# Electroweak Initial and Final States

**Axel Maas**

5th of February 2026
Würzburg
Germany

UNI GRAZ
NAWI Graz
Natural Sciences
FWF
Österreichischer
Wissenschaftsfonds

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Review: 1712.04721 Update: 2305.01960

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- Fröhlich-Morchio-Strocchi mechanism
- Standard Model
  - Experimental signatures

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- A subtelty of Brout-Englert-Higgs Physics
- Fröhlich-Morchio-Strocchi mechanism
- Standard Model
    - Experimental signatures
- Beyond the Standard Model
    - Qualitative changes

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# Brout-Englert-Higgs Physics
# -
# The Standard Model

# A toy model

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- Ws $W_{\mu}^{a}$ W

- Coupling $g$ and some numbers $f^{abc}$

# A toy model: Higgs sector of the SM

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$$D^{ij}_{\mu} = \delta^{ij} \partial_{\mu} - i g W^{a}_{\mu} t^{ij}_{a}$$

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- Couplings $g, v, \lambda$ and some numbers $f^{abc}$ and $t_a^{ij}$
- Parameters selected for a BEH effect

# A toy model: Symmetries

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- Local SU(2) gauge symmetry

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+\left(\delta_{b}^{a}\partial_{\mu}-gf_{bc}^{a}W_{\mu}^{c}\right)\phi^{b} \qquad h_{i}\rightarrow h_{i}+gt_{a}^{ij}\phi^{a}h_{j}$$

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a} \qquad h\rightarrow h\Omega$$

# Textbook approach

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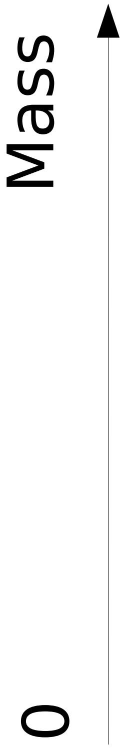
- Choose a suitable gauge and obtain ‘spontaneous gauge symmetry breaking’: SU(2) → 1
- Get masses and degeneracies at tree-level
- Perform perturbation theory

# Physical spectrum

## Perturbation theory



# Physical spectrum

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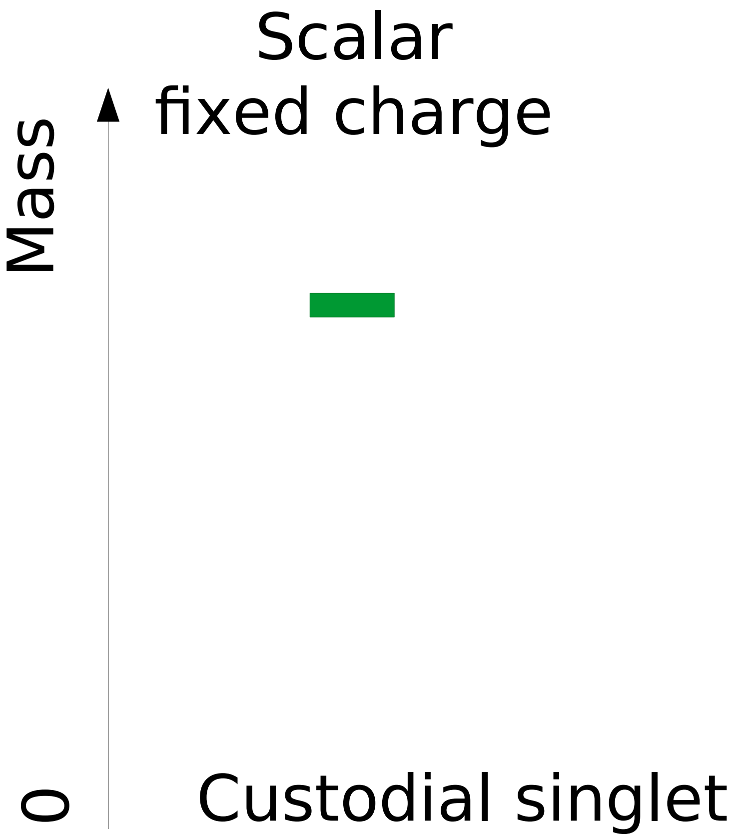

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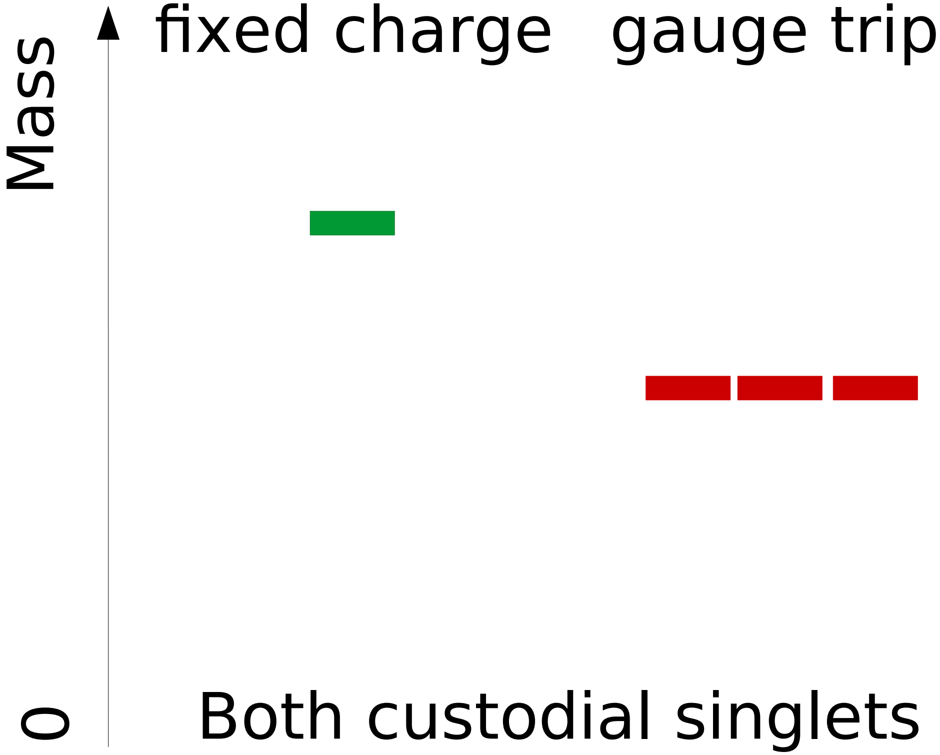

Perturbation theory

Scalar
fixed charge

Vector
gauge triplet

Both custodial singlets

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- **What? How can this be true?**

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- BEH effect does not actually break gauge symmetry
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  - Topologically obstructed [Gribov’78,Singer’78,Fujikawa’82]
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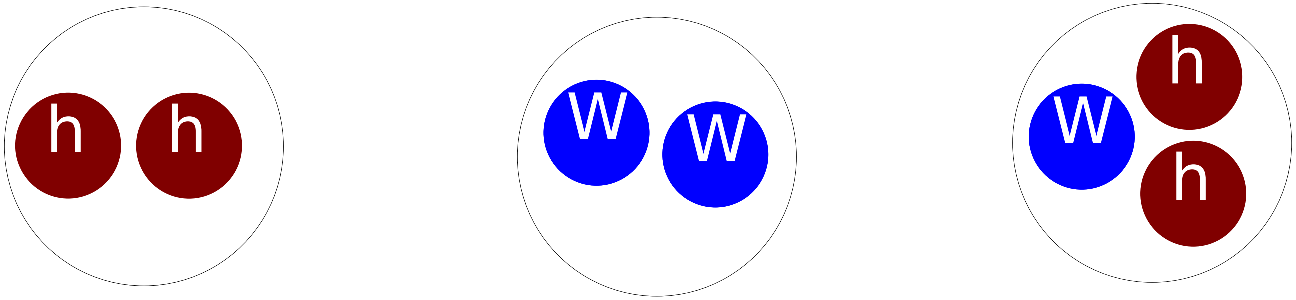
    - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



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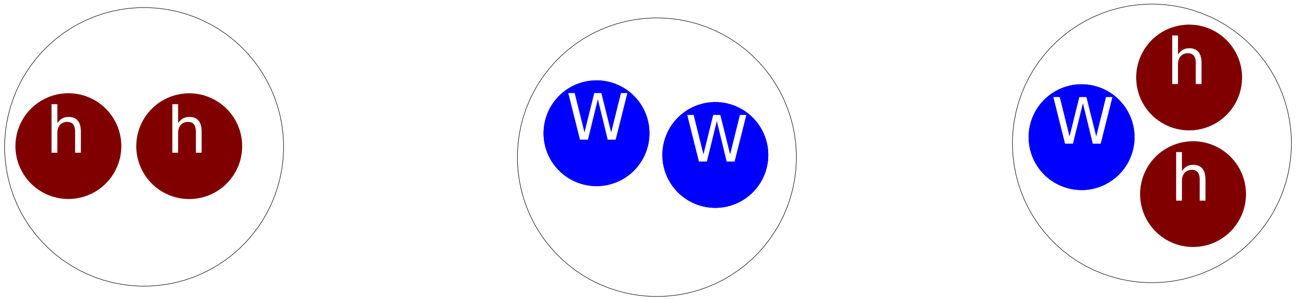
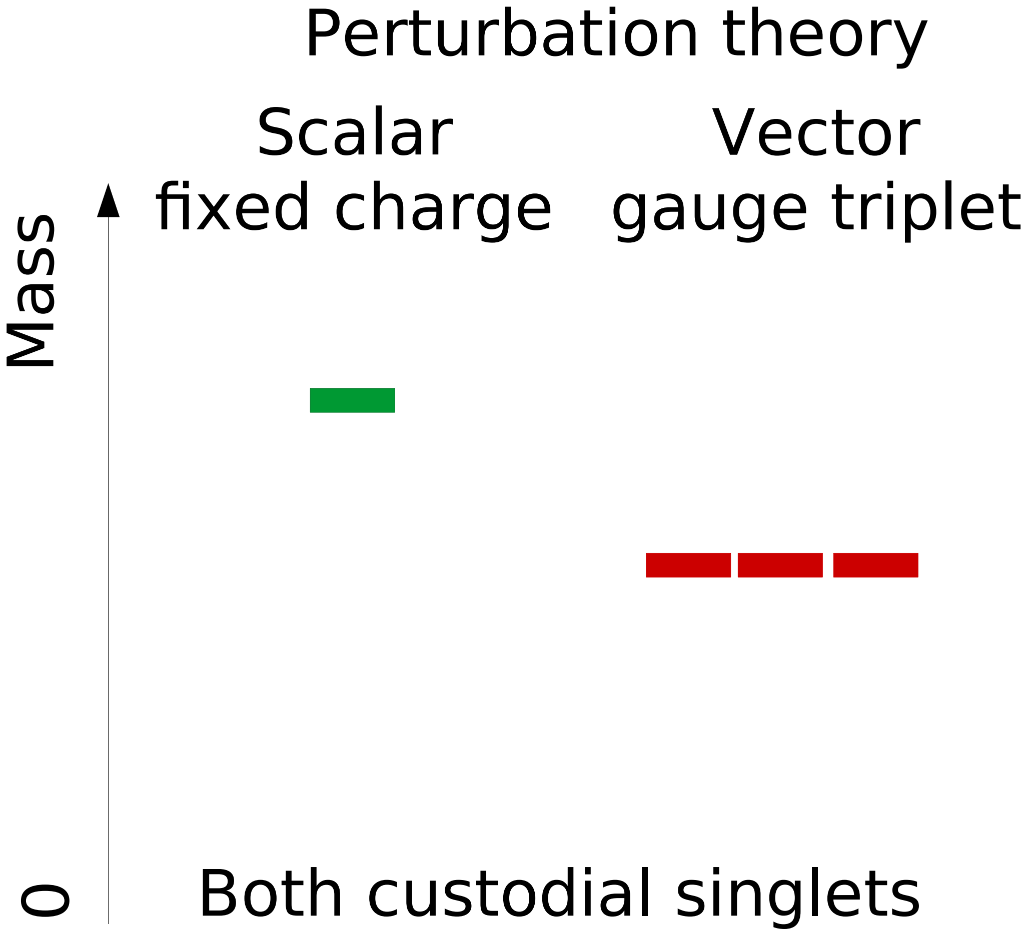


- **What? How can this be true, given the PDG?**

# Physical spectrum



Perturbation theory

Scalar
fixed charge

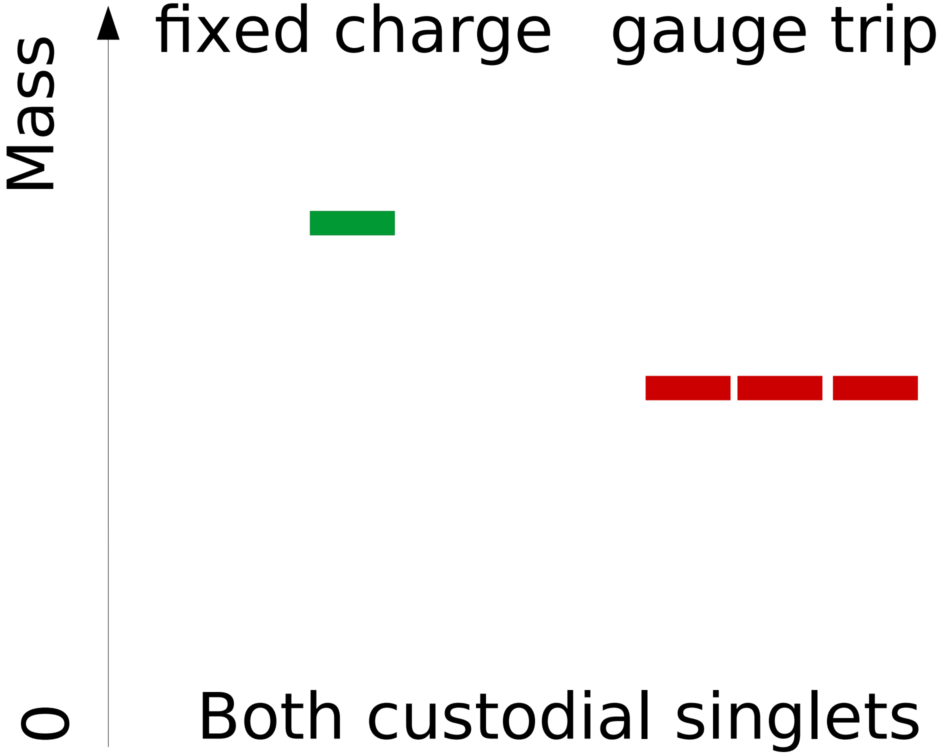
Vector
gauge triplet

Both custodial singlets

Experiment tells that somehow the left is correct

# Physical spectrum

Perturbation theory



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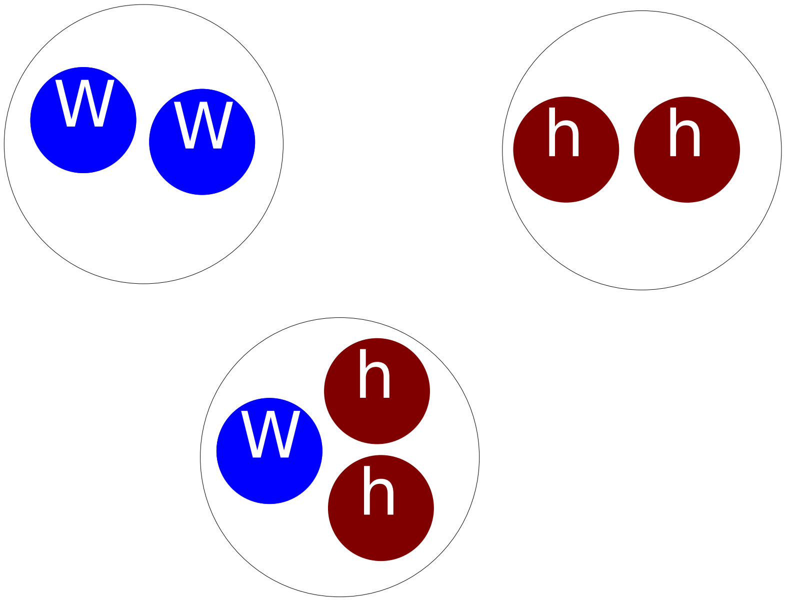
Composite (bound) states



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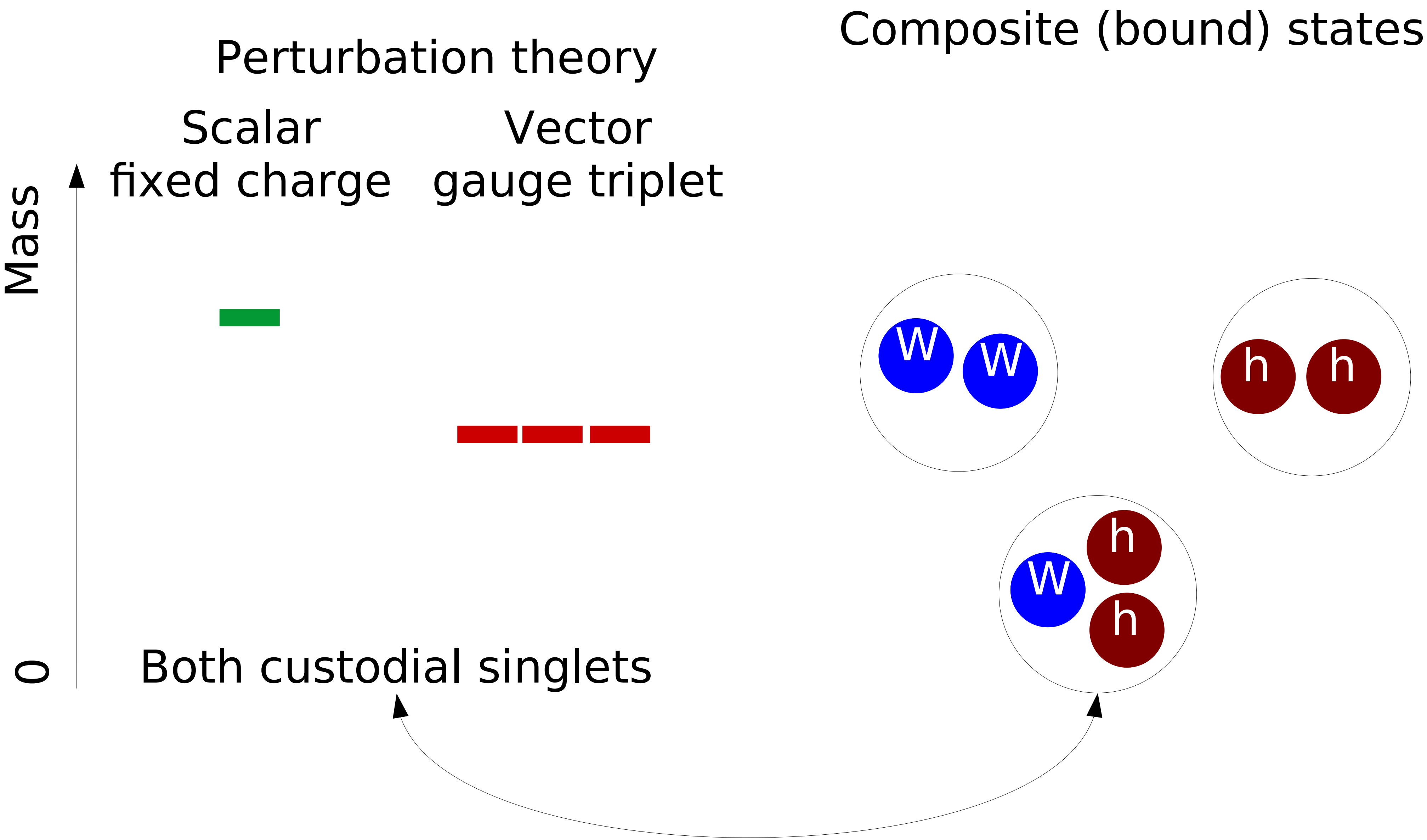
# Physical spectrum



Composite (bound) states

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Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

# Physical spectrum

[Fröhlich et al.’80,’81, Maas’12, Maas & Mufti’14,’15]

- Local composite operators for physical states
  - $J^{PC}$ and custodial charge only quantum numbers
- Same structure as, e.g., hadron operators in QCD

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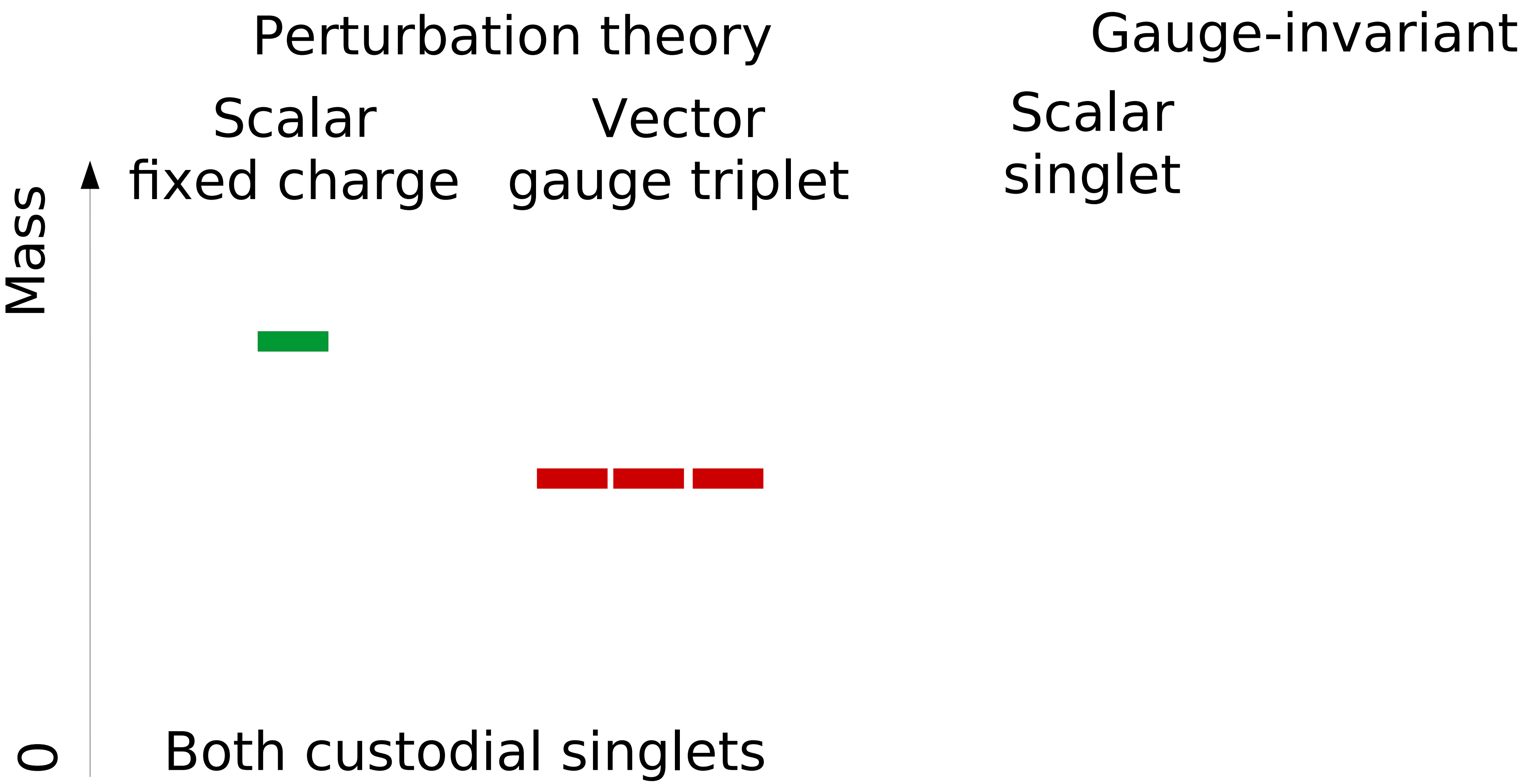
- Same structure as, e.g., hadron operators in QCD
- Could potentially be bound states
  - Non-perturbative methods: Lattice
    - Standard lattice spectroscopy problem
    - Standard methods
      - Smearing, variational analysis, systematic error analysis etc.
    - Very large statistics ($>10^5$ configurations)

# Physical spectrum

[Maas'12, Maas & Mufti'14]



Perturbation theory

Gauge-invariant

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Scalar
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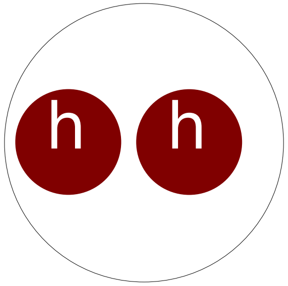
$$h(x)^{+} h(x)$$



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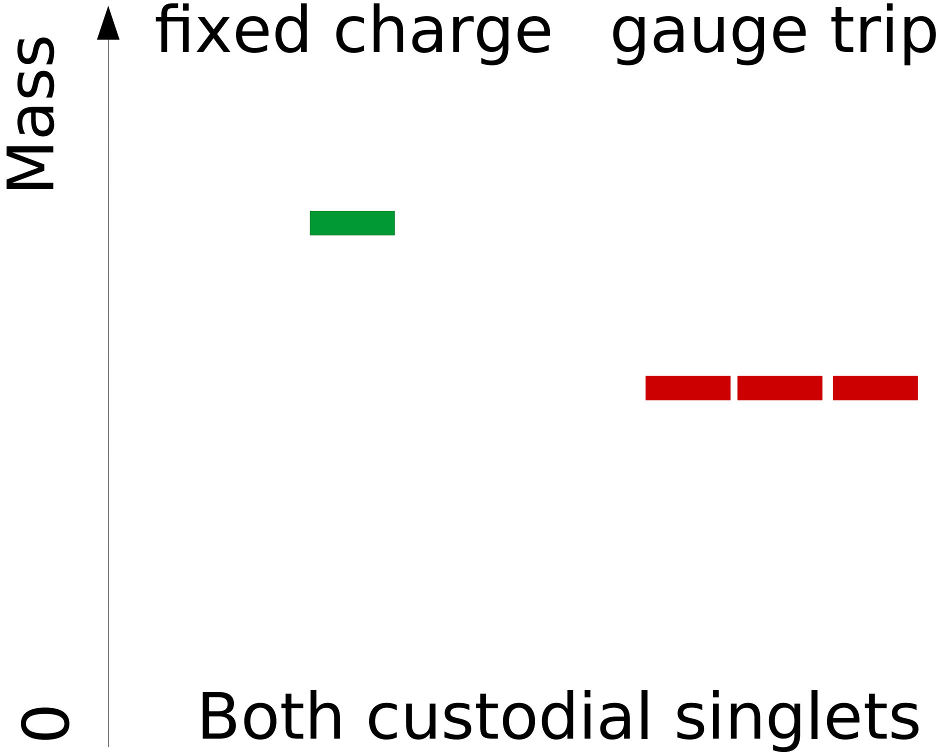

Both custodial singlets

Custodial singlet

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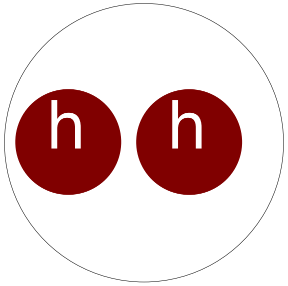

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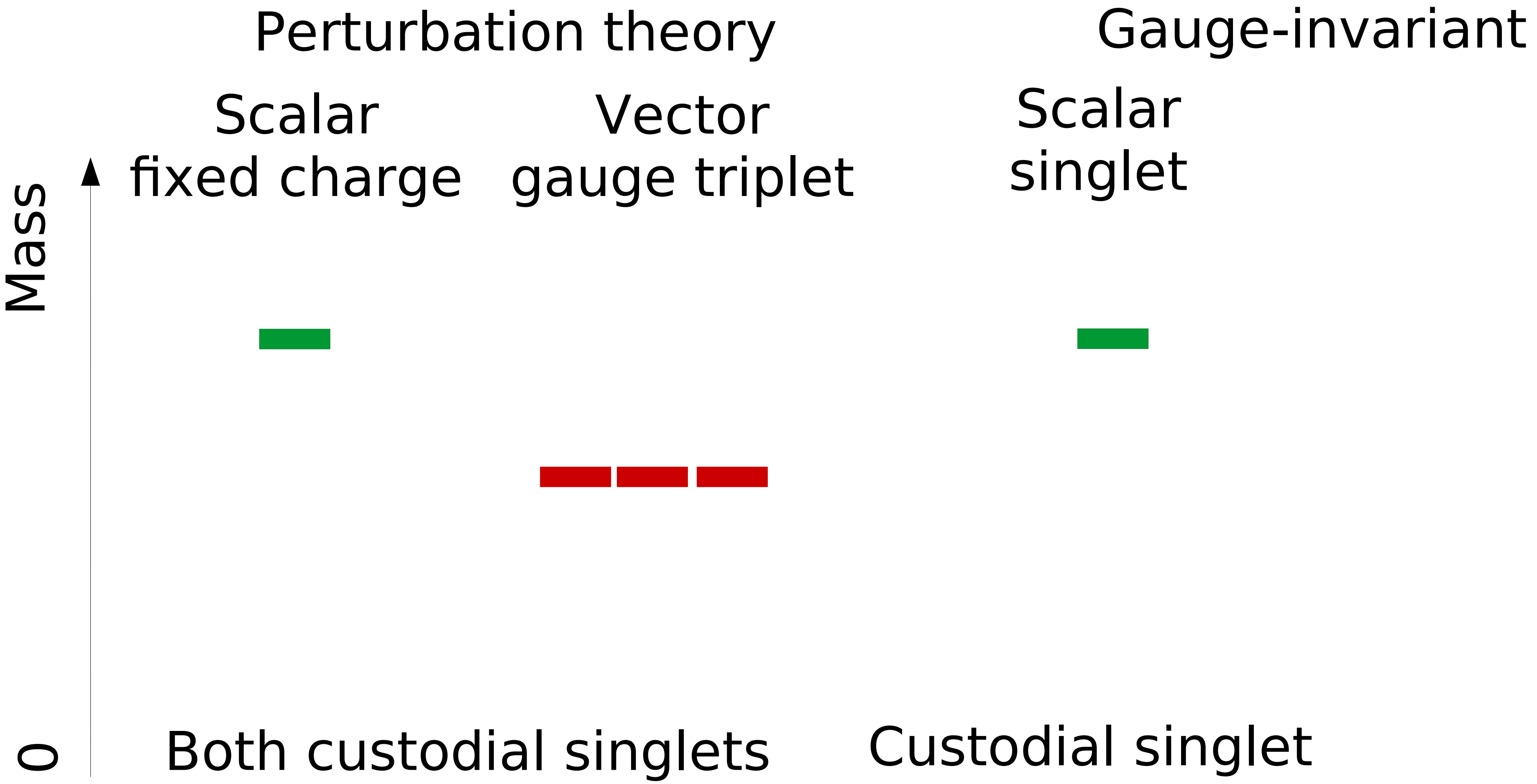

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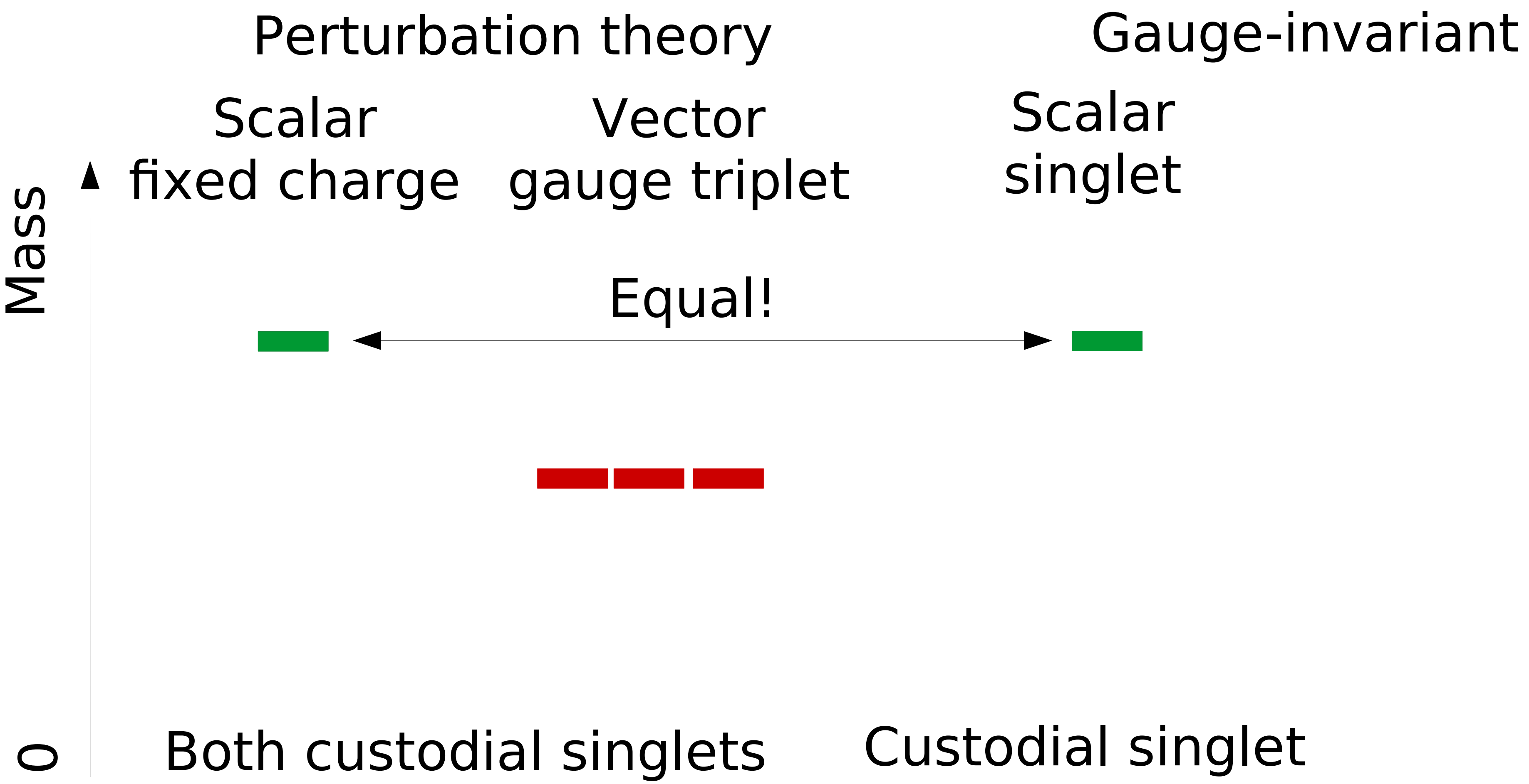

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Mass

Equal!

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Both custodial singlets

Custodial singlet

# Physical spectrum

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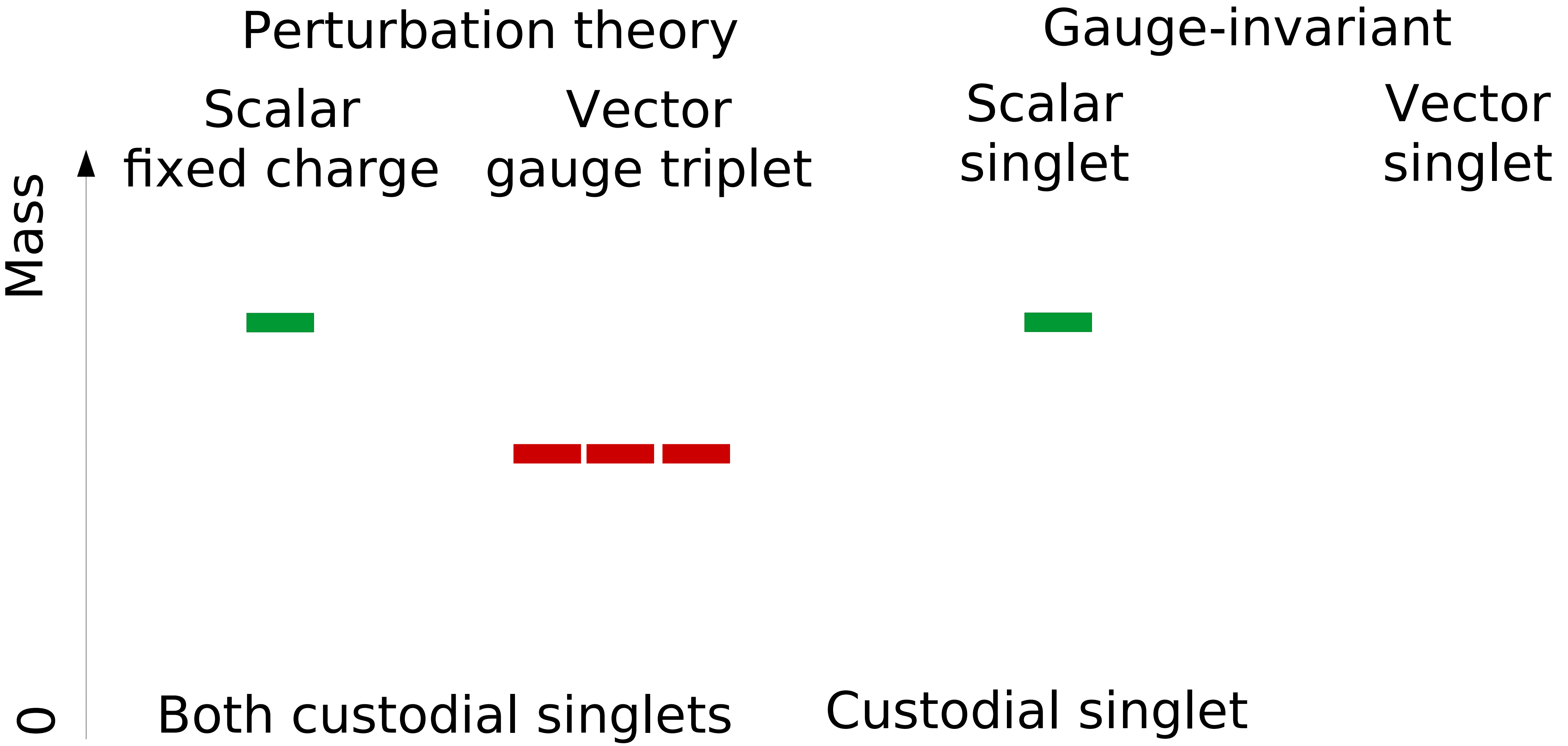

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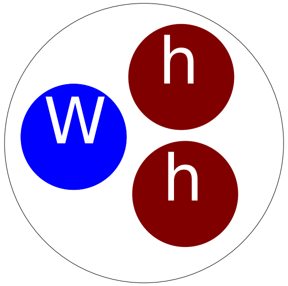

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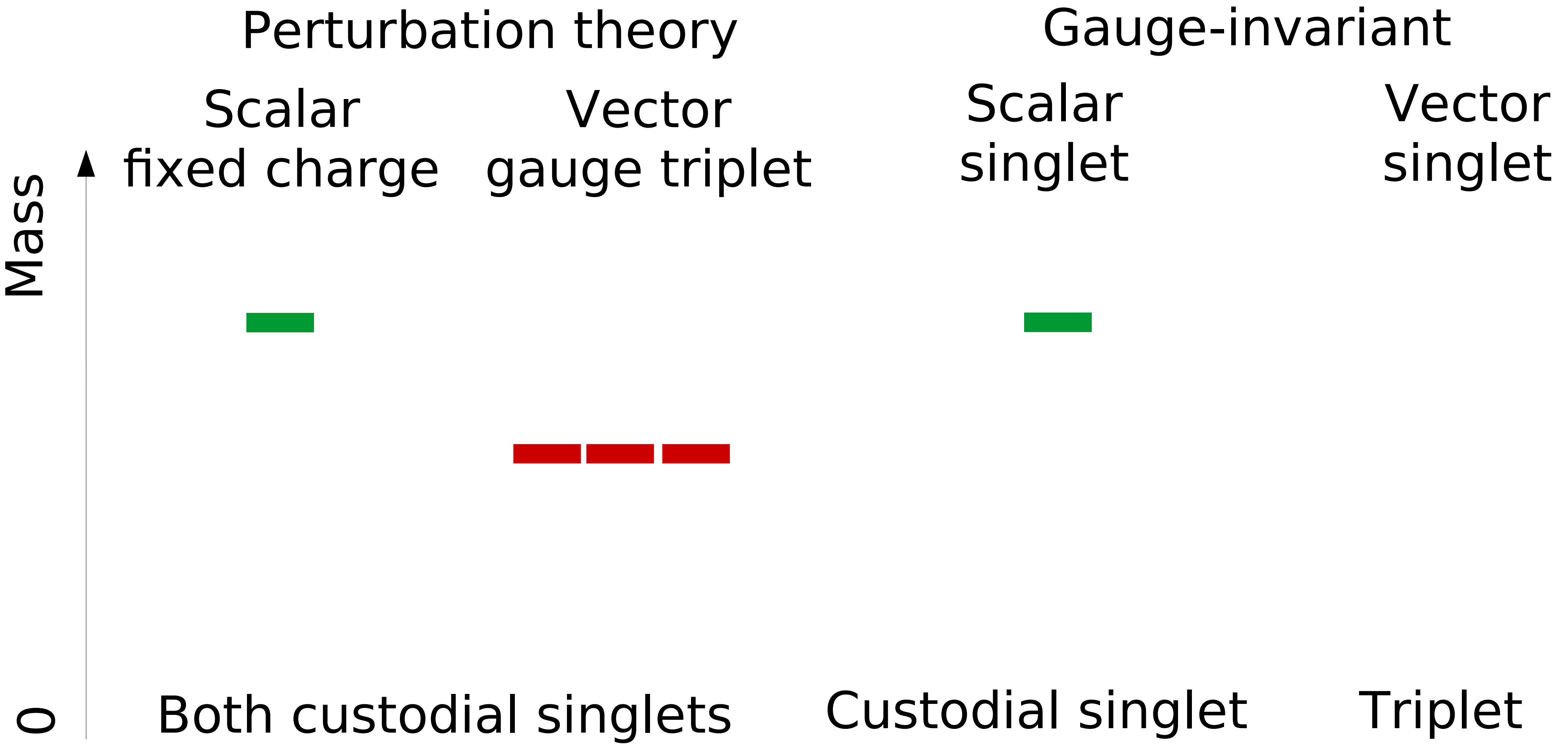

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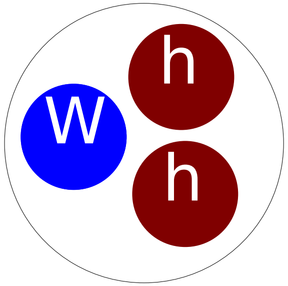

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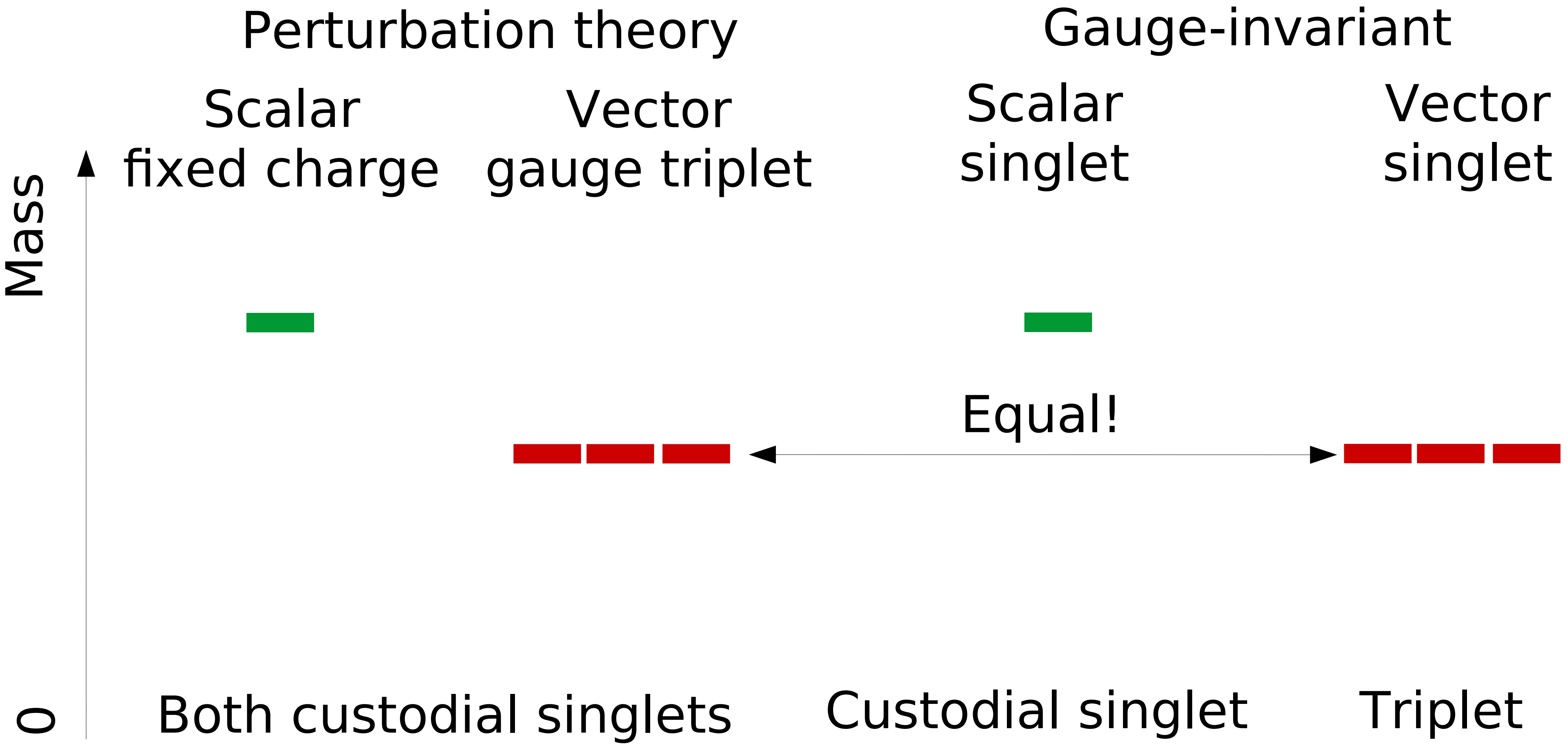

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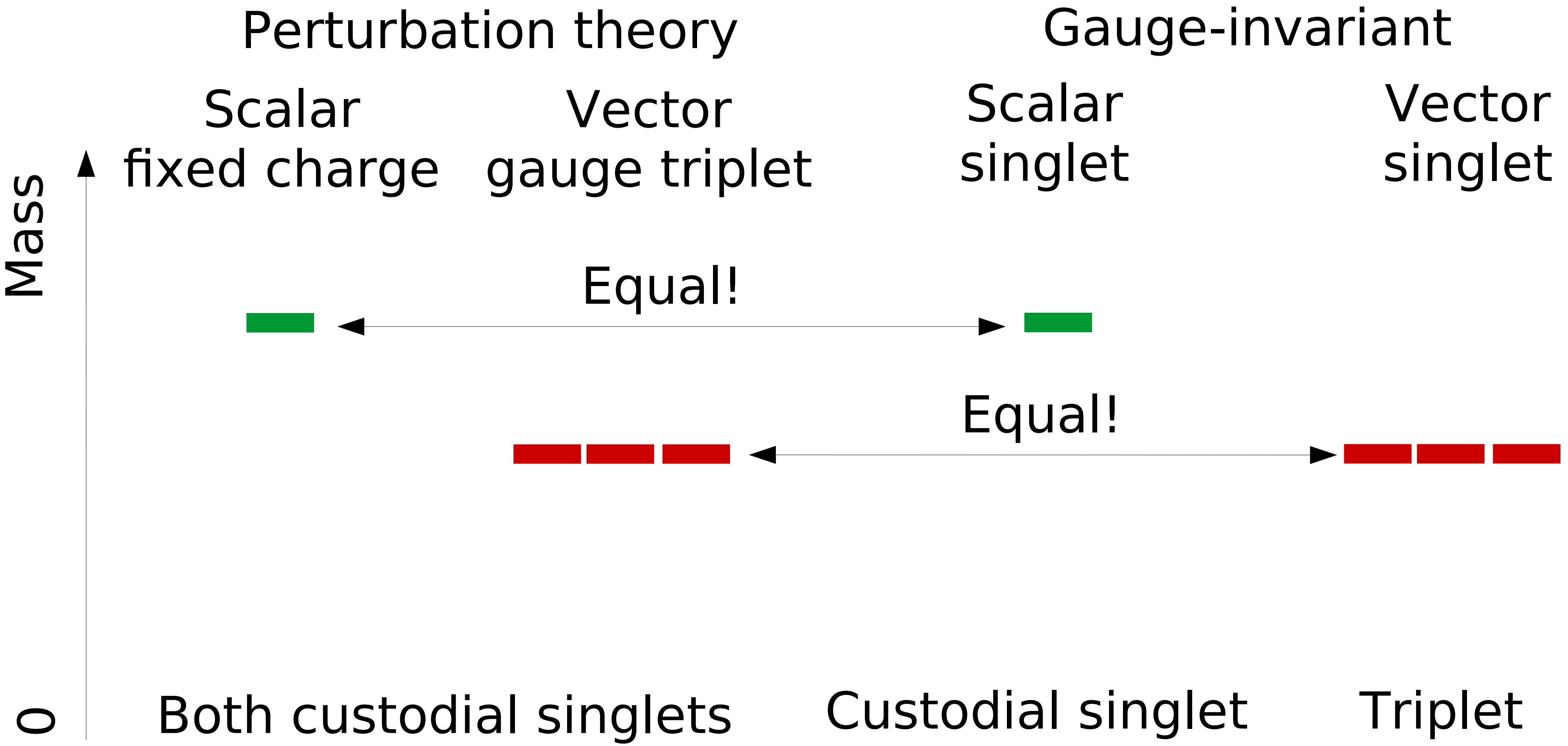

**Why?**

# A microscopic origin
# -
# Fröhlich-Morchio-Strocchi Mechanism
# or
# Utilizing the BEH effect at weak coupling

# Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

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Higgs field

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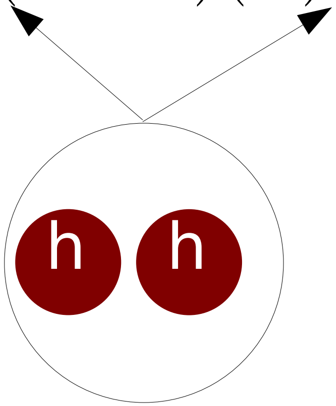

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Manifestly gauge-invariant

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Only sum gauge-invariant,
but not individual terms

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3) Standard perturbation theory

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Bound state mass

$$\boxed{\langle (h^+ h)(x)(h^+ h)(y)\rangle} = v^2 \langle \eta^+ (x)\eta(y)\rangle + \langle \eta^+ (x)\eta(y)\rangle\langle \eta^+ (x)\eta(y)\rangle + O(g,\lambda)$$

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$0^+$ singlet: $\langle (h^{+} h)(x)(h^{+} h)(y)\rangle$

2) Expand Higgs field around fluctuations $h=v+\eta$

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^2 \langle \eta^{+}(x)\eta(y)\rangle + v\langle \eta^{+} \eta^2 + \eta^{+2} \eta\rangle + \langle \eta^{+2} \eta^2\rangle$$

3) Standard perturbation theory

Bound state mass

$$\langle (h^{+} h)(x)(h^{+} h)(y)\rangle = v^2 \langle \eta^{+}(x)\eta(y)\rangle + \langle \eta^{+}(x)\eta(y)\rangle\langle \eta^{+}(x)\eta(y)\rangle + O(g,\lambda)$$

Trivial two-particle state

4) Compare poles on both sides

# Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

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What about this?

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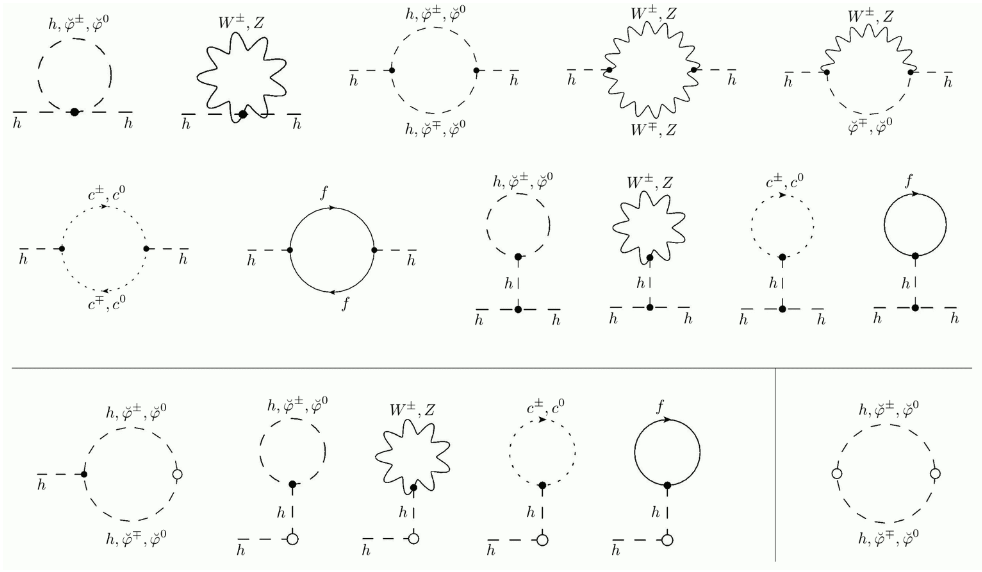

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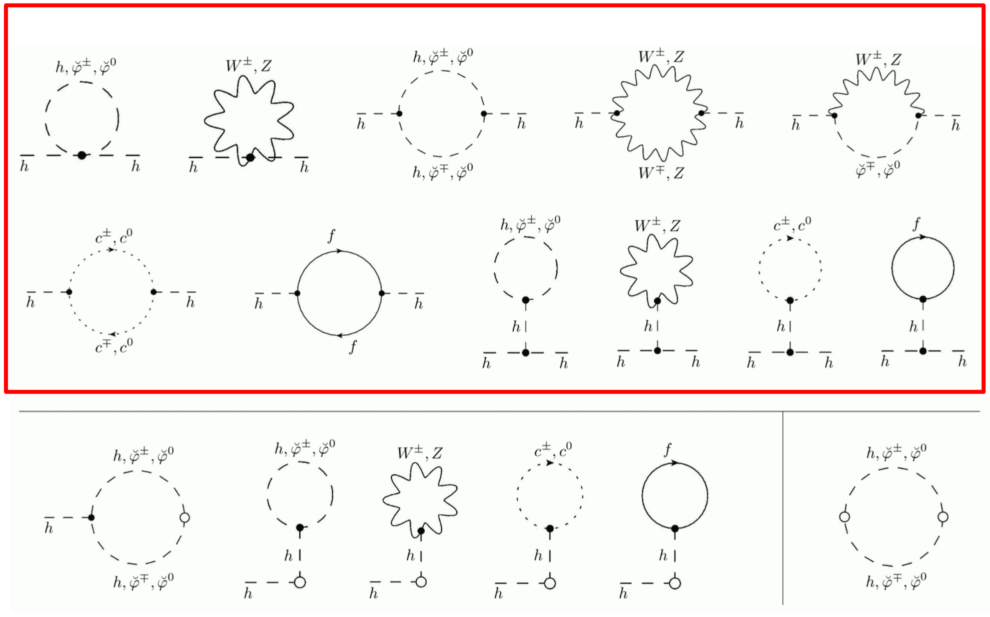

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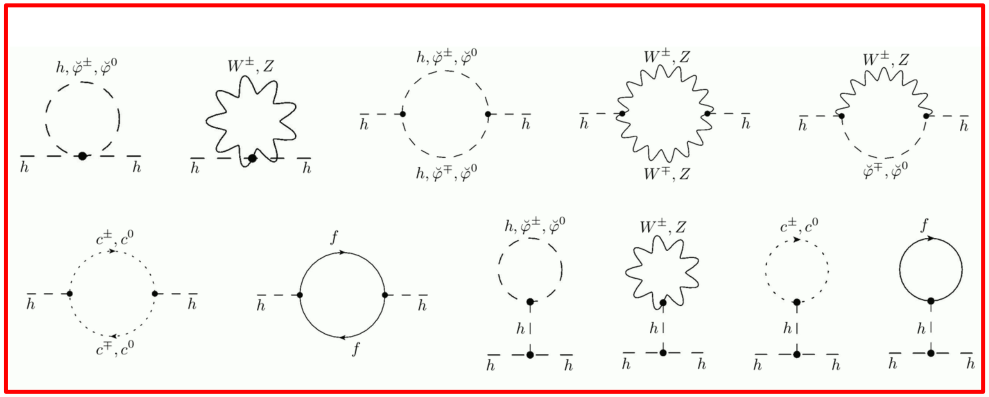

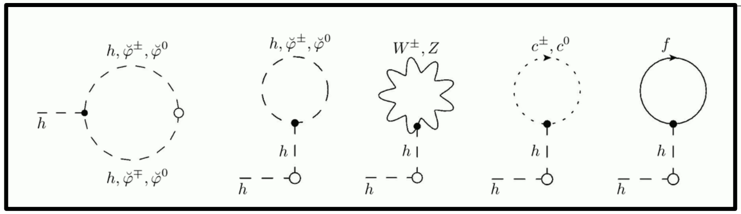

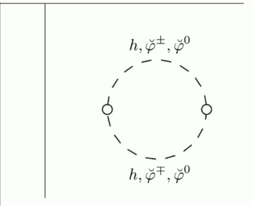

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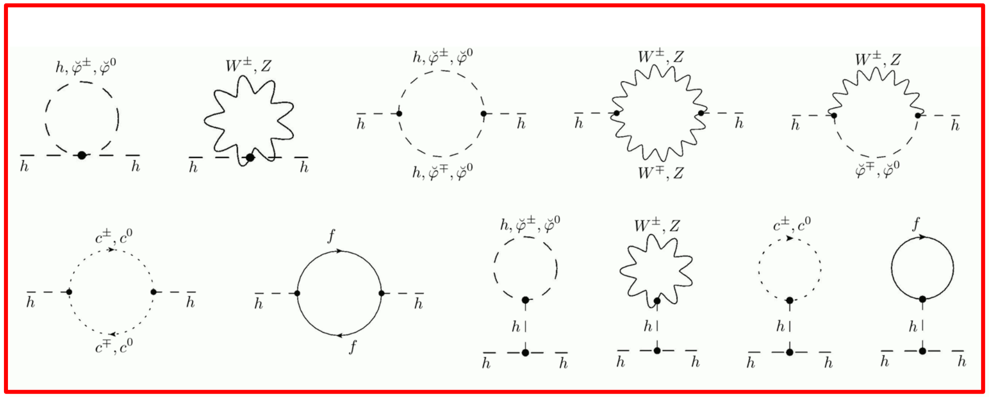

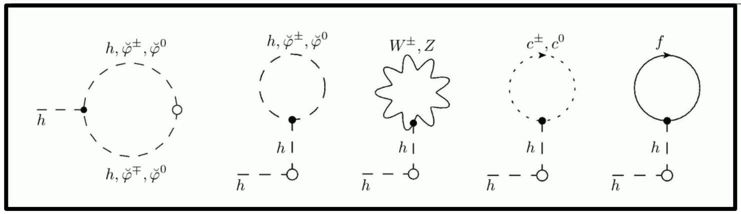

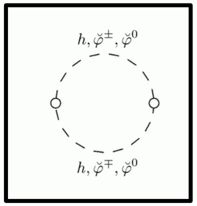

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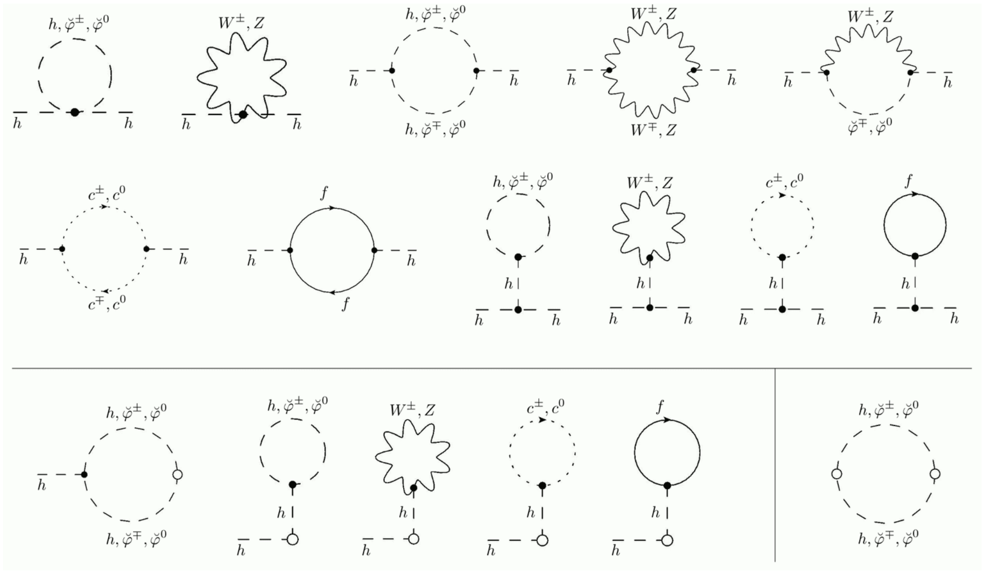

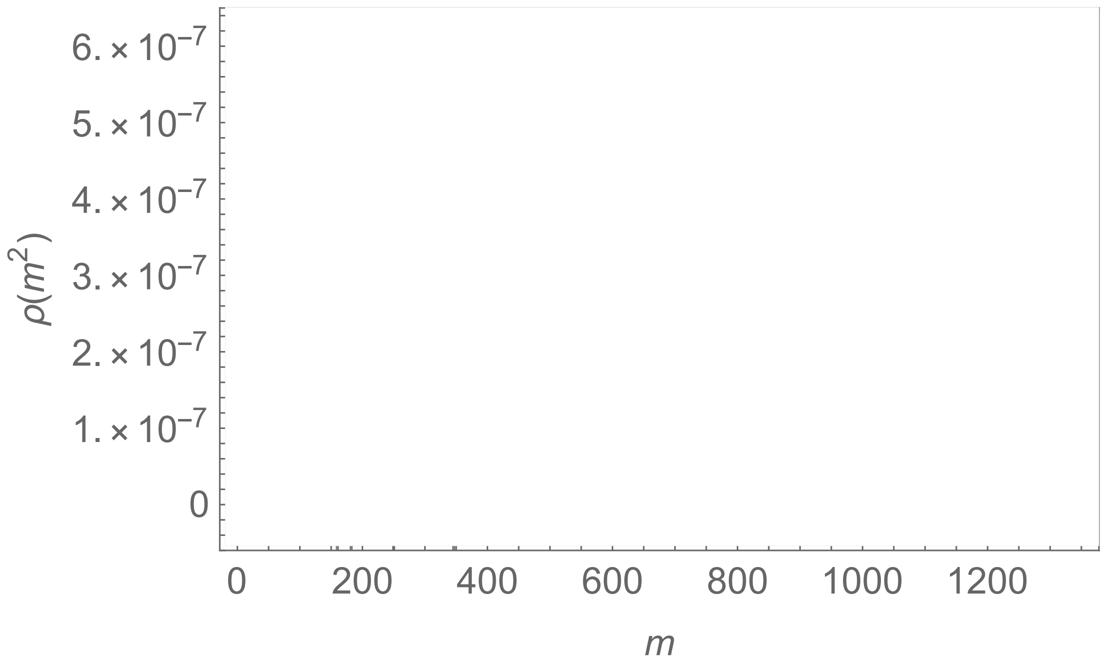

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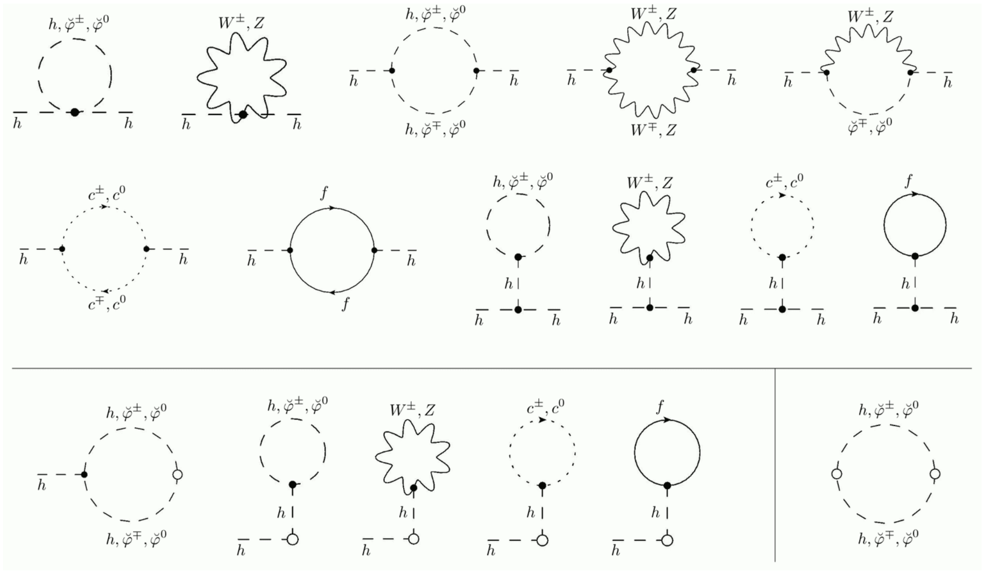

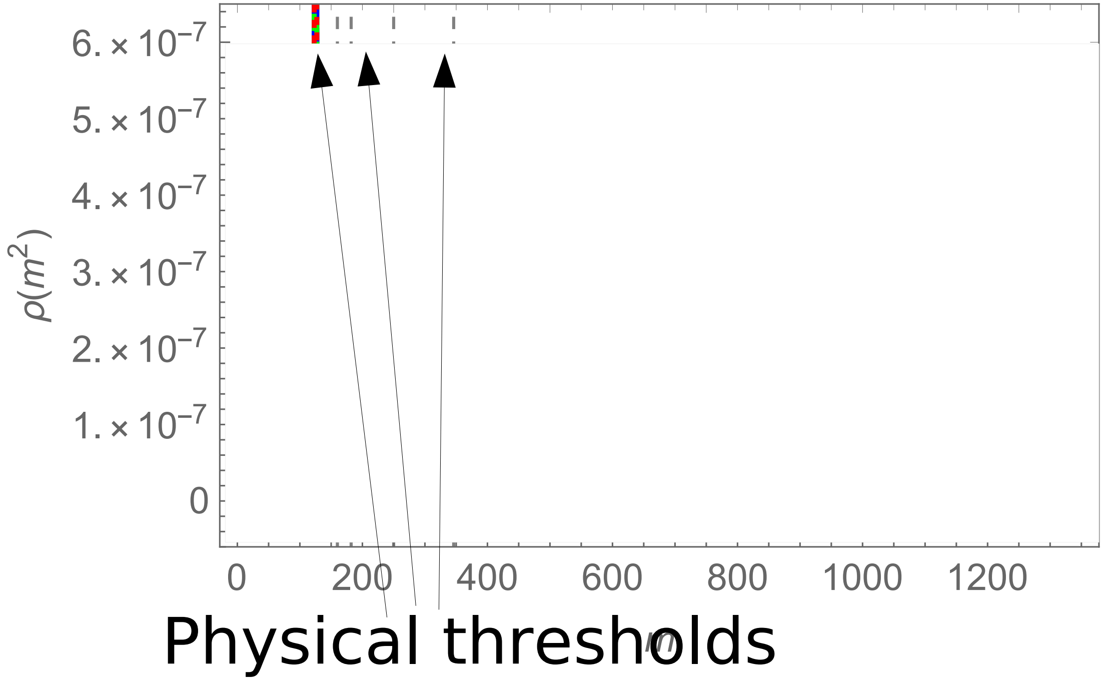

Physical thresholds

# Consequences: The Higgs

[Maas'12,'17
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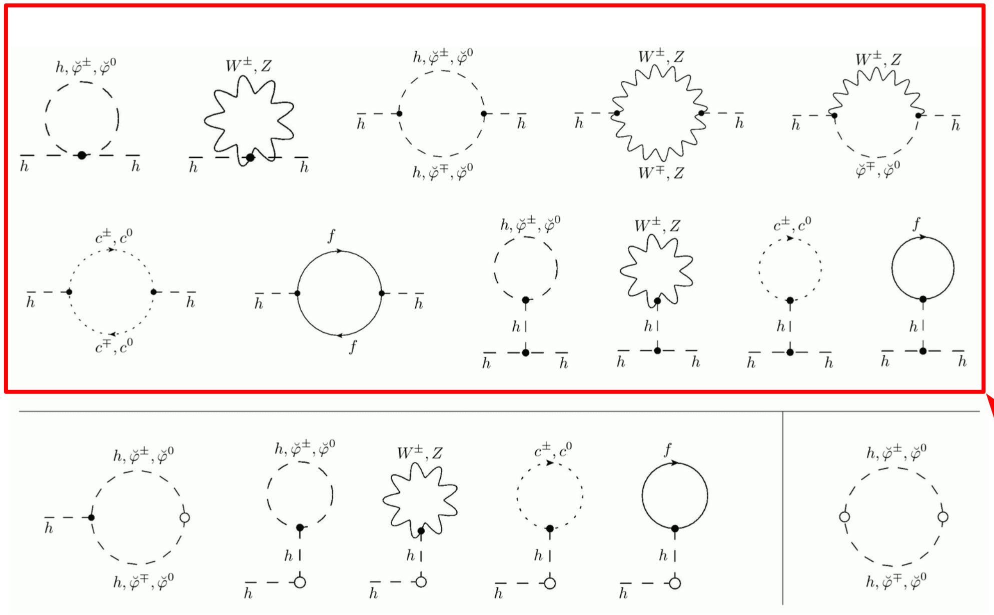

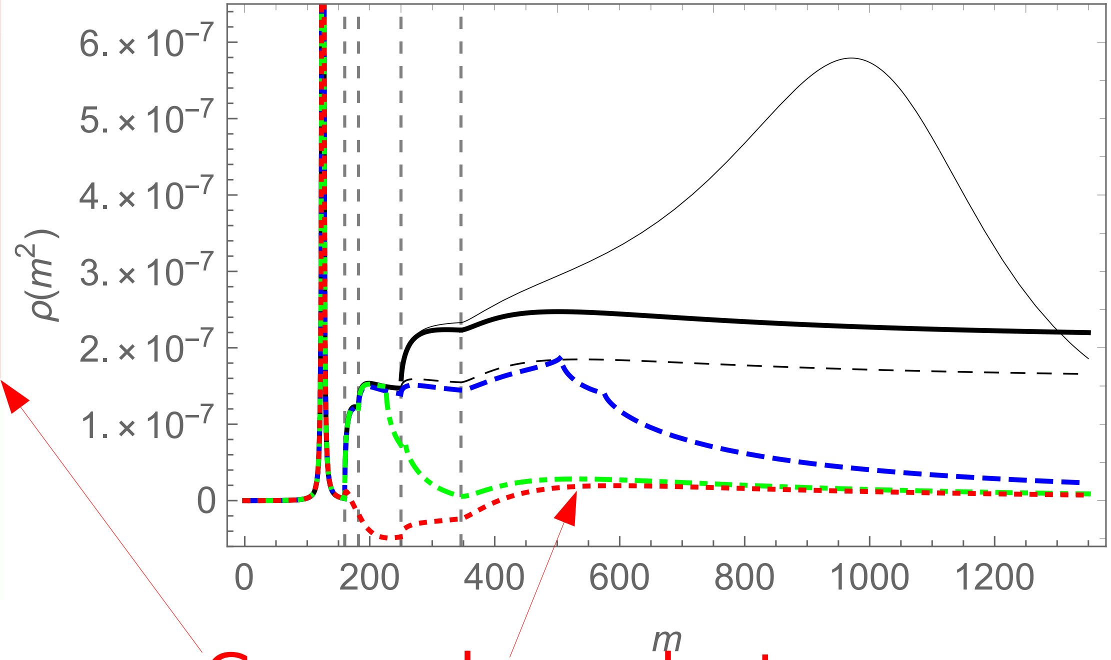

Gauge-dependent

# Consequences: The Higgs

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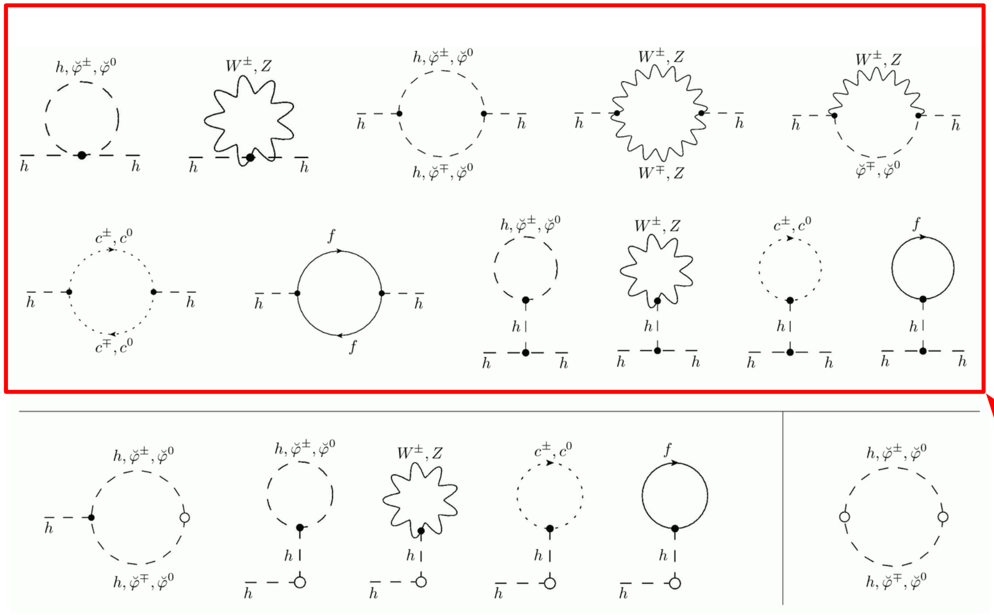

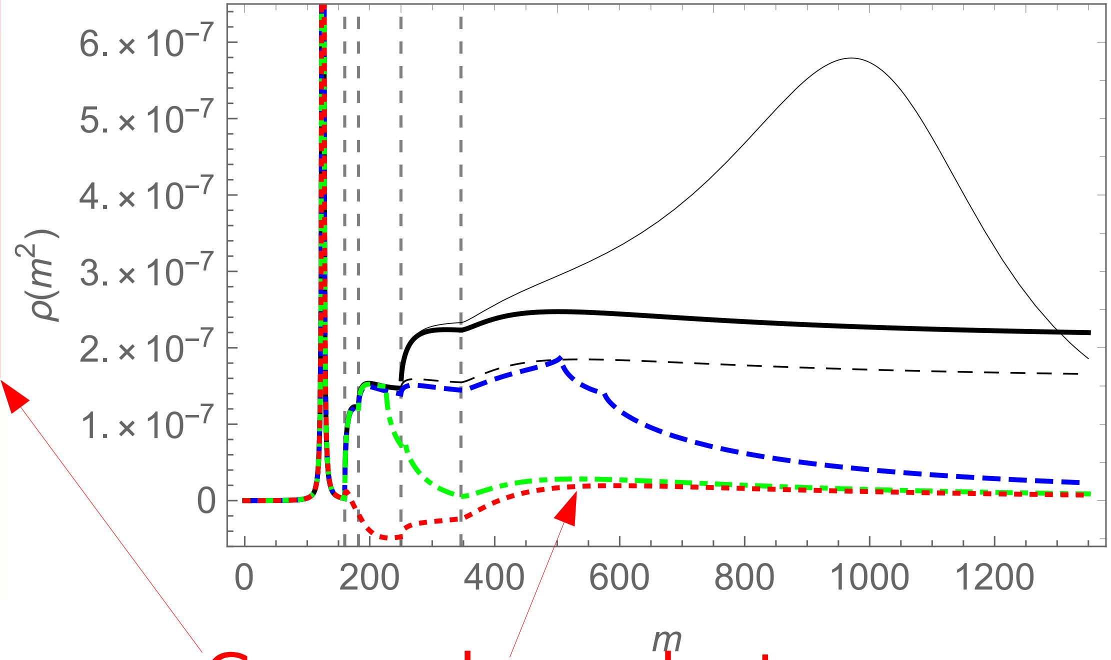

Gauge-dependent

Unphysical features:

Positivity violation

Additional thresholds

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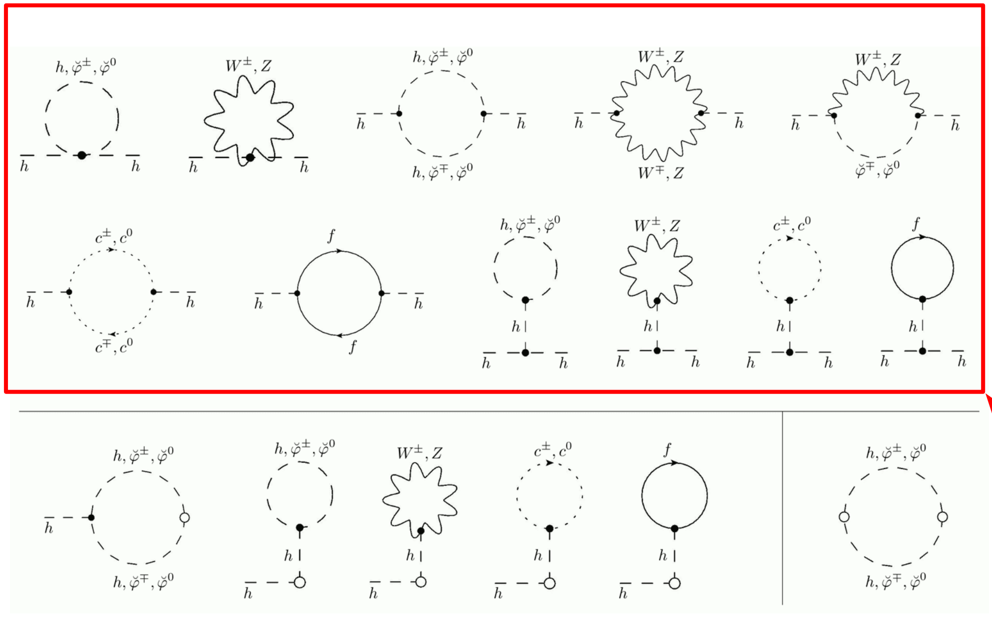

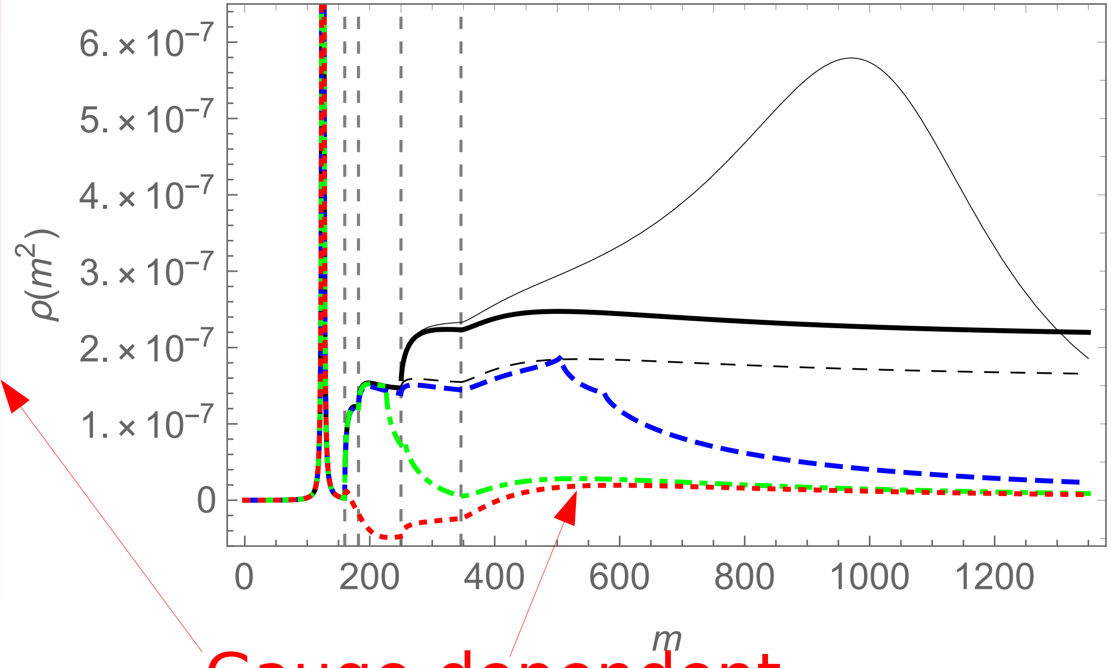

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Not a consequence of instability: Occurs even for an asymptotically stable Higgs in a toy theory

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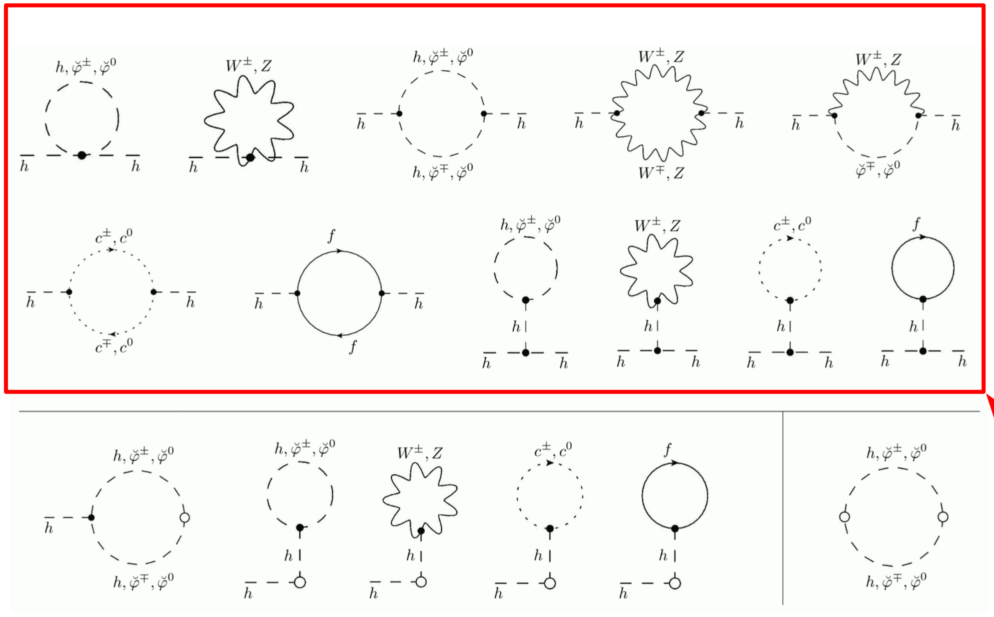

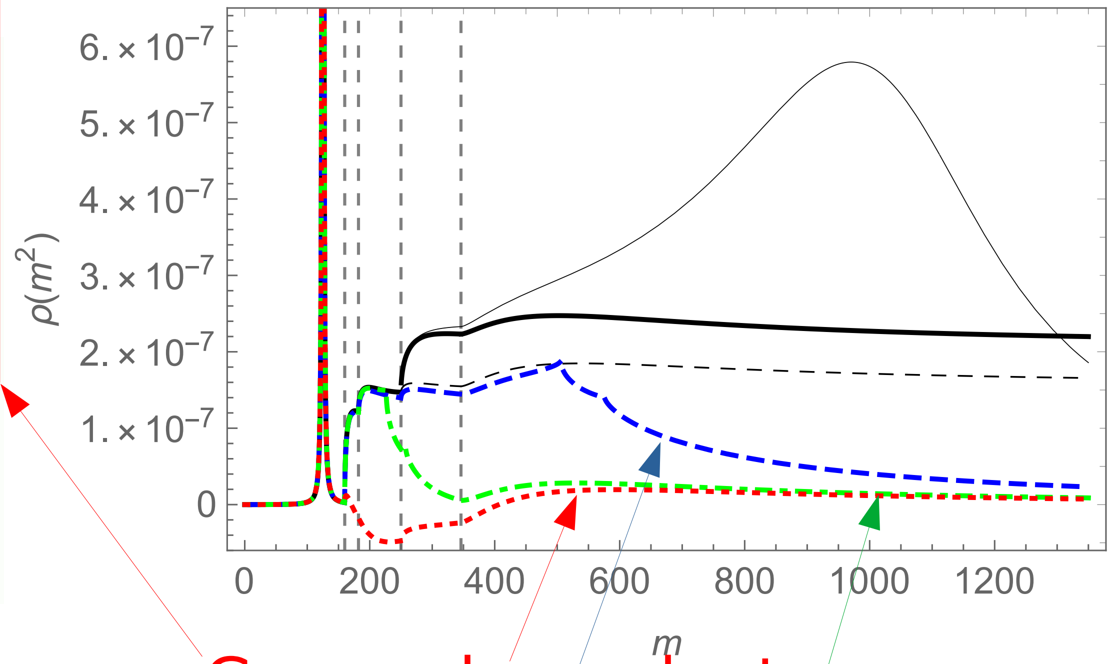

Gauge-dependent

Other gauge choices

# Consequences: The Higgs

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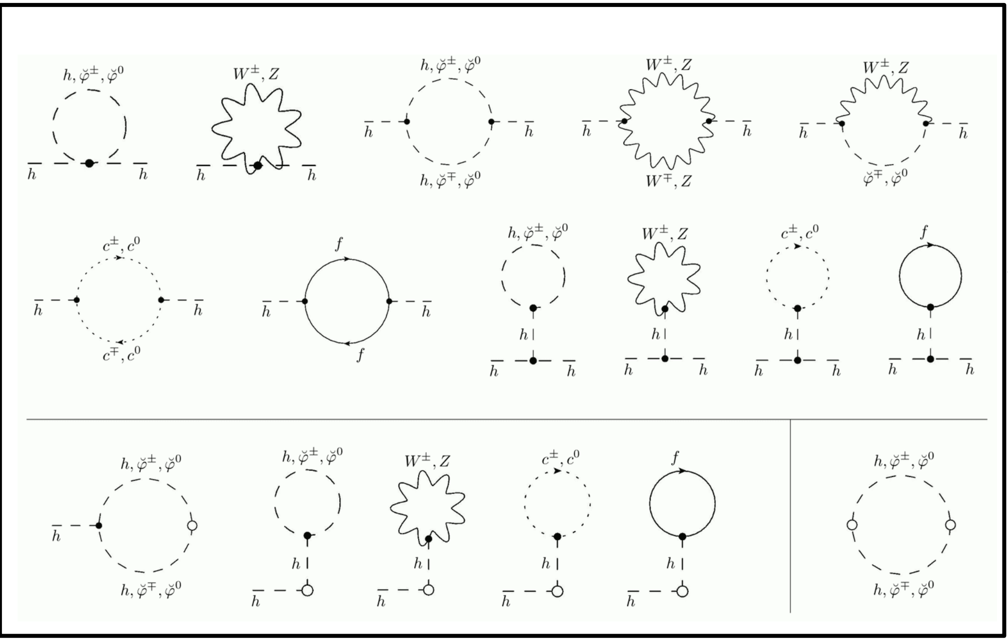

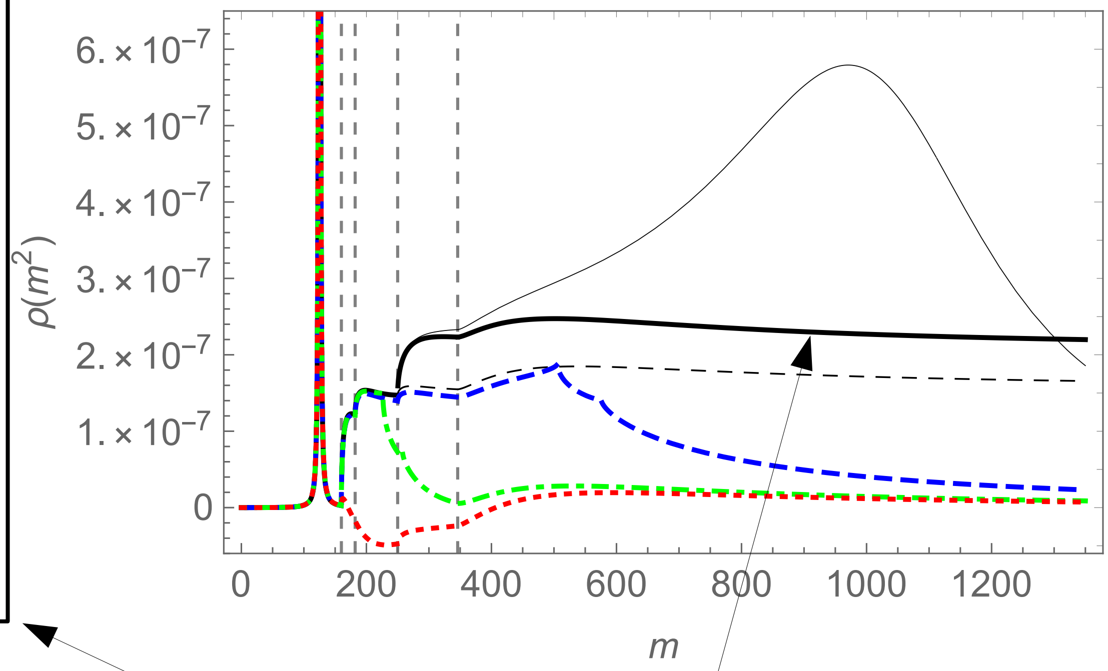

Physical – same for all gauge choices

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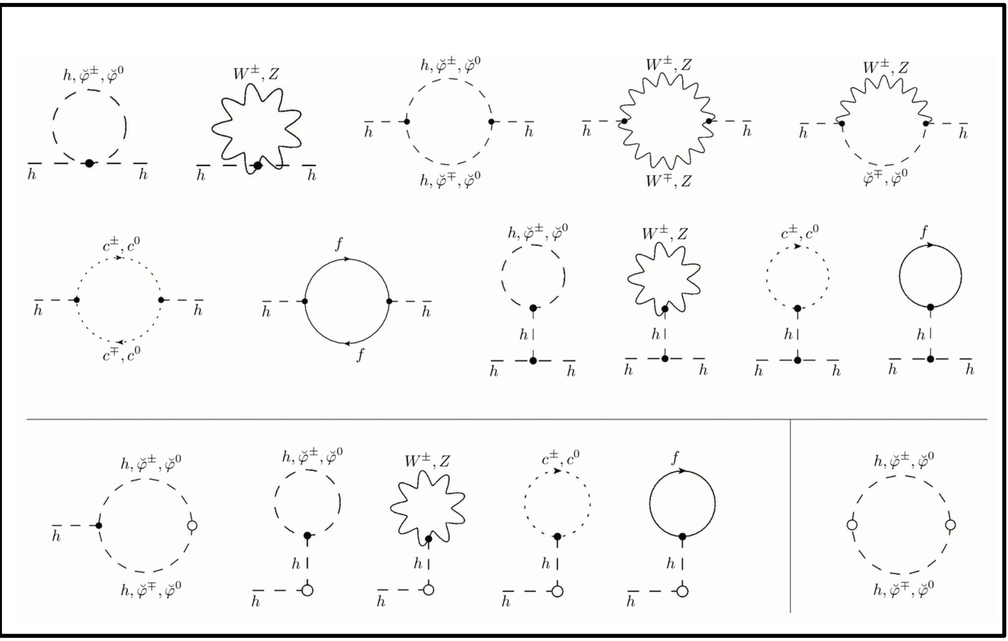

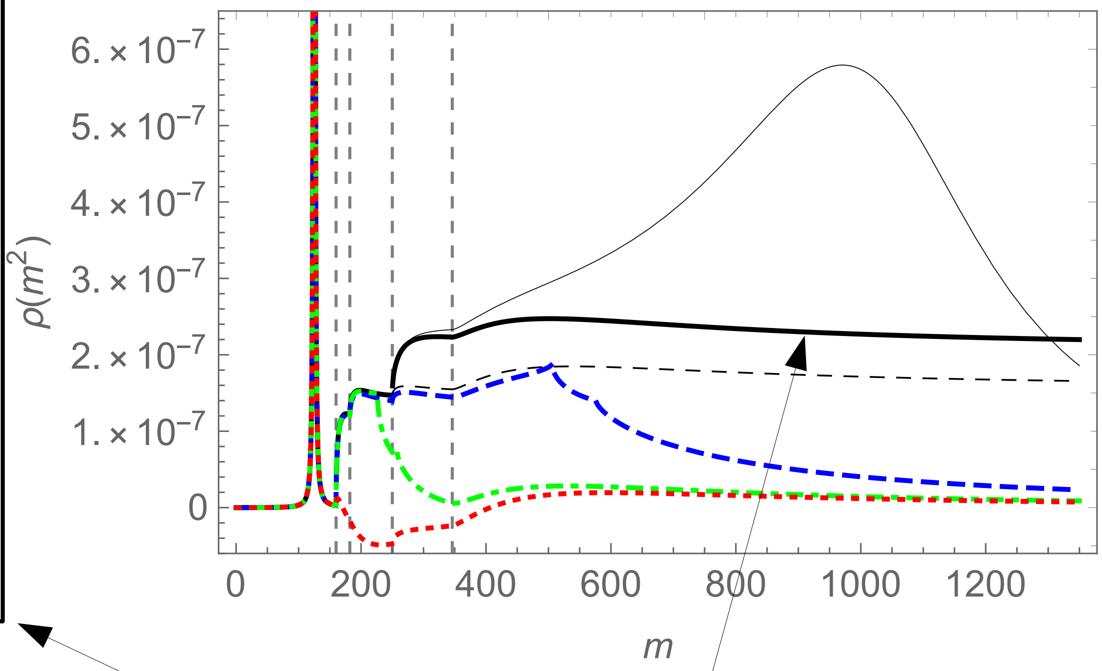

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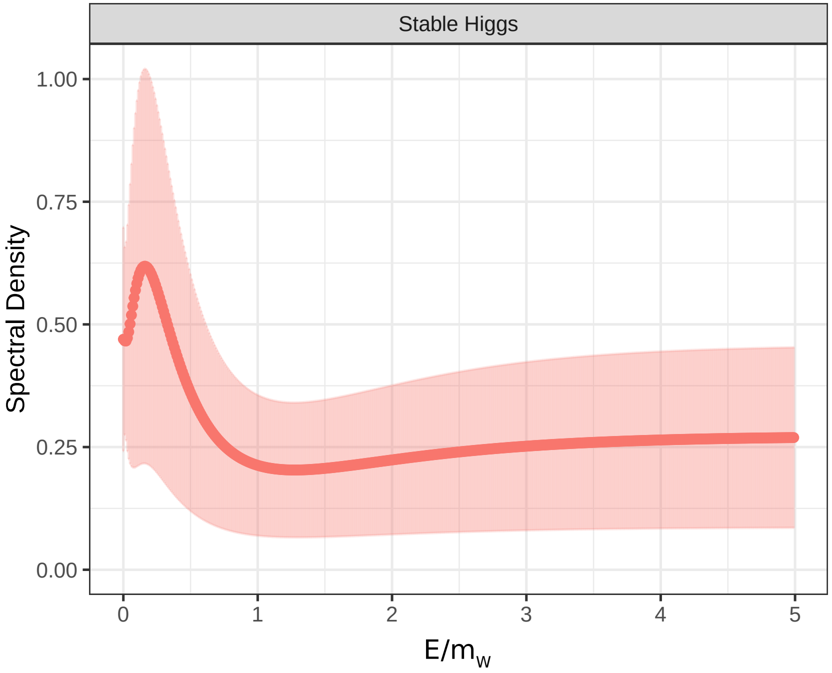

Consistent with lattice results for the composite operator in the toy theory

[Maas'12,'17
Maas & Sondenheimer'20
Dudal et al.'20
Martins et al. unpublished]

# What about the vector?

[Fröhlich et al.’80,’81
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Matrix from
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$c$ projects custodial
states to gauge
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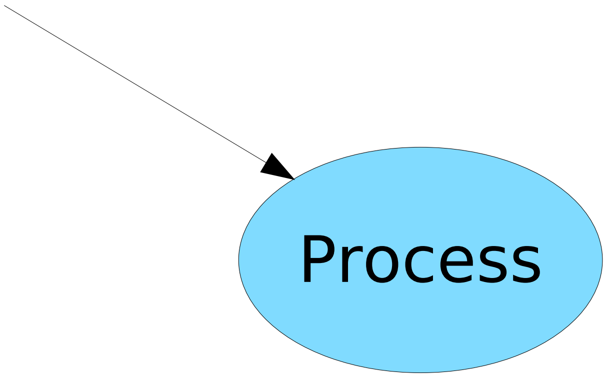
Exactly one gauge boson
for every physical state

# Asymptotic states

[Maas et al.'17
Maas & Reiner '22
Maas, Plätzer et al.'24, unpublished]

Incoming (asymptotic) particle
Standard LSZ: Elementary particle



$$\langle f(p)...\rangle$$

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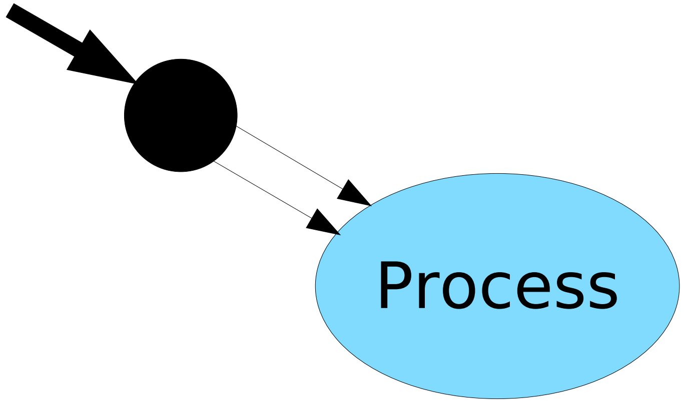

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Gauge-invariant LSZ: Bound state

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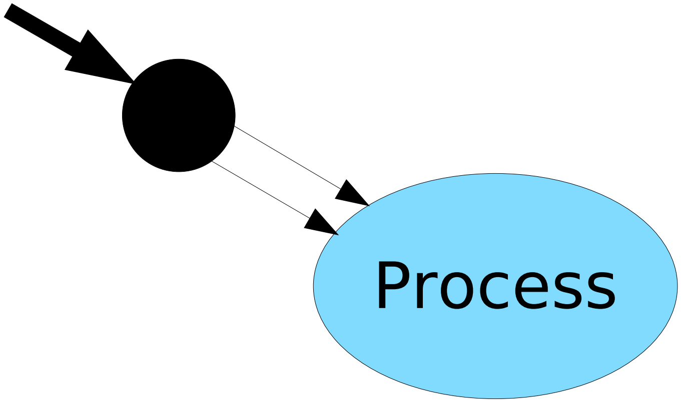

$$\langle (hf)(p)...\rangle$$

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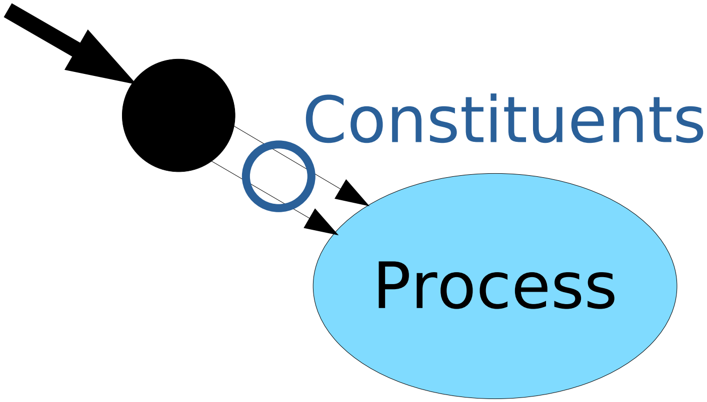

$$\left\langle \left(hf\right)\left(p\right)\ldots\right\rangle$$

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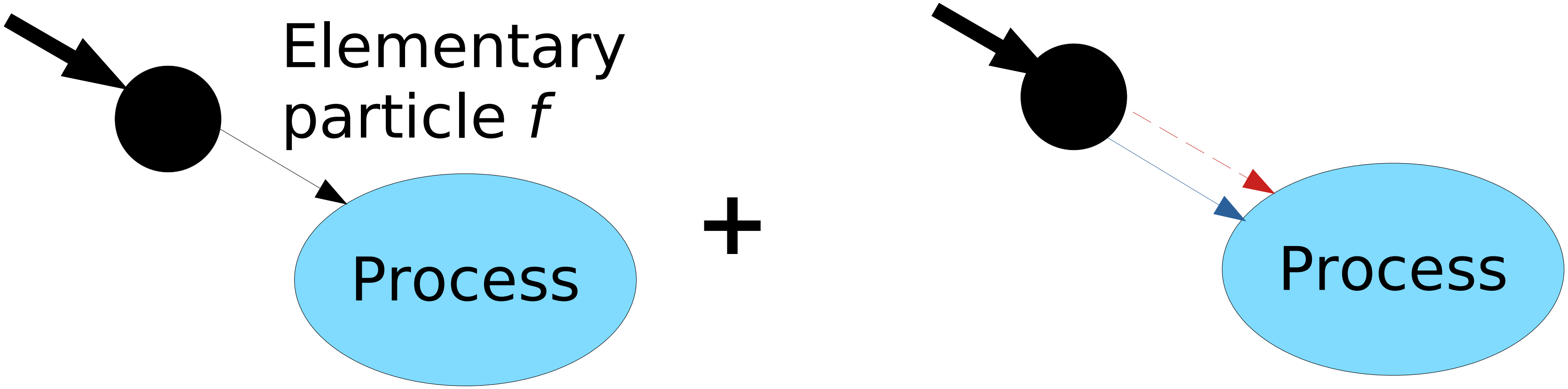
FMS LSZ: Elementary and fluctuations



$$v\langle f(p)\ldots\rangle+\langle(\eta f)(p)\rangle$$

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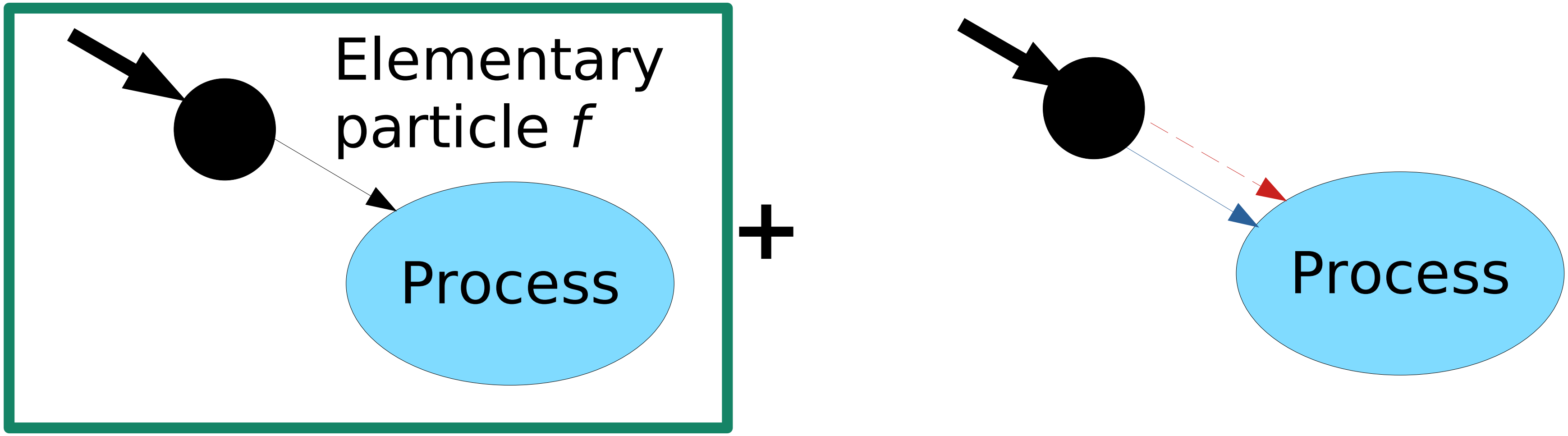

Standard perturbation theory

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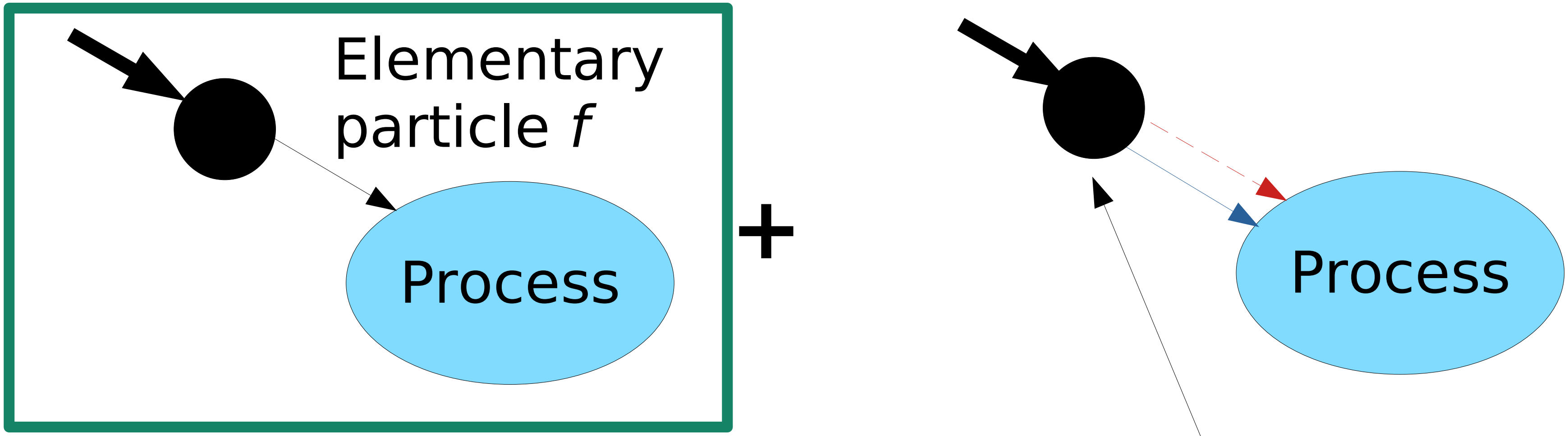

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Can this contribute
after LSZ truncation?

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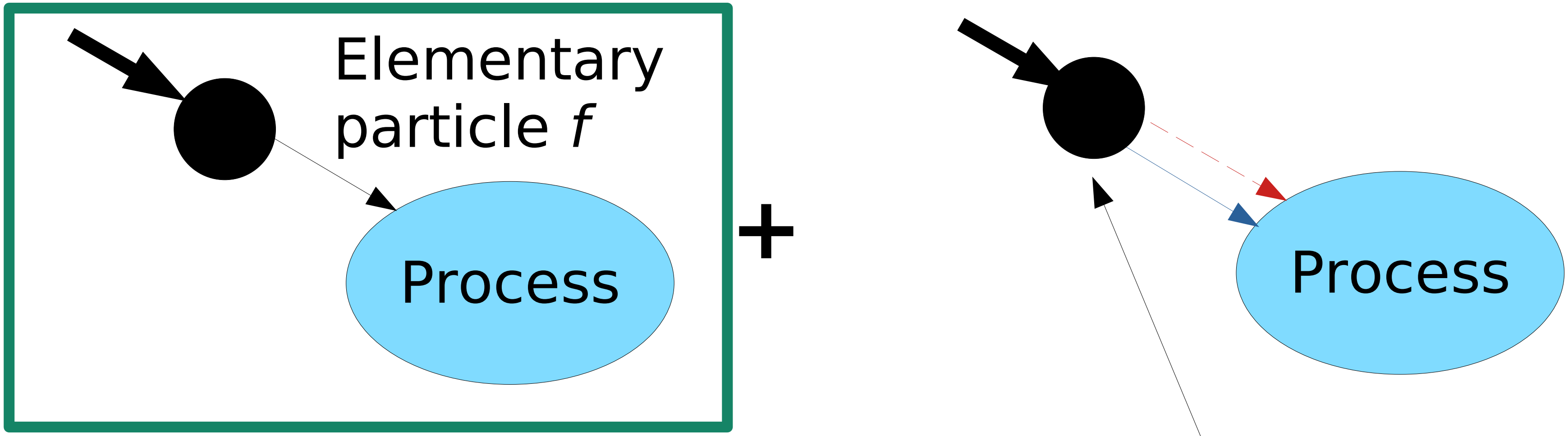

Elementary particle *f*

Process

+

Process

Standard perturbation theory

$$\boxed{v\langle f(p)...\rangle}+\boxed{\langle(\eta f)(p)\rangle}$$

Can this contribute after LSZ truncation?
Yes!

# Bubble sum

[Maas et al.’17
Maas & Reiner ‘22
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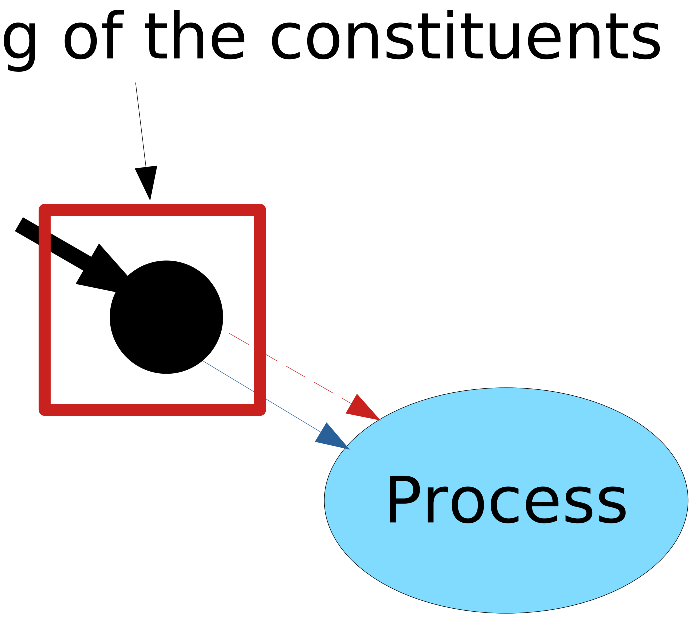

Scattering of the constituents

# Bubble sum

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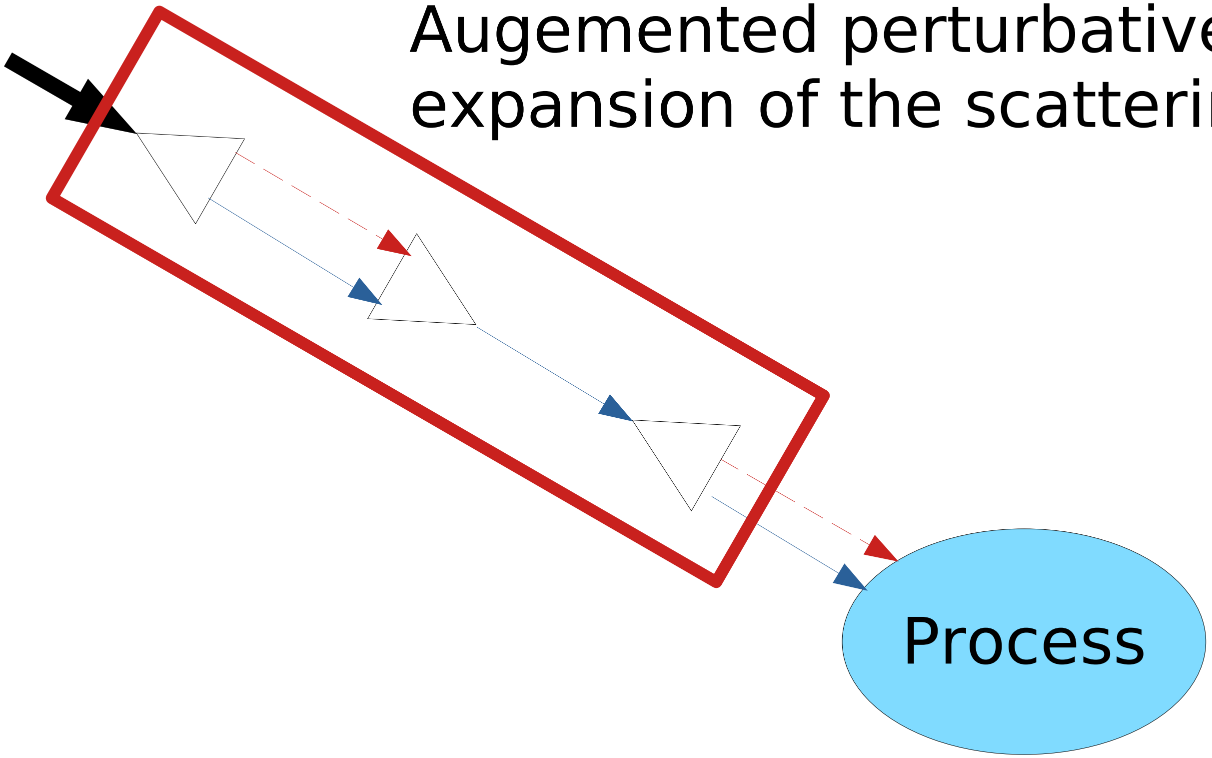

Augemented perturbative
expansion of the scattering kernel

# Bubble sum

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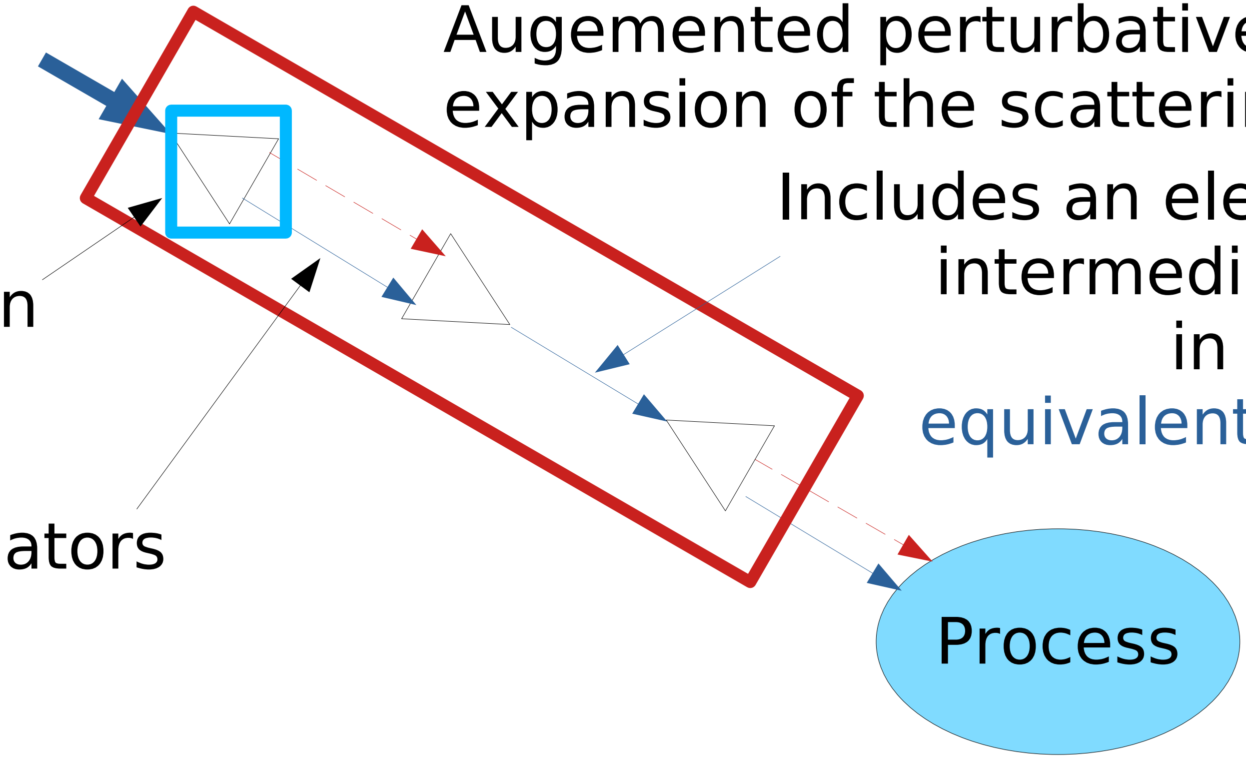

Augemented perturbative expansion of the scattering kernel

Includes an elementary intermediate state in the FMS-equivalent channel

Vertex function

Propagators

Process

# Bubble sum

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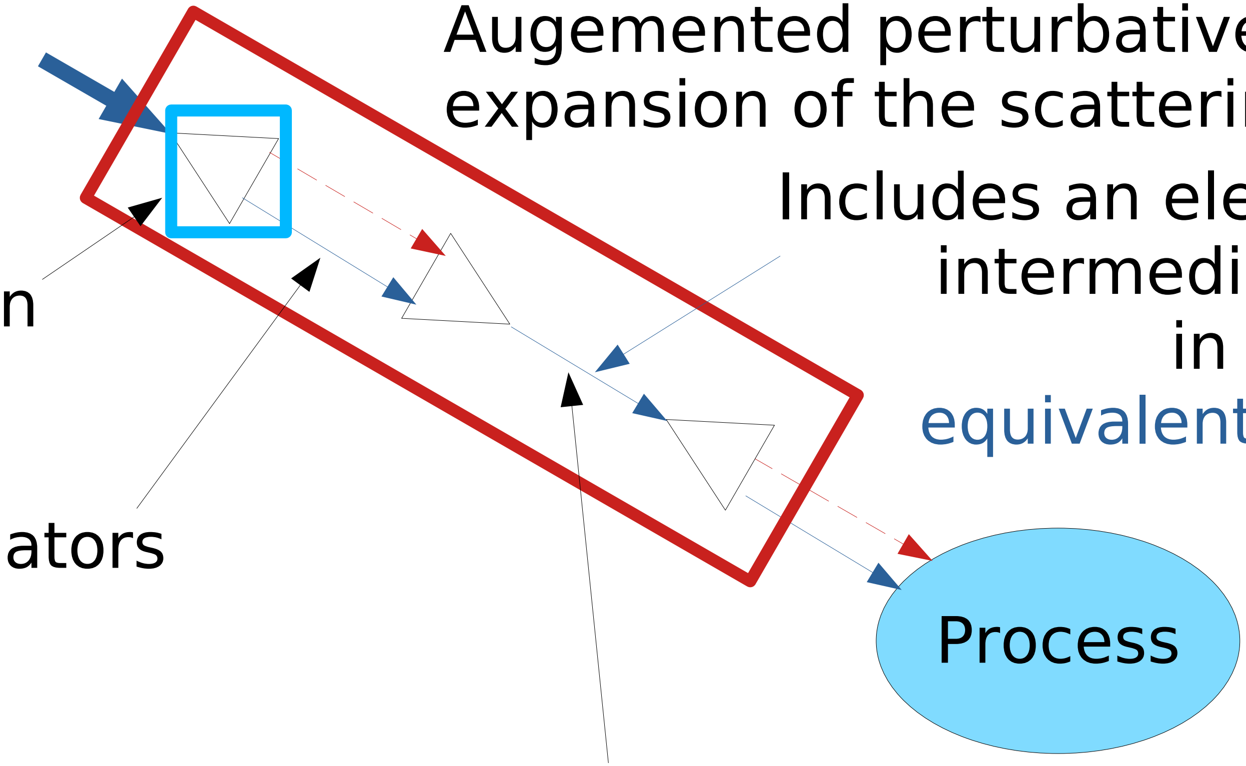

Augemented perturbative
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Includes an elementary
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Vertex function

Propagators

This intermediate state
carries the necessary pole
to let this expression survive the
LSZ truncation

# Bubble sum

[Maas et al.'17
Maas & Reiner '22
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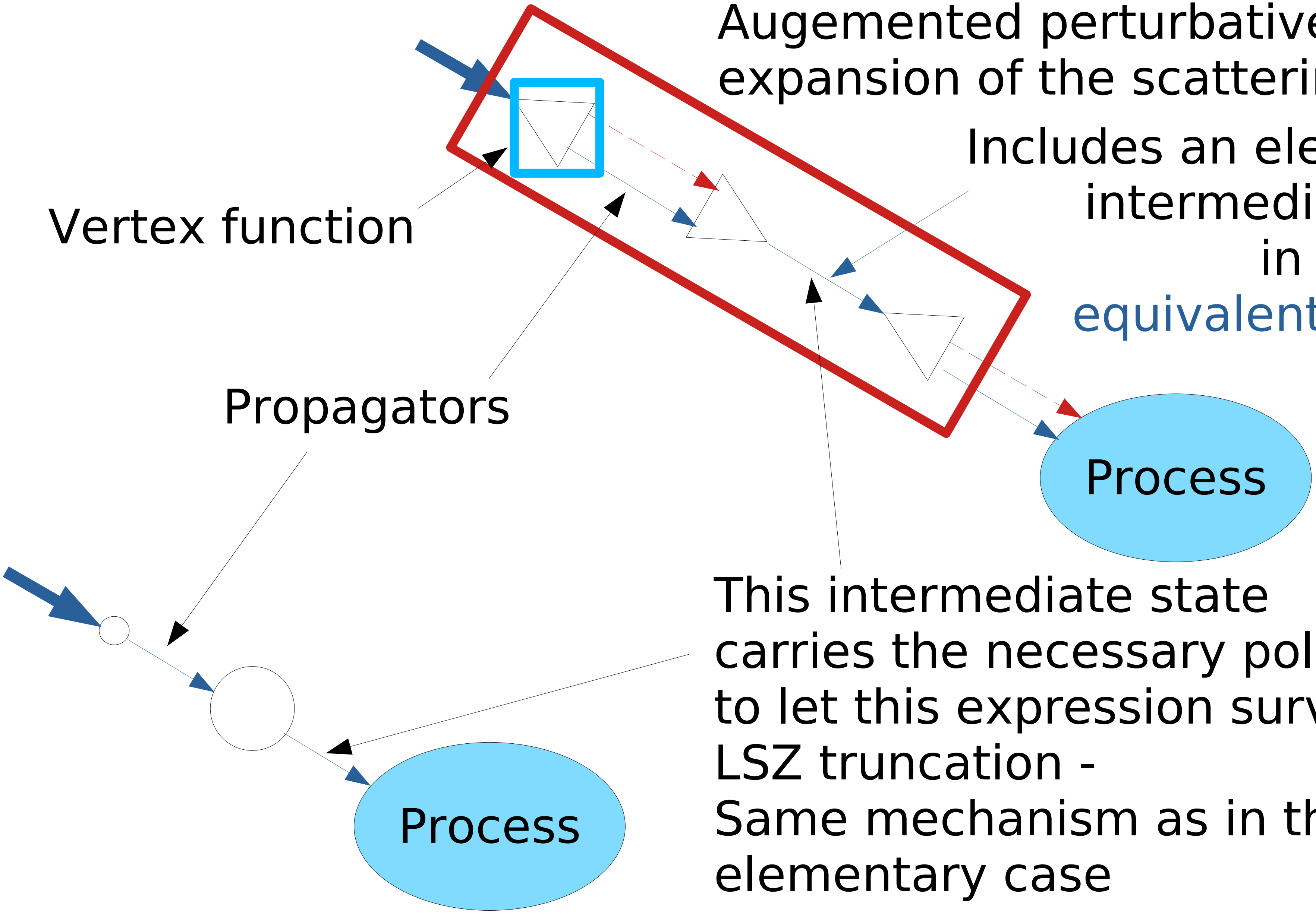

Augemented perturbative expansion of the scattering kernel

Includes an elementary intermediate state in the FMS-equivalent channel

Vertex function

Propagators

This intermediate state carries the necessary pole to let this expression survive the LSZ truncation - Same mechanism as in the elementary case

# Bubble sum

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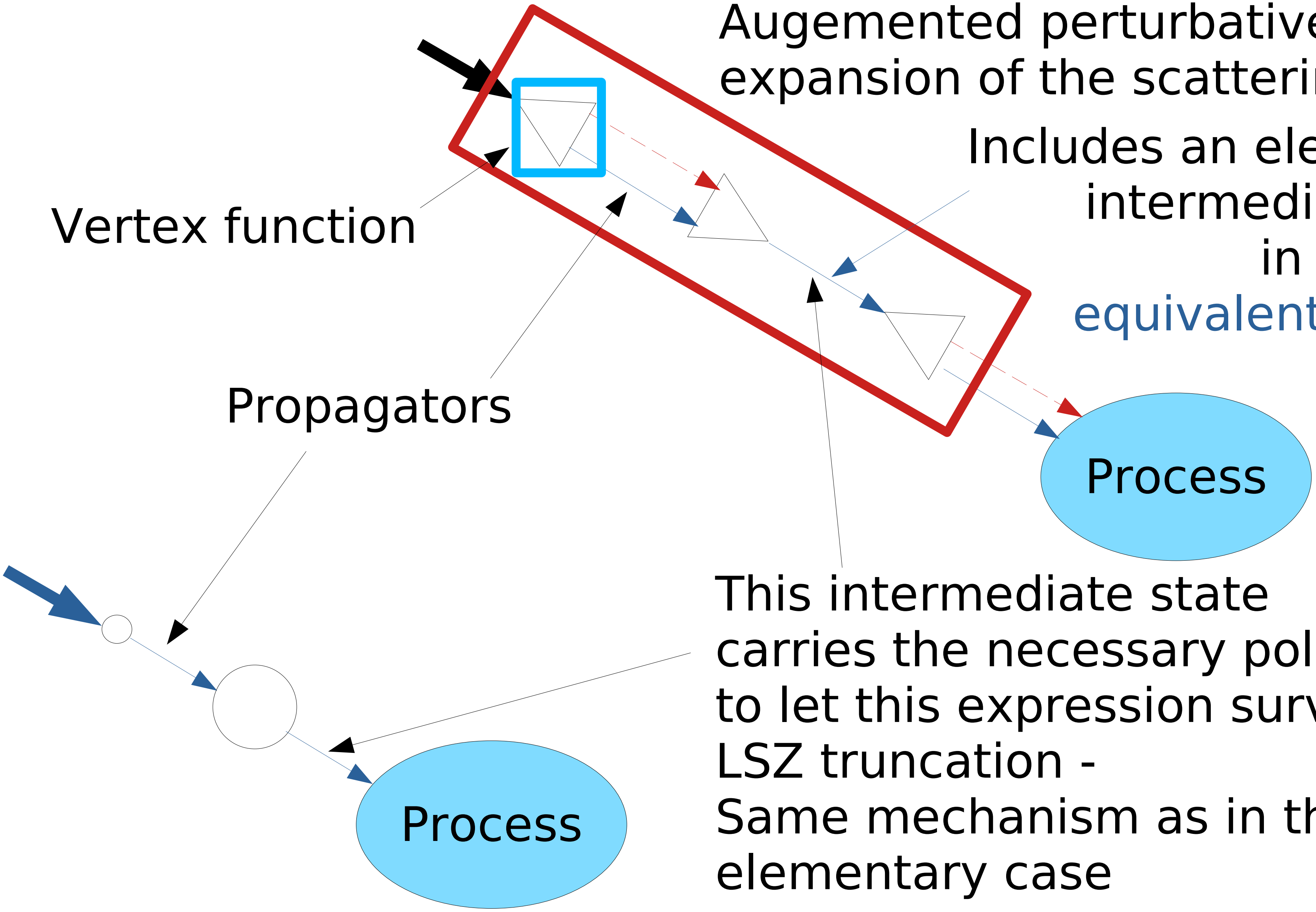

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Can be perturbatively resummed!

# Asymptotic states

[Maas et al.’17
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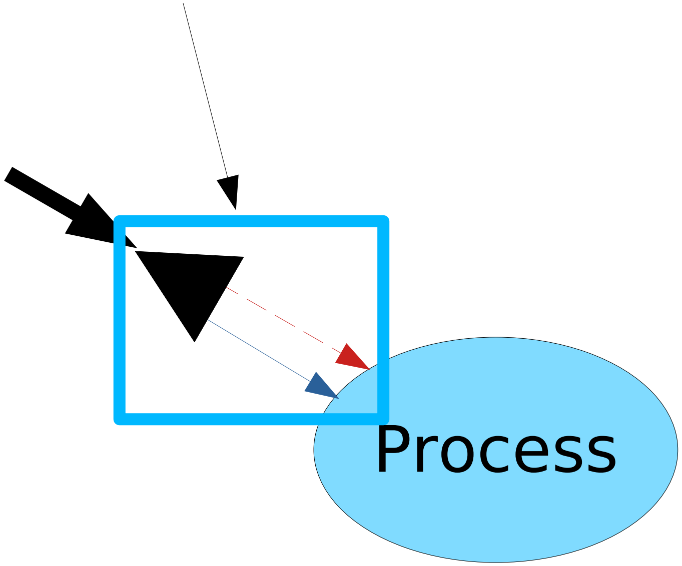
Becomes the perturbatively calculable,
Process-independent Bethe-Salpeter amplitude



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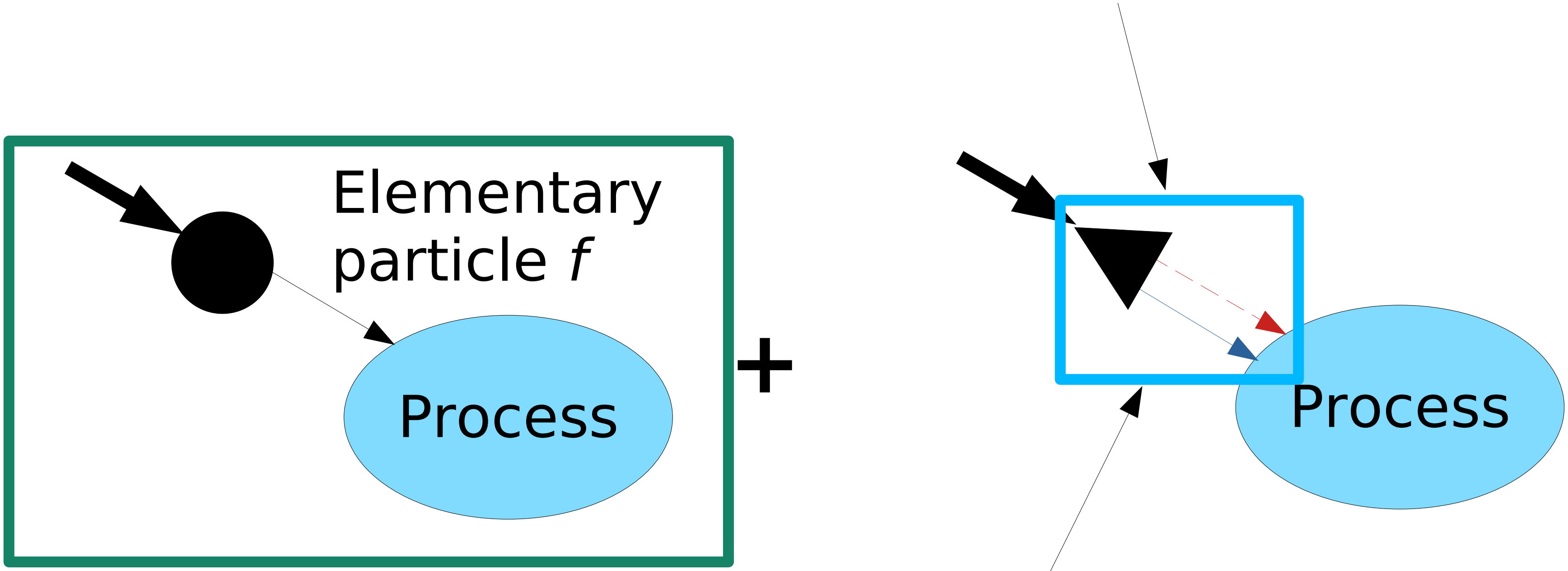

Standard perturbation theory

$$\boxed{v\langle f(p)\ldots\rangle}+\boxed{\int dq\,\Gamma(P,q)\,D_f(p-q)\,D_\eta(q)}\langle\eta(q)\,f(P-q)\ldots\rangle$$

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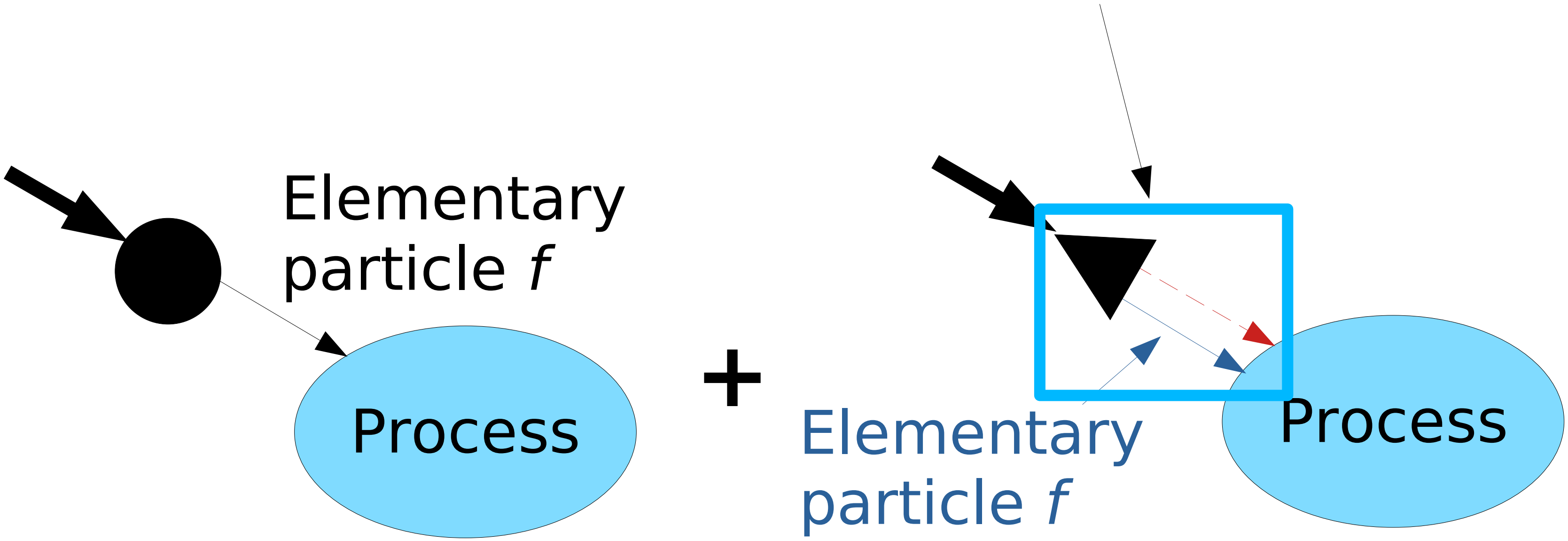

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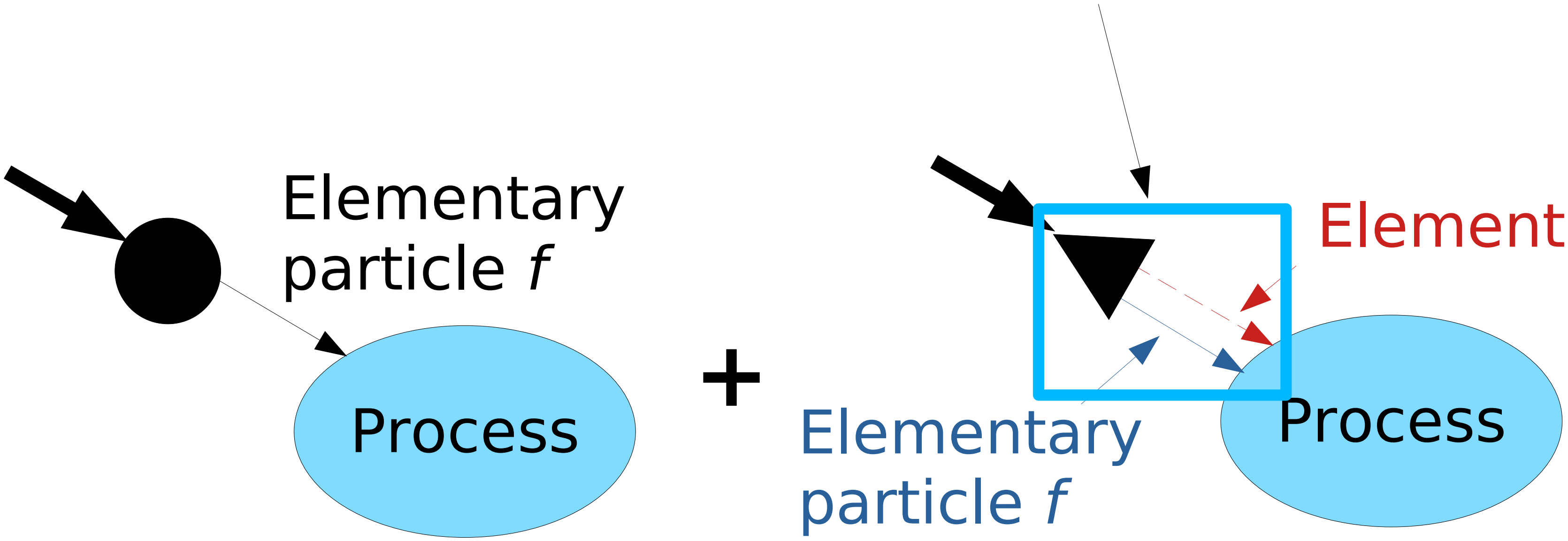

Elementary particle *f*

Process

+

Elementary Higgs

Elementary particle *f*

Process

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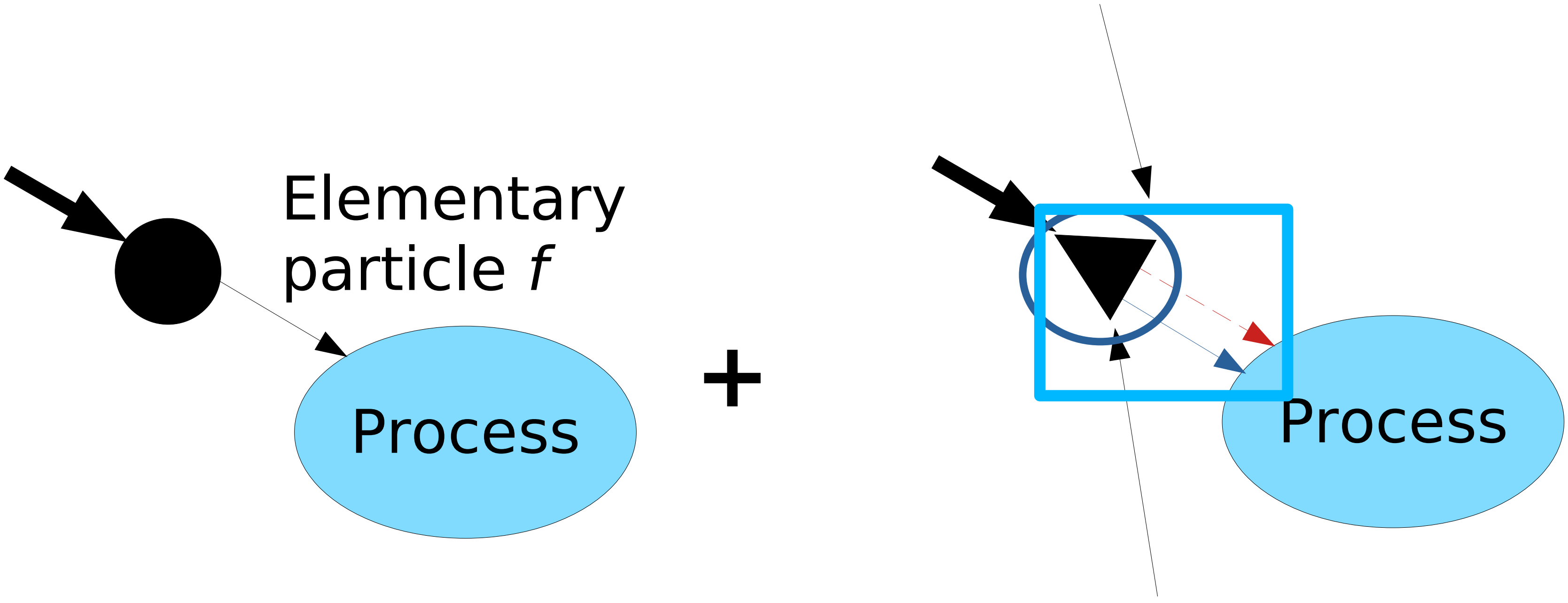

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3-point vertex

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# Phenomenological Implications
-
# Cross sections in the toy theory from the lattice

# Phenomenological Implications
# -
# Cross sections in the toy theory from the lattice
# or: How big is the Higgs?

# Elastic scattering in VBS

- Elastic region: $160/180\,GeV \leqslant \sqrt{s} \leqslant 250\,GeV$
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Scattering length~"size"

Phase shift

→ Lattice Lüscher analysis

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- Consider the Higgs: $J=0$

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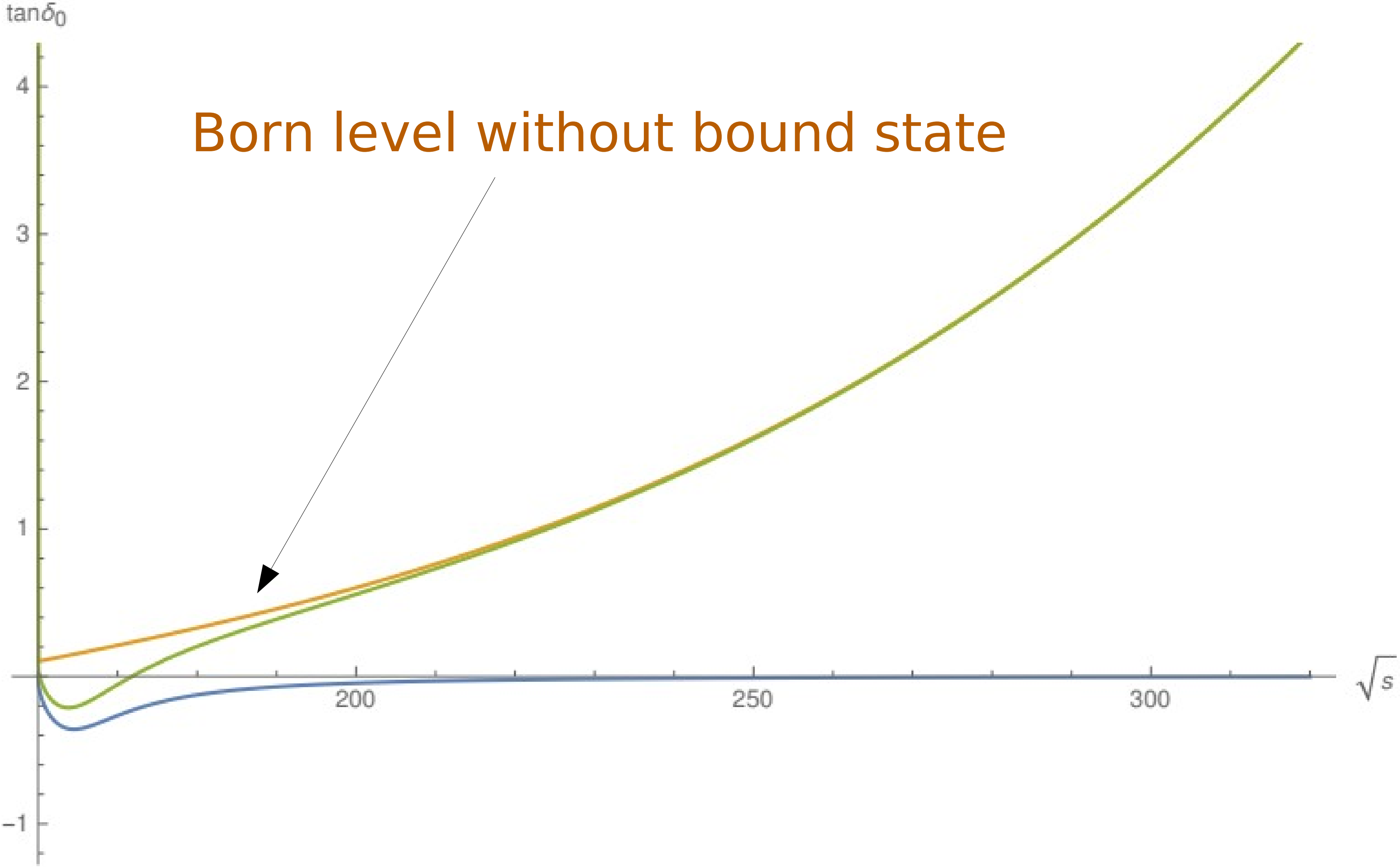

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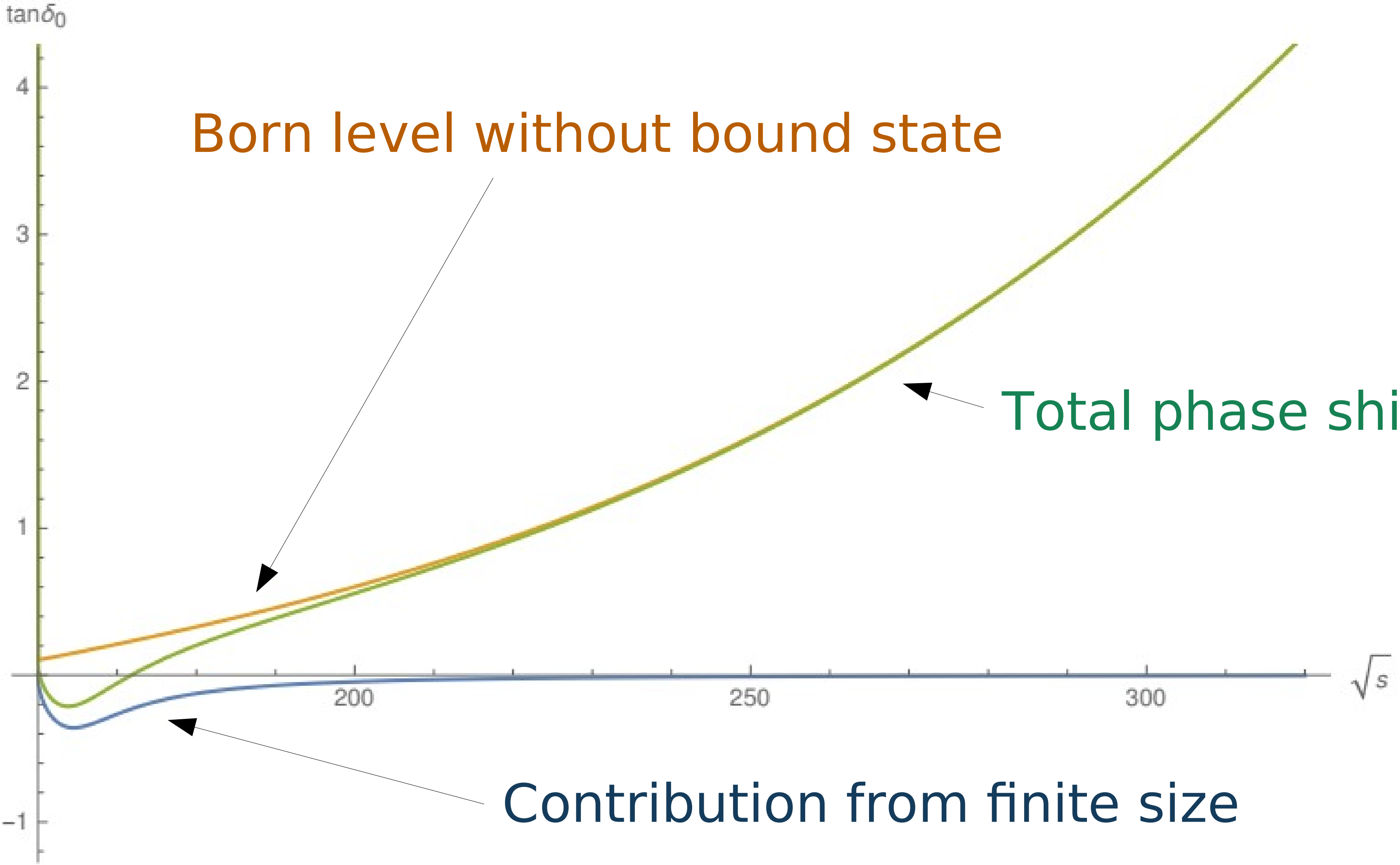

Born level without bound state

Total phase shift

Contribution from finite size

- Consider the Higgs: *J=0*
- Mock-up effect
  - Scattering length 1/(40 GeV)

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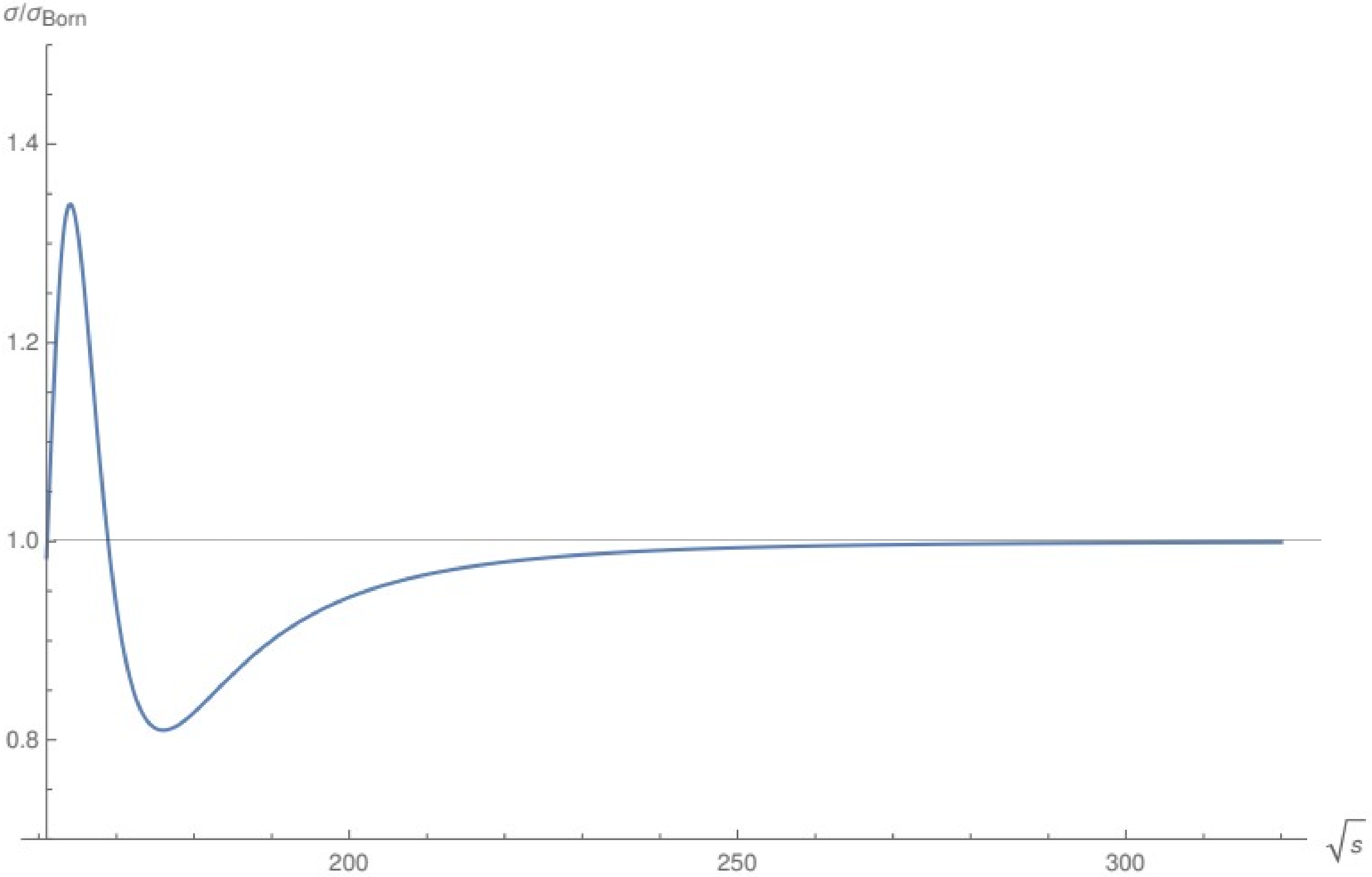

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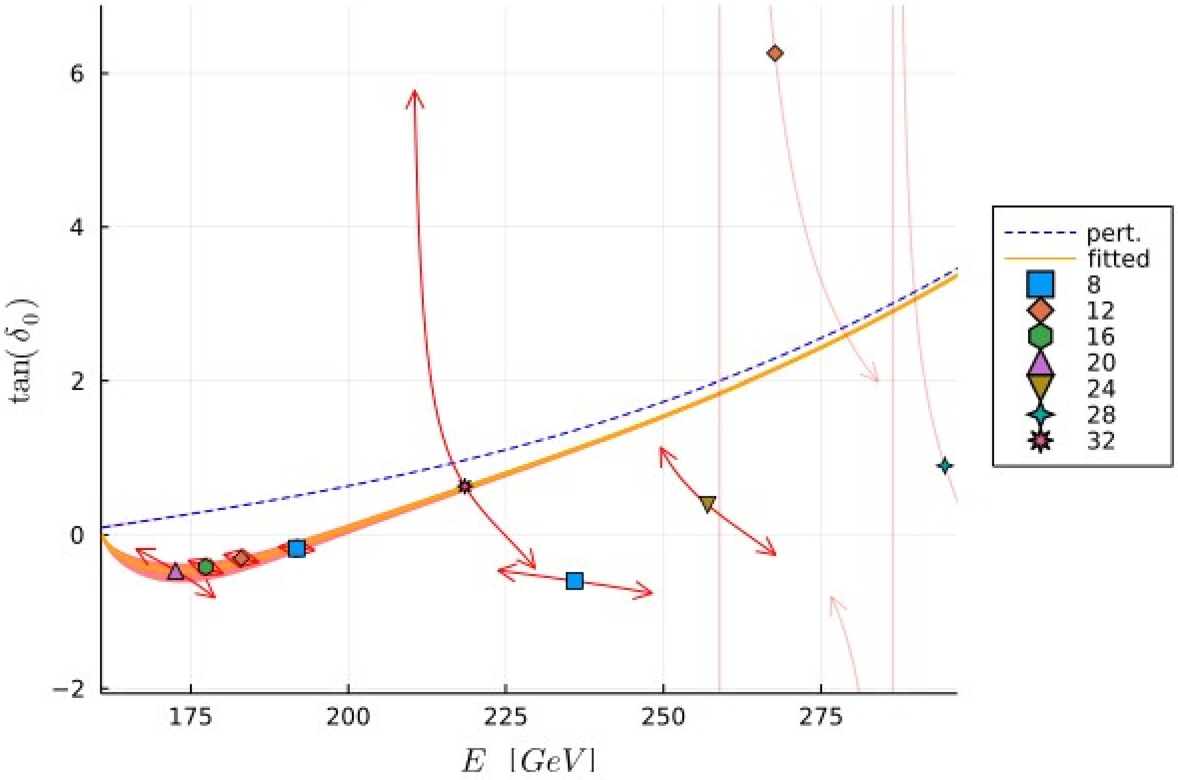

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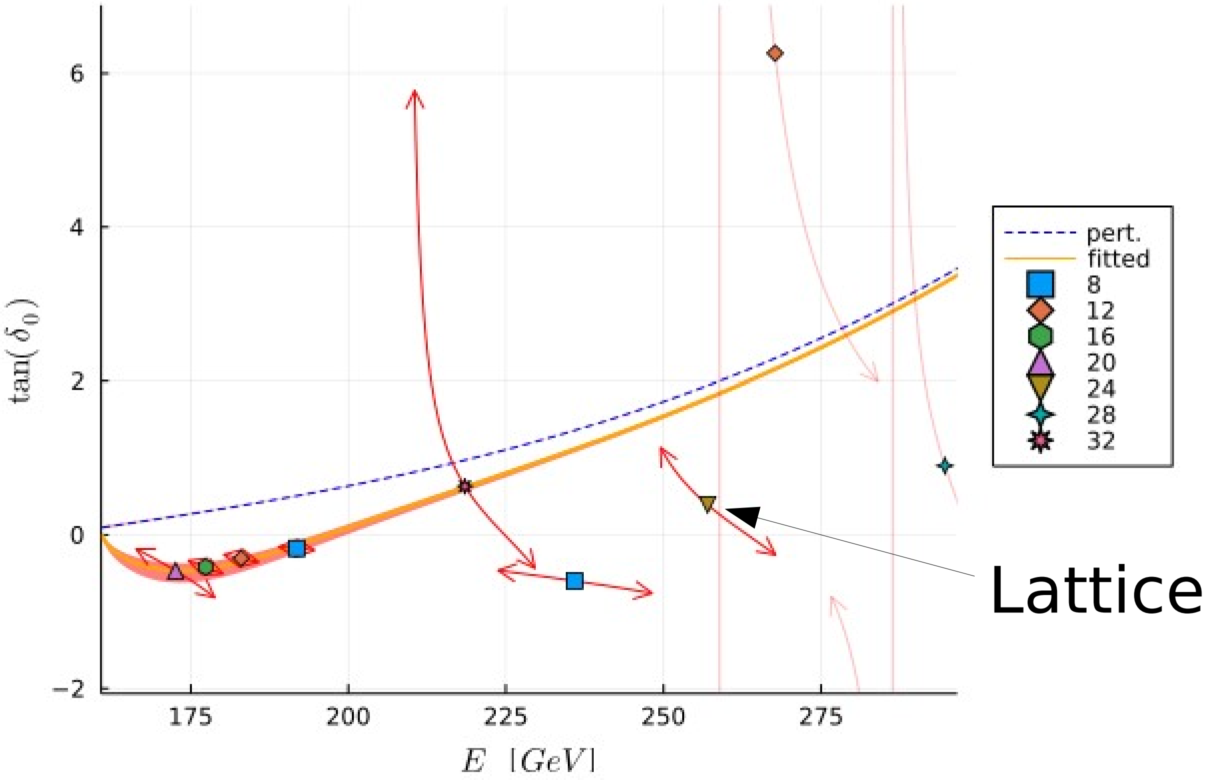

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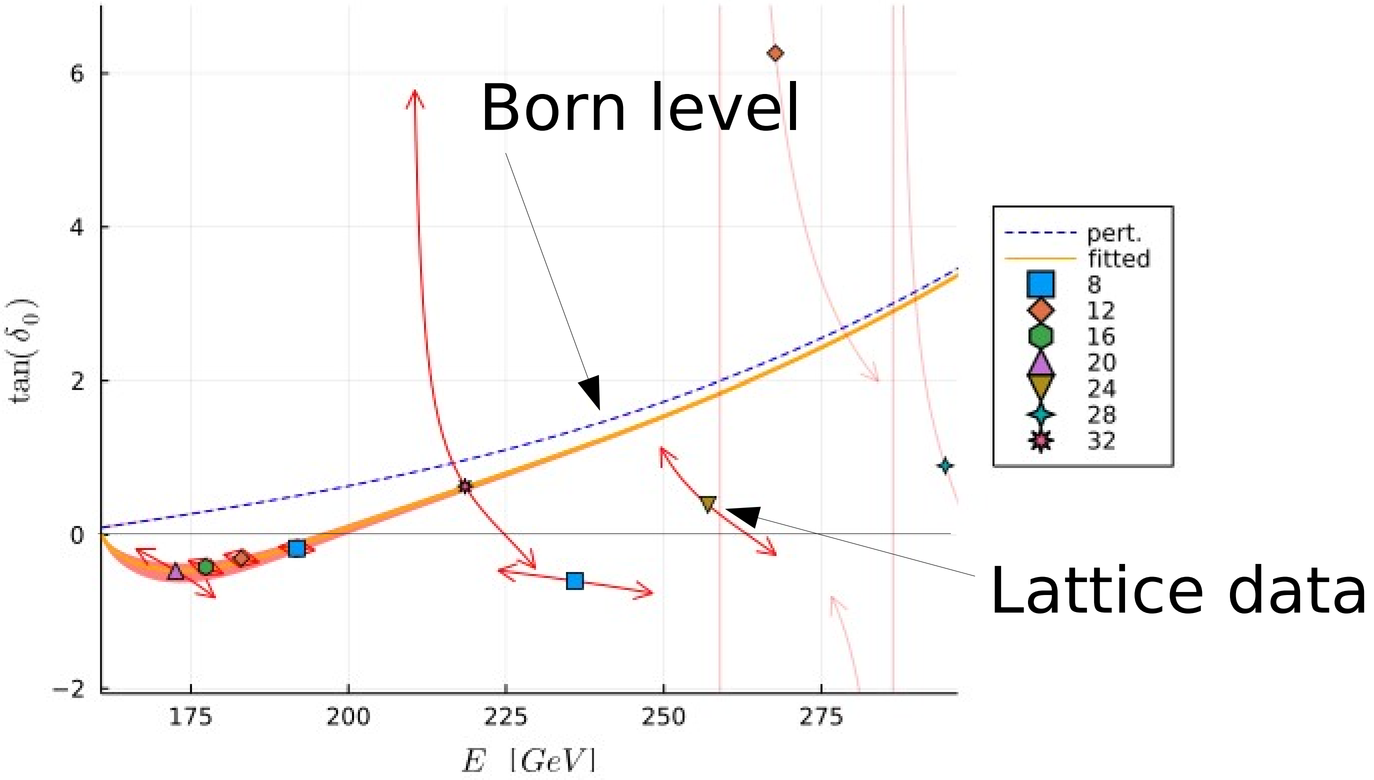

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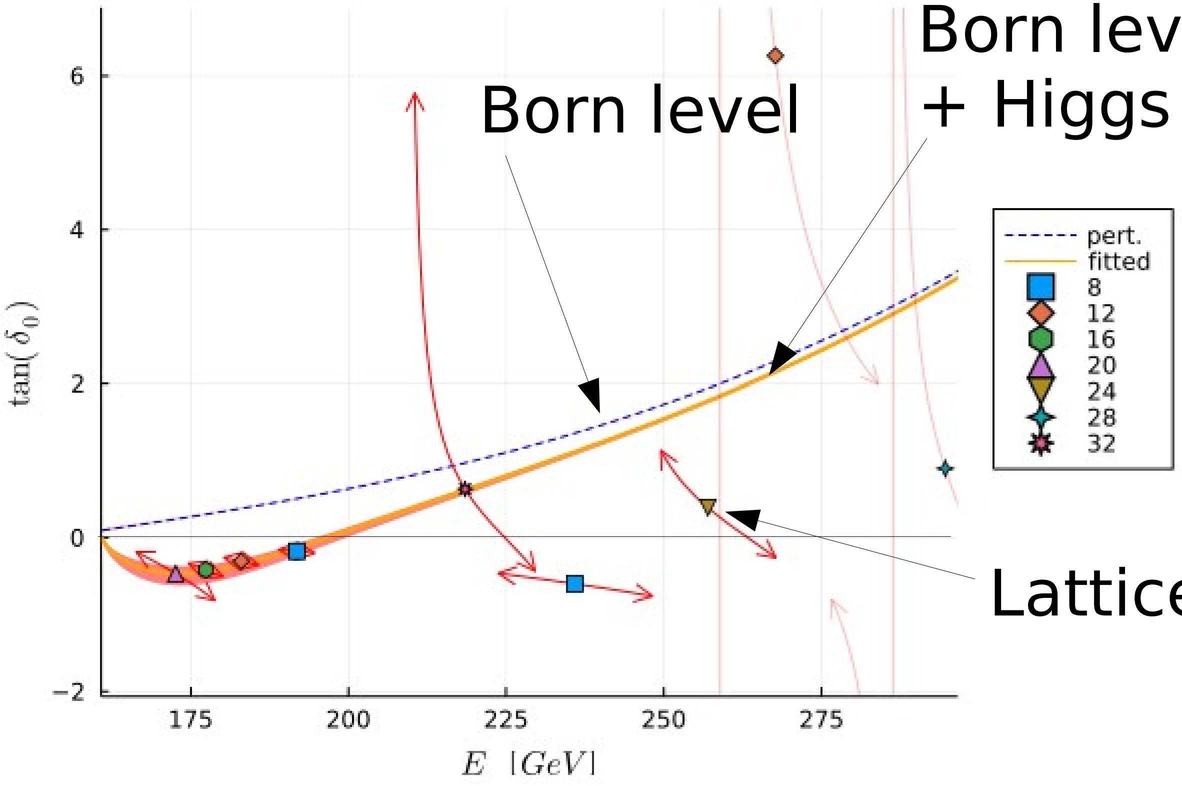

Born level

Born level FMS
+ Higgs size of (40 Gev)$^{-1}$

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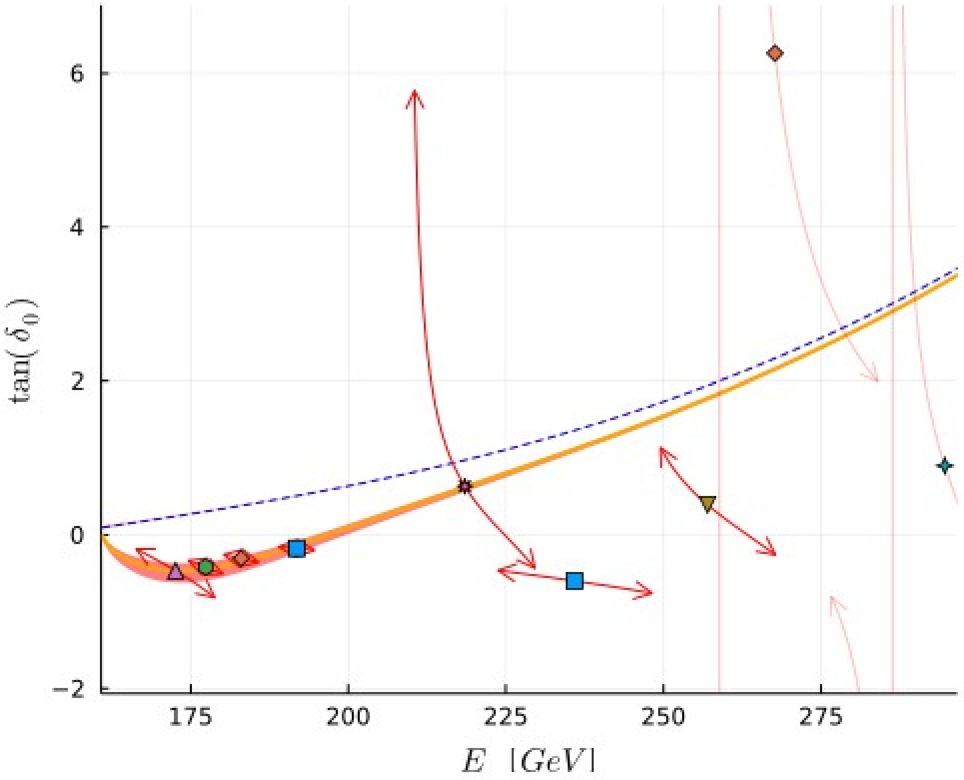

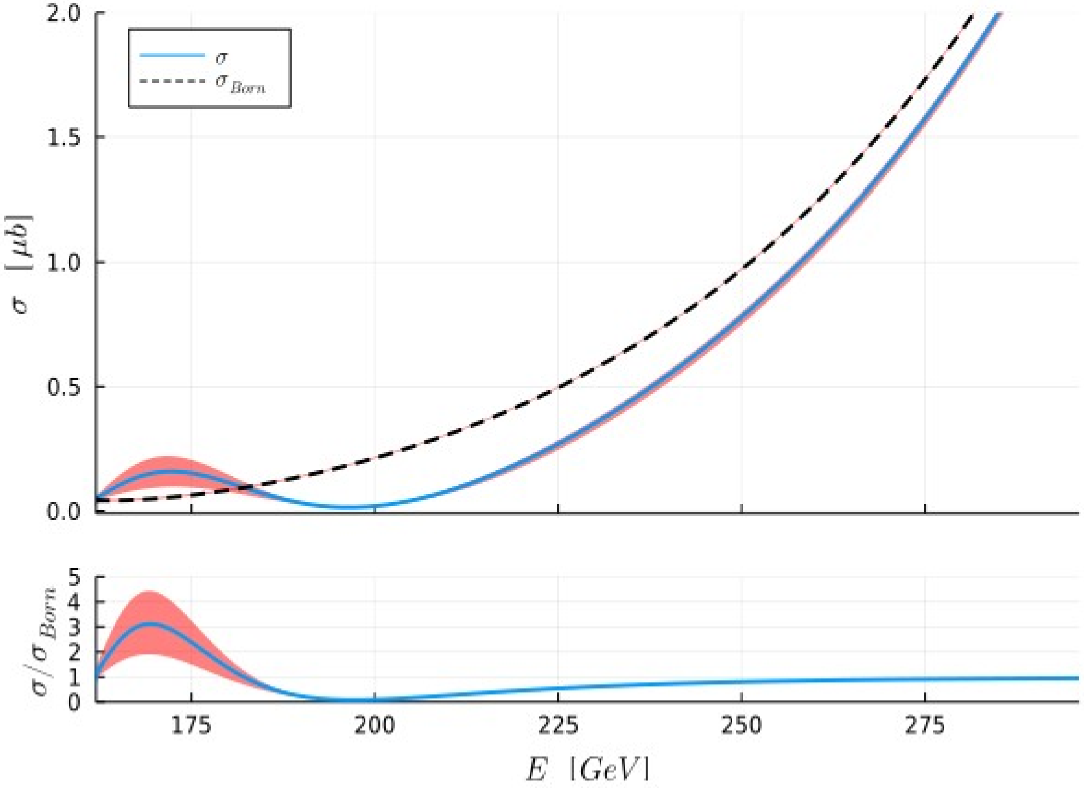

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# Phenomenological Implications
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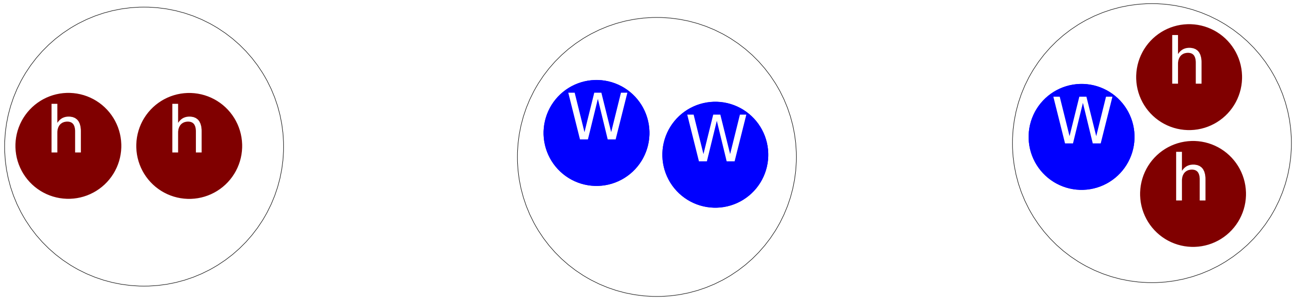
# Adding matter

# A subtelty with impact

- BEH effect does not actually break gauge symmetry [Elitzur’75, Osterwalder & Seiler’77, Fradkin & Shenker’78]
  - A particularilly suitable gauge fixing
- Asymptotic states still need to be gauge-invariant
- Elementary/non-interacting states are not
  - Topologically obstructed [Gribov’78,Singer’78,Fujikawa’82]
  - Dressing by a Wilson line is insufficient [Lavelle, McMullan’97]
  - Only local composite operators can be [Fröhlich et al.’80,’81]
    - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



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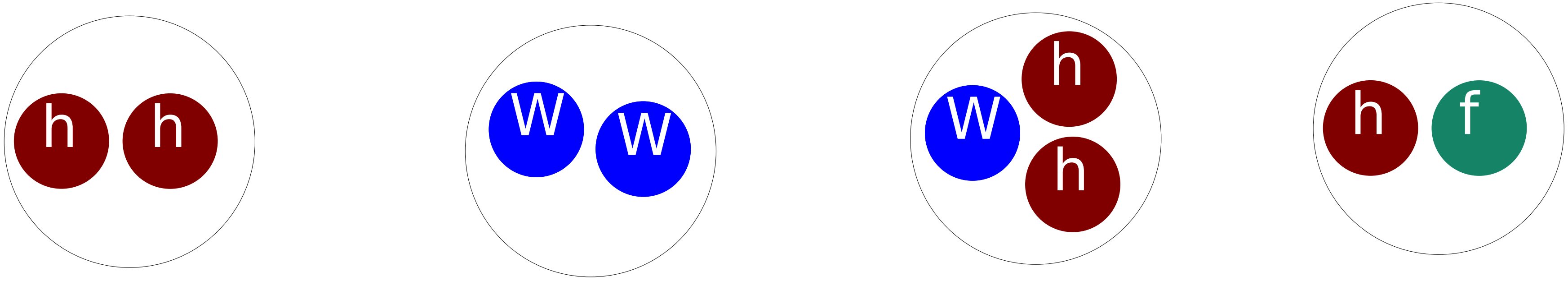

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[Fröhlich et al.'80,
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- (Left-handed) Flavor has two components
    - Global SU(3) generation
    - Local SU(2) weak gauge (up/down distinction)

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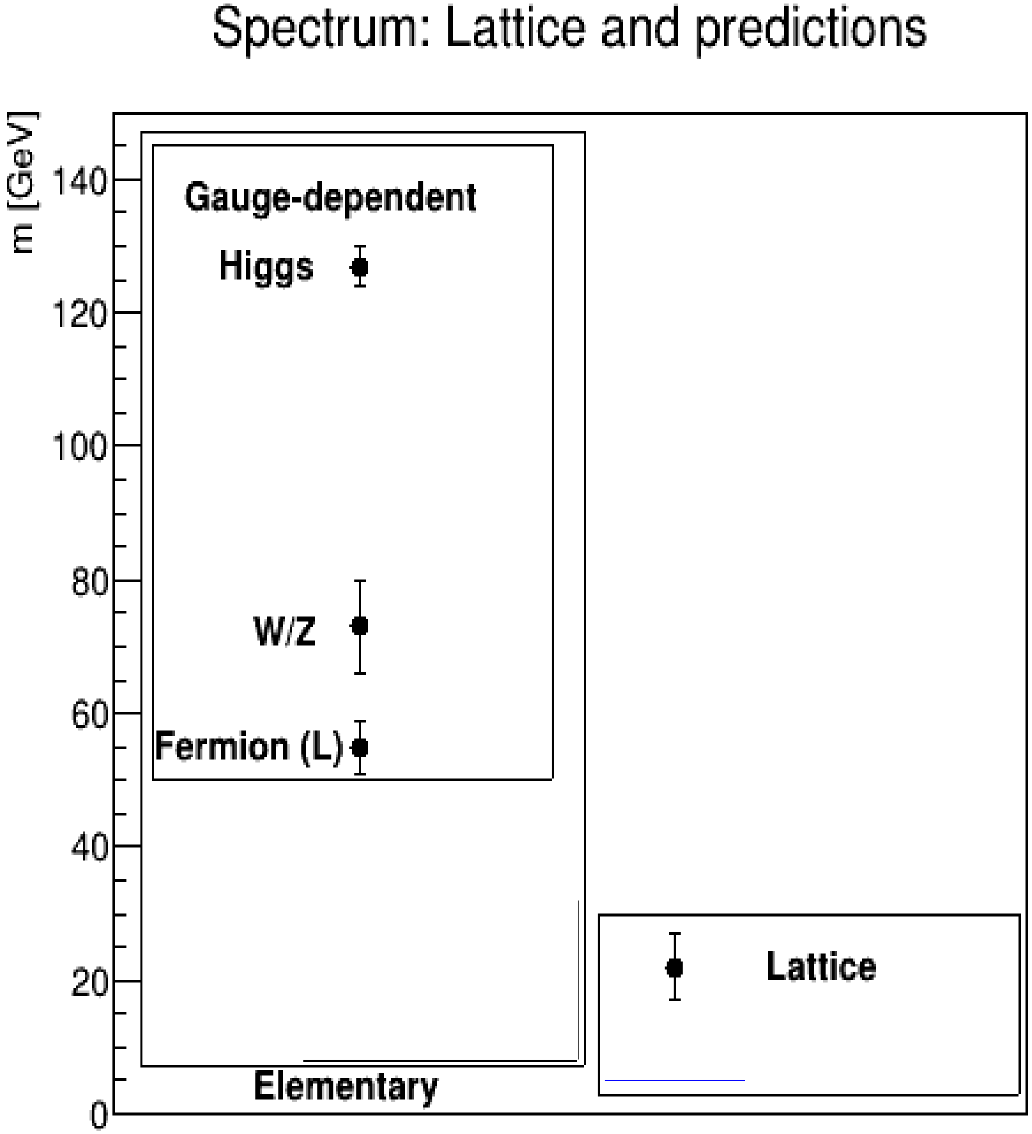

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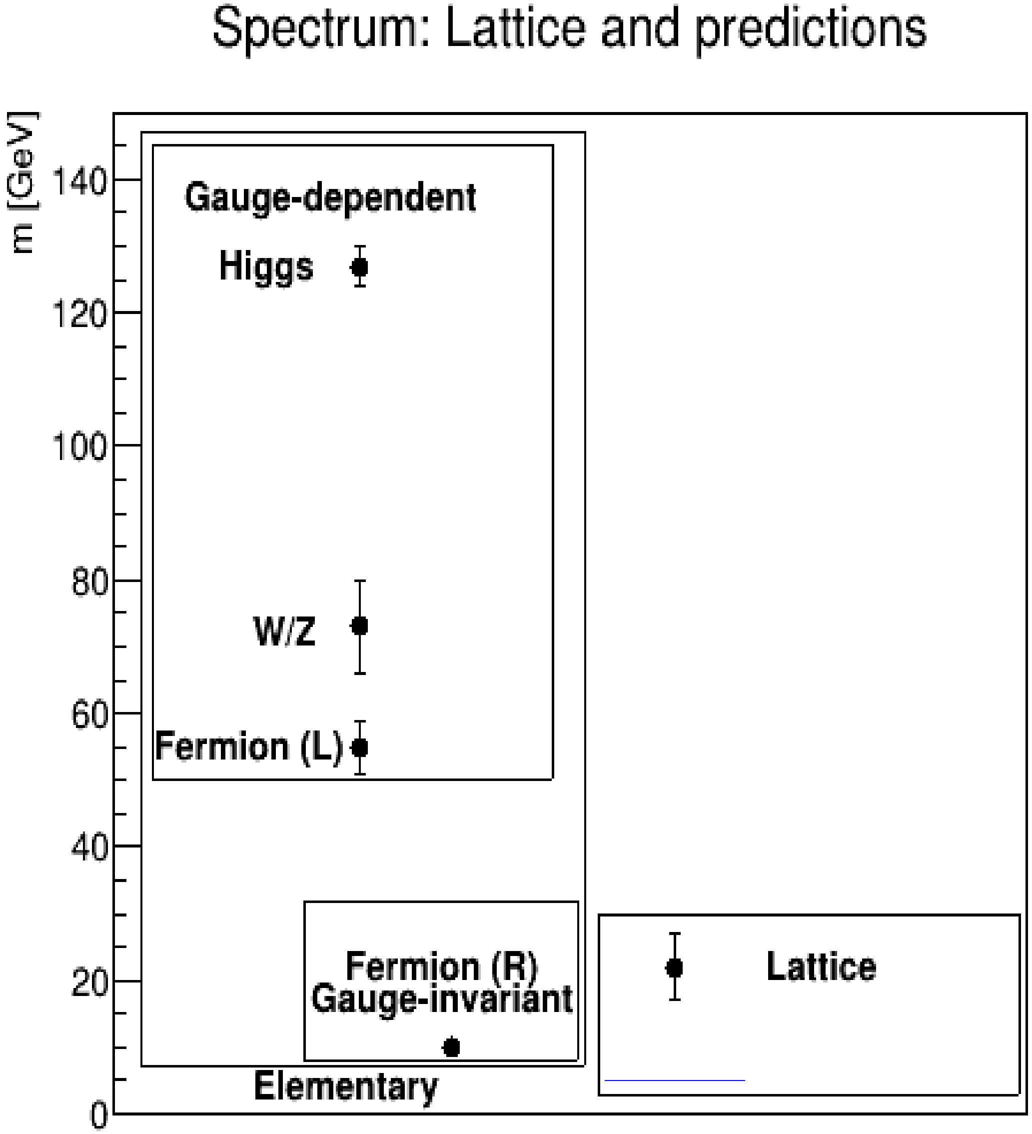

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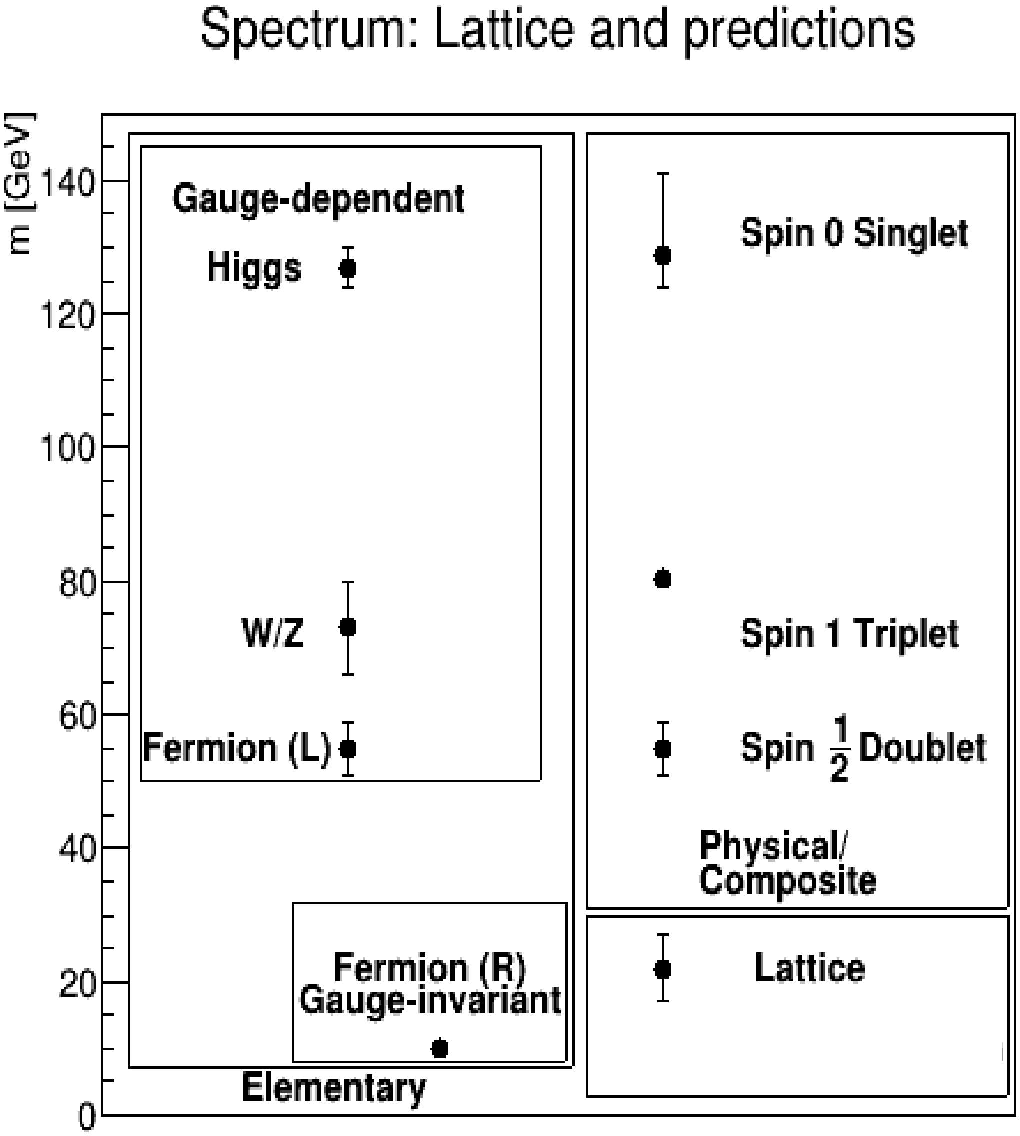
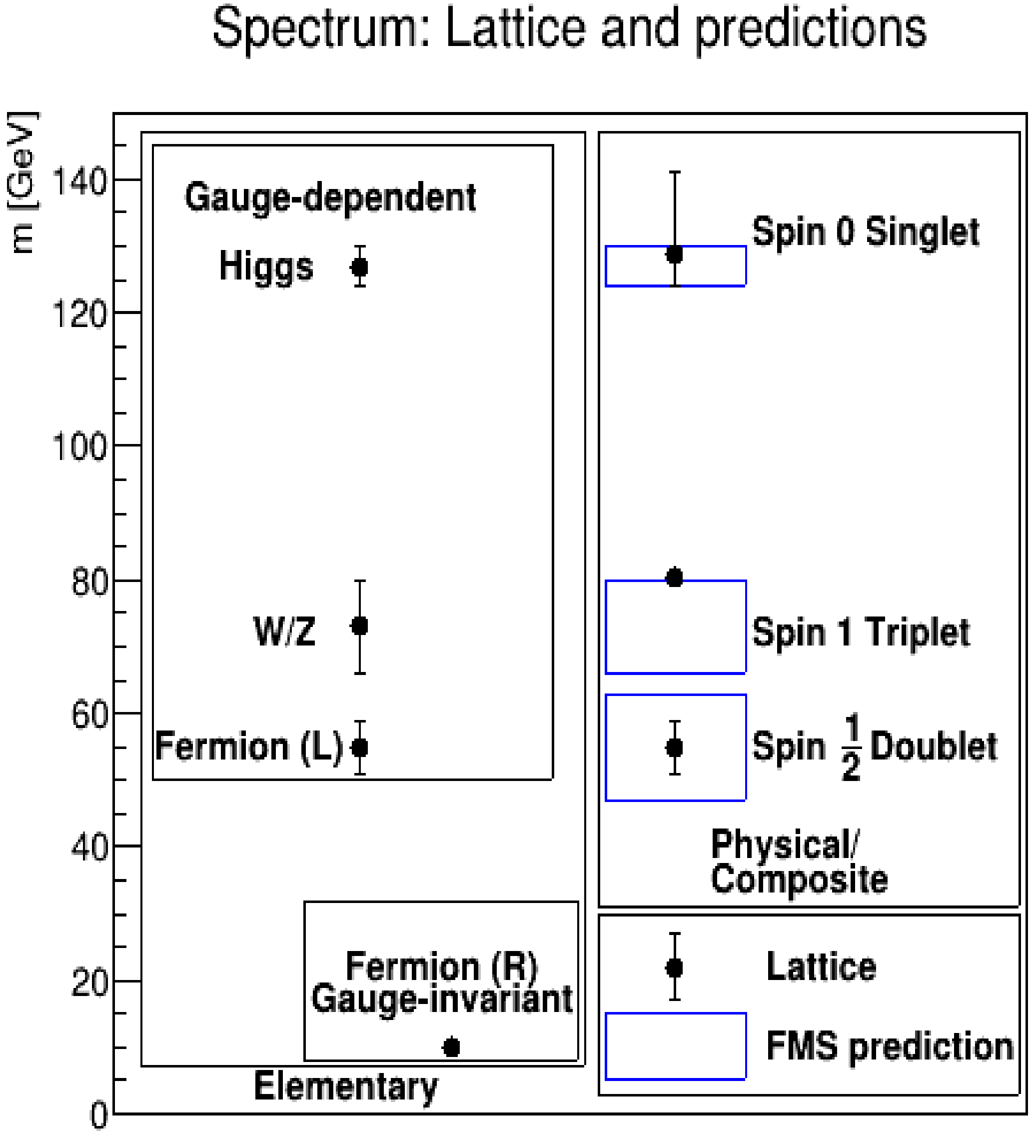


# Flavor on the lattice

[Afferrante,Maas,Sondenheimer,Törek’20]

- Only mock-up standard model
  - Compressed mass scales
  - One generation
  - Degenerate leptons and neutrinos
  - Dirac fermions: left/right-handed non-degenerate
  - Quenched
- Same qualitative outcome
  - FMS construction
  - Mass defect
  - Flavor and custodial symmetry patterns
- Supports FMS prediction



# Flavor on the lattice

[Afferrante,Maas,Sondenheimer,Török'20
Wieland & Maas'25]

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  - One generation
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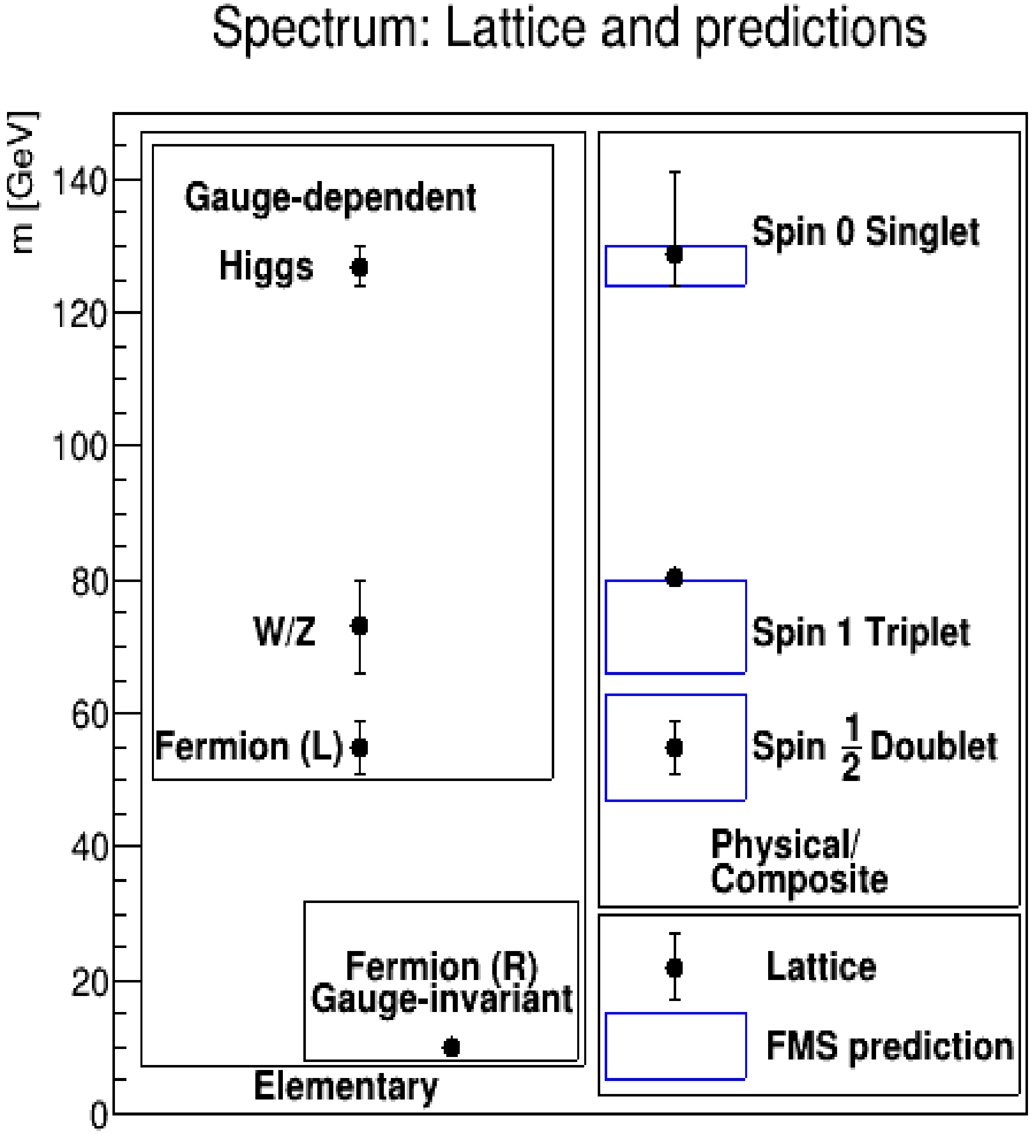
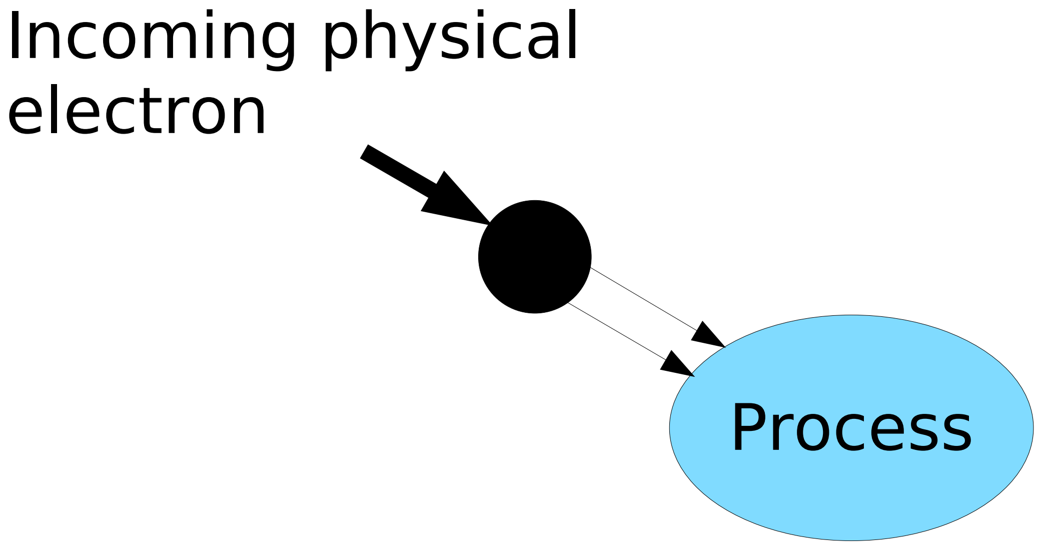
- Same qualitative outcome
  - FMS construction
  - Mass defect
  - Flavor and custodial symmetry patterns
- Supports FMS prediction – unquenching ongoing



# Impact at fixed order

[Maas et al.'17
Maas & Reiner '22
Maas, Plätzer et al.'24, unpublished]



Incoming physical electron

Process

# Impact at fixed order

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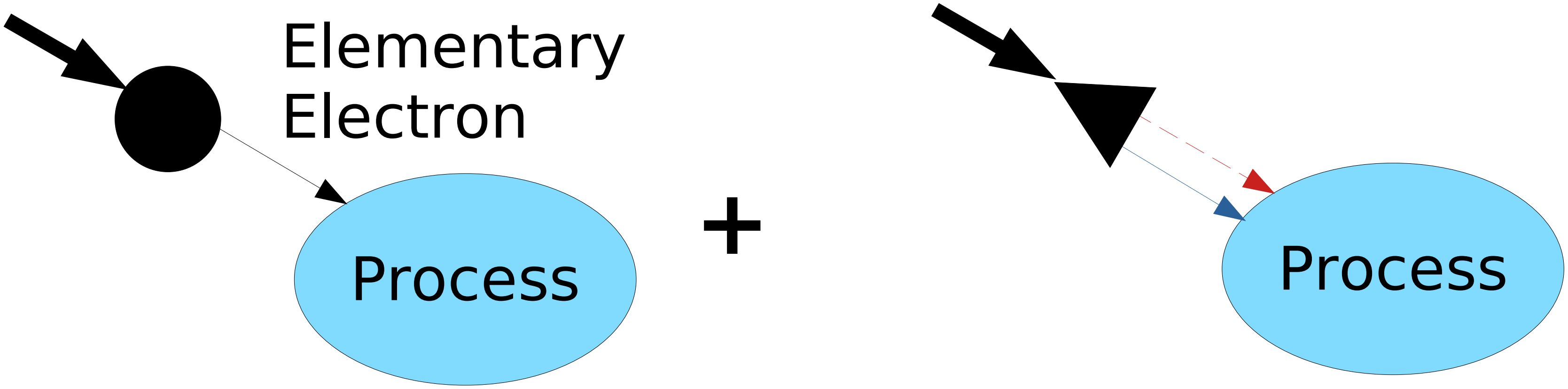

Elementary
Electron

Process

+

Process

$$v\langle f(p)\dots\rangle + \int dq\,\Gamma(P,q)\,D_f(p-q)\,D_\eta(q)\langle \eta(q) f(P-q)\dots\rangle$$

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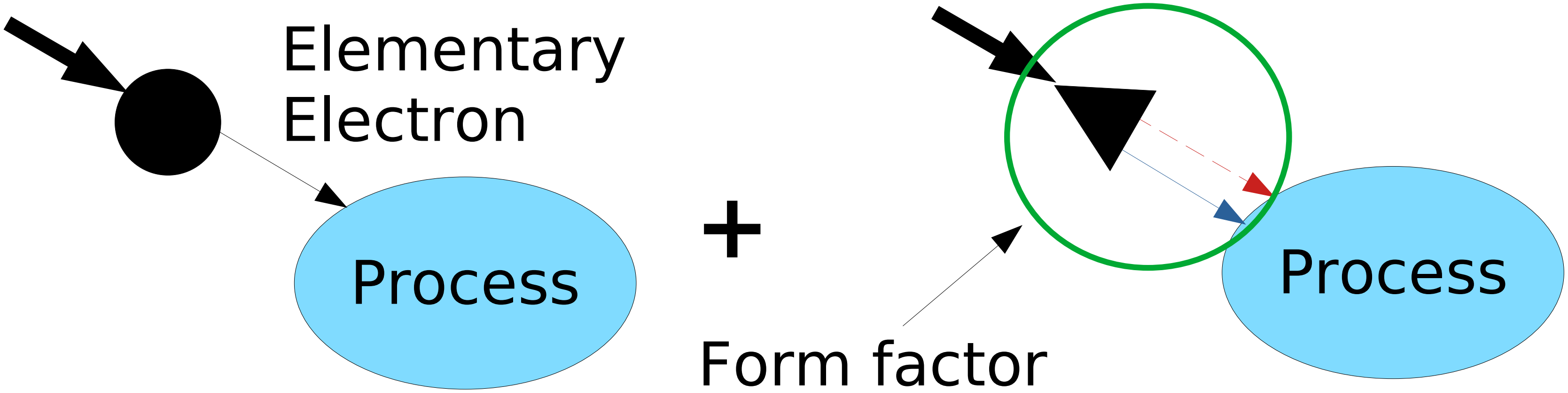

Elementary
Electron

Process

+

Form factor

Process

$$v\langle f(p)\ldots\rangle+\int dq\,\Gamma(P,q)\,D_f(p-q)\,D_\eta(q)\langle\eta(q)f(P-q)\ldots\rangle$$

$$=v\langle f(p)\ldots\rangle+\boxed{\int dq\,G(q)}\langle\eta(q)f(P-q)\ldots\rangle$$

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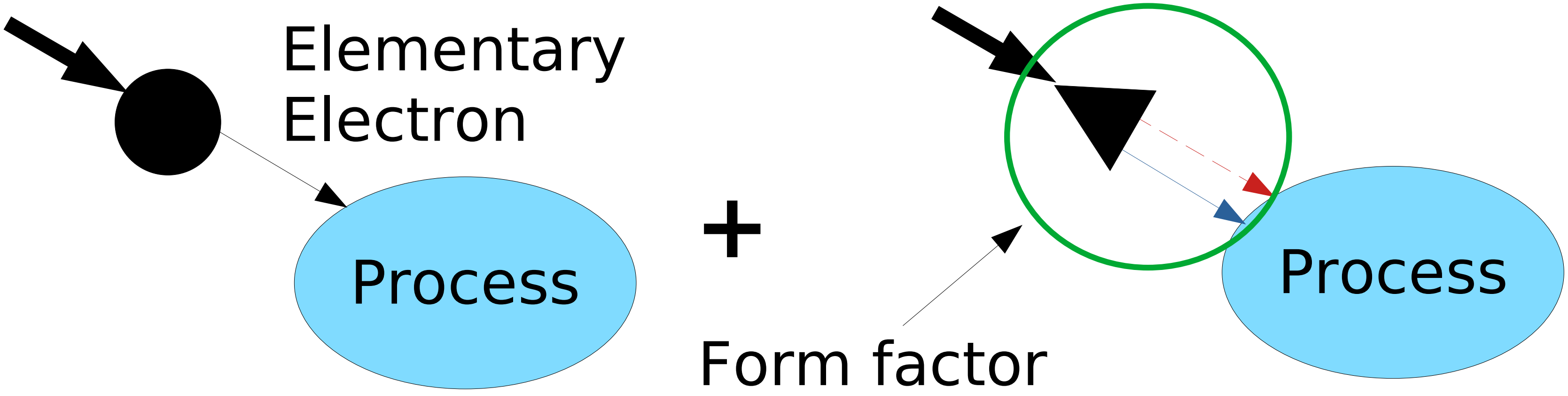

Elementary
Electron

Process

+

Process

Form factor

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Ordinary Feynman diagrams

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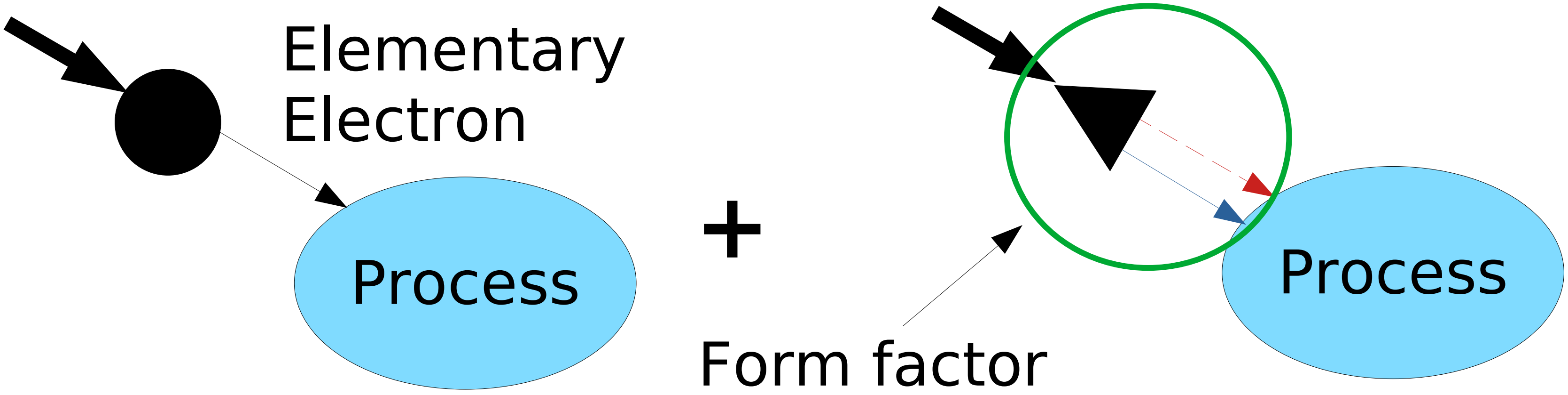

Elementary
Electron

Process

+

Process

Form factor

$$v\langle f(p)\dots\rangle+\int dq\,\Gamma(P,q)D_f(p-q)D_\eta(q)\langle\eta(q)f(P-q)\dots\rangle$$

$$=v\langle f(p)\dots\rangle+\int dq\,G(q)\langle\eta(q)f(P-q)\dots\rangle$$

Ordinary Feynman diagrams
May appear also here: Different weight

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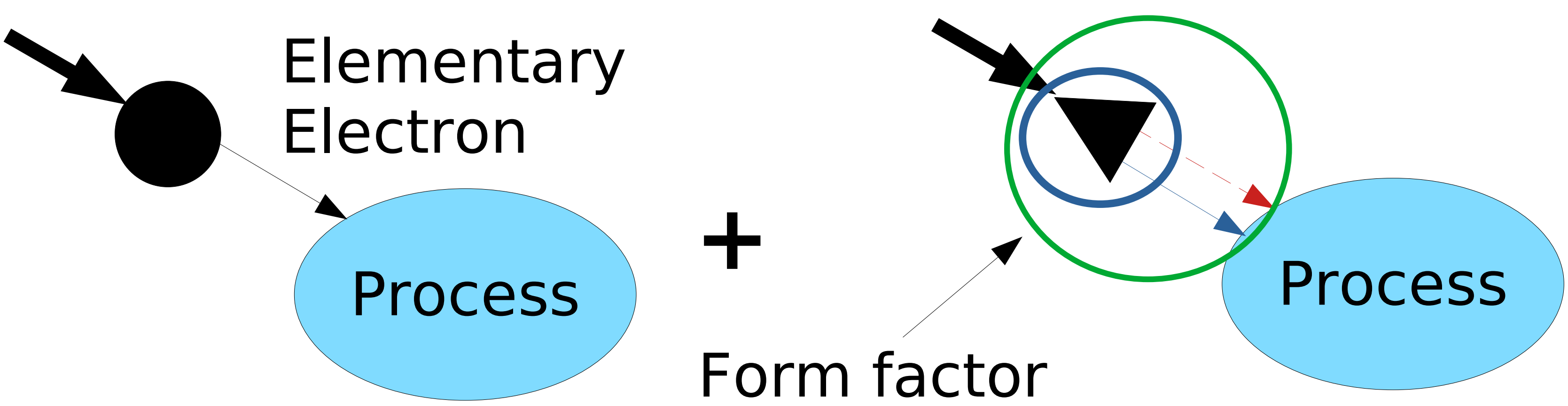

Elementary
Electron

Process

+

Form factor

Process

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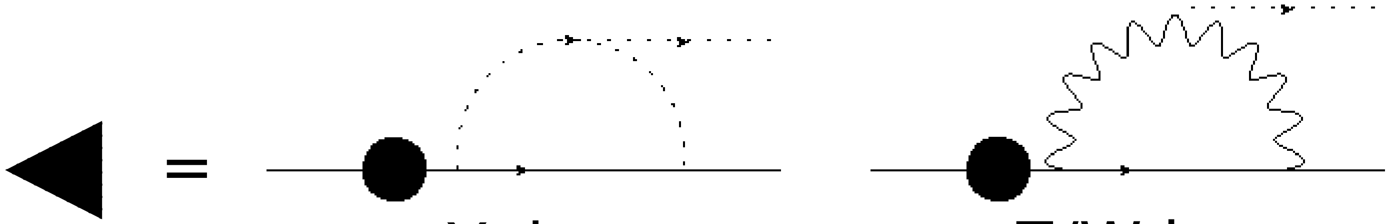

Yukawa

Z/W boson

NLO EW

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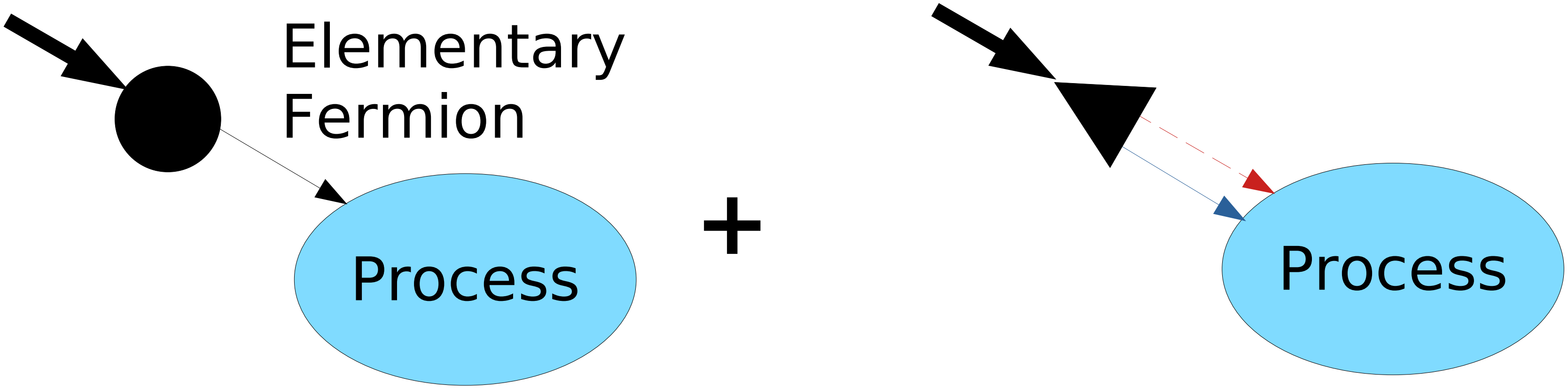

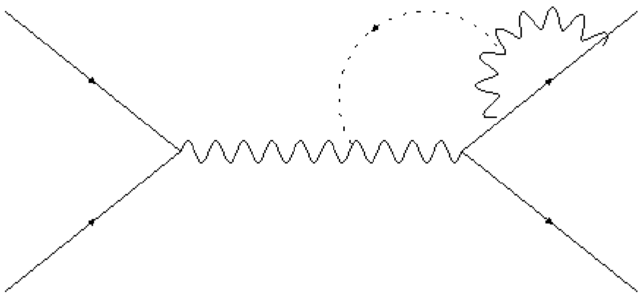

Process ff → ff: 2-loop suppressed contribution

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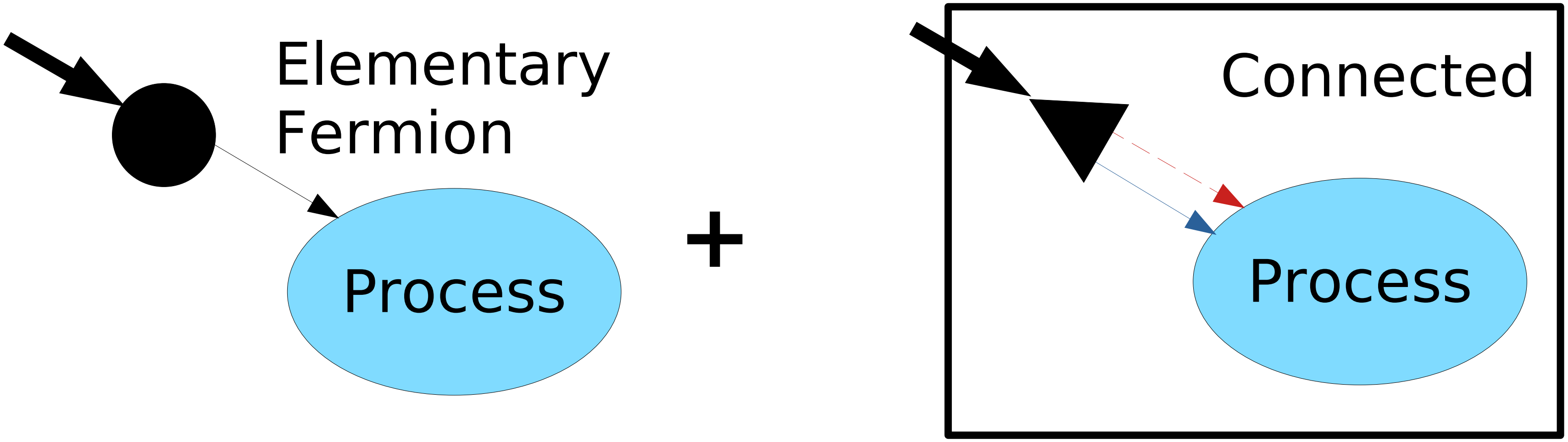

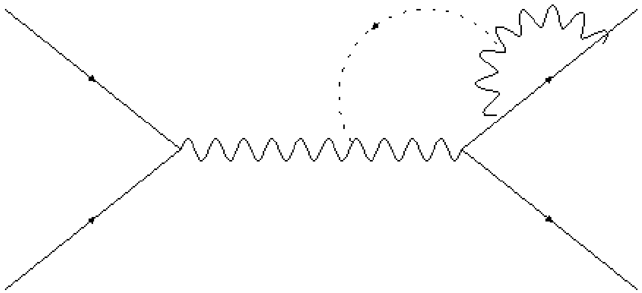

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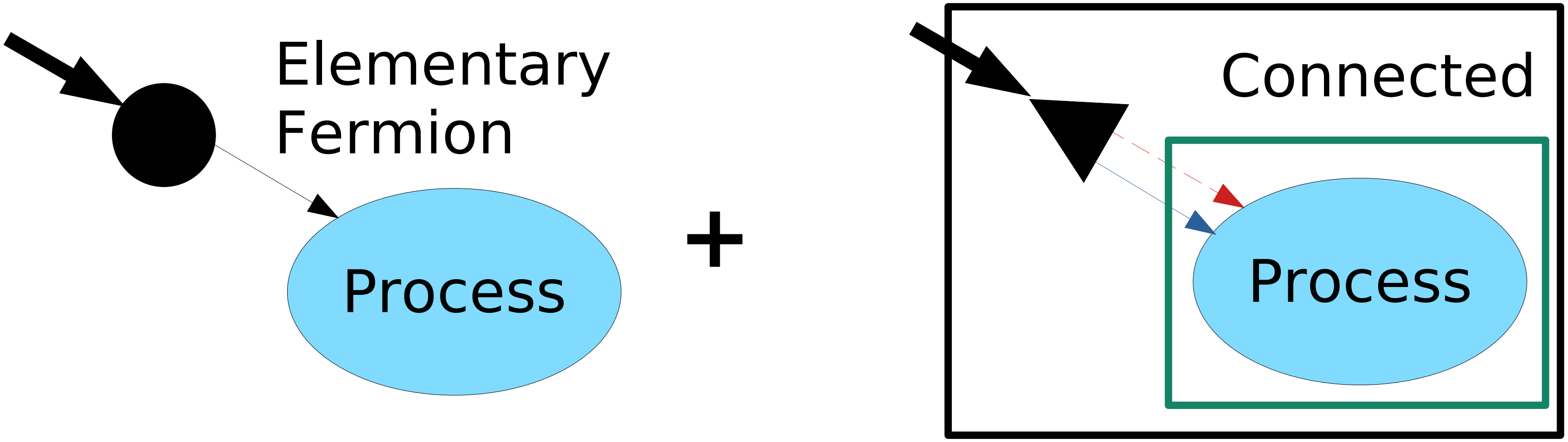


Elementary
Fermion

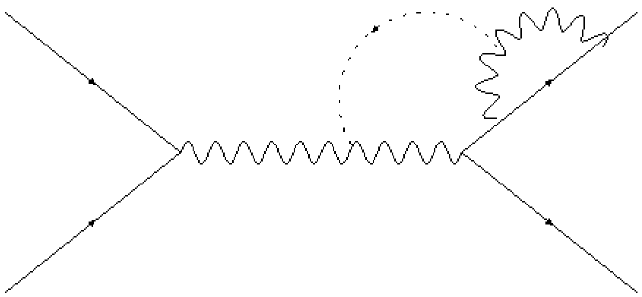
Process

+

Connected

Process

Need not to be connected



Process ff → ff: 2-loop
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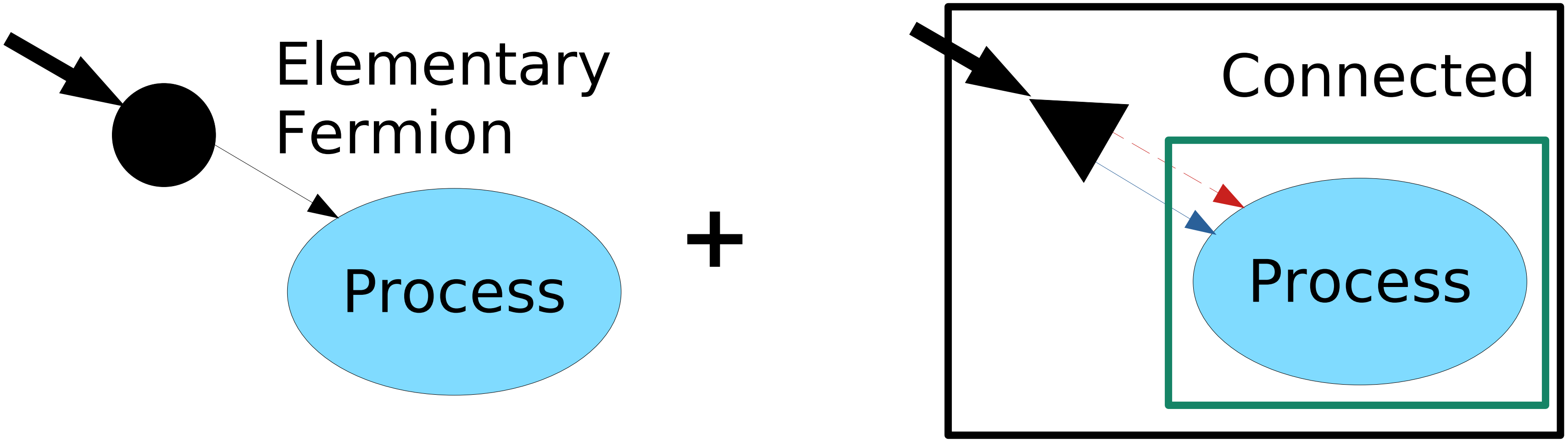
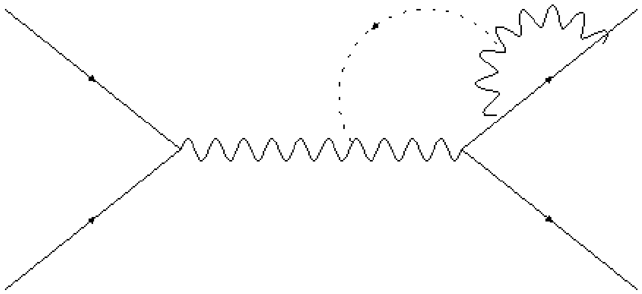
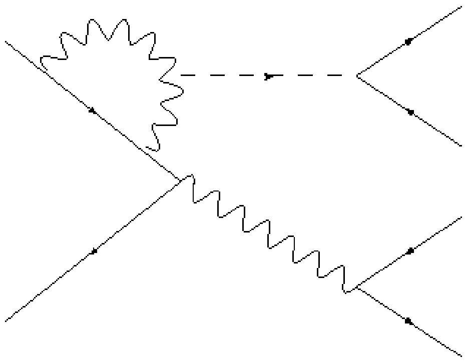


Need not to be connected



Process ff → ff: 2-loop
suppressed contribution



Process ff → ffff 1-loop
suppressed contribution

# Resummation effects at $s \gg M_Z$

[Ciafaloni et al. ‘00]

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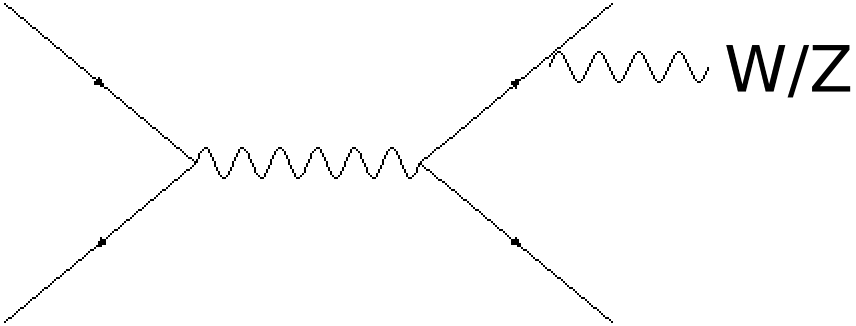
Standard perturbation theory



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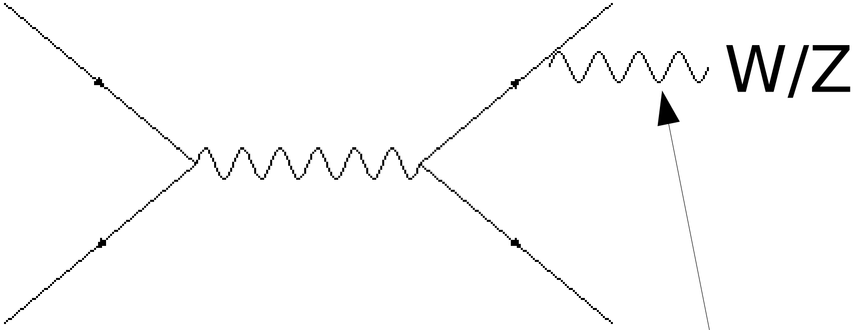

Resumming real emission

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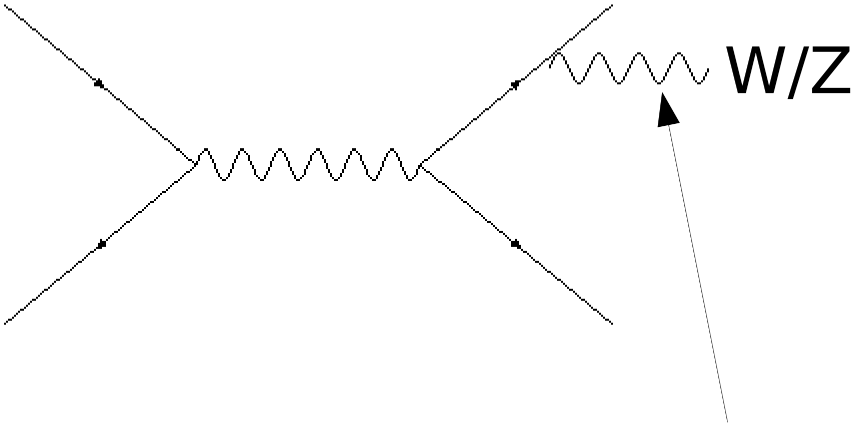

Resumming real (almost collinear) emission: Weak jet

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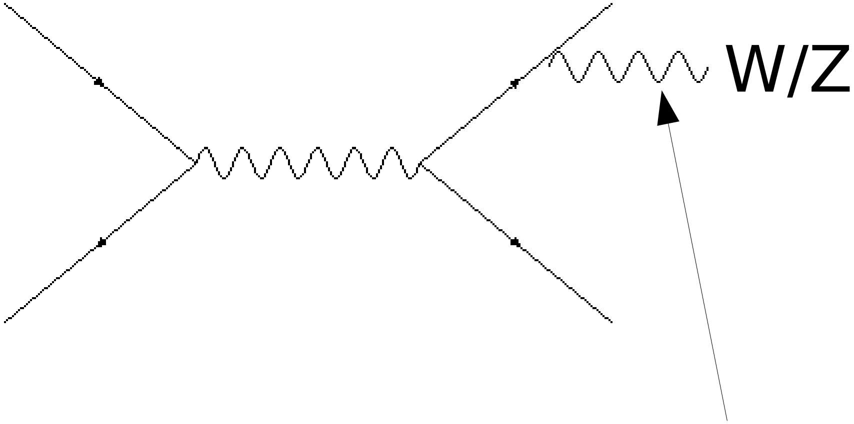

Resumming real (almost collinear) emission: Weak jet

$$\frac{\Delta\sigma}{\sigma_{no\,resummation}} \sim \ln^2 \frac{s}{m_W^2}$$

at 1 TeV of the same order as strong corrections

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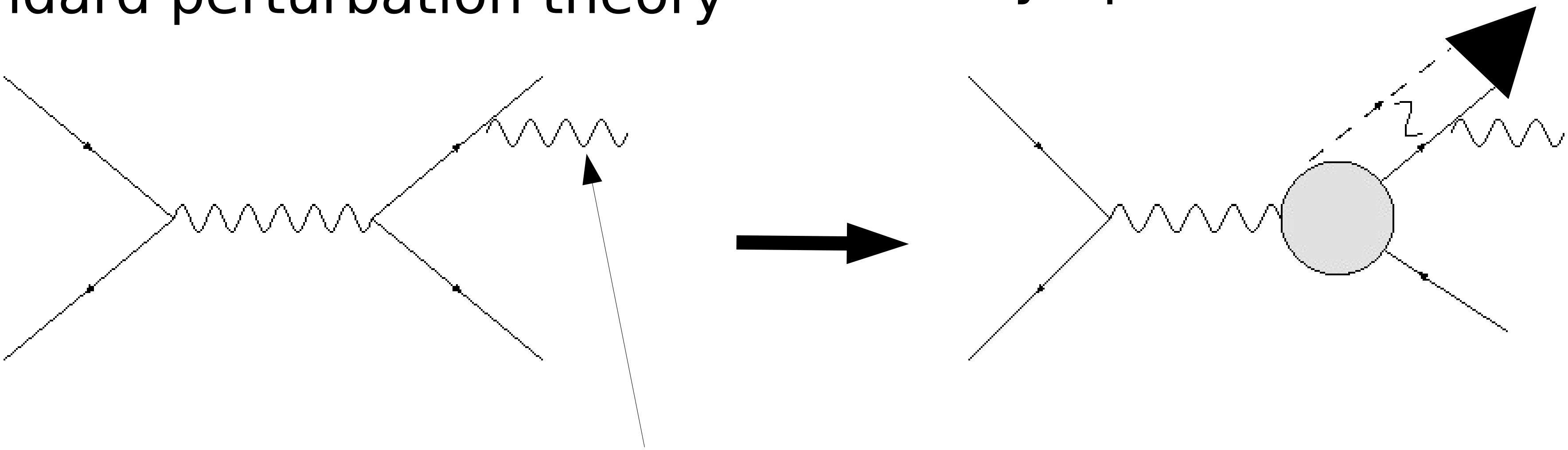
Standard perturbation theory

Augmented by correct asymptotic state



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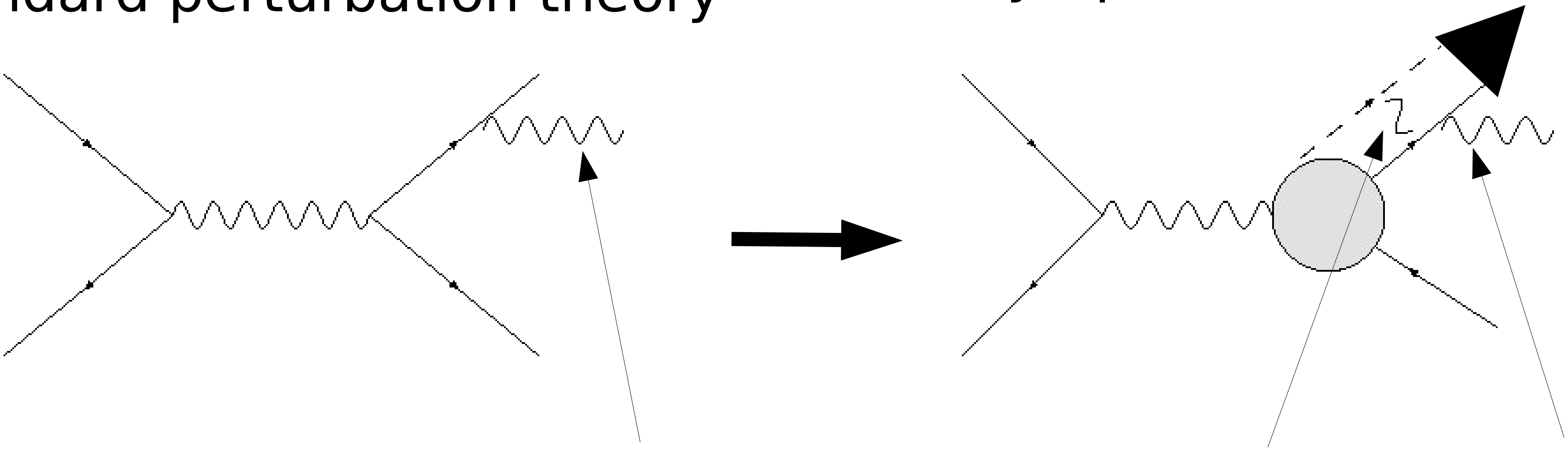

Resumming real emission

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Virtual and real emissions compensate (BN/KLN theorems)

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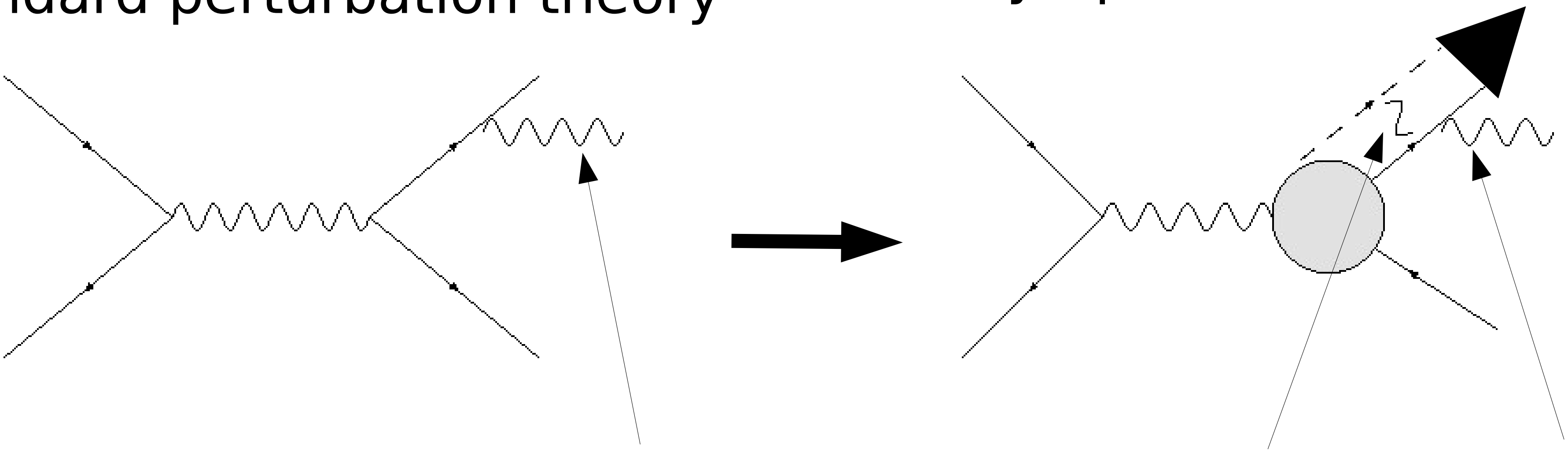

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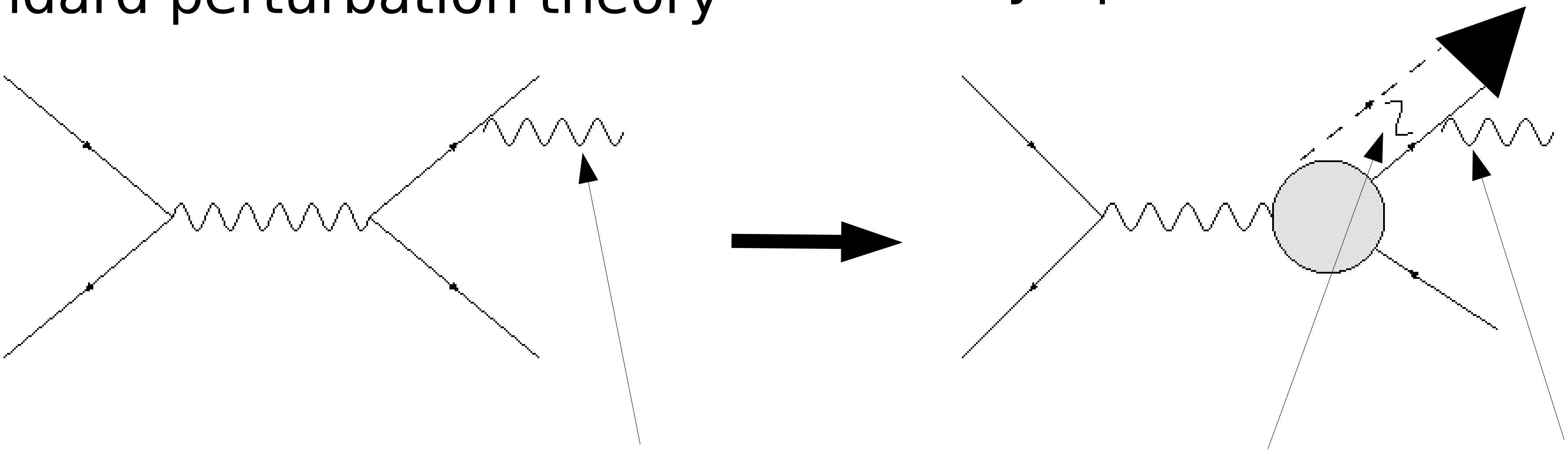

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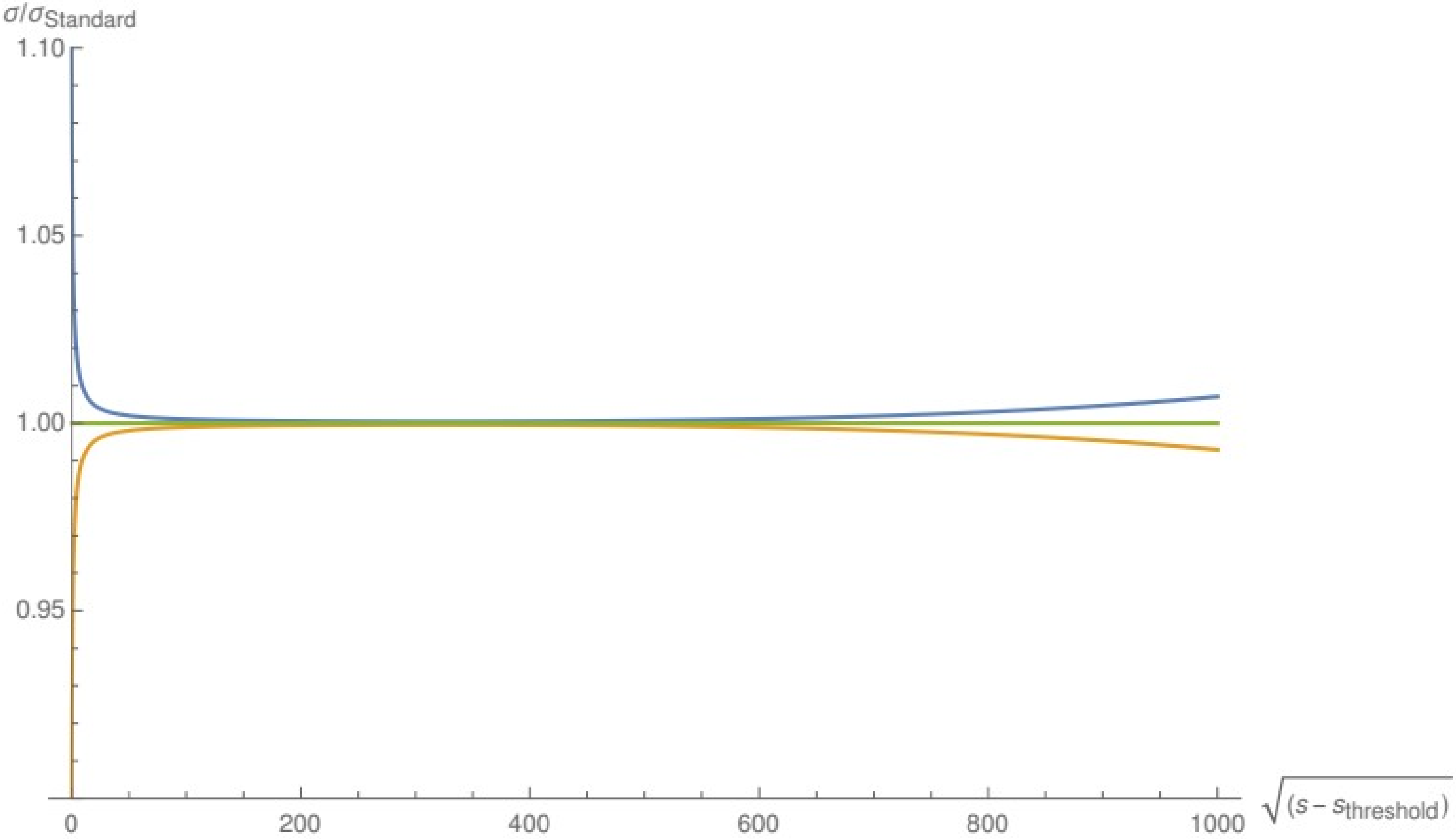
at 1 TeV of the same order as strong corrections

Virtual and real emissions compensate (BN/KLN theorems) - substantial change: Cancel this effect: Less weak jets

# Generic behavior

Has been done for several observables

# Generic behavior: DIS-like



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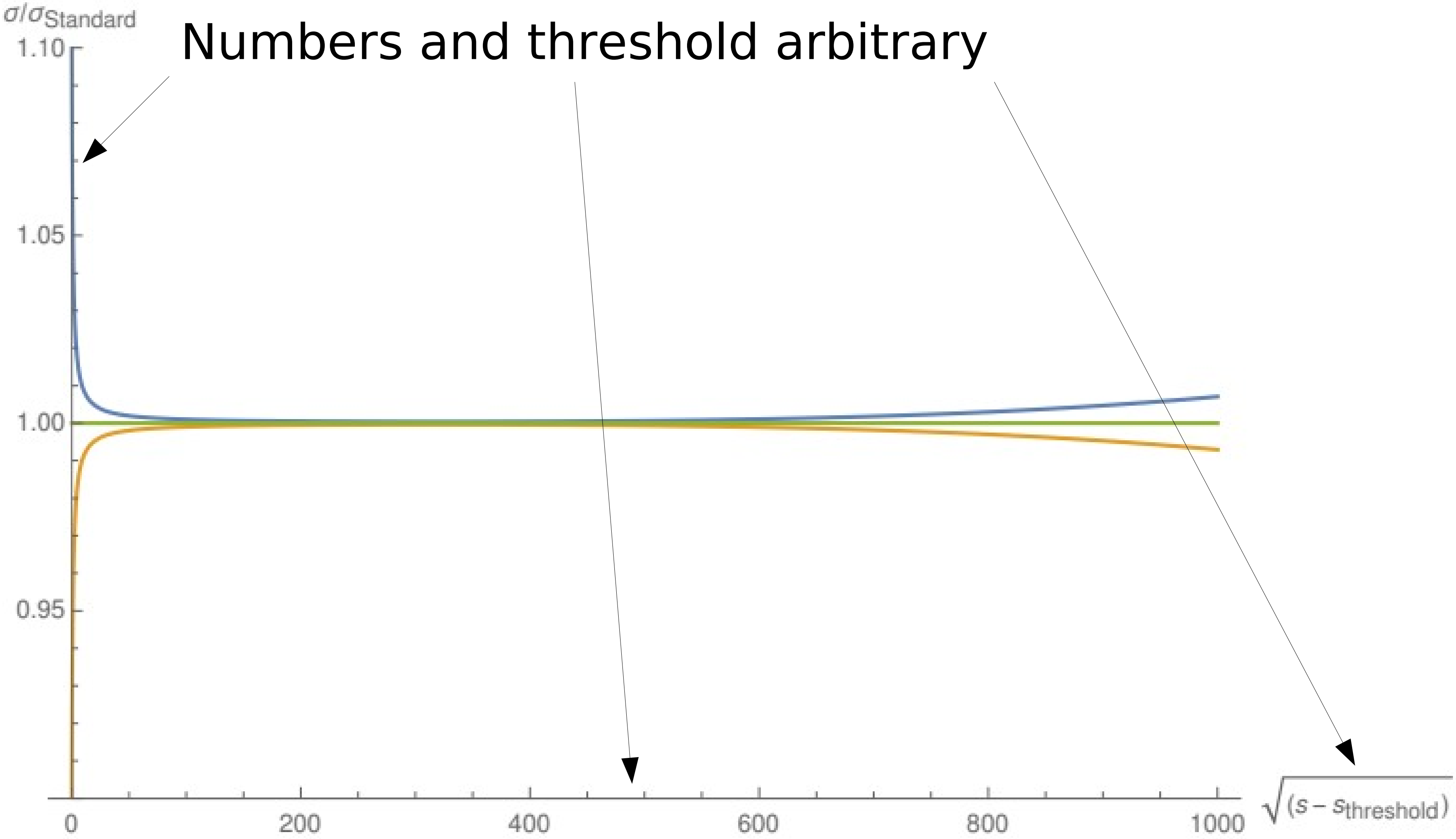

Numbers and threshold arbitrary

Has been done for several observables

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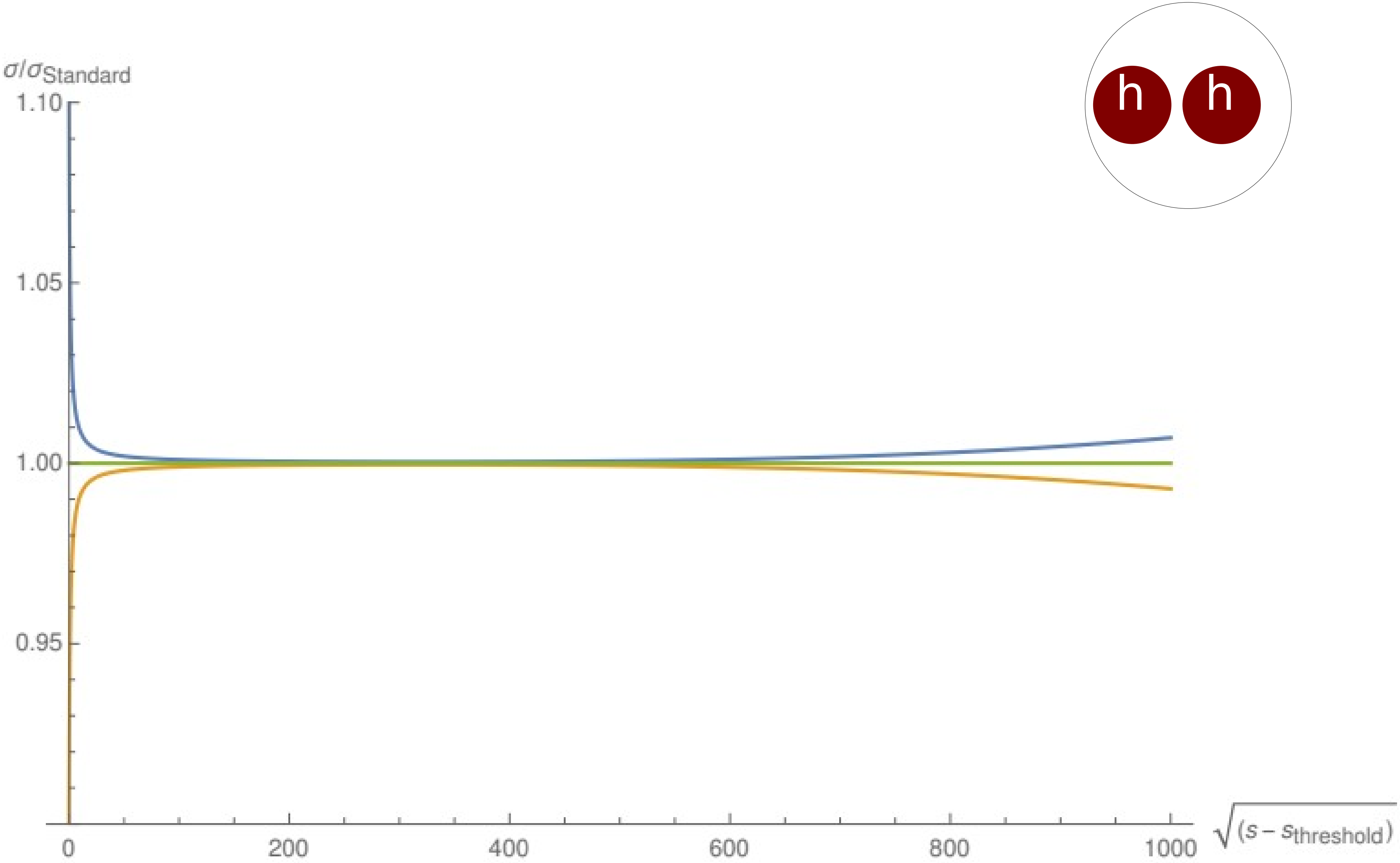

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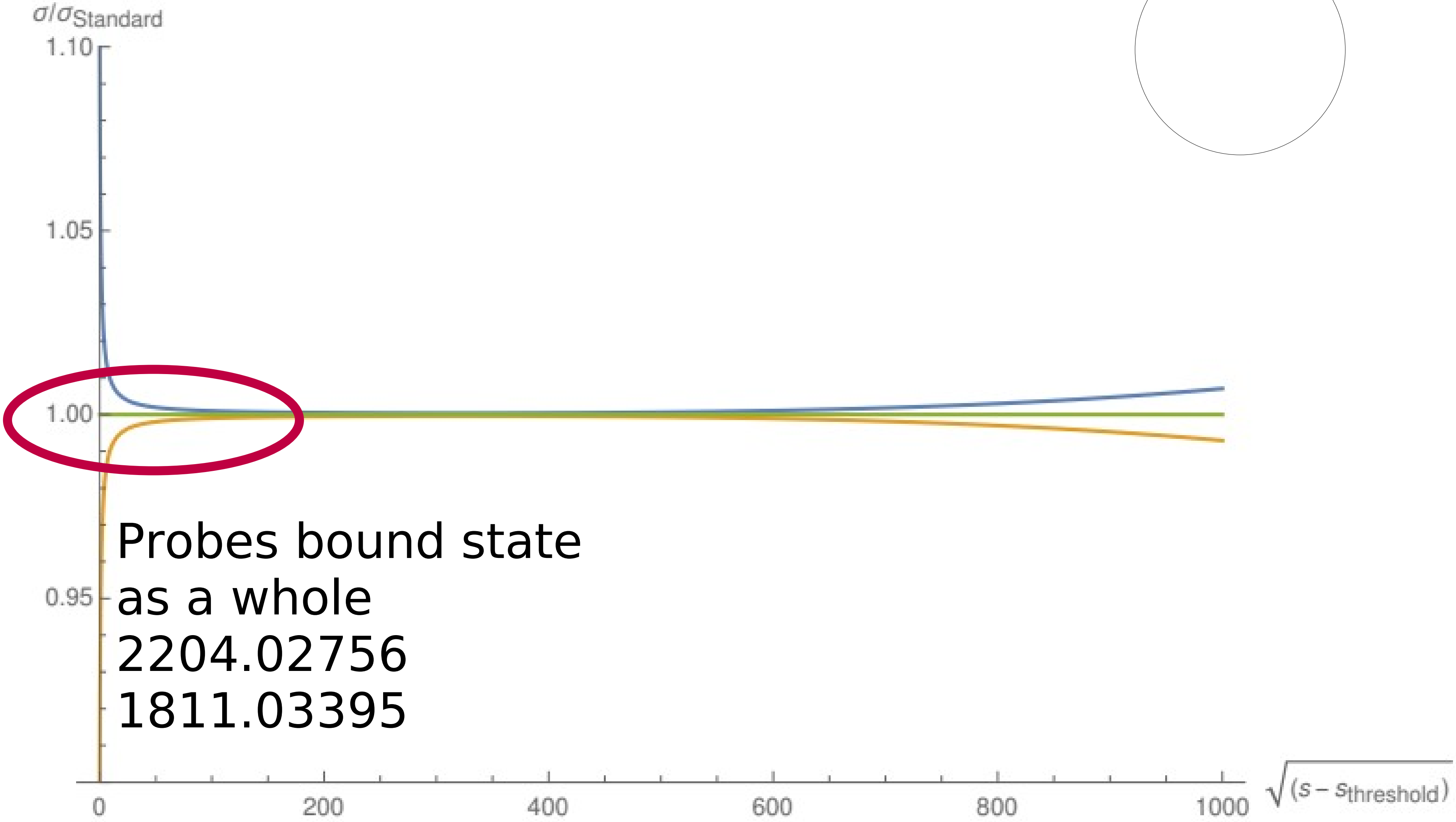
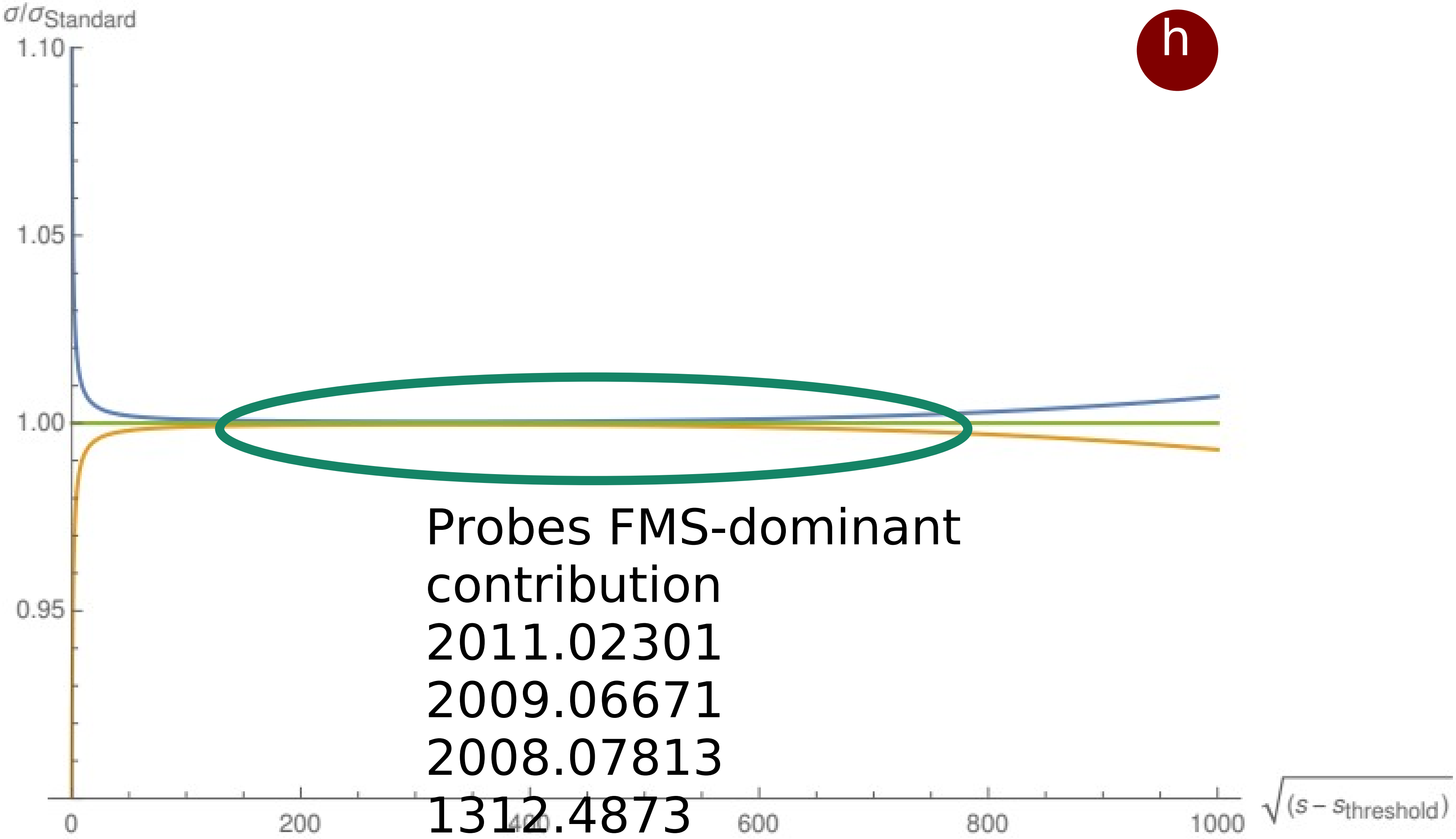


Probes bound state
as a whole
2204.02756
1811.03395

Has been done for several observables

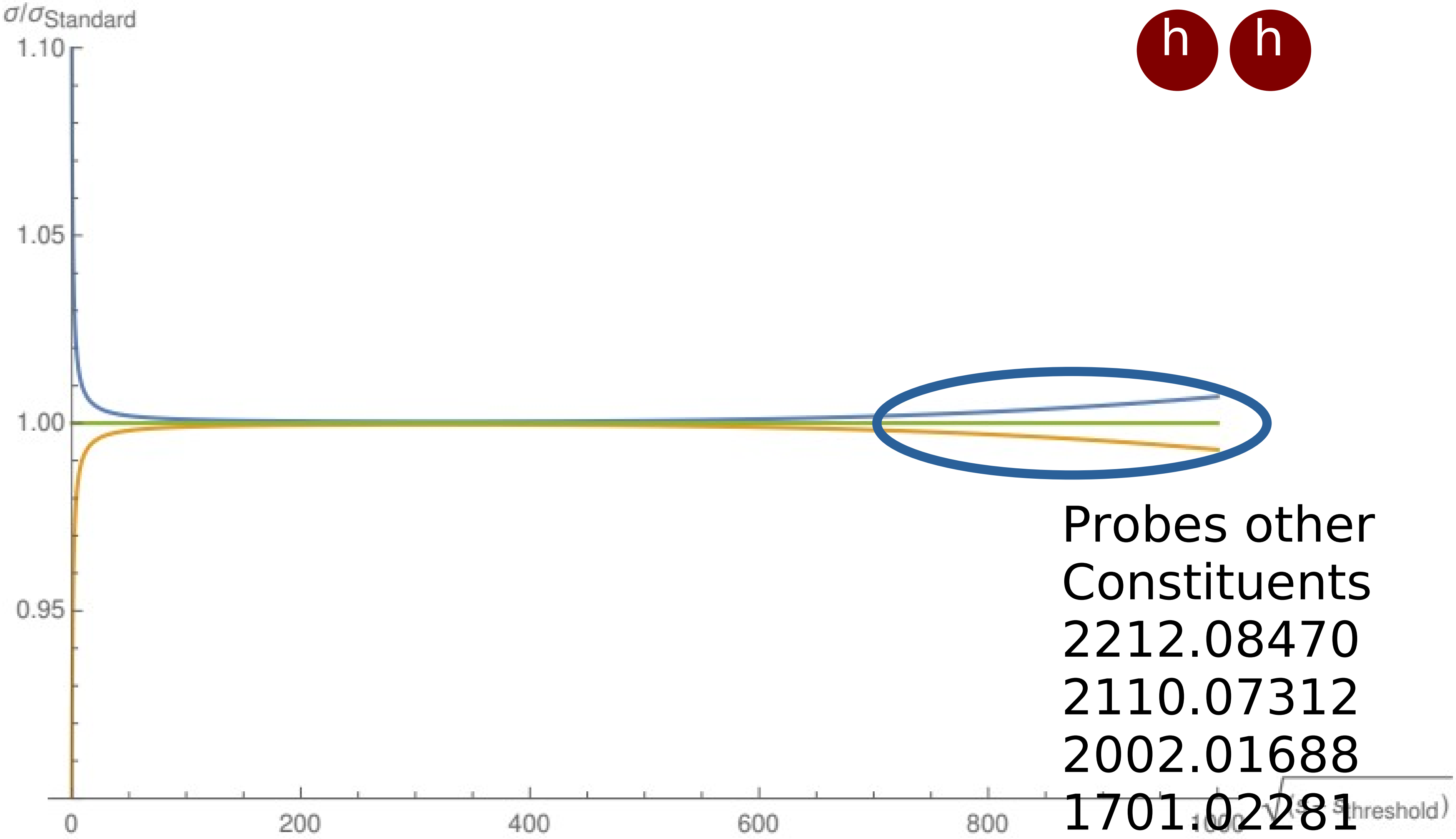
# Generic behavior: DIS-like



Probes FMS-dominant contribution
2011.02301
2009.06671
2008.07813
1312.4873

Has been done for several observables

# Generic behavior: DIS-like



Probes other
Constituents
2212.08470
2110.07312
2002.01688
1701.02281

Has been done for several observables

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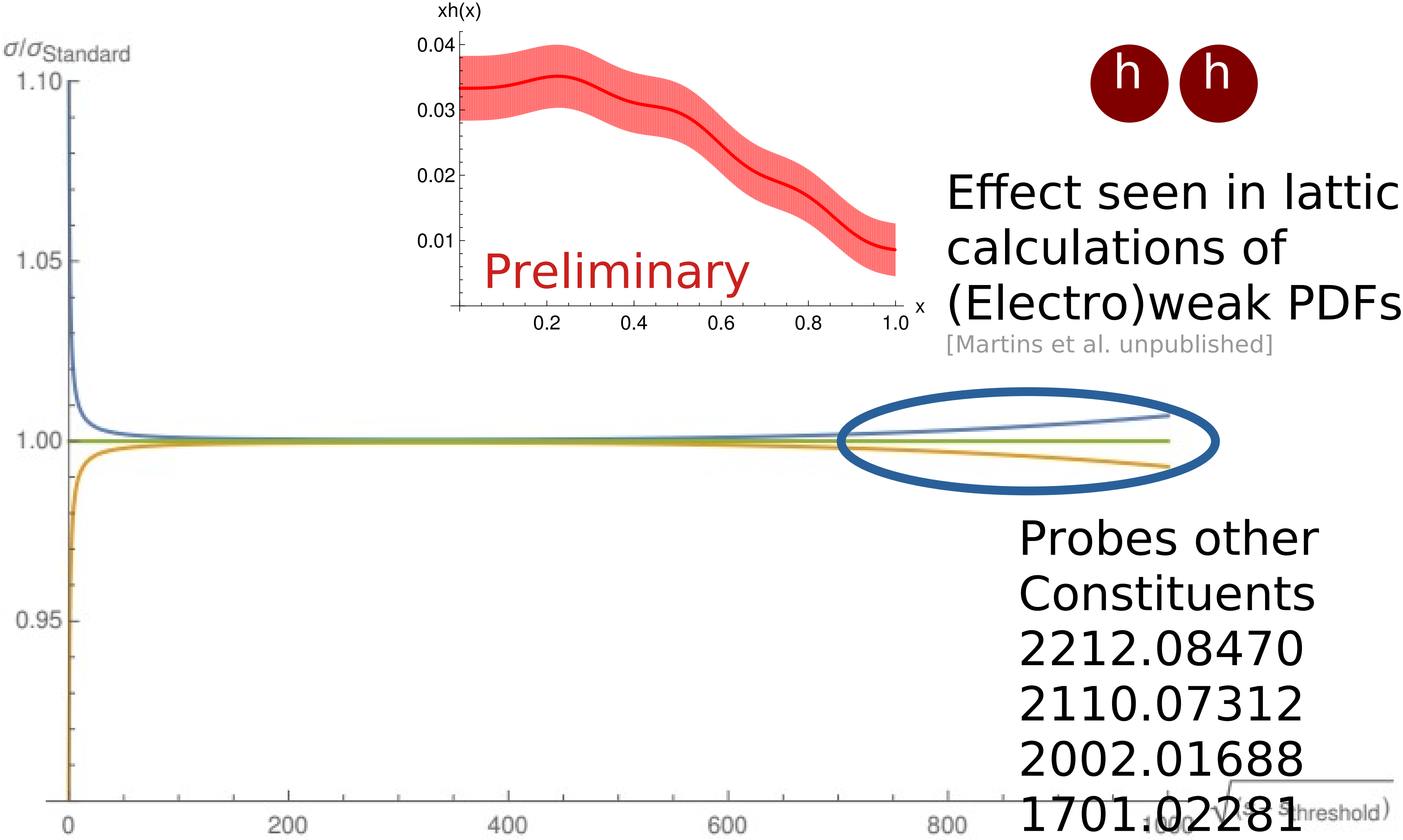

Effect seen in lattice calculations of (Electro)weak PDFs

[Martins et al. unpublished]

Probes other Constituents
2212.08470
2110.07312
2002.01688
1701.02281

Has been done for several observables

# Is the smallness generic?

Is the smallness generic?

-

No

Qualitative changes at tree-level possible

# Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

- Standard model is special

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  - No: Depends on gauge group, representations, and custodial groups
  - Can work sometimes (2HDM,MSSM) [Maas,Pedro’16, Maas,Schreiner’23]
  - Generally qualitative differences
  - Note: Applicable to quantum gravity [Maas’19]

# A toy model for unification

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$$L=-\frac{1}{4}W^a_{\mu\nu}W_a^{\mu\nu}$$

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- Ws $W^a_\mu$ W

- Coupling $g$ and some numbers $f^{abc}$

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$$D^{ij}_\mu = \delta^{ij} \partial_\mu - i g W^a_\mu t^{ij}_a$$

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$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Ws $W_{\mu}^{a}$ W
- Higgs $h_i$ h
- Couplings $g, v, \lambda$ and some numbers $f^{abc}$ and $t_a^{ij}$
- Parameters selected for a BEH effect

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- Local SU(3) gauge symmetry

$$W^a_\mu\to W^a_\mu+(\delta^a_b\partial_\mu-gf^a_{bc}W^c_\mu)\phi^b \qquad h_i\to h_i+g\,t^{ij}_a\phi^a h_j$$

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- Local SU(3) gauge symmetry

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-gf_{bc}^{a}W_{\mu}^{c})\phi^{b}$ $h_{i}\rightarrow h_{i}+g\,t_{a}^{ij}\phi^{a}h_{j}$

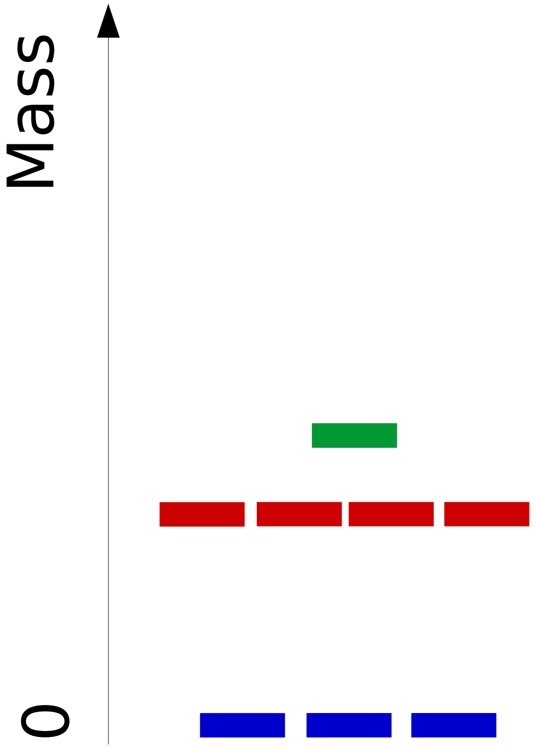
- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$W_{\mu}^{a}\rightarrow W_{\mu}^{a}$ $h\rightarrow\exp\left(ia\right)h$

# Spectrum

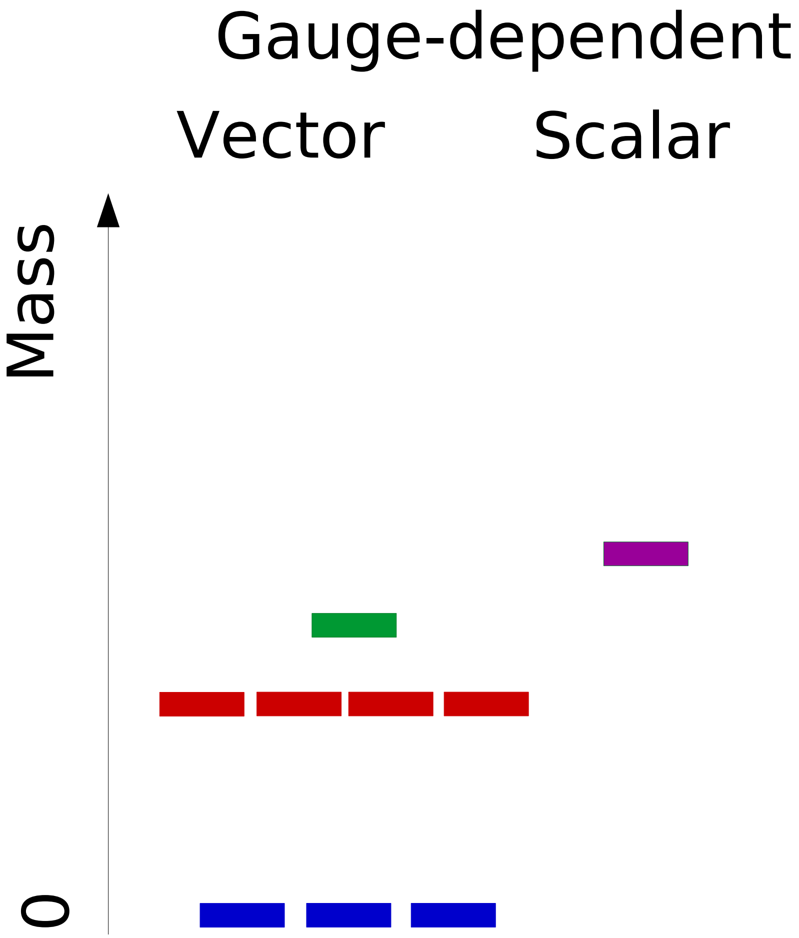
Gauge-dependent

Vector



‘SU(3)→ SU(2)’

# Spectrum



Gauge-dependent

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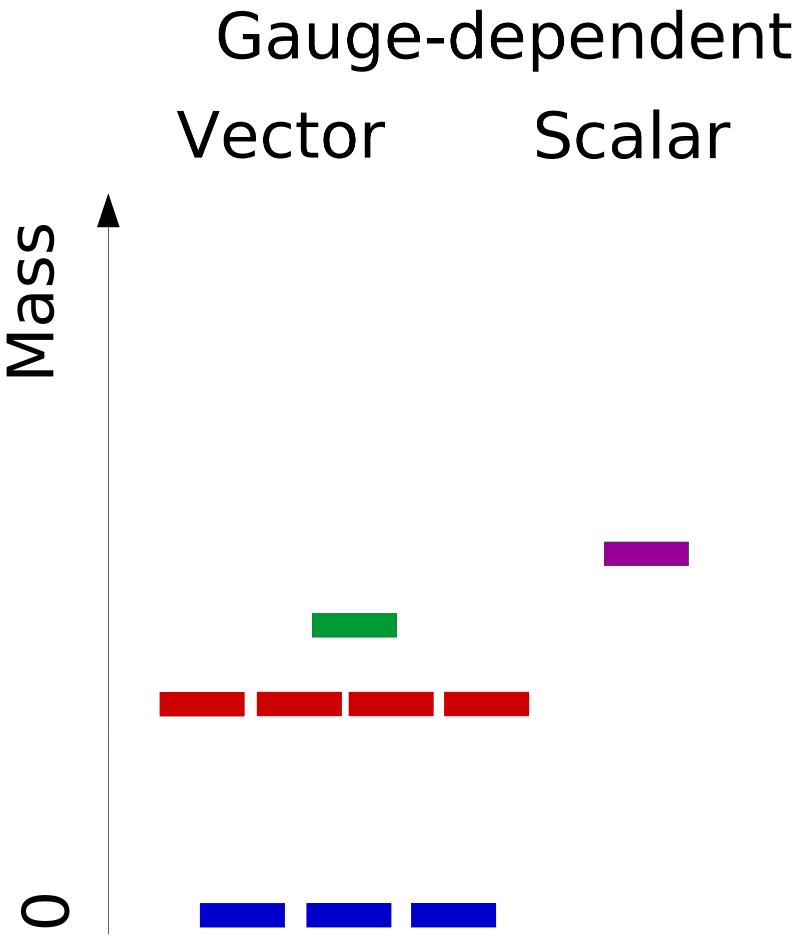

Gauge-dependent

Vector Scalar

Confirmed in gauge-fixed
lattice calculations [Maas et al.’16]

# Spectrum

[Maas & Törek’16,'18
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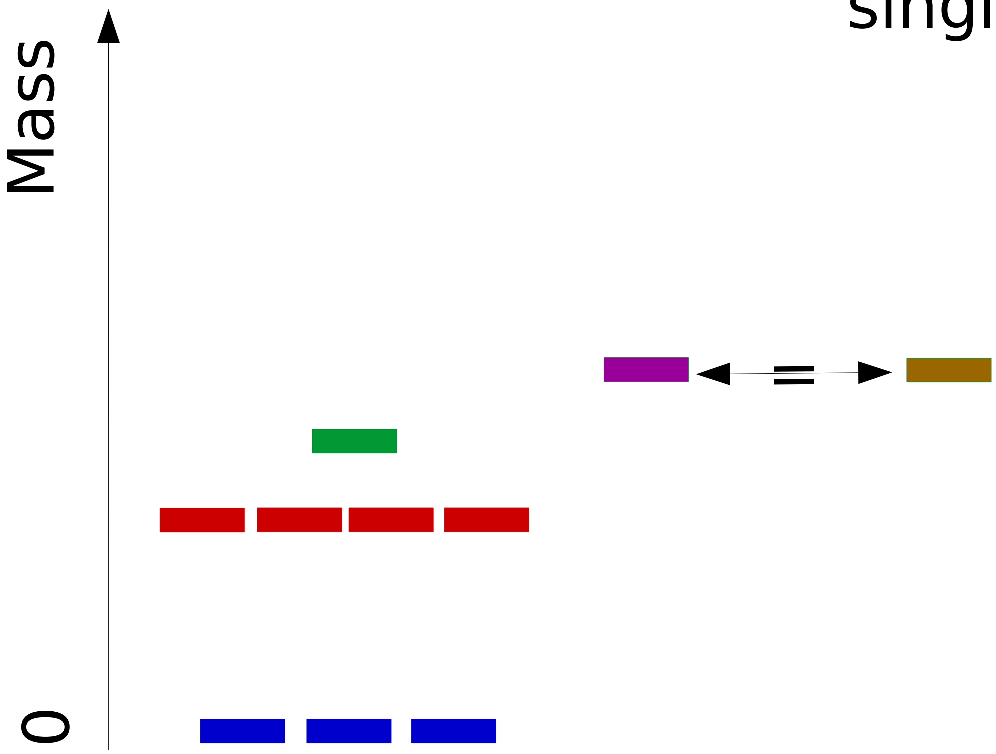

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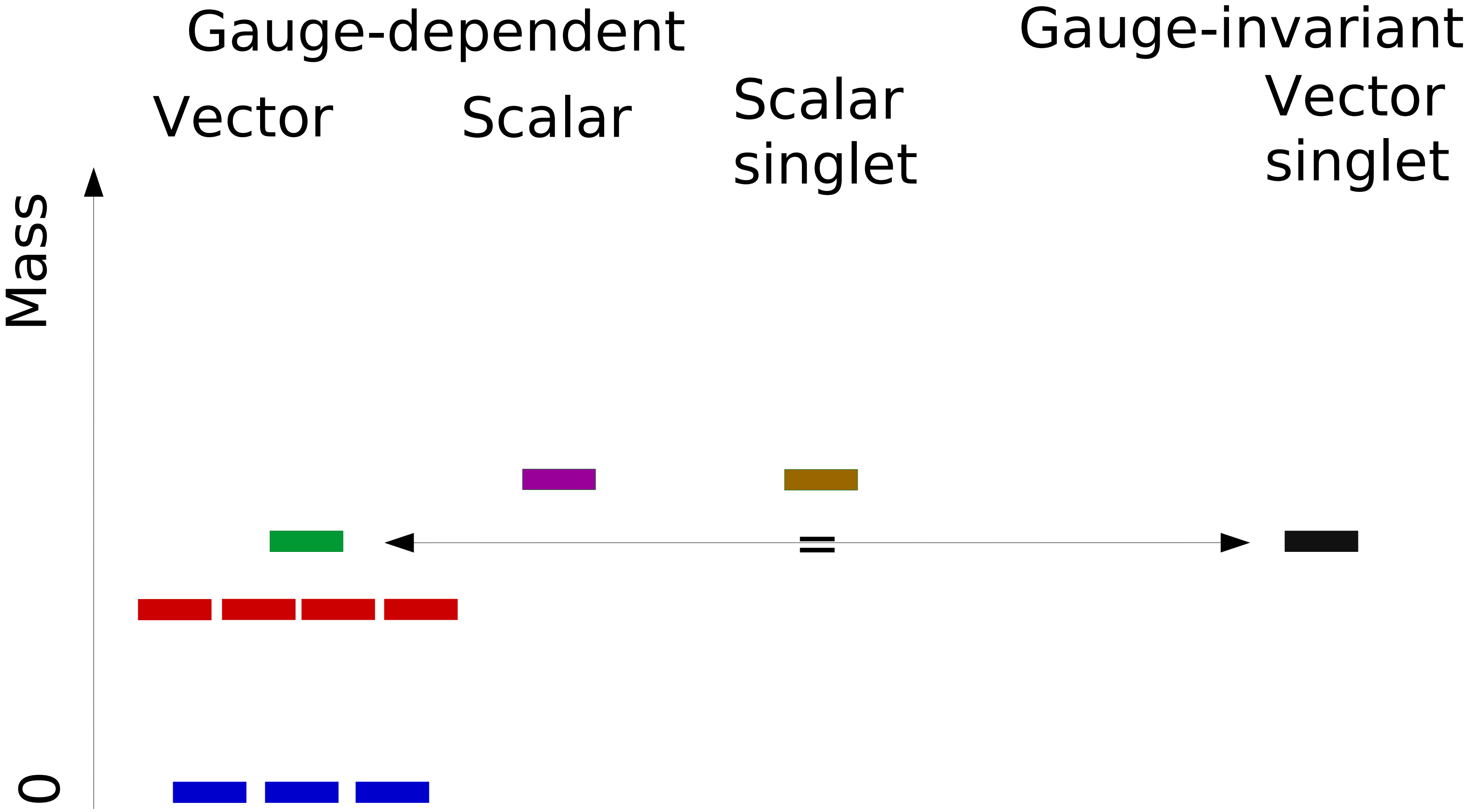

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=

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Matrix from group structure

$c^{ab}$ projects out only one field

Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

# A different kind of states

- SM: Group theory forced same gauge multiplets and custodial multiples for SU(2)
  - Because Higgs is bifundamental
  - Remainder is bound state/resonance or not

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- Now: States without elementary analouge
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue – U(1) acts as baryon number
  - Lightest must exist and be absolutely stable

# Possible new states

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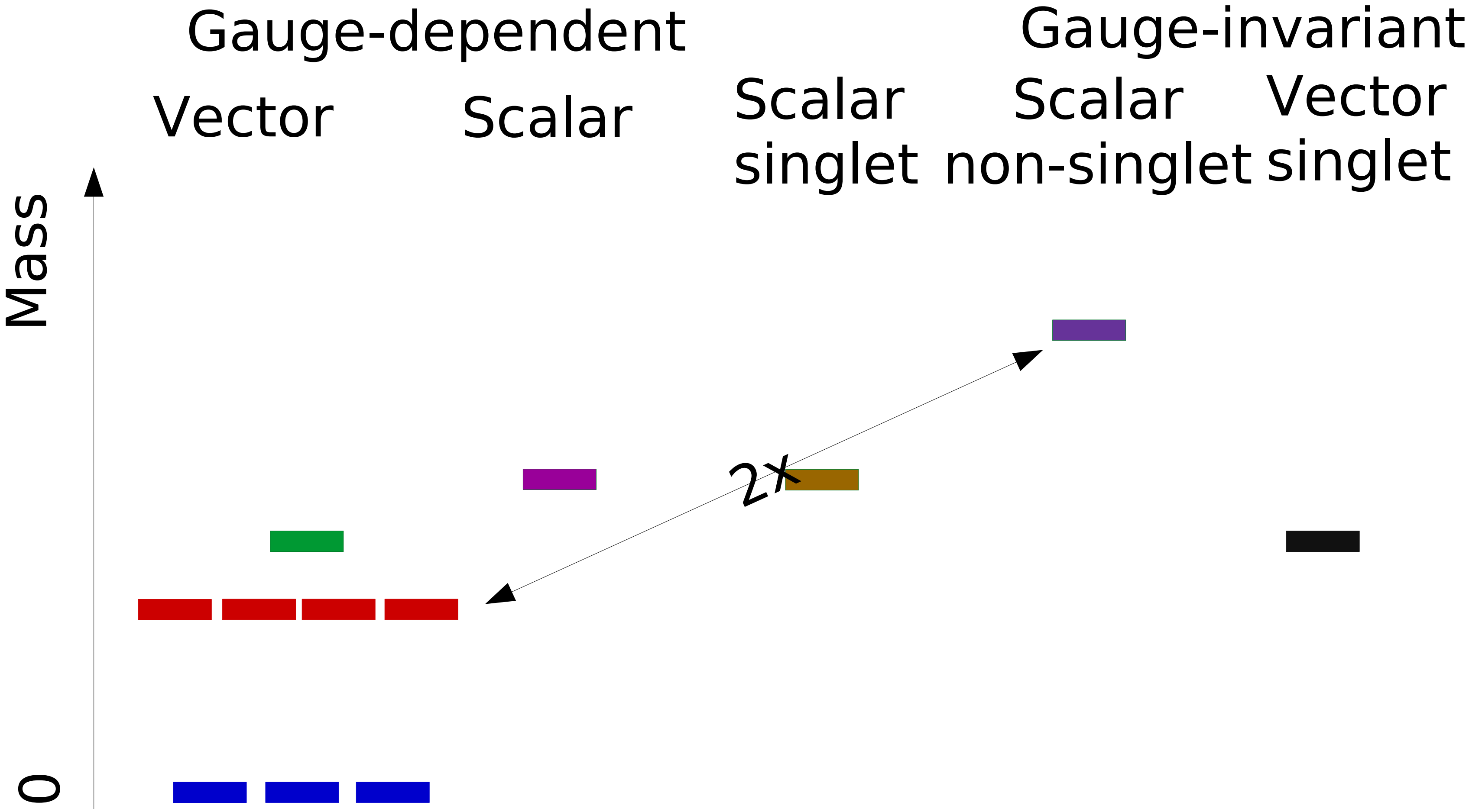

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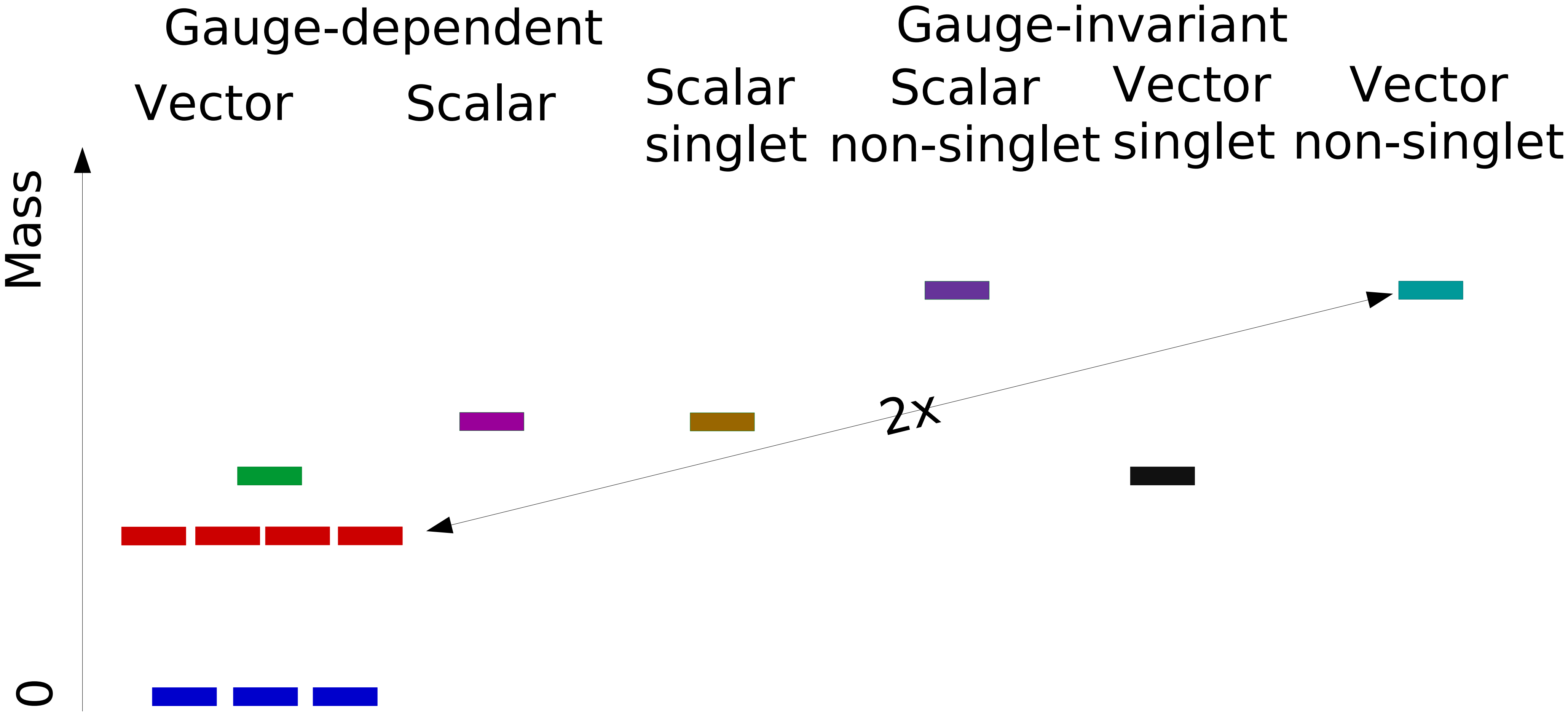

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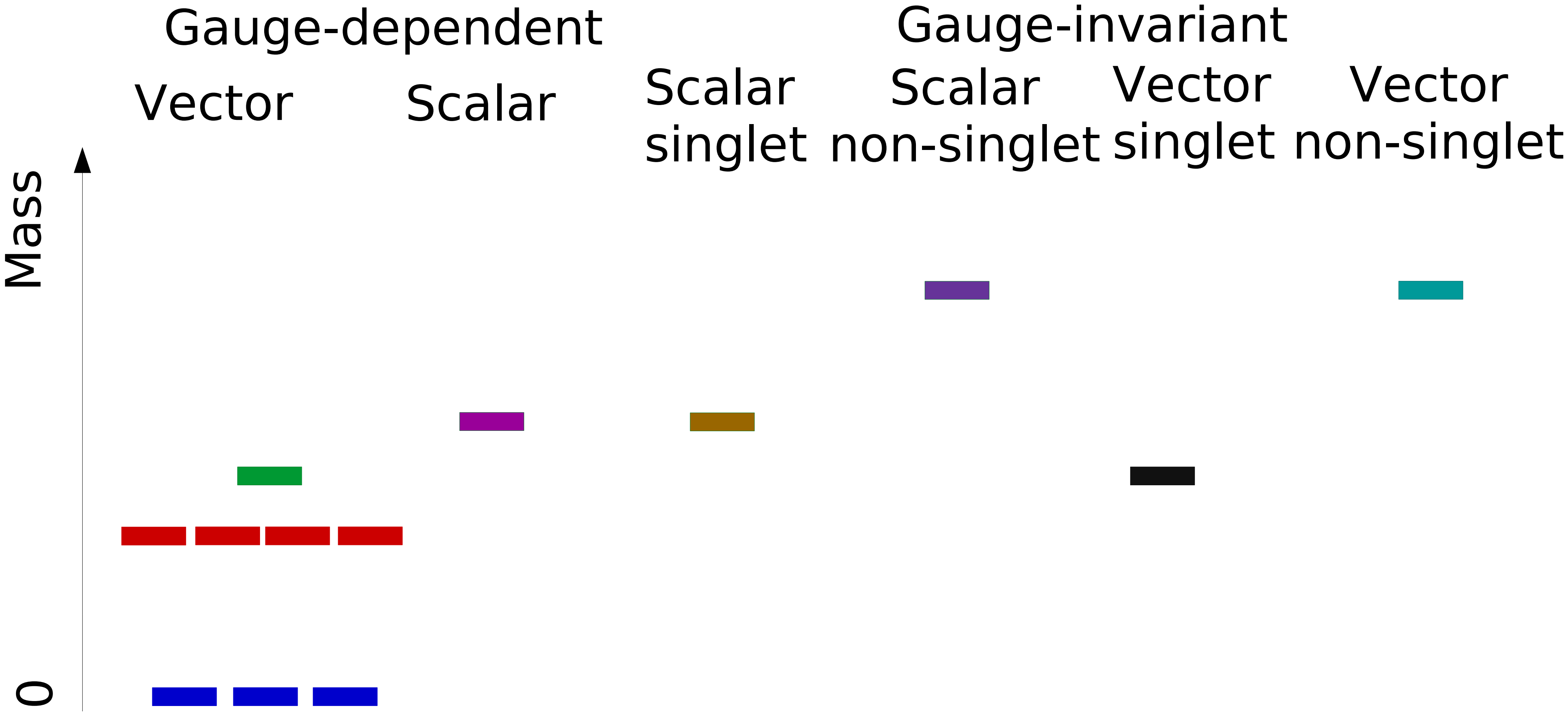

- Qualitatively different spectrum
- No mass gap!

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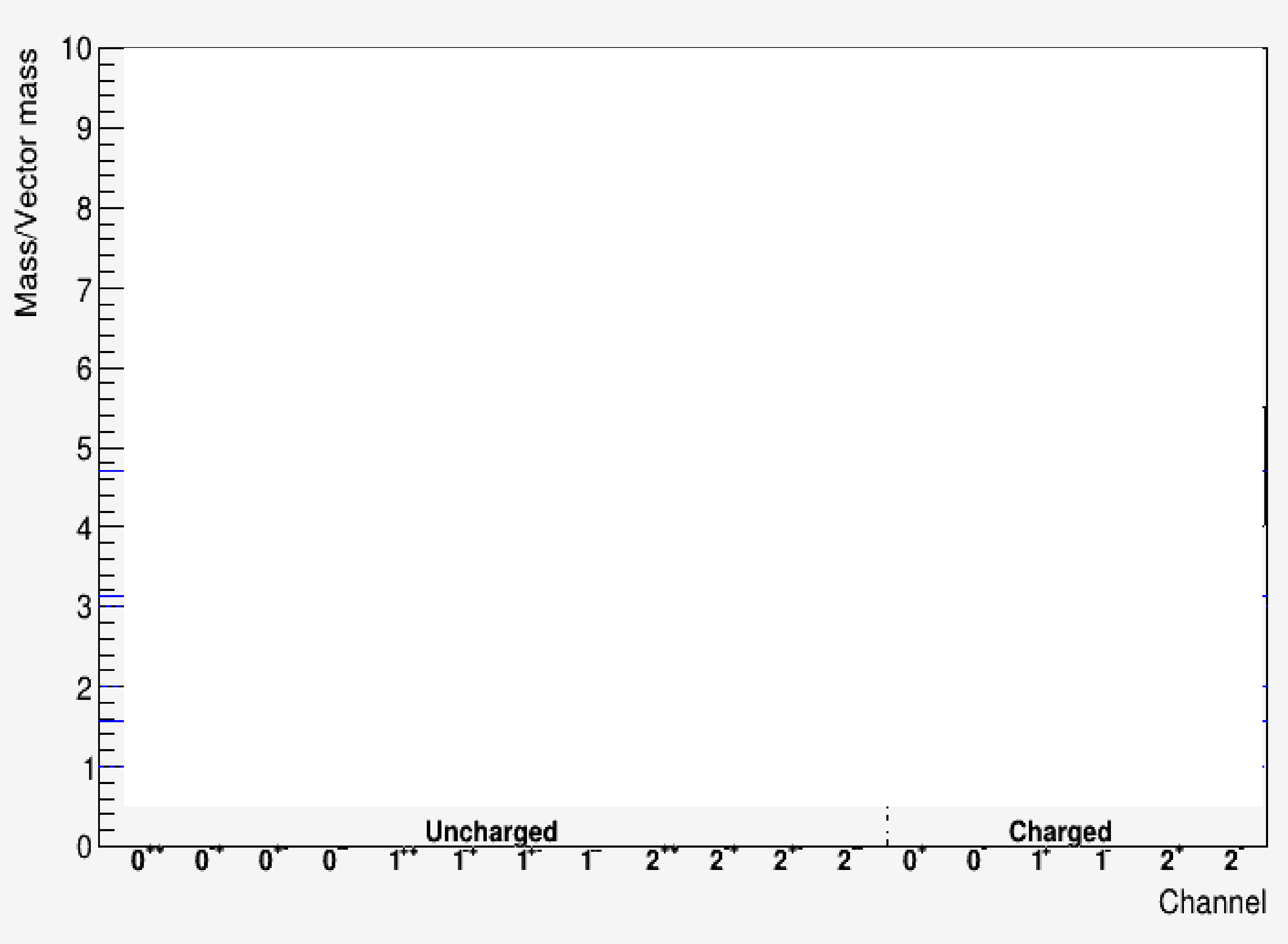
- What is the lightest state?
  - Prediction with constituent model
  - Lattice calulations
  - All channels: J<3
  - Aim: Ground state for each channel
    - Characterization through scattering states

# Typical spectrum

**PRELIMINARY**

[Dobson et al.’25]



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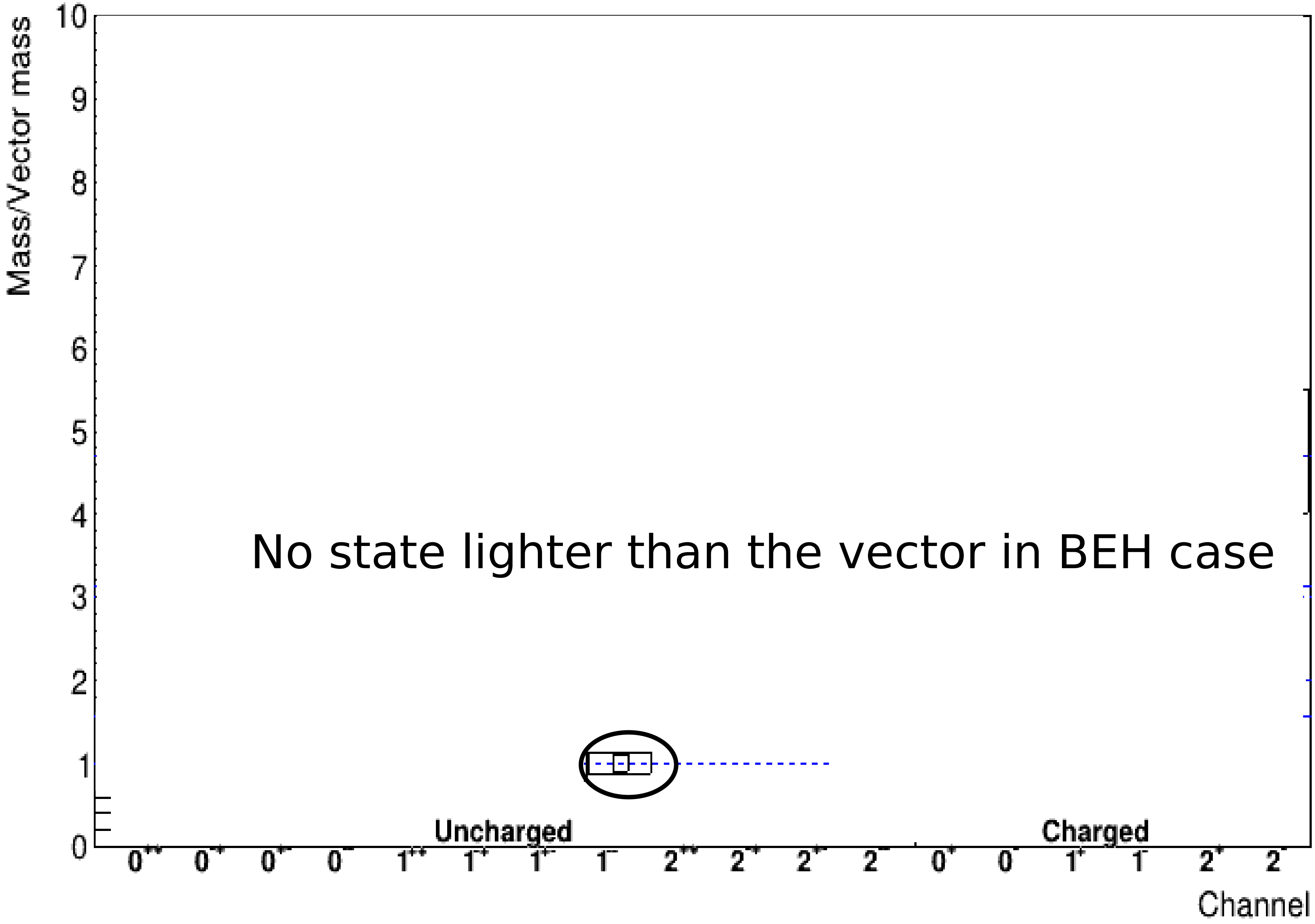

No state lighter than the vector in BEH case

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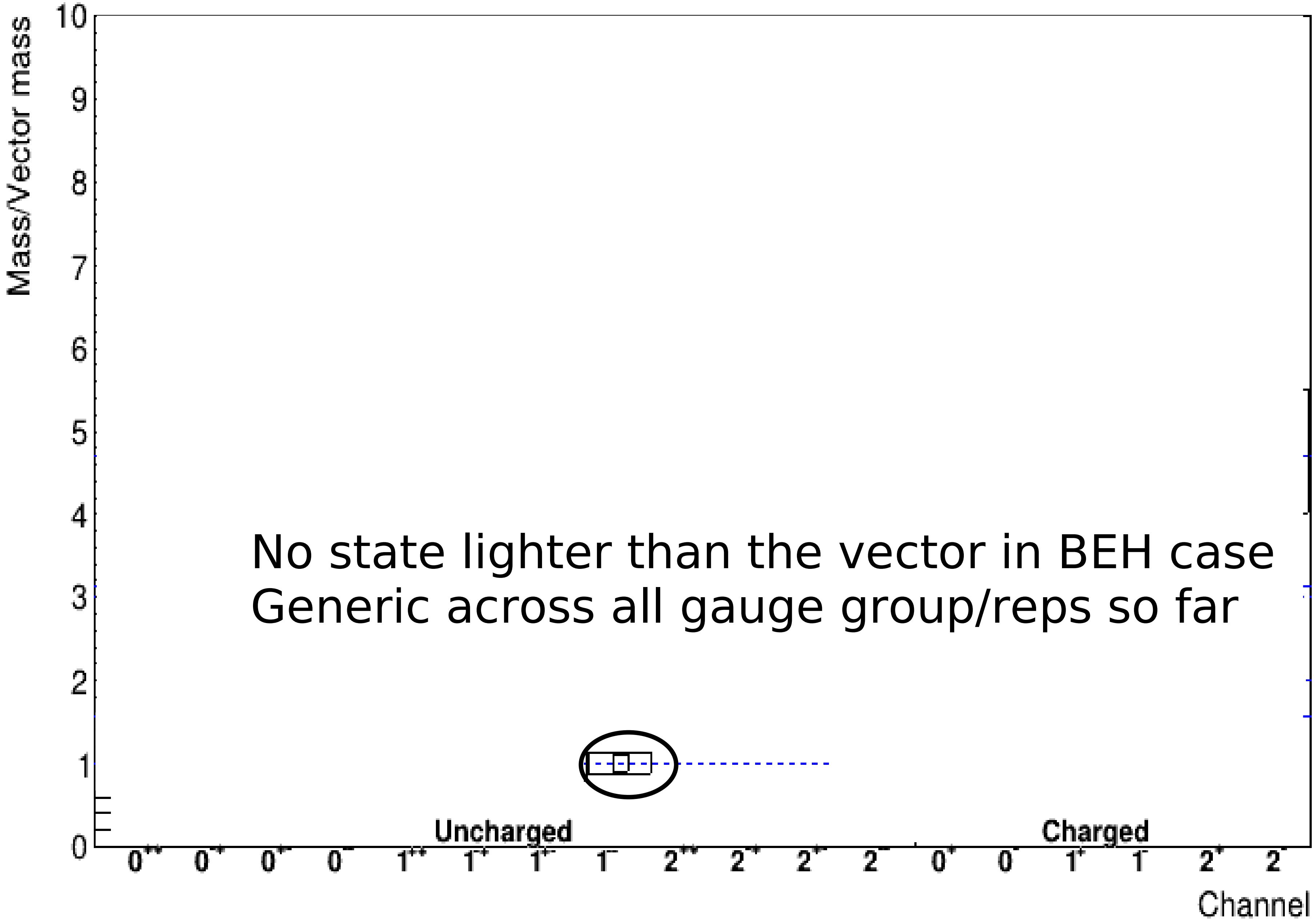

No state lighter than the vector in BEH case

Generic across all gauge group/reps so far

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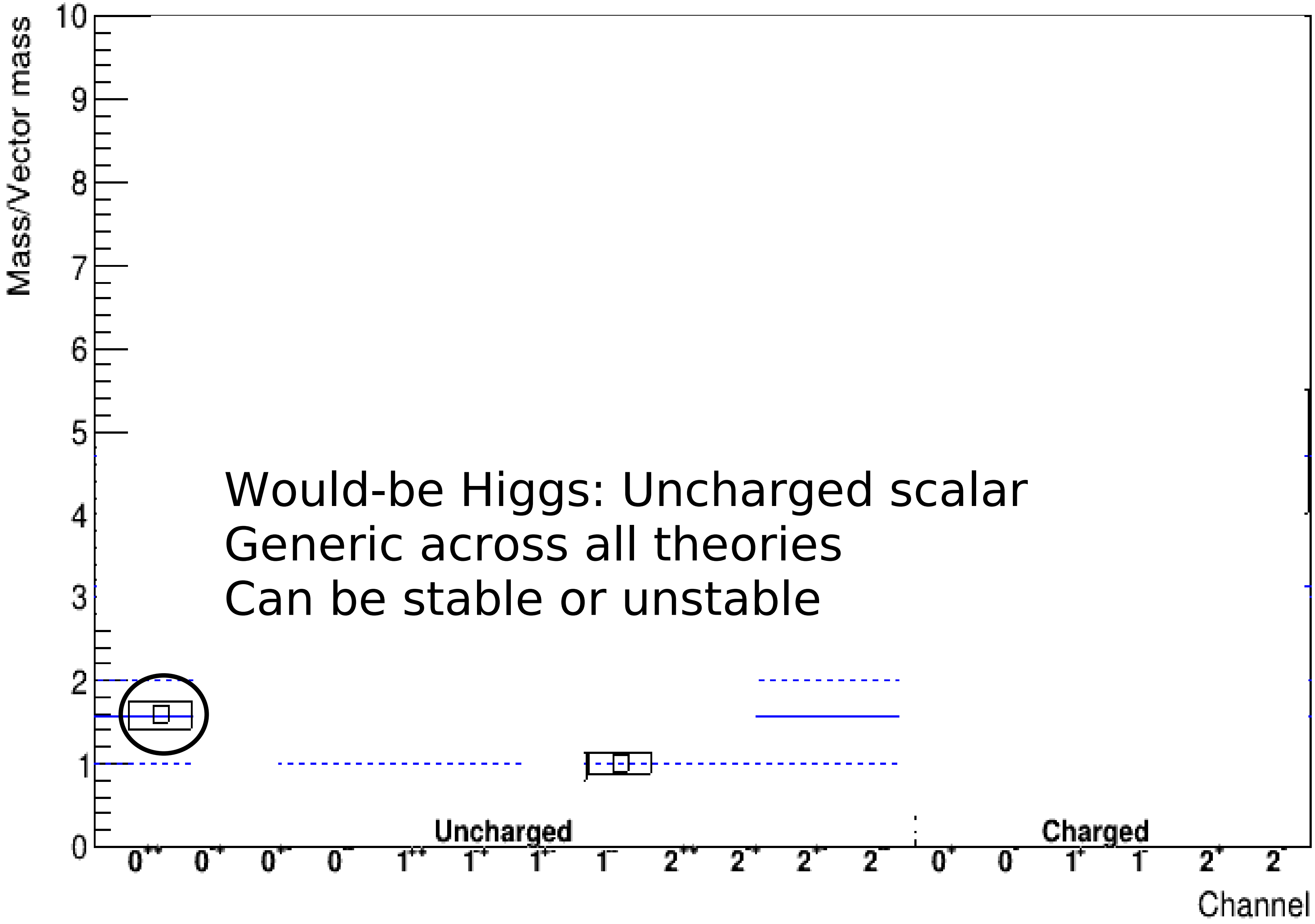

Would-be Higgs: Uncharged scalar
Generic across all theories
Can be stable or unstable

# Typical spectrum

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Maas ‘15
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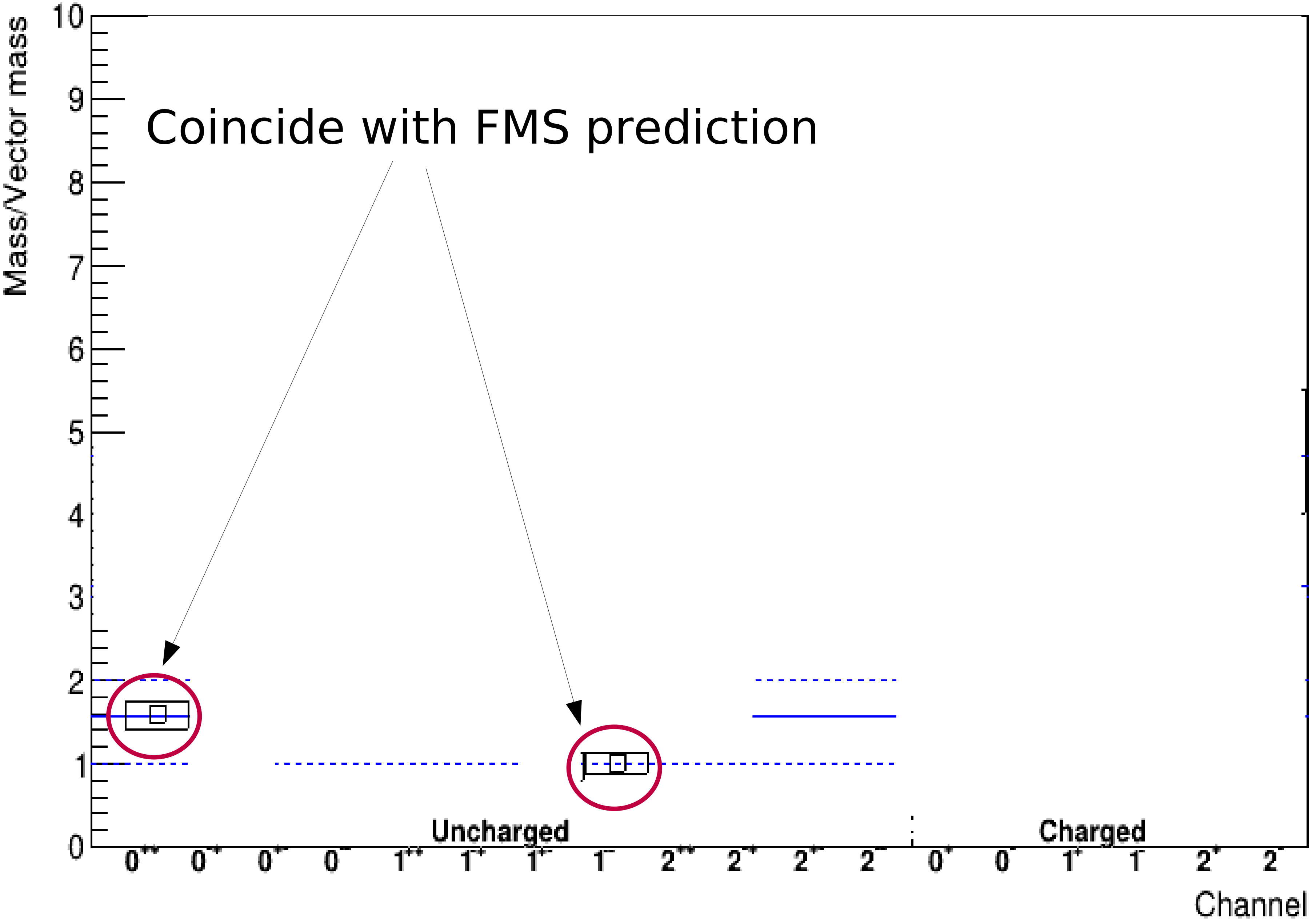

Coincide with FMS prediction

# Typical spectrum

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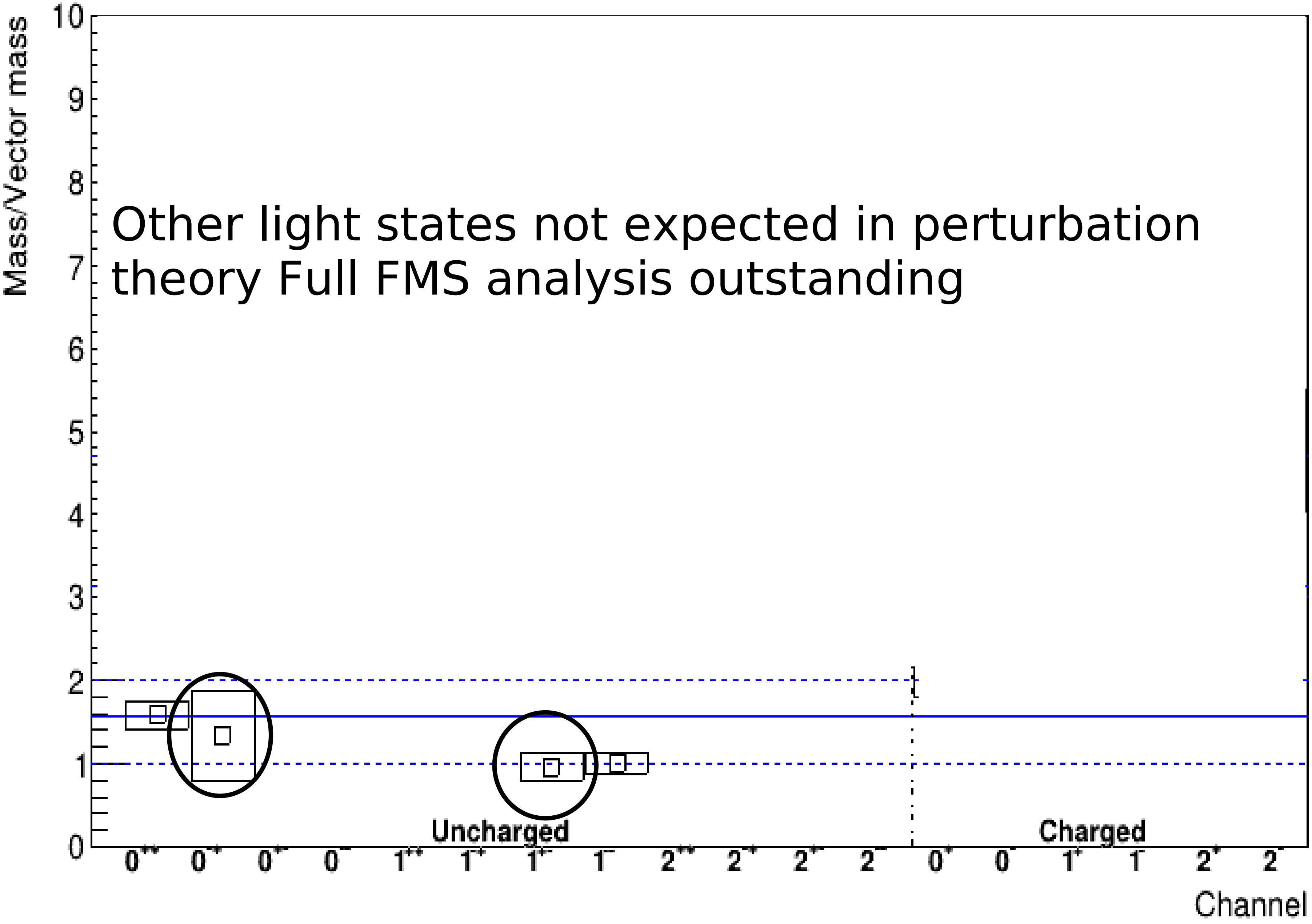

Other light states not expected in perturbation theory Full FMS analysis outstanding

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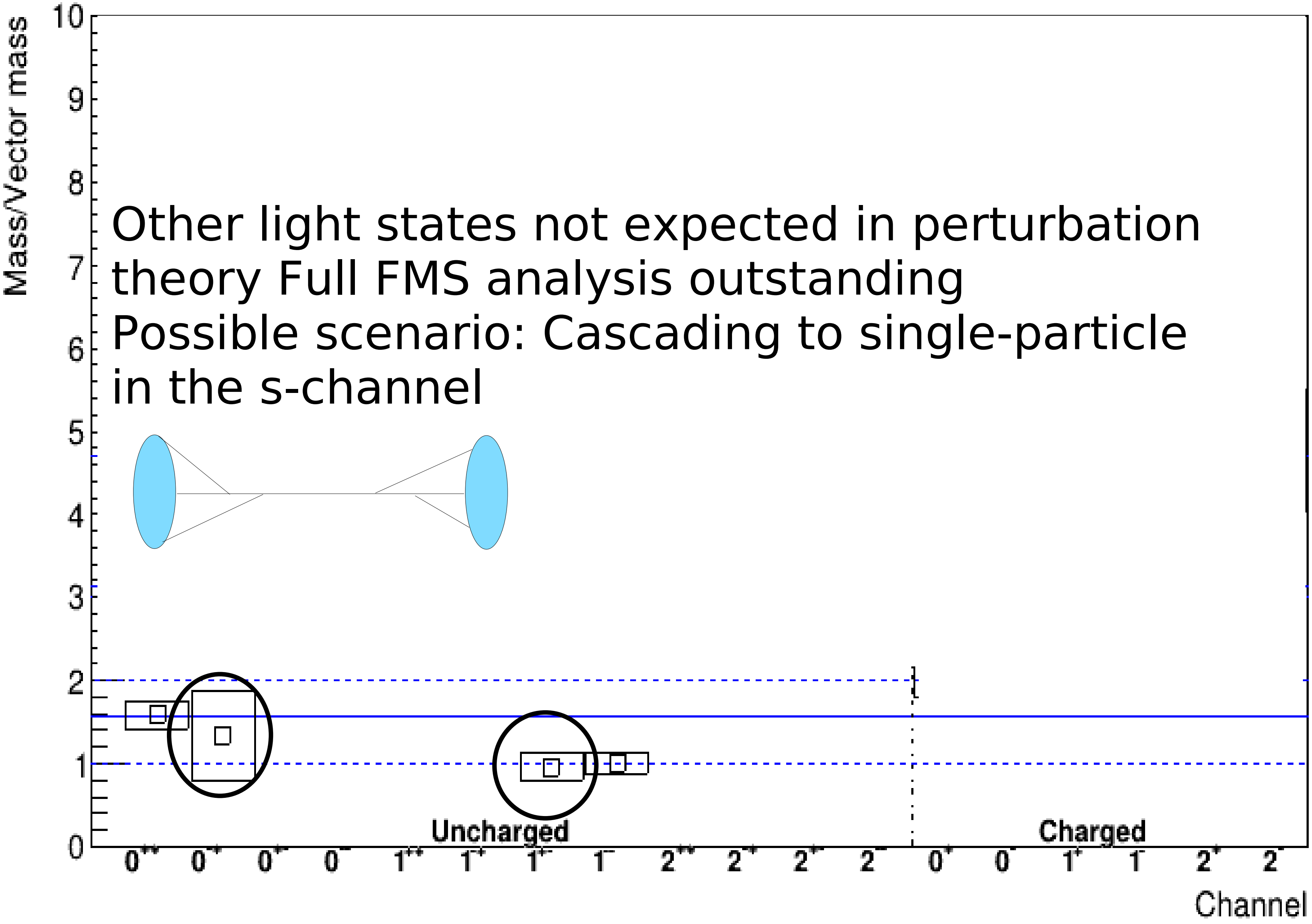

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Possible scenario: Cascading to single-particle in the s-channel

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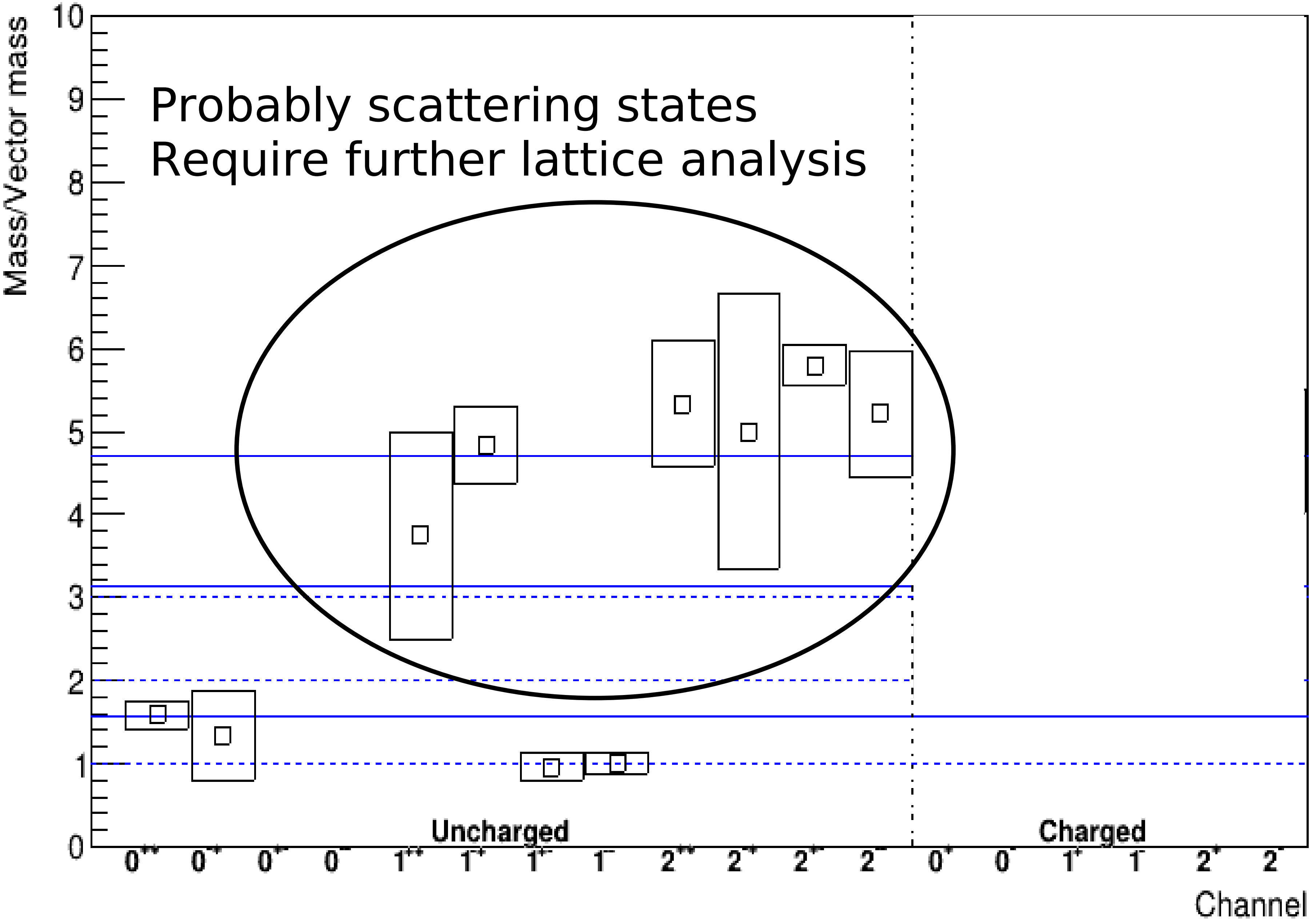

Probably scattering states
Require further lattice analysis

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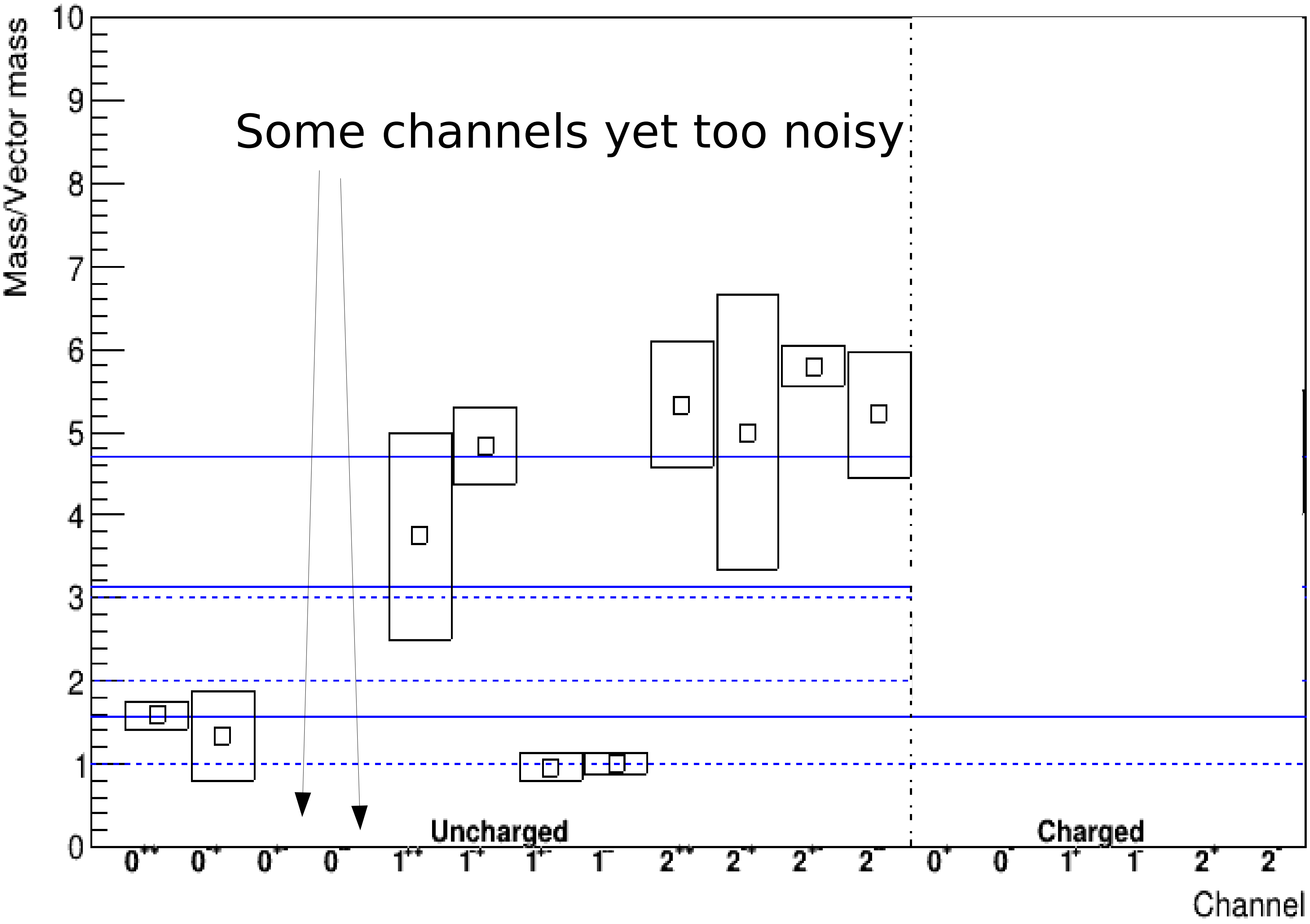

Some channels yet too noisy

# Typical spectrum

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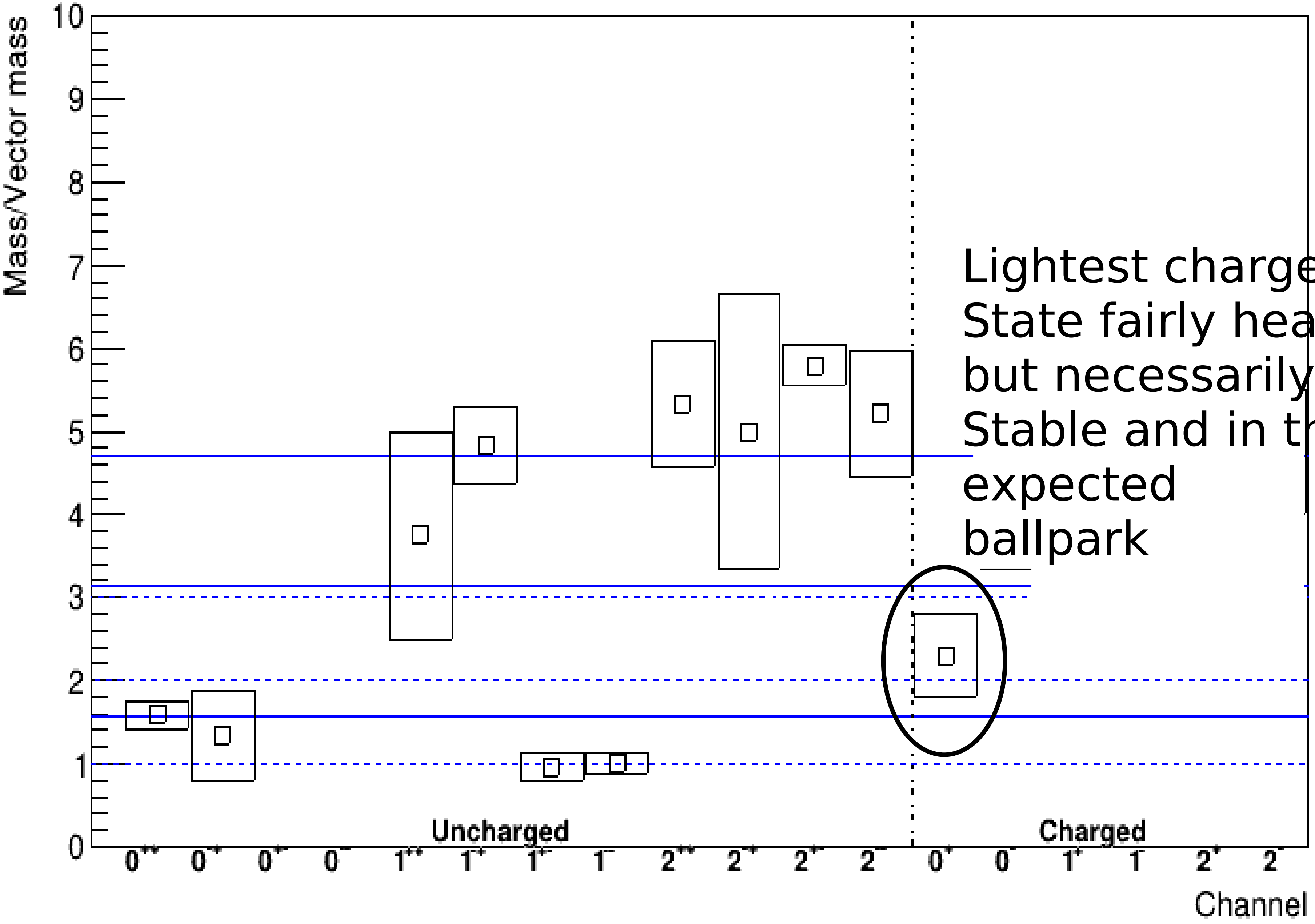

Lightest charged
State fairly heavy,
but necessarily
Stable and in the
expected
ballpark

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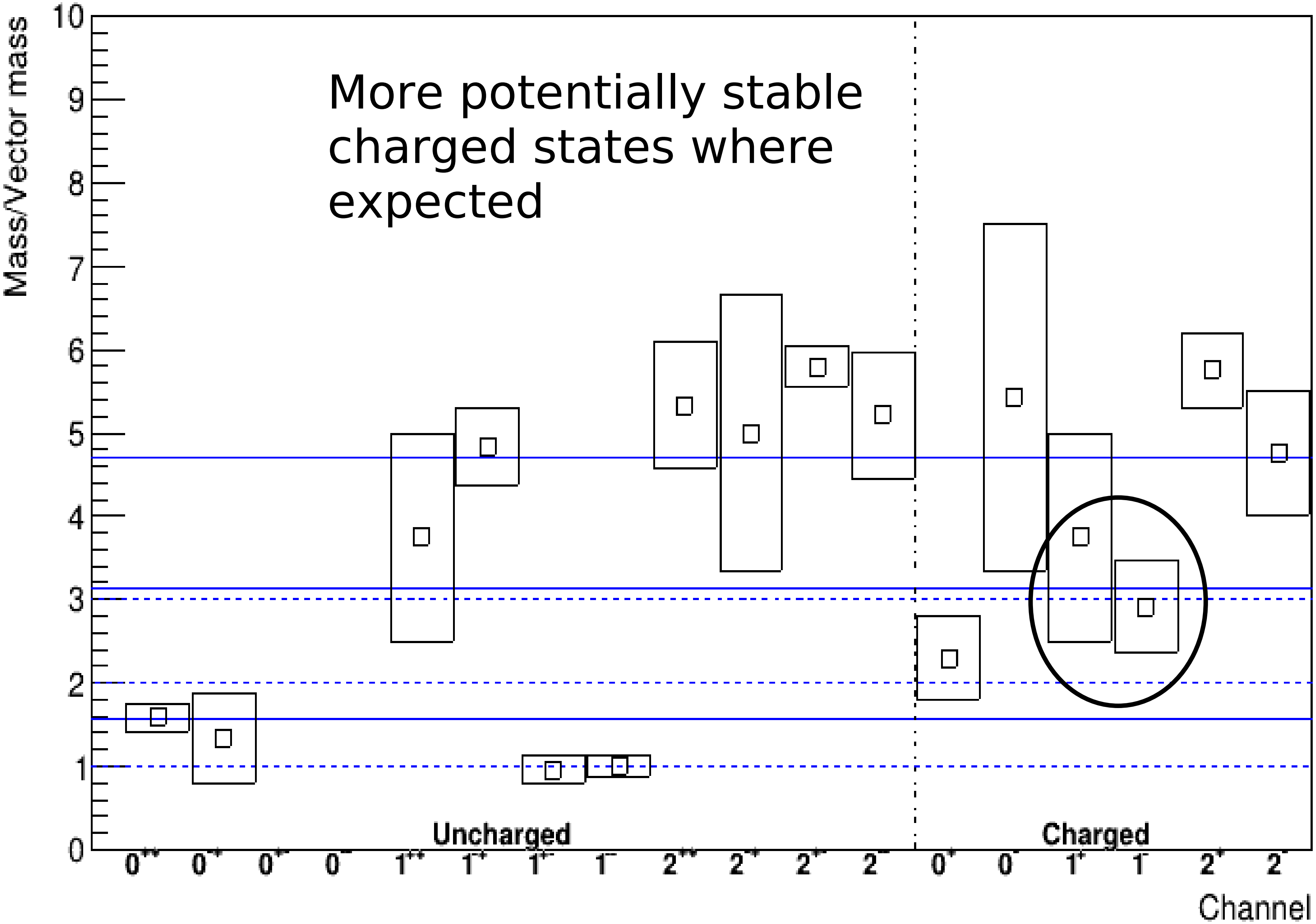

More potentially stable charged states where expected

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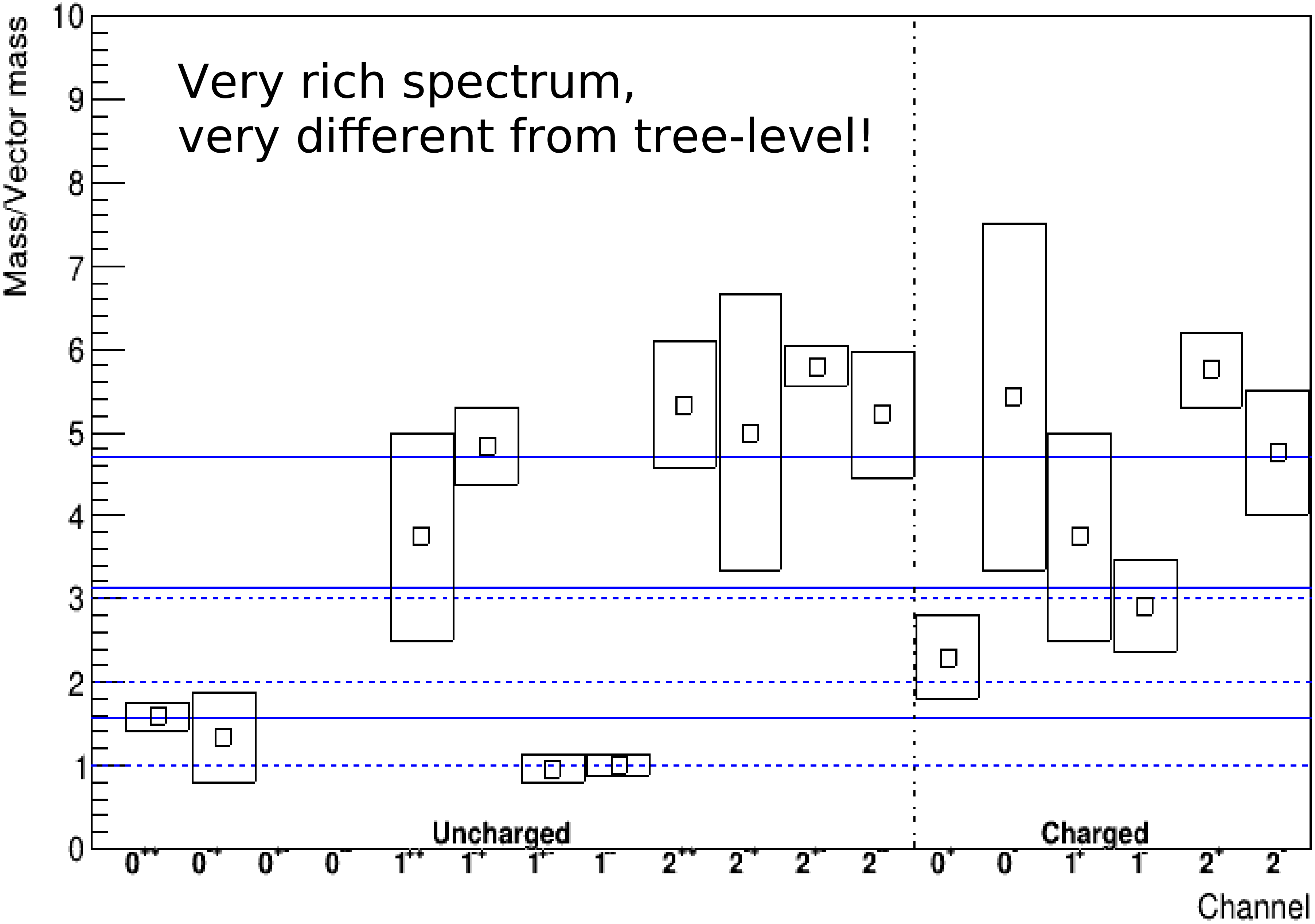

Very rich spectrum,
very different from tree-level!

# Experimental consequences

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[Maas & Törek'18
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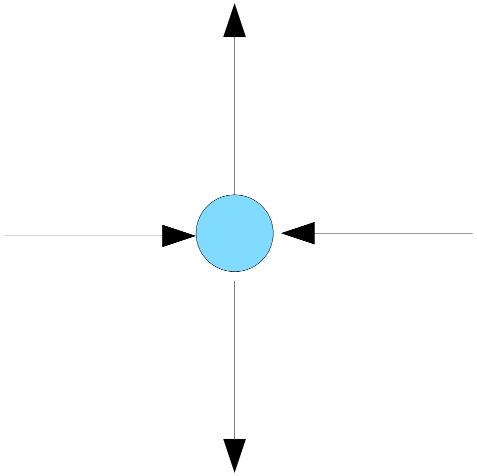

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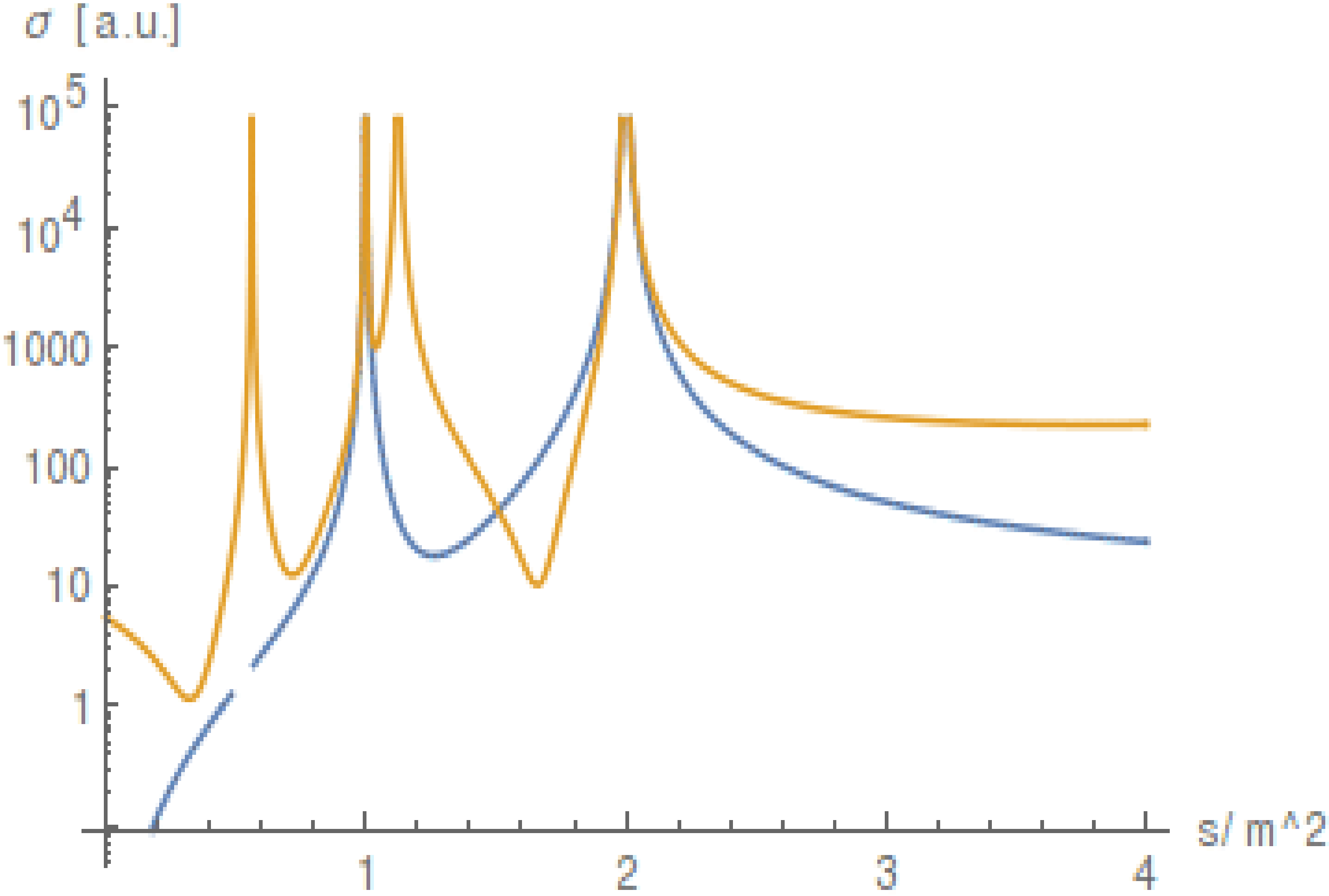

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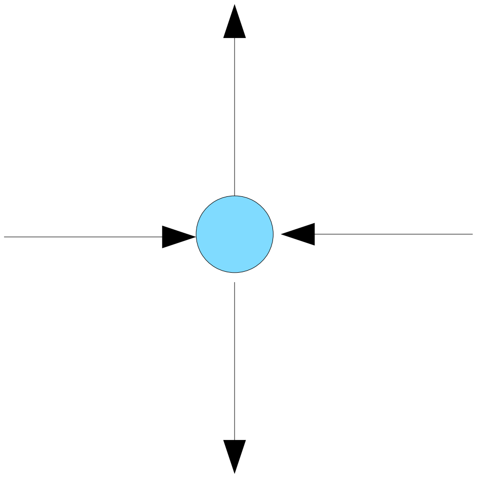

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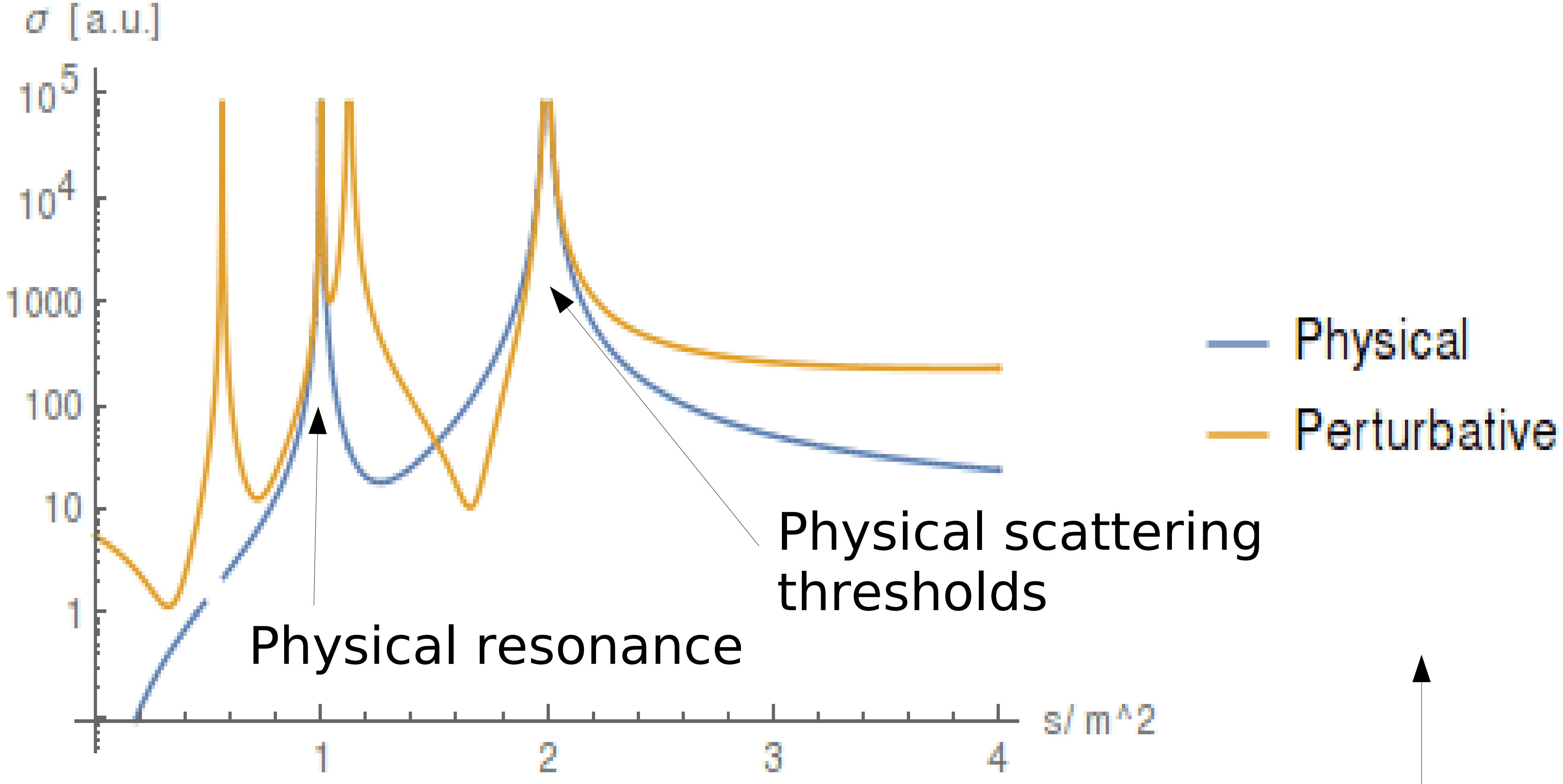

Physical scattering thresholds

Physical resonance

- Add fundamental fermions
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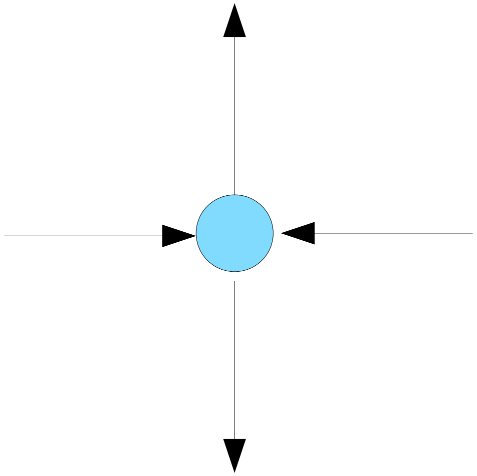

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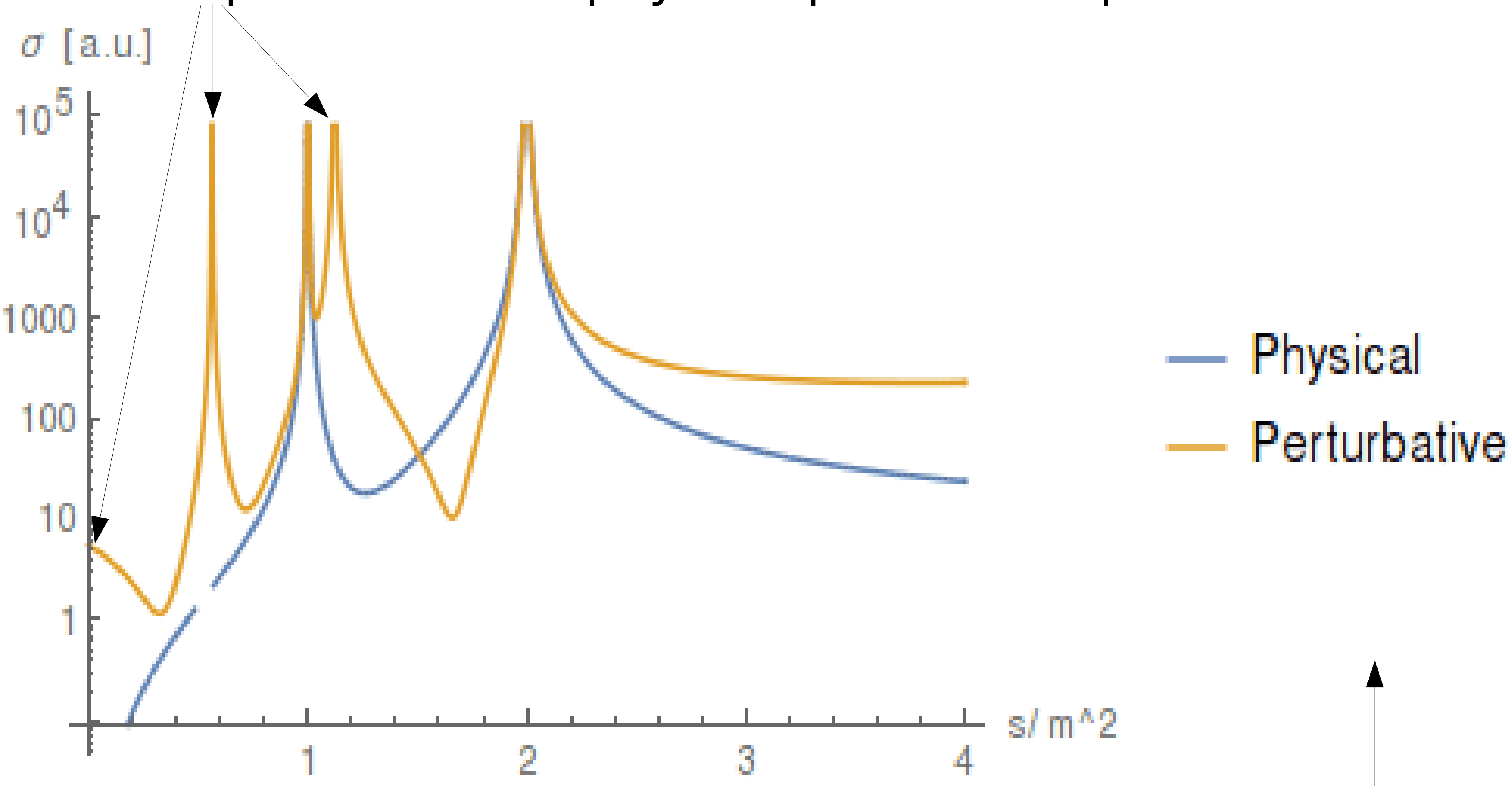
Ghost peaks from unphysical particles in perturbation theory

- Add fundamental fermions
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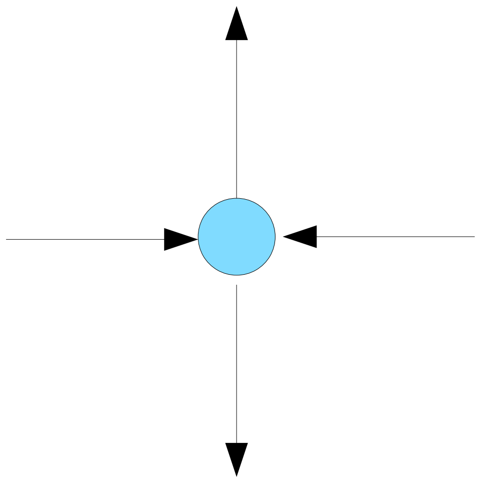

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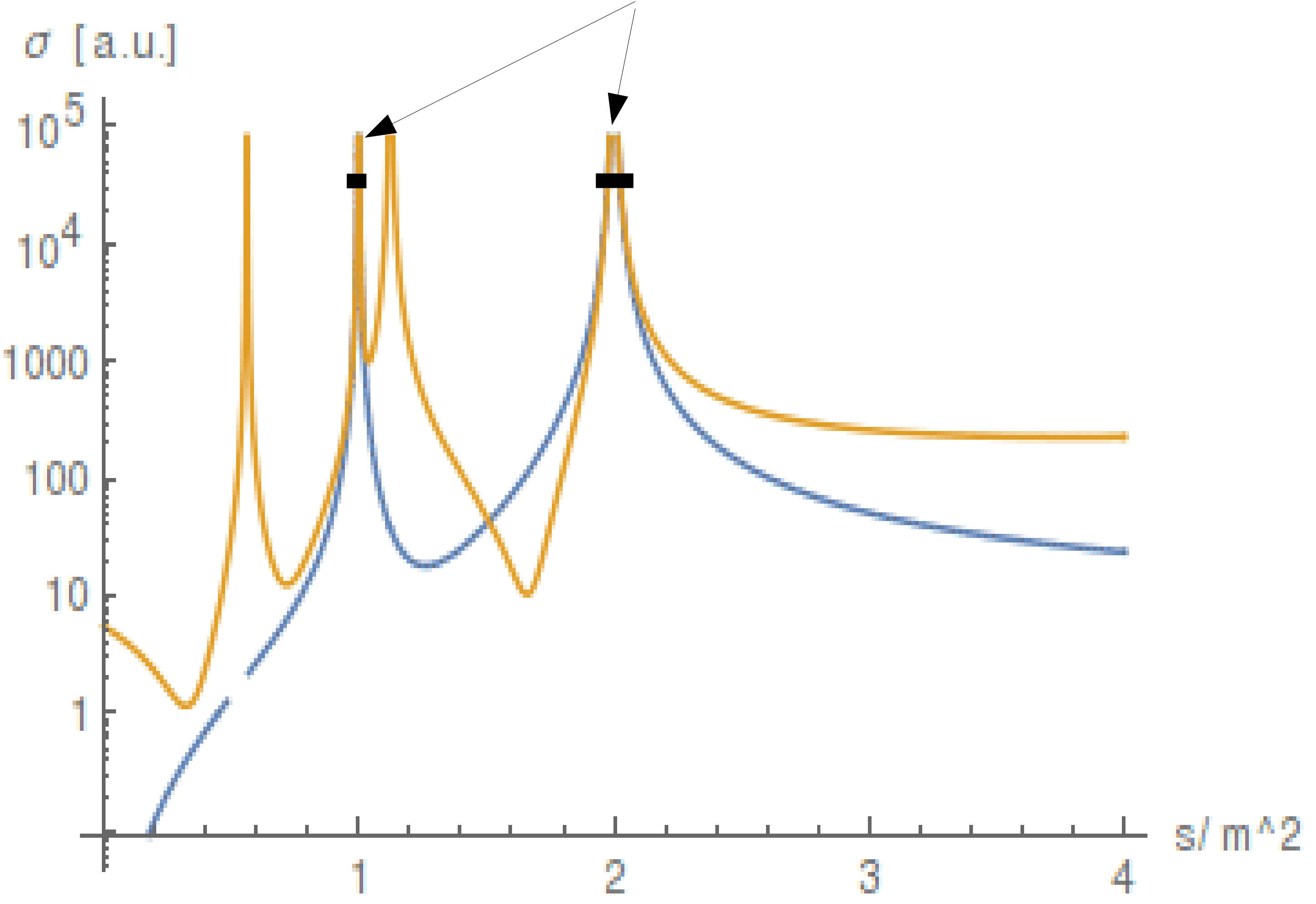

Close to true structures identical!

- Add fundamental fermions
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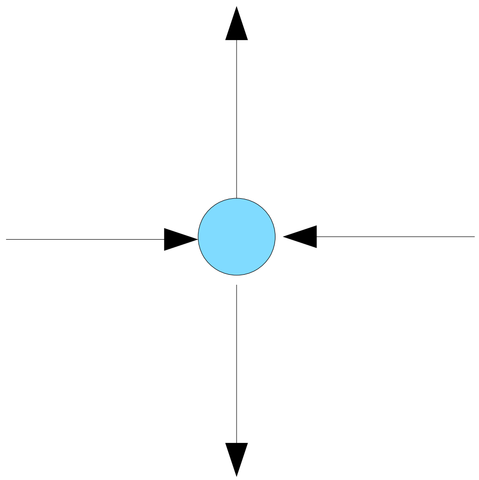

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Review: 1712.04721 Update: 2305.01960

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Review: 1712.04721 Update: 2305.01960