Formulation of electroweak pion decays in functional methods

Walid Ahmed Mian
in collaboration with A. Maas, M. Mitter, J. M. Pawlowski, H. Sanchis-Alepuz and N. Wink

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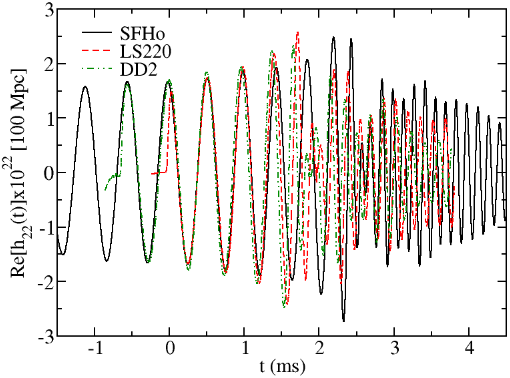
Der Wissenschaftsfonds.

NAWI Graz
Natural Sciences

Motivation

- System of binary neutron stars mergers
- Source of gravitational waves
- Very high neutrino flux
- Back coupling of neutrinos influences gravitational waves
- Measurement shows the inner structure of neutron star mergers



(Foucart et al. arXiv:1510.06398v2 [astro-ph],
Rosswog et al., Mon. Not. Roy. Astron. Soc. 342, 673 (2003),
Y. Sekiguchi et al., PRL 107, 051102 (2011),
J. A. Faber et al., Living Rev. Rel. 15, 8 (2012),
D. Neilsen et al., PRD 89, 104029 (2014),
C. Palenzuela et al., PRD 92, 044045 (2015),
O. L. Caballero arXiv:1603.02755 [nucl-th], . . .)

Back Coupling Neutrinos

- Very dense matter $\Rightarrow$ opaque for neutrinos
- Reaction inside the core (Foucart et al. arXiv:1510.06398v2 [astro-ph])

$$\nu_e + n \longleftrightarrow p + e^-$$
$$\overline{\nu}_e + p \longleftrightarrow n + e^+$$
$$\nu_e + \overline{\nu}_e \longleftrightarrow e^+ + e^-$$
$$\nu_e + \overline{\nu}_e \longleftrightarrow \gamma$$

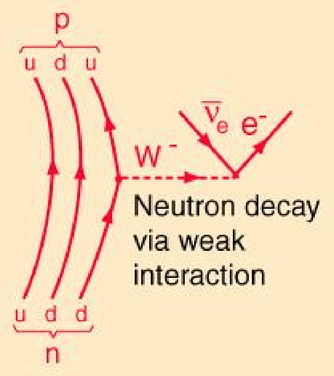
- Electroweak interactions play an important role
- Consider non-perturbative QCD + electroweak interactions

β-decay

- Full resolution of electroweak interactions is complicated
- β-decay captures the main features
- Look at the $\pi^{\pm}$-decay
- Electroweak interactions approximated by 4-Fermi-interaction
- Electroweak interactions violate parity

$$\mathcal{L}_{\text{4-Fermi}} = g_w \left\{ \left[\overline{\psi}_\nu^L \gamma^\mu \psi_e^L\right] \left[\overline{\psi}_u^L \gamma^\mu \psi_d^L\right] + \left[\overline{\psi}_e^L \gamma^\mu \psi_\nu^L\right] \left[\overline{\psi}_d^L \gamma^\mu \psi_u^L\right] \right\}$$

- ψ^L: Left-handed fermion fields



http://hyperphysics.phy-astr.gsu.edu/hbase/particles/proton.html

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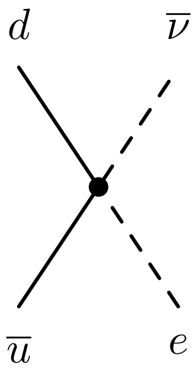

$$\mathcal{L}_{\text{4-Fermi}} = g_w \left\{ \left[\overline{\psi}^L_\nu \gamma^\mu \psi^L_e\right] \left[\overline{\psi}^L_u \gamma^\mu \psi^L_d\right] + \left[\overline{\psi}^L_e \gamma^\mu \psi^L_\nu\right] \left[\overline{\psi}^L_d \gamma^\mu \psi^L_u\right] \right\}$$

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Functional Methods

- Very different energy scales + parity violation
  $\Rightarrow$ Lattice calculation unfeasible
- Functional approaches (Dyson-Schwinger-Equations (DSEs), Bethe-Salpeter-Equations (BSEs), Functional Renormalization Group (FRG), ...) suitable methods
  1. Continuum, covariant and non-perturbative formulation
  2. High and low energy scales accessible at the same time
- Drawback: Infinite tower of coupled, nonlinear integral equations
- Need truncations
- Correlation functions in Minkowski-space: Access to dynamical observables
- Mass and decay-width of the particle: Need poles of the propagator in Minkowski-space
- Extend method to complex momenta

Resonances in the 2nd Riemann sheet

- Resonances: Pole in the 2nd Riemann sheet

  (Haag, Local Quantum Physics Fields, Particles, Algebras)

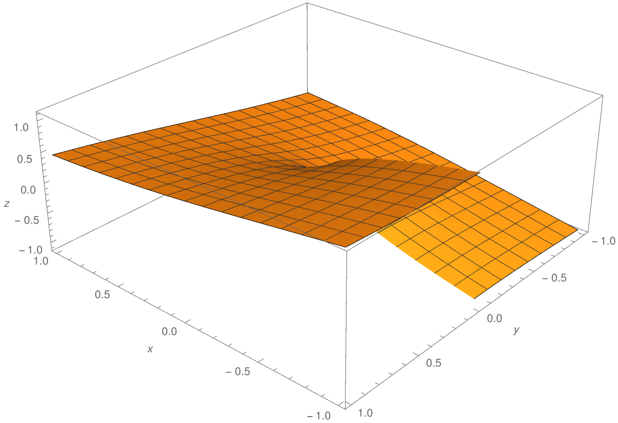
- Minkowski-space

$$s = P^2$$

- Euclidean-space

$$P^2 \to -P^2 = s_E$$

$$z = \pm \mathrm{Im}(\sqrt{x + \mathrm{i}\, y}) = \pm\sqrt{\rho}\sin(\tfrac{\phi}{2})$$



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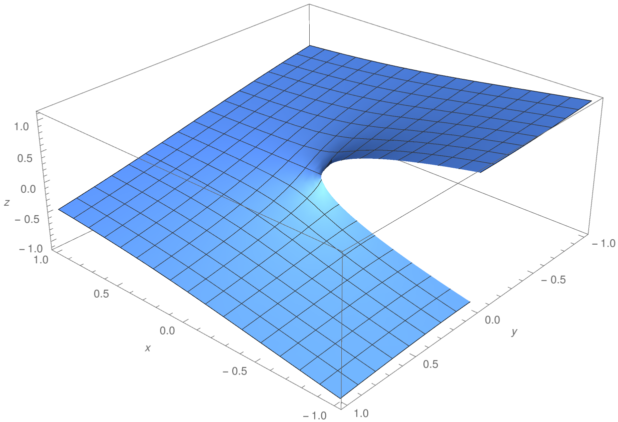

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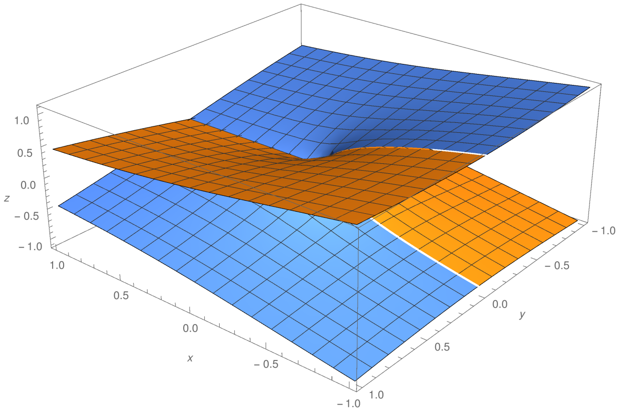

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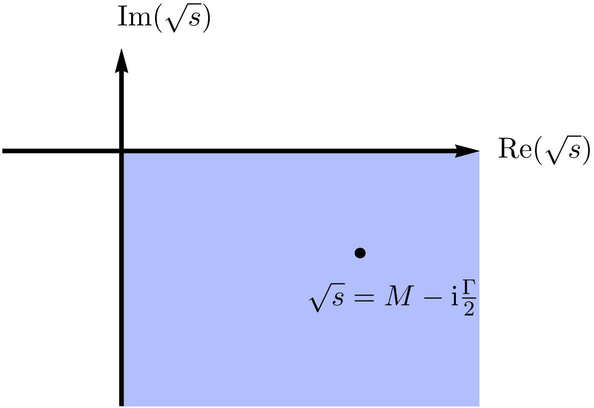

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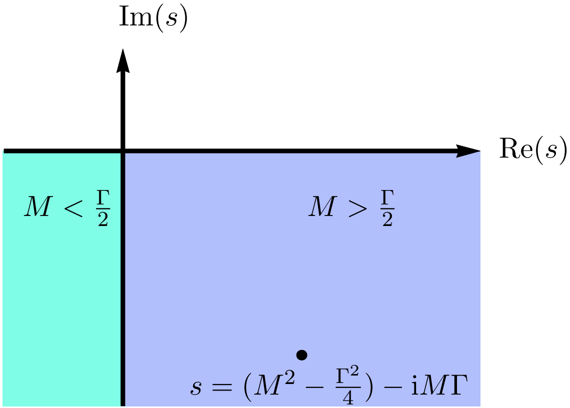

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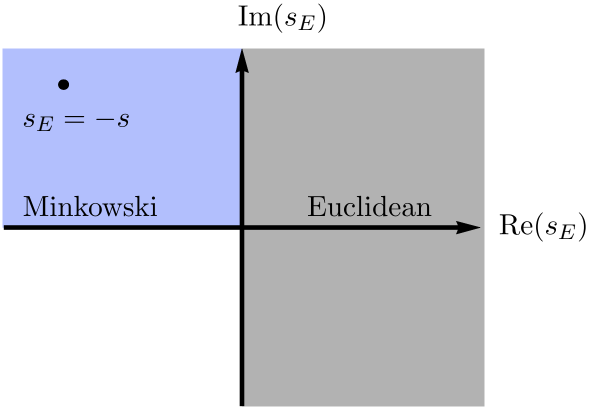
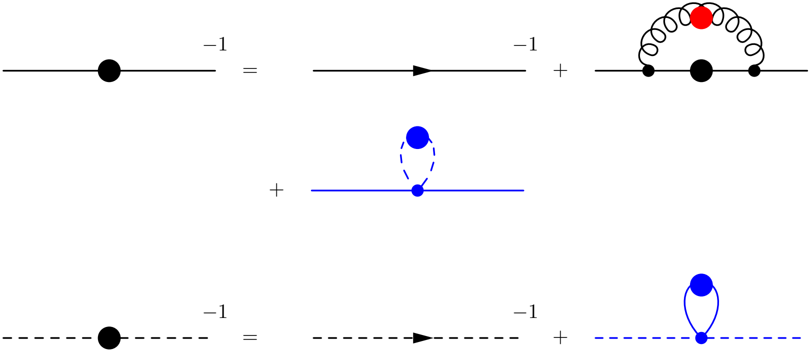


DSEs for the Propagators



- Charge conservation ⇒ Vanishing contribution from 4-Fermi interactions
- No Influence on the propagator level

BSEs

- BSEs: Bound state equations derived from DSEs and evaluated on the pole.
- Total momenta

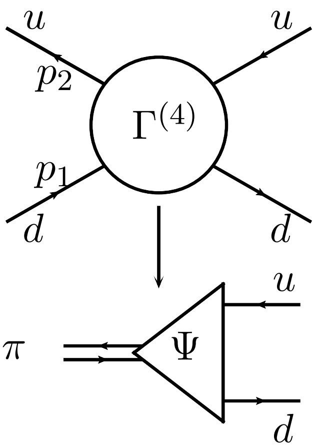
$$P = p_1 - p_2$$

- At the pole M_{Pole}

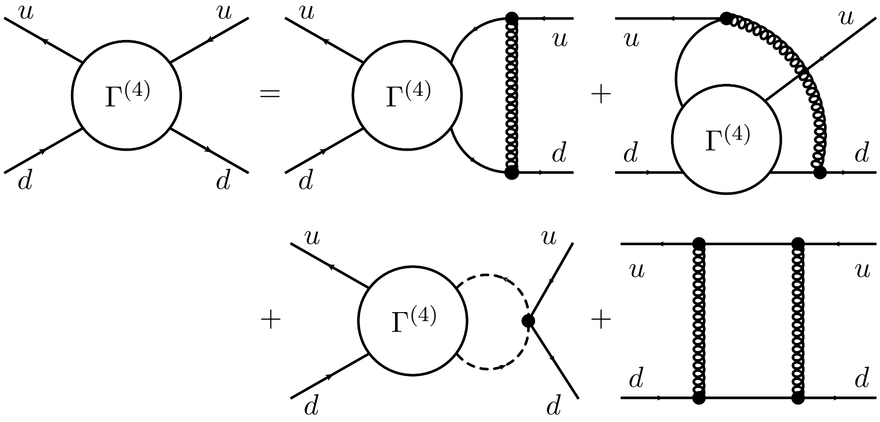
$$\Gamma^{(4)} \propto \frac{\Psi\overline{\Psi}}{P^2 + M_{\text{Pole}}^2}$$

- Ψ: Bethe-Salpeter-Amplitude

$$\Psi = \Gamma^{(3)}\Big|_{\text{Pole}}$$



BSEs



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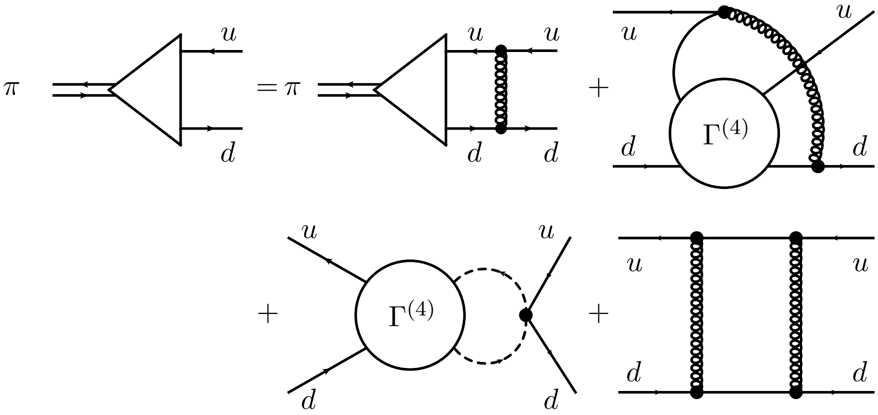

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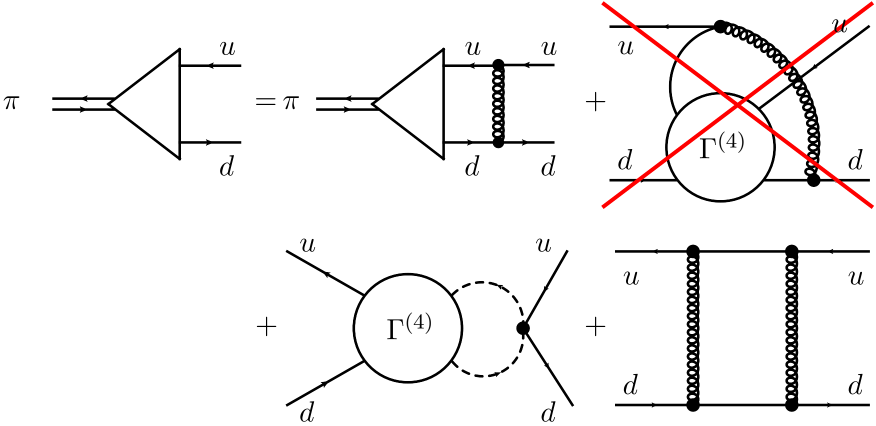
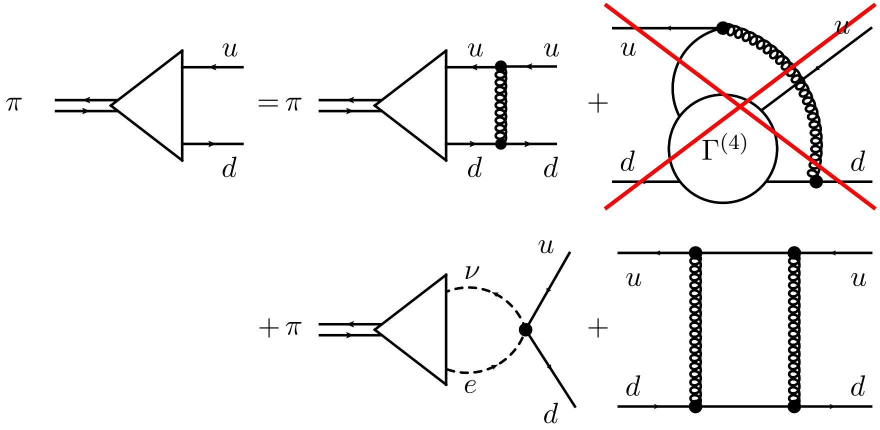


BSEs



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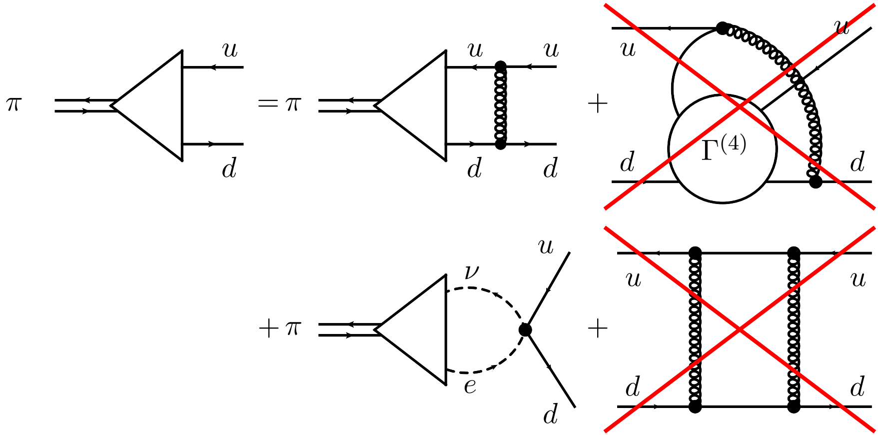
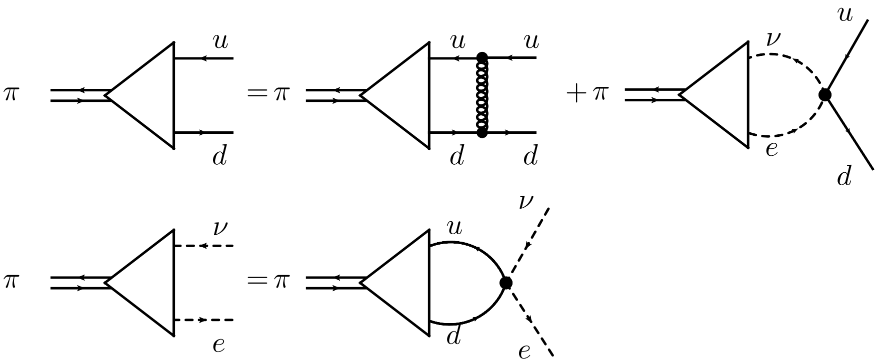


Coupled system of BSEs



- Possibility of pion decay in electron and neutrino: Additional contribution to the Bethe-Salpeter-Amplitude
- Pure QCD: M_{Pole} of pion real in Minkowski-space (stable particle)
- QCD + electroweak interaction + light leptons: Open decay channel for pion ⇒ Searching for poles in the 2nd Riemann sheet

FRG

Wetterich equation

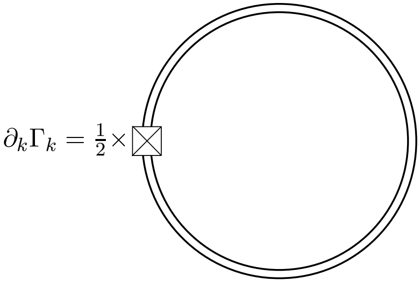
(C. Wetterich, Phys. Lett. B 301, 90 (1993))

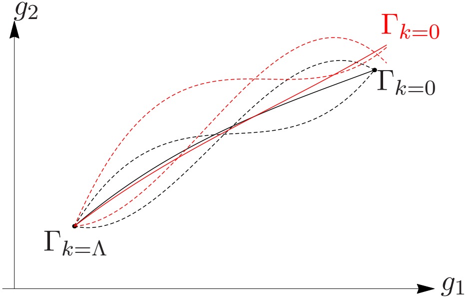
$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{STr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)}[\phi] + R_k}\right]$$

$$\lim_{k\to\Lambda} \Gamma_k = S,$$

$$\lim_{k\to 0} \Gamma_k = \Gamma$$

- Interpolation between the microscopic action S in the UV at $k = \Lambda$ and the full effective action Γ in the IR at $k = 0$





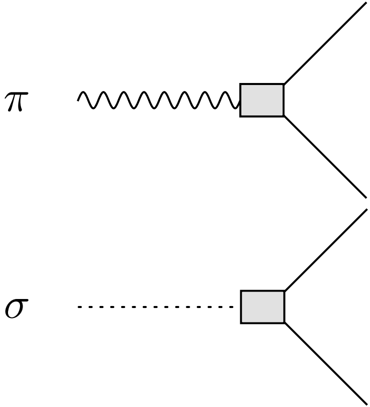
Dynamical Hadronization

- Unified description of fundamental and composite degrees of freedom (see also Talk of Jordi Paris Lopez)

  (H. Gies and C. Wetterich, PRD 65, 065001 (2002))

- E.g.: Transform from fundamental fermions (up and down quark) to bound states (pion and sigma) at each scale

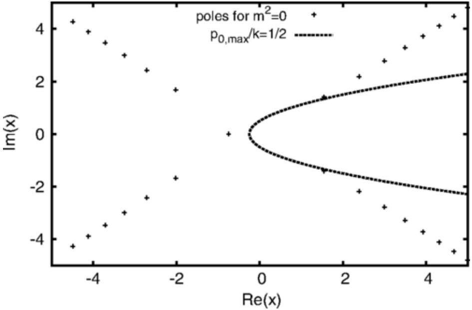
$$\partial_k \pi_k = \partial_k A_0 \left(\overline{\psi}\, \mathrm{i}\, \gamma_5 \tau \psi\right)$$
$$\partial_k \sigma_k = \partial_k A_1 \left(\overline{\psi} \psi\right)$$



Bosonic Propagator:

$$\Pi(p^2) = \frac{1}{p^2 + m^2 + R_k^B(p^2)}$$

- Additional unphysical poles in the propagators due to the regulator
- Modify regulator by additional ΔM_r term: Shifting poles outside



J. M. Pawlowski and N. Strodthoff, PRD 92, 094009 (2015)

$$\Pi(p^2) = \frac{1}{p^2 + m^2 + R_k^B(p^2)} = \frac{1}{p^2 + m^2 + \tilde{R}_k^B(p^2) + \Delta M_r^2(k)}$$

$$x = \frac{p^2}{k^2}, \qquad M^2 = m^2 + \Delta M_r^2$$

Real Time FRG

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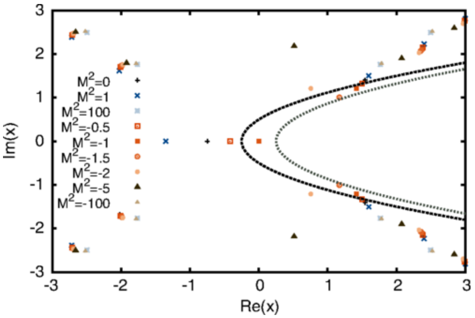

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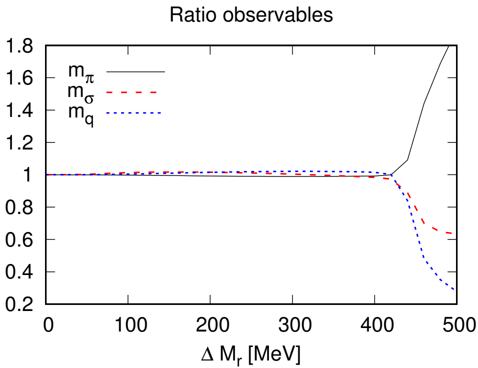
Low-Energy Effective Model: Quark-Meson Model

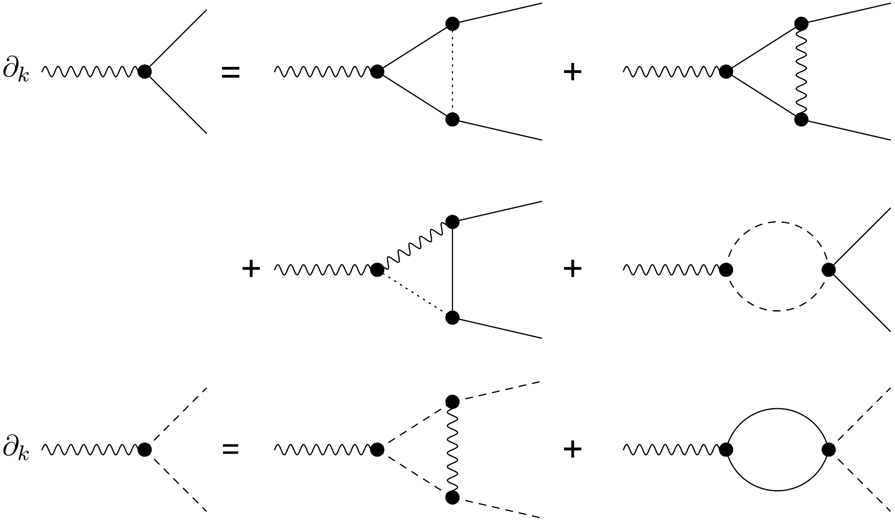
- Analyze regulator dependence on ΔM_r for the real time calculation
- Ratio masses

$$\frac{m_x(\Delta M_r)}{m_x(\Delta M_r = 0)}$$

- Negligible dependence for $\Delta M_r \leq 400$ MeV
- **Pion resonance accessible in real time**



Flow Equation



Bethe-Salpeter-Amplitude

- General form of the pion Bethe-Salpeter-Amplitude (C. H. Llewellyn-Smith, Ann. Phys. (NY) 53, 521 (1969))

$$\Psi_{\pi,ud} = \tau\gamma_5 \left(\mathrm{i}\, f_1 \mathbb{1} + f_2 \not{P} + f_3 \not{k} + \frac{\mathrm{i}}{2} f_4 [\not{P}, \not{k}] \right)$$

- P: Total momenta of the fermions
- k: Relative momenta of the fermions
- Pion Bethe-Salpeter-Amplitude for the lepton decay

$$\Psi_{\pi,e\nu} = \tau\gamma_5 \left(g_2 \not{P} + g_3 \not{k} \right)$$

- Only left-handed fermions contribute to the 4-Fermi-interaction: Vanishing g_1 and g_4

Summary & Outlook

- Consider QCD and electroweak interactions non-perturbative
- Goal:
  1. β-decay in neutron stars
  2. Dynamical decay process in functional approach
- Requirement: Real time calculation
- Building numerical setup in functional methods for real time calculation
- Access to mass and decay-width of the particle
- Self-consistent backcoupling at the level of the Pion
- **Developed all components for the dynamical decay process in FRG and BSEs**
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Thank you for your attention.