[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

Investigating Vector Boson Scattering

A Lattice study

Bernd Riederer
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Lattice22, Bonn

FWF Der Wissenschaftsfonds.

NAWI Graz Natural Sciences

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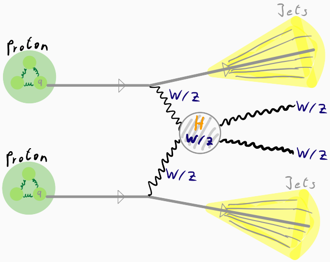
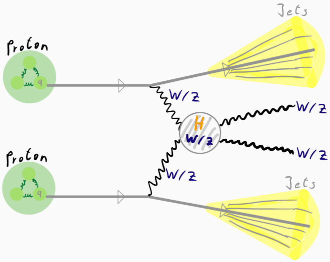
Vector boson scattering

- **Scattering of weak gauge bosons**

  → central electroweak process
- Heavily studied in experiments
- Especially for BSM searches
- **This talk**: First investigation of VBS on the lattice
- Why bother?



Vector boson scattering



- **Scattering of weak gauge bosons**

  $\rightarrow$ central electroweak process

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- **This talk**: First investigation of VBS on the lattice
- Why bother?
- Allows to gain a nonperturbative understanding of the Higgs
  - Study of finite extent/compositeness $\rightarrow$ BSM physics?
  - Maybe additional background from within the SM?

Scattering processes

[Sakurai, Napolitano **Modern Quantum Mechanics** (2020)]

- **Experiment**: cross-sections in specified decay channel
- **Theory**: cross-sections from fundamental theory

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \qquad \mathcal{M} \ldots \text{Transition matrix}$$

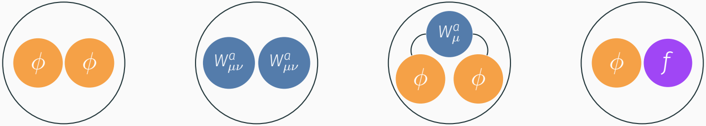
- Specify channel e.g. Higgs: $J_C^P = 0_0^+ \leftrightarrow$ partial wave analysis

$$\mathcal{M} = \underbrace{\langle \text{out}|T|\text{in}\rangle}_{\text{Transition element}} \propto \sum_J (2J+1) \underbrace{e^{i\delta_J} \sin(\delta_J)}_{f_J \ldots \text{partial transition amplitude}} P_J(\cos\theta)$$

- Phase shift δ_J contains full scattering information
- **Perturbative approach**: $\mathcal{M}$ from (augmented) PT
- **Lattice approach**: δ_J from Lüscher analysis

Perturbative approach to VBS

- PT: asymptotic states are fully specified e.g. $W^+W^- \to ZZ$
- **But** on the Lattice:
  - Specific state unknown $\to$ superposition of possible states
    $\to$ Lattice process: $W^\pm W^\mp/ZZ \to W^\pm W^\mp/ZZ$
  - Asymptotic states need to be gauge-invariant
    $\to$ Even weakly interacting particles are bound states



Field theory **requires** physical states to be gauge-invariant

Weak bound states in the SM

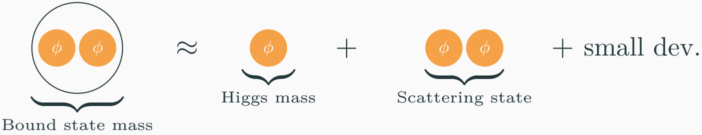
[Fröhlich et al., PL **B97** (1980) and NP **B190** (1981) / **REVIEW:** Maas **PPNP** 106 (2019), **arXiv:** **1712.04721**]

- "Augmented perturbation theory" (APT)

  Maps: **bound state** correlators ↔ **perturbative** correlators

- Masses are unaffected by bound state structure

  [Maas and Sondenheimer **PRD** 102 (2020) / Dudal et al. **EPJ** C81 (2021)]



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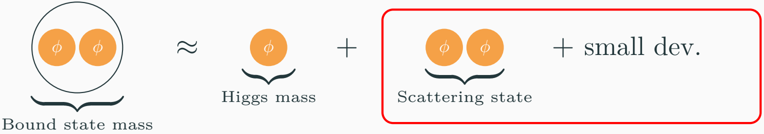
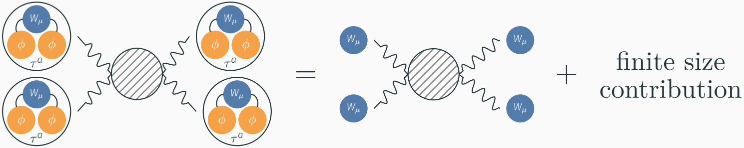


- Scattering processes are sensitive to the change



Lattice setup and technicalities

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

$$\mathcal{L}_{EWH} = -\frac{1}{4} \underbrace{W^a_{\mu\nu}} W_a^{\mu\nu} + (\underbrace{D_\mu} \phi)^\dagger D^\mu \phi - \underbrace{V\left(\phi^\dagger \phi\right)}$$

$$\partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu \qquad \partial_\mu - i g W^a_\mu t_a \qquad \lambda\left(\phi^\dagger\phi - f^2\right)^2$$

- Full EWH-sector not possible → only weak interaction:
  - Gauge group: $\mathrm{SU}(2)_W \times \mathrm{U}(1)_\gamma \to \mathrm{SU}(2)_W$
  - no fermions & photon, $m_W = m_Z$
  - Gauged scalar field: $\mathrm{O}(4) \to \mathrm{SU}(2)_W \times \mathrm{SU}(2)_\mathcal{C}$
  - Additional global $\mathrm{SU}(2)_\mathcal{C}$ symmetry of the Higgs field

Lattice setup and technicalities

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**/ M. Lüscher, NP **B354** (1991) and **B364** (1991)]

- Necessary energy spectra in involved channels:
  - Ground state in decay product channel: vector (1_1^-)
  - Ground and excited states in scattering channel: scalar (0_0^+)
- Lüscher analysis:

  lattice spectra $E_n(V) \leftrightarrow$ **continuum phase shift** $\delta_J(E)$
- Problems:
  - very noisy $\rightarrow$ huge statistics: $\mathcal{O}(10^6)$ confs.
  - excited states are hard to disentangle: variational analysis

    $\rightarrow$ operator-basis including 180 interpolators see appendix

    $\rightarrow$ pre- and post-selection for spectra see appendix

When NP meets PT

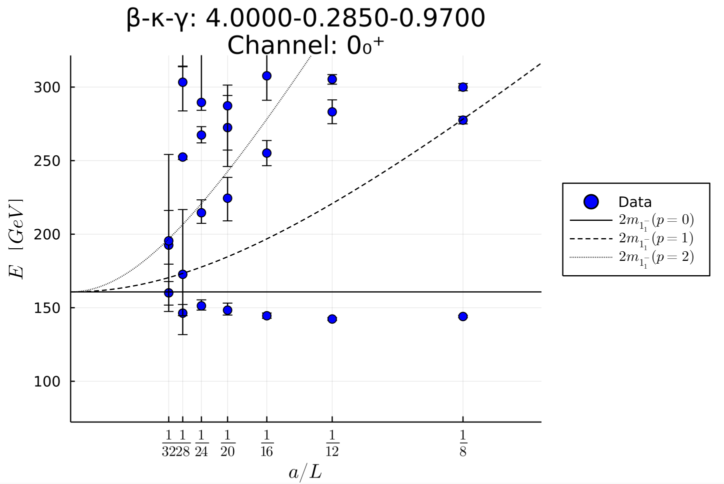
[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

- Checking for deviations/agreements of (A)PT and NP
- (A)PT: analytic prediction for $\tan(\delta_0) = f(E; g_W, m_W, m_H)$
- Lattice: need to choose correct parameter sets
- Previous studies categorized parameter sets as below

Range	Physics	m_W	m_H
$m_H < 2m_W$	stable "Higgs"	✔	✔
$2m_W \leq m_H \leq 4m_W$	"Higgs"-Resonance	✔	✘
$4m_W \ll m_H$	Heavy "Higgs"	✔	$\to \infty$

Heavy Higgs

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]



Spectrum: **Lattice result (w. statistical errors) + expected states**

No bound state below threshold; $\alpha_{W,200\text{ GeV}} = 0.219$

Heavy Higgs

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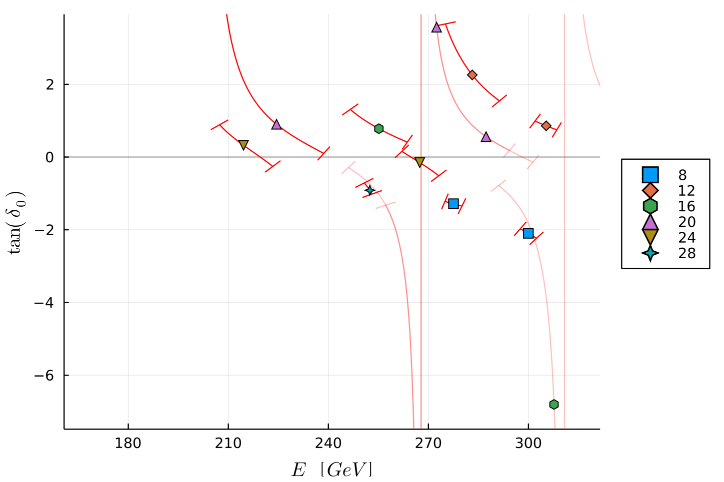

Tangent of phase shift: **Lattice result (w. statistical errors)**

$m_H \to \infty$; $\alpha_{W,200\text{ GeV}} = 0.219$

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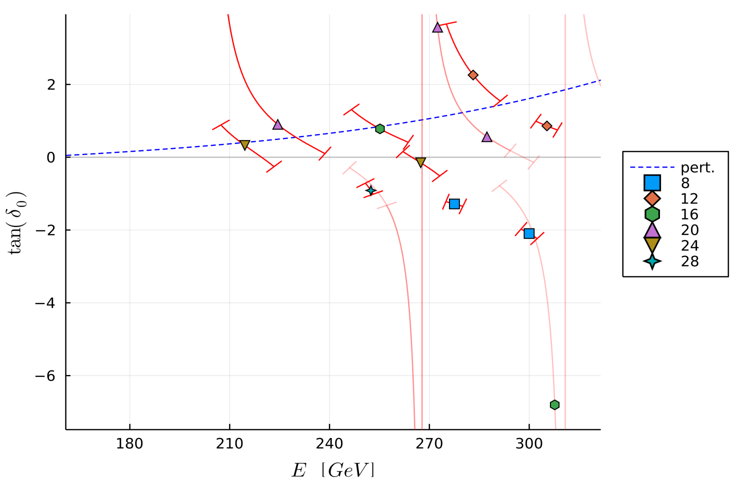

Tangent of phase shift: **Lattice result (w. statistical errors) + Born-level PT**

$m_H \to \infty$; $\alpha_{W,200\text{ GeV}} = 0.219$

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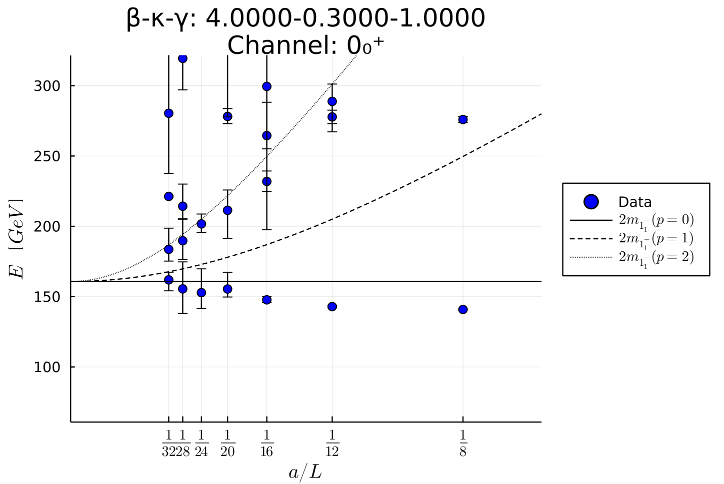
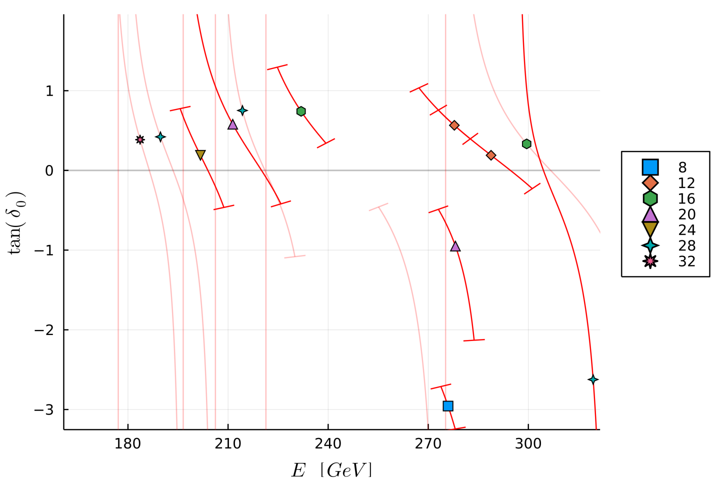


Spectrum: **Lattice result (w. statistical errors) + expected states**

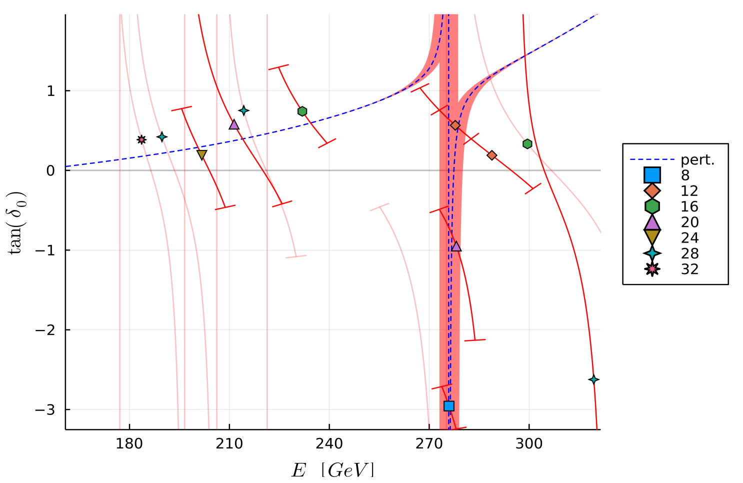
No bound state below threshold; $\alpha_{W,200\text{ GeV}} = 0.211$



Tangent of phase shift: **Lattice result (w. statistical errors)**

$m_H \approx ?$; $\alpha_{W,200\ \mathrm{GeV}} = 0.211$

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]



Tangent of phase shift: **Lattice result (w. statistical errors) + Born-level PT**

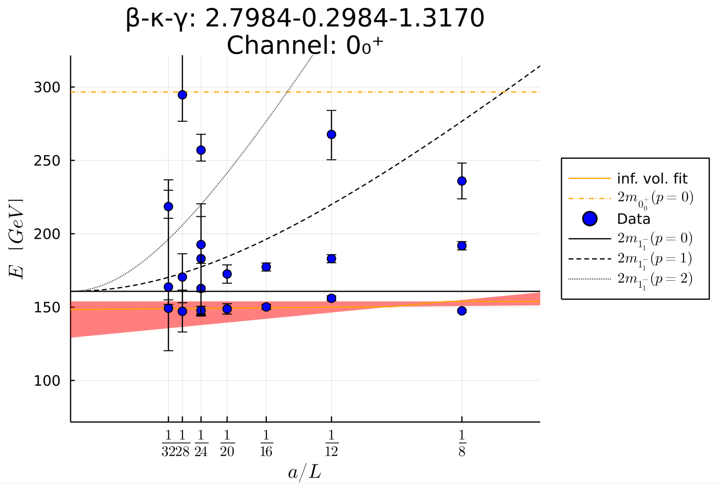
$m_H \approx 275$ GeV; $\alpha_{W,200\text{ GeV}} = 0.211$

Resonance

- Using perturbative prediction as a tool
- Successful although Breit-Wigner fit **failed**
- Reinserting to BW-fit $\rightarrow$ compatible with data
- Provides a useful alternative to access resonances
- **Disadvantages**:
  - No information about particle width
  - May be more involved beyond tree-level

Stable Higgs

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]



Spectrum: **Lattice result (w. statistical errors) + expected states**

$m_H \approx 148$ GeV; $\alpha_{W,200\,\text{GeV}} = 0.492$

Stable Higgs

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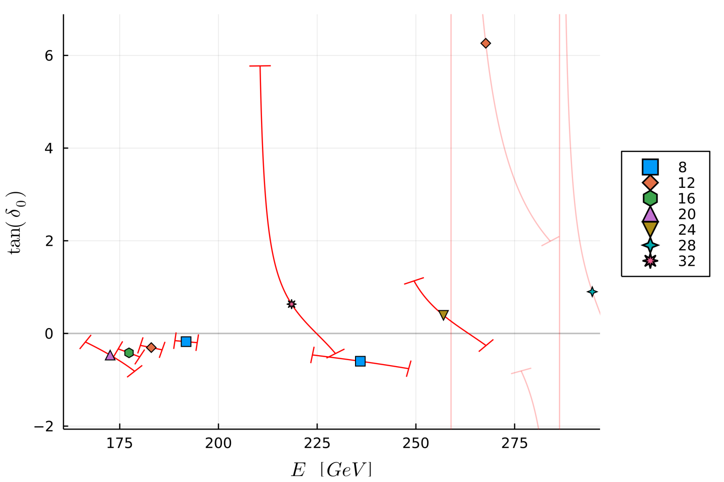

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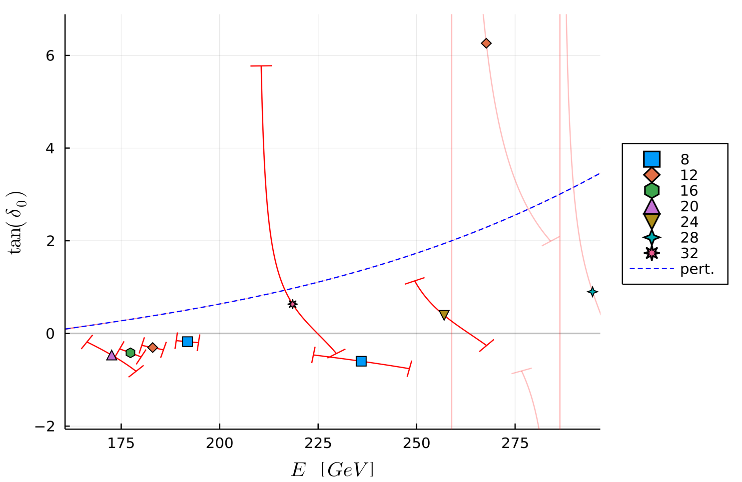

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Stable Higgs

[Sakurai, Napolitano **Modern Quantum Mechanics** (2020) / Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

- Large discrepancy near threshold for "stable Higgs"-sets
- **Because**: Tree-level (A)PT loses bound-state-like nature

  $\rightarrow$ particle extent modifies interaction radius

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  $\rightarrow$ particle extent modifies interaction radius
- Scattering length a_0: $$\tan(\delta_J) \approx \frac{\sqrt{s-4m_W^2}}{a_0^{-1}}$$ @ elastic threshold
  - **Negative** scattering length $\leftrightarrow$ **Bound state** of some size
  - Close to elastic threshold $\rightarrow$ attractive component
  - Inside the elastic region $\rightarrow$ contribution vanishes

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  - **Negative** scattering length $\leftrightarrow$ **Bound state** of some size
  - Close to elastic threshold $\rightarrow$ attractive component
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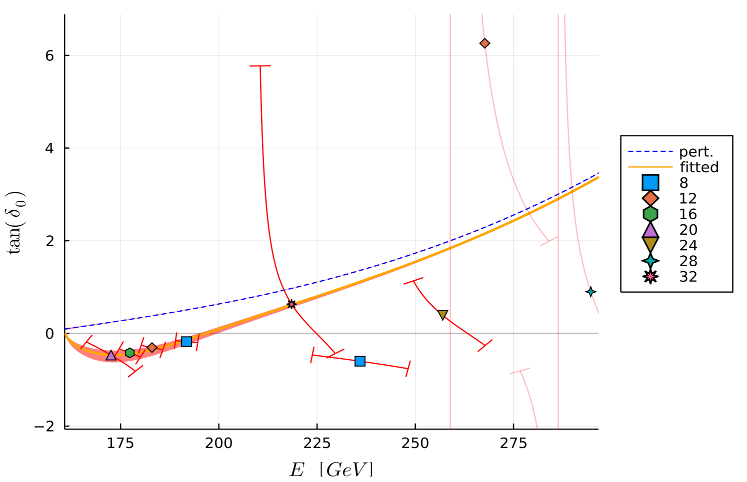
- Modification of perturbative approximation:

$$\tan(\delta_J) \rightarrow \tan(\delta_{J,\text{PT}}) - \Delta f(a_0, s)$$

Stable Higgs



Tangent of phase shift: **Lattice result (w. statistical errors) + Born-level PT + finite size contribution**

$m_H \approx 148$ GeV; $\alpha_{W,200\text{ GeV}} = 0.492$; $a_0^{-1} \approx -39$ GeV

Impact on cross-sections

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

- Modification of $\tan(\delta_J) \leftrightarrow$ direct impact on cross-section
- Ratio of modified differential cross-sections to usual PT

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{mod.}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{PT}} = \left|1 - \frac{2\mathcal{M}_{\Delta f}}{\mathcal{M}_{\text{PT}}}\left(1 - \frac{\mathcal{M}_{\Delta f}}{2\mathcal{M}_{\text{PT}}}\right)\right|$$

see appendix

- Deviations expected only for $\mathcal{M}_{\Delta f} \gg \mathcal{M}_{\text{PT}}$
  - for E close to elastic threshold E_{th}
  - requires $E_{th} \lesssim m_H + |a_0^{-1}|$
- In other words: **Bound state needs to be big enough**

What do these deviations look like?

Impact on cross-sections

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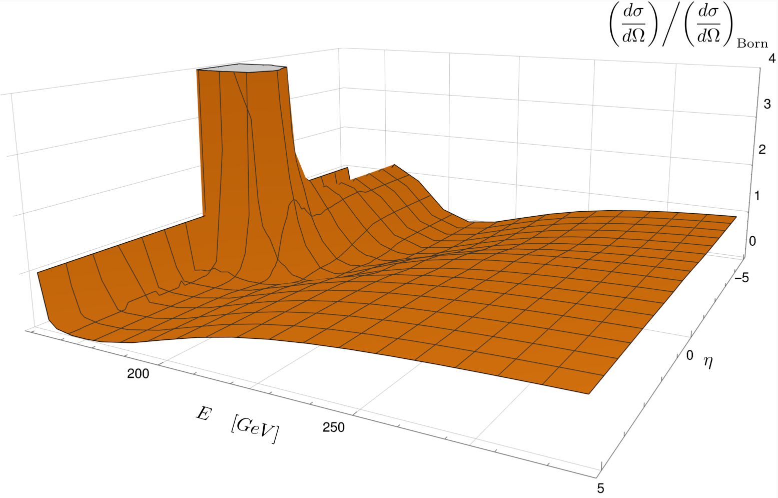

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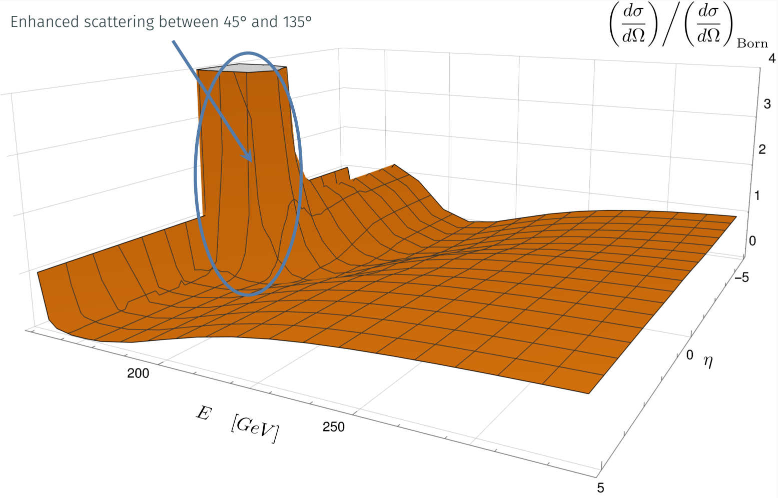

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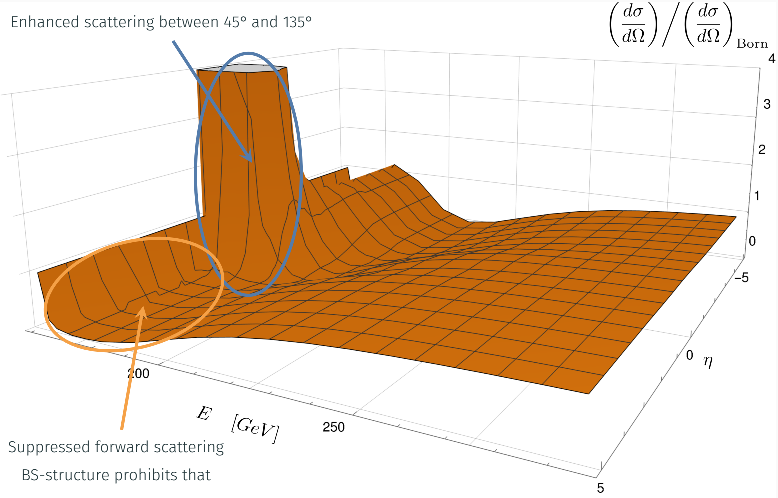

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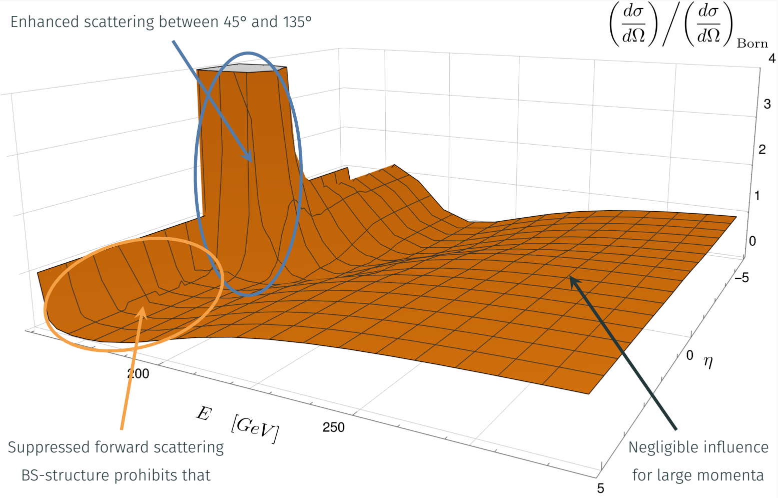

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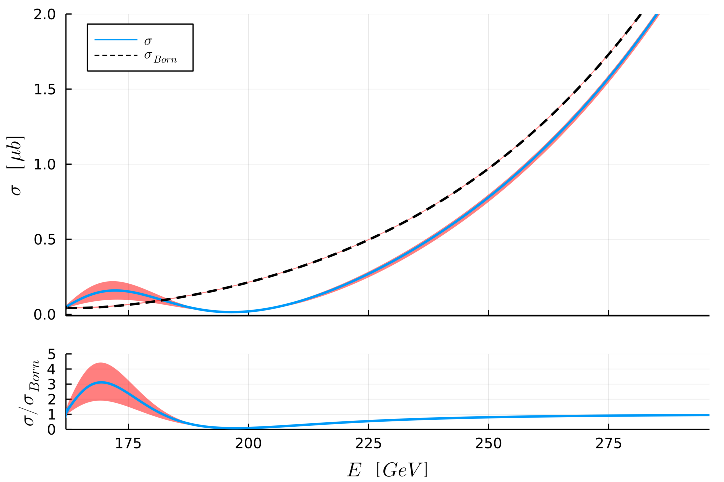

Integrated cross-sections (top) and ratio of these (bottom)

$m_H \approx 148$ GeV; $\alpha_{W,200\text{ GeV}} = 0.492$; $a_0^{-1} \approx -39$ GeV

Summary & Outlook

- VBS process allows to probe "**composite-ness**" of the Higgs
  - SM $\rightarrow$ fully gauge-invariant objects are physical d.o.f.
  - BSM $\rightarrow$ often intrinsically composite
- **Heavy Higgs**
  - does not influence elastic VBS $\rightarrow$ (A)PT and NP agree
- **Resonance**
  - (A)PT yields alternative fit-approach for resonances
- **Stable Higgs**
  - disagreement of (A)PT and NP close to threshold
  - can be explained with BS-like nature of the Higgs
  - measurable modifications to cross-sections are expected

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]

Thank you!

FWF Der Wissenschaftsfonds.

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Augmented Perturbation Theory (Masses)

[Fröhlich et al., PL **B97** (1980) and NP **B190** (1981) / Maas, PPNP **106** (2019), **arXiv: 1712.04721**]

1. Formulate gauge-invariant operator and correlator e.g.:

$$0^+ \text{ singlet:} \quad H(x) = \left(\phi_i^\dagger \phi_i\right)(x)$$

$$\langle H(x)H(y)\rangle = \left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle$$

2. Expand Higgs field in fixed gauge $\phi_i = vn_i + h_i$

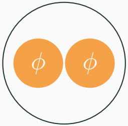
$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = dv^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger (x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle +$$
$$+2v\left\{\left\langle \left(h_i^\dagger h_i\right)(x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle + (x \leftrightarrow y)\right\} + \left\langle \left(h_i^\dagger h_i\right)(x)\left(h_j^\dagger h_j\right)(y)\right\rangle$$

3. Perform standard Perturbation Theory

$$\left\langle \left(\phi_i^\dagger \phi_i\right)(x)\left(\phi_j^\dagger \phi_j\right)(y)\right\rangle = d'v^4 + 4v^2 \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger (x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}} +$$
$$+ \left\langle \Re\left[n_i^\dagger h_i\right]^\dagger (x)\Re\left[n_j^\dagger h_j\right](y)\right\rangle_{\text{tl}}^2 + \mathcal{O}(g^2, \lambda)$$

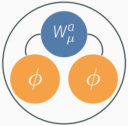
Gauge invariant weakly interacting particles

Physical Higgs: $\mathcal{O}^{0^+_0} =$



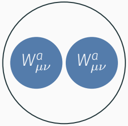
$= \phi^\dagger \phi = \frac{1}{2} \operatorname{tr}\left\{\Phi^\dagger \Phi\right\}$

Physical W^a_μ: $\mathcal{O}^{1^-_1\,a}_\mu =$



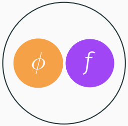
$= \operatorname{tr}\left\{\tau^a \Phi^\dagger D_\mu \Phi\right\}$

W-Ball: $\mathcal{O}^{0^+_0} =$



$= 1 - W^a_{\mu\nu} W^{\mu\nu\,a}$

Physical fermion: $\mathcal{O}^F =$



$= \Phi \begin{pmatrix} \nu^L \\ e^L \end{pmatrix}$

Lattice Action

SU(2)-Gauge-Higgs-Theory on the Lattice

[I. Montvay and G. Münster, Quantum Fields on a Lattice (1994)]

$$S = \sum_{x \in \Lambda} \Bigg[\beta \left(1 - \frac{1}{2} \sum_{\mu < \nu} \mathrm{Re}\{\mathrm{Tr}\,\{U_{\mu\nu}(x)\}\} \right) + \\ + \gamma \left(\phi^\dagger(x)\phi(x) - 1 \right)^2 + \phi^\dagger(x)\phi(x) - \\ - \kappa \sum_{\pm\mu} \phi^\dagger(x) U_\mu(x) \phi(x + e_\mu) \Bigg]$$

Lattice parameters: β, γ and κ

Continuum parameters: g, λ and f

Variational Analysis

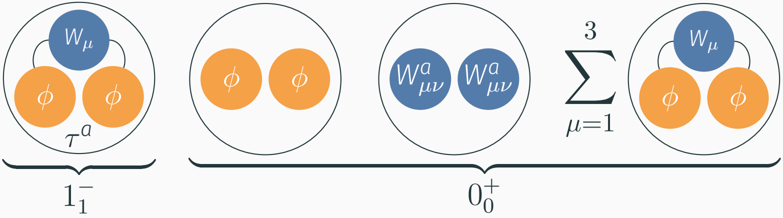
[M. Lüscher et al., NP **B339** (1990) / C. Michael, NP **B259** (1985)]

$$C_{ij}(\Delta t) = \left\langle \mathcal{O}_i^\dagger(\Delta t)\mathcal{O}_j(0) \right\rangle$$

Diagonalization disentangles physical states

$$\lambda_n(\Delta t) \propto e^{-E_n \Delta t}\left(1 + \mathcal{O}\left(e^{-\Delta E_n \Delta t}\right)\right)$$

Basis-Interpolators in the 1_1^- and 0_0^+ (physical Higgs) channel



Variational Analysis

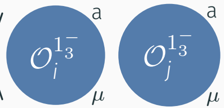
$$C_{ij}^{1_3^-} = \left\langle \mathcal{O}_i^{1_3^-}{}^{a}_{\mu} \; \mathcal{O}_j^{1_3^-}{}^{a}_{\mu} \right\rangle \quad \text{for} \quad i = j = 1, \ldots, 10\,(2 \times 5 \text{ smear})$$

$$C_{ij}^{0_1^+} = \left\langle \begin{matrix} \mathcal{O}_i^{0_1^+}\mathcal{O}_i^{0_1^+} & \mathcal{O}_j^{0_1^+}\mathcal{O}_j^{0_1^+} \\ \mathcal{O}_i^{1_3^-}\mathcal{O}_i^{1_3^-} & \mathcal{O}_j^{1_3^-}\mathcal{O}_j^{1_3^-} \end{matrix} \right\rangle \quad \text{for} \quad |\vec{\mathbf{p}}|^2 \neq 0$$

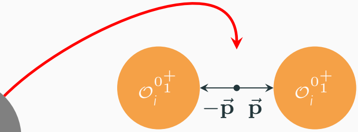
$$\text{for} \quad i = j = 1, \ldots, 180 \left((4 + 2) \times 3\,|\vec{\mathbf{p}}|^2 \times 5 \text{ smear} \times 2 \text{ squared} \right)$$

Variational Analysis

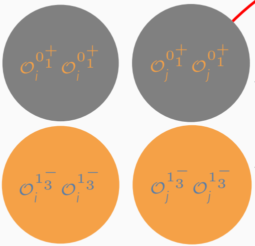
$C_{ij}^{1_3^-} = \Big\langle$



$\Big\rangle$ for $\quad i = j = 1, \ldots, 10\,(2 \times 5 \text{ smear})$



$C_{ij}^{0_1^+} = \Big\langle$



$\Big\rangle$ for $\quad |\vec{\mathbf{p}}|^2 \neq 0$

for $\quad i = j = 1, \ldots, 180 \left((4 + 2) \times 3\, |\vec{\mathbf{p}}|^2 \times 5 \text{ smear} \times 2 \text{ squared} \right)$

Operator Basis

$$\mathcal{O}_H(x) = \phi^\dagger(x)\phi(x) \qquad \mathcal{O}_W(x) = \mathrm{Tr}\left\{U_\mu(x)U_\nu(x+e_\mu)U^\dagger_\mu(x+e_\nu)U^\dagger_\nu(x)\right\}$$

$$\mathcal{O}_{0^+}(x) = \sum_{\mu=1}^{3}\mathrm{Tr}\left\{X^\dagger(x)U_\mu(x)X(x+e_\mu)\right\} \qquad \mathcal{O}_{0^+_n}(x) = \sum_{\mu=1}^{3}\mathrm{Tr}\left\{\frac{X^\dagger(x)}{\sqrt{\det(X(x))}}U_\mu(x)\frac{X(x+e_\mu)}{\sqrt{\det(X(x+e_\mu))}}\right\}$$

$$\mathcal{O}^a_{1^-\,\mu}(x) = \mathrm{Tr}\left\{\tau^a X^\dagger(x)U_\mu(x)X(x+e_\mu)\right\} \qquad \mathcal{O}^a_{1^-_n\,\mu}(x) = \mathrm{Tr}\left\{\tau^a\frac{X^\dagger(x)}{\sqrt{\det(X(x))}}U_\mu(x)\frac{X(x+e_\mu)}{\sqrt{\det(X(x+e_\mu))}}\right\}$$

$$\mathcal{O}^{0^+_0}_{1-30} = \begin{Bmatrix} \mathcal{O}^{(0-4)}_W(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_H(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_{0^+}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_{0^+_n}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)a}_{1^-\,\mu}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)a}_{1^-\,\mu}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)a}_{1^-_n\,\mu}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)a}_{1^-_n\,\mu}(\vec{\mathbf{p}}) \end{Bmatrix} \text{ such that } |\vec{\mathbf{p}}|^2 = 0$$

$$\mathcal{O}^{0^+_0}_{30-90} = \begin{Bmatrix} \mathcal{O}^{(0-4)}_W(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)}_W(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_H(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)}_H(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_{0^+}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)}_{0^+}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)}_{0^+_n}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)}_{0^+_n}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)a}_{1^-\,\mu}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)a}_{1^-\,\mu}(\vec{\mathbf{p}}) \\ \mathcal{O}^{(0-4)a}_{1^-_n\,\mu}(-\vec{\mathbf{p}})\,\mathcal{O}^{(0-4)a}_{1^-_n\,\mu}(\vec{\mathbf{p}}) \end{Bmatrix} \text{ such that } |\vec{\mathbf{p}}|^2 = 1, 2$$

$$\mathcal{O}^{0^+_0}_{91-180} = \left(\mathcal{O}^{0^+_0}_{1-90}\right)^2$$

$$\mathcal{O}^{1^-_1\,a}_{1-10\mu} = \begin{Bmatrix} \mathcal{O}^{(0-4)a}_{1^-\,\mu}\left(\vec{\mathbf{0}}\right) \\ \mathcal{O}^{(0-4)a}_{1^-_n\,\mu}\left(\vec{\mathbf{0}}\right) \end{Bmatrix}$$

Pre- and post-processing of spectra

- Correlator on lattice with periodic boundary

  $C(t) = \sum_k A_k \cosh\left[E_k\left(t - \frac{L_t}{2}\right)\right]$

- Noisiest point at $t = L_t/2 \rightarrow$ contains A_0

- **Idea**: Obtain E_0 and A_0 by assuming $A_0 = \text{const.}$

  $f(A_0(t), t) = \left[\frac{C(t)}{A_0(t)} + \sqrt{\left(\frac{C(t)}{A_0(t)}\right)^2 - 1}\right]^{\frac{1}{t - \frac{L_t}{2}}} = e^{-E_{eff}(t)}$

- Search for plateau in $f(A_0(t), t)$

  $\rightarrow$ gives value for A_0 and thus E_{eff}

Pre- and post-processing of spectra

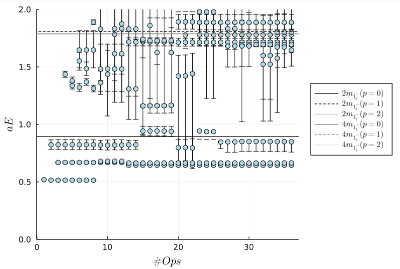
[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**/ Jenny **Master's thesis**]

- Quality of variational analysis depends strongly on basis
- Choose operator basis depending on a signal-to-noise ratio

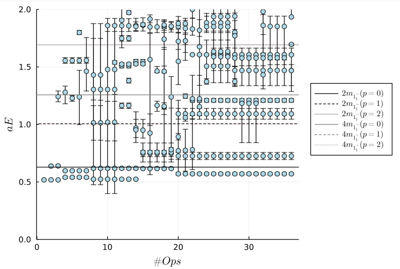
  $\text{SNR}_i = \sum_{t=0}^{L_t/2} \frac{\Delta C_{ii}(t)}{C_{ii}(t)}$
- Adding operators iteratively starting with smallest SNR_i
- First step gives an estimate of ground state
- Further steps should add states but leave ground state
- Repeat until analysis becomes unstable/too noisy

Pre- and post-processing of spectra

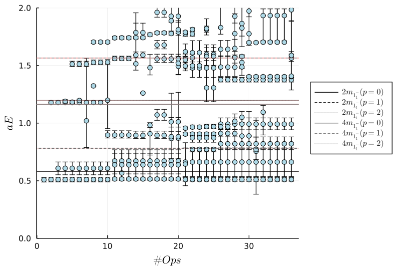
[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**/ Jenny **Master's thesis**]



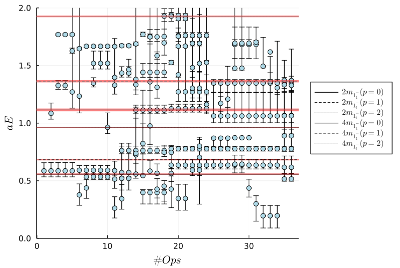
$L = 8$



$L = 16$



$L = 24$



$L = 32$

Pre- and post-processing of spectra

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**/ Jenny **Master's thesis**]

- For volume dependence one needs to choose the basis for each lattice size independently
- Choose an operator basis with:
  - stable behaviour
  - overlap with expected states
  - continuous changes across different lattice sizes

Modified cross-section

- Assuming: phaseshift δ_J can be obtained from PT
- Reunitarized partial transition amplitude:

$$F_J = 1/\,\mathrm{Re}(1/f_J) \to F_J = \tan(\delta_J)/(1 - \mathrm{i}\tan(\delta_J))$$

- Adding finite size contribution: $\tan(\delta_J) \to \tan(\delta_J) - \Delta f_J$

$$\Rightarrow F_J \to F_J - \frac{\Delta f_J}{(\Delta f_J - \mathrm{i} - \tan(\delta_J))(\mathrm{i} + \tan(\delta_J))} = F_J - \Delta F_J$$

$$\mathcal{M}_{\mathsf{PT}} \propto \sum_J (2J+1) F_J P_J(\cos\theta) \quad \mathcal{M}_{\Delta f} \propto \sum_J (2J+1) \Delta F_J P_J(\cos\theta)$$

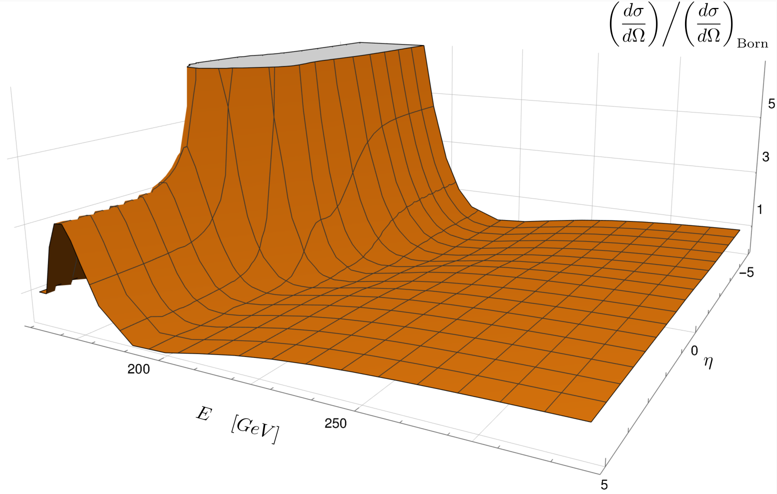
$$\mathcal{M}_{\mathsf{mod.}} = \mathcal{M}_{\mathsf{PT}} - \mathcal{M}_{\Delta f}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathsf{mod.}} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\mathsf{PT}} = \frac{\left|\mathcal{M}_{\mathsf{PT}} - \mathcal{M}_{\Delta f}\right|^2}{\left|\mathcal{M}_{\mathsf{PT}}\right|^2} = \left|\frac{\left(\mathcal{M}_{\mathsf{PT}} - \mathcal{M}_{\Delta f}\right)^2}{\mathcal{M}_{\mathsf{PT}}^2}\right|$$

$$= \left|1 - \frac{2\mathcal{M}_{\Delta f}}{\mathcal{M}_{\mathsf{PT}}}\left(1 - \frac{\mathcal{M}_{\Delta f}}{2\mathcal{M}_{\mathsf{PT}}}\right)\right|$$

Another stable Higgs

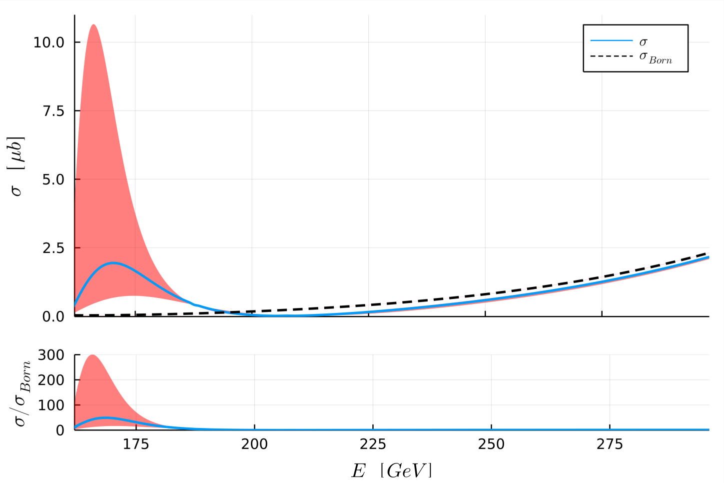
[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]



Modification of differential cross-section: $m_H \approx$ 149 GeV; $\alpha_{W,200\,\text{GeV}} = 0.448$; $a_0^{-1} \approx -12$ GeV

Another stable Higgs

[Jenny, Maas and BR **PRD** 105 (2022), **arXiv: 2204.02756**]



Integrated cross-sections (top) and ratio of these (bottom)
$m_H \approx 149$ GeV; $\alpha_{W,200\,\mathrm{GeV}} = 0.448$; $a_0^{-1} \approx -12$ GeV