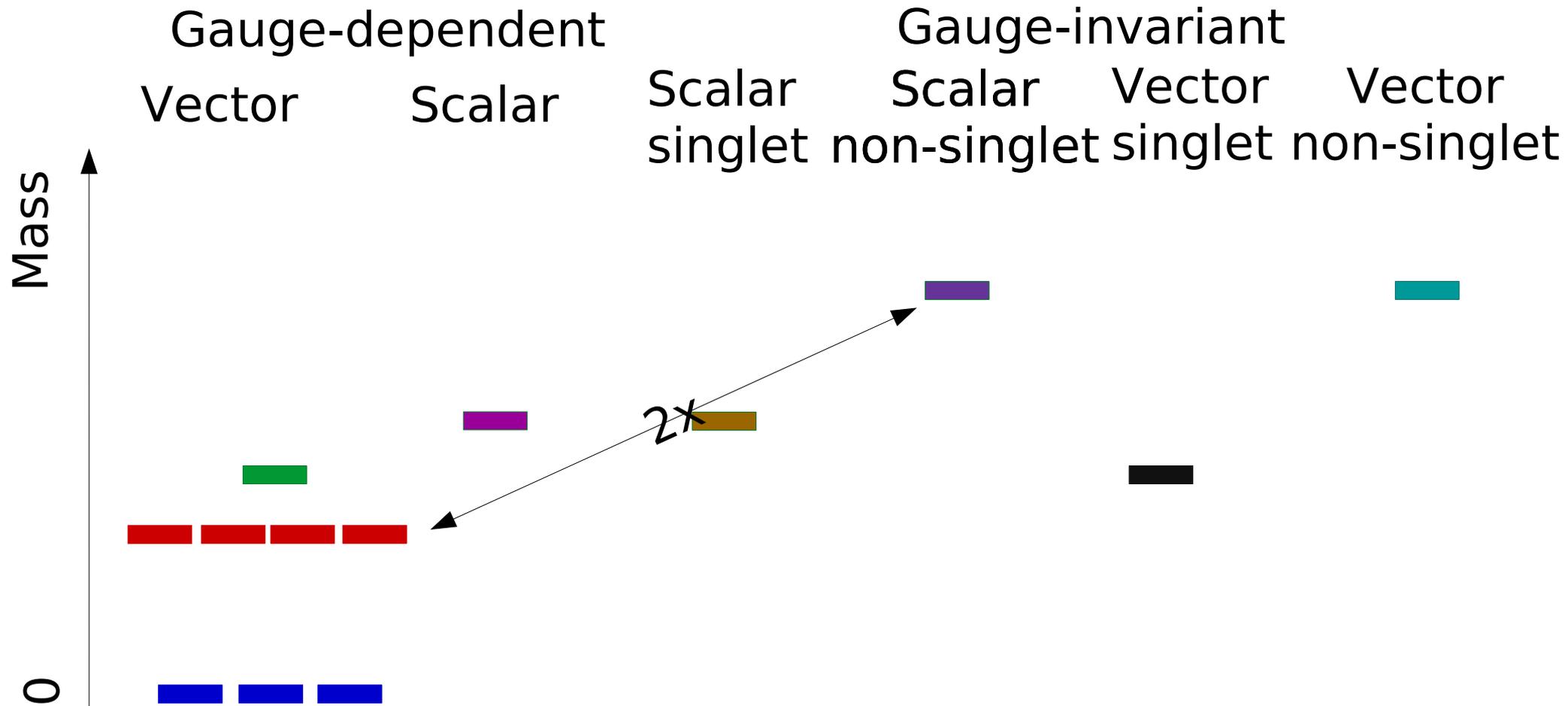
Matrix from  
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## Charged states need additional assumptions

[Maas, Sondenheimer, Törek'17]

# Spectrum

[Maas & Törek'16,'18  
Maas, Sondenheimer & Törek'17]

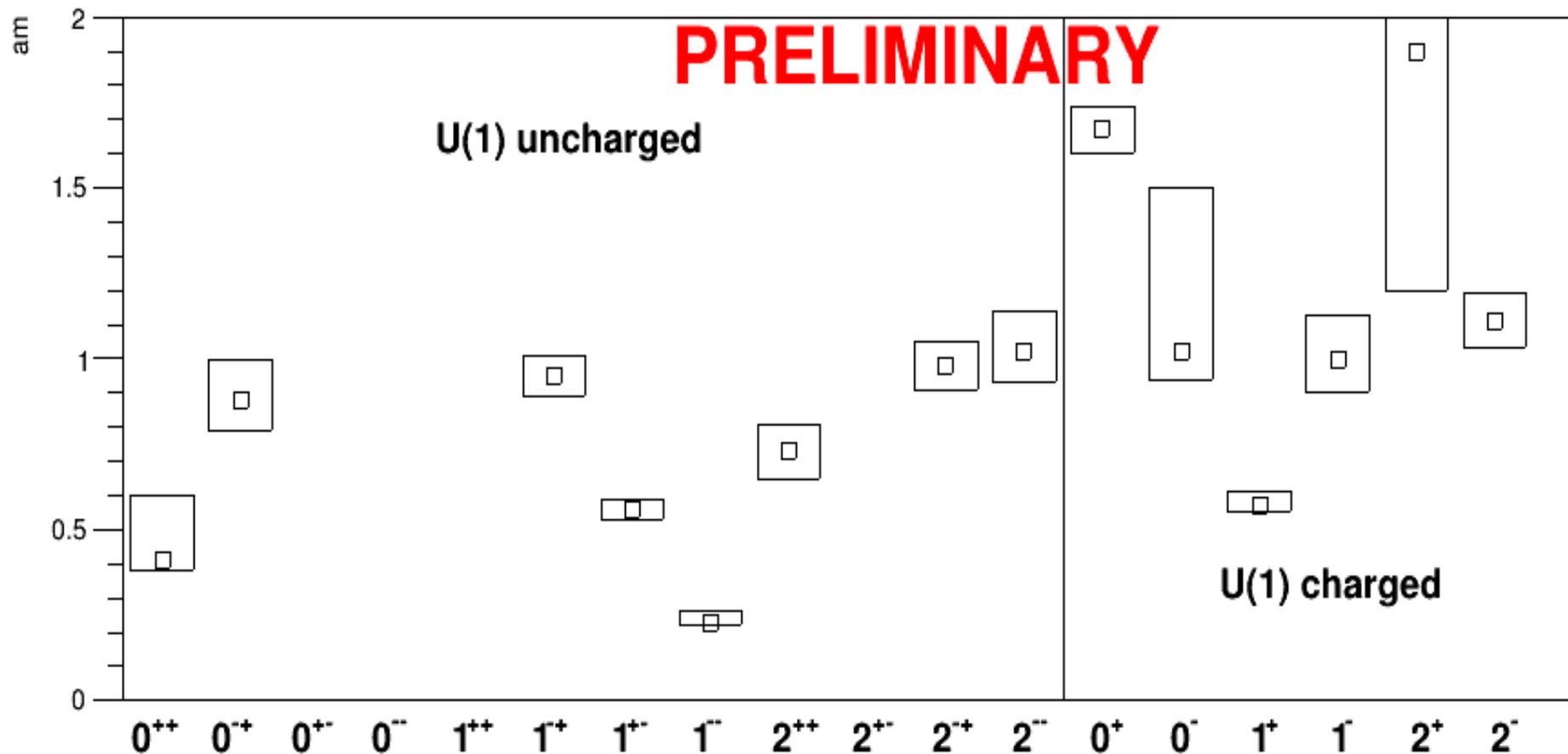


- Qualitatively different spectrum
- Gauge-dependent particles can also be calculated

# Spectrum

[Maas & Törek'16,'18  
Maas, Sondenheimer & Törek'17  
Dobson, Maas, Riedere, unpublished]

Spectrum for SU(3)+fundamental Higgs



- Full spectroscopy will check further FMS predictions
- Results so far show no additional light levels
- U(1) charged: Do not exist in perturbation theory

# Experimental consequences

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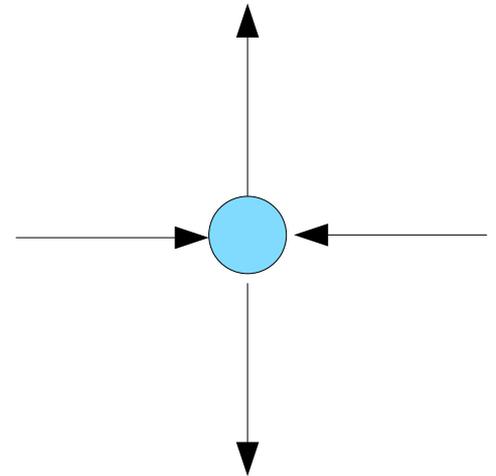
[Maas & Törek'18  
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- Add fundamental fermions

# Experimental consequences

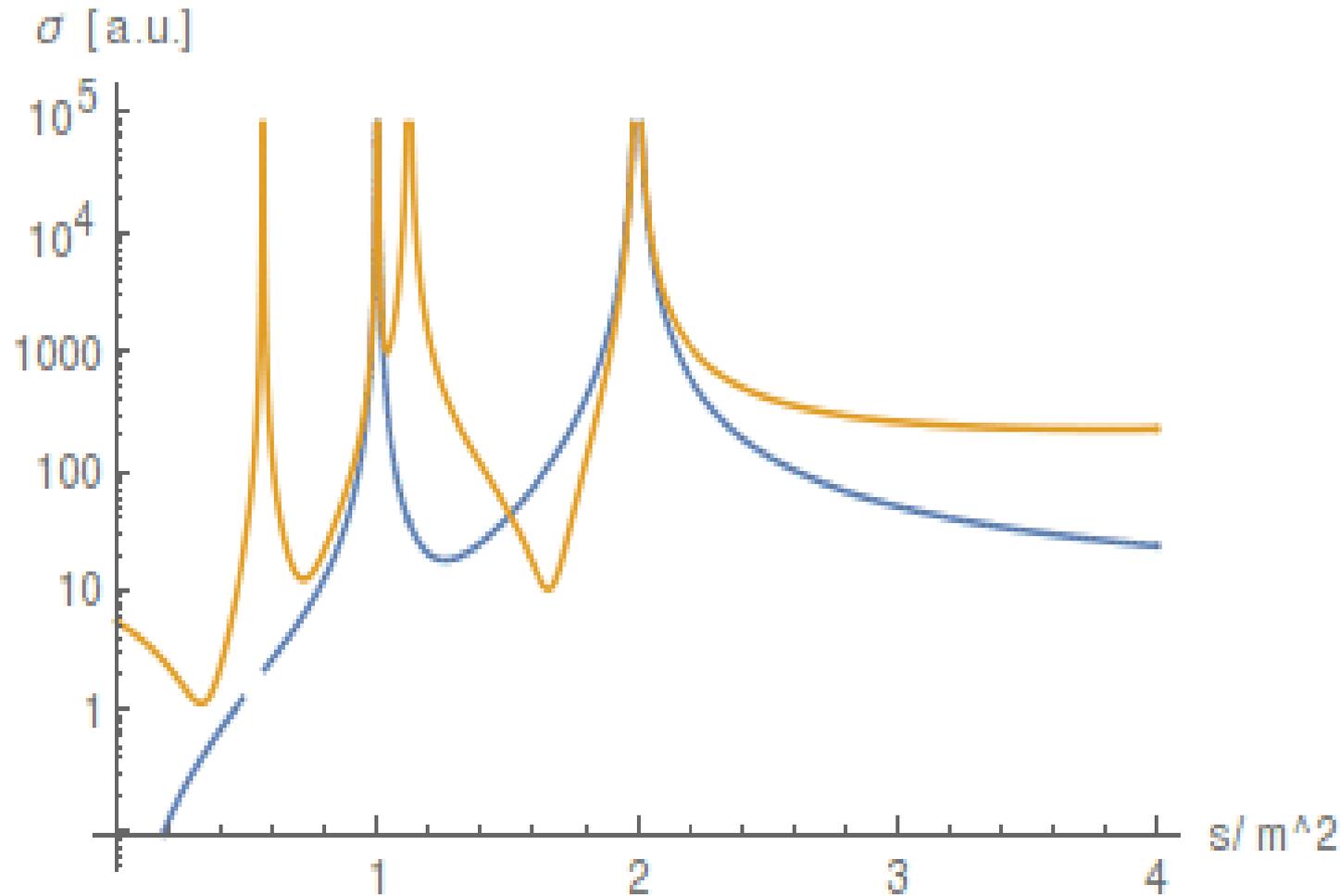
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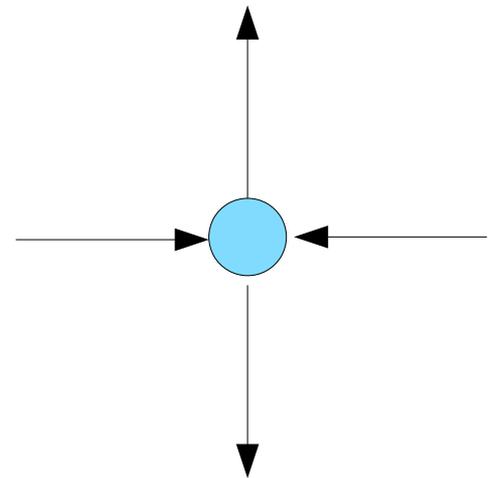
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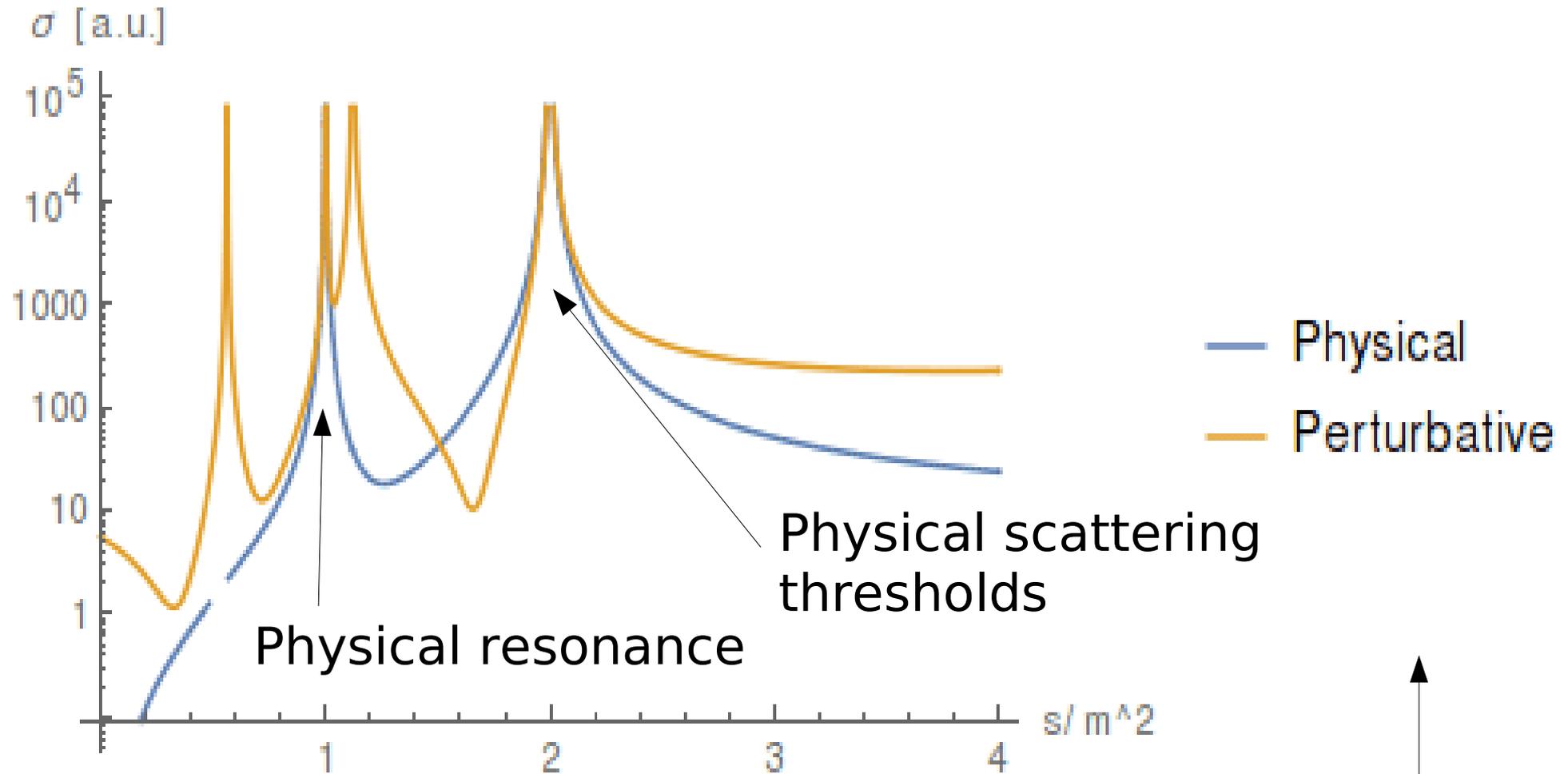
— Physical  
— Perturbative

- Add fundamental fermions
- Bhabha scattering

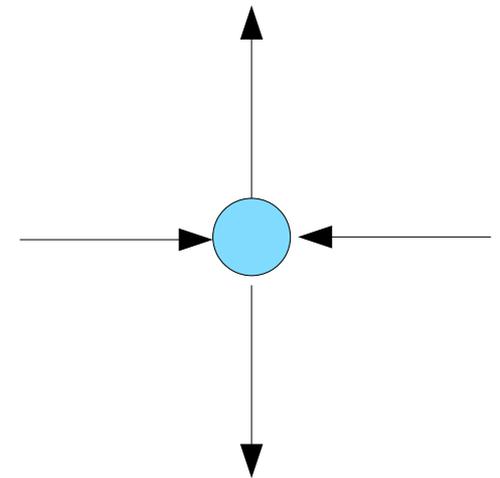


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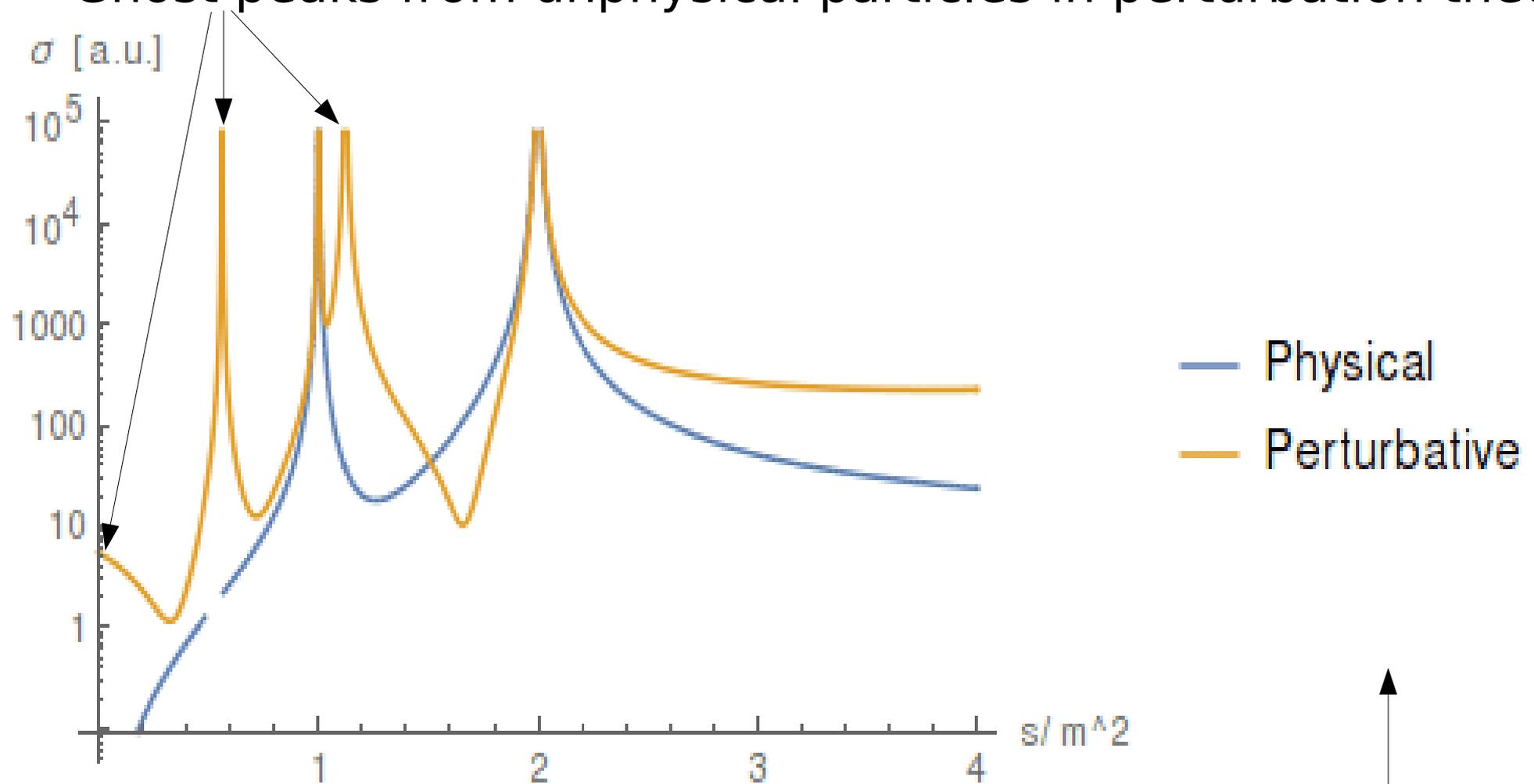
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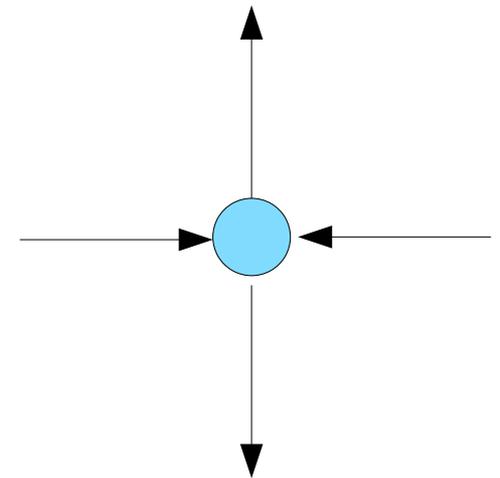
# Experimental consequences

[Maas & Törek'18  
Maas'17]

Ghost peaks from unphysical particles in perturbation theory



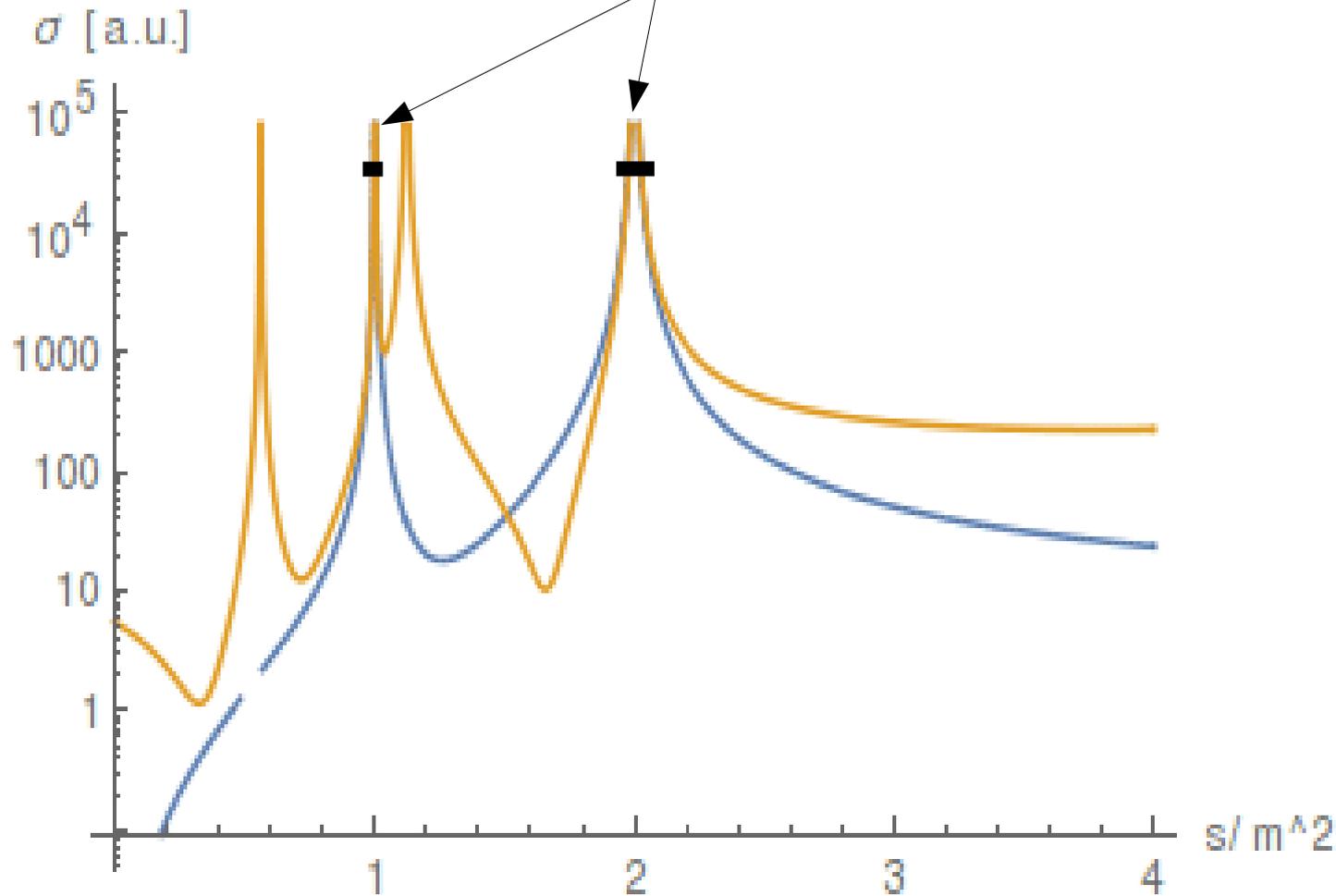
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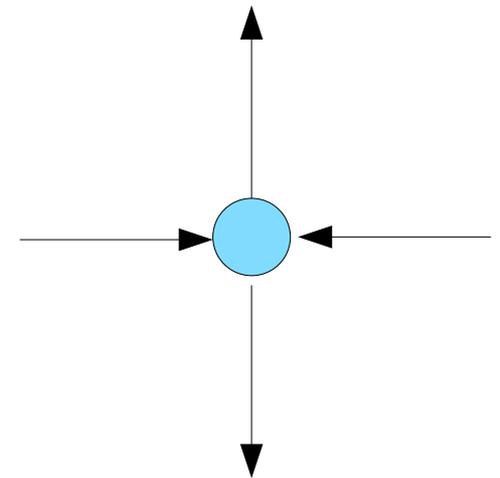
# Experimental consequences

[Maas & Törek'18  
Maas'17]

Close to true structures identical!



- Add fundamental fermions
- Bhabha scattering



# Beyond the toy model

[Maas, Sondenheimer & Törek'17,  
Sondenheimer'19]

- Generic problem in GUT scenarios [Sondenheimer'19]
  - Many standard scenarios are ruled out
  - Too few or too many particles at low mass
  - Includes popular scenarios like SU(5), SO(10), Pati-Salam

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  - Standard model has a special structure – protects the spectrum [Fröhlich et al.'80,'81]
- Requires to rebuild GUT phenomenology
  - Photon as composites possible [Afferrante et al.'20]

# Summary

- Perturbative methods to determine GUT spectra fail qualitatively
- Fröhlich-Morchio-Strocchi mechanism yields a suitable, practical alternative
- Phenomenology of GUTs needs to be redone