Gauge invariance and the composite nature of asymptotic states in weak physics

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Quark Confinement & Hadron Spectrum
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June 02, 2026

Brout-Englert-Higgs (BEH) effect

- Weak sector of the Standard Model

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - \lambda \left(\phi^\dagger \phi - f^2\right)^2$$

  - Local gauge symmetry $\mathcal{G} = \mathrm{SU}(2)$ & global custodial symmetry $\mathcal{C} = \mathrm{SU}(2)$

- Perturbative approach to Higgs physics

  1. $V(\phi)$ allows for a non-trivial degenerate minimum at $\phi \neq 0 \implies$ BEH effect active
  2. Shift $\phi(x) = vN + \varphi(x)$ with $\langle \phi \rangle = vN \implies m_W$ for gauge bosons & Goldstone fields
  3. Quantize theory using the path integral formalism $\implies$ 't Hooft gauge & ghost fields
  4. BRST construction $\implies$ ghosts/Goldstones cancel & gauge bosons/Higgs are singlets

Böhm, Denner, Joos, *Gauge theories of the strong and electroweak interaction* (Teubner, Stuttgart, 2001)
Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

What's the problem?

- Elitzur's theorem
  - A gauge symmetry can never be spontaneously broken
  - Without gauge fixing, all expressions with open gauge indices have zero expectation value, especially $\langle \phi \rangle = 0$

Elitzur, Phys. Rev. D 12 (1975)

- Gribov-Singer ambiguity
  - Gauge condition not uniquely satisfied beyond perturbation theory
  - Explicitly breaks the perturbative BRST construction

Gribov, Nucl. Phys. B 139 (1978)
Singer, Commun. Math. Phys. 60 (1978)
Fujikawa, Nucl. Phys. B 223 (1983)

- Gauge-dependent construction
  - The vev is a gauge-dependent quantity and there are gauges in which $\langle \phi \rangle = 0$
  - Also related quantities, e.g. m_W, depend on the gauge choice in perturbation theory

Lee, Zinn-Justin, Phys. Rev. D 5 (1972)

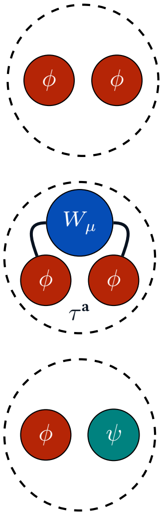
Gauge-invariant approach

- Asymptotic states must be described by **gauge-invariant objects**, i.e., composite operators
- One needs to construct gauge-invariant objects with the same global quantum numbers as the elementary fields
  - Gauge-invariant scalar: $\phi(x) \to \left(\phi^\dagger \phi\right)(x)$
  - Gauge-invariant vector boson: $W_\mu^a(x) \to \left(\tau^{\mathbf{a}} \phi^\dagger D_\mu \phi\right)(x)$
  - Gauge-invariant left-handed fermion: $\psi(x) \to \left(\phi^\dagger \psi\right)(x)$

Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)
Maas, Prog. Part. Nucl. Phys. 106 (2019), 1712.04721

Compare to QCD

- Asymptotic states are color singlets (protons, neutrons, ...) and elementary states (quarks, gluons) are unphysical
- We require ”color” **confinement** in weak physics



Fröhlich-Morchio-Strocchi (FMS) mechanism

- **Weak SM:** FMS mechanism predicts a **one-to-one mapping** between gauge-dependent and gauge-invariant states and explains why perturbation theory is successful
- **Mechanism:** Representations of the gauge group $\mathcal{G}$ are mapped onto representations of the global custodial group $\mathcal{C}$
- Masses of bound states are non-perturbative quantities, but as long as the BEH effect is active, analytical calculations are possible $\rightarrow$ **Augmented perturbation theory (APT)**
  - Consider the scalar bound state $\mathcal{O}_{0^+}(x) = \phi^\dagger(x)\phi(x)$
  - Rewrite the propagator using $\phi(x) = vN + \varphi(x)$ and $h(x) = \text{Re}\{N^\dagger\varphi(x)\}$
  - Perform a perturbative tree-level expansion

$$\langle \mathcal{O}_{0^+}(x)\mathcal{O}^\dagger_{0^+}(y)\rangle = 4v^2\langle h(x)h(y)\rangle_{\text{tl}} + \langle h(x)h(y)\rangle^2_{\text{tl}} + \mathcal{O}(g^2, \ldots)$$

  - Mass pole of bound state coincides with the mass pole of the elementary correlator

Fröhlich, Morchio, Strocchi, Phys. Lett. B97 (1980)
Fröhlich, Morchio, Strocchi, Nucl. Phys. B190 (1981)
Dudal *et al.*, Eur. Phys. J. C 81 (2020), 2008.07813

Deviations at higher orders & internal structure

- ▷ Differences between perturbation theory and a gauge-invariant approach have been identified for vector boson scattering (VBS)

  Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756

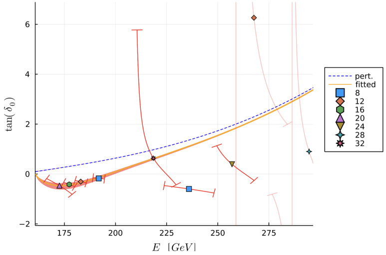
- ▷ Form factor of the leading tensor structure for WWZ in the space-like domain

  Maas, Raubitzek, Törek, Phys. Rev. D 99 (2019), 1811.03395

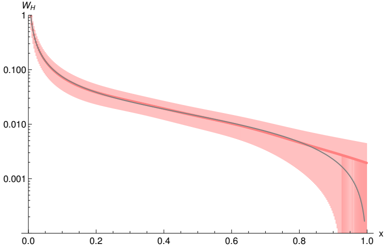
- ▷ Substructure of composite objects: Quasi-PDFs obtained from lattice calculations for the SU(2) gauge-scalar theory

  Martins *et al.*, PoS LATTICE2025 (2026), 2603.12882

Phase shifts for VBS from lattice calculations



W boson PDF in H from lattice calculations



Why should we care?

- Deepen understanding of QFT
  - Learn more about the fundamental field-theoretical effects related to gauge invariance
  - Guaranteed discovery: What if the FMS mechanism is not the answer?
- Implications for future experiments
  - Set baseline for future measurements
  - Avoid false positive regarding new physics
- Phenomenological implications
  - Weak sector of the SM special, since $\mathcal{G} = \mathcal{C} = \mathrm{SU}(2)$
  - Model building should focus on global custodial group $\mathcal{C}$
  - Explicit example: $\mathrm{SU}(n)$ for $n > 2$ with one fundamental scalar

    Maas, Sondenheimer, Törek, Ann. Phys. 402 (2019), 1709.07477
    Maas, Törek, Phys. Rev. D 95 (2017), 1607.05860
    Maas, Törek, Ann. Phys. 397 (2018), 1804.04453

SU(2) scalar-fermion-gauge system

Two generations of leptons

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} + \frac{1}{2} \text{tr}\left[\left(D_\mu X\right)^\dagger \left(D^\mu X\right) \right] - \lambda \left(\frac{1}{2} \text{tr}\left[X^\dagger X\right] - f^2 \right)^2$$
$$+ \sum_g \bar{\psi}^L_g i \not{D} \psi^L_g + \sum_f \bar{\chi}^R_f i \not{\partial} \chi^R_f - \sum_{\bar{f}f} Y_{\bar{f}f} \left[\left(\bar{\psi}^L X\right)_{\bar{f}} \chi^R_f + \bar{\chi}^R_f \left(X^\dagger \psi^L\right)_{\bar{f}} \right]$$

- Introduce matrix-valued field X to replace the usual vector representation

$$X := X_i^{\mathbf{i}} = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}$$

i … gauge index
$\mathbf{i}$ … custodial index

- **Example:** Mapping gauge indices to custodial indices for the spin-1/2 doublet

$$\Psi^L_{\alpha,\mathbf{i}}(x) = \left(X^\dagger\right)_{\mathbf{i}}^{\;i} \psi^L_{\alpha,i}(x) \qquad \Longrightarrow \qquad \begin{pmatrix} N_e^L \\ E^L \end{pmatrix}^\alpha_{\mathbf{i}} = \left(X^\dagger\right)_{\mathbf{i}}^{\;i} \begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix}^\alpha_i$$

Proxy theory & lattice setup

- Chiral nature of the weak gauge theory poses conceptual problems on the lattice
- SM-like proxy that replaces chiral fermions with **vectorial leptons**

$$\mathcal{L}_F = \sum_g \bar{\psi}_g \left(i\not{D} - m_{\psi_g} \right) \psi_g + \sum_f \bar{\chi}_f \left(i\not{\partial} - m_{\chi_f} \right) \chi_f - \sum_{\bar{f}f} Y_{\bar{f}f} \left[\left(\bar{\psi} X \right)_{\bar{f}} \chi_f + \bar{\chi}_f \left(X^\dagger \psi \right)_{\bar{f}} \right]$$

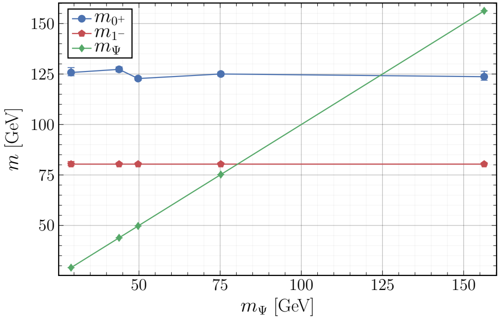
- The terminology *left-* and *right-handed* no longer works since parity is restored
- Parity violation is not an important part of the FMS mechanism $\rightarrow$ "FMS is parity blind"

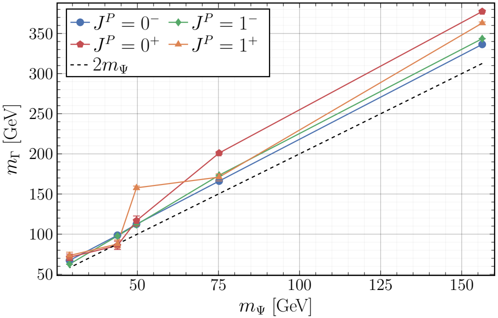
First simulations

- Yukawa couplings $Y_{\bar{f}f} = 0 \longrightarrow$ ungauged fermions χ_f decouple
- Two degenerate generations of gauged fermions, $m_{\psi_1} = m_{\psi_2}$
- Publicly available **HiRep** simulation code

Del Debbio, Patella, Pica, Phys. Rev. D 81 (2010), 0805.2058
Hansen *et al.*, EPJ Web Conf. 175 (2018), 1710.10831

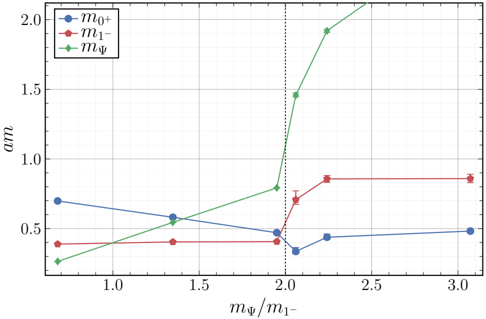
Higgs-like domain

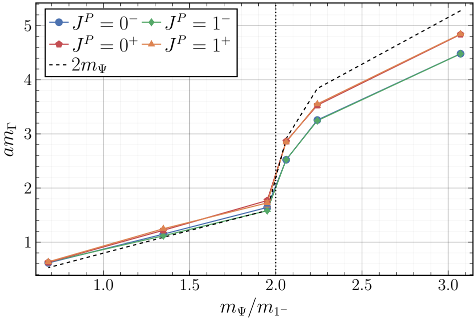




- ▷ The system can be controlled via the fermion mass without changing the overall dynamics of the gauge-scalar subsystem
- ▷ Leptonium channels above the $2m_\Psi$ threshold $\rightarrow$ scattering state of two physical leptons

QCD-like domain

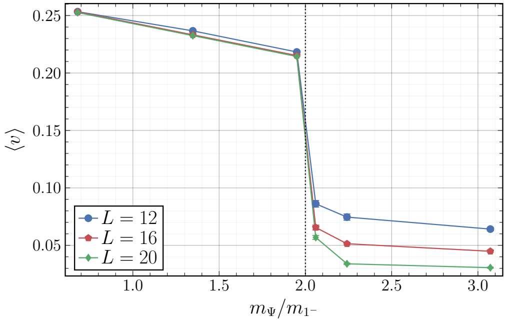




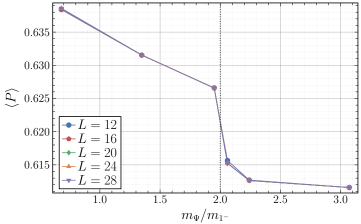
- ▷ Starting from a gauge-scalar theory in the QCD-like domain shows qualitative differences: changes in the mass hierarchy around $m_\Psi = 2m_{1^-}$
- ▷ Transition in all leptonium channels: below vs. above the $2m_\Psi$ threshold

Transition from QCD-like to Higgs-like

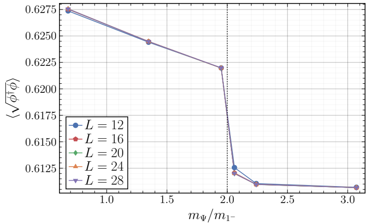
Vacuum expectation value



Plaquette

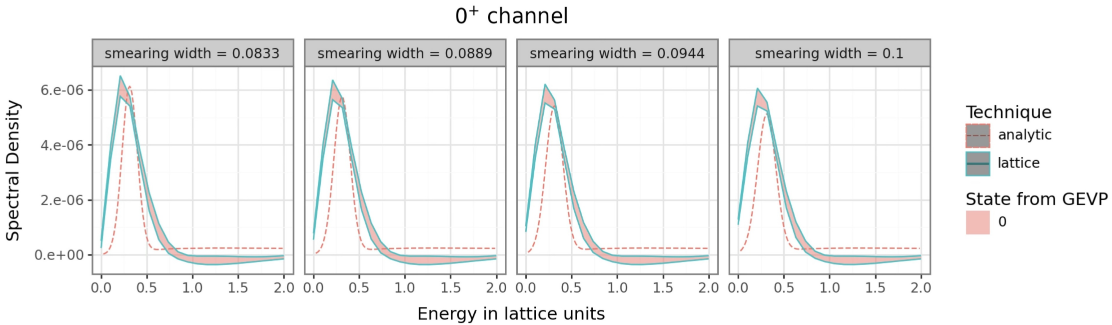


Scalar length



- Volume dependence in the QCD-like domain hints towards $v \to 0$ for $L \to \infty$
- Only small finite size effects in the Higgs-like domain

Spectral densities



- ▷ Higgs spectral densities obtained via the Hansen-Lupo-Tantalo method with $\frac{1}{L} \ll \sigma \ll m_{1^-}$

  Hansen, Lupo, Tantalo, Phys. Rev. D 99 (2019), 1903.06476
- ▷ Compared with an analytic computation smeared with the same smearing parameter

  Maas, Sondenheimer, Phys. Rev. D 102 (2020), 2009.06671

Summary

- Simulation of a proxy theory to the weak sector of the SM in a fully gauge-invariant setup
- The physical spectrum of the theory in the Higgs-like domain supports an interpretation of weak physics based on the FMS mechanism
- First results for spectral densities are compatible with analytic calculations from APT

Outlook

- Perform scattering analysis
  - Scalar singlet: $e^+e^- \to H \to \mu^+\mu^-$
  - Vector triplet: $e^+e^- \to Z \to \mu^+\mu^-$
  - Fermion bound state: excited states of e/μ
- Compare simulations with analytical results from APT

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Thank you for your attention!

Backup slides

Composite operators on the lattice

- Scalar singlet: $\mathcal{O}_{0^+}(x) = \frac{1}{2}\text{tr}\left[X^\dagger(x)X(x)\right]$
- Vector triplet: $\mathcal{O}^{\mathbf{a},\mu}_{1^-}(x) = -\frac{i}{2}\text{tr}\left[\tau^{\mathbf{a}}X^\dagger(x)U^\mu(x)X(x+e_\mu)\right]$

  Evertz *et al.*, Phys. Lett. B 171 (1986)

- Fermion bound state: $\Psi^L_{\alpha,\mathbf{i}}(x) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\,\psi^L_{\alpha,i}(x)$
  - Correlator constructed by Wick contraction and a trace in the Dirac structure

$$M_{\mathbf{ij}}(x|y) = \left(X^\dagger\right)^i_{\mathbf{i}}(x)\ \left(D^{-1}\right)_{ij}(x|y)\ (X)^j_{\mathbf{j}}(y)$$

  Afferrante *et al.*, SciPost Phys. 10 (2021), 2011.02301

- Leptonium operator: $\mathcal{O}_\Gamma(x) = \bar{\psi}_1\Gamma\psi_2$ with 1 and 2 generational indices

Γ	γ_5	$\mathbb{1}$	γ_k	$\gamma_5\gamma_k$
J^P	0^-	0^+	1^-	1^+

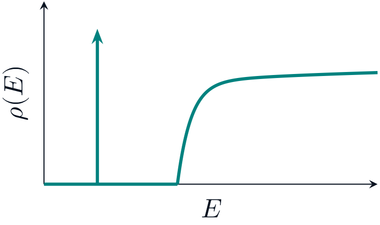
Lattice spectroscopy

1. Construct cross-correlator matrix
   - Time-slice averaging for bosonic correlators: $\mathbf{C}^{\mathsf{B}}_{ij}(t) = \frac{1}{L_t}\sum_{\tau=0}^{L_t-1}\langle \mathcal{O}_i(\tau+t)\mathcal{O}_j^\dagger(\tau)\rangle$
   - Standard correlators for operators including fermion fields: $\mathbf{C}^{\mathsf{F}}_{ij}(t) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0)\rangle$
2. Solve eigenvalue problem: $\lambda^{(k)}(t) \propto e^{-E_k t}\left(1+\mathcal{O}\left(e^{-\Delta E_k t}\right)\right)$
3. Fit resulting correlators via (double-)cosh to obtain E_k and identify lowest energy level E_0
4. Perform an infinite volume fit to find E_0 for $L \to \infty$

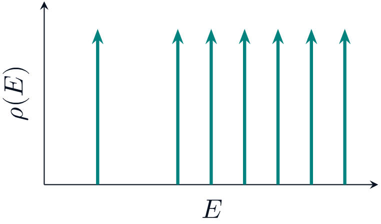
Channel	Scalar singlet 0^+	Vector triplet 1^-	Fermion doublet Ψ	Leptonium Γ
Basis elements	$\mathcal{O}_{0^+}$ $\mathcal{O}_{0^+}\,\mathcal{O}_{0^+}$ $\mathcal{O}_{1^-}^{\mathbf{b},\nu}\,\mathcal{O}_{1^-}^{\mathbf{b},\nu}$	$\mathcal{O}_{1^-}^{\mathbf{a},\mu}$ $\mathcal{O}_{0^+}\,\mathcal{O}_{1^-}^{\mathbf{a},\mu}$ $\mathcal{O}_{1^-}^{\mathbf{b},\nu}\mathcal{O}_{1^-}^{\mathbf{b},\nu}\,\mathcal{O}_{1^-}^{\mathbf{a},\mu}$	$\mathcal{O}_{\Psi}^{\mathbf{i},\alpha}$ $\mathcal{O}_{0^+}\,\mathcal{O}_{\Psi}^{\mathbf{i},\alpha}$ $\mathcal{O}_{1^-}^{\mathbf{b},\nu}\mathcal{O}_{1^-}^{\mathbf{b},\nu}\,\mathcal{O}_{\Psi}^{\mathbf{i},\alpha}$	$\mathcal{O}_{\Gamma}$ $\mathcal{O}_{0^+}\,\mathcal{O}_{\Gamma}$ $\mathcal{O}_{1^-}^{\mathbf{b},\nu}\mathcal{O}_{1^-}^{\mathbf{b},\nu}\,\mathcal{O}_{\Gamma}$

Spectral densities from lattice calculations

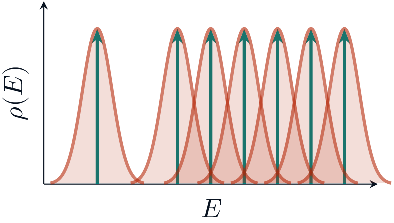
Continuum



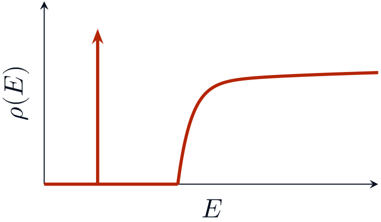
Lattice / Finite Volume



Smearing procedure with width σ



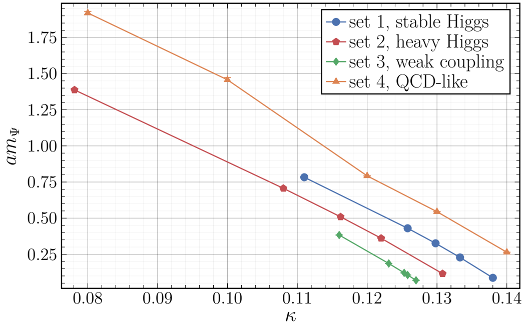
Take limits $L \to \infty$ and $\sigma \to 0$



Backus, Gilbert, Geophys. J. R. Astron. Soc. 16 (1968)
Backus, Gilbert, Phil. Trans. R. Soc. A266 (1970)

Data sets

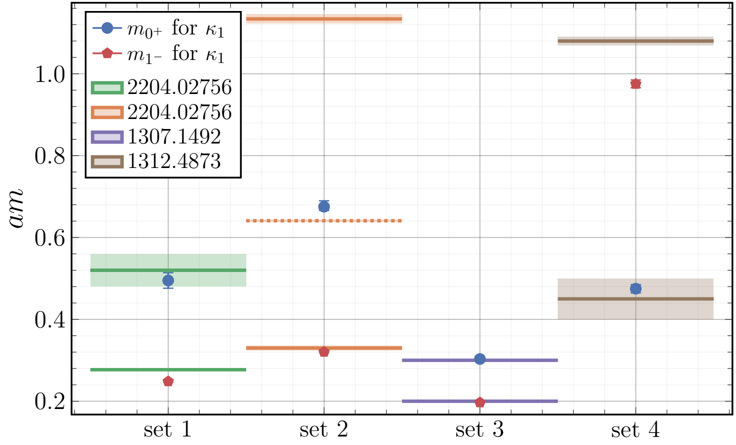
- 4 parameter sets from gauge-scalar simulations
  - **Set 1:** stable Higgs
    Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
  - **Set 2:** heavy Higgs
    Jenny, Maas, Riederer, Phys. Rev. D 105 (2022), 2204.02756
  - **Set 3:** weak coupling
    Wurtz, Lewis, Phys. Rev. D 88 (2013), 1307.1492
  - **Set 4:** QCD-like domain
    Maas, Mufti, JHEP 1404 (2014), 1312.4873



- **Basic idea:** start with a parameter set from gauge-scalar simulations and slowly reduce the fermion mass by increasing the hopping parameter $\kappa = [2(am_0 + 4)]^{-1}$
- Search for points of interest to simulate various scenarios (scattering analysis)

Comparison to gauge-scalar simulations

- Choose lowest κ (largest fermion mass)
- Discrepancies due to different bases and fitting methods



Conversion to physical units

- Set lattice spacing by fixing the mass of the lightest state in the 1^- channel to 80.375 GeV
- Lowest possible error at the sub-percent level