Karl-Franzens University Graz

Master's Thesis (MSc)

The non-violation of the Bloch-Nordsieck theorem for the electroweak sector of the Standard Model

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Abstract

The aim of this master's thesis is to restore the Bloch-Nordsieck theorem for the electroweak sector of the Standard model. For this purpose, we recall that due to Elitzer's theorem, the electroweak gauge symmetry cannot be broken and thus newly constructed bound states take the role of the usual elementary ones as asymptotic states of our theory. We will use augmented perturbation theory (i.e.: perturbation theory augmented by the FMS mechanism) to show that this eradicates any infrared singularities in electroweak processes and discuss the effects in the limits of high and low energies. Furthermore, we will present an approach using a PDF formulation, which will prove to be beneficial in the case of hadron colliders.

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Affidavit

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly indicated all material which has been quoted either literally or by content from the sources used. The text document uploaded to TUGRAZonline is identical to the present master‘s thesis.

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1. Introduction

"The infinitely many atoms move in the void since eternity and from them arise countless worlds with their composite individual objects."

(Demokrit and Leukipp; 4th century BC)

We have come a long way since the first idea of indivisible objects as building blocks of our known world in ancient Greece. The atomistic worldview disappeared from the image of nature until in the 17th century similar thoughts were taken up again by Gassendi and Boyle. But only in the 19th century, the atomism achieved an empirically justified breakthrough due to Dalton and his law of partial pressures and paved the way for modern particle physics. Nowadays, the Standard Model (SM) of particle physics is the theory, that best describes all known phenomena in nature and as a quantum field theory has its origin in the early decades of the last century.

The inception of quantum field theory is usually attributed to Dirac in 1927 when he published his famous paper on "The quantum theory of the emission and absorption of radiation", in which he could rederive the Bohr-Jordan formula without appeal to thermodynamics. Around 1930 arose the awareness of the problem with infinities in quantum field theories. They are usually distinguished on basis of their range into ultraviolet (UV) and infrared (IR) divergences. The issue regarding the latter was solved for the case of QED 1937 by the famous Bloch-Nordsieck (BN) theorem, which was expanded in 1963/64 by Kinoshita and Lee and Nauenberg in form of the KLN theorem named after them. Regarding the former, it took until 1947-49 to finally resolve the UV problem by the renormalization theories of Feynman, Schwinger, Tomonaga and Dyson. The 1960s and 1970s were characterized by the increased study of quantum chromodynamics (QCD) and electroweak (ew) theory induced by the works of Yang and Mills, the Goldstone theorem, and the Brout-Englert-Higgs effect. The term Standard Model of particle physics was first introduced in the early 1970s by Weinberg, who outlined the core of what we today understand under the term.

Bringing into play new aspects of our theory of nature (in form of QCD and ew theory) engendered the old problem with infinities once more and the majority of physicists in the 1980s were convinced that the BN theorem was violated for the QCD case, which shook up the existing understanding of how to deal with IR divergencies. This issue got resolved by considering the quark confinement and it was shown that the infinities cancel for colour-averaged cross-sections. The situation for the electroweak sector of the Standard Model did not seem to work out that beautifully. As a non-abelian theory again IR divergencies were popping up and the argument of colour-averaging could not be transferred equivalently.

This master's thesis aims to show the restoration of the BN theorem for the electroweak sector of the Standard Model by introducing leptonic bound states, justified by the need for gauge-invariance of our theory. Just as in QCD, where we need to include the information about colourless hadrons as actual initial and eventual final states, in the electroweak case we consider gauge-invariant initial states, that force us to sum over the weak isospin and thus lead to a cancellation of IR-divergent terms. The resulting insights of this new treatment will not only be of interest to theorists but also to experimentalists since the validity of the hereafter presented framework can only be put properly to the test at very high energies (several TeV) reached by future colliders.

In the following, I first will be covering some basic concepts including the Bloch-Nordsieck theorem, the coherent state approach and the Fröhlich-Morchio-Strocchi (FMS) mechanism. I will then move on to the treatment of the electroweak case by introducing bound states and as a result, show the non-violation of the BN theorem. Eventually, I am going to summarize the main results and discuss the impact on our understanding of the Standard Model and the effects that can be expected to be observed in future high-energy colliders.

2. Basic Concepts

Since in quantum field theory we are dealing with infinitely many degrees of freedom, the appearance of infinities did not long in coming. In contrast to ultraviolet divergences, infrared divergences arise from the high-energy part of the integration range and appear due to massless fields in the theory. Based on this fact they are also often called mass singularities. When it comes to their characterisation one usually distinguishes between soft and collinear divergences. The former occur if we let the (fictitious) mass of the gauge field go to zero, while the latter are generated if the massless field couples to another massless field or itself, and hence not only the gauge field mass but also the mass of the matter field tends to zero. Although IR divergences are a long-distance phenomenon, they can play an important role in verifying the correctness of the perturbative treatment of short-distance effects.

2.1. The Bloch-Nordsieck theorem

The Bloch-Nordsieck theorem was the first attempt to solve the IR problem and was primarily limited to QED. In their famous paper Felix Bloch and Arnold Nordsieck presented the idea, that in any real experiment that involves charged particles, it is impossible to uniquely specify the final state of the system, due to soft virtual photons. They managed to show that the probability of a finite number of photons escaping detection is in fact zero. Thus, one needs to sum over all indistinguishable final states, which leads to a cancellation between virtual and real infrared divergences and hence one ends up with an IR-finite result. Since then, the principle introduced by Bloch and Nordsieck was also tried to be applied to other theories, such as QCD and electroweak theory. It can be written down more generally in the form:

$$\sum_f |S_{if}|^2 = \text{ finite} \tag{2.1}$$

Here S is the S-matrix and i and f indicate the initial and final states.

The BN theorem represents a special case of the so-called Kinoshita-Lee-Nauenberg (KLN) theorem, which states that all infrared divergences (soft and collinear ones) cancel if one sums over all initial and final states degenerate in energy. In mathematical notation this can be written as:

$$\sum_{i,f} |S_{if}|^2 = \text{ finite,} \tag{2.2}$$

where in contrast to the BN theorem one has an additional sum over the initial states.

To deal with the IR problem -by making use of the above-mentioned theorems- there are two different approaches available:

1. The *inclusive approach*, in which one sums over physically indistinguishable degenerate states and recognises that the sum is then IR-finite.
2. The *coherent state approach*, with the main idea to define a representation of the problematic states other than the Fock representation, without any IR divergences in the S-matrix.

In the first approach, in the calculation of inclusive cross-sections for physical processes, the IR divergences are exactly cancelled at all orders by the divergences arising from real emission.

The second approach is closer to the physical reality due to the introduction of new states (so-called Neumann states) in the theory, which are degenerate in the number of soft particles. Because of that, the KLN-theorem is automatically satisfied, and hence the matrix elements IR-finite.

In the following, I want to describe the coherent state approach in a bit more detail.

2.2. The coherent-state approach

The goal is to obtain an asymptotic Hamiltonian, which describes the long-term evolution and can be simplified via eikonal (infrared) approximation of the interaction vertices. For this purpose, the interaction Hamiltonian is first separated into a soft and a hard part. The former contains vertex frequencies $\nu < \Delta$, where Δ is some upper scale and the energy transfer ν describes the softness of the vertex (i.e.: The smallness of ν specifies the degeneracy of the perturbative states.). The hard part of the Hamiltonian consists of the remainder.

$$H_I^\Delta(t) = H_h^\Delta(t) + H_s^\Delta(t) \tag{2.3}$$

Note that there is no interaction between the hard and the soft subsystem. Consequently, also the Hilbert space needs to be split into a soft and a hard part:

$$\mathcal{H} = \mathcal{H}_h \otimes \mathcal{H}_s, \tag{2.4}$$

where $\mathcal{H}_s$ creates the IR singularities and $\mathcal{H}_h$ screens them.

As a result, the S-matrix can be written in the following form:

$$S = U^{F\dagger}_{\alpha_F \beta_F} \mathrm{S_h} U^I_{\alpha_I \beta_{I'}} \tag{2.5}$$

where U are the coherent state operators and describe the long-term interactions, while S_h describes the short-time hard interactions.

The coherent state operators are defined as:

$$U = \left({}_h \left\langle 0 \left| \Omega^\Delta_\pm \right| 0 \right\rangle_h \right)^{-1} \left\langle h \left| \Omega^\Delta_\pm \right| h \right\rangle \tag{2.6}$$

with $|h\rangle$ hard states and $\Omega^\Delta_\pm$ the soft evolution operators. Note that in this expression also the hard vacuum fluctuations have been subtracted out. From the definition of the Us, the origin of their name becomes apparent, since, applied to the soft vacuum, they yield states which describe the cloud of soft particles surrounding a given set of hard partons (c.f. coherent states). But only in QED they fulfil all mathematical features of coherent operators on a specified algebra. In QCD and electroweak theory, this is not the case due to the lack of a non-trivial classical limit of non-abelian gauge theories.

The coherent state operators possess some essential properties:

1. They are unitary: $\sum_{s\in\Delta} U^\dagger |s\rangle\langle s| U = 1$ (equivalently: $U_{\alpha\beta} U^\dagger_{\beta\alpha'} = U^\dagger_{\alpha\beta} U_{\beta\alpha'} = \delta_{\alpha\alpha'}$)
2. They are operatorially factorised in the hard charges (QED) resp. in the colour space of the hard partons (QCD).
3. They have simple gauge properties: They are in general changed by a phase factor and only gauge-invariant if the total charge vanishes.
4. In QCD, the coherent state operators commute at different colours.

Applying the coherent state approach discussed above to the Bloch-Nordsieck theorem, this means the following:

The basic statement of the BN theorem is that $W^\Delta_f = \sum_{f\in\Delta} |S_{fi}|^2$ is IR-finite. Keeping in mind the unitarity of the coherent state operators one can easily carry out the calculation:

$$\sum_{f\in\Delta} |S_{fi}|^2 = \sum_{f\in\Delta} |\langle f_0|S|i\rangle|^2 = \sum_{f\in\Delta} \left\langle i \left| S^\dagger \right| f_0 \right\rangle \left\langle f_0 \middle| S \middle| i \right\rangle =$$
$$= \sum_{f\in\Delta} \left\langle i \left| U^{I\dagger} S^\dagger_h U^F \right| f_0 \right\rangle \left\langle f_0 \left| U^{F\dagger} S_h U^I \right| i \right\rangle = \left\langle i \left| U^{I\dagger} S^\dagger_h \right| f_0 \right\rangle \left\langle f_0 \left| S_h U^I \right| i \right\rangle = \left\langle i \left| U^{I\dagger} O_h U^I \right| i \right\rangle ,$$

where O_h is the so-called IR-finite final state overlap matrix. Note that the last expression is only finite for abelian theories!

2.2.1. The QCD case

QED, as an abelian theory, satisfies both, the KLN and the BN theorem. For QCD on the other hand we need a little bit more effort to see, that even though it is a non-abelian theory, it does not violate the BN theorem.

Since the gluons in QCD carry a non-trivial charge and therefore show a self-interacting behaviour, the asymptotic Hamiltonian, defined as $H_{as} = H_0 + H_s$, is not solvable due to its non-linearity in the soft fields. But we can still simplify the calculation by applying an eikonal approximation to the trilinear vertices. In the following we will work in the covariant gauge, which allows us to see the Lorentz-invariance more clearly.

We first write the Hamiltonian in the form:

$$H = H_0 + H_{int}, \tag{2.7}$$

with H_0 being the quadratic part and H_{int} the interaction Hamiltonian which consists of trilinear quark-gluon, three-gluon, quadrilinear four-gluon and ghost-gluon vertices:

$$\begin{aligned} H_{int} =& g\left(\boldsymbol{J}_i \cdot \boldsymbol{A}^i - \boldsymbol{J}_0 \cdot \Pi_B\right) \\ & - g\partial_i \boldsymbol{A}_j \cdot \left(A^i \times A^j\right) - g\Pi_i \cdot (\Pi_B \times A_i) \\ & + \frac{1}{4}g^2 \left(A_i \times A_j\right) \cdot \left(A^i \times A^j\right) \\ & - g\left(\Pi_B \times \boldsymbol{c}\right) \cdot \Pi_c - ig\left(A_i \times \boldsymbol{c}\right) \cdot \partial^i \overline{\boldsymbol{c}}, \end{aligned} \tag{2.8}$$

where

$$\Pi_i = -F_{0i}; \quad \Pi_B = -A_0; \quad \Pi_c = i\dot{\bar{c}}; \quad \Pi_{\bar{c}} = -iD_0 c \tag{2.9}$$

are the conjugate momenta and $D_\mu = \partial_\mu + igA_\mu$ is the covariant derivative.

The interaction part is then split up according to the coherent state approach into a soft and a hard part and introducing the (customary) constraint $\lambda < \nu < \Delta$ on the energy transfer[1], we can write the latter in the form:

$$\mathrm{H}_\mathrm{s}^\Delta = \mathrm{H}_\mathrm{f}^\Delta + \mathrm{H}_\mathrm{g}^\Delta + \mathrm{H}_\mathrm{ghost}^\Delta + \mathrm{H}_4^\Delta \tag{2.10}$$

with the fermionic part given by:

$$H_f^\Delta = g\sum_\sigma \int d[p] \int_\lambda^\Delta d[q]\rho_a^f(\mathbf{p})\hat{p}_\mu A_a^{\mu\sigma}(\mathbf{q})e^{-i\sigma\hat{p}qt}\Theta(\Delta - \hat{p}q) \tag{2.11}$$

[1] Here $\nu = \left|\sum_i \sigma_i \omega\left(q_i\right)\right|$ with $\sum_i \sigma_i \mathbf{q}_i = 0$, where q_i are the vertex momenta and $\sigma_i = {}_{(}{}^{+}_{-}{}_{)}$ describe the energy signs for outgoing (ingoing) partons.

the gluon part is given by :

$$\begin{aligned} H_g^{\Delta} = -ig_s \frac{1}{3!} f_{a_1 a_2 a_3} \sum_{\sigma_i} \int d\,[q_1]\, d\,[q_2]\, d\,[q_3]\, (2\pi)^3 \delta \left(\sum_i \sigma_i \mathbf{q}_i \right) \cdot \\ A_{a_1}^{\sigma_1 \mu_1} A_{a_2}^{\sigma_2 \mu_2} A_{a_3}^{\sigma_3 \mu_3} \Gamma_{\mu_1 \mu_2 \mu_3} \left(\sigma_1 q_1, \sigma_2 q_2, \sigma_3 q_3 \right) e^{-i \sum_i \sigma_i \omega_i t} \Theta \left(\Delta - |\Sigma \sigma_i \omega_i| \right) \end{aligned} \tag{2.12}$$

and the ghost part:

$$\begin{aligned} H_{ghost}^{\Delta} =& g f_{a_1 a_2 a_3} \sum_{\sigma_i} \mathrm{d}\,[q_1]\, \mathrm{d}\,[q_2]\, \mathrm{d}\,[q_3]\, (2\pi)^3 \delta^3 \left(\sum \sigma_i \boldsymbol{q}_i \right) \exp \left(-i \sum \sigma_j \omega_j t \right) \cdot \\ & \Theta \left(\Delta - \left| \sum_j \sigma_j \omega_j \right| \right) \bar{c}_{a_1}^{\sigma_1} c_{a_2}^{\sigma_2} \sigma_1 q_1^{\mu} A_{\mu a_3}^{\sigma_3} \left(\mathbf{q}_3 \right) \end{aligned} \tag{2.13}$$

where we used:

$$d[p] = \frac{d^3 p}{2E_p (2\pi)^3}, \quad \hat{p}_\mu = p_\mu / E_p, \quad \rho_a^f \left(\mathbf{p} \right) = \sum_{i=q,\bar{q}} b_{i\alpha}^{\dagger} \left(t_a^i \right)_{\alpha\beta} b_{i\beta}, \quad t_a^q = t_a, \quad t_a^{\bar{q}} = -t_a^{\intercal}$$

as well as the three-gluon vertex Γ_μ:

$$\Gamma^{\mu_1 \mu_2 \mu_3} \left(q_1, q_2, q_3 \right) = \left(q_1^{\mu_3} - q_2^{\mu_3} \right) g^{\mu_1 \mu_2} + \left(q_2^{\mu_1} - q_3^{\mu_1} \right) g^{\mu_2 \mu_3} + \left(q_3^{\mu_2} - q_1^{\mu_2} \right) g^{\mu_3 \mu_1}$$

We omit the 4-gluon contribution, since it does not contribute up to the first sub-leading level.[2]

The constraint $\nu < \Delta$ does not imply that necessarily all energies occurring at the vertex need to be soft, i.e.: H_s connects states of $\mathcal{H}_s$ and $\mathcal{H}_s$ as well as $\mathcal{H}_s$ and $\mathcal{H}_h$. As a result one of the partons may be soft and the other two fast ($\omega_1 \approx \omega_2 \gg \omega_3$) or all three partons are soft ($\omega_1, \omega_2, \omega_3 < \Delta$). In the strong ordering region:

$$\lambda < \omega_1 \ll \omega_2 \ll \cdots \ll \omega_n < \Delta$$

the eikonal form of the three-gluon vertex should be used:

$$\begin{aligned} \Gamma^{\mu_1 \mu_2 \mu_3} \left(\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3 \right) &= \Gamma_{\lambda\mu\nu} (\boldsymbol{p}, -\boldsymbol{p} + \boldsymbol{q}, -\boldsymbol{q}) = \\ &= 2p_{\mu_3} g_{\mu_1 \mu_2} - p_{\mu_1} g_{\mu_2 \mu_3} - p_{\mu_2} g_{\mu_3 \mu_1} + \mathrm{O}(\boldsymbol{q}) \quad = \\ &= 2p_{\mu_3} g_{\mu_1 \mu_2} + (\text{non-eikonal terms}) \end{aligned} \tag{2.14}$$

and we can rewrite the soft Hamiltonian by distinguishing between ordered and non-ordered energy regions in (2.10):

[2] Its exact form can be found in [CC87]

$$H_{\rm s} = H_{\rm eik} + K + H_{\rm s}' \tag{2.15}$$

Here the simple eikonal term gives rise to the leading IR singularities, since K, which includes non-eikonal terms due to the scalar gluon and ghost interactions, does not possesses any IR leading contributions between physical states. The third term in the above equation is of sub-leading type and involves only soft parton energies of the same order.

We note that the partons involved in $\mathcal{H}_h$ always interact with the ones in $\mathcal{H}_s$ via eikonal vertices and thus we can write down another decomposition of H_s, which in contrast to (2.15) is valid to all IR orders:

$$\begin{aligned} H_s &= H_{eik}(E_{partons} > \Delta) + H_s(\omega_i < \Delta) \\ &= H_{eik}^{>} + H_s^{<} = \\ &= g \sum_{i \in h} t_a^i \hat{p}_\mu^i A^{\mu a}\left(t, \boldsymbol{\nu}_i t\right) + H_s^{<} \end{aligned} \tag{2.16}$$

In the strong ordering region, using the eikonal form of the three-gluon vertex (2.14), we can approximate $H_s^<$ by:

$$H_s^< \approx \sum_\sigma \int_\lambda^\Delta d[q']d[q]\ \rho_g^a(q')\ \hat{q}'_\mu\ A_a^{\mu\sigma}(\boldsymbol{q})\ e^{-i\sigma\hat{q}'\cdot qt}\ \Theta(\omega' - \omega), \tag{2.17}$$

where q_μ is the softest gluon and $\rho_{g,a}(q') = i f_{abc} A_\mu^{b\dagger}(q') A_c^\mu(q')$.

We can thus calculate the eikonal coherent state operators to be:

$$U_{eik}^{\Delta,h}(\Pi) = exp\left[ig \int_\lambda^\Delta d[q] \sum_{i=1}^{h} \frac{\hat{p}_i^\mu t_i^a}{\hat{p}_i q} \Pi_{a\mu}^\omega(q) \right] \tag{2.18}$$

with the gluon fields, dressed up to energy ω, defined as:

$$\Pi_{a\mu}^\omega(q) = -i\big(A_{\mu a}^\omega - A_{\mu a}^{\omega\dagger}\big) \tag{2.19}$$

Since $U_{eik}^{\Delta,h}(\Pi)$ are functionals of $\Pi_{a\mu}^\omega(q)$ only it can be shown that they commute at different colors:

$$\left[U_{\alpha\beta}^\Delta(\Pi), U_{\alpha'\beta'}^\Delta(\Pi) \right] = 0 \tag{2.20}$$

As a result, if we perform the calculation of the BN theorem and additionally sum over colours, we end up with an IR-finite result:

$$
\begin{aligned}
\sum_{\text{colour}} \sum_{f\in\Delta} |S_{fi}|^2 &= \sum_{\text{colour}} \sum_{f\in\Delta} \left\langle i \left| U_\Delta^{I\dagger} S_h^\dagger U_\Delta^F \right| f_0 \right\rangle \left\langle f_0 \left| U_\Delta^{F\dagger} S_h U_\Delta^I \right| i \right\rangle = \\
&= \sum_{\text{colour}} \left\langle i \left| U_\Delta^{I\dagger} S_h^\dagger S_h U_\Delta^L \right| i \right\rangle = \\
&= \sum_{\text{colour}} U_{\alpha_I \beta_I'}^{\Delta\dagger} \left(S_h^\dagger S_h \right) U_{\beta_I \alpha_I}^{\Delta} = \\
&= \sum_{\text{colour}} \left(S_h^\dagger S_h \right) U_{\alpha_I \beta_I'}^{\Delta\dagger} U_{\beta_I \alpha_I}^{\Delta} = \\
&= \mathrm{Tr}_{\text{colour}} \left(S_h^\dagger S_h \right)
\end{aligned}
\tag{2.21}
$$

Summarising we can say that using the coherent state approach and simplifying the terms in the interaction Hamiltonian via eikonal approximation we obtain an expression for the coherent state operators, which commutes at different colours and hence ensures the validity of the Bloch-Nordsieck theorem for colour-averaged cross-sections in the QCD case.

2.3. Augmented perturbation theory

Gauge theories are based on the requirement of invariance to local phase transformations (i.e. gauge invariance) of a field theory and due to the principle of causality a necessity of a quantum field theory. The hereby associated gauge symmetry is local and a redundant degree of freedom, since the gauge choice must not affect the result of any physical measurement. Nowadays the electromagnetic, weak and strong interactions can be formulated as gauge theories, whereas gravitation theory can on the one hand also be obtained by a gauge symmetry but on the other hand only on a classical level.

In the classical case, it is rather easy to formulate the theory gauge independent since the Lagrangian itself is gauge-invariant and all physical observables are derived from it. For quantised gauge theories a different picture emerges which is, in fact, twofold: In the abelian case a gauge independent formulation can still be achieved without any great difficulty, but in the non-abelian case the field-strength tensor is not gauge-invariant anymore and hence the usual treatment cannot be applied. However, just these non-abelian theories turned out to be impressively successful in describing the real world.

Focussing now on the quantisation of non-abelian gauge theories, in general one has to differentiate between a perturbative and non-perturbative treatment. Using a path integrational approach, in perturbation theory the Faddeev-Popov-DeWitt method fixes the gauge, which hides the underlying (local) gauge theory and thus leads to a problem proving the renormalizability of a theory. This issue was solved by Becchi, Rouet and

Stora (as well as independently by Tyutin) in 1975 by recognising that even after choosing a gauge the path integral remains invariant under a global symmetry related to gauge invariance. This so-called BRST symmetry ensures the renormalizability and gauge-invariance of the theory in question. Non-perturbatively we cannot do the same trick, due to the Gribov-Singer ambiguity.

For non-abelian gauge theories, the local gauge group imposes additional constraints. In contrast to the abelian case where the gauge is fixed (essentially) uniquely by the gauge condition $\partial \cdot A = 0$, in the non-abelian case there exist distinct transverse configurations, so-called Gribov-copies, $A \neq A'$ with $\partial \cdot A = \partial \cdot A' = 0$ related by a large gauge transformation $A' = {}^{U}A$, where U represents the local gauge transformation. Since gauge theories must obey the constraint that A and A' (i.e. the Gribov copies) can be identified physically, one has to restrict the physical configuration space to the so-called fundamental modular region, which is free of Gribov-copies. One can identify the physical configuration corresponding to one specific configuration A with the gauge orbit through A, $A_{phys} = \left\{A' : A' = {}^{U}A\right\}$. The physical configuration is thus the equivalence class including all gauge orbits, $\mathcal{P} = \{A_{phys}\}$. $\mathcal{P}$ can be written as a quotient space in the following way:

$$\mathcal{P} = \mathcal{A}/\mathcal{U},$$

where $\mathcal{A} = \{A\}$ is the space of all configurations and $\mathcal{U} = \{U\}$ is the group of local gauge transformations. For a gauge to be completely fixed, a parametrisation of this quotient space by selecting a single representative from each equivalence class is needed. For a non-abelian gauge theory, it is impossible to choose a unique representative on each gauge orbit by a linear and continuous gauge condition.

This apparent dead-end can be circumvented considering bound-states and hence restore the gauge invariance of the non-abelian theory under investigation. Instead of unphysical single particles, one thus uses products of fields. The actual construction of these for the electroweak section of the SM is subject of section (3.2).

At first, this might not sound like an all too great relief giving the fact that composite objects require the use of non-perturbative methods to describe them, but there is a method, which allows using the composite fields similar to the elementary ones in standard perturbation theory. This method is called augmented perturbation theory and was developed by Fröhlich, Morchio and Strocchi. It requires an active Brout-Englert-Higgs (BEH) effect and allows for a formulation of the correlation functions entirely in terms of the gauge-invariant composite fields. Those can then be expanded around the vacuum expectation value (v.e.v.) of the Higgs field and thus lead to a result that is computable with standard perturbative methods. At the heart of this is the so-called Fröhlich-Morchio-Strocchi (FMS) mechanism, which will be detailed in the following subsection.

2.3.1. The Fröhlich-Morchio-Strocchi mechanism

The FMS mechanism that together with the structure of the SM only allows the use of bound-states in the electroweak case without contradicting the experimental results, can be basically divided into four steps:

1. Write down the gauge-invariant operators (O) and with these form the required correlation functions (e.g.: propagator $\langle O^\dagger O \rangle$).
2. Fix the gauge with a non-vanishing vacuum expectation value (e.g.: 't Hooft).
3. Split the Higgs field in the operators into the vacuum expectation value and the fluctuation field! This one calls the FMS mechanism.
4. Expand the resulting correlation functions in their usual perturbative series.

This double expansion is called augmented perturbation theory. Step one is the topic of section (3.2) and the actual application of the FMS expansion as well as the justification of the use of composite fermion states mentioned above will be discussed in chapter (3).

2.4. Verification of the FMS mechanism

The attentive reader has certainly already wondered about the validity or verifiability of augmented perturbation theory and the FMS mechanism explained above. From the theoretical side, due to the necessity of using non-perturbative methods [Maa19] we will focus on the mainly used technique in this case, which is lattice simulations. In this context, it must be taken into account that the entire standard model cannot be used for simulations at this point, but the FMS mechanism can be tested systematically in lattice calculations for parts of the full theory (e.g. solemnly the Higgs sector) as well as non-standard model theories.

Lattice gauge theories are a non-perturbative renormalisation method of quantum field theories and are widely used in quantum chromodynamics. The space-time continuum is hereby replaced by a finite lattice with discrete points. Due to the finite distance between the lattice points, it acts as a regulator of the theory in the ultraviolet by providing the necessary cut-off. The legitimacy of non-linear formulations on the other hand takes care of the infrared divergences by making the volume of the gauge transformations finite. The algebra-valued gauge fields are thus mapped to the gauge group, with the advantage that the gauge does not need to get fixed to perform calculations since integrals over compact groups are finite. This does not mean that it is impossible to fix the gauge. It is still necessary whenever we want to calculate gauge-dependent quantities.

A huge advantage of lattice simulations is that you can get actual results within a reasonable time since the CPU time in the majority of the cases is proportional to a fraction of the number of lattice points used for the simulation. Also, it is possible to sample all information including non-perturbative effects, because the numerical evaluation is exact up to some error sources which can still be improved. Taking into account the limitations of current computer resources (e.g.: finite memory, processing power etc.) only mass hierarchies of about one order of magnitude can be covered.

In the latest lattice simulations, the validity of the FMS mechanism and gauge-invariant perturbation theory could be verified for the Higgs sector, whereby there exist only a few calculations where simultaneously also the gauge-fixed propagators have been obtained. A major obstacle in the verification of the FMS mechanism for leptons is the chiral nature of the weak gauge theory since the chiral symmetry is broken explicitly by the lattice regularisation. It would be theoretically possible assuming the chiral symmetry is a global symmetry, to find a replacement which restores the original symmetry in the limit, but up to now, no such replacement has been found. Hence the usual procedure is to use a toy theory, where the Weyl spinors get replaced by Dirac spinors, mimicking the gauged Higgs-Yukawa structure of the standard model by imposing for example similar internal symmetries [AMST21]. It should be noted furthermore that in several lattice simulations the calculations were simplified by the so-called quenching. In this process, the determinant of the dynamic fermion matrix is replaced by a constant during the complex generation of the field configurations, so that only the gauge action remains. For testing the FMS mechanism this is no problem since the dynamics of the fermions will not alter the basic principles of the FMS mechanism, but has to be taken into account whenever one wants to compare results.

The remaining sectors of the standard model can be described by augmented perturbation theory, but due to large CPU times, no tests of these sectors have been performed using lattice calculations. It might be more promising in these cases to make use of functional methods to investigate larger parts of the standard model.

However, the available calculations strongly suggest the validity of the FMS mechanism in the full theory since in all tested scenarios the FMS results have been confirmed.

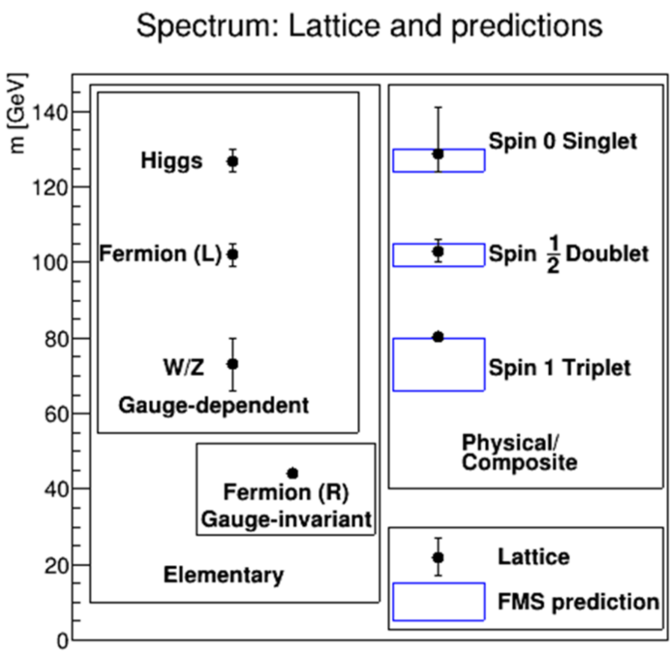

Figure 2.1.: We can see the spectrum of a toy theory where the lattice results (black data points) are depicted in comparison to the FMS predictions (blue boxes).

3. Lepton scattering

In section (2.2.1) we saw how the Bloch-Nordsieck theorem was restored in the case of quantum chromodynamics by using the coherent state approach and summing over the colour. This chapter shall be solemnly dedicated to the treatment of the electroweak case using the example of lepton scattering. To do so we start by looking at the standard perturbative treatment with elementary states and how this leads to an apparent violation of the theorem at question. Next we shall construct gauge-invariant bound states and discuss the so-called custodial symmetry. Finally we use our new states to show the non-violation of the Bloch-Nordsieck theorem for a sample lepton scattering process with the Higgs as spectator, which coincides with the low-energy case.

3.1. The standard perturbative treatment

This section is based on the treatment described in the paper of Ciafaloni and Comelli ([CCC00]). For a non-abelian theory the BN theorem is usually violated, due to the non-cancellation of the initial state interaction:

$$\sum_{f\in\Delta} |S_{fi}|^2 = \left\langle i \left| U^{I\dagger} O_h U^I \right| i \right\rangle = (S_h^\dagger S_h) + \Delta\sigma,$$

where $\Delta\sigma$ stands for the non-vanishing IR-singular part. In order to calculate the non-canceling double logs, we start by looking at ew corrections to the overlap matrix O_h. The lower order soft ew contributions are $\Delta\sigma = \sigma - \sigma_H$.

Considering two partons as initial states and assuming the kinematic invariants to be much larger than the gauge boson masses, the structure of O_h in isospin space is determined by the SU(2) symmetry. We can distinguish between three cases[1]:

- **RR**: Both initial partons are right-handed, they do not carry any non-abelian charges and thus O^h is simply a scalar.

$$O^h = A_0$$

- **LR,RL**: One parton is left- the other right-handed and as a result O^h carries two (left) isospin indices.

$$O^h_{\beta\alpha} = B_0 \delta_{\beta\alpha}$$

[1] Note that for cross-sections $\alpha_i = \beta_i$ holds.

- **LL**: Both initial partons are left-handed. In this case O^h carries four isospin indices.

$$O^h_{\beta_1\beta_2,\alpha_1\alpha_2} = C_0\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + C_1 4t^a_{\beta_2\alpha_2}t^a_{\beta_1\alpha_1} \qquad \text{(part-part)}$$

$$\bar{O}^h_{\beta_1\beta_2,\alpha_1\alpha_2} = \bar{C}_0\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + \bar{C}_1 4t^a_{\alpha_2\beta_2}t^a_{\beta_1\alpha_1} \qquad \text{(part-antipart)}$$

If we now apply the above to write down a generic hard cross-section for e^-_L and e^+_L (in the following denoted as e and $\bar{e}$), resp. ν and $\bar{\nu}$, where particle and anti-particle share the same isospin index, we get:

$$\sigma^h_{e\bar{e}} = \sigma^h_{\nu\bar{\nu}} \propto \bar{O}^h_{11,11} = \bar{O}^h_{22,22} = \bar{C}_0 + \bar{C}_1 \tag{3.1}$$

$$\sigma^h_{e\bar{\nu}} = \sigma^h_{\nu\bar{e}} \propto \bar{O}^h_{12,12} = \bar{O}^h_{21,21} = \bar{C}_0 - \bar{C}_1 \tag{3.2}$$

As a next step we want to dress the hard matrix elements with soft interactions. We again distinguish between the three cases:

- **RR**: The weak interactions become purely abelian and hence there does not exist any BN-violating effect in this case.

$$O^h \xrightarrow{\text{dress}} O = A_0$$

- **LR,RL**: Also in this case no BN-violating effect is present and the dressed overlap matrix coincides with the hard one.

$$O^h_{\alpha\beta} \xrightarrow{\text{dress}} O_{\alpha\beta} = {}_S\left\langle 0 \left| \mathcal{U}^\dagger_{\alpha\alpha'} O^h_{\alpha'\beta'}, \mathcal{U}_{\beta'\beta} \right| 0 \right\rangle_S = {}_S\left\langle 0 | B_0\delta_{\beta\alpha} | 0 \right\rangle_S = B_0\delta_{\beta\alpha}$$

  (Here we used the unitarity property of the coherent states operators.)

- **LL**: The only interesting case remains the one with two left-handed partons in the initial state:

$$O^h_{\beta_1\beta_2,\alpha_1\alpha_2} \xrightarrow{\text{dress}} O_{\beta_1\beta_2,\alpha_1\alpha_2} = {}_S\left\langle 0 \left| \mathcal{U}^{I\dagger}_{\beta_1\beta_2,\beta_1',\beta_2'} (O_h)_{\beta_1'\beta_2',\alpha_1'\alpha_2'} \mathcal{U}^I_{\alpha_1'\alpha_2',\alpha_1,\alpha_2} \right| 0 \right\rangle_S$$

  At the leading log level we can factorize $U^{I\dagger}$ into two leg operators:

$$\mathcal{U}^I_{\alpha_1'\alpha_2',\alpha_1,\alpha_2} = U^{(1)}_{\alpha_1'\alpha_1} U^{(2)}_{\alpha_2'\alpha_2} \qquad \text{(part-part)}$$

$$\mathcal{U}^I_{\alpha_1'\alpha_2',\alpha_1,\alpha_2} = U^{(1)}_{\alpha_1'\alpha_1} U^{(2)\dagger}_{\alpha_2\alpha_2'} \qquad \text{(part-antipart)}$$

  and write down for the case of one particle and one anti-particle:

$$\begin{aligned}\bar{O}_{\beta_1\beta_2,\alpha_1\alpha_2} &\equiv \bar{C}_0(s)\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + \bar{C}_1(s)4t^a_{\beta_1\alpha_1}t^a_{\alpha_2\beta_2} \\ &= \bar{C}_0\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2} + 4\bar{C}_1\ {}_S\left\langle 0 \left| \left(U^{(1)\dagger}t^a U^{(1)}\right)_{\beta_1\alpha_1} \left(U^{(2)\dagger}t^a U^{(2)}\right)_{\alpha_2\beta_2} \right| 0 \right\rangle_S ,\end{aligned}$$

where we can write the expression:

$$_S\left\langle 0\left|\left(U_A^{(2)\dagger}U_A^{(1)}\right)_{ab}\right|0\right\rangle_S t^a_{\beta_1\alpha_1}t^b_{\alpha_2\beta_2} = F_A(s,M^2)\, t^a_{\beta_1\alpha_1}t^b_{\alpha_2\beta_2}$$

with help of the Sudakov form factor in the adjoint representation, which reduces in the case of the isospin SU(2) symmetry to:

$$F_A(s,M^2) = e^{-2L_W(s)} \quad \text{with } L_W(s) = \frac{g^2}{8\pi^2} log^2 \frac{E^2}{M^2}$$

Here $L_W(s)$ denotes the eikonal radiation factor for W exchange. As a result for the dressed overlap matrix we obtain the factors:

$$\bar{C}_0(s) = \bar{C}_0 \quad \text{and} \quad \bar{C}_1(s) = \bar{C}_1 e^{-2L_W(s)}$$

We are now able to write down the dressed cross-sections as:

$$\sigma_{11} = \sigma_{22} \propto \bar{C}_0 + \bar{C}_1 e^{-2L_W(s)} = \frac{(\sigma_{11}+\sigma_{12})^h}{2} + \frac{(\sigma_{11}-\sigma_{12})^h}{2} e^{-2L_W(s)} \tag{3.3}$$

$$\sigma_{12} = \sigma_{21} \propto \bar{C}_0 - \bar{C}_1 e^{-2L_W(s)} = \frac{(\sigma_{11}+\sigma_{12})^h}{2} - \frac{(\sigma_{11}-\sigma_{12})^h}{2} e^{-2L_W(s)} \tag{3.4}$$

The relative effects in double log approximation are thus:

$$\left(\frac{\Delta\sigma}{\sigma}\right)_{11} \equiv \frac{\sigma_{11}-\sigma_{11}^H}{\sigma_{11}^H} = \left(\frac{\sigma_{11}^H-\sigma_{12}^H}{\sigma_{11}^H}\right)\left(\frac{1-e^{-2L_W(s)}}{2}\right) \tag{3.5}$$

$$\left(\frac{\Delta\sigma}{\sigma}\right)_{12} \equiv \frac{\sigma_{12}-\sigma_{12}^H}{\sigma_{12}^H} = \left(\frac{\sigma_{12}^H-\sigma_{11}^H}{\sigma_{12}^H}\right)\left(\frac{1-e^{-2L_W(s)}}{2}\right) \tag{3.6}$$

We can use the above derived knowledge to calculate the effects in simple processes that are of phenomenological relevance for collider experiments. In the following we will restrict ourselves to the case $l\bar{l} \to q\bar{q}$ (s-channel), which is of main interest for NLCs and will be discussed in greater detail in the next sections. Here we will simply state the main results of ([CCC00]):

For the polarized case of $e_L\bar{e}_L \to$ hadrons we get:

$$\left(\frac{\Delta\sigma_{e\bar{e}}}{\sigma_{e\bar{e}}^H}\right)^L = \left(\frac{\sigma_{\nu\bar{e}}^H-\sigma_{e\bar{e}}^H}{\sigma_{e\bar{e}}^H}\right)^L\left(\frac{1-e^{-2L_W(s)}}{2}\right) \approx 0.8\ L_W(s) \tag{3.7}$$

and for unpolarized ones the effect is slightly reduced:

$$\left(\frac{\Delta\sigma}{\sigma}\right)_{e\bar{e}}^{EW} \simeq 0.58 L_W(s) = 0.58\frac{\alpha_W}{4\pi}\log^2\frac{s}{M^2} \quad \text{with} \quad \alpha_W = \frac{g^2}{4\pi} \tag{3.8}$$

but still the radiative corrections of weak origin exceed at the TeV scale the QCD ones.

3.2. Construction of bound states

As was mentioned in section (2.3) the existence of Gribov-copies forces us to use bound-states as asymptotic states in order to restore gauge-invariance of our non-abelian gauge theory. To construct our physical fermion states we start by writing down the standard model Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}W^a_{\mu v}W^{a\mu v} + \frac{1}{2}\operatorname{tr}\left[(D_\mu X)^\dagger (D^\mu X)\right] \\ &-\frac{\lambda}{4}\left(\operatorname{tr}\left(X^\dagger X\right) - v^2\right)^2 + \bar{\psi}^L i\not{D}\psi^L + \bar{\chi}^R_f i\not{\partial}\chi^R_f \\ &-\sum_f y_f\left(\bar{\chi}^R_f\left(X^\dagger\psi^L\right)_f + \left(\bar{\psi}^L X\right)_f \chi^R_f\right),\end{aligned} \tag{3.9}$$

where we left out all terms, that are irrelevant for us in order to keep it simple. The Lagrangian is invariant under the full global group $\mathcal{G}$.

$W^a_{\mu\nu}$ is the field strength tensor for the weak gauge bosons (W and Z), D_μ is the covariant derivative and X is hereby a matrix-valued field constructed from the usual Higgs scalar doublet:

$$X = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix} \tag{3.10}$$

To specify the gauge group G we have 2 possibilities: Either we say G is the full global group $\mathcal{G}$ or we say G is a subgroup of $\mathcal{G}$. Either way we must ensure that $G \subseteq \mathcal{G}$ holds, so that the Lagrangian stays gauge-invariant. For the second possibility it is easy if G is a product group and hence we can write $\mathcal{G} = G \times C$. (Otherwise, the remainder might be just a coset or the group $\mathcal{G}$ even contains additional subgroups.) If C is not just a coset and thus $C \subseteq \mathcal{G} \setminus G$ holds, we call C the custodial symmetry of the group, which is a global symmetry group of the theory if the potential in the Lagrangian is invariant under G and C. (Otherwise, the custodial symmetry is explicitly broken completely or to a subgroup.) Luckily, the Higgs sector of the standard model, whose full global group is SO(4) can be written as a product in the following way:

$$SO(4) \sim (SU(2) \times SU(2)) \setminus Z_2$$

i.e. more loosely speaking, we can split $\mathcal{G}$ of the Standard model into a product of two SU(2) groups.

This new custodial symmetry is conceptionally nothing different from e.g. the flavour symmetry. It simply means that the scalar field possesses more degrees of freedom than would be minimally necessary to write down the theory.

Recall that we usually would have the two elementary fields:

$$\psi^L = \begin{pmatrix} \nu^L \\ e^L \end{pmatrix}$$
$$\chi^R = \begin{pmatrix} \nu^R \\ e^R \end{pmatrix}$$

containing the four physical fermionic states.

Considering an active BEH effect and ‘t Hooft gauge we can group them together into two chiral doublets in the following way:

$$\Psi^L = X^\dagger \psi^L \tag{3.11}$$

$$\chi^R = \begin{pmatrix} \nu^R \\ e^R \end{pmatrix} \tag{3.12}$$

If we expand these states according to the FMS mechanism and look at the leading order (LO) expression we notice that the fields reduce to the elementary ones:

$$\Psi^L = X^\dagger \psi^L = \left(\frac{v}{\sqrt{2}}\mathbb{1} + \eta\right)\psi^L = \frac{v}{\sqrt{2}}\begin{pmatrix} \nu^L \\ e^L \end{pmatrix} + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_1^L}\Psi_2^L \right\rangle = \left\langle \left(\frac{v}{\sqrt{2}}\overline{\psi_1^L} + \mathcal{O}(\eta)\right)\left(\frac{v}{\sqrt{2}}\psi_2^L + \mathcal{O}(\eta)\right)\right\rangle = \frac{v}{2}\left\langle \overline{\psi_1^L}\psi_2^L \right\rangle + \mathcal{O}(\eta)$$

3.3. The FMS mechanism for scattering processes

In order to show the validity of the Bloch-Nordsieck theorem for the electroweak case, we need to apply the FMS mechanism to the following type of correlation functions: $\left\langle \overline{\Psi_i^L}\Psi_j^L\overline{F_k}F_l \right\rangle$, which is the matrix element describing the scattering of two left-handed particles. With F being an arbitrary final state, e.g.: muon or quark, and the indices $i, j, k, l = 1, 2$.

Using equations (3.10) and (3.11) we can formerly write down the left-handed composite fields as:

$$\Psi^L = X^\dagger \psi^L = \begin{pmatrix} \phi_2\nu^L - \phi_1 e^L \\ \phi_1^*\nu^L + \phi_2^* e^L \end{pmatrix} \tag{3.13}$$

and respectively,

$$\bar{\Psi}^L = \bar{\psi}^L X = \begin{pmatrix} \phi_2^*\bar{\nu}^L - \phi_1^*\bar{e}^L \\ \phi_1\bar{\nu}^L + \phi_2\bar{e}^L \end{pmatrix} \tag{3.14}$$

For the following demonstration of the calculation we furthermore choose the final state to be:

$$F^L = X^\dagger f^L \quad \text{with} \quad f^L_{muon} = \begin{pmatrix} \nu_\mu^L \\ \mu^L \end{pmatrix} \tag{3.15}$$

We will explicitly apply the procedure of the FMS mechanism for the case $\left\langle \overline{\Psi_1^L}\Psi_1^L\overline{F_1}F_1 \right\rangle$, the other custodial combinations can be obtained analogously:

$$\begin{aligned}
\left\langle \overline{\Psi_1}\Psi_1\overline{F_1}F_1 \right\rangle =& \left\langle \left(\phi_2^*\bar{\nu}^L - \phi_1^*\bar{e}^L\right)\left(\phi_2\nu^L - \phi_1 e^L\right)\left(\phi_2^*\bar{\nu}_\mu^L - \phi_1^*\bar{\mu}^L\right)\left(\phi_2\nu_\mu^L - \phi_1\mu^L\right)\right\rangle = \\
=& \Big\langle (|\phi_2|^2\overline{\nu}^L\nu^L - 2\,\mathrm{Re}(\phi_1\phi_2^*\overline{\nu}^L e^L) + |\phi_1|^2\overline{e}^L e^L)\Big(|\phi_2|^2\,\bar{\nu}_\mu^L\nu_\mu^L - 2\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}_\mu^L\mu^L\right) \\
&+ |\phi_1|^2\,\bar{\mu}^L\mu^L\Big)\Big\rangle = \\
=& \Big\langle |\phi_2|^4\,\bar{\nu}^L\nu^L\bar{\nu}_\mu^L\nu_\mu^L - 2\,|\phi_2|^2\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}_\mu^L\mu^L\right)\bar{\nu}^L\nu^L + |\phi_1|^2\,|\phi_2|^2\,\bar{\nu}^L\nu^L\bar{\mu}^L\mu^L \\
&- 2\,|\phi_2|^2\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}^L e^L\right) + 4\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}^L e^L\right)\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}_\mu^L\mu^L\right) \\
&- 2\,|\phi_1|^2\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}^L e^L\right)\bar{\mu}^L\mu^L + |\phi_1|^2\,|\phi_2|^2\,\bar{e}^L e^L\bar{\nu}_\mu^L\nu_\mu^L \\
&-2\,|\phi_1|^2\,\mathrm{Re}\left(\phi_1\phi_2^*\bar{\nu}_\mu^L\mu^L\right)\bar{e}^L e^L + |\phi_1|^4\,\bar{e}^L e^L\bar{\mu}^L\mu^L \Big\rangle
\end{aligned}$$

applying step three of the FMS mechanism using:

$$X = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix} = \begin{pmatrix} \frac{v}{\sqrt{2}} & 0 \\ 0 & \frac{v}{\sqrt{2}} \end{pmatrix} + \eta,$$

the third and $6^{th} - 9^{th}$ terms in the last line vanish since $\langle\eta\rangle = 0$. And we are left to leading order (LO) with the expression:

$$\left\langle \overline{\Psi_1}\Psi_1\overline{F_1}F_1 \right\rangle = \frac{v^4}{4}\left\langle \bar{\nu}^L\nu^L\bar{\nu}_\mu^L\nu_\mu^L \right\rangle + \mathcal{O}(\eta) \tag{3.16}$$

It is straight forward to show then that:

$$\begin{aligned}
\left\langle \overline{\Psi_1}\Psi_1\overline{F_1}F_1 \right\rangle &= \frac{v^4}{4}\left\langle \bar{\nu}^L\nu^L\bar{\nu}_\mu^L\nu_\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_2}\Psi_2\overline{F_2}F_2 \right\rangle &= \frac{v^4}{4}\left\langle \bar{e}^L e^L\bar{\mu}^L\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_1}\Psi_2\bar{F}_1F_2 \right\rangle &= \frac{v^4}{4}\left\langle \bar{\nu}^L e^L\bar{\nu}_\mu^L\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_2}\Psi_1\overline{F_2}F_1 \right\rangle &= \frac{v^4}{4}\left\langle \bar{e}^L\nu^L\bar{\mu}^L\nu_\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_1}\Psi_1\overline{F_2}F_2 \right\rangle &= \frac{v^4}{4}\left\langle \bar{\nu}^L\nu^L\bar{\mu}^L\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_1}\Psi_1\overline{F_1}F_2 \right\rangle &= \frac{v^4}{4}\left\langle \bar{\nu}^L\nu^L\bar{\nu}_\mu^L\mu^L \right\rangle + \mathcal{O}(\eta) \\
\left\langle \overline{\Psi_1}\Psi_1\bar{F}_2F_1 \right\rangle &= \frac{v^4}{4}\left\langle \bar{\nu}^L\nu^L\bar{\mu}^L\nu_\mu^L \right\rangle + \mathcal{O}(\eta)
\end{aligned}$$

$$\left\langle \overline{\Psi_2}\Psi_2\overline{F_1}F_1 \right\rangle = \frac{v^4}{4}\left\langle e^L e^L \bar{\nu}_\mu^L \nu_\mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_2}\Psi_2\overline{F_1}F_2 \right\rangle = \frac{v^4}{4}\left\langle \bar{e}^L e^L \bar{\nu}_\mu^L \mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_2}\Psi_2\overline{F_2}F_1 \right\rangle = \frac{v^4}{4}\left\langle \bar{e}^L e^L \bar{\mu}^L \nu_\mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_1}\Psi_2\overline{F_1}F_1 \right\rangle = \frac{v^4}{4}\left\langle \bar{\nu}^L e^L \bar{\nu}_\mu^L \nu_\mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_1}\Psi_2\overline{F_2}F_2 \right\rangle = \frac{v^4}{4}\left\langle \bar{\nu}^L e^L \bar{\mu}^L \mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_2}\Psi_1\overline{F_1}F_1 \right\rangle = \frac{v^4}{4}\left\langle \bar{e}^L \nu^L \bar{\nu}_\mu^L \nu_\mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_2}\Psi_1\overline{F_2}F_2 \right\rangle = \frac{v^4}{4}\left\langle \bar{e}^L \nu^L \bar{\mu}^L \mu^L \right\rangle + \mathcal{O}(\eta)$$
$$\left\langle \overline{\Psi_2}\Psi_1\overline{F_1}F_2 \right\rangle = \frac{v^4}{4}\left\langle \bar{e}^L \nu^L \bar{\nu}_\mu^L \mu^L \right\rangle + \mathcal{O}(n)$$

3.4. The BN restoration

The primary objective of this section will be the restoration of the Bloch-Nordsieck theorem for the electroweak case using our newly constructed bound-states. Our main focus to achieve this purpose will be on an s-channel scattering process of the form: $(e^+, H)(e^-, H) \rightarrow (q, H)(\bar{q}, H)$, depictured in figure (3.1) since this type is most interesting from the experimental side of view.

Depending on the particular energy scale different interactions will eventually be observable. For low energies, the aforesaid process will appear as a standard scattering process with elementary fermion states and it is not before reaching the high energy region that the presence of the Higgs becomes apparent.

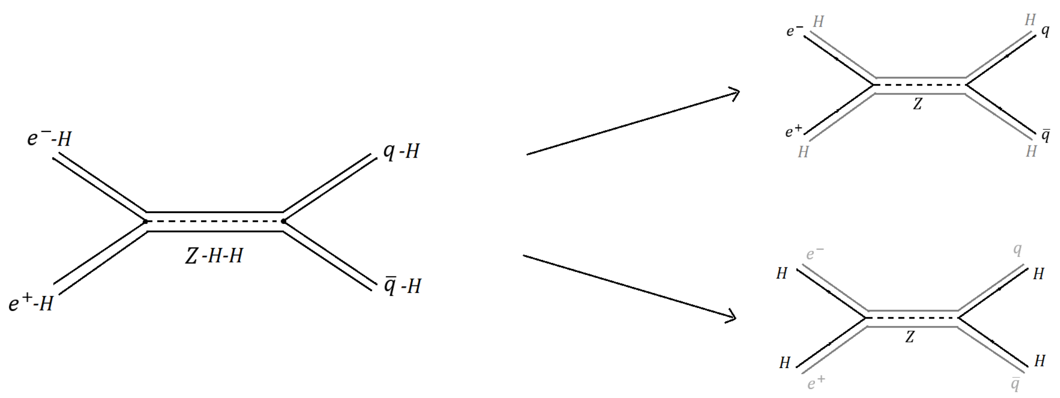

Figure 3.1.: Bound state scattering process divided into the possible observable cases at low energy and high energy (The grey lines indicate particles not involved in the actual scattering process.)

If we want to describe such a bound-state process in accordance with QCD by a parton model approach, it is necessary to know the Higgs PDF. Its specific form was exploited by Simon Fernbach in his Master's thesis [Fer19] and further studied in my Bachelor's thesis [Rei19], where I used the event generator HERWIG to simulate the relevant scattering processes. As a result, we ended up with a possible shape for the Higgs PDF, which can be seen in figure (3.2).

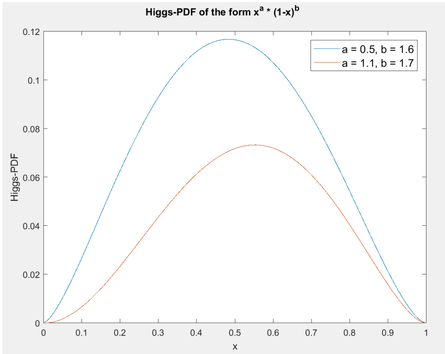

Figure 3.2.: possible forms of the Higgs PDF

By studying the picture of the Higgs PDF and comparing it with quark and gluon PDFs we notice the extraordinary size difference. While the former is still in the tenths of a percent range even at its peak, the probability for the latter is substantially larger. It is thus valid to assume that the Higgs' proportion is vanishing in the limit and to treat the Higgs boson as mere spectator. But by treating the Higgs fields as spectators for a fully exclusive measurement it is crucial in the following to sum the cross-sections of all possible pairings, i.e.:

$$\sigma_{\overline{\Psi_L^2}\Psi_L^2 \to X} \sim \sigma_{\bar{l}_L l_L \to X} + \sigma_{\bar{l}_L \nu_L \to X} + \sigma_{\overline{\nu_L} l_L \to X} + \sigma_{\overline{\nu_L} \nu_L \to X}, \tag{3.17}$$

where in our case $l \mathrel{\widehat{=}} e^-$ and $\bar{l} \mathrel{\widehat{=}} e^+$.

In appendix A we derived the differential cross-section for the process $e^+e^- \to q\bar{q}$ with photon and Z-boson exchange. Including also W-boson exchange we can rewrite our result in the form used by Ciafaloni and Comelli [CCC00]:

$$\begin{aligned} \frac{d\sigma_{ij}}{d\cos\theta} = \frac{N_c N_f}{128\pi s} \Big[& \left(A_0^R + C_0^L \pm C_1^L e^{-2L_W(s)} \right) (1 + \cos\theta)^2 + \\ & + \left(A_0^L + C_0^R \pm C_1^R e^{-2L_W(s)} \right) (1 - \cos\theta)^2 \Big], \end{aligned} \tag{3.18}$$

where N_f is the number of flavours included (for simplicity we set it to 1), N_c is the number of colours (i.e.: $N_c = 3$), the $\pm$ sign refers to σ_{11} and σ_{12} respectively and A_0, C_0, C_1 are the dressed overlap matrix components, which in this case read:

$$\begin{aligned} A_0^R &= \delta_{ij} g'^4 y_R^2 \sum Y_R^2 & A_0^L &= \delta_{ij} g'^4 y_R^2 \sum Y_L^2 \\ C_0^L &= \frac{3}{16} g^4 + \frac{1}{2} g'^4 y_L^2 \sum Y_L^2 & C_1^L &= -\frac{1}{16} g^4 + \frac{1}{2} g'^4 y_L^2 \sum Y_L^2 \\ C_0^R &= \frac{1}{2} g'^4 y_L^2 \sum Y_R^2 & C_1^R &= \frac{1}{2} g'^4 y_L^2 \sum Y_R^2 \end{aligned} \tag{3.19}$$

The factors $\frac{3}{16}g^4$ in C_0^L and $-\frac{1}{16}g^4$ in C_1^L are due to the W exchange and the delta function in A_0 ensures that the g'^4 contribution is only present for the case of an initial particle and its own antiparticle.

Considering the smallness of g', which is suppressed by a factor $\tan^4\theta_W = \frac{g'^4}{g^4}$, we can take the limit $g' \to 0$ and are left only with the left-handed components:

$$\lim_{g'\to 0} \frac{d\sigma_{ij}}{d\cos\theta} = \frac{d\sigma_{ij}^{LL}}{d\cos\theta} = \frac{\pi N_c \alpha_W^2}{32s} \left(\frac{1+\cos\theta}{2}\right)^2 \left(3 \mp e^{-2L_W(s)}\right) \tag{3.20}$$

with $\alpha_W = \frac{g^2}{4\pi}$.

The total cross-section thus reads:

$$\sigma_{ij}^{LL} = \frac{\pi^2 N_c \alpha_W^2}{24s} \left(3 \mp e^{-2L_W(s)}\right) \tag{3.21}$$

and we can see immediately that on average ($\sigma_{e\bar{e}}+\sigma_{\nu\bar{e}}$) the double logarithms cancel each other out and the result will be free of any IR divergences stemming from the Sudakov terms.

3.4.1. Coherent-state approach for the ew case

Formulating the coherent state approach for the electroweak case, we first need the according interaction Hamiltonian density:

$$\mathcal{H}_{int}^{ew} = \mathcal{H}_{\text{fermion}} + \mathcal{H}_{\text{3int}} + \mathcal{H}_{\text{4int}} + \mathcal{H}_{\text{Higgs}} + \mathcal{H}_{\text{ghost}} \tag{3.22}$$

where:

$$\mathcal{H}_{\text{fermion}} = \sum_i \bar{\Psi}_i'^L \left(-g_2 I_w^a W_\mu^a + g_1 \frac{Y_w}{2} B_\mu \right) \Psi_i'^L + \sum_{i,\sigma} \bar{\psi}_{i,\sigma}^R \left(g_1 \frac{Y_w}{2} B_\mu \right) \psi_{i,\sigma}^R \tag{3.23}$$

$$\mathcal{H}_{\text{3int}} = g_2 \epsilon^{abc} (\partial_\mu W_\nu^a) W_\mu^b W_\nu^c \tag{3.24}$$

$$\mathcal{H}_{\text{4int}} = \frac{1}{4} g_2^2 \epsilon^{abc} \epsilon^{ade} W_\mu^b W_\nu^c W^{d,\mu} W^{c,\nu} \tag{3.25}$$

$$\begin{aligned}\mathcal{H}_{\text{Higgs}} = &\left(i g_2 I_w^a W_\mu^a - i g_1 \frac{Y_w}{2} B_\mu \right) \left[\Phi^\dagger \partial_\mu \Phi - \Phi \left(\partial_\mu \Phi \right)^\dagger \right] + \\ &- \left[\left(-i g_2 I_w^a W_\mu^a + i g_1 \frac{Y_w}{2} B_\mu \right) \Phi \right]^\dagger \left[\left(-i g_2 I_w^a W^{a\mu} + i g_1 \frac{Y_w}{2} B^\mu \right) \Phi \right] + \\ &+ \frac{\lambda}{4} \left(\Phi^\dagger \Phi \right)^2 + \sum_{i,j} (g_{ij} \bar{\Psi}_i^L \psi_{j,-}^R \Phi + \tilde{g}_{ij} \bar{\Psi}_i^L \psi_{j,+}^R \Phi^c + hc)\end{aligned} \tag{3.26}$$

$$\begin{aligned}\mathcal{H}_{\text{ghost}} = &\bar{u}^1 (\partial^i W_\mu^3) u^2 - \bar{u}^1 (\partial^i W_\mu^2) u^3 - \bar{u}^2 (\partial^i W_\mu^3) u^1 + \\ &+ \bar{u}^2 (\partial^i W_\mu^1) u^3 + \bar{u}^3 (\partial^i W_\mu^2) u^1 - \bar{u}^3 (\partial^i W_\mu^1) u^2\end{aligned} \tag{3.27}$$

are the fermion, 3-point interaction, 4-point interaction, Higgs and ghost part accordingly. In the following, we will omit $\mathcal{H}_{\text{4int}}$ as well as the 4-Higgs interaction since they are not relevant at LO.

Next, we want to split the Hamiltonian into a soft and a hard part:

$$H_{int} = H_s^\Delta + H_h^\Delta \tag{3.28}$$

To do so we first quantise (3.28) in the interaction picture around $t = 0$ and introduce the customary restriction on the energy transfer: $\lambda < \nu = |\sum_i \sigma_i \omega_i| < \Delta$. It is important to point out that in the ew case the lower energy scale is characterised by the mass of the W boson. But since we assume here to be at sufficiently high energies we can neglect this nonetheless and continue our calculation analogously to the QCD case. Thus we obtain the soft interaction Hamiltonian:

$$H_s^{\Delta}(t) = H_{fermion} + H_{3int} + H_{Higgs} + H_{ghost}, \tag{3.29}$$

with its individual terms:

$$\begin{aligned}
H_{fermion} = & H_f^R + H_f^L = \\
= & + g_1 \frac{Y_w^R}{2} \sum_{\sigma} \int d[p_1] d[q_1] \rho_{f,R}\, \hat{p}_\mu B^{\mu\sigma}(q_1) e^{-i\sigma \hat{p}_1 q_1 t} \Theta(E_p - \omega_q) \Theta(\Delta - \hat{p}_1 q_1) + \\
& + g_1 \frac{Y_w^L}{2} \sum_{\sigma} \int d[p_2] d[q_2] \rho_{f,L}\, \hat{p}_\mu B^{\mu\sigma}(q_2) e^{-i\sigma \hat{p}_2 q_2 t} \Theta(E_p - \omega_q) \Theta(\Delta - \hat{p}_2 q_2) + \\
& - g_2 \sum_a I_w^a \sum_{\sigma} d[p_2] d[q_3] \rho_{f,L}\, \hat{p}_\mu W_a^{\mu\sigma}(q_3) e^{-i\sigma \hat{p}_2 q_3 t} \Theta(E_p - \omega_q) \Theta(\Delta - \hat{p}_2 q_3)
\end{aligned} \tag{3.30}$$

$$\begin{aligned}
H_{3int} = & -\frac{i g_2}{3!} \epsilon^{abc} \sum_{\sigma_i} \int d[q_1] d[q_2] d_[q_3] (2\pi)^3 \delta^3 \left(\sum \sigma_i q_i \right) e^{-i \sum \sigma_j \omega_j t} \cdot \\
& \cdot \Theta \left(\Delta - | \sum_j \sigma_j \omega_j | \right) \Gamma_{\mu_1 \mu_2 \mu_3}(\sigma_i q_i) W_a^{\mu_1 \sigma_1} W_b^{\mu_2 \sigma_2} W_c^{\mu_3 \sigma_3}
\end{aligned} \tag{3.31}$$

$$\begin{aligned}
H_{ghost} = & i \epsilon_{abc} \sum_{\sigma_i} \int d[q_1] d[q_2] d[q_3] (2\pi)^3 \delta^3 \left(\sum \sigma_i q_i \right) e^{-i \sum \sigma_j \omega_j t}\, \Theta \left(\Delta - | \sum_j \sigma_j \omega_j | \right) \cdot \\
& \cdot \bar{u}_a^{\sigma_1} u_b^{\sigma_2} q_3^{\mu} W_{\mu,c}^{\sigma_3}
\end{aligned} \tag{3.32}$$

$$
\begin{aligned}
H_{Higgs} = & ig_2\epsilon_{abc}\sum_{d=1}^{3} I_w^d \sum_{\sigma_i}\int d[q_1]d[q_2]d[q_3](2\pi)^3\delta^3\left(\sum\sigma_i q_i\right)e^{-i\sum\sigma_j\omega_j t}\cdot\\
& \cdot\Theta\left(\Delta - |\sum_j \sigma_j\omega_j|\right)\bar{h}_a^{\sigma_1}h_b^{\sigma_2}q_1^{\mu}W_{\mu,c}^{\sigma_3,d}+\\
& +ig_1\frac{Y_w}{2}\epsilon_{abc}\sum_{\sigma_i}\int d[q_1]d[q_2]d[q_3](2\pi)^3\delta^3\left(\sum\sigma_i q_i\right)e^{-i\sum\sigma_j\omega_j t}\cdot\\
& \cdot\Theta\left(\Delta - |\sum_j \sigma_j\omega_j|\right)\bar{h}_a^{\sigma_1}h_b^{\sigma_2}q_1^{\mu}B_{\mu,c}^{\sigma_3}+\\
& +\left[g_{11}\sum_{\sigma}\int d[p]d[q]\rho_f\hat{p}_\mu\Phi^{\sigma}(q)e^{-i\sigma\hat{p}qt}\Theta(E_p-\omega_q)\Theta(\Delta-\hat{p}q)+\right.\\
& \left.+\,\tilde{g}_{11}\sum_{\sigma}\int d[p]d[q]\rho_f\hat{p}_\mu(\Phi^c)^{\sigma}(q)e^{-i\sigma\hat{p}qt}\Theta(E_p-\omega_q)\Theta(\Delta-\hat{p}q)+h.c.\right]
\end{aligned}
\tag{3.33}
$$

where $\sigma = \pm$ and $\Gamma_{\mu_1\mu_2\mu_3}$ is the VVV vertex:

$$
\Gamma^{\mu_1\mu_2\mu_3}(q_1,q_2,q_3) = (q_1^{\mu_3}-q_2^{\mu_3})\,g^{\mu_1\mu_2} + (q_2^{\mu_1}-q_3^{\mu_1})\,g^{\mu_2\mu_3} + (q_3^{\mu_2}-q_1^{\mu_2})\,g^{\mu_3\mu_1}, \tag{3.34}
$$

which can be approximated in the strong ordering region by the eikonal form analogously to equation (2.14). Furthermore:

$$
\hat{p}_\mu = \frac{p_\mu}{E_p}; \qquad \rho_f = \sum_f b_f^\dagger b_f. \tag{3.35}
$$

and notice that we used a short hand notation, where e.g.: b^- is the annihilation operator for particles, b^+ the creation operator for anti-particles, $(b^+)^\dagger$ the annihilation operator for anti-particles and $(b^-)^\dagger$ is the creation operator for particles.

As the next step, we want to use the method of eikonal approximation to differentiate between soft and hard energies occurring at the vertex. Looking at a usual scattering experiment we have 3 types of first order radiative corrections: real, virtual (electroweak) and QCD ones. The first includes initial and final state radiations of a photon as well as interference between those (c.f.: fig. 3.3), the second consists of vertex and propagator corrections and box diagrams with two massive boson exchanges (c.f.: fig. 3.4), while the third covers gluon radiation (c.f.: fig. 3.5). Since we study in this section only the electroweak interaction we can omit the QCD correction in the following. The eikonal approximation thus enables us to factorize the scattering amplitude squared into the hard Born amplitude (without any radiation) and a factor containing all radiation and any virtual corrections coincide with the real ones up to a sign. As a result, for the VVV-interaction vertex one can define in the strongly ordered energy region an emission current similar to QCD:

$$j^a_\mu(q) = \left(\frac{\bar{p}_\mu}{\bar{p} \cdot q + \mathrm{i}\varepsilon} t^a_{\bar{p}} + \frac{p_\mu}{p \cdot q + \mathrm{i}\varepsilon} t^a_p \right), \tag{3.36}$$

with $t^a = \sigma^a/2$ the SU(2) generators, which can be used to write down the amplitude squared.

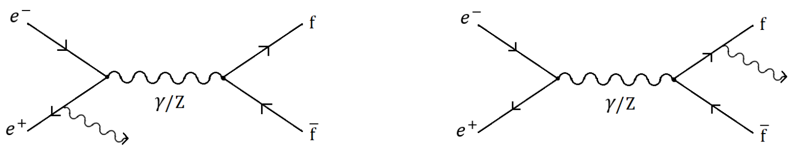

Figure 3.3.: Feynman diagrams of QED corrections

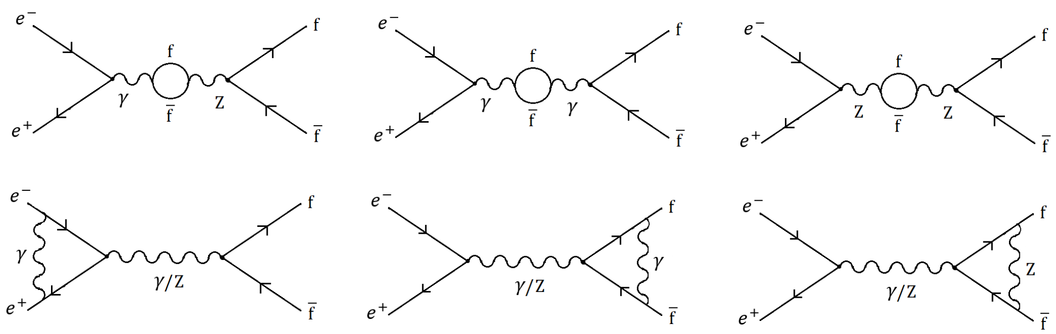

Figure 3.4.: Feynman diagrams of virtual (electroweak) corrections

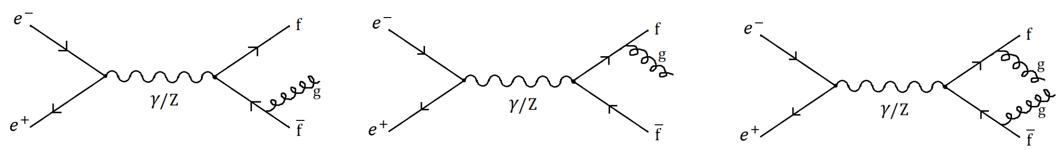

Figure 3.5.: Feynman diagrams of QCD corrections

Applying this to our soft Hamiltonian and acting on a set of states $|h\rangle = |p_1\alpha_1, ..., p_h\alpha_h\rangle$ with it, we obtain the decomposition for the transition $h_i \to h_f$ [2]:

[2] The ghost and VVVV-interaction can be omitted.

$$\begin{aligned} H_s &= H_{eik}(E_{partons} > \Delta) + H_s(\omega_i < \Delta) \\ &= H_{eik}^{>} + H_s^{<} = \\ &= g_i \sum_{i \in h} t_a^i \hat{p}_\mu^i A^{\mu,a}\left(t, \boldsymbol{\nu}_i t\right) + g_{11} \sum_{i \in h} t_a^i \hat{p}_\mu^i \Phi + \tilde{g}_{11} \sum_{i \in h} t_a^i \hat{p}_\mu^i \Phi^c + H_s^{<} \end{aligned} \tag{3.37}$$

where

$$g_i = g_1 \frac{Y_w^R}{2} + g_1 \frac{Y_w^L}{2} - g_2 \sum_a I_w^a$$

$$A^{\mu,a} = B^\mu + W^{\mu,b} \qquad b = 1, 2, 3$$

and

$$H_s^{<} \approx \sum_\sigma \int_\lambda^\Delta d[p]d[q]\ \rho_{3int}^a(p)\ \hat{p}_\mu\ W_a^{\mu\sigma}(\boldsymbol{q})\ e^{-i\sigma \hat{p}\cdot qt}\ \Theta(E_p - \omega), \tag{3.38}$$

with

$$\rho_{3int} = i\epsilon^{abc} W_b^{\mu\sigma}(p) W_c^{\mu\sigma}(p).$$

Eventually, we can calculate the eikonal coherent state operators:

$$U_{eik}^{\Delta,h}(\Pi) = exp\left[ig \int_\lambda^\Delta d[q] \sum_a \sum_{i=1}^h \frac{\hat{p}_i^\mu t_i^a}{\hat{p}_i q} \Pi_{a,\mu}^\omega(q)\right] \tag{3.39}$$

with the vector fields, dressed up to energy ω, defined as:

$$\Pi_{a,\mu}^\omega(q) = -i\left(A_{\mu,a}^\omega - A_{\mu,a}^{\omega\dagger}\right) \tag{3.40}$$

Since $U_{eik}^{\Delta,h}(\Pi)$ are functionals of $\Pi_{a\mu}^\omega(q)$ only they commute at different isospin indices:

$$\left[U_{\alpha\beta}^\Delta(\Pi), U_{\alpha'\beta'}^\Delta(\Pi)\right] = 0 \tag{3.41}$$

and thus by summing over the weak isospin in our calculation to prove the BN theorem we end up with an IR-finite result.[3]

[3]Compare with e.q. (2.21) but instead of the sum over different colours we have in this case the sum over different weak isospins.

4. Hadron scattering and PDF formulation

Having discussed the case of lepton scattering in great detail in the previous chapter, we now turn our focus on the hadron scattering case, with quarks as initial states. These kind of processes are of special interest for hadron colliders such as the LHC (or future FCC).

Due to the fact that the initial proton states are not SU(2) gauge singlets, the BN theorem is violated in the standard approach. The effect of the appearing Sudakov logarithms gets worse as the center of mass energy increases ([CCC00], [BPW18]). Trying to fix it analogously to the procedure described previously, we encounter some difficulties arising from the need of using PDFs to describe the internal structure of the proton. In the standard approach the cross-section is defined as:

$$\sigma_{PP\to X} = \sum_{ij} \int_0^1 dx \int_0^1 dy f_i(x) f_j(y) \sigma_{\bar{i}j\to X}\left(xp_1, yp_2\right), \tag{4.1}$$

where i, j run over all hadron constituents and f_i and f_j are their according PDFs. For the moment we leave the final state undetermined and denoted by X.

Restricting us for the moment to strong interactions only, we want to explore how the violation of the BN theorem is resolved in the QCD case. As we have already explored, we need to sum over all colours in the hard cross-section to truly include all the information of all possible final states. This is represented in the PDF formulation via three separate PDFs for each quark. However, this way we encounter a seemingly unavoidable contradiction, namely the coloured quark states are physical states in the BRST construction, where we are in the realm of perturbation theory. In contrast to that, PDFs are non-perturbative quantities, which breaks down the BRST construction. Also, quarks cannot act as physical initial states. The answer to the resulting question, how such a factorisation can still work, lies in the one-to-one correspondence between each quark and its flavour.

4.1. Construction of gauge-invariant quark fields

Switching back to our starting situation including weak interactions and restricting ourselves to first generation quarks only, the isospin sum rules:

$$\int dx\left(f_{u_L}(x)-f_{\bar{u}_L}(x)+f_{u_R}(x)-f_{\bar{u}_R}(x)\right)=2 \tag{4.2a}$$

$$\int dx\left(f_{d_L}(x)-f_{\bar{d}_L}(x)+f_{d_R}(x)-f_{\bar{d}_R}(x)\right)=1 \tag{4.2b}$$

$$f_{i_L}(x)=f_{i_R}(x) \tag{4.2c}$$

dictate different PDFs for up- and down-type quarks. Having left-handed up- and down-quarks as members of the multiplets the BN violation appears unavoidable. Especially since the possibility of loosening the equality of left-handed and right-handed quarks, which after all becomes necessary at high energies, and completely shifting the isospin to the right-handed partons seems rather unlikely since the proton is a parity eigenstate.

The way out of this apparent dead end will open up by the nature of eq.(4.1), which represents only the LO of the FMS expansion, i.e.: we will need to consider higher orders of the expansion as well. The usual elementary left-handed quark field, $q^L=(u^L,d^L)$ can be dressed equivalently to the lepton case in eq.(3.11) and thus the physical flavour of the left-handed quarks equals the global symmetry of the Higgs field. The difference to the left-handed leptons is consequently entirely determined by the remaining quantum numbers, such as hyper charge, baryon number and lepton number.

Eventually we can construct our weak gauge-invariant quark fields as Dirac spinors of the form:

$$\begin{aligned} U&=\begin{pmatrix} \left((X)^\dagger q^L\right)_1 \\ u^R \end{pmatrix}\\ D&=\begin{pmatrix} \left((X)^\dagger q^L\right)_2 \\ d^R \end{pmatrix}. \end{aligned} \tag{4.3}$$

4.2. The BN theorem restoration for hadrons

Furthermore we can construct a nucleon operator needed for lattice simulations:

$$\begin{aligned} N=&\frac{1}{2}\left(1+\gamma_0\right)\epsilon^{IJK}U_I\left(U_J^T C\gamma_5 D_K\right)=\\ =&\frac{\epsilon^{IJK}}{2}\begin{pmatrix} u_R-\left((X)^\dagger q_L\right)_1 \\ \left((X)^\dagger q_L\right)_1-u_R \end{pmatrix}^I \times\left(\left(\left((X)^\dagger q_L\right)_1^J\right)^T\tau^2\left((X)^\dagger q_L\right)_2^K-\left(u_R^J\right)^T\tau^2 d_R^K\right)=\\ =&\frac{\epsilon^{IJK}}{2}\left(\begin{pmatrix} u_R-\phi_2 u_L \\ \phi_2 u_L-u_R \end{pmatrix}^I+\phi_1\begin{pmatrix} d_L \\ -d_L \end{pmatrix}^I\right)\times\\ &\times\left(|\phi_2|^2\left(u_L^J\right)^T\tau_2 d_L^K-|\phi_1|^2\left(d_L^J\right)^T\tau^2 u_L^j-\left(u_R^J\right)^T\tau^2 d_R^K\right.\\ &\left.+\ \phi_2\phi_1^*\left(u_L^J\right)^T\tau^2 u_L^K-\phi_1\phi_2^*\left(d_L^J\right)^T\tau_2 d_L^K\right), \end{aligned} \tag{4.4}$$

where we follow the construction described in ([Maa19], [EMS17], [GL10]). In eq.(4.4) C is the charge conjugation matrix, τ^2 the second Pauli matrix and the capital indices run over the different colours. The projector in front excludes positive parity as is required for a proton. This nucleon operator collapses to its usual QCD form [GL10], assuming LO in the Higgs v.e.v., $\phi_1 = 0$ and $\phi_2 = v$.

Taking a closer look at eq.(4.4) we recognise that U solemnly carries all of the spin and flavour and due the preceding projector mixes left-handed with right-handed components. It is easily apparent that in a manifestly gauge-invariant description additional valence Higgs degrees of freedom exist, which carry the proton flavour.

This allows for the interpretation of the would-be quark PDFs in eq.(4.1) as the probability to encounter a flavour carrier eq.(4.3) in the proton. At low energies we have the case that the structure of the flavour carriers is not resolved and thus we have quark-Higgs bound states as initial states in our hard cross-section, which can be FMS expanded.

Assuming the entire internal weak structure of the proton, eq.(4.4), to be part of the weak bound state, eq.(4.3), enables us to switch at high energies from LO to the full expression just like in eq.(3.17) and hence restore the BN theorem by reinterpreting the quark PDFs as physical flavour PDFs. However, the consideration of multi-parton interactions or in the case of the impossibility to separate between strong and weak bound states, we rely on the introduction of explicit valence Higgs PDFs to maintain the isospin sum rules.

This can be done by defining four Higgs PDFs, f_{hij}, where i and j denote the weak and custodial contributions. Note that the flavour is carried by the custodial symmetry and therefore does not appear as extra index. To preserve gauge invariance as well as the BN theorem the following equation must hold for all PDFs:

$$f_{h\frac{1}{2}j} = f_{h-\frac{1}{2}j}, \tag{4.5}$$

where $j = u, d$ for up-type and down-type quark flavour respectively. As a result, we can now rewrite the sum rules of eq. (4.2a) and (4.2b) as:

$$\int dx \left(f_{u_R}(x) - f_{\bar{u}_R}(x) + f_{h\frac{1}{2}u} + f_{h-\frac{1}{2}u} \right) = 2 \tag{4.6a}$$

$$\int dx \left(f_{d_R}(x) - f_{\bar{d}_R}(x) + f_{h\frac{1}{2}d} + f_{h-\frac{1}{2}d} \right) = 1 \tag{4.6b}$$

In R_ξ gauges this would mean possibly having unphysical initial states, which, however, can be at high energies due to the Goldstone boson equivalence theorem be seen as longitudinally polarized vector boson matrix elements. The remaining sum rule of eq. (4.2c) still contains left-handed quark PDFs, but they relate via:

$$f_{u_L} = f_{d_L}. \tag{4.7}$$

Note that the sum rules for momentum and electromagnetic charge in the new setting contain all types of PDFs, also the Higgs ones. This new structure preserves the BN theorem automatically also for hadron collisions. The challenge, however, is the determination of all the necessary PDFs. Nevertheless, we can view two different scenarios, where we can use approximations to avoid this tedious work and still get rather satisfying results.

In the first one, we consider the low energy case, where we can ignore the weak sub-structure and thus approximate:

$$\begin{aligned}&\sum_{u_L,d_L,h_{ij}} \int_0^1 dx \int_0^1 dy\; f_i(x) f_j(y) \sigma^{elementary}_{\bar{i}j\to X}(xp_1, yp_2) \approx \\ &\approx \sum_{u_L,d_L} \int_0^1 dx \int_0^1 dy\; \tilde{f}_i(x) \tilde{f}_j(y) \sigma^{elementary}_{\bar{i}j\to X}(xp_1, yp_2)\,,\end{aligned} \tag{4.8}$$

where $\tilde{f}_i$ are the standard BN theorem violating PDFs and the "elementary" superscripts imply that they are the usual perturbative cross-sections, i.e.: the ones in LO in the Higgs v.e.v. of perturbation theory. From the high energy point of view, what in reality happens is that either a combination of left-handed quarks from a valence bound state, eq.(4.3), or a radiation of left-handed quarks from the valence Higgs take part in the hard interaction, and the Higgs becomes off-shell suppressed. Ignoring the Higgs part in the process will lead to the ususal BN theorem violating terms, whose effect is sufficient small to be neglected at low energies.

In the second scenario, we treat the weak sub-structure solemnly with augmented perturbation theory and use eq.(3.17). This might take care of the weak sub-structure, but not of the strong one. The consequent sub-processes due to the interaction of the QCD cores of the physical proton need to be approached via a PDF formulation again. This harbours the problem of not only having to deal with proton-like states but also QCD-like objects, whose PDFs are in contrast to the former not as easily available. The problem might better be approached by other sources, such as lattice calculations.

As we restricted us in the previous discussion on the first generation quarks only, it is worth noting how the inclusion of the other generations can possibly be treated. The basic concepts of course remain the same, but get further complicated by the off-diagonal inter-generation elements of the CKM matrix. These can be recast as inter-generation Yukawa interactions of the Higgs, which suggests a PDF scheme to be the formulation of choice here. The exact implementation would however go beyond the scope of this work and will be left for future researchers.

5. Conclusion

Let us recall the basic statement of the BN theorem once again: In massless theories, we are getting confronted with IR singularities because emission and absorption probabilities for massless particles do not decrease sufficiently fast enough in the limit of large distances and long times and thus interactions cannot be neglected at remote future or past. The infinite number of possible indistinguishable final states must be considered by summing over them and as a result, the virtual and real IR divergences cancel each other.

Reminiscing the initial problem we were confronted with, namely the violation of the Bloch-Nordsieck theorem for the electroweak sector of the Standard model due to the existence of a non-Abelian charge, which destroys the necessary gauge-invariance of the asymptotic states, we were able to restore its validity by introducing compound operators as physical states in our theory. At low energies and an active BEH effect by using the FMS expansion and only keeping terms at LO, we obtain the same results as standard perturbation theory. At energies $s >> m_w^2$ the Higgs part of the bound state only acts as a spectator to the dominant FMS part and since the scattering process is inclusive with respect to the weak doublet, it is necessary to sum over all possible cross-sections, which results in an IR safe result.

Encouraged by the direct calculations, it was only natural to attempt a more general proof in the form of the coherent state approach, where it turned out that the coherent state operators commute at different weak isospin indices and thus the S-matrix becomes IR safe by summing over the different isospin indices of the bound-states.

A more sophisticated situation presents itself for situations where the influence of the Higgs becomes inevitable (at the TeV scale of future lepton colliders) as well as in the case of non-trivial bound-state contributions. Either way, we will have to switch to a PDF language, which is more complicated and arises the need for further studies, whether it be to analyse the Higgs PDF in more detail or to include further lepton generations. The latter gets complicated due to inter-generational entries of the CKM matrix and might be a good candidate for the use of lattice methods. Furthermore, at very high energies we expect fragmentation to happen analogously to QCD.

In the end, the augmented perturbation theory using the FMS mechanism may be an efficient way to describe the potential impact for predictions of effects happening at future colliders without too much extra effort. Moreover, the new gauge-invariant description of weak scattering processes requiring a Higgs proportion in the initial and final states opens up a variety of new research opportunities in theoretical as well as experimental particle physics, which allows us to look into the future with joyful anticipation for what remains to be seen.

A. S-channel cross-section $e^-e^+ \to q\bar{q}$

We want to focus in this section on the derivation of the cross-section for the s-channel process $e^-e^+ \to q\bar{q}$ at tree- level including photon and Z-boson exchange, as well as making a high energy approximation. We start by drawing the relevant Feynman diagrams and recalling the according Feynman rules.

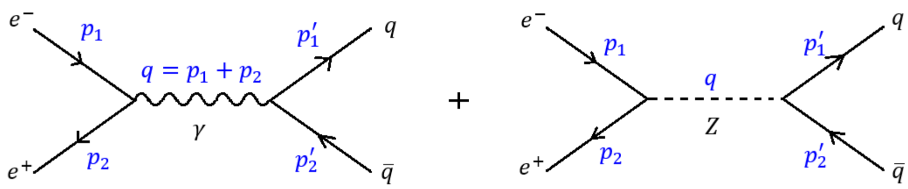

Figure A.1.: Feynman diagrams

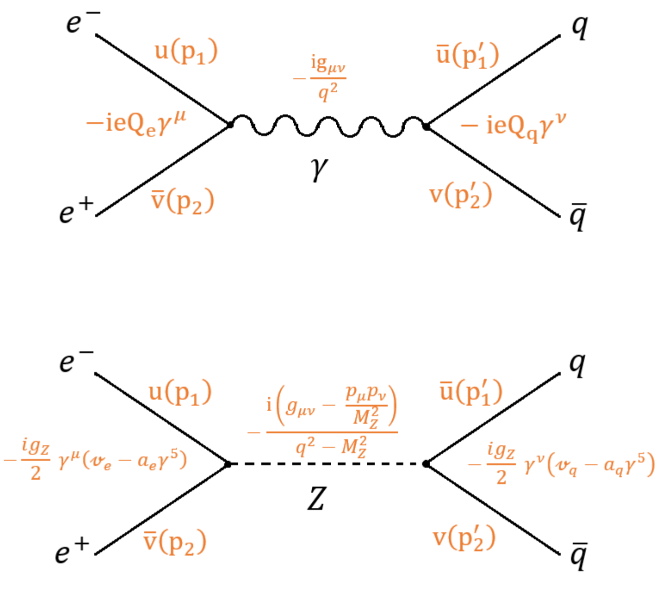

Figure A.2.: Feynman rules for fig.(A.1) with $g_Z = \frac{e}{sin\theta_w cos\theta_w}$, $v_f = I_f^3 - 2Q_f sin^2\theta_w$, $a_f = I_f^3$, where $f = e, q$ in our case, I_f^3 is the weak isospin and Q_f is the electric charge measured in units of positron charge ($e > 0$), i.e.: $Q_e = -1$, $Q_{up} = 2/3$, $Q_{down} = -1/3$. Hence the matrix element reads as follows:

$$\begin{aligned} -i\mathcal{M} =& - i(\mathcal{M}_\gamma + \mathcal{M}_\mathcal{Z}) = \\ &= ie^2 Q_e Q_q \left[\bar{\mathsf{v}}(p_2)\gamma^\mu u(p_1)\right] \frac{g_{\mu\nu}}{q^2} \left[\bar{u}(p_1')\gamma^\nu \mathsf{v}(p_2')\right] + \\ &+ i\left(\frac{g_Z}{2}\right)^2 \left[\bar{\mathsf{v}}(p_2)\gamma^\mu(v_e - a_e\gamma^5)u(p_1)\right] \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}}{q^2 - M_Z^2} \left[\bar{u}(p_1')\gamma^\nu(v_q - a_q\gamma^5)\mathsf{v}(p_2')\right] \end{aligned} \tag{A.1}$$

To continue with the calculation we first need to specify some pre-arrangements. In the ultra-relativistic limit we are working in ($E \gg m \Rightarrow E \approx p$ and $m \approx 0$) the left- and right-handed spinors take the form:

$$\begin{aligned} u_R &\approx \sqrt{E}\left(c, se^{i\phi}, c, se^{i\phi}\right) \\ u_L &\approx \sqrt{E}\left(-s, ce^{i\phi}, s, -ce^{i\phi}\right) \\ \mathsf{v}_R &\approx \sqrt{E}\left(s, -ce^{i\phi}, -s, ce^{i\phi}\right) \\ \mathsf{v}_L &\approx \sqrt{E}\left(c, se^{i\phi}, c, se^{i\phi}\right) \end{aligned} \tag{A.2}$$

where we used the abbreviations $s = sin\frac{\theta}{2}$ and $c = cos\frac{\theta}{2}$. Furthermore we lay the coordinate system in such a way, that the incoming leptons lay on the z-axis and thus with:

e^-	$\theta = 0$ $\phi = \phi$	e^+	$\theta = \pi$ $\phi = \pi - \phi$
q	$\theta = \theta$ $\phi = 0$	$\bar{q}$	$\theta = \pi - \theta$ $\phi = \pi$

we get:

e^-	$u_R = \sqrt{E}\,(1, 0, 1, 0)$ $u_L = \sqrt{E}\,(0, 1, 0, -1)$	e^+	$\mathsf{v}_R = \sqrt{E}\,(1, 0, -1, 0)$ $\mathsf{v}_L = \sqrt{E}\,(0, 1, 0, 1)$
q	$u_R = \sqrt{E}\,(c, s, c, s)$ $u_L = \sqrt{E}\,(-s, c, s, -c)$	$\bar{q}$	$\mathsf{v}_R = \sqrt{E}\,(c, s, -c, -s)$ $\mathsf{v}_L = \sqrt{E}\,(s, -c, s, -c)$

Table A.1.: left-handed and right-handed spinors

Additionally for two arbitrary spinors ψ and ϕ we have:

$$\begin{aligned}
\bar{\psi}\gamma^0\phi &= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\bar{\psi}\gamma^1\phi &= \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\bar{\psi}\gamma^2\phi &= -i\left(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1\right) \\
\bar{\psi}\gamma^3\phi &= \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}
\bar{\psi}\gamma^0\gamma^5\phi &= \psi_1^*\phi_3 + \psi_2^*\phi_4 + \psi_3^*\phi_1 + \psi_4^*\phi_2 \\
\bar{\psi}\gamma^1\gamma^5\phi &= \psi_1^*\phi_2 + \psi_2^*\phi_1 + \psi_3^*\phi_4 + \psi_4^*\phi_3 \\
\bar{\psi}\gamma^2\gamma^5\phi &= +i\left(\psi_1^*\phi_2 - \psi_2^*\phi_1 + \psi_3^*\phi_4 - \psi_4^*\phi_3\right) \\
\bar{\psi}\gamma^3\gamma^5\phi &= \psi_1^*\phi_1 - \psi_2^*\phi_2 + \psi_3^*\phi_3 - \psi_4^*\phi_4
\end{aligned} \tag{A.4}$$

Having all the necessary ingredients together we will split the matrix element into the photon and the Z-boson part and calculate them separately, starting with the former.

A.1. Photon part

The photon part of the matrix element eq. (A.1) reads:

$$\begin{aligned}
-i\mathcal{M}_\gamma &= ie^2 Q_e Q_q \left[\bar{\mathsf{v}}(p_2)\gamma^\mu u(p_1)\right] \frac{g_{\mu\nu}}{q^2} \left[\bar{u}(p_1')\gamma^\nu \mathsf{v}(p_2')\right] \\
&= ie^2 Q_e Q_q \, (j_e^\gamma)^\mu \, \frac{g_{\mu\nu}}{q^2} \, (j_q^\gamma)^\nu
\end{aligned} \tag{A.5}$$

with $(j_e^\gamma)^\mu$ the electron current and $(j_q^\gamma)^\nu$ the quark current. Using eq. (A.3) we are able to write down expressions for the electron and quark currents for different combinations of left-handed and right-handed particles.

electron currents		quark currents	
$e_R^- e_L^+$	$(j_e^\gamma)^\mu = 2E(0, -1, -i, 0)$	$q_R\bar{q}_L$	$(j_q^\gamma)^\nu = 2E(0, -cos\theta, i, sin\theta)$
$e_L^- e_R^+$	$(j_e^\gamma)^\mu = 2E(0, -1, i, 0)$	$q_L\bar{q}_R$	$(j_q^\gamma)^\nu = 2E(0, -cos\theta, -i, sin\theta)$
$e_R^- e_R^+$	$(j_e^\gamma)^\mu = 0$	$q_R\bar{q}_R$	$(j_q^\gamma)^\nu = 0$
$e_L^- e_L^+$	$(j_e^\gamma)^\mu = 0$	$q_L\bar{q}_L$	$(j_q^\gamma)^\nu = 0$

Table A.2.: electron and quark currents in case of a photon as exchange particle

We are thus left with only four non-zero combinations:

$$\begin{aligned} e^-_R e^+_L &\to q_R \bar{q}_L \\ e^-_R e^+_L &\to q_L \bar{q}_R \\ e^-_L e^+_R &\to q_R \bar{q}_L \\ e^-_L e^+_R &\to q_L \bar{q}_R \end{aligned}$$

whose according matrix elements we will in the following denote as $\mathcal{M}_{RR}$, $\mathcal{M}_{RL}$, $\mathcal{M}_{LR}$ and $\mathcal{M}_{LL}$. Their absolute values squared take the forms:

$$\begin{aligned} |\mathcal{M}_{RR}|^2 = |\mathcal{M}_{LL}|^2 &= (4\pi\alpha)^2 Q_f^2 (1+\cos\theta)^2 \\ &= e^4 Q_f^2 (1+\cos\theta)^2 \\ |\mathcal{M}_{RL}|^2 = |\mathcal{M}_{LR}|^2 &= (4\pi\alpha)^2 Q_f^2 (1-\cos\theta)^2 \\ &= e^4 Q_f^2 (1-\cos\theta)^2 \end{aligned} \tag{A.6}$$

For the differential cross-section we also need to perform a spin averaging and sum over the different colors of the final state quarks. Considering this the formula reads:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_1|}{|\vec{p}_1|} \frac{3}{4} |M_{ji}|^2 \tag{A.7}$$

and we can simply plug in the results from eq.(A.6):

$$\begin{aligned} \frac{d\sigma}{d\Omega} =& \frac{1}{64\pi^2 s} \frac{3}{4} 2\, (4\pi\alpha)^2 Q_f^2 \left[(1+\cos\theta)^2 + (1-\cos\theta)^2 \right] = \\ =& \frac{3}{4} \frac{e^4}{(4\pi)^2} \frac{Q_f^2}{s} (1+cos^2\theta) \end{aligned} \tag{A.8}$$

where we used $q^2 = s = 4E^2$ and $e^2 = 4\pi\alpha$. Since we are viewing an azimuthally symmetric scattering process, we can rewrite eq.(A.8) as:

$$\frac{d\sigma}{dcos\theta} = \frac{3}{4} \frac{e^4}{8\pi s} Q_f^2 (1+cos^2\theta) \tag{A.9}$$

A.2. Z-boson part

We continue now with the Z-boson part of the matrix element eq.(A.1):

$$\begin{aligned} -i\mathcal{M}_{\mathcal{Z}} = &+ i\left(\frac{g_Z}{2}\right)^2 \left[\bar{\mathsf{v}}(p_2)\gamma^\mu(v_e - a_e\gamma^5)u(p_1)\right] \cdot \\ &\frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}}{q^2 - M_Z^2} \cdot \left[\bar{u}(p_1')\gamma^\nu(v_q - a_q\gamma^5)\mathsf{v}(p_2')\right] \end{aligned} \tag{A.10}$$

Using the polarisation operators $P_L = \frac{1}{2}(1-\gamma^5)$ and $P_R = \frac{1}{2}(1+\gamma^5)$ and rewriting v_f and a_f with help of a left-handed and a right-handed part: $v_f = c_L^f + c_R^f$, $a_f = c_L^f - c_R^f$, where $c_R^f = -Q_f sin^2\theta_W$ and $c_L^f = I_f^3 - Q_f sin^2\theta_W$, the matrix element eq.(A.10) reads:

$$\begin{aligned} \mathcal{M}_{\mathcal{Z}} = -g_Z^2 \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}}{q^2 - M_Z^2} &\left[c_L^e \bar{\mathsf{v}}(p_2)\gamma^\mu P_L u(p_1) + c_R^e \bar{\mathsf{v}}(p_2)\gamma^\mu P_R u(p_1)\right] \cdot \\ &\cdot \left[c_L^q \bar{u}(p_1')\gamma^\nu P_L \mathsf{v}(p_2') + c_R^q \bar{u}(p_1')\gamma^\nu P_R \mathsf{v}(p_2')\right] \end{aligned} \tag{A.11}$$

Furthermore we know that $P_L u = u_L$, $P_R u = u_R$, $P_L v = v_R$ and $P_R v = v_L$, which leads to the expression:

$$\begin{aligned} \mathcal{M}_{\mathcal{Z}} = &- P_Z g_Z^2 g_{\mu\nu} \left[c_L^e \bar{\mathsf{v}}_R(p_2)\gamma^\mu u_L(p_1) + c_R^e \bar{\mathsf{v}}_L(p_2)\gamma^\mu u_R(p_1)\right] \cdot \\ &\cdot \left[c_L^q \bar{u}_L(p_1')\gamma^\nu \mathsf{v}_R(p_2') + c_R^q \bar{u}_R(p_1')\gamma^\nu \mathsf{v}_L(p_2')\right] = \\ = &- P_Z g_Z^2 g_{\mu\nu} [c_L^e (j_{e,LR}^Z)^\mu + c_R^e (j_{e,RL}^Z)^\mu] \cdot [c_L^q (j_{q,LR}^Z)^\nu + c_R^q (j_{q,RL}^Z)^\nu] \end{aligned} \tag{A.12}$$

since the currents $(j_{e,RR}^Z)^\mu$, $(j_{e,LL}^Z)^\mu$, $(j_{q,RR}^Z)^\nu$ and $(j_{q,LL}^Z)^\nu$ are zero (c.f.: photon part) and we used the abbreviation $P_Z = \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M_Z^2}}{q^2 - M_Z^2} = \frac{g_{\mu\nu}}{q^2 - MZ^2 + iM_Z\Gamma_Z}$. Hence we are left with:

$$\begin{aligned} \mathcal{M}_{RR} &= -P_z g_z^2 c_R^e c_R^q g_{\mu\nu} \left[\bar{\mathsf{v}}_L(p_2)\gamma^\mu u_R(p_1)\right] \cdot \left[\bar{u}_R(p_1')\gamma^\nu \mathsf{v}_L(p_2')\right] \\ \mathcal{M}_{RL} &= -P_z g_z^2 c_R^e c_L^q g_{\mu\nu} \left[\bar{\mathsf{v}}_L(p_2)\gamma^\mu u_R(p_1)\right] \cdot \left[\tilde{u}_L(p_1')\gamma^\nu \mathsf{v}_R(p_2')\right] \\ \mathcal{M}_{LR} &= -P_z g_z^2 c_L^e c_R^q g_{\mu\nu} \left[\bar{\mathsf{v}}_R(p_2)\gamma^\mu u_L(p_1)\right] \cdot \left[\bar{u}_R(p_1')\gamma^\nu \mathsf{v}_L(p_2')\right] \\ \mathcal{M}_{LL} &= -P_z g_z^2 c_L^e c_L^q g_{\mu\nu} \left[\bar{\mathsf{v}}_R(p_2)\gamma^\mu u_L(p_1)\right] \cdot \left[\bar{u}_L(p_1')\gamma^\nu \mathsf{v}_R(p_2')\right] \end{aligned} \tag{A.13}$$

The according absolute values squared are:

$$\begin{aligned} |\mathcal{M}_{RR}|^2 &= |P_z|^2 g_z^4 (c_R^e)^2 \left(c_R^q\right)^2 q^4 (1+\cos\theta)^2 \\ |\mathcal{M}_{RL}|^2 &= |P_z|^2 g_z^4 (c_R^e)^2 \left(c_L^q\right)^2 q^4 (1-\cos\theta)^2 \\ |\mathcal{M}_{LR}|^2 &= |P_z|^2 g_z^4 (c_L^e)^2 \left(c_R^q\right)^2 q^4 (1-\cos\theta)^2 \\ |\mathcal{M}_{LL}|^2 &= |P_z|^2 g_z^4 (c_L^e)^2 \left(c_L^q\right)^2 q^4 (1+\cos\theta)^2 \end{aligned} \tag{A.14}$$

with $|P_Z|^2 = \frac{1}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}$.

Having all the necessary ingredients together we can finally write down the differential cross-section for the Z-boson part of the scattering process:

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{3}{4}\frac{g_Z^4 s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_2^2}\{&\left[(c_R^e)^2(c_R^q)^2+(c_L^e)^2(c_L^q)^2\right](1+cos\theta)^2+\\ &+\left[(c_R^e)^2(c_L^q)^2+(c_L^e)^2(c_R^q)^2\right](1-cos\theta)^2\}\end{aligned} \tag{A.15}$$

resp.:

$$\begin{aligned}\frac{d\sigma}{dcos\theta} = &\frac{3g_z^4}{128\pi s\left(1-\frac{2M_Z^2}{s}+\frac{M_Z^4}{s^2}+\frac{M_Z^2\Gamma_Z^2}{s^2}\right)}\cdot\\ &\cdot\left\{\left[(c_R^e)^2\left(c_R^q\right)^2+(c_L^e)^2\left(c_L^q\right)^2\right](1+\cos\theta)^2+\right.\\ &\left.+\left[(c_R^e)^2\left(c_L^q\right)^2+(c_L^e)^2\left(c_R^q\right)^2\right](1-\cos\theta)^2\right\}\end{aligned} \tag{A.16}$$

A.3. Final cross-section

To obtain the complete cross-section that includes photon as well as Z-boson exchange, we first need to take $\mathcal{M}_\gamma + \mathcal{M}_Z$ and sum, square and average over all spins.

$$\frac{1}{4}\sum_{spins}|\mathcal{M}_\gamma+\mathcal{M}_Z|^2 = \frac{1}{4}\sum_{spins}\left[|\mathcal{M}_\gamma|^2+2Re(\mathcal{M}_\gamma\mathcal{M}_Z^*)+|\mathcal{M}_Z|^2\right] \tag{A.17}$$

From sections (A.1) and (A.2) we already know that:

$$\frac{1}{4}\sum_{spins}|\mathcal{M}_\gamma|^2 = e^4 Q_f^2\frac{1}{2}\left[(1+\cos\theta)^2+(1-\cos\theta)^2\right] \tag{A.18}$$

$$\begin{aligned}\frac{1}{4}\sum_{spins}|\mathcal{M}_Z|^2 = \frac{g_Z^4 s^2}{4(s-M_Z^2)^2}&\left\{\left[(c_R^e)^2\left(c_R^q\right)^2+(c_L^e)^2\left(c_L^q\right)^2\right](1+\cos\theta)^2+\right.\\ &\left.+\left[(c_R^e)^2(c_L^q)^2+(c_L^e)^2(c_R^q)^2\right](1-cos\theta)^2\right\}\end{aligned} \tag{A.19}$$

and with the equations (A.6) and (A.13) we can also calculate:

$$\begin{aligned}\frac{1}{4}\sum_{spins}2Re(\mathcal{M}_\gamma\mathcal{M}_Z^*) = -\frac{e^2Q_f g_Z^2 s}{2(s-M_Z^2)}&\left\{\left[(c_R^e)\left(c_R^q\right)+(c_L^e)\left(c_L^q\right)\right](1+\cos\theta)^2+\right.\\ &\left.+\left[(c_R^e)(c_L^q)+(c_L^e)(c_R^q)\right](1-cos\theta)^2\right\}\end{aligned} \tag{A.20}$$

Eventually, we are able to put everything together:

$$\begin{aligned}\frac{d\sigma}{dcos\theta} =& \frac{2\pi\cdot 3}{64\pi^2 s}\Bigg\{e^4 Q_f^2 \frac{1}{2}\left[(1+\cos\theta)^2+(1-\cos\theta)^2\right]+\\ &+\frac{e^4 s^2}{4(s_W c_W)^4(s-M_Z^2)^2}\left[\beta_1(1+\cos\theta)^2+\beta_2(1-\cos\theta)^2\right]+\\ &+-\frac{e^4 Q_f s}{2(s_W c_W)^2(s-M_Z^2)}\left[\gamma_1(1+\cos\theta)^2+\gamma_2(1-\cos\theta)^2\right]\Bigg\}=\\ =&\frac{3}{128\pi s}\left\{\left[2Q_f^2 e^4-8Q_f e^4 F(s)\gamma_1+4^2 e^4 F^2(s)\beta_1\right](1+\cos\theta)^2+\right.\\ &\left.+\left[2Q_f^2 e^4-8Q_f e^4 F(s)\gamma_2+4^2 e^4 F^2(s)\beta_2\right](1-\cos\theta)^2\right\}\end{aligned} \tag{A.21}$$

where we used the abbreviations:

$$\beta_1=\left[\left(c_R^e\right)^2\left(c_R^q\right)^2+\left(c_L^e\right)^2\left(c_L^q\right)^2\right] \quad \text{and} \quad \beta_2=\left[\left(c_R^e\right)^2\left(c_L^q\right)^2+\left(c_L^e\right)^2\left(c_R^q\right)^2\right]$$

$$\gamma_1=\left[\left(c_R^e\right)\left(c_R^q\right)+\left(c_L^e\right)\left(c_L^q\right)\right] \quad \text{and} \quad \gamma_2=\left[\left(c_R^e\right)\left(c_L^q\right)+\left(c_L^e\right)\left(c_R^q\right)\right]$$

$$F(s)=\frac{s}{4(sin\theta_W\ cos\theta_W)^2(s-M_Z^2)}$$

and the fact that the neutral weak coupling constant is related to the electromagnetic coupling constant via:

$$g_Z=\frac{e}{sin\theta_W\ cos\theta_W} \tag{A.22}$$

The above cross-section can be re-expressed using the weak hypercharge $Y_w = 2(Q - I_w^3)$ instead of the isospin. Using:

$$s_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}; \quad c_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}; \quad e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}; \quad y := Y_e; \quad Y := Y_q \tag{A.23}$$

we thus obtain for the limit $M_Z \to 0$ and inserting the electron's electric charge $Q_e = -1$:

$$\frac{d\sigma}{dcos\theta} = \frac{3}{128\pi s} \left\{ C_1 (1 + \cos\theta)^2 + C_2 (1 - \cos\theta)^2 \right\} \tag{A.24}$$

$$C_1 = \frac{1}{16} \left(-2 g_1^2 g_2^2 \; y \; (2 + y)(2Q_f - Y)Y + g_1^4 (2 + y)^2 (-2Q_f + Y)^2 + g_2^4 (16 Q_f^2 + y^2 Y^2) \right)$$

$$C_2 = \frac{1}{4} g_2^4 \left(Q_f^2 y^2 + Y^2 \right)$$

Summing over the possible final states and inserting the according the electric charges:

$$Q_{up} = 2/3; \quad Q_{down} = -1/3 \quad \Rightarrow \quad Q_q^2 = Q_{up}^2 + Q_{down}^2 = 5/9 \tag{A.25}$$

results in:

$$C_1 = \frac{1}{72} \left(6 g_1^2 g_2^2 \; y \; (2 + y) Y (-1 + 3Y) + g_1^4 (2 + y)^2 \left(10 - 6Y + 9Y^2 \right) + g_2^4 \left(40 + 9 y^2 Y^2 \right) \right)$$

$$C_2 = \frac{1}{36} g_2^4 \left(5y^2 + 18Y^2 \right)$$

B. Electroweak interaction Hamiltonian density

Since for the coherent state approach the interaction part of the electroweak Hamiltonian is needed, we will derive the electroweak Hamiltonian density from the Lagrangian here. Let us start by writing down the electroweak Lagrangian in the non-physical fields [1]:

$$\mathcal{L}_{ew} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{gf} + \mathcal{L}_{ghost}, \tag{B.1}$$

where the gauge part has the form:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \frac{1}{4}W^a_{\mu\nu}(x)W^{a,\mu\nu}(x) \tag{B.2}$$

with

$$B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$$

$$W^a_{\mu\nu}(x) = \partial_\mu W^a_\nu(x) - \partial_\nu W^a_\mu(x) + g_2 \varepsilon^{abc} W^b_\mu(x) W^c_\nu(x), \quad a = 1, 2, 3$$

The fermionic part can be written as:

$$\mathcal{L}_{\text{fermion}} = \sum_i \bar{\Psi}^L_i i \not{D} \Psi^L_i + \sum_{i,\sigma} \bar{\psi}^R_{i,\sigma} i \not{D} \psi^R_{i,\sigma} \tag{B.3}$$

using:

$$D_\mu = \left(\partial_\mu - i g_2 I^a_w W^a_\mu + i g_1 \frac{Y_w}{2} B_\mu\right)$$

$$\psi^2(x) = \frac{1}{2}(1 - \gamma_5)\psi(x)$$

$$\psi^R(x) = \frac{1}{2}(1 + y_5)\psi(x)$$

$$\bar{\Psi}^L_i(x) = \left(\psi^L_{i,t}, \psi^L_{i,-}\right) \quad i = 1, 2, 3 \ldots \text{lepton doublet} \quad \sigma = \pm \text{... member of the doublet}$$

$$I^a_w = \frac{\tau^a}{2} \quad \text{with } \tau^a\text{... Pauli matrices}$$

[1] This is sufficient since we are working in the high energy regime.

The Higgs part (including the Yukawa part) will be denoted as:

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} =&(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi)\\ &-\sum_{i,j}(\mathrm{g}_{ij}\,\bar{\Psi}_i^L\psi_{j,-}^R\Phi + \tilde{g}_{ij}\bar{\Psi}_i^L\psi_{j,+}^R\Phi^c + \text{ h.c.}),\end{aligned} \tag{B.4}$$

where

$$\begin{aligned}&\Phi(x) = \left(\phi^+(x), \phi^0(x)\right) \quad (\text{ for } Y_\Phi = +1)\\ &\Phi^c(x) = i\tau^2\Phi^* = \left(\phi^{0*}(x), -\Phi^-(x)\right)\\ &V(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 - \mu^2\Phi^+\Phi \quad \left(\mu^2, \lambda > 0\right).\end{aligned}$$

The gauge conditions for this Lagrangian read as follows:

$$\begin{aligned}C^0 &= \partial^\mu B_\mu + \tfrac{1}{s_w^2+c_w^2}M_z\xi_z x && = 0\\ C^1 &= \partial^\mu W_\mu^1 + i\tfrac{1}{\sqrt{2}}M_w\xi_w'\phi^+ + i\tfrac{1}{\sqrt{2}}M_w\xi_w'\phi^- && = 0\\ C^2 &= \partial^\mu W_\mu^2 + \tfrac{1}{\sqrt{2}}M_w\xi_w'\phi^- - \tfrac{1}{\sqrt{2}}M_w\xi_w'\phi^+ && = 0\\ C^3 &= \partial^\mu W_\mu^3 + \left(1 - \tfrac{s_w}{s_w^2+c_w^2}\right)\tfrac{1}{c_w}M_z\xi_z\chi && = 0,\end{aligned}$$

which can be rewritten using the definition of Φ and Φ^c, as well as introducing the matrices:

$$\begin{aligned}\varphi_1 &:= \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}}(v+\eta)\end{pmatrix}\\ P^+ &:= \frac{\tau^0+\tau^3}{2}\\ P^- &:= \frac{\tau^0-\tau^3}{2}\end{aligned}$$

in the form:

$$\begin{aligned}C^0 &= \partial^\mu B_\mu - i\sqrt{2}\frac{1}{s_w^2+c_w^2}M_z\xi_z\left(P^-\Phi + \varphi_1\right) && = 0\\ C^1 &= \partial^\mu W_\mu^\mu + \frac{i}{\sqrt{2}}M_w\xi_w'P^+\Phi - \frac{1}{\sqrt{2}}M_w\xi_w'P^-\Phi^c && = 0\\ C^2 &= \partial^\mu W_\mu^2 - \frac{1}{\sqrt{2}}M_w\xi_w'P^-\Phi^c - \frac{1}{\sqrt{2}}M_w\xi_w'P^+\bar{\Phi} && = 0\\ C^3 &= \partial^\mu W_\mu^3 - i\sqrt{2}\left(1 - \frac{s_w}{s_w^2+c_w^2}\right)\frac{1}{c_w}M_z\xi_z\left(P^-\Phi + \varphi_1\right) && = 0\end{aligned}$$

Thus the gauge-fixing part of the Lagrangian can be written as:

$$\mathcal{L}_{gf} = -\frac{1}{2\xi_1}\left(C^1\right)^2 - \frac{1}{2\xi_2}\left(C^2\right)^2 - \frac{1}{2\xi_3}\left(C^3\right)^2 - \frac{1}{2\xi_0}\left(C^0\right)^2 \tag{B.5}$$

For the ghost part of the electroweak Lagrangian we introduce the ghost fields u^0, u^1, u^2, u^3 and write down the formula for the unphysical fields:

$$\begin{aligned}\mathcal{L}_{\text{ghost}} &= -\int d^4zd^4y\bar{u}^a(x)\left(\frac{\delta C^a(x)}{\delta V_\nu^c(z)}\frac{\delta V_v^c(z)}{\delta\theta^b(y)} + \delta_{ba}\delta_{ca}\frac{\delta C^a(x)}{\delta\Phi_1(z)}\frac{\delta\Phi_1(z)}{\delta\theta^b(y)} + \right.\\ &\left. +\delta_{ba}\delta_{ca}\frac{\delta C^a(x)}{\delta\Phi_2(z)}\frac{\delta\Phi_2(z)}{\delta\theta^b(y)}\right)u^b(y),\end{aligned}$$

where $\Phi_1 = \Phi$, $\Phi_2 = \Phi^c$ and $a, b, c \in \{0, 1, 2, 3\}$.

Using the gauge-fixing conditions above and the transformation laws of the fields:

$$\begin{aligned}
&B_\mu(x) \to B_\mu(x) + \frac{1}{g_1}\partial_\mu\delta\theta^y\\
&W_\mu^1(x) \to W_\mu^1 + \frac{1}{g_2}\partial_\mu\delta\theta^1 + W_\mu^3\delta\theta^3 - W_\mu^3\delta\theta^2\\
&W_\mu^2(x) \to W_\mu^2 + \frac{1}{\rho_2^2}\partial_\mu\delta\theta^2 + W_\mu^3\delta\theta^1 - W_\mu^1\delta\theta^3\\
&W_\mu^3(x) \to W_\mu^3(x) + \frac{1}{\rho_2}\partial_\mu\delta\theta^3 + W_\mu^1\delta\theta^2 - W_\mu^2\delta\theta^1\\
&\Phi(x) \to \left[1 - i\frac{1}{2}\delta\theta^y(x) + i\frac{\tau^a}{2}\delta\theta^a(x)\right]\Phi(x)\\
&\Phi^c(x) \to i\tau^2\left[1 + i\frac{1}{2}\delta\theta^y(x) - i\frac{\tau^a}{2}\delta\theta^a(x)\right]\Phi^*
\end{aligned}$$

we can calculate the ghost part of the Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\text{ghost}} = &\left\{-\bar{u}^0\left(\frac{1}{g_1}\partial^\mu\partial_\mu\delta\theta^Y - \frac{1}{\sqrt{2}}\frac{1}{s_w^2 + c_w^2}M_z\xi_zP^-\Phi\right)u^0\right.\\
&\left.-\bar{u}^0\frac{1}{\sqrt{2}}\frac{1}{s_w^2 + c_w^2}M_z\xi_z\left(P^-\tau^1\Phi u^1 + P^-\tau^2\Phi u^2 + P^-\tau^3\Phi u^3\right)\right\} +\\
&+\left\{-\bar{u}^1\frac{1}{2\sqrt{2}}M_w\xi_w'\left(P^+\Phi u^0 - P^+\tau^1\Phi u^1 - P^+\tau^2\Phi u^2 - P^+\tau^3\Phi u^3\right.\right.\\
&\left.+iP^-\tau^2\Phi^*u^0 - iP^-\tau^2\tau^1\Phi^*u^1 - iP^-\tau^2\tau^2\Phi^*u^2 - iP^-\tau^2\tau^3\Phi u^3\right)\\
&\left.-\bar{u}^1\left(\frac{1}{g_2}\partial^\mu\partial_\mu\right)u^1 + \bar{u}^1\left(\partial^\mu W_\mu^3\right)u^2 - \bar{u}^1\left(\partial_\mu W_\mu^2\right)u^3\right\} +
\end{aligned} \tag{B.6}$$

$$
\begin{aligned}
&+\Bigg\{\bar{u}^2\frac{1}{2\sqrt{2}}M_w\xi'_w\left(iP^+\Phi u^0 - iP^+\tau^1\Phi u^1 - iP^+\tau^2\Phi u^2 - iP^+\tau^3\Phi u^3\right.\\
&\quad \left.+P^-\tau^2\Phi^* u^0 - P^-\tau^2\tau^1\Phi^* u^1 - P^-\tau^2\tau^2\Phi^* u^2 - P^-\tau^1\tau^3\Phi^* u^3\right)\\
&\quad -\bar{u}s^2(\partial^\mu W^2_\mu)u^1 - \bar{u}^2\left(\frac{1}{g_2}\partial^\mu\partial_\mu\right)u^2 + \bar{u}^2(\partial^\mu W^1_\mu)u^3\Bigg\}+\\
&+\Bigg\{-\bar{u}^3\frac{1}{\sqrt{2}}\left(1-\frac{s_w}{s_w^2+c_w^2}\right)\frac{1}{c_w^2}M_z\xi_z(P^-\tau^1\Phi u^1 - P^-\Phi u^0 + P^-\tau^2\Phi u^2)\\
&\quad +P^-\tau^3\Phi u^3) + \bar{u}^3(\partial^\mu W^2_\mu)u^1 - \bar{u}^3(\partial^\mu W^1_\mu)u^2 - \bar{u}^3\left(\frac{1}{g_2}\partial^\mu\partial_\mu\right)u^3\Bigg\}
\end{aligned}
$$

The next step is to calculate all the conjugate momenta needed for the Hamiltonian density:

$$
\Pi_B := \frac{\partial\mathcal{L}}{\partial(\partial_0 B_\beta)} = -B^{0\beta} - \frac{1}{\xi_0}\left[\partial^\mu B^0 - i\sqrt{2}\frac{1}{s_w^2+c_w^2}M_z\xi_z(P^-\Phi-\varphi_1)g^{0\beta}\right] \tag{B.7}
$$

$$
\begin{aligned}
\Pi_{W^1} := \frac{\partial\mathcal{L}}{\partial(\partial_0 W^1_\beta)} =& - W^{1,0\beta}\\
&-\frac{1}{\xi_1}\left[\partial^\mu W^{1,0} + \frac{i}{\sqrt{2}}M_w\xi'_w\left(P^+\Phi - P^-\Phi^*\right)g^{0\beta}\right]\\
&+\bar{u}^2 g^{0\beta}\bar{u}^3 - u^3 g^{0\beta}u^2
\end{aligned} \tag{B.8}
$$

$$
\begin{aligned}
\Pi_{W^2} := \frac{\partial\mathcal{L}}{\partial(\partial_0 W^2_\beta)} =& - W^{2,0\beta}\\
&-\frac{1}{\xi_2}\left[\partial^\mu W^{2,0} + \frac{1}{\sqrt{2}}M_w\xi'_w\left(P^-\Phi^* + P^+\Phi\right)g^{0\beta}\right]\\
&-\bar{u}^1 g^{0\beta}u^3 - \bar{u}^2 g^{0\beta}u^1 + \bar{u}^3 g^{0\beta}u^1
\end{aligned} \tag{B.9}
$$

$$
\begin{aligned}
\Pi_{W^3} := \frac{\partial\mathcal{L}}{\partial(\partial_0 W^3_\beta)} =& - W^{3,0\beta}\\
&-\frac{1}{\xi_3}\left[\partial^\mu W^{2,0} + i\sqrt{2}\left(1-\frac{s_w}{s_w^2+c_w^2}\right)\frac{1}{c_w^2}M_z\xi_z\left(P^-\Phi+\varphi_1\right)g^{0\beta}\right]\\
&+\bar{u}^1 g^{0\beta}u^2
\end{aligned} \tag{B.10}
$$

$$\Pi_{\psi_L} := \frac{\partial \mathcal{L}}{\partial(\partial_0 \Psi^L)} = \sum_i \bar{\Psi}_i^L i\gamma^0 \tag{B.11}$$

$$\Pi_{\psi_R} := \frac{\partial \mathcal{L}}{\partial(\partial_0 \psi^R)} = \sum_{i,\sigma} \bar{\psi}_{i,\sigma}^R i\gamma^0 \tag{B.12}$$

$$\Pi_\Phi := \frac{\partial \mathcal{L}}{\partial(\partial_0 \Phi)} = \left(D^0\Phi\right)^\dagger \tag{B.13}$$

$$\Pi_{u^0} := \frac{\partial \mathcal{L}}{\partial(\partial_0 u^0)} = -\frac{1}{g_1}\theta^Y \bar{u}^0 \partial_0 \tag{B.14}$$

$$\Pi_{u^a} := \frac{\partial \mathcal{L}}{\partial(\partial_0 u^a)} = -\frac{1}{g_2}\bar{u}^a \partial_0 \quad \text{for} \quad a = 1, 2, 3 \tag{B.15}$$

We can now write down the electroweak Hamiltonian density:

$$\begin{aligned}
\mathcal{H}_{ew} &= \Pi_B \partial_0 B_\beta + \Pi_{W^1}\partial_0 W^1_\beta + \Pi_{W^2}\partial_0 W^2_\beta + \Pi_{W^3}\partial_0 W^3_\beta \\
&\quad + \Pi_{\psi^2}\partial_0 \Psi^L + \Pi_{\psi^2}\partial_0 \psi^R + \Pi_\Phi \partial_0 \Phi + \\
&\quad + \Pi_{u^0}\partial_0 u^0 + \Pi_{u^a}\partial_0 u^a - \mathcal{L}_{\text{ew}} = \\
&= \left\{ -B^{0\beta} - \frac{1}{\xi_0}\left[\partial^\mu B^0 - i\sqrt{2}\frac{1}{s_w^2 + c_w^2} M_z \xi_z (P^-\Phi - \varphi_1) g^{0\beta}\right]\right\} \partial_0 B_\beta + \\
&+ \left\{ -W^{1,0\beta} - \frac{1}{\xi_1}\left[\partial^\mu W^{1,0} + \frac{i}{\sqrt{2}} M_w \xi'_w \left(P^+\Phi - P^-\Phi^*\right) g^{0\beta}\right] + \bar{u}^2 g^{0\beta} u^3 - \bar{u}^3 g^{0\beta} u^2 \right\} \partial_0 W^1_\beta + \\
&+ \left\{ -W^{2,0\beta} - \frac{1}{\xi_2}\left[\partial^\mu W^{2,0} + \frac{1}{\sqrt{2}} M_w \xi'_w \left(P^-\Phi^* + P^+\Phi\right) g^{0\beta}\right] - \bar{u}^1 g^{0\beta} u^3 + \bar{u}^3 g^{0\beta} u^1 \right\} \partial_0 W^2_\beta + \\
&+ \Bigg\{ -W^{3,0\beta} - \frac{1}{\xi_3}\left[\partial^\mu W^{2,0} + i\sqrt{2}\left(1 - \frac{s_w}{s_w^2 + c_w^2}\right)\frac{1}{c_w^2} M_z \xi_z \left(P^-\Phi + \varphi_1\right) g^{0\beta}\right] \\
&\qquad + \bar{u}^1 g^{0\beta} u^2 - \bar{u}^2 g^{0\beta} u^1 \Bigg\} \partial_0 W^3_\beta + \\
&+ \left(\sum_i \bar{\Psi}_i^L i\gamma^0\right)\partial_0 \Psi^L + \left(\sum_{i,\sigma} \bar{\psi}_{i,\sigma}^R i\gamma^0\right)\partial_0 \psi^R + \left(D^0\Phi\right)^\dagger \partial_0 \Phi + \\
&+ \left(-\frac{1}{g_1}\theta^Y \bar{u}^0 \partial_0\right)\partial_0 u^0 + \sum_a \left(-\frac{1}{g_2}\bar{u}^a \partial_0\right)\partial_0 u^a - \mathcal{L}_{ew}
\end{aligned} \tag{B.16}$$

The Hamiltonian density in (B.16) consists of the quadratic part and the interaction part. Since we are only interested in the latter, we can dispose the unnecessary terms. Additionally we can further simplify the expression by taking into account, that in contrast to the QCD case discussed in section (2.2.1) the mass scale in the electroweak case is bounded by the mass of the W and Z bosons ($\sim$ 100 GeV). With this in mind it becomes apparent that the extra mass terms in (e.q.: (B.16)) are irrelevant for the further discussion. Eventually the interaction part of the electroweak Hamiltonian density takes the form:

$$
\begin{aligned}
\mathcal{H}^{ew}_{int} = & \left\{+\bar{u}^2 g^{0\beta} u^3 - \bar{u}^3 g^{0\beta} u^2\right\} \partial_0 W^1_\beta + \left\{-\bar{u}^1 g^{0\beta} u^3 + \bar{u}^3 g^{0\beta} u^1\right\} \partial_0 W^2_\beta + \\
& + \left\{\bar{u}^1 g^{0\beta} u^2 - \bar{u}^2 g^{0\beta} u^1\right\} \partial_0 W^3_\beta + g_2 \epsilon^{abc} (\partial_\mu W^a_\nu) W^b_\mu W^c_\nu + \\
& + \frac{1}{4} g_2^2 \epsilon^{abc} \epsilon^{ade} W^b_\mu W^c_\nu W^{d,\mu} W^{c,\nu} + \\
& + \sum_i \bar{\Psi}'^L_i \left(-g_2 I^a_w W^a_\mu + g_1 \frac{Y_w}{2} B_\mu\right) \Psi'^L_i + \sum_{i,\sigma} \bar{\psi}^R_{i,\sigma} \left(g_1 \frac{Y_w}{2} B_\mu\right) \bar{\psi}^R_{i,\sigma} + \\
& + \left(i g_2 I^a_w W^a_\mu - i g_1 \frac{Y_w}{2} B_\mu\right) \left[\Phi^\dagger \partial_\mu \Phi - \Phi \left(\partial_\mu \Phi\right)^\dagger\right] + \\
& - \left[\left(-i g_2 I^a_w W^a_\mu + i g_1 \frac{Y_w}{2} B_\mu\right) \Phi\right]^\dagger \left[\left(-i g_2 I^a_w W^{a\mu} + i g_1 \frac{Y_w}{2} B^\mu\right) \Phi\right] + \\
& + \frac{\lambda}{4} \left(\Phi^\dagger \Phi\right)^2 + \sum_{i,j} (g_{ij} \bar{\Psi}^L_i \psi^R_{j,-} \Phi + \tilde{g}_{ij} \bar{\Psi}^L_i \psi^R_{j,+} \Phi^c + hc) + \\
& + \left[-\bar{u}^1 \left(\partial^\mu W^3_\mu\right) u^2 + \bar{u}^1 \left(\partial^\mu W^2_\mu\right) u^3\right] + \left[+\bar{u}^2 \left(\partial^\mu W^3_\mu\right) u^1 - \bar{u}^2 \left(\partial^\mu W^1_\mu\right) u^3\right] + \\
& + \left[-\bar{u}^3 \left(\partial^\mu W^2_\mu\right) u^1 + \bar{u}^3 \left(\partial^\mu W^1_\mu\right) u^2\right],
\end{aligned}
\tag{B.17}
$$

whose terms can be rearranged and combined to the form which is used in section (3.4.1):

$$
\begin{aligned}
\mathcal{H}_{int}^{ew} =& \mathcal{H}_{\text{fermion}} + \mathcal{H}_{\text{3int}} + \mathcal{H}_{\text{4int}} + \mathcal{H}_{\text{Higgs}} + \mathcal{H}_{\text{ghost}} = \\
=& \sum_i \bar{\Psi}_i'^L \left(-g_2 I_w^a W_\mu^a + g_1 \frac{Y_w}{2} B_\mu \right) \Psi_i'^L + \sum_{i,\sigma} \bar{\psi}_{i,\sigma}^R \left(g_1 \frac{Y_w}{2} B_\mu \right) \bar{\psi}_{i,\sigma}^R + \\
&+ g_2 \epsilon^{abc} (\partial_\mu W_\nu^a) W_\mu^b W_\nu^c + \\
&+ \frac{1}{4} g_2^2 \epsilon^{abc} \epsilon^{ade} W_\mu^b W_\nu^c W^{d,\mu} W^{c,\nu} + \\
&+ \left(i g_2 I_w^a W_\mu^a - i g_1 \frac{Y_w}{2} B_\mu \right) \left[\Phi^\dagger \partial_\mu \Phi - \Phi \left(\partial_\mu \Phi \right)^\dagger \right] + \\
&- \left[\left(-i g_2 I_w^a W_\mu^a + i g_1 \frac{Y_w}{2} B_\mu \right) \Phi \right]^\dagger \left[\left(-i g_2 I_w^a W^{a\mu} + i g_1 \frac{Y_w}{2} B^\mu \right) \Phi \right] + \\
&+ \frac{\lambda}{4} \left(\Phi^\dagger \Phi \right)^2 + \sum_{i,j} (g_{ij} \bar{\Psi}_i^L \psi_{j,-}^R \Phi + \tilde{g}_{ij} \bar{\Psi}_i^L \psi_{j,+}^R \Phi^c + hc) + \\
&+ \bar{u}^1 \left(\partial^i W_\mu^2 \right) u^3 - \bar{u}^1 \left(\partial^i W_\mu^3 \right) u^2 + \bar{u}^2 \left(\partial^i W_\mu^3 \right) u^1 - \bar{u}^2 \left(\partial^i W_\mu^1 \right) u^3 + \\
&+ \bar{u}^3 \left(\partial^i W_\mu^1 \right) u^2 - \bar{u}^3 \left(\partial^i W_\mu^2 \right) u^1
\end{aligned}
\tag{B.18}
$$

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