Patrick Jenny

Improving the determination of energy levels in lattice simulations

to achieve the university degree of
Bachelor of Science

submitted to

University of Graz
Graz University of Technology

Supervisor
Univ.-Prof. Dipl.-Phys. Dr.rer.nat. Axel Maas
Co-Supervisor
Bernd Riederer, MSc

Institute of Physics

Graz, December 2021

AFFIDAVIT

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly indicated all material which has been quoted either literally or by content from the sources used.

Date, Signature

ABSTRACT

In the search for new physics or physics beyond the Standard Model, a fundamental objective is to make predictions from a theory under test. A general quantity one would like to obtain are the masses of the particles, which are included in the theory. In the case of Quantum Field Theory one of the used methods is Lattice Field Theory, which assumes a space-time lattice to make predictions. In this context correlation two-point functions can be defined, which contain the sought energy spectrum. The aim of this work is to develop and test a new method to obtain the Ground-State energy from the correlator. This new method assumes a region in which the correlator is dominated by a single energy level and can therefore be calculated. The algorithm was tested with self-generated mock-up data as well as correlators obtained by physical simulations. The tests yielded promising results, which led us to the conclusion that the new algorithm provides a sufficient tool to obtain the Ground-State energy of the correlator.

Contents

1. Introduction

In the past several decades, the theory of Quantum Fields has played a major role in the foundations of physics. So much that besides Einstein's Theory of General Relativity, all elementary aspects of physics manifest themselves as a Quantum Field Theory, known as the Standard Model of particle physics. As for a variety of reasons it is known that this theory of the elementary particles is incomplete, the search for new physics beyond the Standard Model is part of ongoing research.

The general aim in this field of study is to obtain predictions from a theory under test. Probably one of the most interesting question that arises is: "What particles are included in my theory?". A common technique to make predictions is the use of Lattice Field Theory. This is not particularly new and has been used for many years in the low energy regime of Quantum Chromo Dynamics. Of particular interest in this context are the two-point correlation or short correlator functions, which contain the particle spectrum. The aim of this work is to develop and test a new method to obtain the Ground-State level from the correlator (two-point correlation function).

The remainder of the thesis is organized as follows. In section 2 the theoretical background is covered by introducing the general concepts of Lattice Field Theory and two-point correlation functions. In section 3 the problem is again presented in greater detail with the insights from the chapter before. The new method is introduced in section 4. Section 5 shows some results which were obtained by using this new method. Finally, the conclusion and the outlook are reported in section 6.

2. Theoretical Background

In this chapter, the necessary theoretical background for this thesis is introduced. Before starting with the more specific aspects, the general physical framework in which this thesis takes place is briefly explained. As already mentioned in the Introduction, the Standard Model of particle physics is the best description of the universe at small scales we currently have. Although very successful, it still has some shortcomings both in theoretical and experimental nature. A famous experimental deviation from the Standard Model is the Muon $g-2$ anomaly [1]. Regarding the theoretical limitation, the absence of a unified treatment of the forces that is expected in a complete theory is probably the biggest deficit. A postulated solution is the approach of a Great Unified Theory (GUT), which, in the most general description, combines the Electroweak Force with the Strong Nuclear Force. This was done likewise in the unification of the Weak and the Electromagnetic interaction to the Electroweak interaction. However, so far none of the candidates for a GUT could be verified. [4]

With the introduction of the general setting, we can now continue with the presentation of the theoretical quantities and concepts necessary for this thesis.

2.1. Lattice Field Theory

This subsection is based on [5].

As the work of this thesis targets a problem that emerges in the context of Lattice Field Theory or Lattice Gauge Theory, a short introduction is given. As a full explanation of the matter would go beyond the scope of this work, only some key features necessary to understand the form of the correlator are covered. In Quantum Field Theory, problems are normally solved with perturbation theory. However, in some areas of modern physics (e.g. the low energy regime of QCD) this approach fails. The introduction of a space-time lattice now yields the possibility of using non-perturbative techniques. Thus, instead of a continuous variable x a discretized coordinate x with the lattice spacing a is used.[1]

$$x = an \qquad n \in \mathbb{Z} \tag{1}$$

This means for a quantity, for example a (scalar) field ϕ, that it is only defined on the lattice points.

$$\phi(x), \quad x \in \text{lattice} \tag{2}$$

Furthermore, if for practical reasons only a finite volume is considered, boundary conditions for the appearing quantities need to be introduced. In this work, periodic boundary

[1]In this thesis it is assumed that $a = 1$

conditions are assumed, which are a popular choice. Thus, with $n = 0, 1, 2, \ldots, L-1$ this leads to.

$$\phi(x) = \phi(x + aL) \tag{3}$$

Although it is only shown for one coordinate, these statements hold for all three space dimensions as well as the time dimension.

2.2. The Two-Point Correlation Function

In this section, the correlator function is introduced. Since the focus of this work is on the mathematical evaluation of the correlator, an exact physical derivation is omitted. The major part of this chapter is dedicated to connecting the form of the correlator in the Minkowski space to the correlator on the lattice, which is the quantity under study.

The correlator can be defined within Quantum Field Theory and "[...] can be interpreted physically as the amplitude for propagation of a particle or excitation between y and x."[6] In the context of this work, the two-point correlation function takes the form of an expectation value between an operator at t: $\mathcal{O}(t)$ and an operator at t_0: $\mathcal{O}(t_0)$.

$$C(t, t_0) = \langle \mathcal{O}(t)^\dagger | \mathcal{O}(t_0) \rangle \tag{4}$$

This expectation value $C_{mink}(t, t_0)$ is given by a sum of phases with the energy states E_k and the coefficients A_k. For a more detailed derivation of the form of the correlator we suggest [10]. The subscript *"mink"* stands for Minkowski space and is used to distinguish it from the correlator in Euclidean space, which is derived in the following.

$$C_{mink}(t, t_0) = \sum_k A_k e^{-iE_k(t-t_0)} \tag{5}$$

To get the form of the correlator on the lattice, first it is chosen that all times are purely imaginary $t = -i\tau$. This variable transformation is normally called Wick-Rotation and transforms the four-dimensional Minkowski space to a four-dimensional Euclidean space, as seen in the comparison of the metrics. [3]

$$ds^2_{mink} = -(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2 \quad \rightarrow \quad ds^2_{eucl} = (d\tau)^2 + (dx)^2 + \ldots \tag{6}$$

The correlator transforms therefore too, from the Minkowski correlator to the Euclidean correlator $C_{mink}(t, t_0) \rightarrow C_{eucl}(t, t_0)$. Thus, the correlator is now expressed by a sum of exponential decays, as seen in Equation 7.

$$C_{eucl}(t, t_0) = \sum_k A_k e^{-E_k(t-t_0)} \tag{7}$$

Finally, the effects of the lattice can be taken into account. First, the time variable becomes discretized with a lattice spacing $a = 1$ and therefore $t \in \mathbb{N}$. Furthermore, as mentioned in subsection 2.1, the periodic boundary conditions need to be considered as well as the time reversal symmetry, which generally holds in physics. To satisfy this, the same exponential functions must be added with opposite signs in the exponent. Considering the definition of the cosh function $\cosh(x) = e^x + e^{-x}$, the following form of the correlator follows.

$$C_{lat.}(t) = \sum_k A_k \cosh\left[E_k \left(t - \frac{t_{max}}{2}\right)\right] \tag{8}$$

Here, the variable t_{max} represents the lattice size. Equation 8 shows the final form of the correlator on the lattice, which is used in this form for the rest of this work. From here on $C(t)$ refers to the Lattice version of the correlator, unless stated otherwise. For context Figure 1 displays a correlator function with two cosh terms.

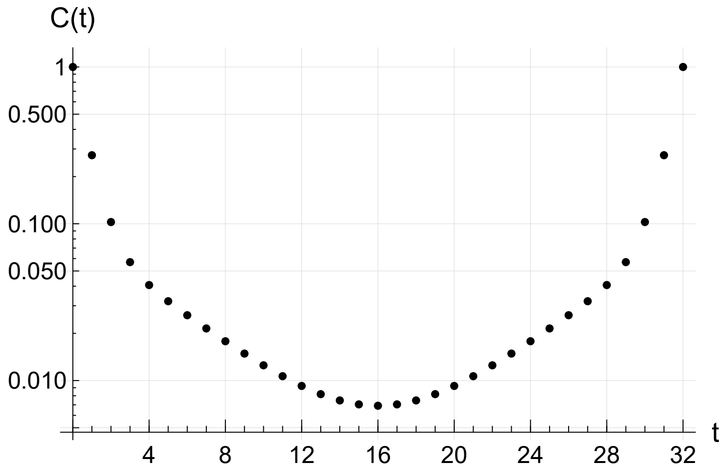


Figure 1: Example Correlator $C(t)$ with two cosh terms

3. Presentation of Problem

As in subsection 2.2 the correlator has been introduced, the goal of this work can now be presented in greater detail. The correlator on the lattice $C(t)$ can be obtained by simulation. Furthermore, the correlator contains the wanted lattice spectrum in form of the E_k-values (see Equation 8). Thus, the problem can be reformulated to finding the parameters A_k and E_k that fit the correlator points. However, this task yields some problems.

Before this is explained further, the input data will be specified here first. The lattice size t_{max} is a predefined parameter and can take values from 8 to 32. The correlator values themselves are normalized so that $C(0) = 1$ and exactly symmetric around $\frac{t_{max}}{2}$. Furthermore, the correlator data points $C(t)$ have an uncertainty in the same order of magnitude for all times t. This leads to an exponential increase in the relative uncertainty towards the value at $\frac{t_{max}}{2}$. This will come up again later.

The obvious solution of trying to fit the correlator function directly has some problems. The number of cosh terms which should be used in the fit is not obvious, and every additional term considered also increases the parameters by two. Already in the case of two terms this leads to three fit parameters.[2] The generic problem that arises in such cases is that the underlying uncertainties of the data points make it hard for the fit algorithm to optimize the different fit parameters to the individual points. This as a whole makes the cosh fits quite unreliable.

One of the simplest ways of tackling this problem is the use of effective mass curves with the definition of the effective mass as follows. This definition will lead to a constant value if the correlator is described by a single exponential term.

$$m_{eff}\left(t + \frac{1}{2}\right) = \log\left[\frac{C_{lat.}(t)}{C_{lat.}(t+1)}\right] \tag{9}$$

It is now argued that in some region between $t = 0$ and $\frac{t_{max}}{2}$ this approximation holds and in this region $m_{eff}(t) \approx E_0$, with E_0 being the Ground-State energy. Thus, if this region is known, at least one of the E_k's can be calculated. For the correlator displayed in Figure 1, the effective mass curve takes the following form.

[2] With the normalization $C(0) = 1$ one parameter can be eliminated

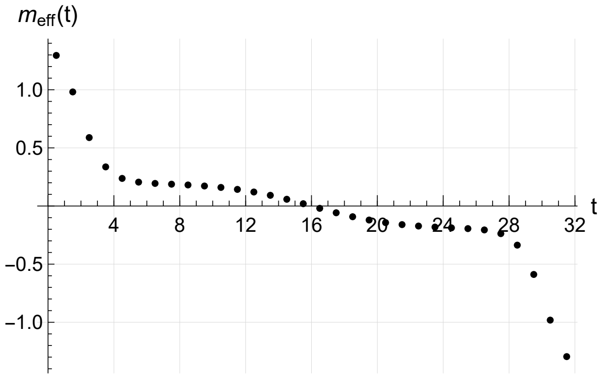


Figure 2: Example effective mass curve $m_{eff}(t)$

The issue that appears here is that this region can be very tight in time and therefore only include individual points. This work tries to solve this problem by taking a similar approach, but instead of utilizing the region that can approximately be described by a single exponential term, a region that can be described by a single cosh term is used. Since the correlator follows a sum of cosh curves, we argue that this approach will lead to better results. This adaption will be explained in more detail in the next chapter.

4. Algorithm

In this chapter, the new method is introduced and explained.

4.1. General

The starting point is again the form of the correlator function on the lattice, as seen in Equation 8.

$$C(t) = \sum_k A_k \cosh\left[E_k\left(t - \frac{t_{max}}{2}\right)\right]$$

The cosh function can also be written in exponential form with $\cosh(x) = e^x + e^{-x}$, and thus the correlator can also be seen as a sum of exponential decays and growths for which the following two limits are introduced (with $t_0 = \frac{t_{max}}{2}$).

$$\lim_{(t-t_0)\to\infty} \sum_k e^{-E_k(t-t_0)} = e^{E_0(t-t_0)} \quad \text{(i)} \qquad \lim_{(t-t_0)\to-\infty} \sum_k e^{E_k(t-t_0)} = e^{E_0(t-t_0)} \quad \text{(ii)}$$

It is now assumed that for the considered correlators both of these limits hold in some symmetric region around $\frac{t_{max}}{2}$. This region is further referred to as the plateau region, in which the correlator can be described by a single cosh term with the Ground-State energy E_0. This means:

$$C(t)\Big|_{t\in\text{Plateau}} = A_k \cosh\left[E_0\left(t - \frac{t_{max}}{2}\right)\right] \tag{10}$$

The following formula is now introduced to interrelate the parameters $A_k(t)$, $C(t)$ and $E_{eff}(t)$ to each other, whereby $E_{eff}(t)$ serves a similar purpose as the effective mass $m_{eff}(t)$ explained before.

$$f(A_k(t), t) = \left[\frac{C(t)}{A_k(t)} + \sqrt{\left(\frac{C(t)}{A_k(t)}\right)^2 - 1}\right]^{\frac{1}{t-\frac{t_{max}}{2}}} = e^{-E_{eff}(t)} \tag{11}$$

Thus, $E_{eff}(t \in \text{Plateu})$ should lead to a constant value. This can be seen by inserting Equation 10 for $C(t)$ in Equation 11, which is shown in the following.

$$f(t) = \left[\cosh\left[E_0\left(t - \frac{t_{max}}{2}\right)\right] - \sqrt{\cosh\left[E_0\left(t - \frac{t_{max}}{2}\right)\right]^2 - 1}\right]^{\frac{1}{t-\frac{t_{max}}{2}}}$$

$$= \left[\cosh\left[E_0\left(t-\frac{t_{max}}{2}\right)\right]-\sqrt{\sinh\left[E_0\left(t-\frac{t_{max}}{2}\right)\right]^2}\right]^{\frac{1}{t-\frac{t_{max}}{2}}}$$

$$= \left[\exp\left[-E_0\left(t-\frac{t_{max}}{2}\right)\right]\right]^{\frac{1}{t-\frac{t_{max}}{2}}} = \exp[-E_0]$$

The result e^{-E_0} can be identified as $e^{-E_{eff}(t)}$. Furthermore, this leads to $f(A_k(t), t) =$ const. in the plateau region, which introduces an interesting possibility. By setting

$$f(A_k(t+\frac{1}{2}), t) \stackrel{!}{=} f(A_k(t+\frac{1}{2}), t+1) \tag{12}$$

a common value of $A_k(t+\frac{1}{2})$ between two correlator points can be calculated. Since the addend $\frac{1}{2}$ in $A_k(t+\frac{1}{2})$ only serves the notation, it is omitted in the further course. Although not needed in the further calculation, for each point the uncertainty is calculated by using the values $C(t) \pm \delta C(t)$ to gain $A_k(t) \pm \delta A_k(t)$. This is done to make sure that all values used for the further analysis have defined error intervals. If $A_k(t)$ is in the plateau region, this should lead to a constant region for $A_k(t)$.

The remaining task is to find an algorithm which defines the points that are part of the plateau, or in other words, to find the region in which $A_k(t)$ is constant. This algorithm is further explained in subsection 4.2. After determining the plateau region, a common value for $A_k(t \in \text{Plateau})$ describing the single cosh term can be calculated. This value, further referred to as $\bar{A}_k$, is computed by taking the mean of the data points that are part of the plateau. To assign an uncertainty to $\bar{A}_k$, the standard deviation was used.

$$\bar{A}_k = \text{Mean}[A_k(t \in \text{Plateau})] \qquad \delta\bar{A}_k = \text{Std}[A_k(t \in \text{Plateau})] \tag{13}$$

To obtain the value of $\bar{A}_k$ as well as the uncertainty in a sophisticated manner, it was defined that the plateau must contain at least three independent points. If the calculation did not lead to that number of points, which fulfil the requirements for being defined as a plateau, the output was defined to be not valid. This condition resulted in another restriction. Since the correlator consists of $\frac{t_{max}}{2}+1$ independent points and calculating $A_k(t)$ reduces the independent points by one, as well as the Equation 11 is not defined at $t = \frac{t_{max}}{2}$, this leads to $\frac{t_{max}}{2}-1$ independent points for $A_k(t)$. Since in the case of $t_{max} = 8$ only three values of $A_k(t)$ can be calculated, $t_{max} \geq 10$ was set as a lower limit of the algorithm.

The part described above leads to a common value for $A_k(t)$ in the plateau region. The goal now is to obtain a single energy state for the plateau region. This value is further referred to as $\bar{E}_{Plat.}$. As shown in Equation 11, the value of $f(A_k(t), t)$ already corresponds to $E_{eff}(t)$, thus by inserting the obtained value of $\bar{A}_k$, the values of $f(\bar{A}_k = \text{const}, t)$ could be calculated.[3] In the plateau region similar argumentation as before

[3]The sign of the exponent in Equation 11 in this calculation is chosen so that the symmetry of $f(\bar{A}_k, t)$ around $\frac{t_{max}}{2}$ is preserved.

holds: if the region can be described by a single cosh term, this also means for $f(\bar{A}_k = \text{const}, t \in \text{Plateau})$ that it leads to a constant region. The plateau region ($t \in$ Plateau) was already defined within the context of the $A_k(t)$ values. The calculation of the sought $\bar{E}_{Plat.}$ value is then given by: [4]

$$\bar{f} = \text{Mean}[f(\bar{A}_k, t \in \text{Plateau})] \quad \rightarrow \quad \bar{E}_{\text{Plat.}} = -\log[\bar{f}] \tag{14}$$

The uncertainty interval $[\bar{f}_-, \bar{f}_+]$ is given by:

$$\bar{f}_\pm = \text{Mean}[f(\bar{A}_k \pm \delta\bar{A}_k, t \in \text{Plateau})]$$

It should be noted here that for all error propagations which were not explicitly mentioned to be calculated otherwise, the built-in *Mathematica* functionality of the "Around" command was used, which uses first-order series approximation, as stated in the official documentation: "When Around is used in computations, uncertainties are by default propagated using a first-order series approximation, assuming no correlations."[7]

[4] The sign of the exponent in Equation 11 is chosen so that the symmetry of $f(\bar{A}_k, t)$ around $\frac{t_{max}}{2}$ is preserved.

4.2. Defining the Plateau Region

As mentioned in subsection 4.1, an algorithm defining the points that are part of the plateau was needed, and therefore a statistical quantity describing the compatibility between the data points and a constant fit was sought. In the case of a linear fit, a simple measure of the goodness of the fit is the coefficient of determination R^2, which is defined as. [2]

$$R^2 = 1 - \frac{\mathrm{SSE}}{\mathrm{SST}} \tag{15}$$

With SST standing for "**S**um of **S**quares **T**otal" and SSE for "**S**um of **S**quares **E**rror". These quantities arise from the data points y_i, the mean of the data $\bar{y}$ and the points in the linear regression model $\hat{\mu}_i$ in the following way.

$$\mathrm{SST} = \|\boldsymbol{y} - \bar{y}\mathbf{1}\|^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2 \qquad \mathrm{SSE} = \|\boldsymbol{y} - \hat{\boldsymbol{\mu}}\|^2 = \sum_{i=1}^{n}(y_i - \hat{\mu}_i)^2 \tag{16}$$

In the case of a constant fit, which corresponds to calculating the mean of the data points, this definition of the R^2 value leads to $R^2 = 0$ for all cases. This is due to the fact that $\hat{\mu}_i = \bar{y}$ and therefore SST = SSE. So a slightly different definition for the R^2 value was used, which is also implemented in the *Mathematica* function "NonLinearModelFit" (see Wolfram Language & System Documentation Center [8]). Any further reference to the coefficient of determination or R^2 is defined by the following formula, in which instead of SST being related to the mean of the data, the quantity is obtained by taking the sum over all values.

$$R^2 = 1 - \frac{\sum_i (\hat{\mu}_i - y_i)^2}{\sum_i y_i^2} \tag{17}$$

The algorithm to define the region of the plateau works as follows. By successively decreasing the number of points symmetrically around $\frac{t_{max}}{2}$, the coefficient of determination is increased until it reaches the minimum value of R^2_{min} and does not change its value in one iteration by more than ε_{R^2}. After some initial trials, the parameters used for the algorithm were set to $R^2_{min} = 0.8$ and $\varepsilon_{R^2} = 0.01$. As the results, especially in the test-run (see subsection 5.1), did not evoke any reason to change the starting parameters, they were kept.

5. Results

5.1. Test-Run with self-generated Mock-Up Data

To test the algorithm for systematic errors and investigate the influence of uncertainties of the correlator data points themselves on the results obtained by the program, a test-run with self-generated mock-up data was done. The evaluation of these data led to many insights and further to code improvements and restrictions for the input data. Thus, this chapter continues as follows. First, the generation of the mock-up data is explained, followed by some results obtained by the algorithm. After that, the insights gained during this process are introduced and it is explained to which changes or restrictions this led.

For the correlator data double cosh terms were presumed, thus:

$$C(t) = A_1 \cosh[E_1(t - t_{max}/2)] + A_2 \cosh[E_2(t - t_{max}/2)] \tag{18}$$

The parameters A_1, A_2, E_1, E_2 and t_{max} were now randomly chosen, with the following restrictions. For the parameters E_1 and E_2 random real numbers within the interval of $[0.01; 1]$ were used, arranged so that $E_1 < E_2$. Since only the ratio between A_1 and A_2 and not the values of the A_i themselves are important, this ratio was randomly selected out of the following set.

$$\frac{A_2}{A_1} \in \left\{ \frac{1.}{10^5}, \frac{5.}{10^6}, \frac{1.}{10^6}, \frac{5.}{10^7}, \frac{1.}{10^7}, \frac{5.}{10^8}, \frac{1.}{10^8}, \frac{5.}{10^9}, \frac{1.}{10^9} \right\}$$

Furthermore, t_{max} was randomly selected from:

$$t_{max} \in \{8, 12, 16, 20, 24, 28, 32\}$$

It should be noted here that even though it was stated before that $t_{max} \geq 10$ was set as the lower limit of the lattice size, this restriction was only set after this test-run. Thus, this test-run also covers data sets with $t_{max} = 8$. To introduce randomness as well as a level of uncertainty to the generated data, a normally distributed quantity was added to each correlator point. The uncertainties of the correlator were then directly derived from the standard deviation σ, which was used in the generation of the additive shift. In this test-run, 50 different sets of parameters were generated. Furthermore, for each set 10 different values for σ were used with $\sigma_i = 2^{i-1} \cdot \sigma_0$ with i from 1 to 10. With $C\left(t = \frac{t_{max}}{2}\right)$ being the value of the correlator at t_{max} before the addition of uncertainties, the value of σ_0 was defined in the following way:

$$\sigma_0 = 0.01 \cdot C\left(t = \frac{t_{max}}{2}\right) \cdot \frac{t_{max}}{32} \tag{19}$$

The last factor $\frac{t_{max}}{32}$ was introduced to compensate for the dependence of the correlator middle point on t_{max}, as the point tends to have a higher value for lower t_{max}. The sets of parameters as well as the σ_0 value obtained are shown in Table 1 and Table 2. The following figure shows the generated correlator for Parameter-Set-1 and σ_4 as an example.

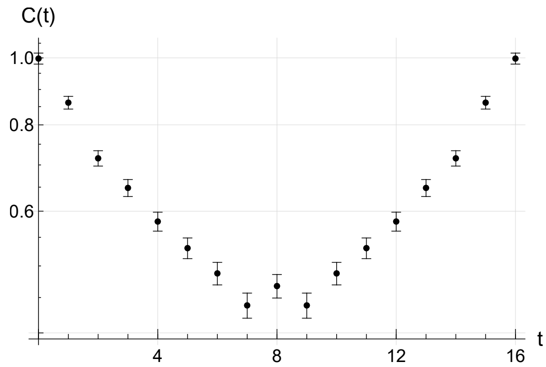
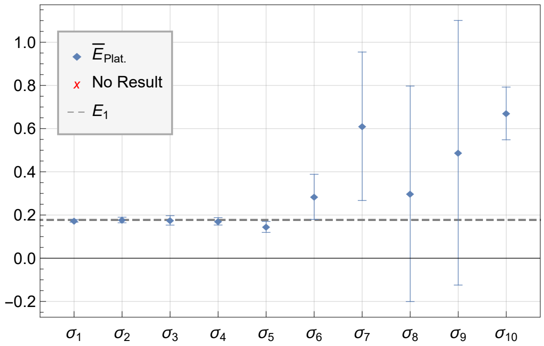
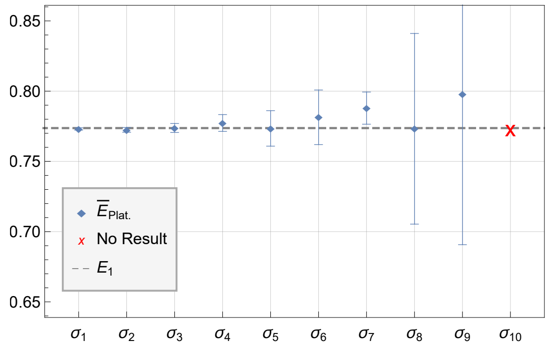


Figure 3: Correlator for Parameter-Set-1 with $\sigma = \sigma_4$

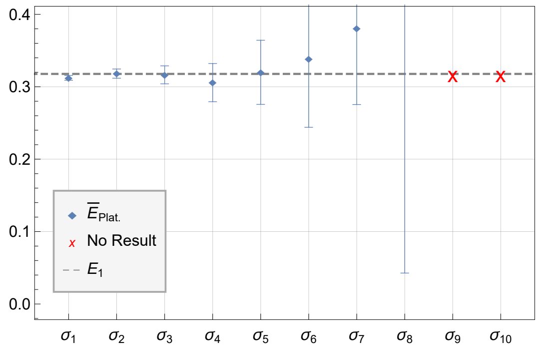
Thus, a total of 500 such correlator functions were generated and evaluated. It should be noted here that these results should be seen as a proof of concept and not exact numerical values, as the code was further improved after this test-run. However, in the following the results for $\bar{E}_{\text{Plat.}}$ for some sets of parameters in dependence of the value used for σ are displayed.



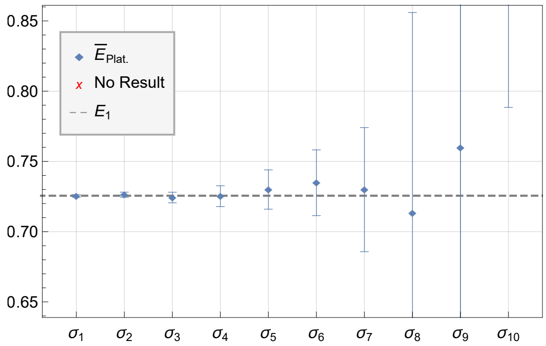
(a) Parameter-Set-1



(b) Parameter-Set-26



(c) Parameter-Set-39

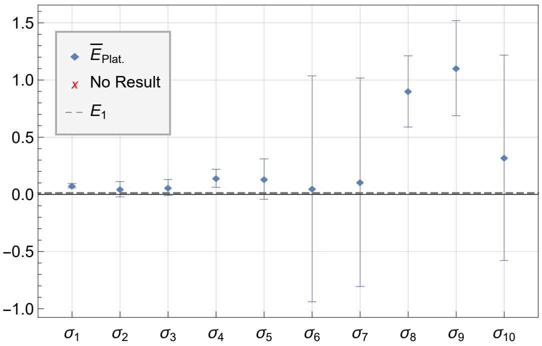


(d) Parameter-Set-50

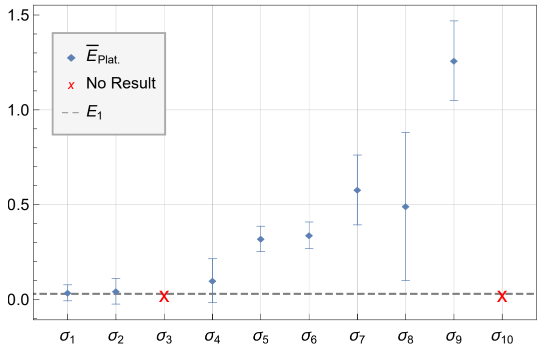
Figure 4: $\bar{E}_{Plat.}$ plotted over σ for different Parameter-Sets (sufficient Examples)

In Figure 4 some examples are shown for which the algorithm fulfils the desired behaviour. In these examples, increasing the uncertainties for the correlator points leads to an increase of the errors in the final result of the $\bar{E}_{Plat.}$ value, whereby the error intervals still include the input value for E_1. Also, the non-computable values in Figure 4c comply with the expectations, as this appears in cases with a high σ. In total, around 35 of the 50 Parameter sets followed this behaviour.

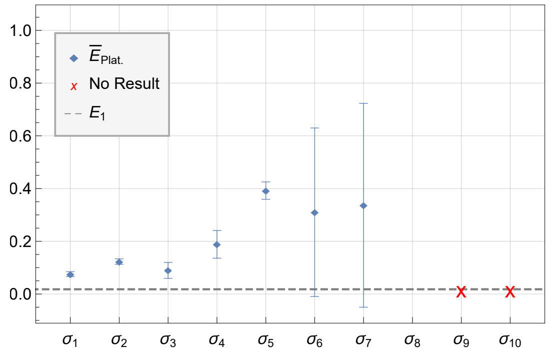
Some cases led to poor results, as seen in Figure 5. In these Parameter-Sets, only a few results for $\bar{E}_{Plat.}$ comply with the parameter E_1 within their uncertainties. These sets also show the tendency to increase their mean without increasing their uncertainty for higher values for σ (especially for Parameter-Set-41). All three of these sets have the problem of giving poor results with small uncertainties, which would mislead false accuracy. If we look at the results at low sigma for Parameter-Set-5 and Parameter-Set-28, we see that the values which match the parameter E_1 also include the value 0 in their error interval, which also corresponds to an insufficient result.



(a) Parameter-Set-5



(b) Parameter-Set-28



(c) Parameter-Set-41

Figure 5: $\bar{E}_{Plat.}$ plotted over σ for different Parameter-Sets (insufficient Examples)

A common property of the Parameter-Sets with insufficient results is the tendency to have low values for the parameters E_1 and t_{max}. This leads to a flatter course of the correlator function and hence to larger values for σ, as can be seen in Equation 19. However, the higher uncertainties should only be reflected by larger error values in the final result.

The further analysis of these cases showed a source of errors in the algorithm, which is the numerical calculation of $A_k(t)$ using Equation 12. This part proved to be very demanding in terms of computing resources and furthermore, for some $A_k(t)$ simply no solutions could be found. As σ increased, these unsolvable cases became more frequent. It should also be mentioned here that as the relative uncertainties of the correlator are highest in the plateau region, the unsolvable cases also appear mostly there. If the number of non-calculable points becomes too large, it is no longer possible to define a meaningful plateau and the algorithm returns invalid or wrong results. In some extreme cases only one of the values for $A_k(t)$ could be calculated. This is the case for Parameter-Set-5 in which for each sigma only one or two data points could be calculated and therefore, the plateau contained only one (independent) point.

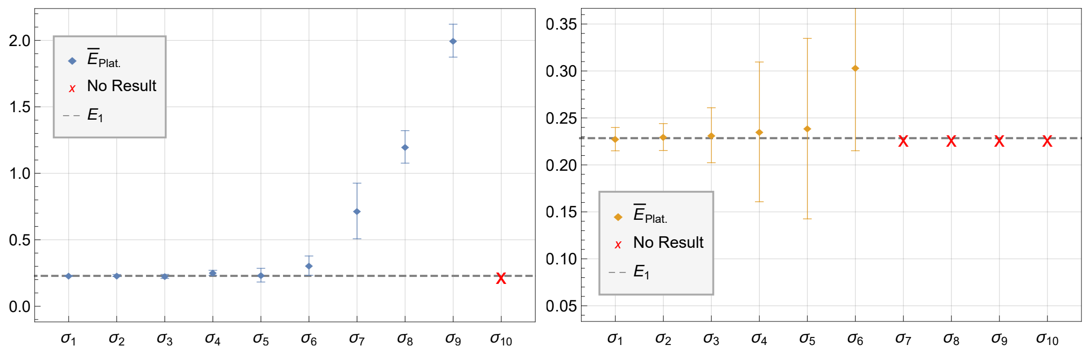
The gained insights led to general code improvements and some restrictions for the input data. As mentioned below:

1. The use of a more sophisticated calculation method for $A_k(t)$.
2. The plateau for $A_k(t \in \text{Plateau})$ must contain at least three independent points.
3. Due to (2.) the minimal lattice size was set as $t_{max} \geq 10$.

The improved calculation method was mainly reflected by multiple numerical solving with different starting parameters and the enhanced checking of the individual intermediate results.

For the last two restrictions, another justification occurred. During the process of this work, the calculation for the error for $\bar{A}_k$ was changed to using the standard derivation, which in this context was set to only be statistically significant for at least three independent points. This is already covered in section 4.

With these restrictions in place, most of the poor results could not meet the set conditions and were thus avoided. This is especially true for Parameter-Set-5 and Parameter-Set-28, as they both have a lattice size of $t_{max} = 8$. Likewise, only isolated valid results could be found for large values of σ (approx. from σ_5). This is seen in the following plot, which compares the results of the used algorithm for the test-run to the final version of the algorithm for Parameter-Set-45. The obtained values for σ_7, σ_8 and σ_9 no longer meet the specifications and the poor results from the test-run are avoided.



(a) Test-Run Version (b) Final Version

Figure 6: $\bar{E}_{Plat.}$ over σ for Parameter-Set-45

In the following, a short overview of the results obtained with the final version of the algorithm is given. Of the 500 correlators, 60 fell outside the specification of the algorithm (as they have a lattice size of $t_{max} = 8$). Furthermore, out of these 440 correlators, 147 could not be calculated because no meaningful plateau could be defined. This leaves a total number of 293 results for which 253 included the parameter E_1 in their error interval. This corresponds to 86%.

5.2. Evaluation of Physical Correlator Functions

Besides the self-generated data, two other sets of correlator data were evaluated. The source of these is the work of Bernd Riederer, especially in the context of his Master Thesis [9]. These correlators were calculated in the Weak-Higgs-Sector of the Standard Model using Variational Analysis.

These two datasets are further referred to as Dataset-1 (shown in Table 3 and Table 4) and Dataset-2 (shown in Table 5 and Table 6). They include respectively 14 correlator functions. These 14 correlators are composed of two physical correlators (Ground-State & Excited-State) for seven different lattice sizes.

For both of these datasets, the individual calculation steps are shown using two examples. Afterwards, an overview of all results is given. The selected examples each include one case that led to a result and one case that could not be calculated successfully.

Dataset-1

The first example is the correlator of the Excited-State for Dataset-1 with a lattice size $t_{max} = 16$. In Figure 7 the correlator function for this case is shown.

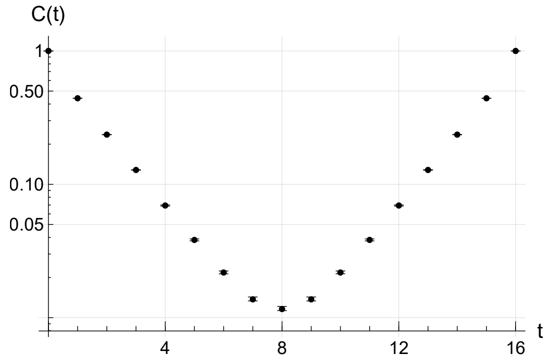


Figure 7: Excited-State correlator $C(t)$ with $t_{max} = 16$ (Dataset-1)

With the correlator points, the values of $A_k(t)$ are calculated. In the next step, the plateau is identified, and the common value $\bar{A}_k$ is computed. The values for $A_k(t)$, the identified plateau region and $\bar{A}_k$ are shown in Figure 8.

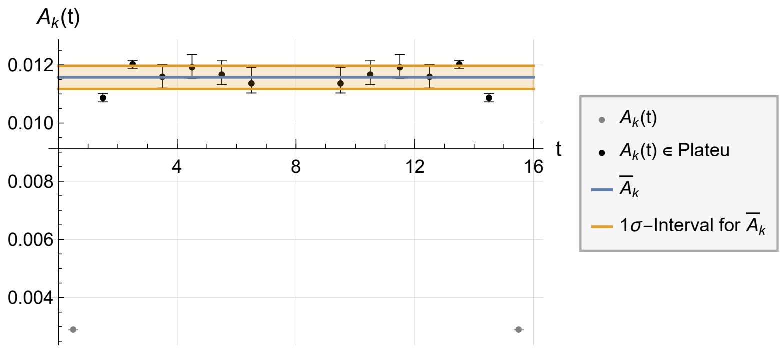


Figure 8: $A_k(t)$-Plot for Excited-State with $t_{max} = 16$ (Dataset-1)

Inserting the value of $\bar{A}_k$ into Equation 11 leads to the values of $f(\bar{A}_k, t)$ for which the common value $\bar{f}$ in the plateau region can also be computed. This is displayed in Figure 9.

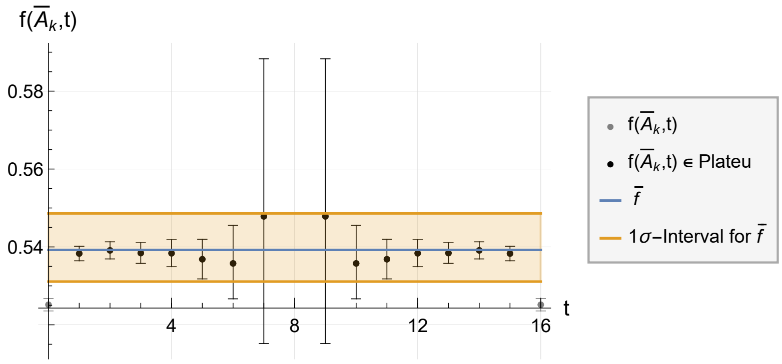


Figure 9: $f(\bar{A}_k, t)$-Plot for Excited-State with $t_{max} = 16$ (Dataset-1)

Finally, Equation 14 is used to obtain the final result:

$$\bar{E}_{\text{Plat.}} = 0.618^{+0.016}_{-0.018}$$

Not all correlators of Dataset-1 returned a result. This is demonstrated in the following using the example of the correlator of the Excited-State with $t_{max} = 32$. The plot of this correlator is shown in Figure 10.

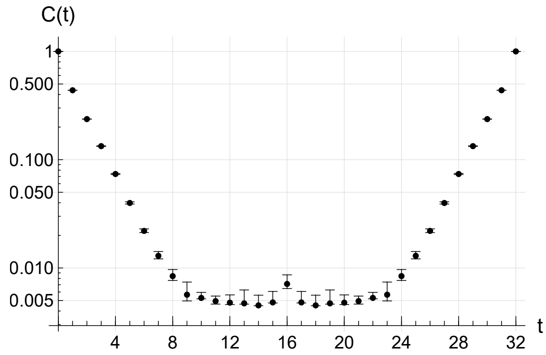


Figure 10: Excited-State correlator $C(t)$ with $t_{max} = 32$ (Dataset-1)

The $A_k(t)$'s were calculated In the same way as in the previous example. However, as seen in Figure 11, there are some values missing which could not be computed. The algorithm for the plateau then outputs only two independent points (four total). This is outside the minimum requirements and therefore no valid $\bar{A}_k$ & $\bar{E}_{Plat.}$ is returned.

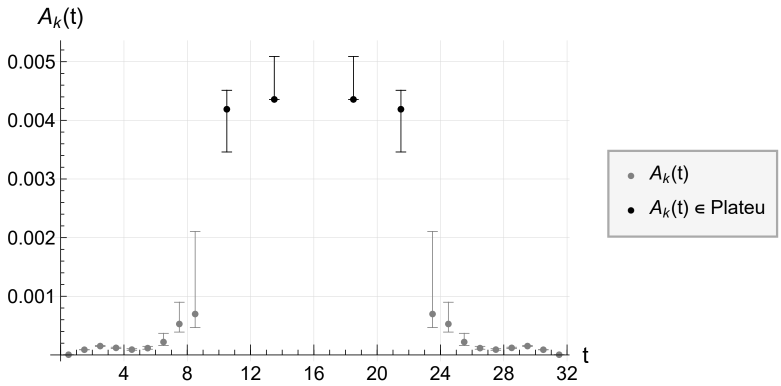


Figure 11: $A_k(t)$-Plot for Excited-State with $t_{max} = 32$ (Dataset-1)

The other results for Dataset-1 over the inverse lattice size $\frac{1}{t_{max}}$ are shown in the following graph. The two correlators with $t_{max} = 8$ were not evaluated, as they lie outside the specification of the algorithm. For the Excited-State, the evaluation with $t_{max} = 32$ and $t_{max} = 28$ led to no result.

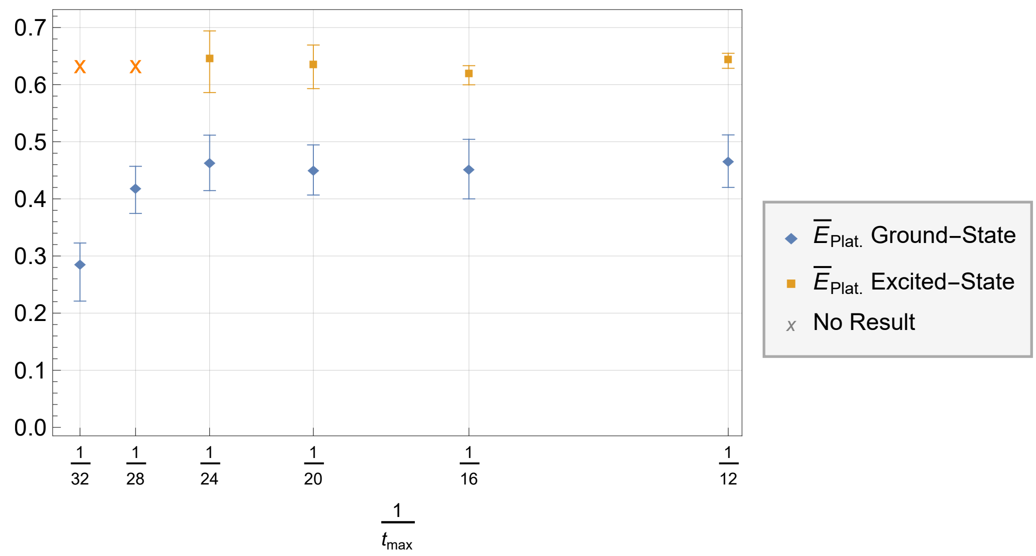
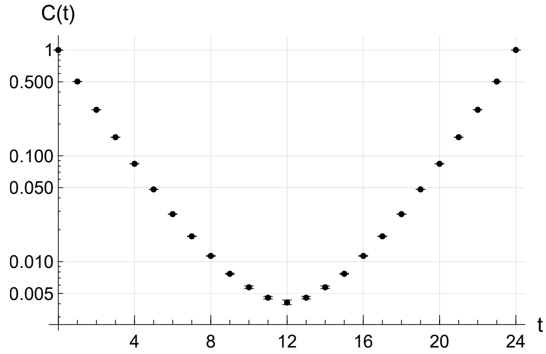


Figure 12: Results for $\bar{E}_{Plat.}$ over inverse lattice size $1/t_{max}$ for Dataset-1

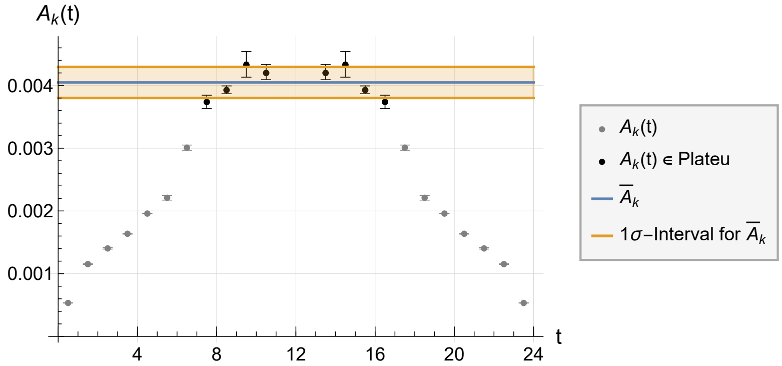
As seen in Figure 12, the obtained values are nearly constant over the lattice size, except for the value of the Ground-State correlator with $t_{max} = 32$. It is also possible to see that they correspond to distinguishable values for $\bar{E}_{\text{Plat.}}$. Furthermore, the relative errors of the values do not exceed 20%.

Dataset-2

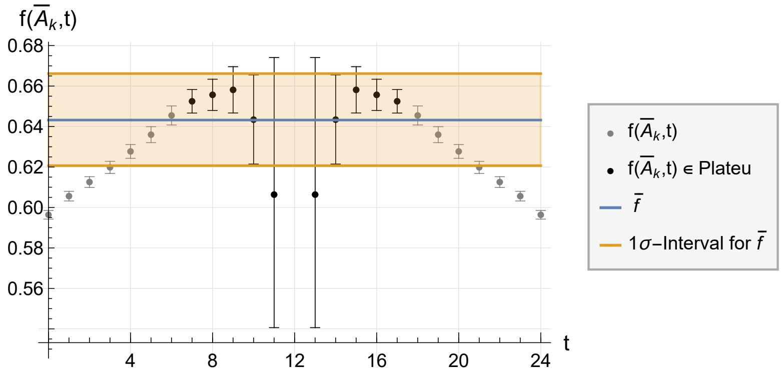
For Dataset-2 the evaluation for the Ground-State correlator with $t_{max} = 24$ is shown. Like in the previous examples, the plots for $C(t)$, $A_k(t)$ and $f(\bar{A}_k, t)$ are displayed in Figure 13. As can be seen in Figure 13b, the algorithm defined the plateau region to be between $t = 7$ and $t = 17$. In this region, four independent points for $A_k(t)$ are included. With these, the common value $\bar{A}_k$ for the plateau could be calculated. The calculated values for $f(\bar{A}_k, t)$ are displayed in Figure 13c, along with the obtained value for $\bar{f}$.



(a) Correlator $C(t)$



(b) $A_k(t)$-Plot



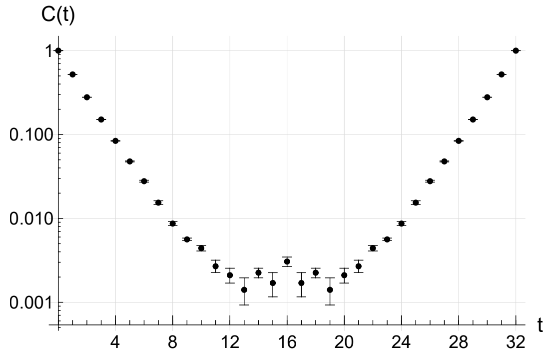
(c) $f(\bar{A}_k, t)$-Plot

Figure 13: Evaluation Ground-State Correlator with lattice size $t_{max} = 24$ (Dataset-2)

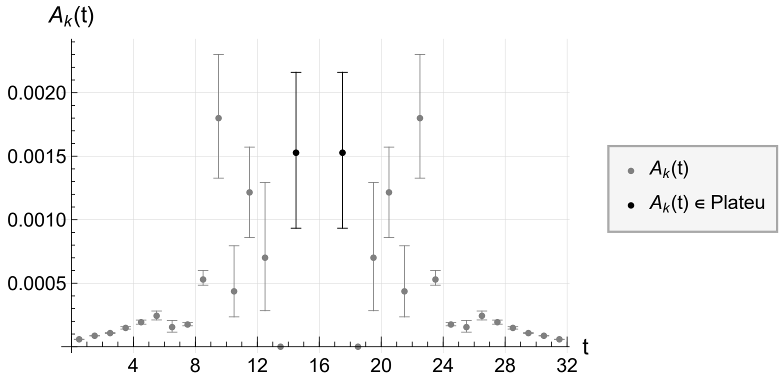
With the value for $\bar{f}$, the result for $\bar{E}_{\text{Plat.}}$ could again be calculated using Equation 14. This led to:

$$\bar{E}_{\text{Plat.}} = 0.44^{+0.04}_{-0.04}$$

As the last example, the evaluation of the Ground-State with $t_{max} = 32$ is shown. Figure 14 shows the plot of the correlator and the plot for $A_k(t)$.



(a) Correlator $C(t)$



(b) $A_k(t)$-Plot

Figure 14: Evaluation Ground-State Correlator with lattice size $t_{max} = 32$ (Dataset-2)

In this case, the $C(t)$ points around $\frac{t_{max}}{2}$ fluctuate a lot. This leads to a similar behaviour for the $A_k(t)$ values. It is not possible to define a plateau for such distributed values. Therefore, for this correlator no result could be obtained.

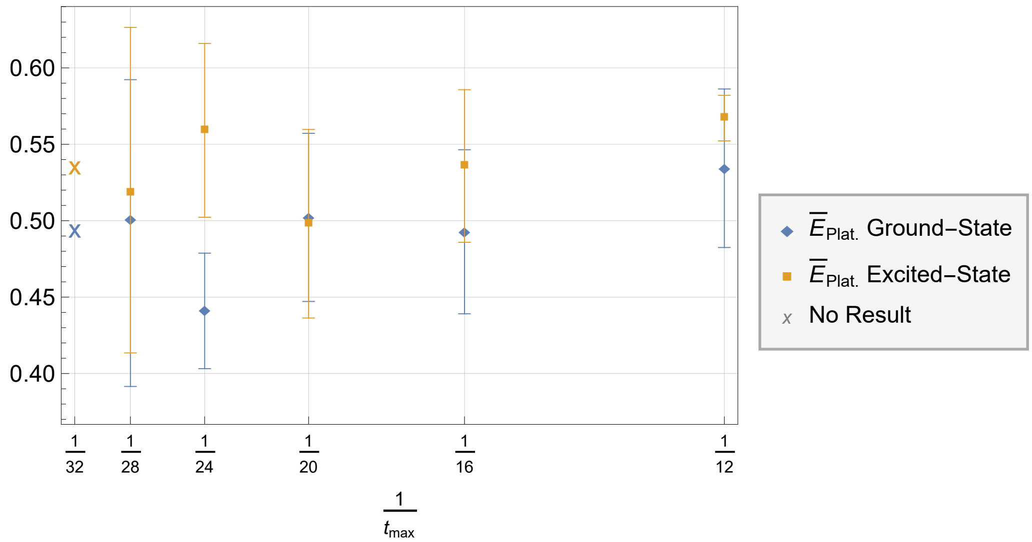


Figure 15: Results for $\bar{E}_{Plat.}$ over inverse lattice size $1/t_{max}$ for Dataset-2

Figure 15 shows again the results for $\bar{E}_{Plat.}$ for Dataset-2. As before, the correlators with $t_{max} = 8$ were not evaluated. For this set of correlators, the Ground-State as well as the Excited-State for $t_{max} = 32$ lead to no result. In contrast to the results of Dataset-1, the obtained values for the Ground-State and the Excited-State are on top of each other. However, in good approximation they show a constant value over the different lattice sizes. Furthermore, the relative errors of the values do not exceed 20%.

6. Conclusion & Outlook

6.1. Conclusion

In this thesis, a new algorithm for obtaining the Ground-State energy from the two-point correlator function was developed and tested. The method is built on the assumption that the correlator can in some region be described by a single cosh term with the energy level $\bar{E}_{Plat.}$.

As described in subsection 4.2, an algorithm for defining the plateau region was developed using the coefficient of determination. Although the method proved to be effective, it has some underlying shortcomings. Firstly, as used in this thesis, the coefficient of determination is of limited value as a statistical measure. Furthermore, the uncertainties of the data points themselves do not influence the definition of the plateau. So, while this part is sufficient for this work, it might need further improvements if the settings change (e.g. bigger lattice size).

The algorithm was then tested by using self-generated mock-up data, which were generated by assuming double cosh terms. For a large part of the cases considered, the results showed sufficient behaviour in which declining statistics for the data points led to an increase in the uncertainty of the result with similar mean. This is seen in Figure 4. However, some cases did not match this behaviour, as seen in Figure 5. The analysis of these cases helped to gain further insights and improve the algorithm, which avoided most of the poor results.

With the reviewed algorithm, two sets of correlators derived from physical simulations were evaluated. These results are shown in Figure 12 and Figure 15. In total, the algorithm returned a result for 20 of the 24 correlators[5]. The relative errors of the obtained values for $\bar{E}_{Plat.}$ were smaller than 20%. The results suggest that the algorithm works best until $t_{max} = 24$, as for larger lattice sizes only two out of four results could be obtained. However, given the small number of evaluated correlators, this statement needs further verification.

In conclusion, the new algorithm provides a sufficient tool in obtaining the Ground-State energy level of the correlator on the conditions that the plateau region is well-developed enough to justify the approximation of a single cosh term, and the statistics of the correlator allows the calculation of an adequate number of values for $A_k(t)$.

[5]Excluding the correlators with $t_{max} = 8$.

6.2. Outlook

Since the scope of this work was limited, it was not possible to review and test all options that arose. Therefore, two starting points for future improvements and extensions are given here.

Using the $C(t_{max}/2)$-Point

The algorithm, as implemented, does not use the correlator point at $\frac{t_{max}}{2}$. This is due to the fact that $f(A_k(t), t)$ (see Equation 11) is not defined at $t_{max}/2$. However, as seen in Equation 8, the value $C(t_{max}/2)$ corresponds directly to the value of $A_k(t_{max}/2)$. This can simply be used by inserting this value into the calculated $A_k(t)$.

$$A_k(t_{max}/2) = C(t_{max}/2)$$

This results in an immediate advantage. With the additional value for $A_k(t)$, a plateau with a meaningful statistics for $\bar{A}_k$ can also be defined for correlators with $t_{max} = 8$. Figure 16 displays this for the Ground-State correlator from Dataset-1 (with $t_{max} = 8$).

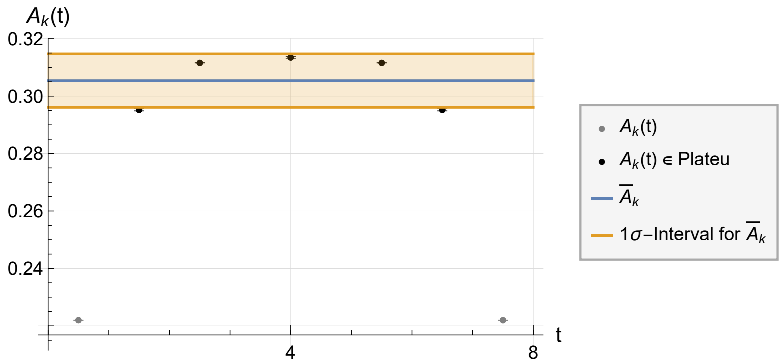


Figure 16: $A_k(t)$-Plot for Ground-State with $t_{max} = 8$ and $C(t_{max}/2)$-Point (Dataset-1)

However, it also worsens some results, which is especially true for bigger lattice sizes. A detailed analysis of whether and when it makes sense to use this point has been omitted. Nevertheless, it definitely offers the possibility to improve the method and to extend the range in terms of the lattice size.

Application on more than one Plateau

Another interesting finding that occurred is the formation of further plateaus in the $A_k(t)$ plots. As the only assumption on which the algorithm is based is that the correlator can be described by a single cosh term in some region, these plateaus could correspond to further terms of the correlator. The following figure shows this for the Ground-State correlator from Dataset-1 with a lattice size $t_{max} = 32$ and a second plateau between $t = 5$ and $t = 9$.

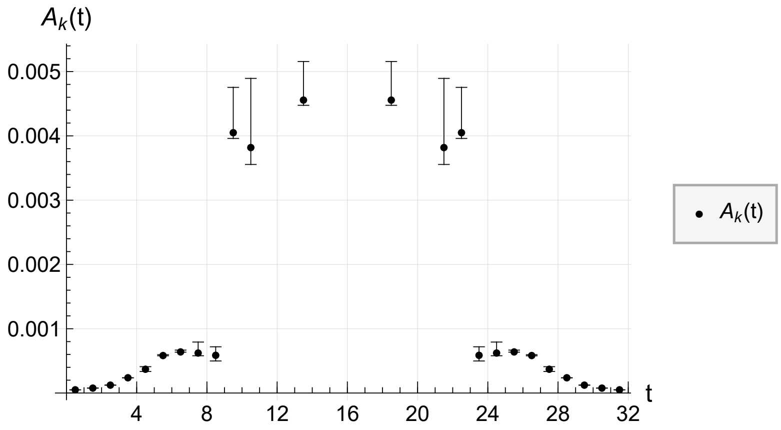


Figure 17: $A_k(t)$-Plot for Ground-State with $t_{max} = 32$ (Dataset-1)

However, further work needs to be performed to establish whether this hypothesis is true.

A. References

[1] B. Abi et al. "Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm". In: *Phys. Rev. Lett.* 126 (14 Apr. 2021), p. 141801. DOI: 10.1103/PhysRevLett.126.141801.

[2] Tilo Arens et al. *Mathematik*. 4th ed. Springer-Verlag, 2018. ISBN: 978-3-662-56740-1. DOI: 10.1007/978-3-662-56741-8.

[3] nLab authors. *Wick rotation*. Revision 24. Oct. 2021. URL: http://ncatlab.org/nlab/show/Wick%20rotation.

[4] Kenneth S. Krane. *Modern physics*. 3rd ed. John Wiley & Sons, 2012. ISBN: 978-1-118-06114-5.

[5] Gernot Münster and Markus Walzl. "Lattice gauge theory-a short primer". In: *arXiv: High Energy Physics - Lattice* (2000).

[6] Michael E. Peskin and Daniel V. Schroeder. *An introduction to quantum field theory*. CRC press, 1995. ISBN: 0-201-50397-2.

[7] Wolfram Reserach. *Around*. Accessed: 01-November-2021. 2019. URL: https://reference.wolfram.com/language/ref/Around.html.

[8] Wolfram Reserach. *NonlinearModelFit*. Accessed: 01-November-2021. 2008. URL: https://reference.wolfram.com/language/ref/NonlinearModelFit.html.

[9] Bernd Riederer. "Scattering Phases in the $H \to WW^*$ channel". MA thesis. Graz: University of Graz & Graz University of Technology, Feb. 2020.

[10] Jun J. Sakurai and Jim Napolitano. *Modern quantum mechanics*. 2nd ed. Pearson Education Limited, 2014. ISBN: 978-1-292-02410-3.

B. Code

```
ClearAll["Global`*"];

(*Empty lists for error and warning messages*)
warnings = {};
errors = {};

(*Option to use C(tmax/2) as value for Ak(tmax/2*)
optionAktmax2 = False;

(*Input correlator values for t=0 until tmax/2*)
Cor = {1.0000000000001232, 0.4409447180355774, 0.23132195469510042,
      0.12446643074340222, 0.06948477112729584, 0.042736568101142375,
      0.03505537623756215};
(*Uncertainties for corr dCor[[t,pm] pm=1 -> -dCor pm=2 -> +dCor*)
dCor = {
  {4.43045600206915*10^-12, 1.667554982986985*10^-13},
  {0.000555878614993010б, 0.0005579342793681219},
  {0.0007696064024982074, 0.0007773783527582068},
  {0.0008165312757394683, 0.0008453708052333364},
  {0.0007715860858287654, 0.0008186093711772674},
  {0.0005932584236703412, 0.0006652480377861494},
  {0.00046365718232631997, 0.0006715113022606051}};

nN = (Length[Cor] - 1)*2;

(*Definition of f(Ak(t),t)*)
func[paramCorr_, paramAk_, paramT_] :=
  ((paramCorr/paramAk)+Sqrt[(paramCorr/paramAk)^2-1])^(1/(paramT-nN/2))

(*Input Corr to Formats which are needed during calculation*)
corr = Cor
corrMin = corr - Transpose[dCor][[1]]
corrMax = corr + Transpose[dCor][[2]]
corrErr = Table[Around[corr[[i]],
    {(Transpose[dCor][[1]])[[i]],
      (Transpose[dCor][[2]])[[i]]}], {i, Length[corr]}];
corrSymErrT = Transpose[{Range[nN + 1] - 1,
            Transpose[Join[Transpose[corrErr],
              Drop[Reverse[Transpose[corrErr]], 1]]]}]

(*
Get indices of all correlators which are negative within their errors
To avoid calc of f(Ak(t+1/2),t)==f(Ak(t+1/2),t+1) in this cases
*)
negIndCorr = Position[corrMin, _?Negative];
delIndFunc = Partition[DeleteCases[
    Flatten[Union[negIndCorr, (negIndCorr - 1)]],
    x_ /; (x > nN/2 - 1) || (x < 1)], 1];
```

```
(*
Gen. List of f(Ak(t+1/2),t)==f(Ak(t+1/2),t+1) for Mean, Max & Min val.
*)
funcAk = Transpose[
    {
    (*Time*)
    Range[nN/2 - 1] - 0.5,
    (*Mean*)
    Table[
      func[corr[[i]], A, i - 1] ==
      func[corr[[i + 1]], A, i], {i, nN/2 - 1}],
    (*Max*)
    Table[
      func[corrMax[[i]], A, i - 1] ==
      func[corrMax[[i + 1]], A, i], {i, nN/2 - 1}],
    (*Min*)
    Table[
      func[corrMin[[i]], A, i - 1] ==
      func[corrMin[[i + 1]], A, i], {i, nN/2 - 1}]
    }
    ];

funcAk = Delete[funcAk, delIndFunc]; (*Delete invalid positions*)

(*Empty Lists for Solution of A_k(t+1/2)*)
Akbyfunc = {};
AkbyfuncMin = {};
AkbyfuncMax = {};

(*Calculate A_k(t+1/2) using f(Ak(t+1/2),t)==f(Ak(t+1/2),t+1)*)
(*-------------------------------------------------------------------*)
(*
For finding values for A_k(t+1/2) FindRoot is applied for which two
different approaches are used the second only if the first does not
lead to a result):
(1) FindRoot[lhs==rhs,{x,x0,x1}] searches for a solution using
  x0 and x1 as the first two values of x,avoiding the use of
  derivatives.
(2) FindRoot[lhs==rhs,{x,xstart,xmin,xmax}] searches for a solution,
  stopping the search if x ever gets outside the range xmin to xmax.
*)

(*Start values for Approach (1) *)
(*50% and 75% of lowest value of Corr CorrMax & CorrMin *)
guessA = Min[Delete[corr, negIndCorr]]*{0.5, 0.75};
guessAMax = Min[Delete[corrMax, negIndCorr]]*{0.5, 0.75};
guessAMin =  Min[Delete[corrMin, negIndCorr]]*{0.5, 0.75};
```

```
(*Limits for Approach (2) *)
(* Lower Bound is Min of CorrMin times 10^-12*)
lowerLimA =
  Min[Delete[corrMin,
    negIndCorr]]*10^(-12);
(* Upper Bound is corr(t), corrMax(t) & corrMin(t)*)
upperLimA =
  Table[Min[{corr[[i]], corr[[i + 1]]}], {i, nN/2 - 1}];
upperLimA = Delete[upperLimA, delIndFunc];
upperLimAMax =
  Table[Min[{corrMax[[i]], corrMax[[i + 1]]}], {i, nN/2 - 1}];
upperLimAMax = Delete[upperLimAMax, delIndFunc];
upperLimAMin =
  Table[Min[{corrMin[[i]], corrMin[[i + 1]]}], {i, nN/2 - 1}];
upperLimAMin = Delete[upperLimAMin, delIndFunc];

(*
Solving for A_k(t+1/2). Check if Approach 1 leads to a result if not
try Approach 2.
*)
For[i = 1, i <= Length[funcAk], i++,
  errCheck =
  Quiet[
    Check[
    (*Approach 1*)
    Ak = FindRoot[funcAk [[i, 2]], {A, guessA[[1]], guessA[[2]]},
      MaxIterations -> 1000];
    AkMax =
      FindRoot[funcAk [[i, 3]], {A, guessAMax[[1]], guessAMax[[2]]},
      MaxIterations -> 1000];
    AkMin =
      FindRoot[funcAk [[i, 4]], {A, guessAMin[[1]], guessAMin[[2]]},
      MaxIterations -> 1000];
    errCheck = False,
    True,
    (*Errors which are checked directly in calculation (1)*)
    Power::infy, FindRoot::cvmit, FindRoot::nlnum , FindRoot::lstol ,
    Infinity::indet
    ],
    {Power::infy, FindRoot::cvmit, FindRoot::nlnum , FindRoot::lstol ,
    Infinity::indet}
    ];
  (*Check for complex solution*)
  AkSorted = Sort[{(A /. Ak), ( A /. AkMax), (A /. AkMin)}];
  If[Length[Position[AkSorted, z_ /; Im[z] != 0]] != 0,
  errCheck = True];
```

```
If[errCheck,
errCheck =
  Quiet[
  Check[
    (*Approach 2*)
    Ak =
    FindRoot[
      funcAk [[i, 2]], {A, upperLimA[[i]]*0.5, lowerLimA,
      upperLimA[[i]]}, MaxIterations -> 100000];
    AkMax =
    FindRoot[
      funcAk [[i, 3]], {A, upperLimAMax[[i]]*0.5, lowerLimA,
      upperLimAMax[[i]]}, MaxIterations -> 100000];
    AkMin =
    FindRoot[
      funcAk [[i, 4]], {A, upperLimAMin[[i]]*0.5, lowerLimA,
      upperLimAMin[[i]]}, MaxIterations -> 100000];
    errCheck = False,
    True,
    FindRoot::reged, FindRoot::lstol, Infinity::indet (*(1)*)
    ],
  {FindRoot::reged, FindRoot::lstol, Infinity::indet}
  ];
(*Check for complex solution*)
AkSorted = Sort[{(A /. Ak), ( A /. AkMax), (A /. AkMin)}];
If[Length[Position[AkSorted, z_ /; Im[z] != 0]] != 0,
  errCheck = True];
];

If[! errCheck,
(*if*)
AppendTo[Akbyfunc, {funcAk[[i, 1]], AkSorted[[2]]}];
AppendTo[AkbyfuncMax, {funcAk[[i, 1]], AkSorted[[3]]}];
AppendTo[AkbyfuncMin, {funcAk[[i, 1]], AkSorted[[1]]}],
(*else*)
AppendTo[warnings,
  {
    50, "Warning: Could not calculate Ak(t) for t=", funcAk[[i, 1]]
  }
  ]]]

(* List for Ak(t) and min, max values*)
akT = Union[Akbyfunc,
  Reverse[Table[{nN - Akbyfunc[[i]][[1]], Akbyfunc[[i]][[2]]}, {i,
    Length[Akbyfunc]}]]];
akTMax = Union[AkbyfuncMax,
  Reverse[Table[{nN - AkbyfuncMax[[i]][[1]],
    AkbyfuncMax[[i]][[2]]}, {i, Length[AkbyfuncMax]}]]];
akTMin = Union[AkbyfuncMin,
  Reverse[Table[{nN - AkbyfuncMin[[i]][[1]],
    AkbyfuncMin[[i]][[2]]}, {i, Length[AkbyfuncMin]}]]];
```

```
If[Length[akT] == 0,
   AppendTo[errors,
    {30,"For none of the times a value for Ak(t) was found."}
   ]
  ];

(*Add value of C(tmax/2) to Ak(t) if optionAktmax2 is true*)
If[optionAktmax2,
  akT = Insert[akT,{nN/2,Cor[[nN/2+1]]},Length[akT]/2+1];
  akTMax = Insert[akTMax,{nN/2,corrMax[[nN/2+1]]},Length[akTMax]/2+1];
  akTMin = Insert[akTMin,{nN/2,corrMin[[nN/2+1]]},Length[akTMin]/2+1]
]

(*List of Ak(t) with errors (only used for plotting*)
akTEr = Table[
  {
   akT[[i]][[1]],
   Around[
    akT[[i]][[2]], {akTMin[[i]][[2]] - akT[[i]][[2]],
     akTMax[[i]][[2]] - akT[[i]][[2]]}]
   },
  {i, Length[akT]}
  ]

(*Find Plateau*)
(*-------------------------------------------------------------------*)
(*Parameters for Finding the Plateau*)
epsMin = 0.01;
rSquaredMin = 0.8;
eps = 1;
rSquaredOld = 0;
indPlateu = 1;
akPlateuConsFit = NonlinearModelFit[akT[[indPlateu ;; -indPlateu]],
   consAk, consAk, x];
rSquared = akPlateuConsFit["RSquared"];

While[
 (*Condition*)
 (eps > epsMin || rSquared <= rSquaredMin ) &&
  indPlateu < Length[akT]/2,
 (*Loop*)
 indPlateu++;
 rSquaredOld = rSquared;
 akPlateuConsFit = NonlinearModelFit[
   akT[[indPlateu ;; -indPlateu]],
   consAk, consAk, x];
 rSquared = akPlateuConsFit["RSquared"];
 eps = rSquared - rSquaredOld;
 ]
If[eps < epsMin, indPlateu--];
```

```
  If[Length[akTPlateauEr] <= 4,
  AppendTo[
   errors, {60,
    "The plateau consists of less than 3 independent points."}]];

(*Value & Uncertainty for Ak for t in Plateau*)
aKFitValue = Mean[Transpose[akT[[indPlateu ;; -indPlateu]]][[2]]];
akSigma =
  StandardDeviation[Transpose[akT[[indPlateu ;; -indPlateu]]][[2]]];

(*Final Result for Ak for t in Plateau*)
akFitValueEr = Around[aKFitValue, akSigma]

(*Get Time borders of Plateau with integer times*)
plateuTimeBorder =
  {IntegerPart[Transpose[akT][[1]][[indPlateu]]], nN/2};

(*Calculate f(A_k = const, t)*)
(*-------------------------------------------------------------------*)
funcByAkFit = Range[nN/2];
funcByAkFitBadInd = {};

For[i = 1, i <= nN/2, i++,
 (*Check if term is not 0 otherwise this would lead to division by 0*)
 If[(corrSymErrT[[i,-1]]/akFitValueEr +
    Sqrt[(corrSymErrT[[i,-1]]/akFitValueEr)^2 - 1])["Value"] != 0.0,
      (*THEN*)
    funcByAkFit[[i]] =
     {
      corrSymErrT[[i, 1]],
      func [corrSymErrT[[i, -1]], akFitValueEr, corrSymErrT[[i, 1]]]
      },
     (*ELSE*)
    AppendTo[funcByAkFitBadInd, i]
  ]
 ]

(*Delete Bad Values*)
funcByAkFitBadInd = Partition[funcByAkFitBadInd, 1];
funcByAkFit = Delete[funcByAkFit, funcByAkFitBadInd];
funcByAkFit = DeleteCases[funcByAkFit, x_ /; Im[ x[[2]]] != 0];

indexFunc = Flatten[Position[Transpose[funcByAkFit][[1]],
            n_ /; n >= plateuTimeBorder[[1]] &&
              n <= plateuTimeBorder[[2]]]][[1]];

funcByAkFitSym = Union[funcByAkFit,
  Reverse[Table[{nN - funcByAkFit[[i]][[1]], funcByAkFit[[i]][[2]]},
    {i, Length[funcByAkFit]}]]];
```

```
(*Values of f(Ak,t in Plateau)*)
funcByAkFitPlatEr =  funcByAkFitSym[[indexFunc ;; -indexFunc]];
(*Values of f(Ak,t in Plateau) without errors*)
funcByAkFitPlat = Transpose[{
  Transpose[funcByAkFitSym][[1]][[indexFunc ;; -indexFunc]],
    Table[m["Value"], {m, Transpose[funcByAkFitSym][[2]]}][[
      indexFunc ;; -indexFunc]]}];

(*Calculate mean of f(Ak,t) for t in Plateau*)
consFuncFit =  NonlinearModelFit[funcByAkFitPlat, consF, consF, x];
funcFitValue = consF /. consFuncFit["BestFitParameters"];

If[consFuncFit["RSquared"] < 0.80,
  AppendTo[warnings,
    {61,"f(A_k,t) might not be const. in plateau region (R^2<0.8)."}]]

(*Calculate Errors of f(Ak,t) for t in Plateau*)
funcFitValueMax = Mean[Table[Max[m["Interval"]],
    {m, Transpose[funcByAkFitSym][[2]]}][[indexFunc ;; -indexFunc]]];
funcFitValueMin = Mean[Table[Min[m["Interval"]],
    {m, Transpose[funcByAkFitSym][[2]]}][[indexFunc ;; -indexFunc]]];

(*Final Result for f for t in Plateau*)
funcFitValueEr = Around[funcFitValue,
    {funcFitValueMin - funcFitValue, funcFitValueMax - funcFitValue}];

(*Final value for E_Plat. for t in plateau*)
Eplat = -Log[funcFitValueEr];
```

C. Appendix

Table 1: Random parameters for Test-Run (1)

No.	...	Number of Parameter-Set
t_{max}	...	Parameter for lattice size
E_1, E_2, $\frac{A_2}{A_1}$	...	Parameters according to Equation 18
σ_0	...	Parameter for normally distributed shift (see Equation 19)

No.	$\boldsymbol{t_{max}}$	$\boldsymbol{E_1}$	$\boldsymbol{E_2}$	$\boldsymbol{\frac{A_2}{A_1}}$	$\boldsymbol{\sigma_0}$
1	16	0.177	0.531	5.0E-07	2.3E-03
2	24	0.220	0.282	1.0E-07	1.1E-03
3	28	0.360	0.749	1.0E-08	1.1E-04
4	8	0.351	0.494	1.0E-08	1.2E-03
5	8	0.011	0.884	1.0E-09	2.5E-03
6	16	0.516	0.709	5.0E-08	1.6E-04
7	8	0.652	0.653	1.0E-07	3.7E-04
8	32	0.337	0.884	1.0E-06	9.1E-05
9	20	0.902	0.963	5.0E-06	1.5E-06
10	24	0.100	0.671	1.0E-07	4.1E-03
11	32	0.782	0.830	5.0E-08	7.3E-08
12	24	0.024	0.454	5.0E-09	7.2E-03
13	24	0.109	0.143	1.0E-08	3.8E-03
14	16	0.676	0.974	1.0E-07	4.5E-05
15	28	0.521	0.671	1.0E-06	1.2E-05
16	12	0.688	0.825	5.0E-07	1.2E-04
17	12	0.358	0.393	5.0E-09	8.6E-04
18	16	0.525	0.604	1.0E-08	1.5E-04
19	8	0.044	0.541	5.0E-08	2.5E-03
20	24	0.038	0.050	5.0E-06	6.8E-03
21	12	0.158	0.596	1.0E-06	2.5E-03
22	16	0.685	0.696	1.0E-09	4.2E-05
23	20	0.087	0.122	1.0E-09	4.5E-03
24	24	0.253	0.898	1.0E-06	7.1E-04
25	16	0.915	0.983	5.0E-07	6.6E-06

Table 2: Random parameters for Test-Run (2)

No.	...	Number of Parameter-Set
t_{max}	...	Parameter for lattice size
E_1, E_2, $\frac{A_2}{A_1}$	...	Parameters according to Equation 18
σ_0	...	Parameter for normally distributed shift (see Equation 19)

No.	$\boldsymbol{t_{max}}$	$\boldsymbol{E_1}$	$\boldsymbol{E_2}$	$\boldsymbol{\frac{A_2}{A_1}}$	$\boldsymbol{\sigma_0}$
26	24	0.774	0.872	1.0E-06	1.4E-06
27	24	0.630	0.703	1.0E-06	7.8E-06
28	8	0.030	0.052	1.0E-09	2.5E-03
29	24	0.223	0.852	1.0E-09	1.0E-03
30	24	0.525	0.695	5.0E-08	2.7E-05
31	32	0.014	0.850	5.0E-06	3.3E-03
32	12	0.107	0.898	5.0E-06	3.1E-03
33	32	0.373	0.884	5.0E-06	5.1E-05
34	32	0.384	0.669	5.0E-08	4.3E-05
35	32	0.323	0.596	5.0E-06	1.1E-04
36	32	0.119	0.363	1.0E-07	2.9E-03
37	12	0.420	0.797	1.0E-05	6.0E-04
38	16	0.573	0.947	1.0E-05	1.0E-04
39	12	0.318	0.932	1.0E-09	1.1E-03
40	24	0.281	0.469	5.0E-06	5.1E-04
41	20	0.018	0.279	5.0E-06	6.2E-03
42	20	0.099	0.765	1.0E-05	4.1E-03
43	20	0.262	0.530	1.0E-08	9.1E-04
44	28	0.844	0.927	5.0E-07	1.3E-07
45	20	0.228	0.581	5.0E-06	1.3E-03
46	16	0.457	0.564	1.0E-05	2.6E-04
47	20	0.485	0.989	5.0E-08	9.8E-05
48	8	0.120	0.721	1.0E-08	2.2E-03
49	24	0.061	0.612	1.0E-05	5.8E-03
50	20	0.726	0.796	1.0E-07	8.8E-06

Table 3: Dataset-1 Ground-State *(CorDat_4-0.285-0.97_L-8-4-32)*

$C_N(t)$ ... Correlator values with $t_{max} = N$
$\delta C_N(t)$ ... Uncertainties for correlator values with $t_{max} = N$

$C_8(t)$	$\delta C_8(t)$	$C_{12}(t)$	$\delta C_{12}(t)$	$C_{16}(t)$	δC_{16}	$C_{20}(t)$	$\delta C_{20}(t)$	$C_{24}(t)$	$\delta C_{24}(t)$	$C_{28}(t)$	δC_{28}	$C_{32}(t)$	$\delta C_{32}(t)$
1.	$^{+2.8\times10^{-11}}_{-3.7\times10^{-11}}$	1.	$^{+2.\times10^{-11}}_{-8.9\times10^{-11}}$	1.	$^{+1.8\times10^{-10}}_{-8.2\times10^{-11}}$	1.	$^{+1.2\times10^{-10}}_{-8.1\times10^{-11}}$	1.	$^{+8.3\times10^{-11}}_{-5.3\times10^{-11}}$	1.	$^{+3.8\times10^{-12}}_{-4.3\times10^{-12}}$	1.	$^{+1.5\times10^{-11}}_{-3.8\times10^{-13}}$
0.593326	$^{+2.7\times10^{-5}}_{-3.1\times10^{-5}}$	0.534507	$^{+7.2\times10^{-5}}_{-7.\times10^{-5}}$	0.522050	$^{+1.8\times10^{-4}}_{-1.8\times10^{-4}}$	0.517781	$^{+1.8\times10^{-4}}_{-1.8\times10^{-4}}$	0.515322	$^{+8.7\times10^{-5}}_{-8.6\times10^{-5}}$	0.514832	$^{+6.2\times10^{-5}}_{-6.1\times10^{-5}}$	0.515673	$^{+4.1\times10^{-4}}_{-4.1\times10^{-4}}$
0.417603	$^{+2.4\times10^{-4}}_{-2.5\times10^{-4}}$	0.313902	$^{+7.\times10^{-5}}_{-6.7\times10^{-5}}$	0.290008	$^{+1.7\times10^{-4}}_{-1.7\times10^{-4}}$	0.280557	$^{+3.\times10^{-4}}_{-3.\times10^{-4}}$	0.275229	$^{+1.3\times10^{-4}}_{-1.3\times10^{-4}}$	0.273280	$^{+8.3\times10^{-5}}_{-8.1\times10^{-5}}$	0.273590	$^{+2.9\times10^{-4}}_{-2.9\times10^{-4}}$
0.337042	$^{+5.3\times10^{-5}}_{-6.2\times10^{-5}}$	0.202885	$^{+9.9\times10^{-5}}_{-9.3\times10^{-5}}$	0.173630	$^{+2.6\times10^{-4}}_{-2.5\times10^{-4}}$	0.161354	$^{+2.4\times10^{-5}}_{-2.\times10^{-5}}$	0.154622	$^{+1.8\times10^{-4}}_{-1.8\times10^{-4}}$	0.151071	$^{+4.4\times10^{-6}}_{-5.5\times10^{-6}}$	0.150133	$^{+2.9\times10^{-4}}_{-2.8\times10^{-4}}$
0.313463	$^{+2.6\times10^{-4}}_{-2.3\times10^{-4}}$	0.145008	$^{+2.\times10^{-4}}_{-1.8\times10^{-4}}$	0.111181	$^{+1.1\times10^{-4}}_{-1.1\times10^{-4}}$	0.098229	$^{+1.3\times10^{-4}}_{-1.2\times10^{-4}}$	0.092022	$^{+1.1\times10^{-4}}_{-1.1\times10^{-4}}$	0.088329	$^{+5.7\times10^{-5}}_{-5.4\times10^{-5}}$	0.086656	$^{+1.6\times10^{-4}}_{-1.4\times10^{-4}}$
-	- -	0.117072	$^{+1.9\times10^{-4}}_{-1.7\times10^{-4}}$	0.076234	$^{+2.2\times10^{-4}}_{-2.1\times10^{-4}}$	0.062587	$^{+5.5\times10^{-5}}_{-4.5\times10^{-5}}$	0.056443	$^{+2.2\times10^{-4}}_{-2.1\times10^{-4}}$	0.052950	$^{+1.4\times10^{-4}}_{-1.3\times10^{-4}}$	0.051909	$^{+5.4\times10^{-4}}_{-5.\times10^{-4}}$
-	- -	0.109404	$^{+4.2\times10^{-4}}_{-3.6\times10^{-4}}$	0.056990	$^{+1.7\times10^{-4}}_{-1.5\times10^{-4}}$	0.041426	$^{+2.5\times10^{-4}}_{-2.3\times10^{-4}}$	0.035655	$^{+8.6\times10^{-5}}_{-7.6\times10^{-5}}$	0.032620	$^{+1.9\times10^{-4}}_{-1.8\times10^{-4}}$	0.032409	$^{+3.7\times10^{-4}}_{-3.1\times10^{-4}}$
-	- -	-	- -	0.046897	$^{+2.7\times10^{-4}}_{-2.3\times10^{-4}}$	0.028777	$^{+2.4\times10^{-4}}_{-2.1\times10^{-4}}$	0.022882	$^{+1.3\times10^{-4}}_{-1.1\times10^{-4}}$	0.020100	$^{+2.3\times10^{-4}}_{-2.\times10^{-4}}$	0.020412	$^{+3.1\times10^{-4}}_{-1.6\times10^{-4}}$
-	- -	-	- -	0.044184	$^{+2.6\times10^{-4}}_{-2.3\times10^{-4}}$	0.021512	$^{+1.9\times10^{-4}}_{-1.4\times10^{-4}}$	0.015205	$^{+1.8\times10^{-4}}_{-1.6\times10^{-4}}$	0.012820	$^{+3.1\times10^{-4}}_{-2.7\times10^{-4}}$	0.012828	$^{+5.3\times10^{-4}}_{-1.9\times10^{-4}}$
-	- -	-	- -	-	- -	0.017553	$^{+2.1\times10^{-4}}_{-1.4\times10^{-4}}$	0.010656	$^{+2.5\times10^{-4}}_{-2.\times10^{-4}}$	0.008871	$^{+3.3\times10^{-4}}_{-2.4\times10^{-4}}$	0.008006	$^{+5.1\times10^{-4}}_{-2.6\times10^{-4}}$
-	- -	-	- -	-	- -	0.016985	$^{+4.2\times10^{-4}}_{-2.5\times10^{-4}}$	0.008015	$^{+2.3\times10^{-4}}_{-1.7\times10^{-4}}$	0.006394	$^{+3.1\times10^{-4}}_{-2.1\times10^{-4}}$	0.006821	$^{+6.1\times10^{-4}}_{-1.5\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	0.006819	$^{+2.9\times10^{-4}}_{-2.\times10^{-4}}$	0.004889	$^{+3.9\times10^{-4}}_{-1.6\times10^{-4}}$	0.005734	$^{+8.8\times10^{-4}}_{-1.1\times10^{-6}}$
-	- -	-	- -	-	- -	-	- -	0.006683	$^{+4.3\times10^{-4}}_{-2.8\times10^{-4}}$	0.003942	$^{+4.\times10^{-4}}_{-2.2\times10^{-4}}$	0.005803	$^{+3.9\times10^{-4}}_{-2.3\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.003931	$^{+2.2\times10^{-4}}_{-4.7\times10^{-5}}$	0.005675	$^{+1.1\times10^{-3}}_{-5.2\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.004166	$^{+4.8\times10^{-4}}_{-3.6\times10^{-5}}$	0.005043	$^{+8.2\times10^{-4}}_{-2.7\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.005416	$^{+1.1\times10^{-3}}_{-2.7\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.007449	$^{+1.7\times10^{-3}}_{-1.5\times10^{-4}}$

Table 4: Dataset-1 Excited-State *(CorDat_4-0.285-0.97_L-8-4-32)*

$C_N(t)$ ... Correlator values with $t_{max} = N$
$\delta C_N(t)$ ... Uncertainties for correlator values with $t_{max} = N$

$C_8(t)$	$\delta C_8(t)$	$C_{12}(t)$	$\delta C_{12}(t)$	$C_{16}(t)$	δC_{16}	$C_{20}(t)$	$\delta C_{20}(t)$	$C_{24}(t)$	$\delta C_{24}(t)$	$C_{28}(t)$	δC_{28}	$C_{32}(t)$	$\delta C_{32}(t)$
1.	$^{+1.1\times10^{-12}}_{-4.2\times10^{-12}}$	1.	$^{+1.7\times10^{-13}}_{-4.4\times10^{-12}}$	1.	$^{+7.5\times10^{-12}}_{-7.\times10^{-12}}$	1.	$^{+2.7\times10^{-12}}_{-6.\times10^{-12}}$	1.	$^{+6.2\times10^{-12}}_{-3.7\times10^{-12}}$	1.	$^{+1.8\times10^{-13}}_{-3.\times10^{-13}}$	1.	$^{+5.3\times10^{-13}}_{-2.5\times10^{-12}}$
0.450683	$^{+5.5\times10^{-4}}_{-5.4\times10^{-4}}$	0.440945	$^{+5.6\times10^{-4}}_{-5.6\times10^{-4}}$	0.441767	$^{+2.6\times10^{-4}}_{-2.6\times10^{-4}}$	0.439929	$^{+1.7\times10^{-4}}_{-1.7\times10^{-4}}$	0.439246	$^{+1.9\times10^{-4}}_{-1.9\times10^{-4}}$	0.438398	$^{+2.6\times10^{-4}}_{-2.6\times10^{-4}}$	0.437677	$^{+4.\times10^{-4}}_{-4.\times10^{-4}}$
0.237271	$^{+5.\times10^{-4}}_{-4.9\times10^{-4}}$	0.231322	$^{+7.8\times10^{-4}}_{-7.7\times10^{-4}}$	0.235768	$^{+5.6\times10^{-4}}_{-5.5\times10^{-4}}$	0.236386	$^{+2.8\times10^{-4}}_{-2.8\times10^{-4}}$	0.237373	$^{+2.8\times10^{-4}}_{-2.8\times10^{-4}}$	0.237250	$^{+3.2\times10^{-4}}_{-3.1\times10^{-4}}$	0.237052	$^{+5.1\times10^{-4}}_{-5.\times10^{-4}}$
0.143744	$^{+5.1\times10^{-4}}_{-4.9\times10^{-4}}$	0.124466	$^{+8.5\times10^{-4}}_{-8.2\times10^{-4}}$	0.128114	$^{+5.1\times10^{-4}}_{-4.9\times10^{-4}}$	0.129326	$^{+5.\times10^{-4}}_{-4.9\times10^{-4}}$	0.131909	$^{+2.9\times10^{-4}}_{-2.9\times10^{-4}}$	0.133281	$^{+5.2\times10^{-4}}_{-5.1\times10^{-4}}$	0.133367	$^{+8.5\times10^{-4}}_{-8.3\times10^{-4}}$
0.117764	$^{+8.7\times10^{-4}}_{-5.6\times10^{-4}}$	0.069485	$^{+8.2\times10^{-4}}_{-7.7\times10^{-4}}$	0.069351	$^{+7.3\times10^{-4}}_{-7.\times10^{-4}}$	0.069935	$^{+5.5\times10^{-4}}_{-5.3\times10^{-4}}$	0.071906	$^{+3.5\times10^{-4}}_{-3.4\times10^{-4}}$	0.073382	$^{+4.8\times10^{-4}}_{-4.7\times10^{-4}}$	0.073744	$^{+9.3\times10^{-4}}_{-8.7\times10^{-4}}$
-	- -	0.042737	$^{+6.7\times10^{-4}}_{-5.9\times10^{-4}}$	0.038288	$^{+6.8\times10^{-4}}_{-6.2\times10^{-4}}$	0.037567	$^{+5.7\times10^{-4}}_{-5.2\times10^{-4}}$	0.038779	$^{+3.1\times10^{-4}}_{-3.\times10^{-4}}$	0.039671	$^{+4.6\times10^{-4}}_{-4.4\times10^{-4}}$	0.039815	$^{+1.\times10^{-3}}_{-8.8\times10^{-4}}$
-	- -	0.035055	$^{+6.7\times10^{-4}}_{-4.6\times10^{-4}}$	0.021814	$^{+5.9\times10^{-4}}_{-4.7\times10^{-4}}$	0.019952	$^{+6.8\times10^{-4}}_{-5.5\times10^{-4}}$	0.020919	$^{+3.1\times10^{-4}}_{-2.8\times10^{-4}}$	0.021534	$^{+5.2\times10^{-4}}_{-4.8\times10^{-4}}$	0.021985	$^{+9.8\times10^{-4}}_{-7.9\times10^{-4}}$
-	- -	-	- -	0.013729	$^{+5.7\times10^{-4}}_{-3.6\times10^{-4}}$	0.010983	$^{+7.6\times10^{-4}}_{-5.2\times10^{-4}}$	0.011456	$^{+3.7\times10^{-4}}_{-3.1\times10^{-4}}$	0.011897	$^{+5.2\times10^{-4}}_{-4.4\times10^{-4}}$	0.012946	$^{+1.2\times10^{-3}}_{-8.1\times10^{-4}}$
-	- -	-	- -	0.011611	$^{+5.3\times10^{-4}}_{-3.2\times10^{-4}}$	0.006777	$^{+7.1\times10^{-4}}_{-3.9\times10^{-4}}$	0.006356	$^{+4.\times10^{-4}}_{-2.7\times10^{-4}}$	0.006672	$^{+5.2\times10^{-4}}_{-3.8\times10^{-4}}$	0.008405	$^{+1.3\times10^{-3}}_{-7.4\times10^{-4}}$
-	- -	-	- -	-	- -	0.005176	$^{+5.8\times10^{-4}}_{-2.4\times10^{-4}}$	0.003793	$^{+6.\times10^{-4}}_{-3.1\times10^{-4}}$	0.004031	$^{+8.\times10^{-4}}_{-4.4\times10^{-4}}$	0.005660	$^{+1.7\times10^{-3}}_{-7.\times10^{-4}}$
-	- -	-	- -	-	- -	0.005687	$^{+1.4\times10^{-3}}_{-3.4\times10^{-4}}$	0.002765	$^{+4.7\times10^{-4}}_{-8.8\times10^{-5}}$	0.002795	$^{+6.8\times10^{-4}}_{-1.8\times10^{-4}}$	0.005286	$^{+6.6\times10^{-4}}_{-9.8\times10^{-5}}$
-	- -	-	- -	-	- -	-	- -	0.002378	$^{+7.4\times10^{-4}}_{-1.3\times10^{-4}}$	0.002652	$^{+5.8\times10^{-4}}_{-4.9\times10^{-5}}$	0.004941	$^{+5.5\times10^{-4}}_{-3.1\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	0.002894	$^{+8.3\times10^{-4}}_{-3.3\times10^{-4}}$	0.002400	$^{+5.8\times10^{-4}}_{-3.\times10^{-5}}$	0.004781	$^{+8.6\times10^{-4}}_{-1.9\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.002480	$^{+5.6\times10^{-4}}_{-1.4\times10^{-4}}$	0.004709	$^{+1.6\times10^{-3}}_{-8.\times10^{-5}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.003988	$^{+1.1\times10^{-4}}_{-5.6\times10^{-5}}$	0.004511	$^{+1.1\times10^{-3}}_{-3.4\times10^{-5}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.004805	$^{+1.3\times10^{-3}}_{-5.7\times10^{-5}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.007117	$^{+1.5\times10^{-3}}_{-6.6\times10^{-4}}$

Table 5: Dataset-2 Ground-State *(CorDat_2.7984-0.2984-1.317_L-8-4-32)*

$C_N(t)$ ... Correlator values with $t_{max} = N$
$\delta C_N(t)$ ... Uncertainties for correlator values with $t_{max} = N$

$C_8(t)$	$\delta C_8(t)$	$C_{12}(t)$	$\delta C_{12}(t)$	$C_{16}(t)$	δC_{16}	$C_{20}(t)$	$\delta C_{20}(t)$	$C_{24}(t)$	$\delta C_{24}(t)$	$C_{28}(t)$	δC_{28}	$C_{32}(t)$	$\delta C_{32}(t)$
1.	$^{+5.7\times10^{-13}}_{-9.7\times10^{-13}}$	1.	$^{+2.2\times10^{-12}}_{-3.8\times10^{-12}}$	1.	$^{+1.3\times10^{-13}}_{-7.3\times10^{-14}}$	1.	$^{+9.1\times10^{-14}}_{-3.4\times10^{-11}}$	1.	$^{+8.2\times10^{-11}}_{-6.8\times10^{-14}}$	1.	$^{+4.\times10^{-15}}_{-1.3\times10^{-15}}$	1.	$^{+5.4\times10^{-11}}_{-2.6\times10^{-11}}$
0.567516	$^{+5.\times10^{-5}}_{-5.2\times10^{-5}}$	0.529520	$^{+2.2\times10^{-4}}_{-2.2\times10^{-4}}$	0.524948	$^{+4.3\times10^{-5}}_{-4.2\times10^{-5}}$	0.521633	$^{+1.3\times10^{-4}}_{-1.3\times10^{-4}}$	0.503696	$^{+1.6\times10^{-4}}_{-1.6\times10^{-4}}$	0.503035	$^{+1.3\times10^{-4}}_{-1.3\times10^{-4}}$	0.520740	$^{+6.2\times10^{-5}}_{-6.2\times10^{-5}}$
0.377723	$^{+6.8\times10^{-5}}_{-7.1\times10^{-5}}$	0.295568	$^{+3.4\times10^{-4}}_{-3.3\times10^{-4}}$	0.287192	$^{+1.5\times10^{-4}}_{-1.5\times10^{-4}}$	0.281714	$^{+1.4\times10^{-4}}_{-1.4\times10^{-4}}$	0.272121	$^{+1.9\times10^{-4}}_{-1.9\times10^{-4}}$	0.271086	$^{+1.7\times10^{-4}}_{-1.7\times10^{-4}}$	0.278299	$^{+2.9\times10^{-4}}_{-2.9\times10^{-4}}$
0.294436	$^{+2.1\times10^{-4}}_{-2.1\times10^{-4}}$	0.176035	$^{+2.2\times10^{-4}}_{-2.2\times10^{-4}}$	0.161951	$^{+1.2\times10^{-4}}_{-1.2\times10^{-4}}$	0.155584	$^{+1.5\times10^{-4}}_{-1.5\times10^{-4}}$	0.149931	$^{+2.1\times10^{-4}}_{-2.1\times10^{-4}}$	0.148933	$^{+1.9\times10^{-4}}_{-1.9\times10^{-4}}$	0.151069	$^{+4.9\times10^{-4}}_{-4.9\times10^{-4}}$
0.271352	$^{+1.2\times10^{-5}}_{-9.3\times10^{-6}}$	0.116455	$^{+6.4\times10^{-5}}_{-6.6\times10^{-5}}$	0.094547	$^{+5.8\times10^{-5}}_{-5.8\times10^{-5}}$	0.087468	$^{+2.2\times10^{-4}}_{-2.2\times10^{-4}}$	0.084037	$^{+1.4\times10^{-4}}_{-1.4\times10^{-4}}$	0.082698	$^{+2.\times10^{-4}}_{-2.\times10^{-4}}$	0.084054	$^{+6.6\times10^{-4}}_{-6.6\times10^{-4}}$
-	- -	0.088877	$^{+2.\times10^{-5}}_{-2.1\times10^{-5}}$	0.058966	$^{+4.9\times10^{-5}}_{-4.8\times10^{-5}}$	0.050883	$^{+1.9\times10^{-4}}_{-1.9\times10^{-4}}$	0.048183	$^{+6.8\times10^{-5}}_{-6.6\times10^{-5}}$	0.045994	$^{+1.9\times10^{-4}}_{-1.8\times10^{-4}}$	0.047814	$^{+6.8\times10^{-4}}_{-6.7\times10^{-4}}$
-	- -	0.081491	$^{+3.8\times10^{-5}}_{-4.2\times10^{-5}}$	0.041240	$^{+1.7\times10^{-5}}_{-1.7\times10^{-5}}$	0.030911	$^{+1.1\times10^{-4}}_{-1.1\times10^{-4}}$	0.028129	$^{+1.\times10^{-4}}_{-1.\times10^{-4}}$	0.026350	$^{+1.9\times10^{-4}}_{-1.8\times10^{-4}}$	0.027778	$^{+7.3\times10^{-4}}_{-7.1\times10^{-4}}$
-	- -	-	- -	0.032540	$^{+3.5\times10^{-5}}_{-3.4\times10^{-5}}$	0.019916	$^{+1.2\times10^{-4}}_{-1.1\times10^{-4}}$	0.017356	$^{+9.4\times10^{-5}}_{-9.4\times10^{-5}}$	0.014989	$^{+1.5\times10^{-4}}_{-1.3\times10^{-4}}$	0.015420	$^{+8.3\times10^{-4}}_{-7.9\times10^{-4}}$
-	- -	-	- -	0.030390	$^{+1.7\times10^{-4}}_{-1.6\times10^{-4}}$	0.014113	$^{+1.1\times10^{-4}}_{-9.7\times10^{-5}}$	0.011326	$^{+1.2\times10^{-4}}_{-1.2\times10^{-4}}$	0.009043	$^{+1.\times10^{-4}}_{-9.2\times10^{-5}}$	0.008680	$^{+4.9\times10^{-4}}_{-4.5\times10^{-4}}$
-	- -	-	- -	-	- -	0.011394	$^{+1.3\times10^{-4}}_{-1.2\times10^{-4}}$	0.007677	$^{+9.7\times10^{-5}}_{-9.2\times10^{-5}}$	0.005556	$^{+9.5\times10^{-5}}_{-8.8\times10^{-5}}$	0.005619	$^{+2.1\times10^{-4}}_{-1.7\times10^{-4}}$
-	- -	-	- -	-	- -	0.010855	$^{+3.5\times10^{-4}}_{-3.2\times10^{-4}}$	0.005727	$^{+1.7\times10^{-4}}_{-1.6\times10^{-4}}$	0.003671	$^{+8.1\times10^{-5}}_{-7.3\times10^{-5}}$	0.004419	$^{+3.4\times10^{-4}}_{-3.3\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	0.004565	$^{+1.4\times10^{-4}}_{-1.2\times10^{-4}}$	0.002243	$^{+5.1\times10^{-4}}_{-1.9\times10^{-4}}$	0.002689	$^{+4.9\times10^{-4}}_{-4.2\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	0.004114	$^{+2.2\times10^{-4}}_{-1.9\times10^{-4}}$	0.001712	$^{+5.5\times10^{-4}}_{-2.7\times10^{-4}}$	0.002103	$^{+4.4\times10^{-4}}_{-4.1\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.001370	$^{+4.8\times10^{-4}}_{-4.1\times10^{-4}}$	0.001408	$^{+5.5\times10^{-4}}_{-4.8\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.001287	$^{+2.1\times10^{-4}}_{-9.\times10^{-5}}$	0.002252	$^{+3.\times10^{-4}}_{-2.9\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.001699	$^{+5.6\times10^{-4}}_{-5.4\times10^{-4}}$
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.003061	$^{+4.\times10^{-4}}_{-3.9\times10^{-4}}$

Table 6: Dataset-2 Excited-State *(CorDat_2.7984-0.2984-1.317_L-8-4-32)*

$C_N(t)$ ... Correlator values with $t_{max} = N$
$\delta C_N(t)$ ... Uncertainties for correlator values with $t_{max} = N$

$C_8(t)$	$\delta C_8(t)$	$C_{12}(t)$	$\delta C_{12}(t)$	$C_{16}(t)$	δC_{16}	$C_{20}(t)$	$\delta C_{20}(t)$	$C_{24}(t)$	$\delta C_{24}(t)$	$C_{28}(t)$	δC_{28}	$C_{32}(t)$	$\delta C_{32}(t)$
1.	$+6.7\times10^{-16}$ -1.1×10^{-16}	1.	$+1.1\times10^{-14}$ -6.7×10^{-16}	1.	$+4.4\times10^{-16}$ -6.7×10^{-16}	1.	$+1.3\times10^{-15}$ -1.3×10^{-15}	1.	$+4.4\times10^{-16}$ -2.2×10^{-15}	1.	$+0.$ -4.4×10^{-16}	1.	$+0.$ -2.2×10^{-16}
0.345951	$+4.9\times10^{-4}$ -4.8×10^{-4}	0.331460	$+5.4\times10^{-4}$ -5.4×10^{-4}	0.328198	$+1.8\times10^{-4}$ -1.8×10^{-4}	0.324771	$+2.4\times10^{-4}$ -2.4×10^{-4}	0.323771	$+2.1\times10^{-4}$ -2.1×10^{-4}	0.323242	$+3.4\times10^{-4}$ -3.4×10^{-4}	0.269793	$+4.6\times10^{-4}$ -4.6×10^{-4}
0.181892	$+7.4\times10^{-4}$ -7.2×10^{-4}	0.167917	$+8.1\times10^{-4}$ -8.1×10^{-4}	0.163192	$+3.6\times10^{-4}$ -3.6×10^{-4}	0.159700	$+3.6\times10^{-4}$ -3.7×10^{-4}	0.157983	$+3.2\times10^{-4}$ -3.2×10^{-4}	0.156716	$+2.7\times10^{-4}$ -2.7×10^{-4}	0.080793	$+5.6\times10^{-4}$ -5.6×10^{-4}
0.117193	$+2.9\times10^{-4}$ -2.8×10^{-4}	0.097343	$+6.4\times10^{-4}$ -6.4×10^{-4}	0.092705	$+4.1\times10^{-4}$ -4.1×10^{-4}	0.089870	$+4.3\times10^{-4}$ -4.4×10^{-4}	0.087771	$+3.4\times10^{-4}$ -3.4×10^{-4}	0.086539	$+3.9\times10^{-4}$ -3.9×10^{-4}	0.042454	$+4.5\times10^{-4}$ -4.6×10^{-4}
0.098453	$+5.3\times10^{-4}$ -5.1×10^{-4}	0.059420	$+8.4\times10^{-4}$ -8.4×10^{-4}	0.055534	$+3.6\times10^{-4}$ -3.6×10^{-4}	0.053708	$+4.9\times10^{-4}$ -4.9×10^{-4}	0.051700	$+3.3\times10^{-4}$ -3.3×10^{-4}	0.050825	$+4.3\times10^{-4}$ -4.3×10^{-4}	0.024276	$+6.\times10^{-4}$ -6.1×10^{-4}
-	- -	0.041188	$+9.3\times10^{-4}$ -9.3×10^{-4}	0.034193	$+5.8\times10^{-4}$ -5.8×10^{-4}	0.032337	$+5.5\times10^{-4}$ -5.6×10^{-4}	0.030538	$+3.7\times10^{-4}$ -3.7×10^{-4}	0.030264	$+4.6\times10^{-4}$ -4.7×10^{-4}	0.013349	$+6.5\times10^{-4}$ -6.6×10^{-4}
-	- -	0.034508	$+4.8\times10^{-4}$ -4.8×10^{-4}	0.021159	$+5.2\times10^{-4}$ -5.2×10^{-4}	0.020045	$+6.4\times10^{-4}$ -6.4×10^{-4}	0.018149	$+4.4\times10^{-4}$ -4.4×10^{-4}	0.018120	$+5.1\times10^{-4}$ -5.2×10^{-4}	0.009086	$+4.8\times10^{-4}$ -5.1×10^{-4}
-	- -	-	- -	0.014227	$+6.3\times10^{-4}$ -6.3×10^{-4}	0.011966	$+6.7\times10^{-4}$ -6.6×10^{-4}	0.010620	$+4.6\times10^{-4}$ -4.6×10^{-4}	0.010664	$+5.6\times10^{-4}$ -5.7×10^{-4}	0.005325	$+6.6\times10^{-4}$ -6.8×10^{-4}
-	- -	-	- -	0.012023	$+5.5\times10^{-4}$ -5.5×10^{-4}	0.008060	$+6.4\times10^{-4}$ -6.5×10^{-4}	0.005686	$+4.4\times10^{-4}$ -4.4×10^{-4}	0.005961	$+5.5\times10^{-4}$ -5.6×10^{-4}	0.003866	$+6.4\times10^{-4}$ -6.4×10^{-4}
-	- -	-	- -	-	- -	0.005387	$+5.9\times10^{-4}$ $-6.\times10^{-4}$	0.003263	$+4.1\times10^{-4}$ -4.1×10^{-4}	0.003909	$+5.\times10^{-4}$ $-5.\times10^{-4}$	0.002227	$+8.1\times10^{-4}$ -7.4×10^{-4}
-	- -	-	- -	-	- -	0.004725	$+6.1\times10^{-4}$ -6.3×10^{-4}	0.001831	$+4.8\times10^{-4}$ -4.8×10^{-4}	0.002532	$+5.2\times10^{-4}$ -5.2×10^{-4}	0.001216	$+5.1\times10^{-4}$ -3.5×10^{-4}
-	- -	-	- -	-	- -	-	- -	0.000854	$+3.8\times10^{-4}$ -3.5×10^{-4}	0.002154	$+1.2\times10^{-4}$ -4.2×10^{-4}	0.001816	$+7.6\times10^{-4}$ -7.2×10^{-4}
-	- -	-	- -	-	- -	-	- -	0.001041	$+2.5\times10^{-4}$ -1.3×10^{-4}	0.001385	$+1.2\times10^{-4}$ -3.9×10^{-4}	0.001150	$+9.9\times10^{-4}$ -8.8×10^{-4}
-	- -	-	- -	-	- -	-	- -	-	- -	0.000882	$+1.6\times10^{-4}$ -1.6×10^{-4}	0.000931	$+5.6\times10^{-4}$ $-4.\times10^{-4}$
-	- -	-	- -	-	- -	-	- -	-	- -	0.000811	$+3.3\times10^{-4}$ -3.6×10^{-4}	0.000989	$+2.7\times10^{-4}$ -2.3×10^{-4}
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.000897	$+3.\times10^{-4}$ -2.1×10^{-4}
-	- -	-	- -	-	- -	-	- -	-	- -	-	- -	0.000260	$+4.8\times10^{-4}$ -2.1×10^{-4}

List of Tables

List of Figures