Gauge-invariant spectrum in the weak sector of the Standard Model

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Outline

I. Motivation & Background
   - Gauge-invariant formulation of Higgs theories
   - The Fröhlich-Morchio-Strocchi mechanism
   - Spectrum of the weak SM

II. Gauge-invariant spectrum from lattice calculations
   - SU(2) scalar-fermion-gauge system as a proxy to the weak sector of the SM
   - Gauge-invariant bound states on the lattice
   - First preliminary results and conclusions

Motivation & Background

Gauge-invariant spectrum from lattice calculations

Perturbative approach

- Consider an SU(2) gauge-scalar theory with Brout-Englert-Higgs (BEH) effect

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \frac{1}{2} \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) + \lambda \left(\phi^\dagger \phi - v^2\right)^2$$

- Classical minimum at $\phi^\dagger \phi = v^2$
- Introduce shift with fluctuations around the minimum

$$\phi(x) = \langle \phi(x) \rangle + \varphi(x) = vN + \varphi(x)$$

- Inserting this into $\mathcal{L}$ results in a mass term for the gauge bosons with $m_W \propto v^2$

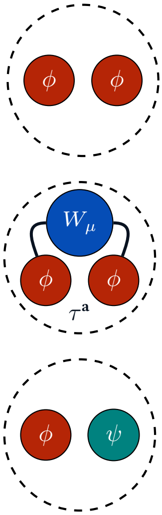
This construction is gauge-dependent

! There are gauges in which $\langle \phi(x) \rangle = 0$ and gauge bosons remain massless

! Elementary fields cannot be treated as physical degrees of freedom

Gauge-invariant approach

- In perturbation theory, elementary fields are treated as asymptotic states, although they are not gauge-invariant
- Asymptotic states must be described by **gauge-invariant composite objects**, i.e., bound-state operators†
- One needs to construct gauge-invariant objects with the same global quantum numbers as the elementary fields
  - Gauge-invariant scalar: $\phi(x) \to \left(\phi^\dagger \phi\right)(x)$
  - Gauge-invariant vector boson: $W_\mu^a(x) \to \left(\tau^{\mathbf{a}} \phi^\dagger D_\mu \phi\right)(x)$
  - Gauge-invariant left-handed fermion: $\psi(x) \to \left(\phi^\dagger \psi\right)(x)$



†Inherently non-perturbative methods are required to obtain physical quantities

FMS mechanism

Open questions

- Why does perturbation theory reproduce the physical spectrum of the weak SM?
- Are there any differences between the two approaches?

▷ Answers for both questions lie in a **gauge-invariant formulation** of quantum field theories → Fröhlich-Morchio-Strocchi (FMS) mechanism

Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)

▷ Example: consider the scalar bound state $\mathcal{O}_{0^+}(x) = \phi^\dagger(x)\phi(x)$

- ◇ Rewrite the propagator using $\phi(x) = vN + \varphi(x)$ and $h(x) = \mathrm{Re}\{N^\dagger\phi(x)\}$

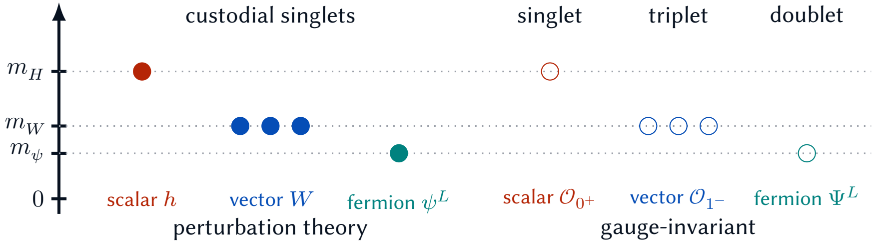
$$\langle \mathcal{O}_{0^+}(x)\mathcal{O}^\dagger_{0^+}(y)\rangle = \text{const} + 4v^2\langle h(x)h^\dagger(y)\rangle_{\text{tl}} + \langle h(x)h^\dagger(y)\rangle^2_{\text{tl}} + \mathcal{O}(g^2, \ldots)$$

- ◇ Mass pole of bound state coincides (to first order) with the mass pole of the elementary correlator $\langle h(x)h^\dagger(y)\rangle$

Maas, Sondenheimer, Phys. Rev. D 102 (2020), 2009.06671
Dudal *et al.*, Eur. Phys. J. C 81 (2020), 2008.07813

Spectrum of the weak SM

- FMS mechanism predicts a **one-to-one mapping** between gauge-dependent and gauge-invariant states and explains why perturbation theory is successful
- A similar construction for the vector boson shows agreement as well
  Dudal *et al.*, Eur. Phys. J. C 81 (2020), 2008.07813
- Confirmed by lattice calculations
  Maas, Mufti, JHEP 1404 (2014), 1312.4873

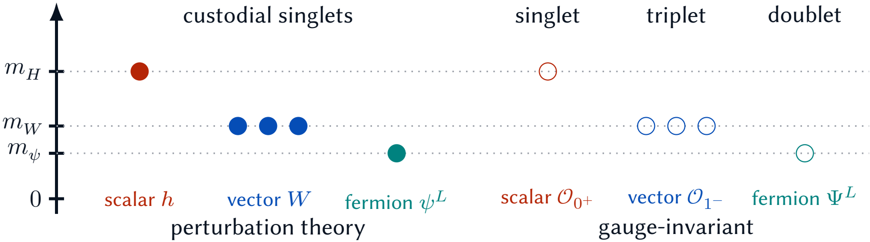


Spectrum of the weak SM

- Left-handed fermions are not gauge-invariant → construct bound state $\Psi^L = \phi^\dagger \psi^L$
- Employ the **FMS mechanism:** $\langle \Psi^L(x) \bar{\Psi}^L(y) \rangle = v^2 |N|^2 \langle \psi^L(x) \bar{\psi}^L(y) \rangle + \mathcal{O}(\varphi)$

  Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
  Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)
- Has been confirmed for vectorial leptons on the lattice

  Afferrante *et al.*, SciPost Phys. 10 (2021), 2011.02301



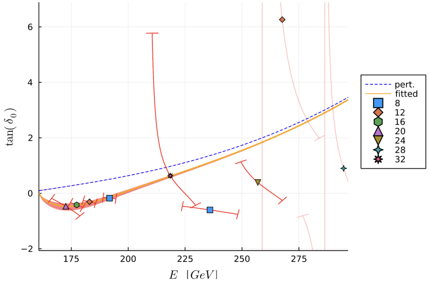
Deviations at higher orders

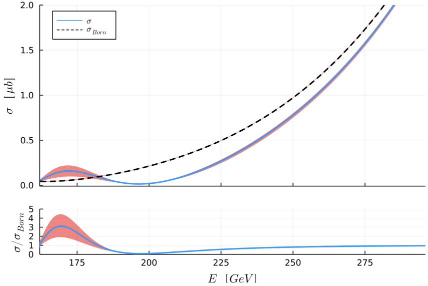
- Differences between perturbation theory and a gauge-invariant approach have been identified for **vector boson scattering** (VBS)

  Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756

- Experimentally, it is easier to access **fermions**
  - Search for signatures related to the bound state structure of observables
  - Determine phase shifts for, e.g., e^+e^- scattering
  - Determine the substructure of bound states via quasi-PDFs and form factors

- Are these signatures measurable?





Why bother?

- Deepen understanding of QFT
  - Learn more about the fundamental field-theoretical effects related to gauge invariance
  - What if the FMS mechanism is not the answer?
- Implications for future experiments
  - Set baseline for future mesurements
  - Avoid false positive regarding new physics
- Phenomenological implications
  - The weak sector of the SM is special, since $\mathrm{SU}(2)_\mathrm{w} \to \mathrm{SU}(2)_\mathrm{c}$
  - Model building should focus on custodial (global) group

Motivation & Background

Gauge-invariant spectrum from lattice calculations

SU(2) scalar-fermion-gauge theory

Two generations of leptons

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \frac{1}{2} \mathrm{tr}\left[\left(D_\mu X\right)^\dagger \left(D^\mu X\right) \right] - \frac{\lambda}{4} \left(\mathrm{tr}\left[X^\dagger X\right] - v^2 \right)^2$$
$$+ \sum_g \bar{\psi}^L_g i \not{D} \psi^L_g + \sum_f \bar{\chi}^R_f i \not{\partial} \chi^R_f - \sum_f \sum_g y_{f,g} \left[\left(\bar{\psi}^L X\right)_{f,g} \chi^R_f + \bar{\chi}^R_f \left(X^\dagger \psi^L\right)_{f,g} \right]$$

- Matrix-valued field X which contains the components of the usual scalar doublet ϕ
- Two generations of left-handed Weyl spinors gauged under the weak interaction

$$\psi^L_{g=1} = \begin{pmatrix} \nu^L_e & e^L \end{pmatrix}^\top \qquad \psi^L_{g=2} = \begin{pmatrix} \nu^L_\mu & \mu^L \end{pmatrix}^\top$$

- Four flavors of ungauged right-handed Weyl spinors $\chi^R_f = \begin{pmatrix} \nu^R_e & e^R & \nu^R_\mu & \mu^R \end{pmatrix}^\top_f$
- Symmetries for $y_{f,g} = 0$: local $SU(2)_w$ & global $SU(2)_c$, $SU(2)_{Lg}$, $SU(4)_{Rf}$

Lattice setup

- Chiral nature of the weak gauge theory poses conceptual problems
- SM-like proxy that replaces the Weyl fermions by Dirac spinors
- Same symmetries with additional possibility to break generation/flavor symmetry explicitly
- Does not interfere with FMS predictions

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \frac{1}{2} \mathrm{tr}\left[\left(D_\mu X \right)^\dagger \left(D^\mu X \right) \right] - \frac{\lambda}{4} \left(\mathrm{tr}\left[X^\dagger X \right] - v^2 \right)^2$$
$$+ \sum_g \bar{\psi}_g \left(i \not{D} - m_{\psi_g} \right) \psi_g + \sum_f \bar{\chi}_f \left(i \not{\partial} - m_{\chi_f} \right) \chi_f$$
$$- \sum_f \sum_g y_{f,g} \left[\left(\bar{\psi}_L X \right)_{f,g} \chi_f + \bar{\chi}_f \left(X^\dagger \psi_L \right)_{f,g} \right]$$

First calculations

- Yukawa couplings $y_{f,g} = 0 \longrightarrow \chi_f$ decouple and symmetries are intact
- Two degenerate generations $m_{\psi_1} = m_{\psi_2}$

Bound states

- Scalar singlet: $\mathcal{O}_{0^+}(x) = \text{tr}\left[X^\dagger(x) X(x)\right]$
- Vector triplet: $\mathcal{O}^{\mathbf{a},\mu}_{1^-}(x) = \text{tr}\left[\tau^{\mathbf{a}} \frac{X^\dagger(x)}{\sqrt{\det X^\dagger(x)}} U^\mu(x) \frac{X(x+e_\mu)}{\sqrt{\det X(x+e_\mu)}}\right]$

  Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721
- Fermion bound state: $\Psi^L_{\alpha,\mathbf{i}}(x) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\, \psi^L_{\alpha,i}(x)$
  - $\mathbf{i}$ ... custodial index, i ... gauge index, α ... Dirac index
  - Correlator constructed by Wick contraction and a trace in the Dirac structure

$$M_{\mathbf{ij}}(x|y) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\, \left(D^{-1}\right)_{ij}(x|y)\, (X)^j_{\mathbf{j}}(y)$$

Afferrante *et al.*, SciPost Phys. 10 (2021), 2011.02301

Simulation details

- Hybrid Monte Carlo (HMC) algorithm with dynamical Wilson fermions
- Publicly available **HiRep** simulation code

Debbio, Patella, Pica, Phys. Rev. D 81 (2010), 0805.2058
Hansen *et al.*, EPJ Web Conf. 175 (2018), 1710.10831

Exploring the parameter space

- 7 parameter sets for small lattice sizes ($L = 8, 10, 12$)
  - 2 sets with stable Higgs, 1 set with heavy Higgs & 1 set with Higgs resonance

    Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
  - 1 SM-like set

    Wurtz, Lewis, Phys. Rev. D 88 (2013), 1307.1492
  - 1 set to compare with quenched results

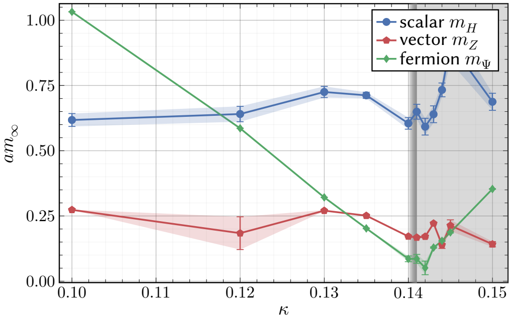
    Afferrante *et al.*, SciPost Phys. 10 (2021), 2011.02301
  - 1 set in QCD-like domain

    Maas, Mufti, JHEP 1404 (2014), 1312.4873
- **Basic idea:** start with given parameter sets and increase the fermion hopping parameter κ

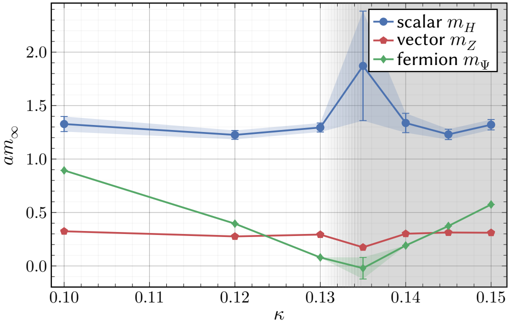
Disclaimer: **preliminary** results!

Exploring the parameter space

Set #2 with stable Higgs



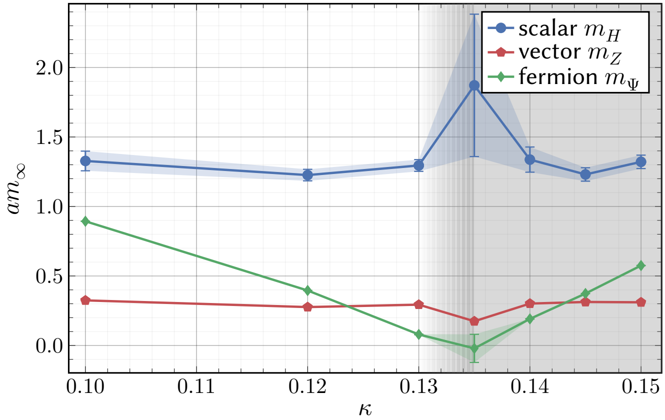
Set #4 with Higgs resonance



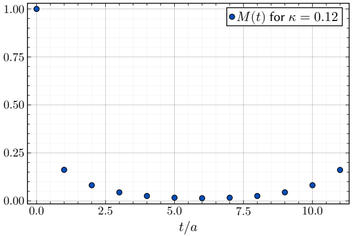
First conclusions

- ! Gauge-invariant bound states are accessible with decent statistics
- ! We are able to controll the system via the fermion mass without changing the overall dynamics of the gauge-scalar subsystem

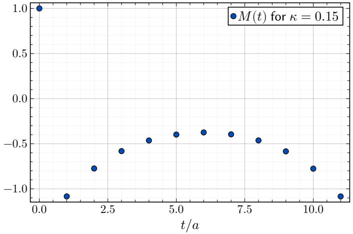
Set #4 with Higgs resonance



Physical phase ($\kappa < \kappa_{\text{crit}}$)

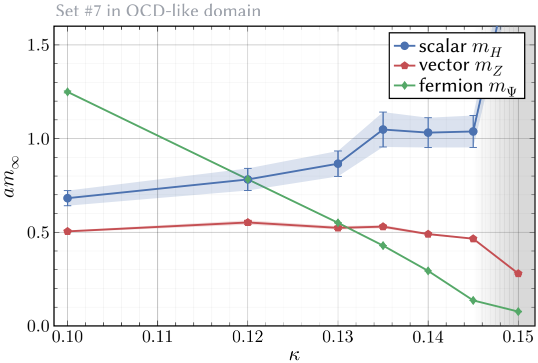


Unphysical phase ($\kappa > \kappa_{\text{crit}}$)



Simulation points for scattering analysis

- Five points of interest to simulate various scenarios
  - $m_\psi \approx 1.25 m_H$
  - $2m_\psi \approx 1.25 m_H$
  - $2m_\psi \approx 0.9 m_H$
  - $2m_\psi \approx (m_H + m_Z)/2$
  - $2m_\psi \approx 0.9 m_Z$



First conclusions

- ! We are able to access all points of interest due to the dynamics of m_H and m_Z
- ! Unphysical phase transition can (most likely) be avoided

Simulation points for scattering analysis

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  - $m_\psi \approx 1.25 m_H$
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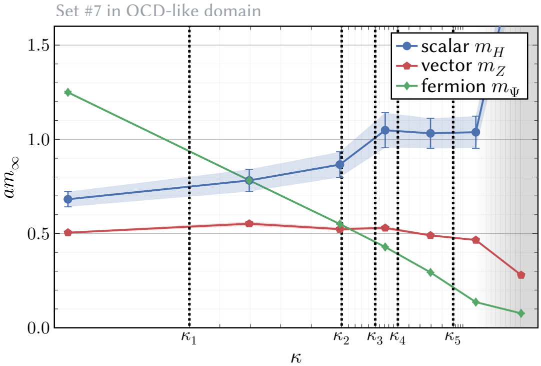

First conclusions

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Summary

- Simulation of a proxy theory to the weak sector of the SM in a fully gauge-invariant setup
  - Gauge-invariant equivalent to the Higgs, Z boson & weakly charged leptons
- Dynamics of the system are consistent for various parameter sets
  - The gauge-scalar dynamics persist with the introduction of fermions and the system can be fully controlled by varying the fermion mass
  - We have found a good proxy theory to describe the weak SM via lattice calculations
- First steps towards testing the FMS mechanism via lepton scattering
  - Suitable simulation points are found and accessible with decent statistics
  - Issues with an unphysical phase transition can (most likely) be avoided

Next steps

- Continue simulations with larger lattices and confirm points of interest
- Start data production for subsequent scattering analysis

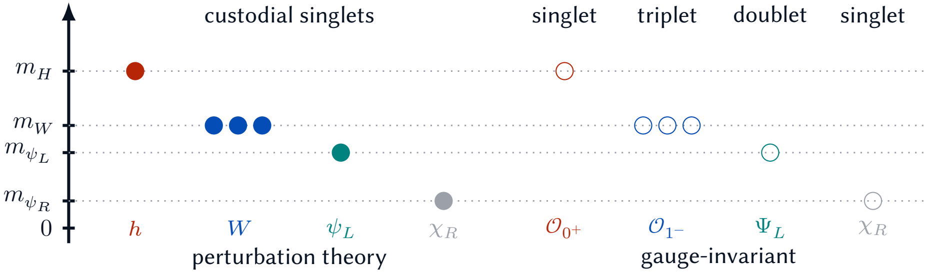
Backup slides

BSM theories

- Weak sector of the SM special, since $SU(2)_w \to SU(2)_c$
- GUT theories: gauge group can be larger than custodial group
- For example, SU(n) with $n > 2$ with one fundamental scalar
- Contradiction in vector channel
  - Perturbation theory: $2(n-1)+1$ massive and $n(n-2)$ massless gauge bosons
  - FMS: only one massive state

  Maas, Sondenheimer, Török, Ann. Phys. 402 (2019), 1709.07477
- For SU(3), lattice supports FMS predictions

  Maas, Török, Phys. Rev. D 95 (2017), 1607.05860
  Maas, Török, Ann. Phys. 397 (2018), 1804.04453

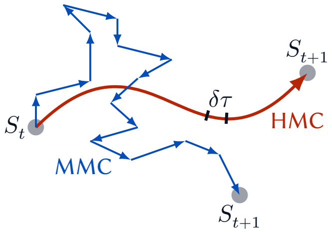


custodial singlets
singlet
triplet
doublet
singlet
m_H
m_W
m_{ψ_L}
m_{ψ_R}
0
h
W
ψ_L
χ_R
$\mathcal{O}_{0^+}$
$\mathcal{O}_{1^-}$
Ψ_L
χ_R
perturbation theory
gauge-invariant

Hybrid Monte Carlo (HMC) algorithm

- Molecular dynamics algorithm with Gaussian distributed conjugate momenta and a Metropolis accept step
- Dirac operator enters quadratically by rewriting $\det\{DD^\dagger\}$ as an integral over pseudo-fermion fields
- Can only simulate an even number of fermions (no sign problem) $\rightarrow$ **two degenerate generations of fermions**
- Rational HMC: assuming $\det\{D\}$ is positive definite, the HMC algorithm can be generalized to an arbitrary number of fermion species

Gattringer, Lang, Quantum chromodynamics on the lattice (2010)



- Publicly available **HiRep** simulation code

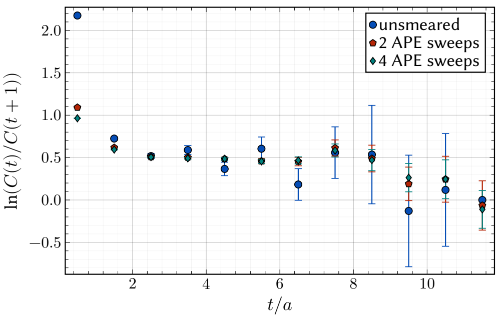
Debbio, Patella, Pica, Phys. Rev. D 81 (2010), 0805.2058
Hansen *et al.*, EPJ Web Conf. 175 (2018), 1710.10831

APE smearing

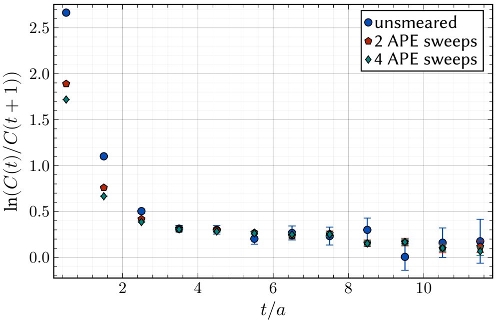
Numerical challenges

- Inversion of Dirac operator is expensive
- Strong statistical fluctuations in the bosonic channels

Effective mass for $\mathcal{O}_{0^+}$

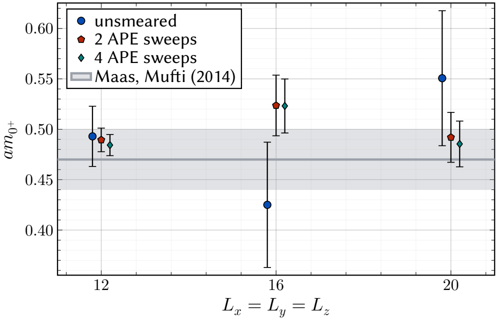


Effective mass for $\mathcal{O}_{1^-}$

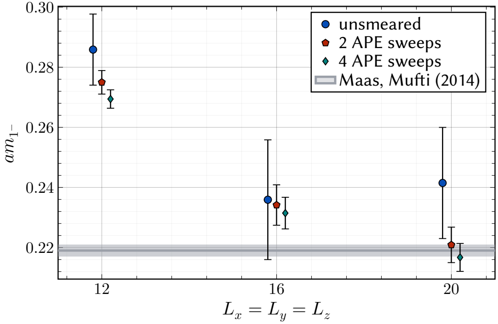


- Does HMC work for gauge-scalar theories in the Higgs-like domain?
- Comparison to Maas, Mufti, JHEP 1404 (2014), 1312.4873

Energy levels for $\mathcal{O}_{0^+}$



Energy levels for $\mathcal{O}_{1^-}$



Fermion bound state

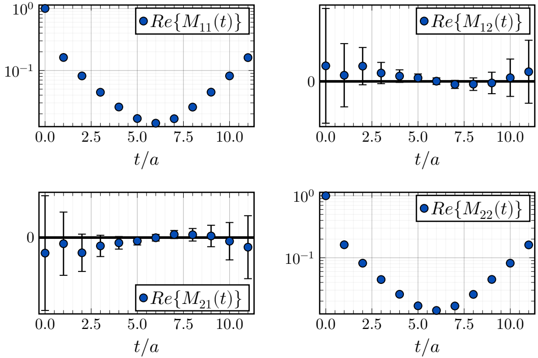
- $M_{\mathbf{ij}}$ has to be proportional to SU(2) group element

$$\left(M_{\mathbf{ji}}\right)^{\dagger} M_{\mathbf{jk}} \propto \delta_{\mathbf{ik}}$$

- Mass matrix can be written as

$$M_{\mathbf{ij}} = c\,\delta_{\mathbf{ij}} + i\tilde{M}_{\mathbf{ij}} \quad c \in \mathbb{R}$$

- $\tilde{M}_{\mathbf{ij}}$ has direction in custodial space, but no direction is preferred
  - ⇨ $\langle \tilde{M}_{\mathbf{ij}} \rangle$ has to vanish
  - ⇨ $\langle M_{\mathbf{ij}} \rangle$ has to be real



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