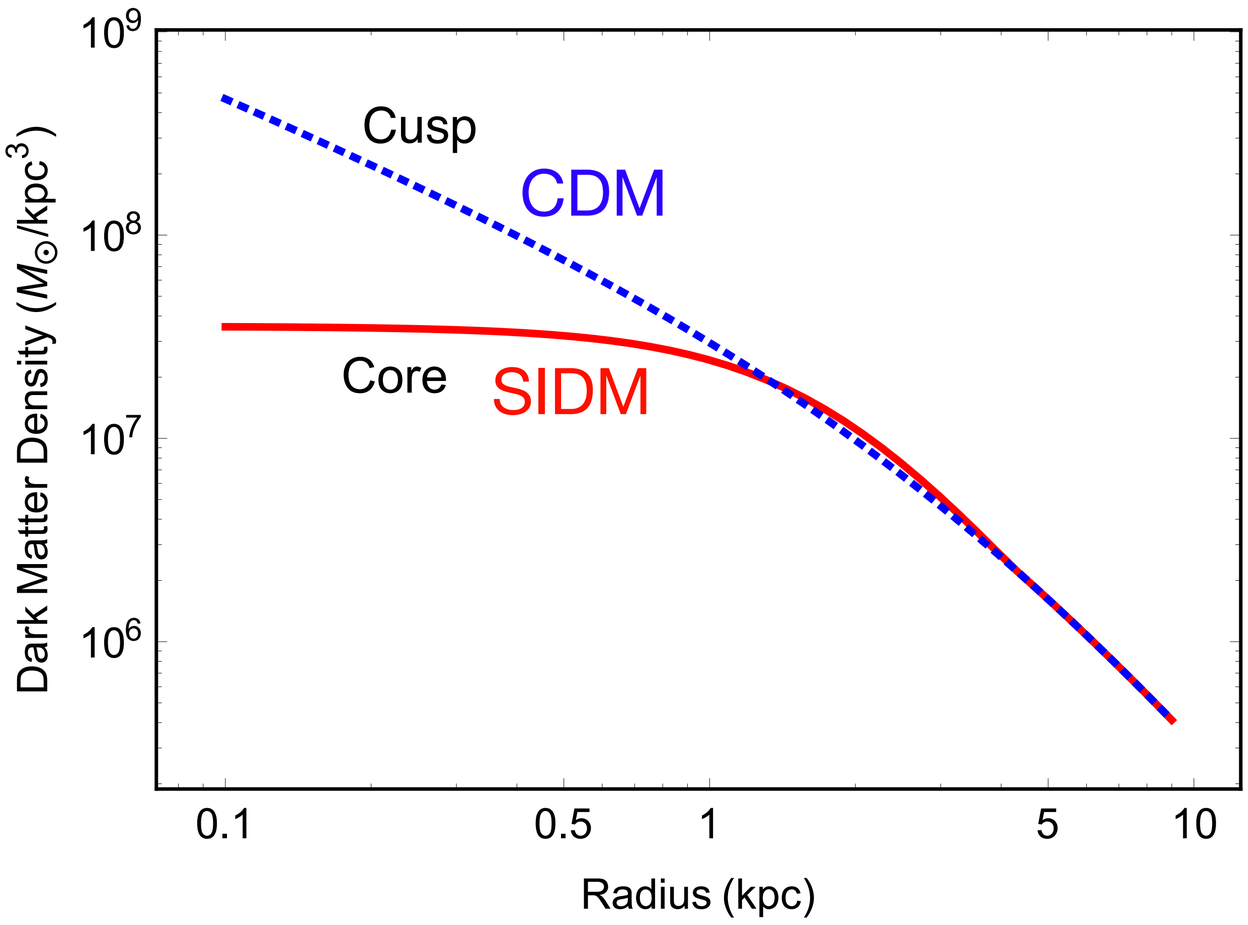
Scattering of symplectic SIMP DM with lattice field theory

Yannick Dengler, 19.3.24

With Axel Maas und Fabian Zierler

Self interaction of Dark Matter

- Observations are in conflict with cold dark matter (CDM) models
  - "cusp vs. core", …
- Possible solution:
  - Self-interacting dark matter (SIDM)



Tulin, Yu: arXiv:1705.02358 (2017)

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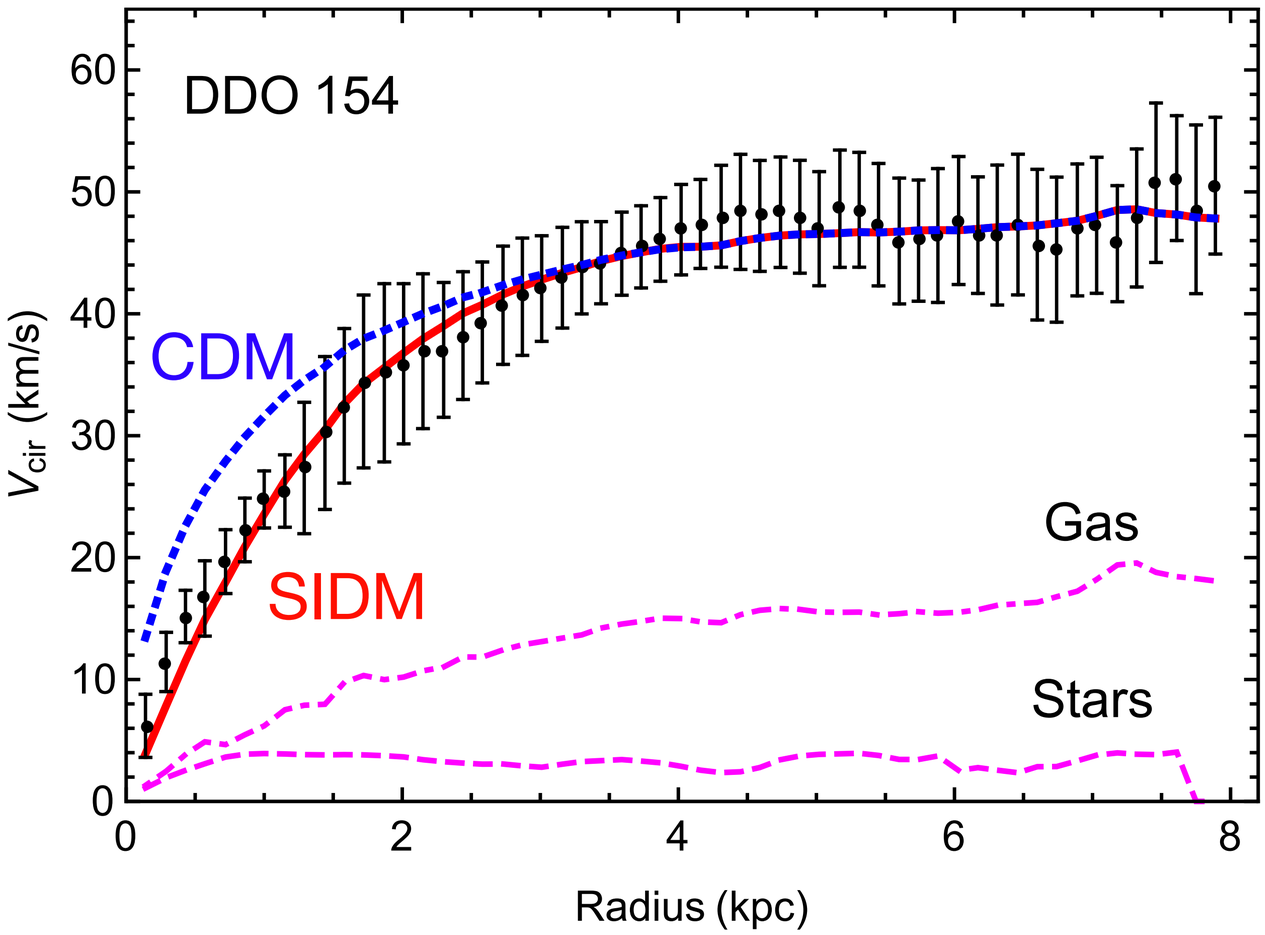

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LSBs - low surface brightness

- Observations are in conflict with cold dark matter (CDM) models
  - "cusp vs. core", …
- Possible solution:
  - Self-interacting dark matter (SIDM)
- "DM halos as particle accelerators"
- Velocity dependence preferred

Dwarf Galaxies

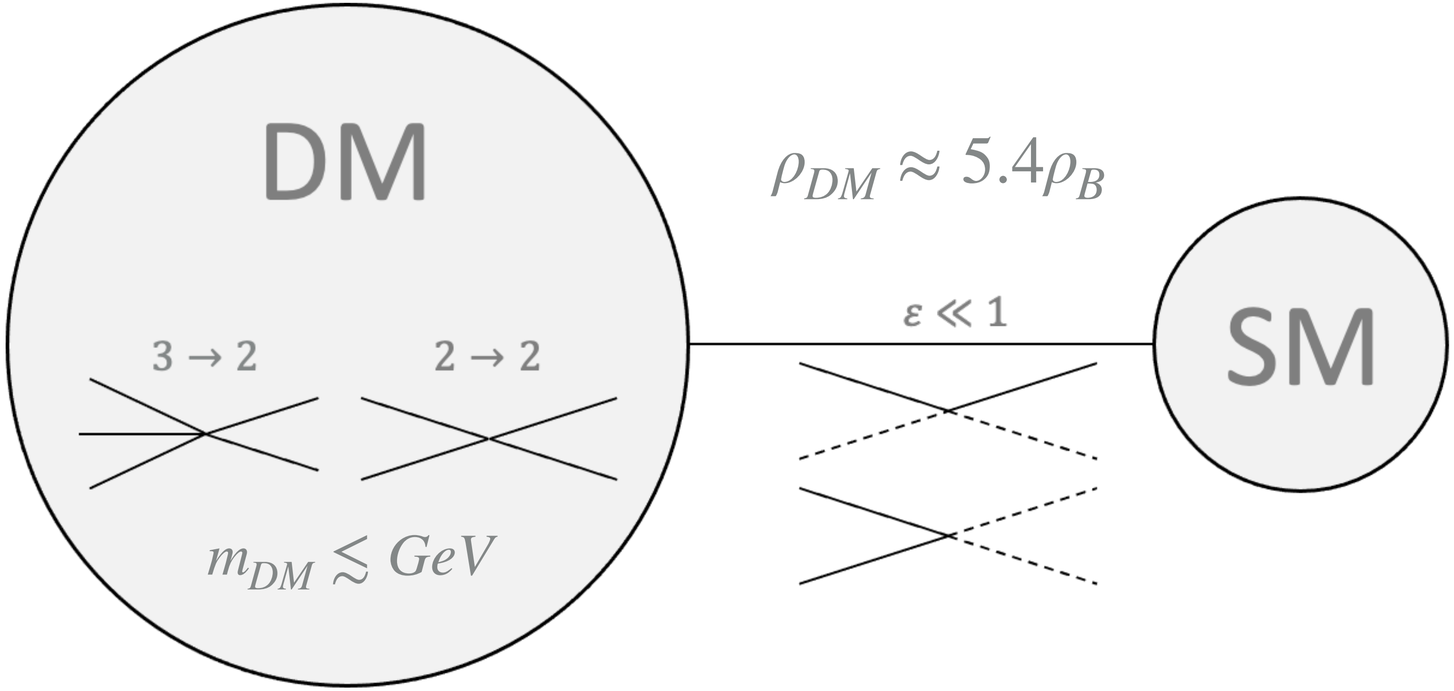
LSBs

Galaxy clusters

Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Strongly-Interacting Massive Particle (SIMP-miracle)

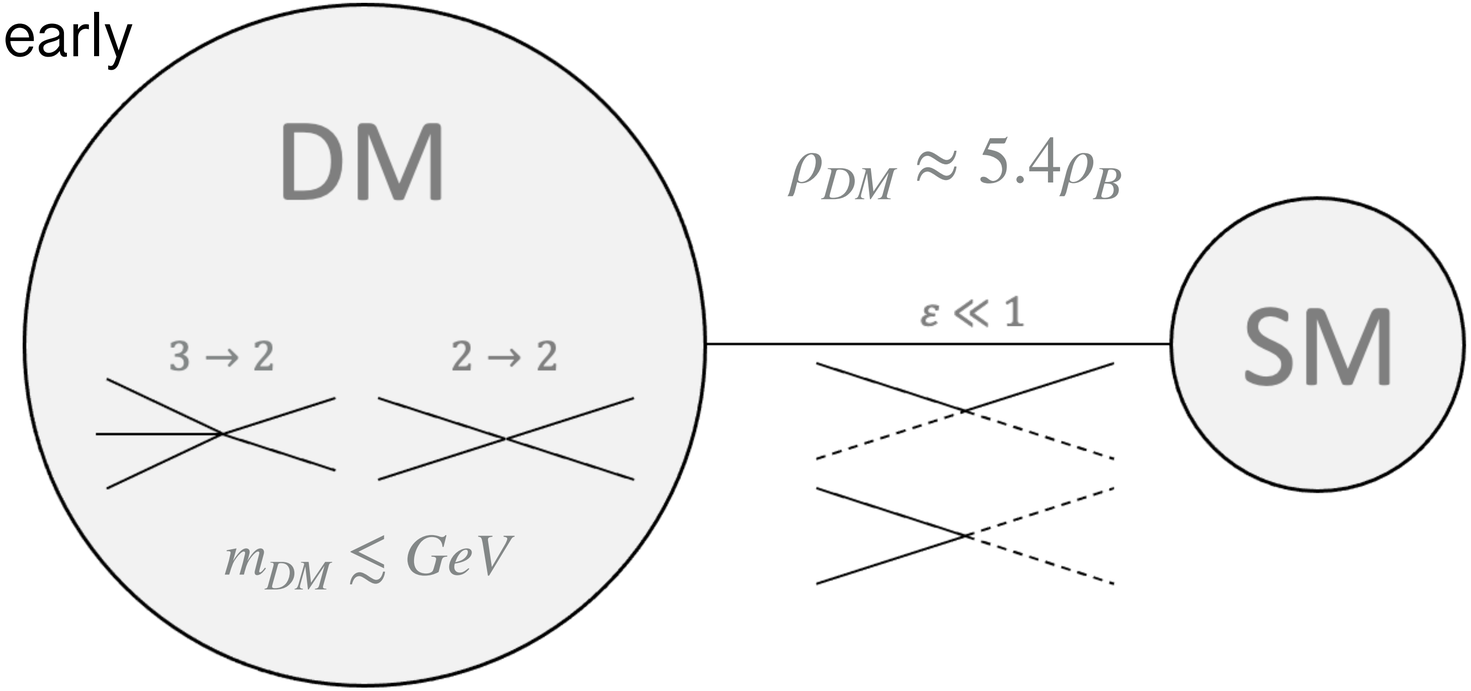
- One possible realization of SIDM
- Dark Matter Candidate:
  - Mesons in confining (strong) dark sector
- DM as a thermal relic of the early universe via freeze-out



Hochberg et al.: Phys. Rev. Lett. 113 (2014)

Strongly-Interacting Massive Particle (SIMP-miracle)

- The mechanism:
- SM & DM in thermal equilibrium in early universe



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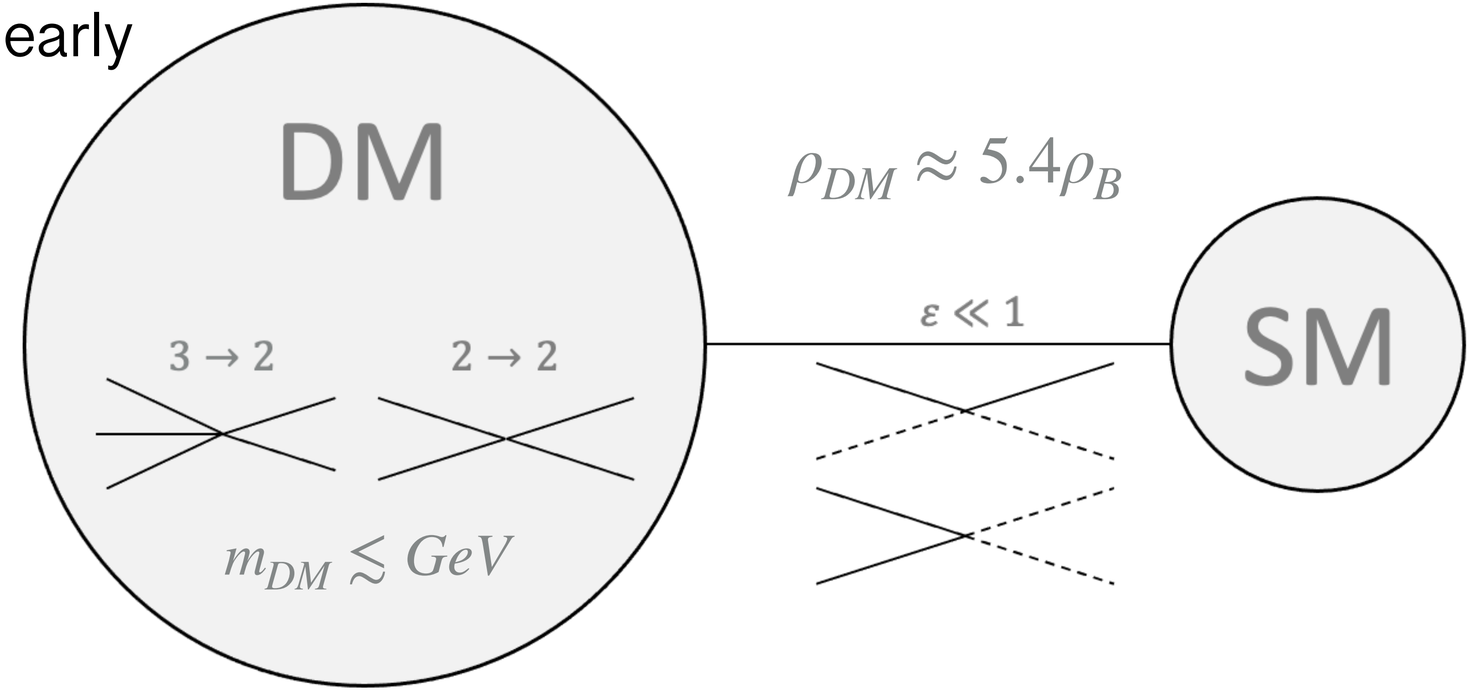
  - Dark sector heats up



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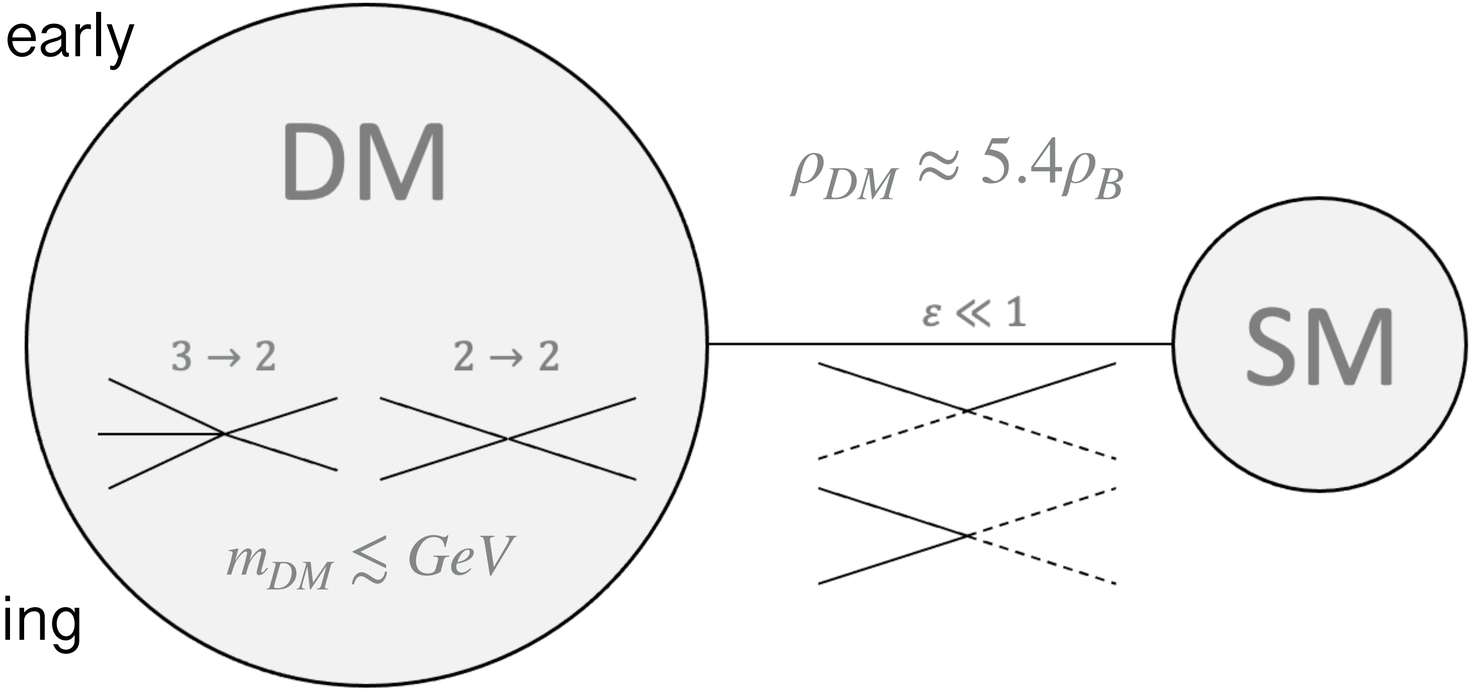
- The mechanism:
- SM & DM in thermal equilibrium in early universe
- Depletion of DM via number-lowering process ($3 \to 2$)
  - Dark sector heats up
- Heat flow from DM to SM via coupling
  - Mediator enables direct detection



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Effective Field Theory

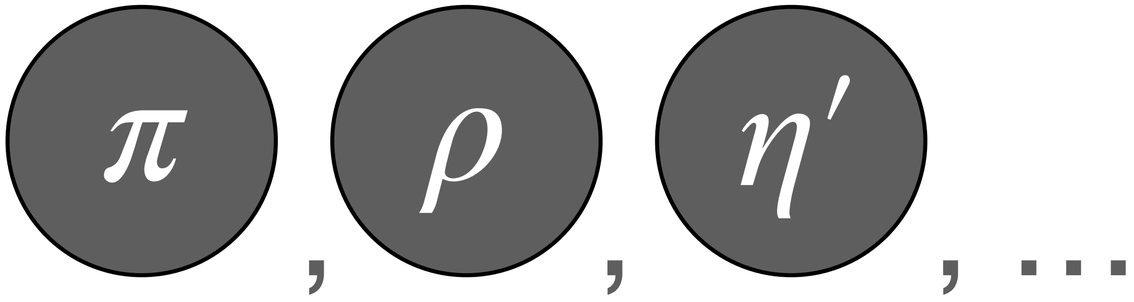
- Low energy description of mesons
- Powerful tool for numerous calculations

π, ρ, η', ...

Effective Field Theory

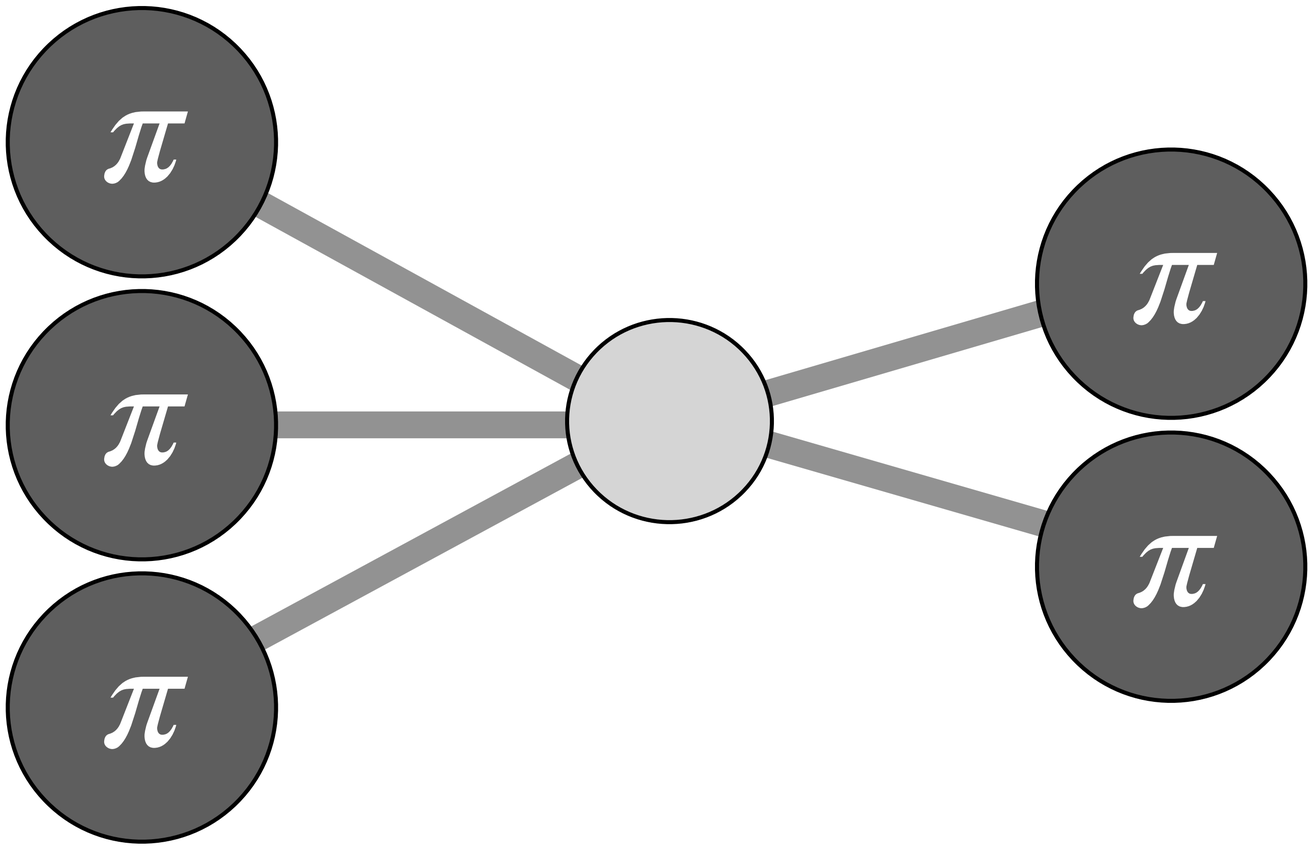
- Low energy description of mesons
- Powerful tool for numerous calculations
- What can we do on the lattice?

  1. Provide low energy constants (masses, decay constants, …)
  2. Test bounds of applicability



Wess Zumino Witten Term

- Nonzero topological term in chiral Lagrangian
  - From axial anomaly
- 5-point interaction
- Can incorporate a $3 \to 2$ process in an effective description



Witten: Nucl. Phys. B, Volume 223 (1978)

The model

- Symplectic gauge group

Kulkarni et al.: SciPost Phys. 14 (2023)

The model

- Symplectic gauge group
- Pseudo-real rep of the gauge group enlarge the flavor symmetry
  - "Weyl Fermions"
  - More pseudo Goldstone-Bosons

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Flavour symmetry breaking pattern for $N_f = 2$

Complex	Pseudo-real
(QCD/SM)	(Our model)
U(2)xU(2)	**U(4)**

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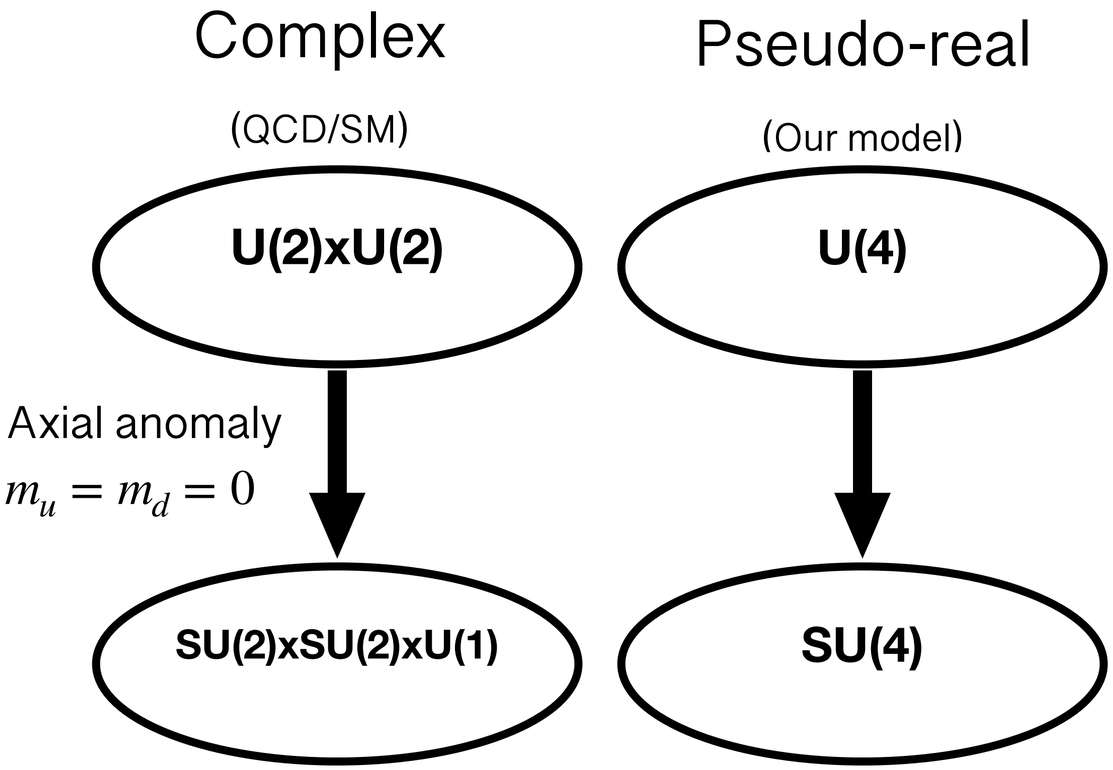

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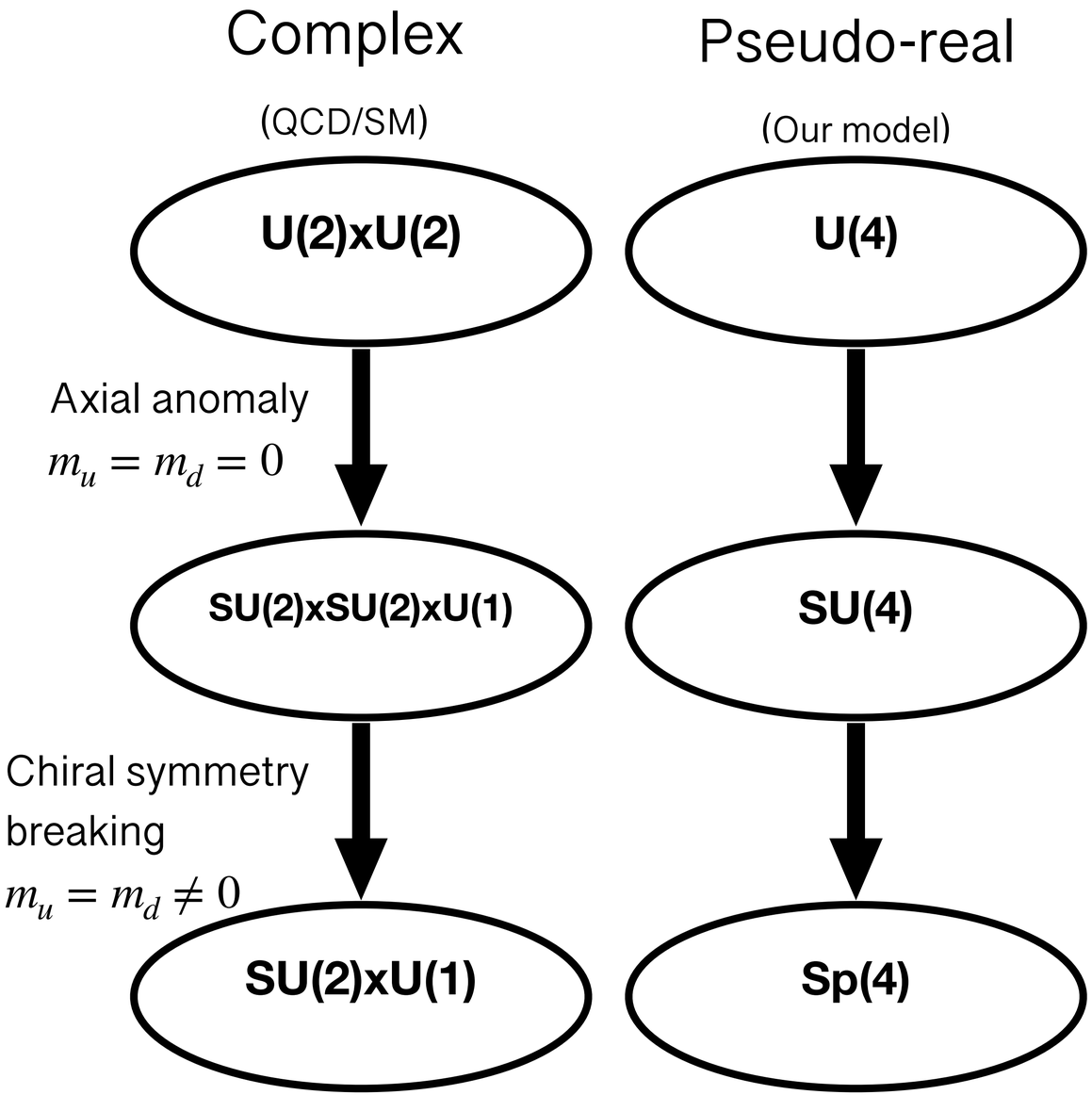

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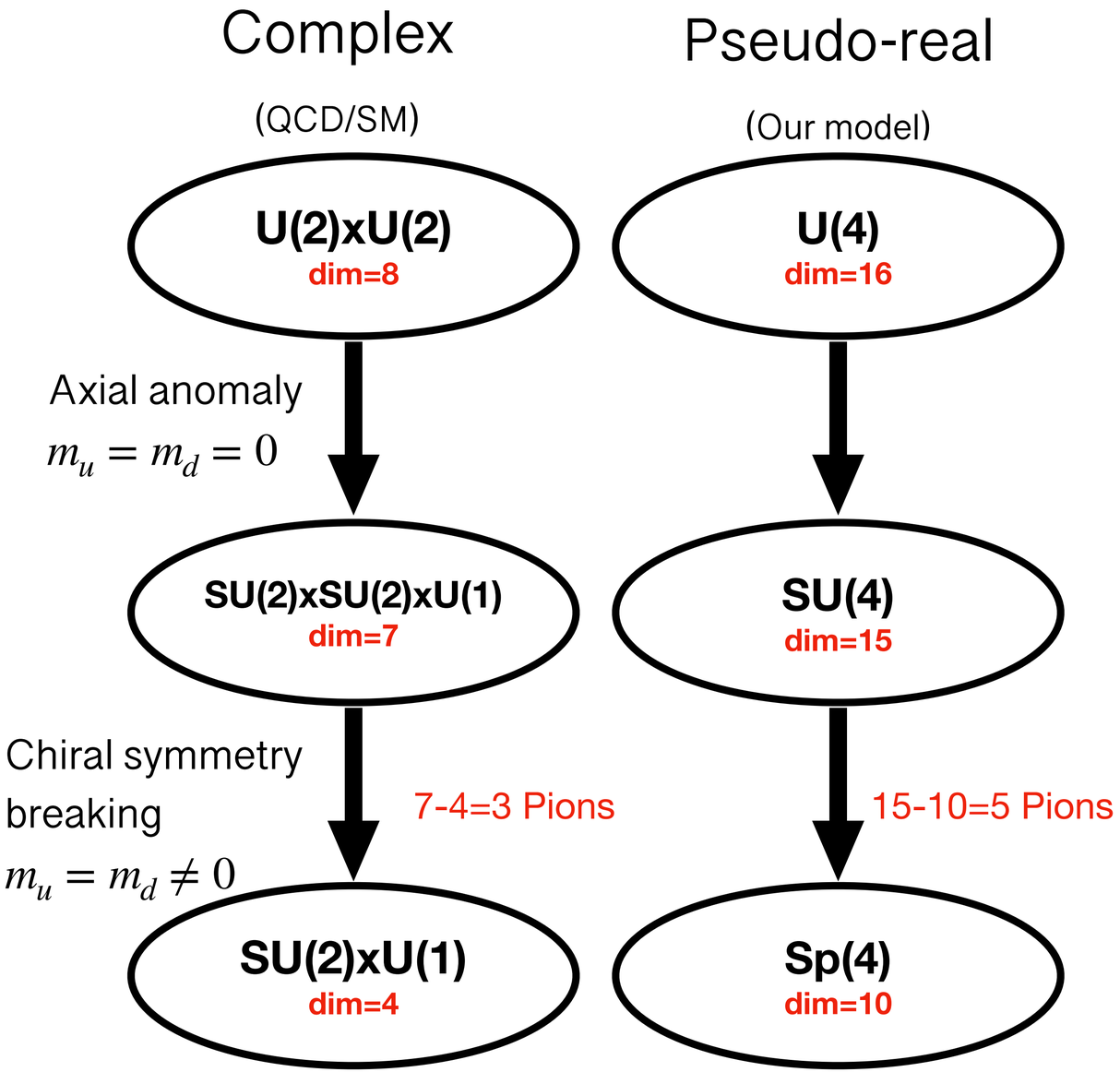
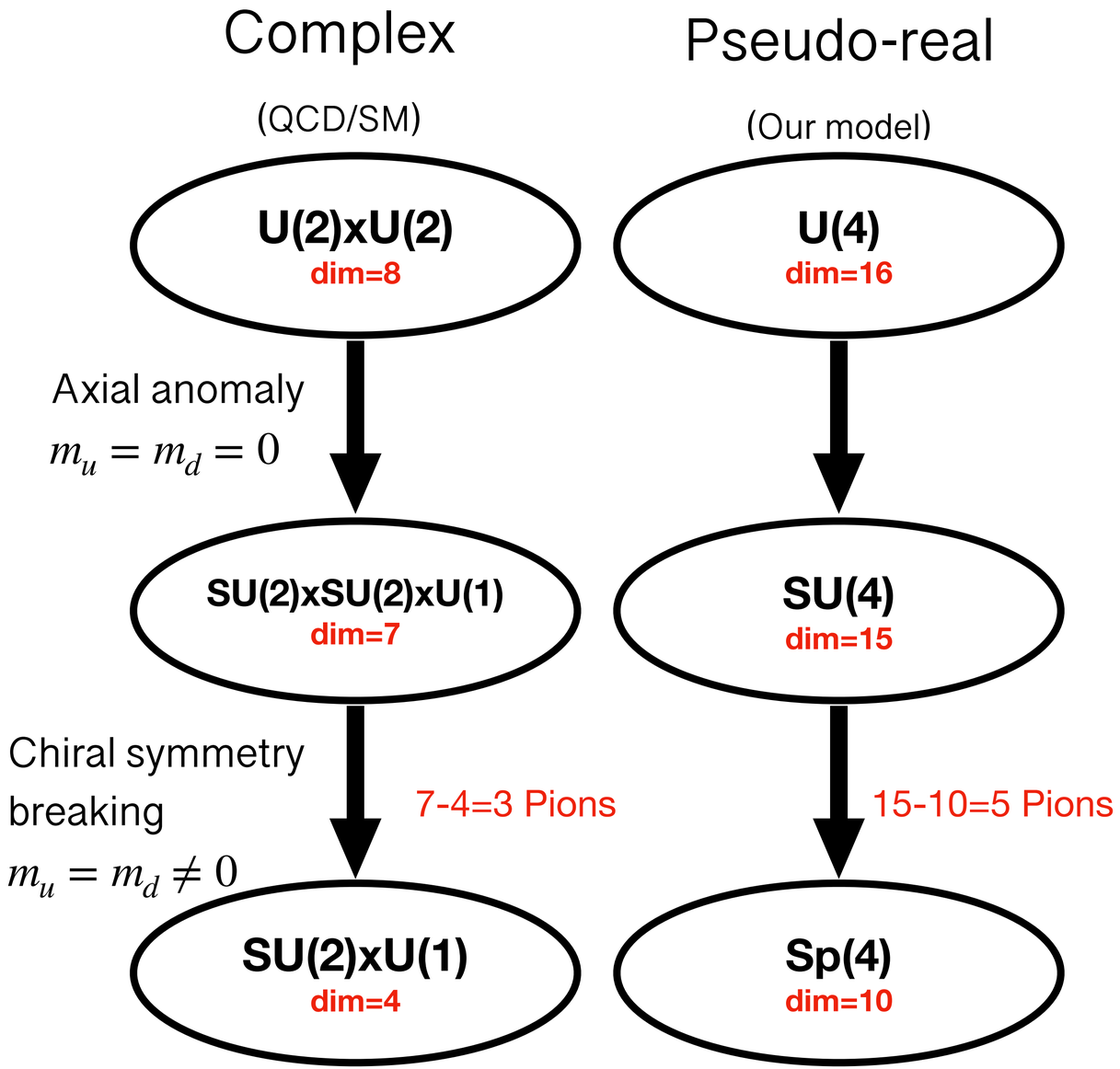


Kulkarni et al.: SciPost Phys. 14 (2023)

The model

- Symplectic gauge group
- Pseudo-real rep of the gauge group enlarge the flavor symmetry
  - "Weyl Fermions"
  - More pseudo Goldstone-Bosons
- $N_f = 2$:
  - 5 Goldstones that are needed for SIMP

Flavour symmetry breaking pattern for $N_f = 2$

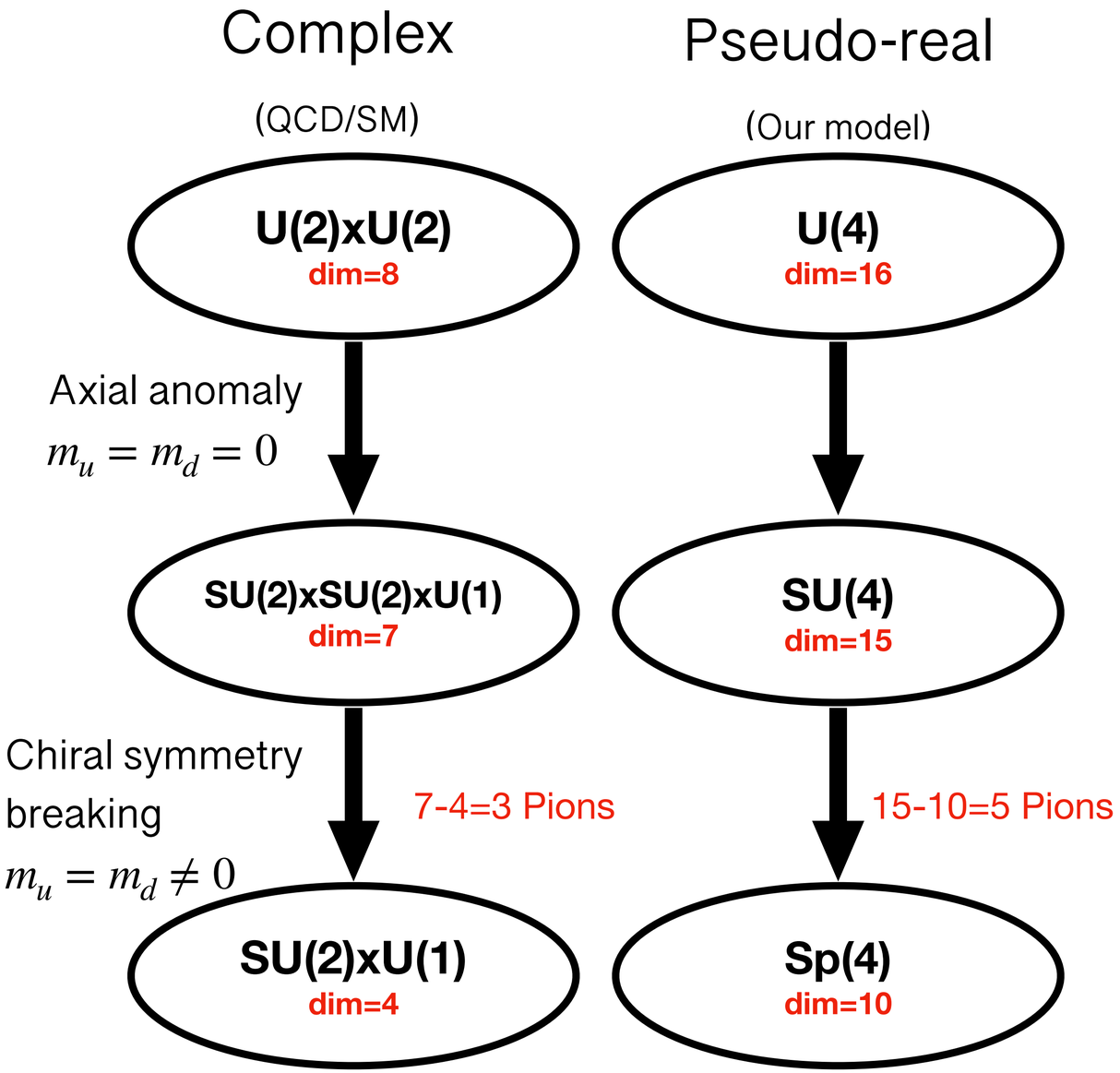


Kulkarni et al.: SciPost Phys. 14 (2023)

Our model - 2 Sp(4)'s

- $\mathrm{Sp}(4)_C$ with $N_f = 2$ fermions is a minimal realization of the SIMP-mechanism!
  - "Induces" a global $\mathrm{Sp}(4)_F$ flavour symmetry
- Rich phenomenology different from $\mathrm{SU}(3)_C$

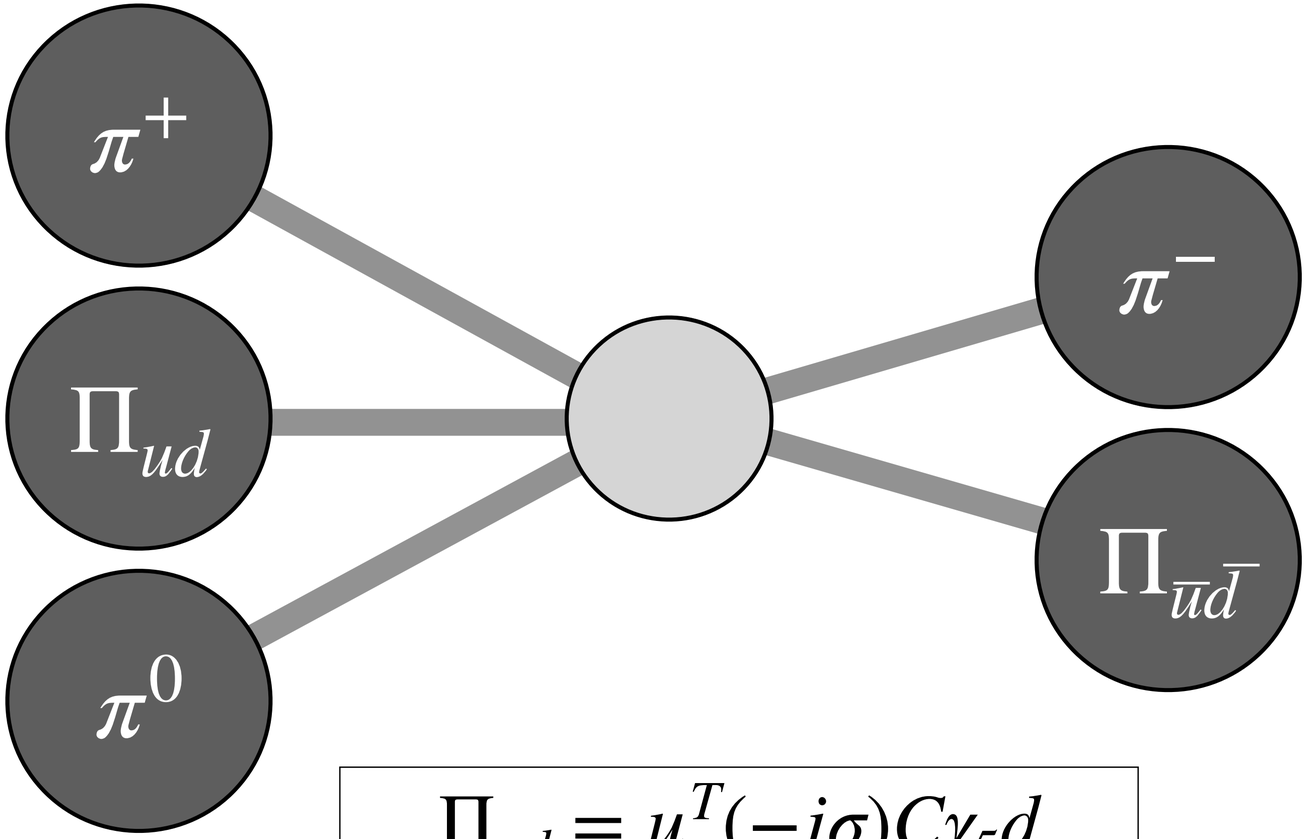
Flavour symmetry breaking pattern for $N_f = 2$



Kulkarni et al.: SciPost Phys. 14 (2023)

Sp(4) gauge with $N_f = 2$

- "Zoo" of dark particles:
  - 5 "dark" Pions
    - 2 gauge invariant di-quarks
  - 10 "dark" Rhos
  - Potentially light η' etc.
- No di-quark vector flavour singlet
- Even number of colours:
  - No fermionic bound states



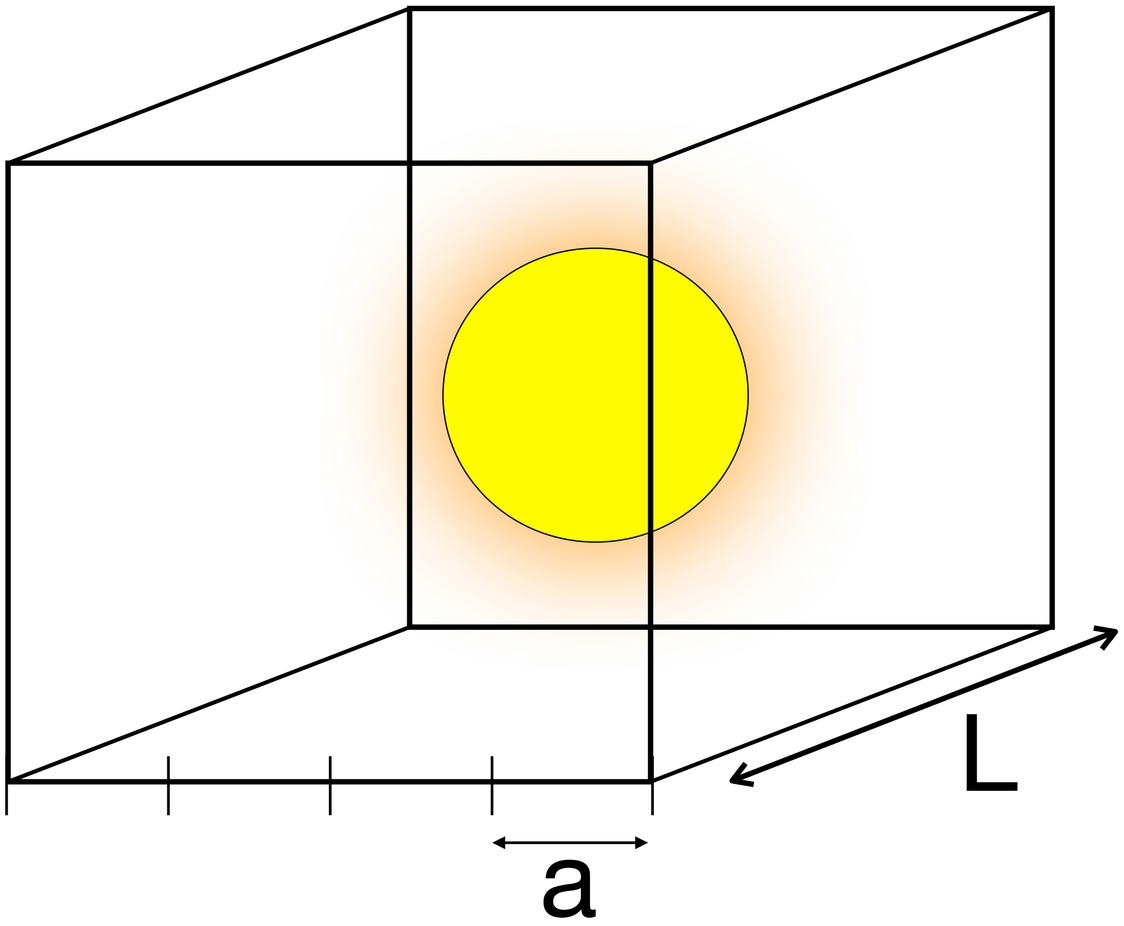
$$\Pi_{ud} = u^T(-i\sigma)C\gamma_5 d$$

$$\Pi_{ud} = \bar{u}(-i\sigma)C\gamma_5 \bar{d}^T$$

Zierler et al. - PhysRevD.109 (2024)
Drach et al. - Eur. Phys. J. C 82 (2022)

Lattice

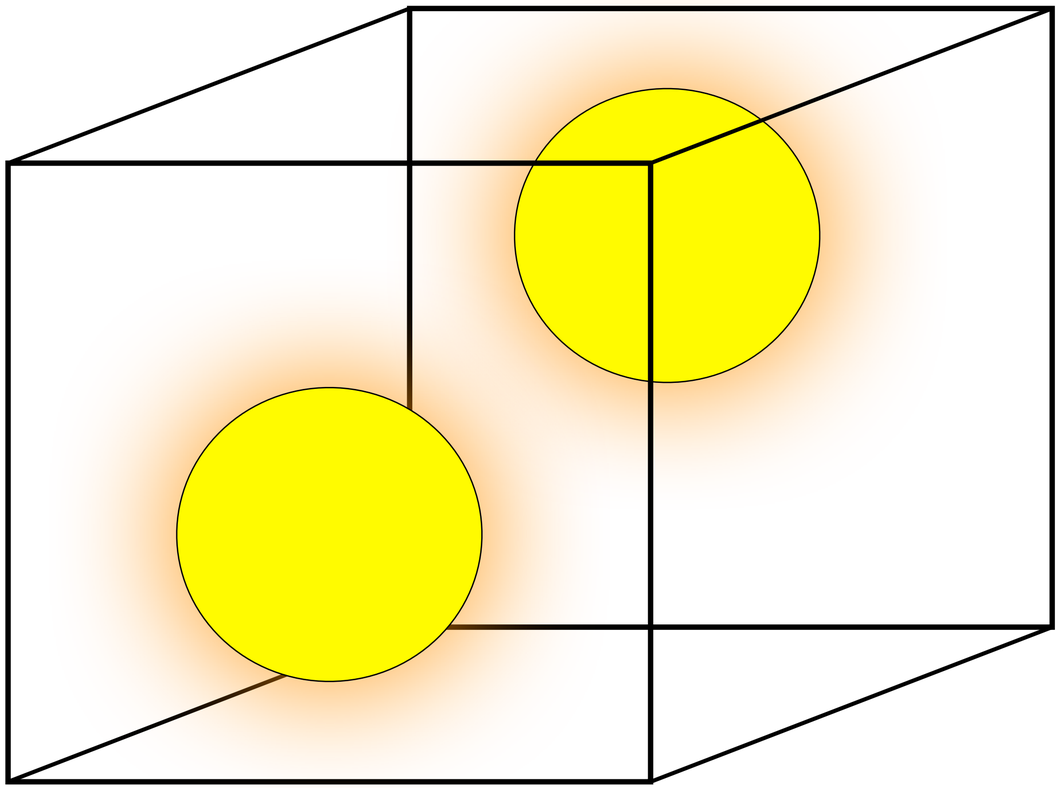
- Sample gauge configurations in a discretized space-time
- Challenges:
  - UV and IR cutoff because of a and L
- First principles
  - Can provide low energy constants for effective theories



Gattringer, Lang (2010)

Lattice - Scattering

- Particles enclosed in a box
  - Energy levels are shifted in finite volume due to scattering effects
- Energy shift ↔ scattering properties
- Full scattering information (phase shift)
- This work: $\pi\pi \to \pi\pi$ scattering in Isospin-2



Briceño et al.: Rev. Mod. Phys. 90 (2018)

Lüscher: Commun. Math. Phys. 104/105 (1986)

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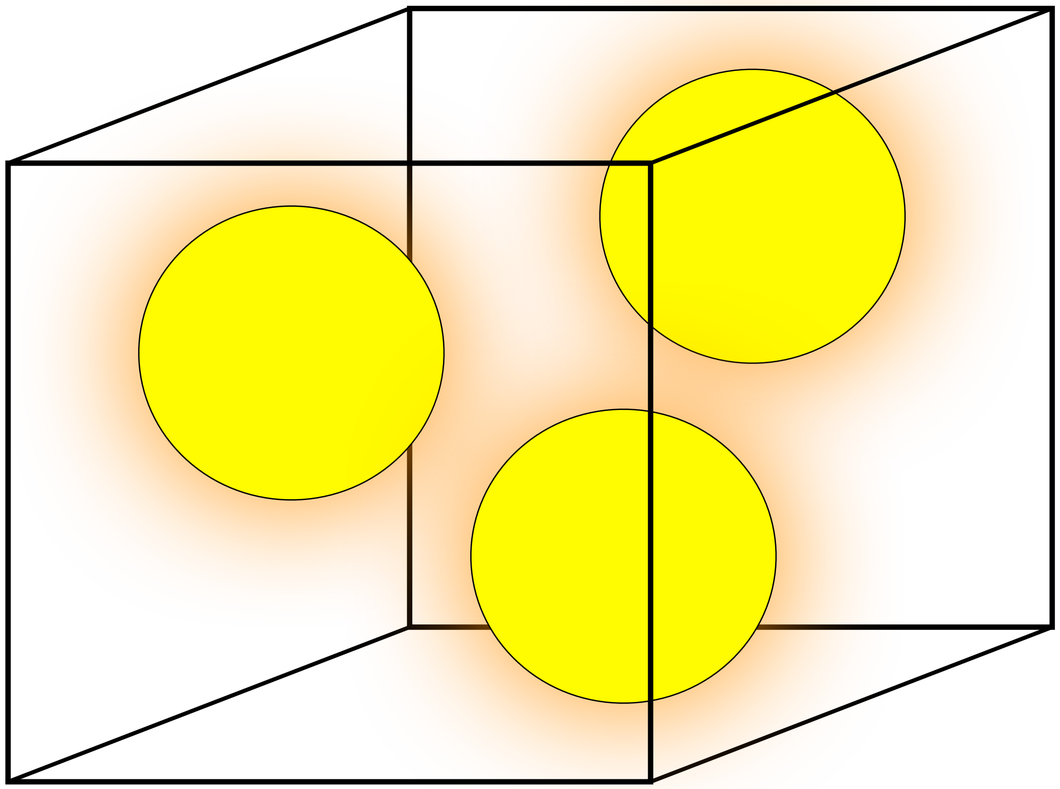
- Full scattering information (phase shift)
- Can be extended to three particles for the $3 \to 2$ process

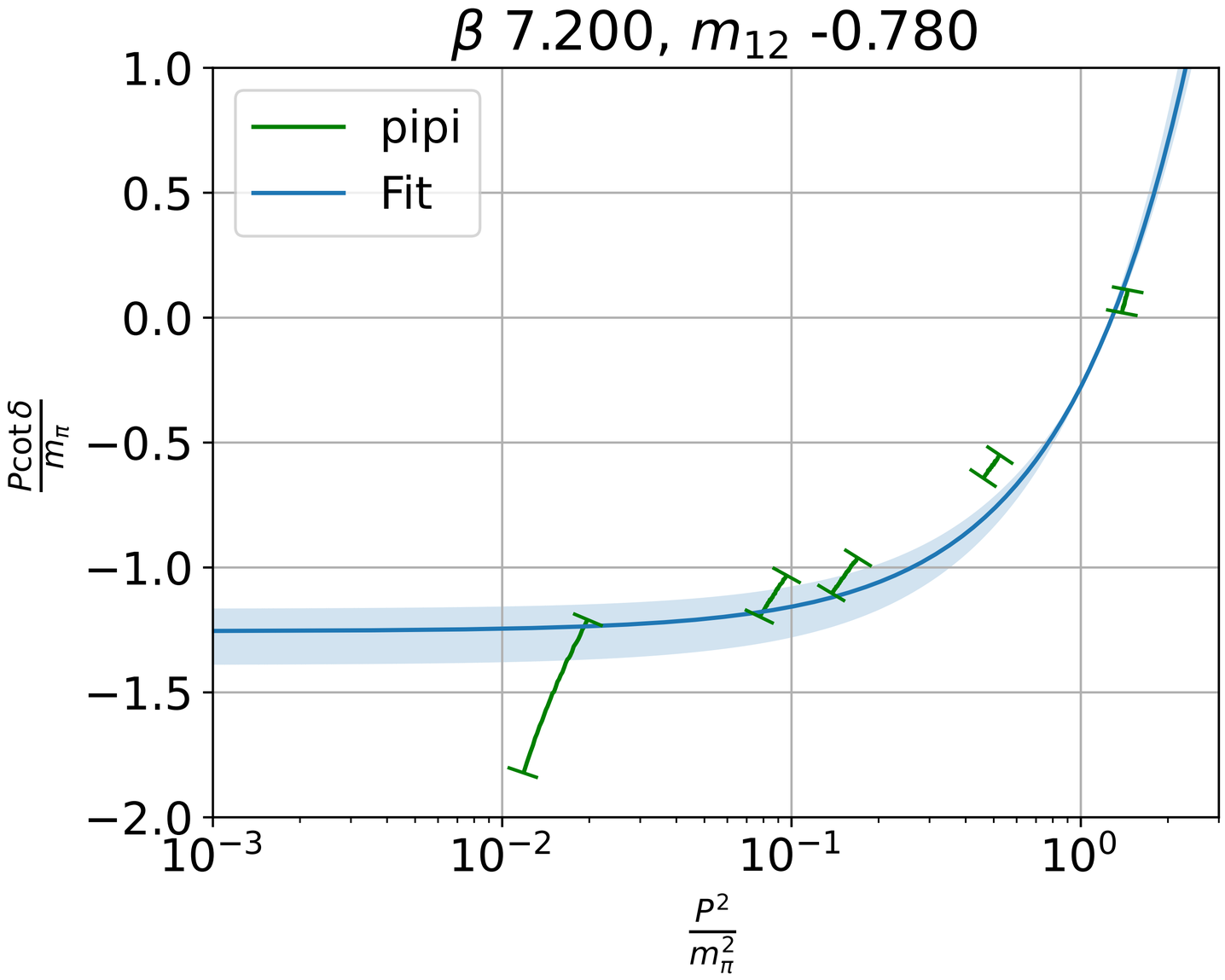


Hansen et al., Phys. Rev. D (2014)

Hansen et al., J. High Energ. Phys. 2021

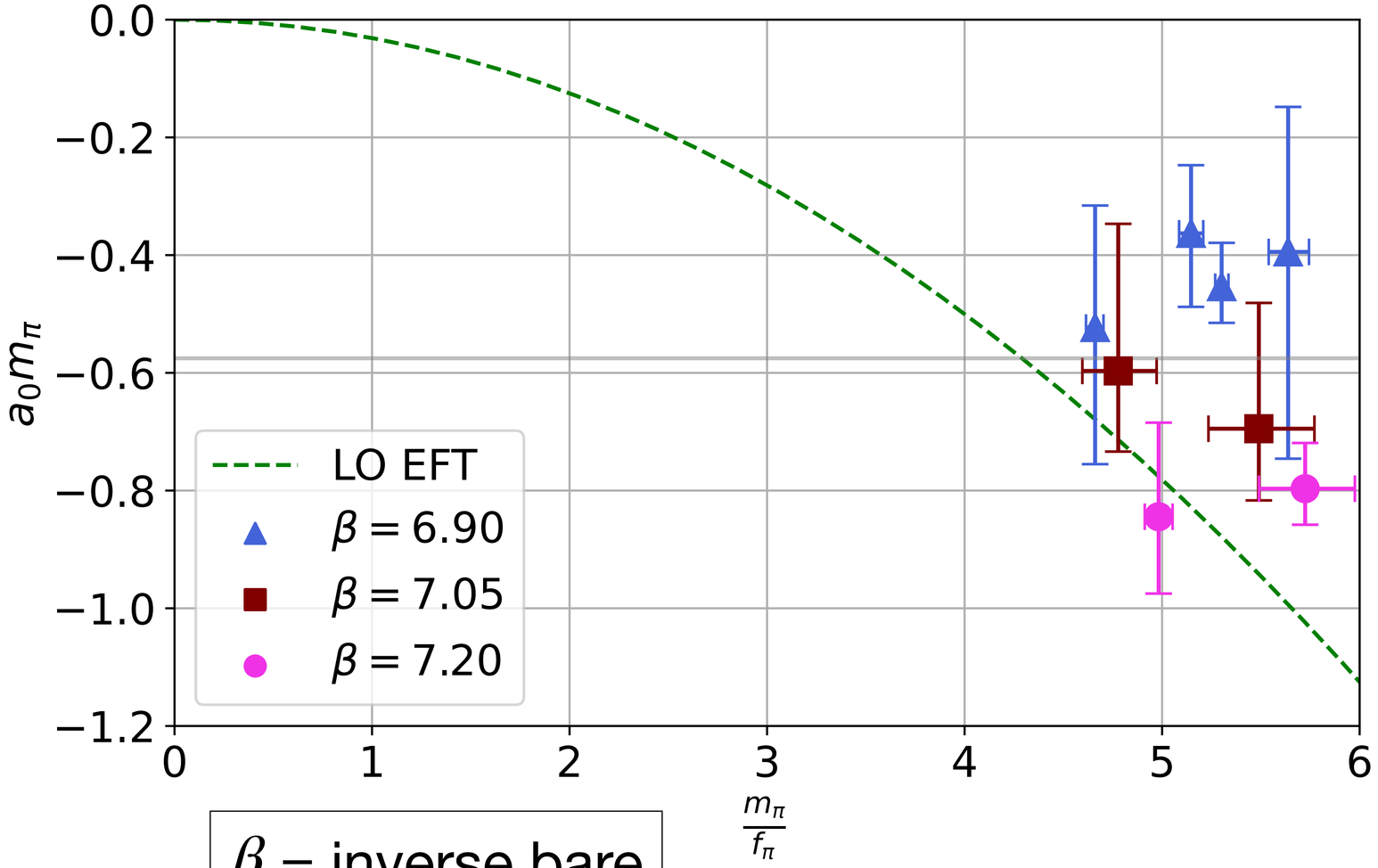
Results

- Main result: Full phase shift $\delta(P)$
  - Scattering length, cross-section, Resonances, etc.



Results - Applicability of χ-pT

- χ-pT prediction for the scattering length:
  - $a_0 m_\pi = -\frac{1}{32}\left(\frac{m_\pi}{f_\pi}\right)^2$
- χ-pT applicable within 1-2σ for $\beta = 7.05, 7.2$

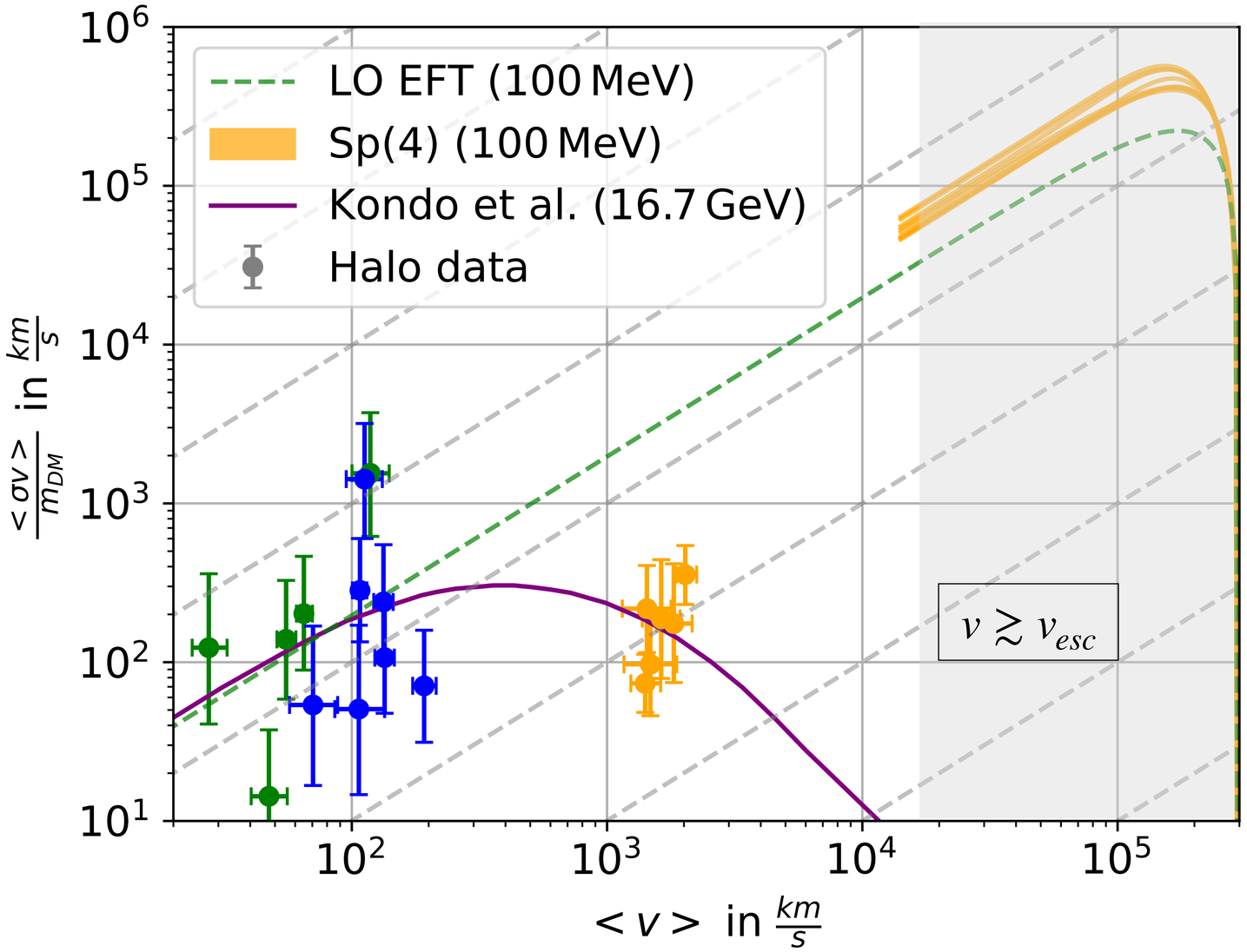


β = inverse bare gauge coupling

Bijens et al.: JHEP (2011)

Velocity dependence σ

- "Velocity weighted cross-section"
- $$\frac{<\sigma v>}{m} = \int_0^\infty v\,\sigma\,f(v)\,dv$$
- No sign for velocity dependence for relevant velocities



Kondo et al: J. High Energ. Phys. (2022)

Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

Summary & Outlook

- Why Sp(4) dark matter?
- Calculation of scattering properties from first principles
- Comparison with different constraints indicate $m_{DM} \gtrsim 100\,MeV$

- Other isospin channels
- Full $\pi\pi\pi \to \pi\pi$ scattering
- Low energy constants for an effective description

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Side Project: Neutron Stars with dark matter!

(With Suchita Kulkarni)

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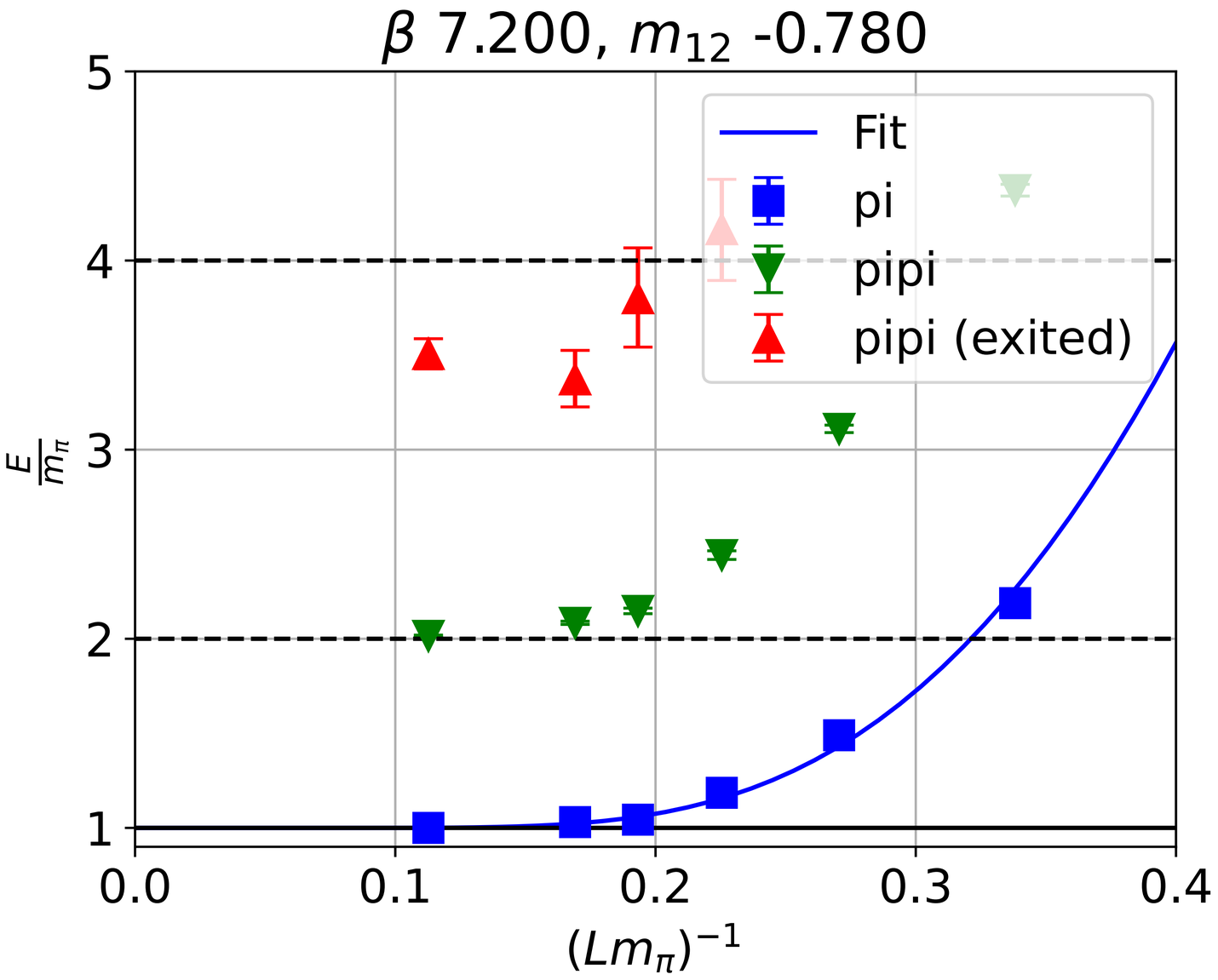
Thank you!

Results

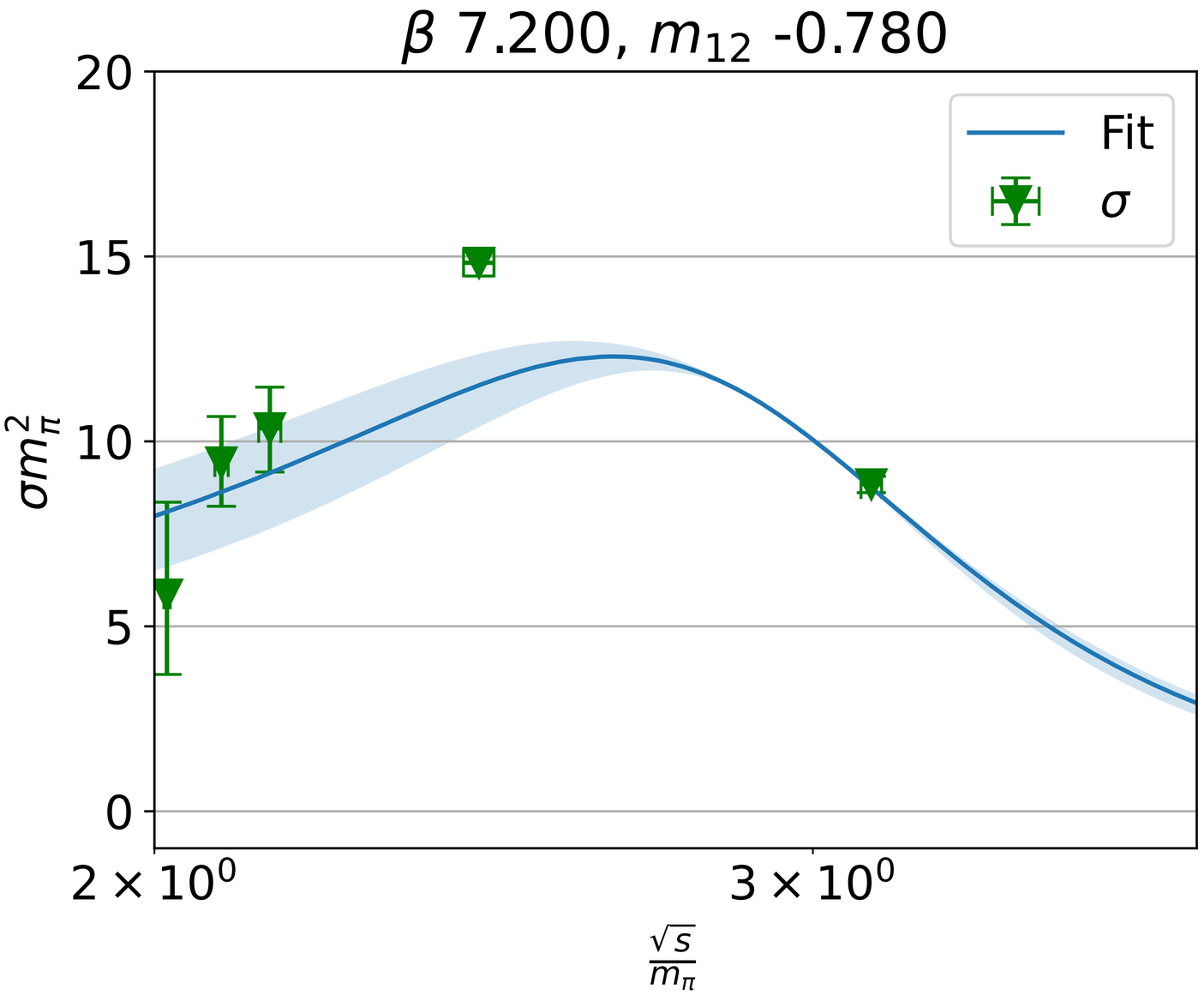
- Energy levels



Bijens et al.: JHEP (2011)

Result - cross-section

- Full cross-section



Bijens et al.: JHEP (03:028, 2011)

Application for dark matter
s-wave scattering cross-section

$$\tan(\delta) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}^{\vec{0}}_{00}(1,q^2)}$$

$$\sigma_s(\delta, k) = \frac{4\pi}{|k\cot(\delta) - ik|^2}$$

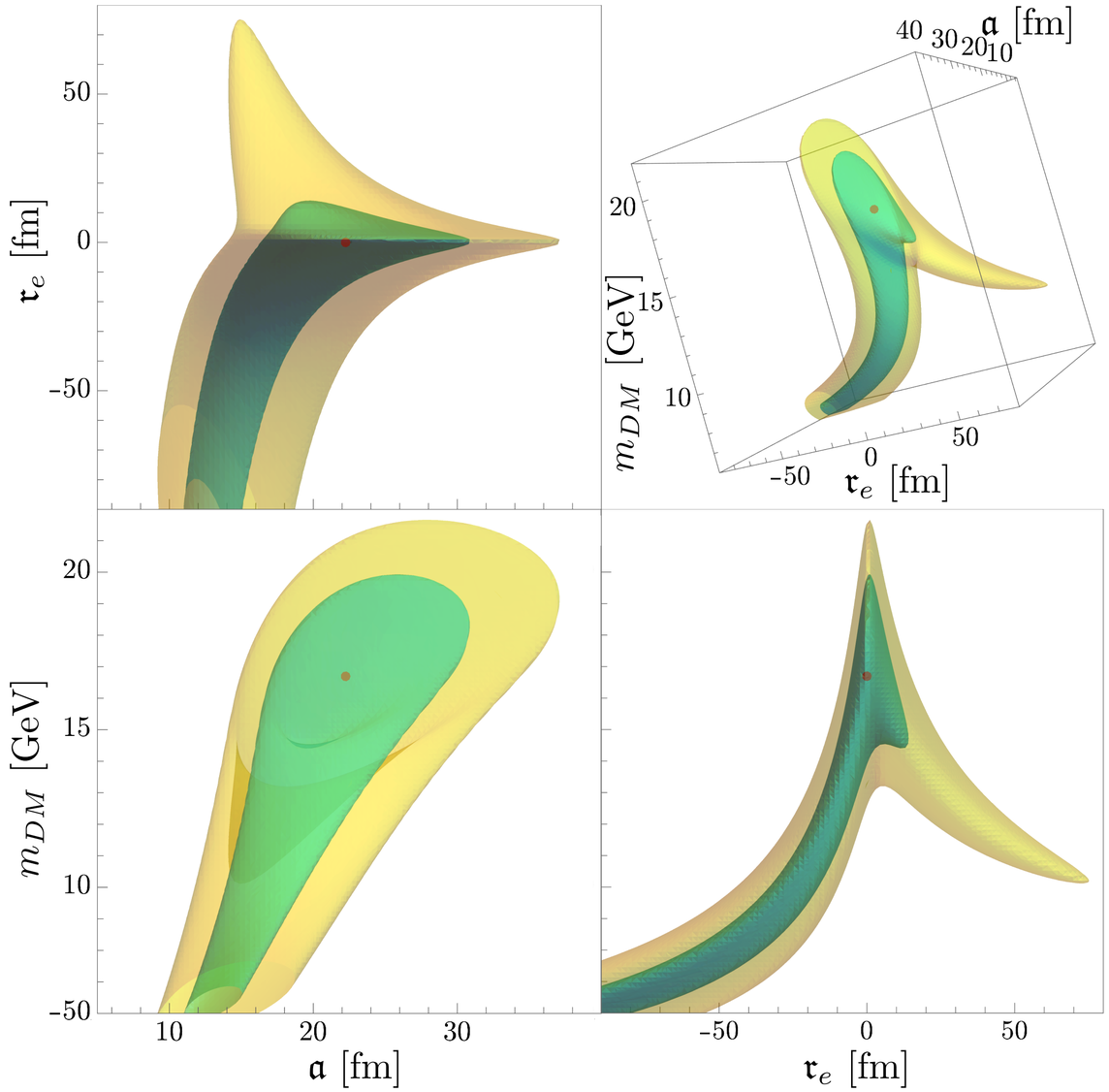
$$\langle v \rangle = \int_0^\infty dv\, v f(v)$$

$$f(v) = \frac{32 v^2}{\pi^2 \langle v \rangle^3} exp\left(-\frac{4v^2}{\pi \langle v \rangle^2}\right)$$

$$\langle \sigma v \rangle = \int_0^\infty dv\, \sigma(v)\, v f(v)$$

Comparison to halo data

- Effective range-expansion (s-wave)
- $k\cot(\delta) = -\frac{1}{a} + \frac{k^2}{2r_e} + \mathcal{O}(k^4)$
- Best fit:
  - $a = 22.2$ fm
  - $r_e = -2.59 \times 10^{-3}$ fm
  - $m_{DM} = 16.7 GeV$



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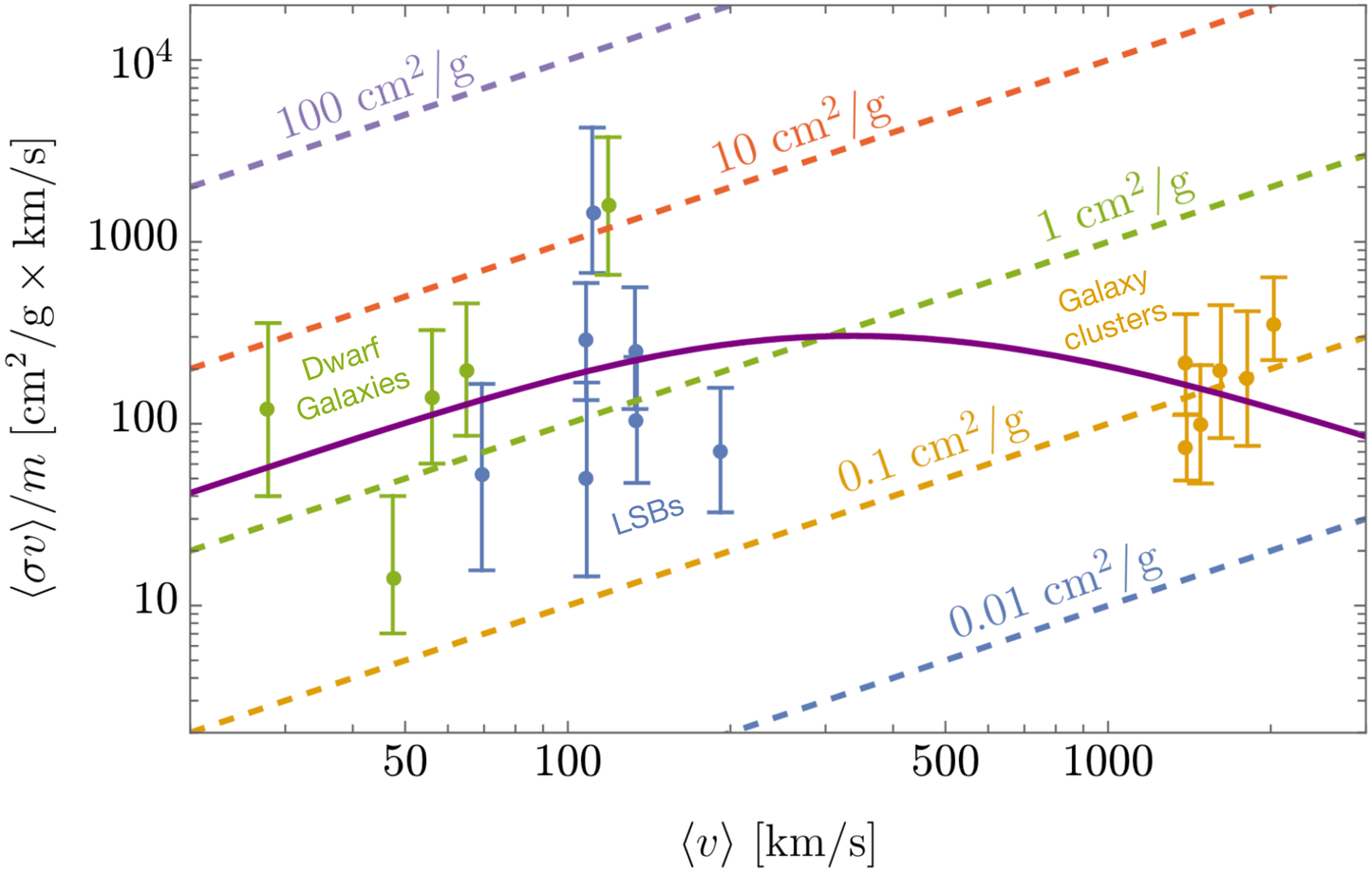

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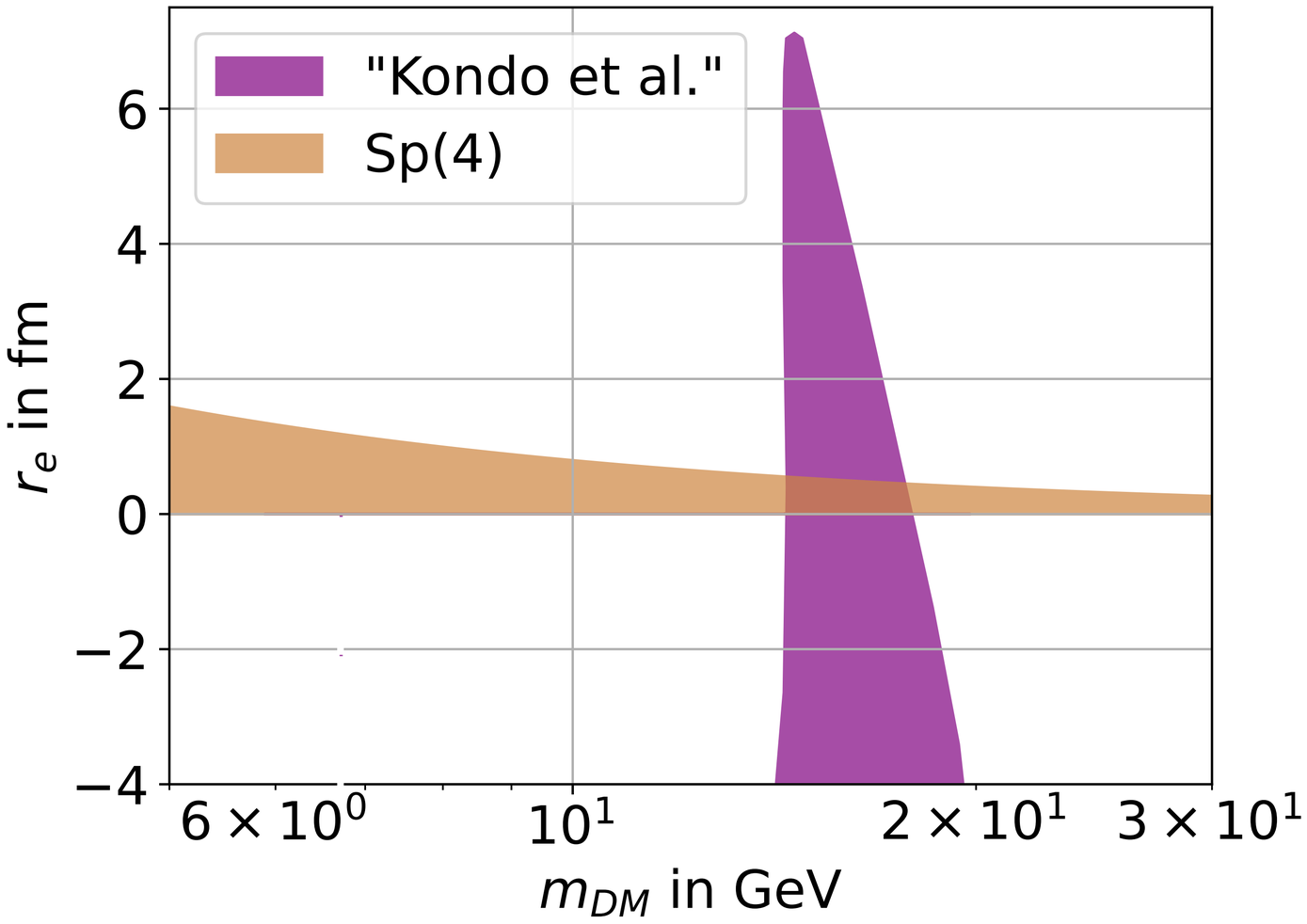

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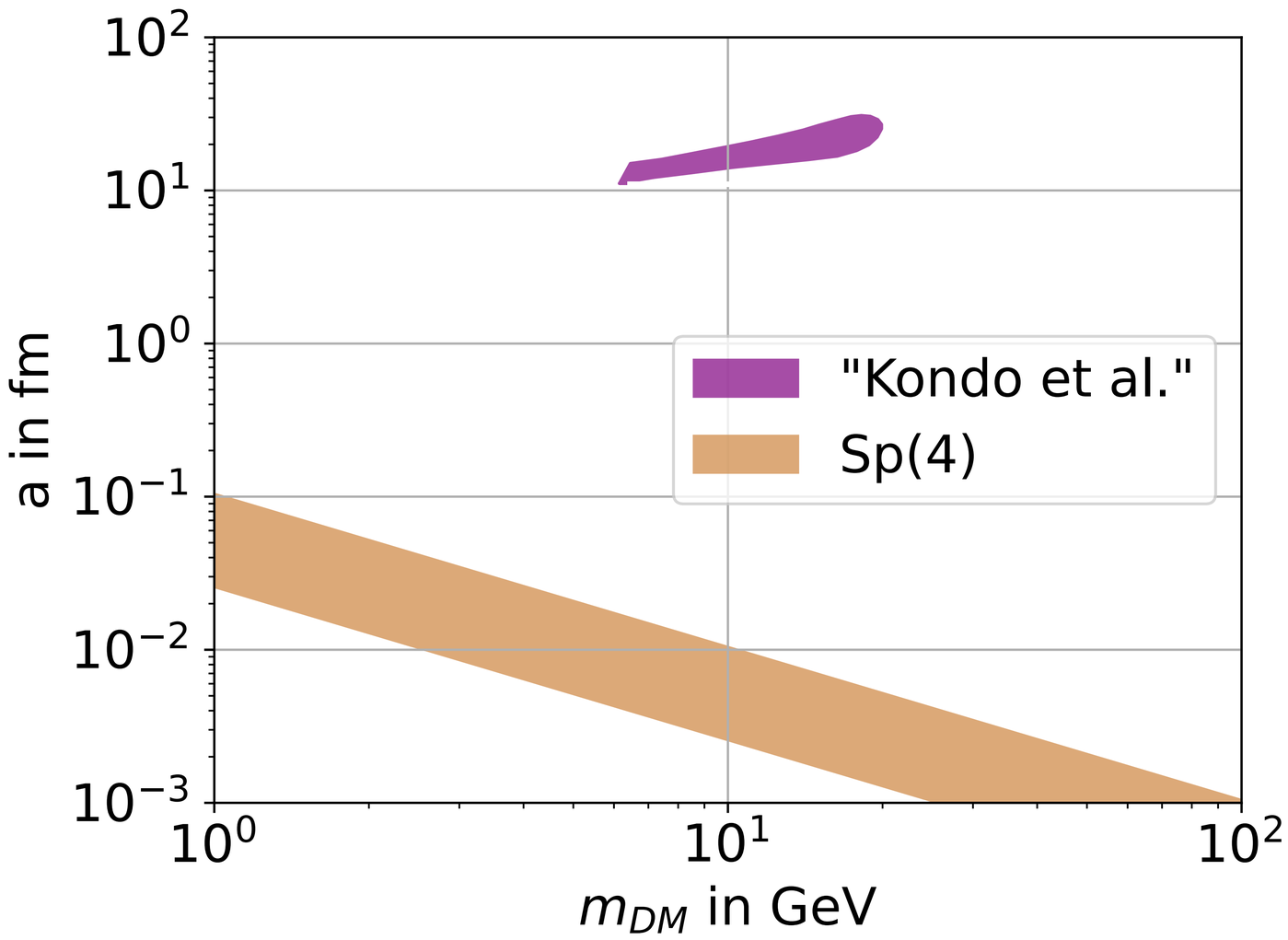
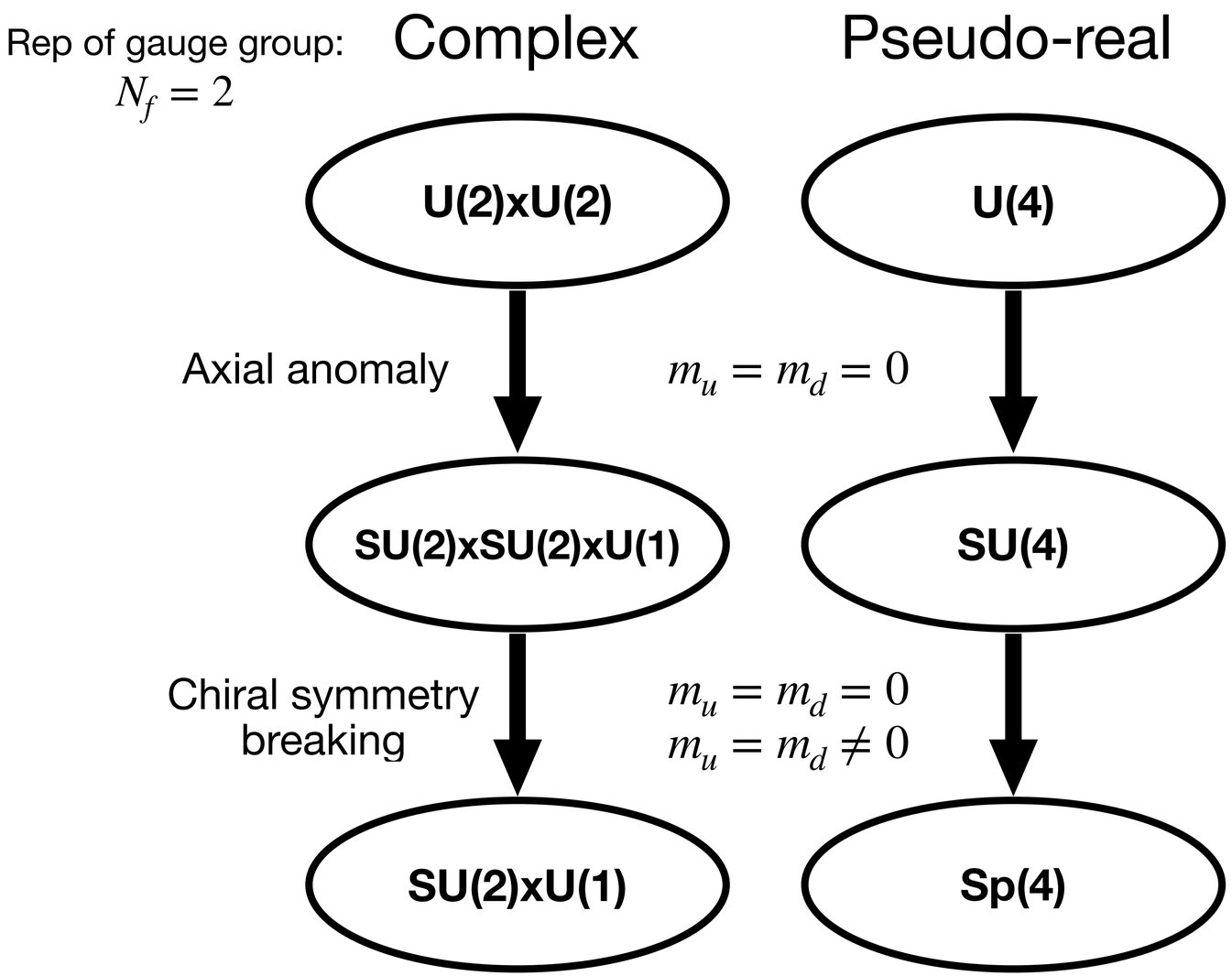


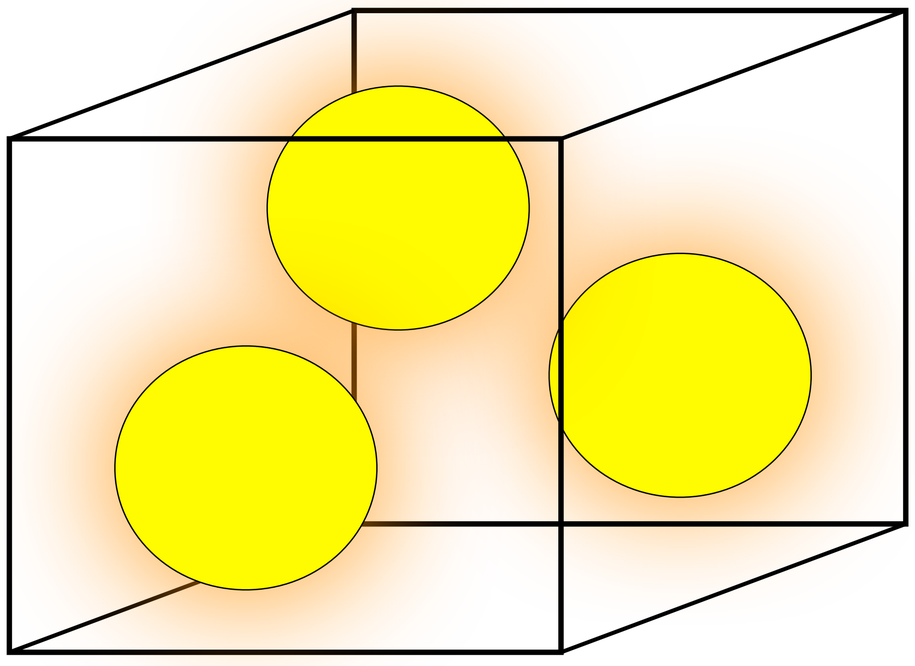
$Sp(4)_c$ vs. $Sp(4)_f$ - clarification

- Symplectic groups always have a even dimension - Sp(2N)
- Flavour symmetry:
  - Needed for symmetry breaking pattern
- Gauge symmetry: Needed for the pseudo-real representation
  - Also SU(2) or Sp(6) for example possible



Lattice - Scattering

- „3 particle quantization condition“
- Exact up to discretization effects
- $\det[F_3^{-1} + \mathscr{K}_3] = 0$
  - Matrix in (l,m) → cut-off in partial wave
  - Full $2 \to 2$ information needed



5 dark Pions

- Pions form a 5-plet of the flavour symmetry
  - $\pi^+, \pi^0, \pi^-, \Pi_{ud}, \Pi_{\bar{u}\bar{d}}$
- What are the possible scattering channels?
- Tensor products of the corresponding representations
- 3 Isospin channels in $\pi\pi$:
- I=0 (1-dim), I=1 (10-dim), I=2 (14-dim)

$$Sp(4)_f$$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$
$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$
$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$\pi\pi \to \pi\pi$ (I=0,1,2)
$\pi\pi \to \rho$ (I=1)
$\pi\pi \to \pi\pi\pi$ (I=1)
$\pi\pi \to \pi\pi\rho$ (I=0,1,2)
etc.

Phenomenology of scattering channels

- I=2 (14-dim):
  - (Probably) contributes most to $\pi\pi$-scattering
  - 14 out of 25 possible combinations of Pions
- Considered in this talk

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etc.

Phenomenology of scattering channels

- I=0 (1-dim):
  - (Probably) no large contribution to $\pi\pi$-scattering
  - Mixing with the „singlet“
  - Numerically challenging („connected diagrams“)
- Not considered in this work

$$Sp(4)_f$$

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etc.

Phenomenology of scattering channels

- I=1 (10-dim):
  - Mixing with the Rho
  - $\pi\pi\pi \to \pi\pi$
  - No contribution to $\pi\pi$-s-wave scattering
- Tackled in the future

$$Sp(4)_f$$

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$\pi\pi \to \pi\pi\rho$ (I=0,1,2)
etc.

Phenomenology of scattering channels

- I=2:
  - Makes up most 2 π scattering (14/25)
  - Easiest on the lattice
- I=1:
  - No s-wave scattering
  - Mixing with dark Rho
  - $\pi\pi\pi \to \pi\pi$
- I=0:
  - Mixing with the flavour singlet

$$Sp(4)_f$$

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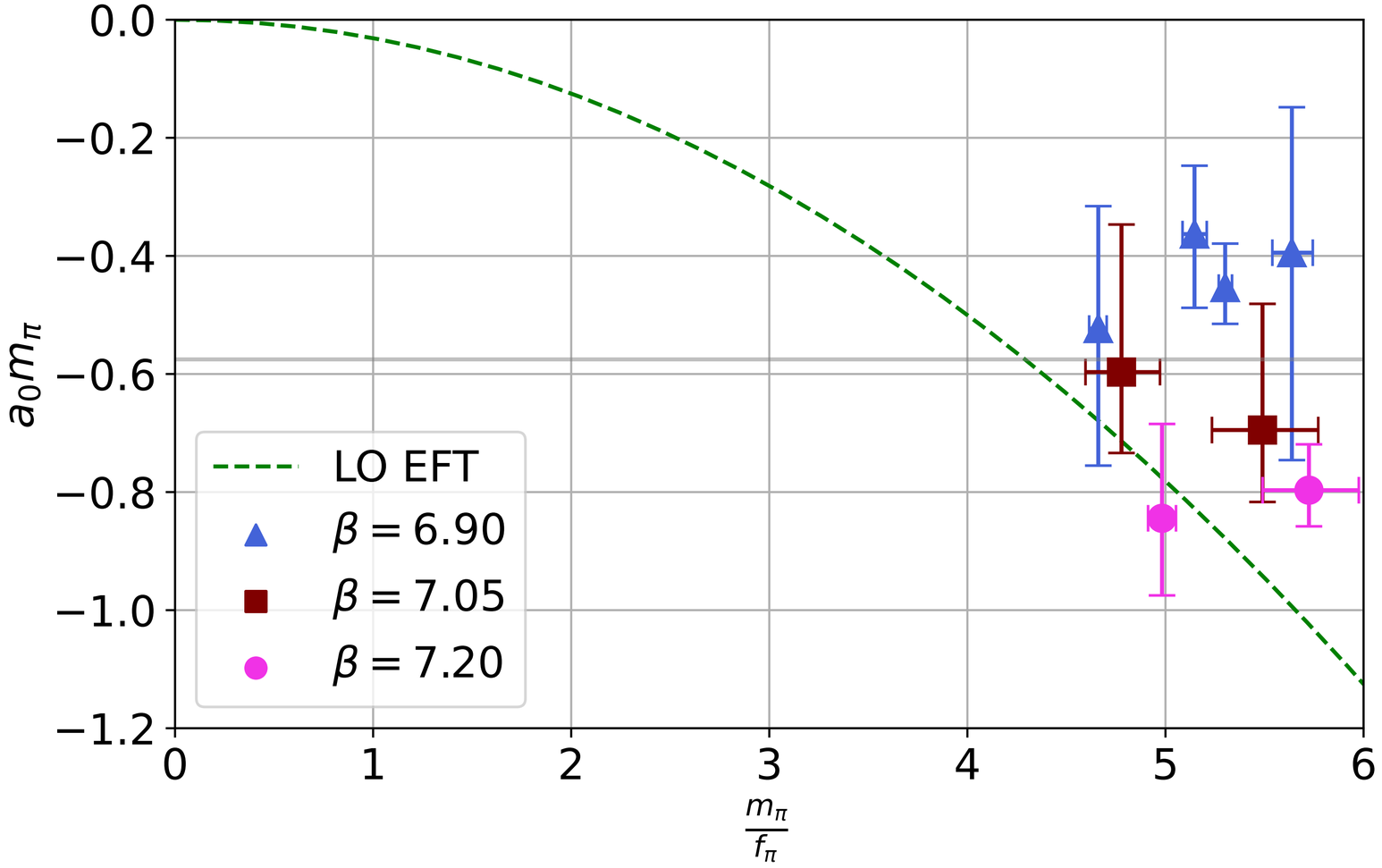
etc.

Comparison to astrophysical constraints

- Scattering length:
- $a_0 m_\pi = -0.6^{+0.2}_{-0.2}$
- $\frac{\sigma}{m} < 0.19 \frac{cm^2}{g}$
- Fixes the lattice constant

➡ $m_{DM} \gtrsim 100\, MeV$



Eckert et al.: A&A 666, A41 (2022)