Gauge Invariance and Particles

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Vienna
Austria

UNI GRAZ

NAWI Graz
Natural Sciences

FWF Österreichischer Wissenschaftsfonds

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Gauge symmetry needs to be respected:
Elementary particles are not observables

Review: 1712.04721 Update: 2305.01960

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- Brout-Englert-Higgs Physics
- Fröhlich-Morchio-Strocchi mechanism
  - Standard Model
    - Experimental signatures

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- Brout-Englert-Higgs Physics
- Fröhlich-Morchio-Strocchi mechanism
    - Standard Model
        - Experimental signatures
    - Beyond the Standard Model
        - Qualitative changes

Review: 1712.04721 Update: 2305.01960

Brout-Englert-Higgs Physics
-
The Standard Model

A toy model

A toy model: Higgs sector of the SM

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- Consider an SU(2) with a fundamental scalar
- Essentially the standard model Higgs

$$L = -\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^a_{bc} W^b_\mu W^c_\nu$$

- Ws $\quad W^a_\mu$ 

- Coupling g and some numbers f^{abc}

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$$L=-\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu}+\left(D_{\mu}^{ij}h^{j}\right)^{+}D_{ik}^{\mu}h_{k}$$

$$W_{\mu\nu}^{a}=\partial_{\mu}W_{\nu}^{a}-\partial_{\nu}W_{\mu}^{a}+gf_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$

$$D_{\mu}^{ij}=\delta^{ij}\partial_{\mu}-igW_{\mu}^{a}t_{a}^{ij}$$

- Ws W_{μ}^{a} W
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- Couplings g, v, λ and some numbers f^{abc} and t_a^{ij}
- Parameters selected for a BEH effect

A toy model: Symmetries

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$$W_{\mu}^{a}\rightarrow W_{\mu}^{a}+(\delta_{b}^{a}\partial_{\mu}-g\,f_{bc}^{a}W_{\mu}^{c})\phi^{b}$$

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- Global SU(2) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_{\mu}^{a}\rightarrow W_{\mu}^{a} \qquad h\rightarrow h\Omega$$

Textbook approach

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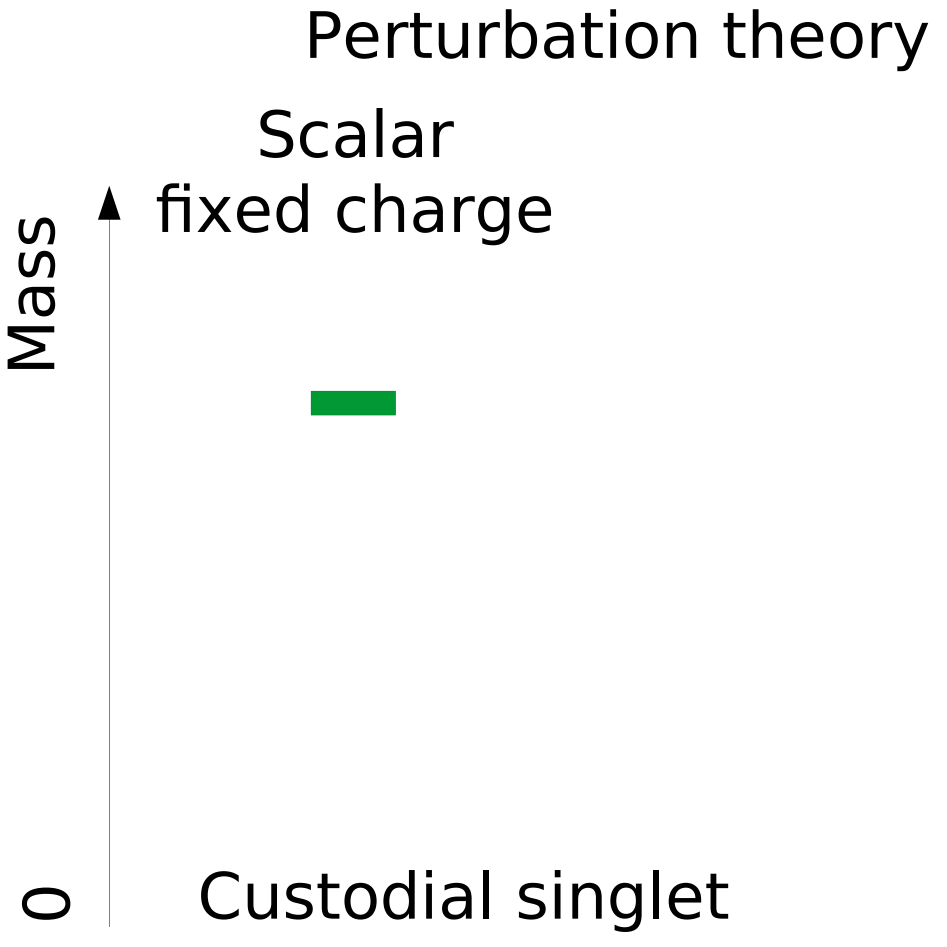
- Get masses and degeneracies at tree-level
- Perform perturbation theory

Physical spectrum

Perturbation theory

Physical spectrum



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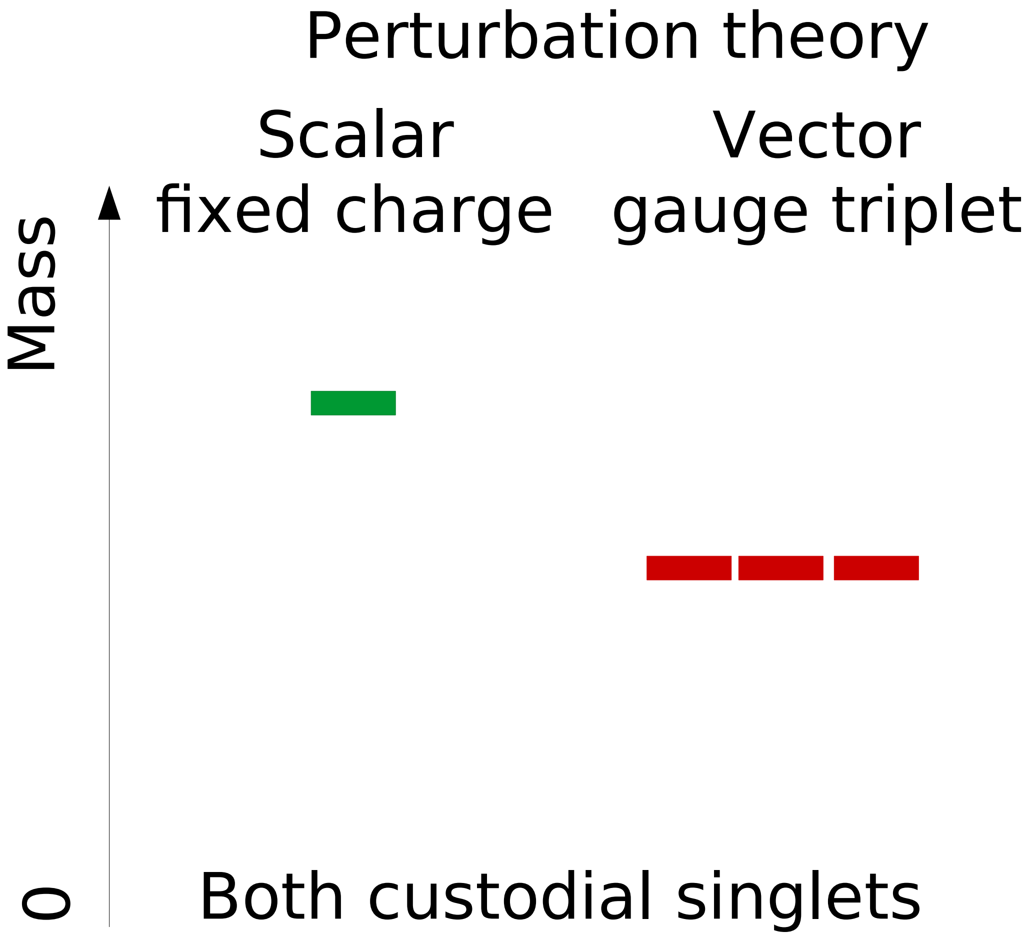

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[Fröhlich et al.'80,
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  - And this includes non-perturbative aspects...
  - ...even at weak coupling [Gribov'78,Singer'78,Fujikawa'82]

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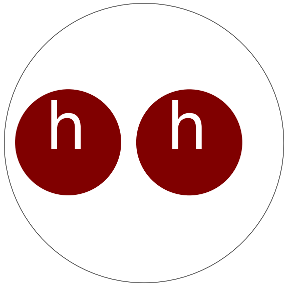

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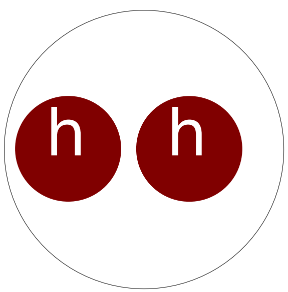

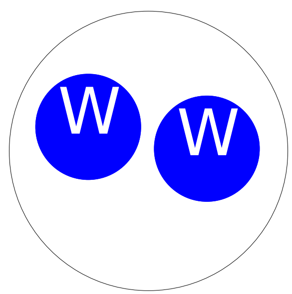

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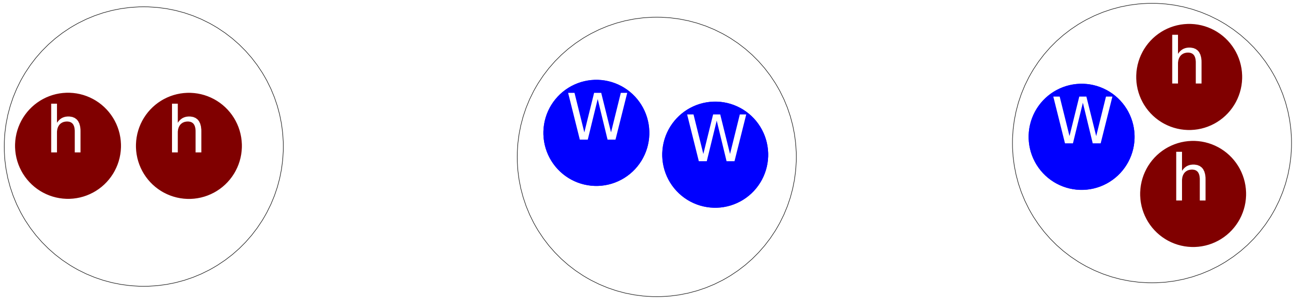

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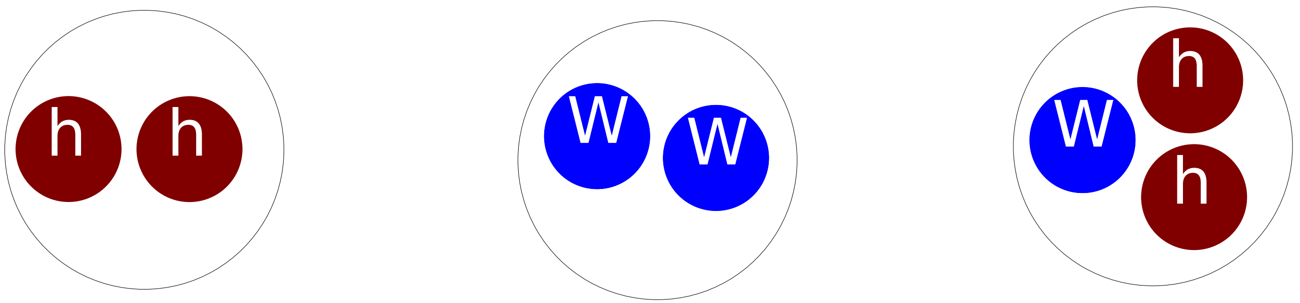

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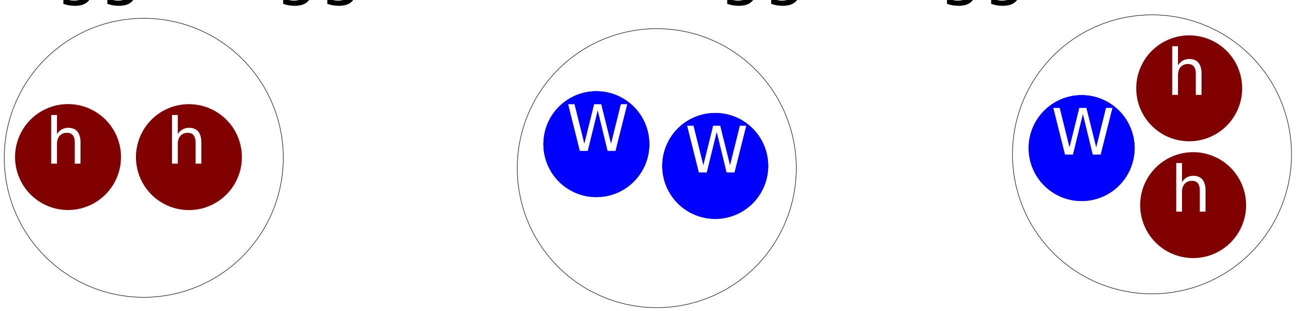

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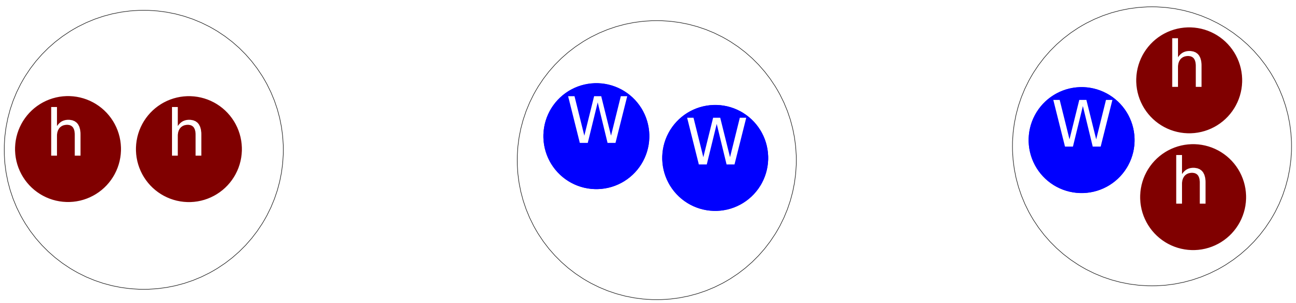
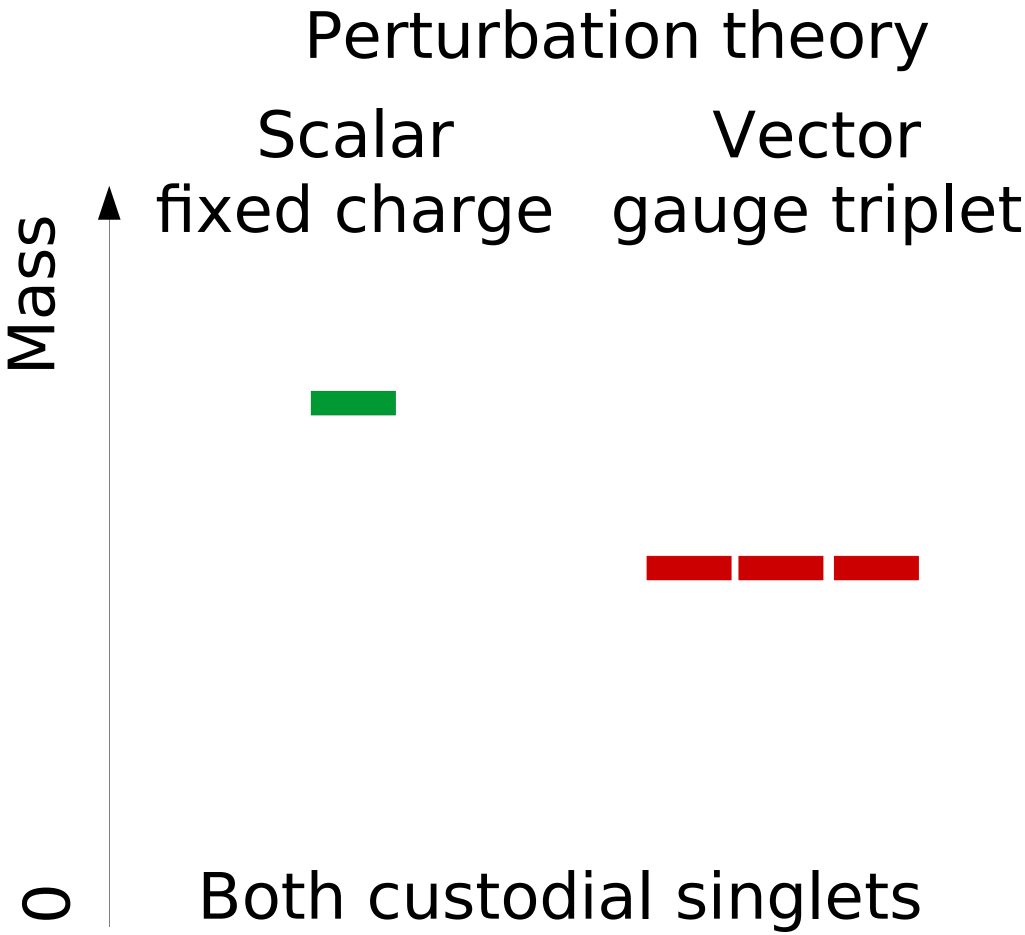


- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)
- Can this matter?

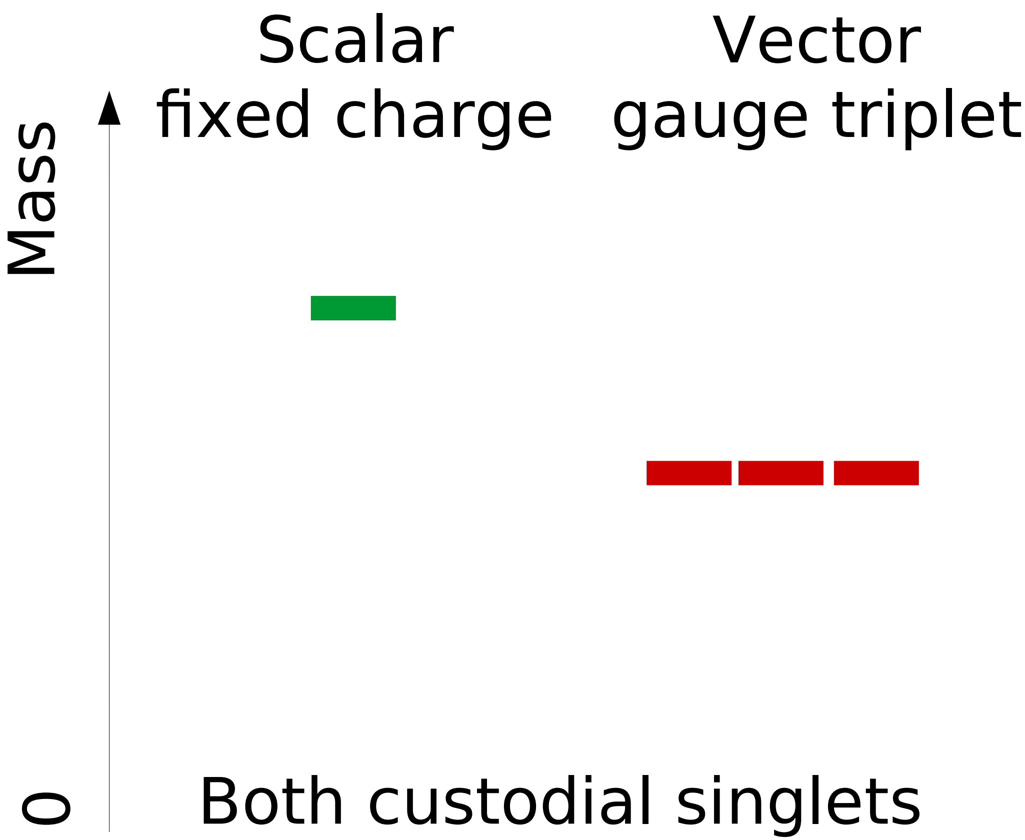
Physical spectrum



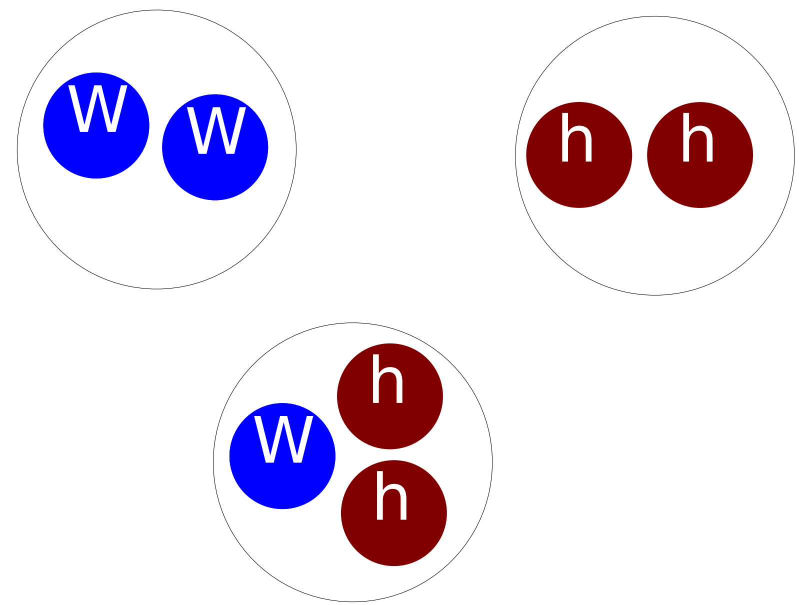
Remember: Experiment tells that somehow the left is correct!

Physical spectrum

Perturbation theory



Composite (bound) states



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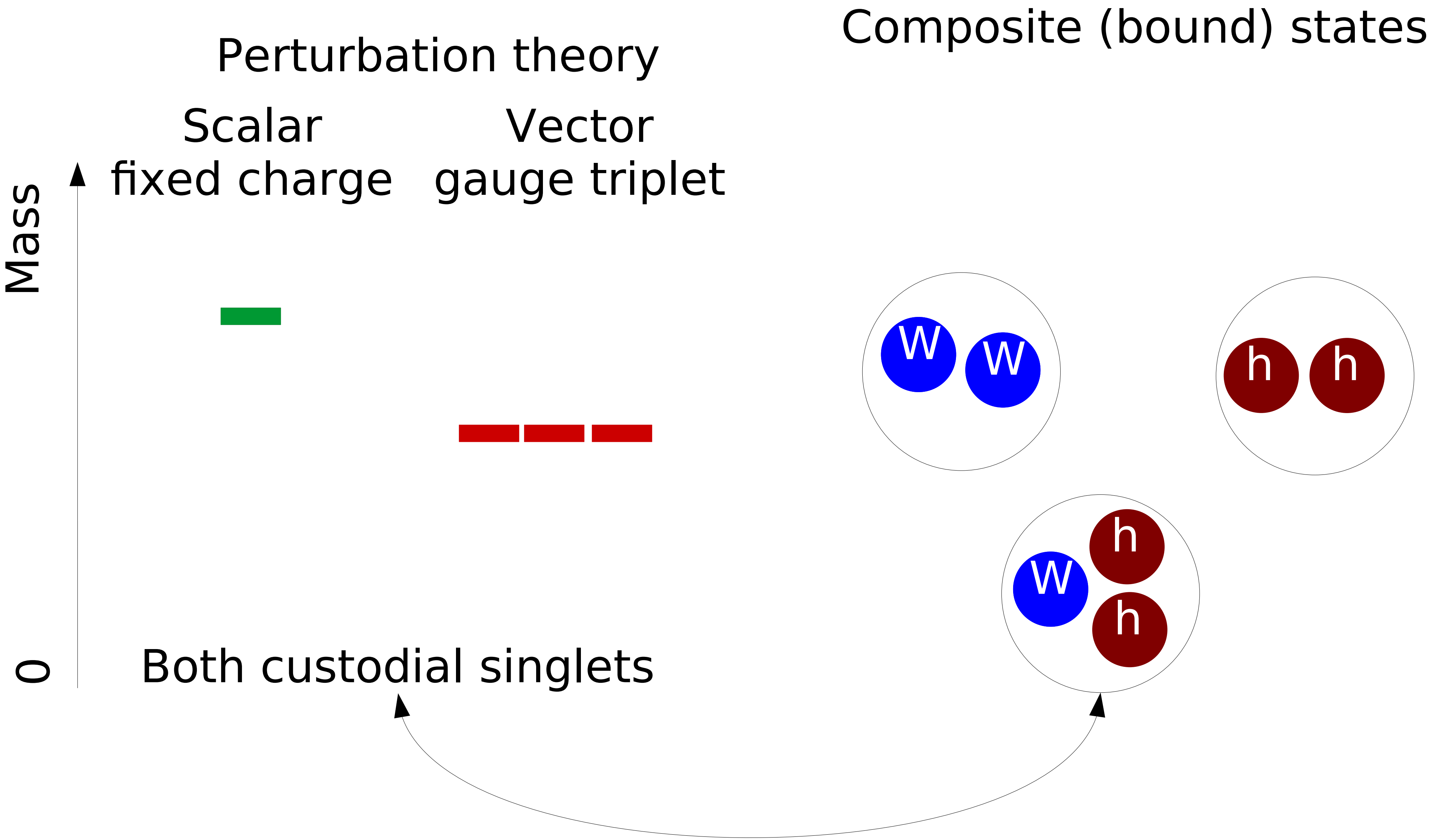

Experiment tells that somehow the left is correct
Theory say the right is correct
There must exist a relation that both are correct

Physical particles

[Fröhlich et al.'80,'81, Maas & Törek'16,'18, Maas, Sondenheimer & Törek'17]

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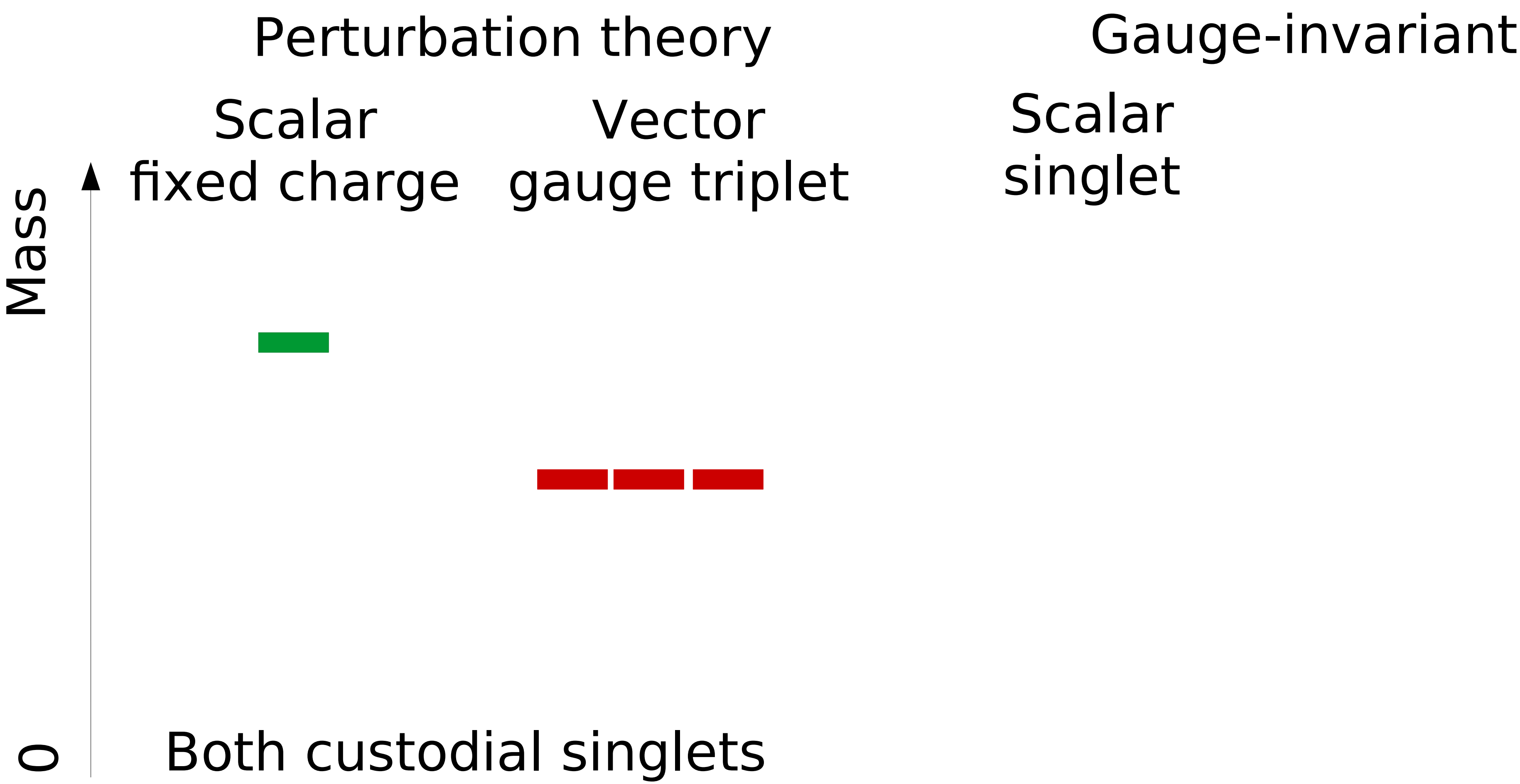
    - Standard lattice spectroscopy problem
    - Standard methods
      - Smearing, variational analysis, systematic error analysis etc.
    - Very large statistics (>10^5 configurations)

Physical spectrum

[Maas'12, Maas & Mufti'14]



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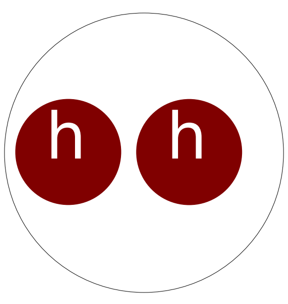

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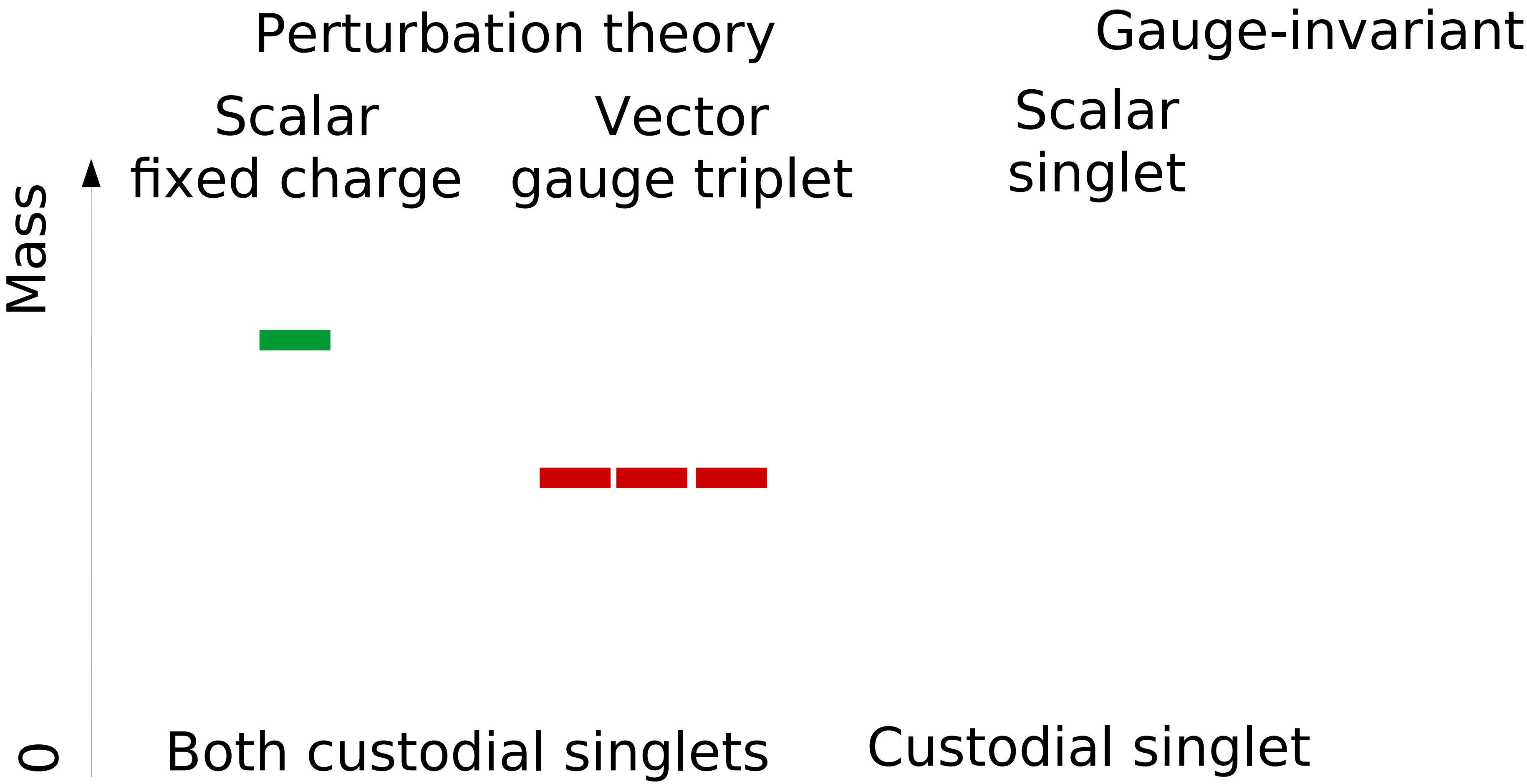

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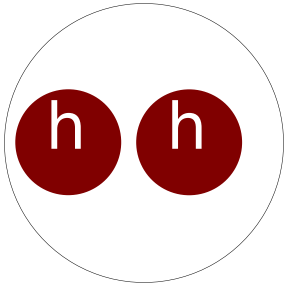

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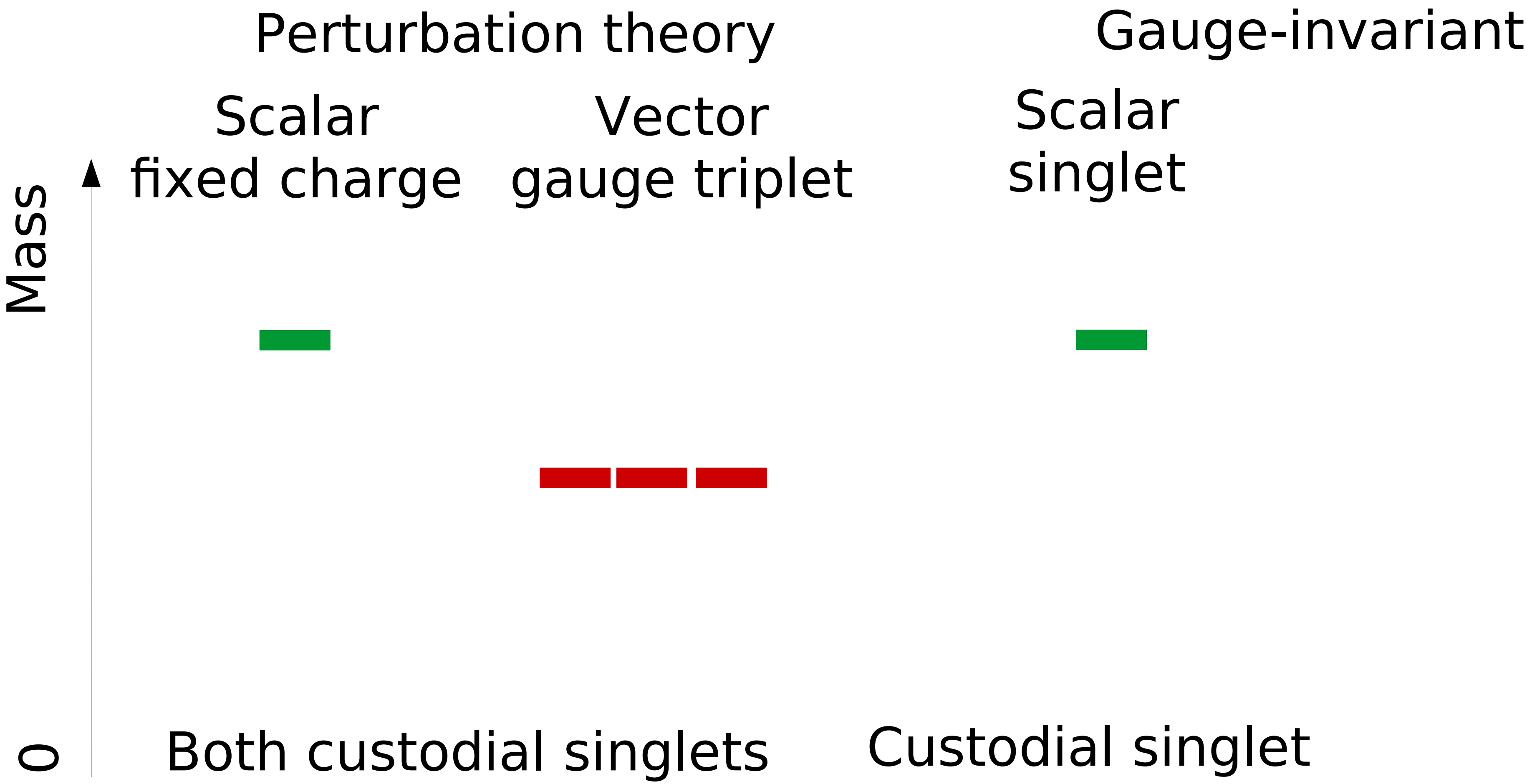

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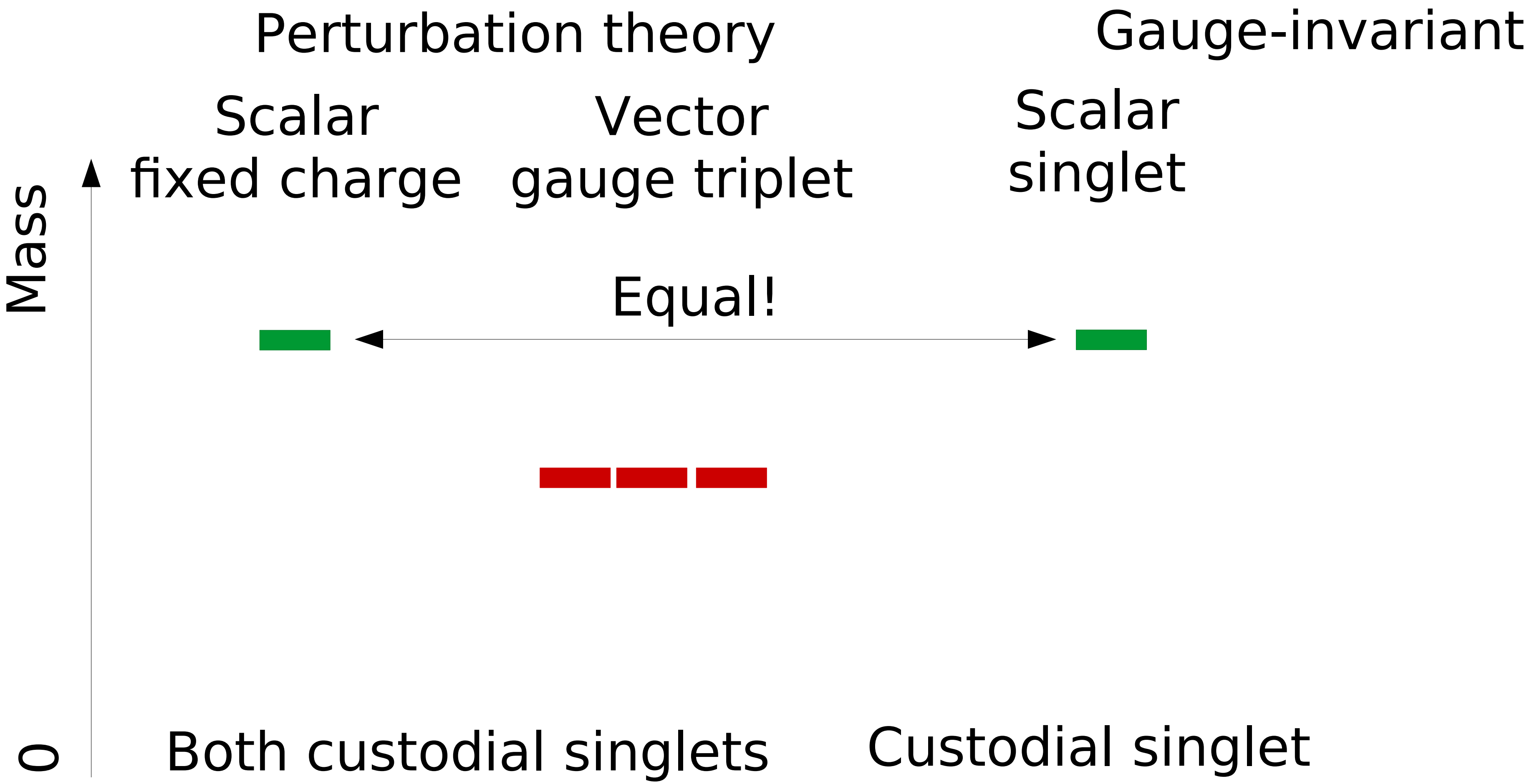

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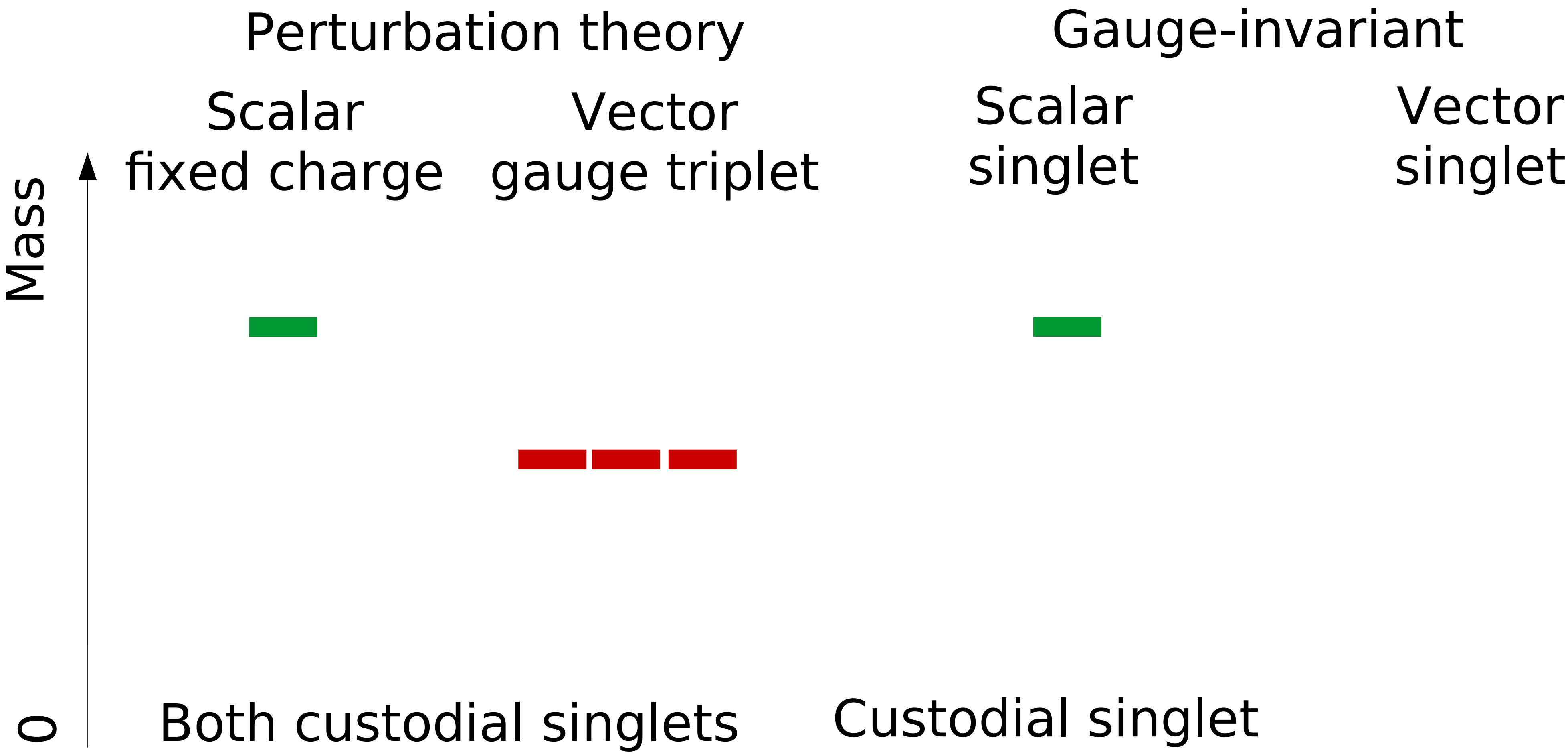
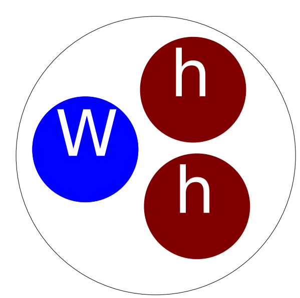


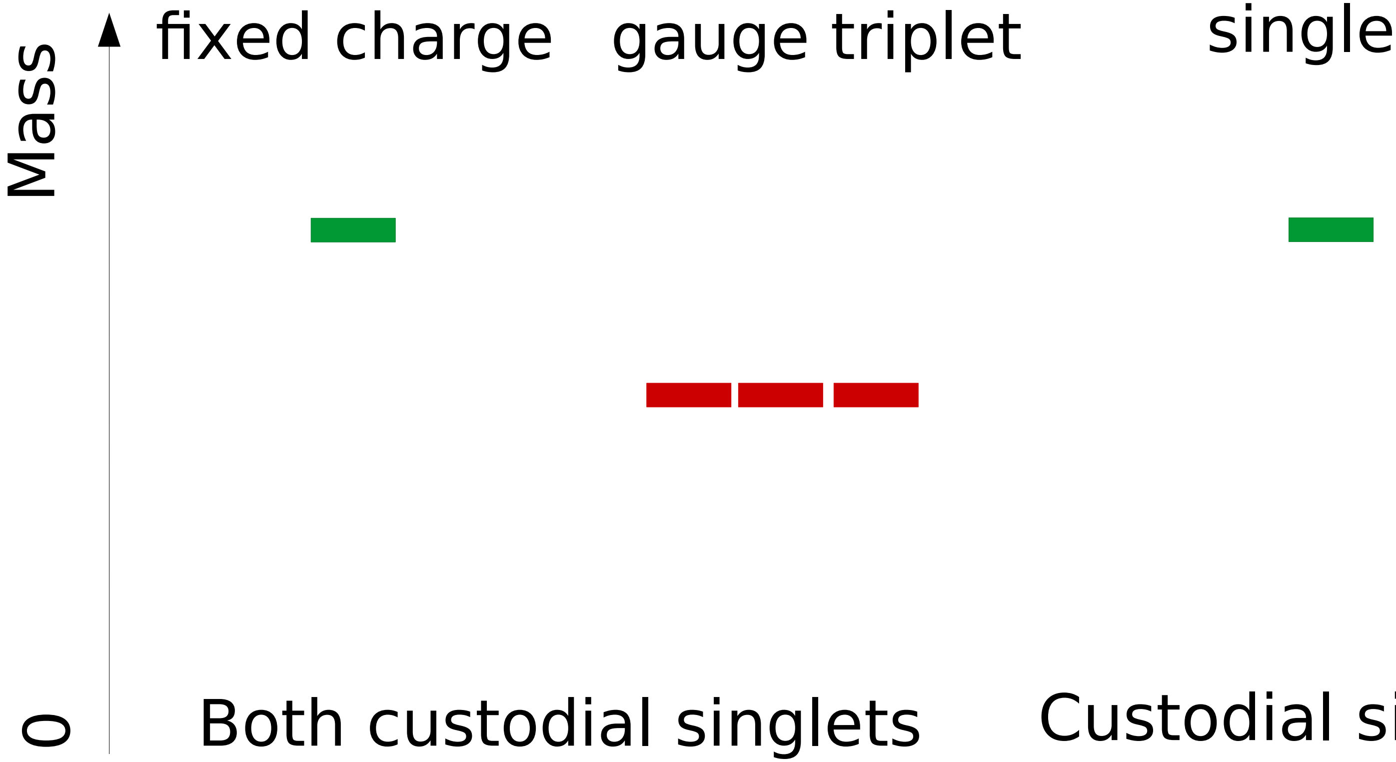
$$tr\, t^a \frac{h^+}{\sqrt{h^+ h}} D_\mu \frac{h}{\sqrt{h^+ h}}$$



Physical spectrum

[Maas'12, Maas & Mufti'14]

Perturbation theory		Gauge-invariant	
Scalar fixed charge	Vector gauge triplet	Scalar singlet	Vector singlet



Both custodial singlets — Custodial singlet — Triplet

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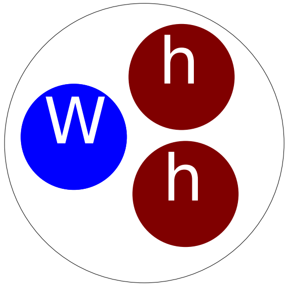

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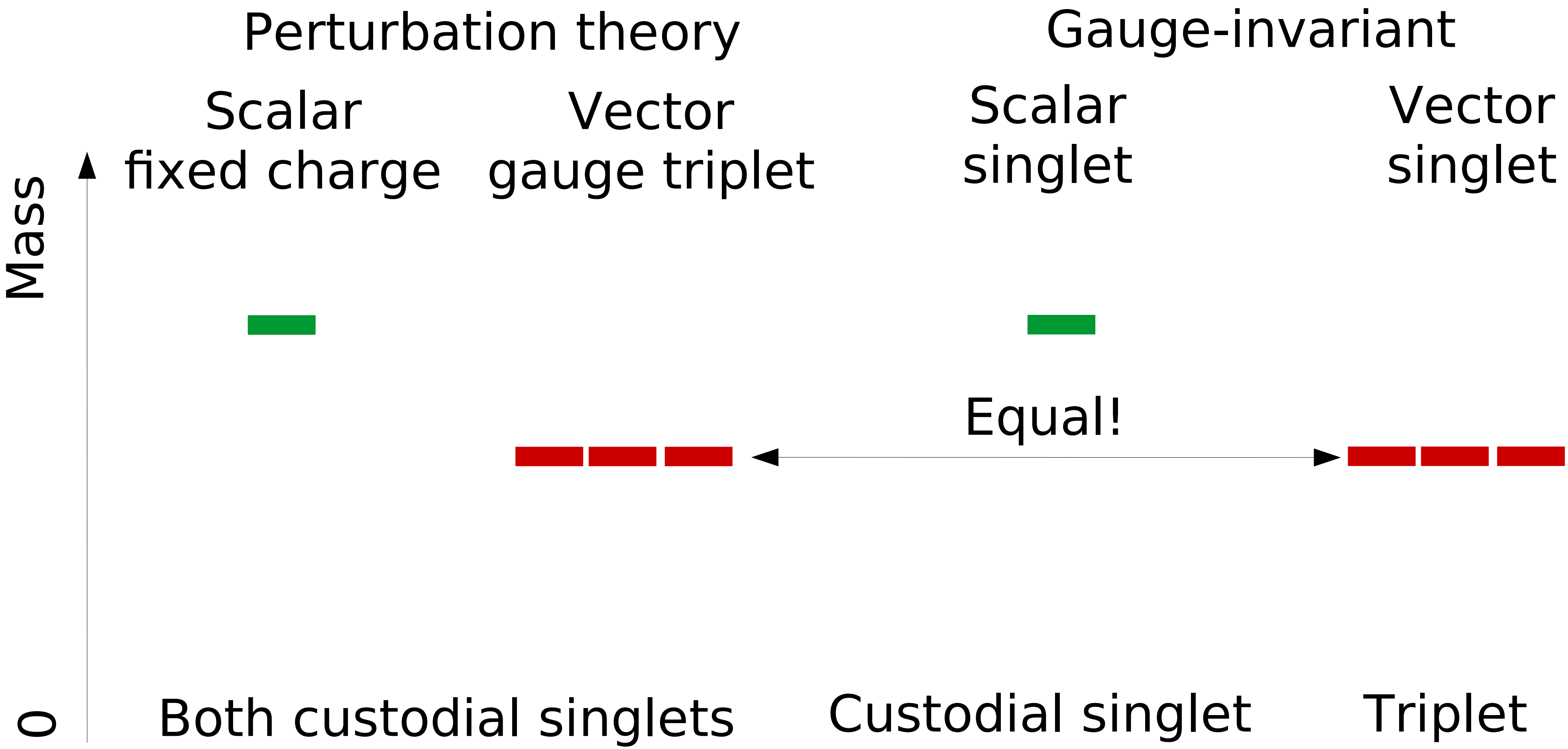

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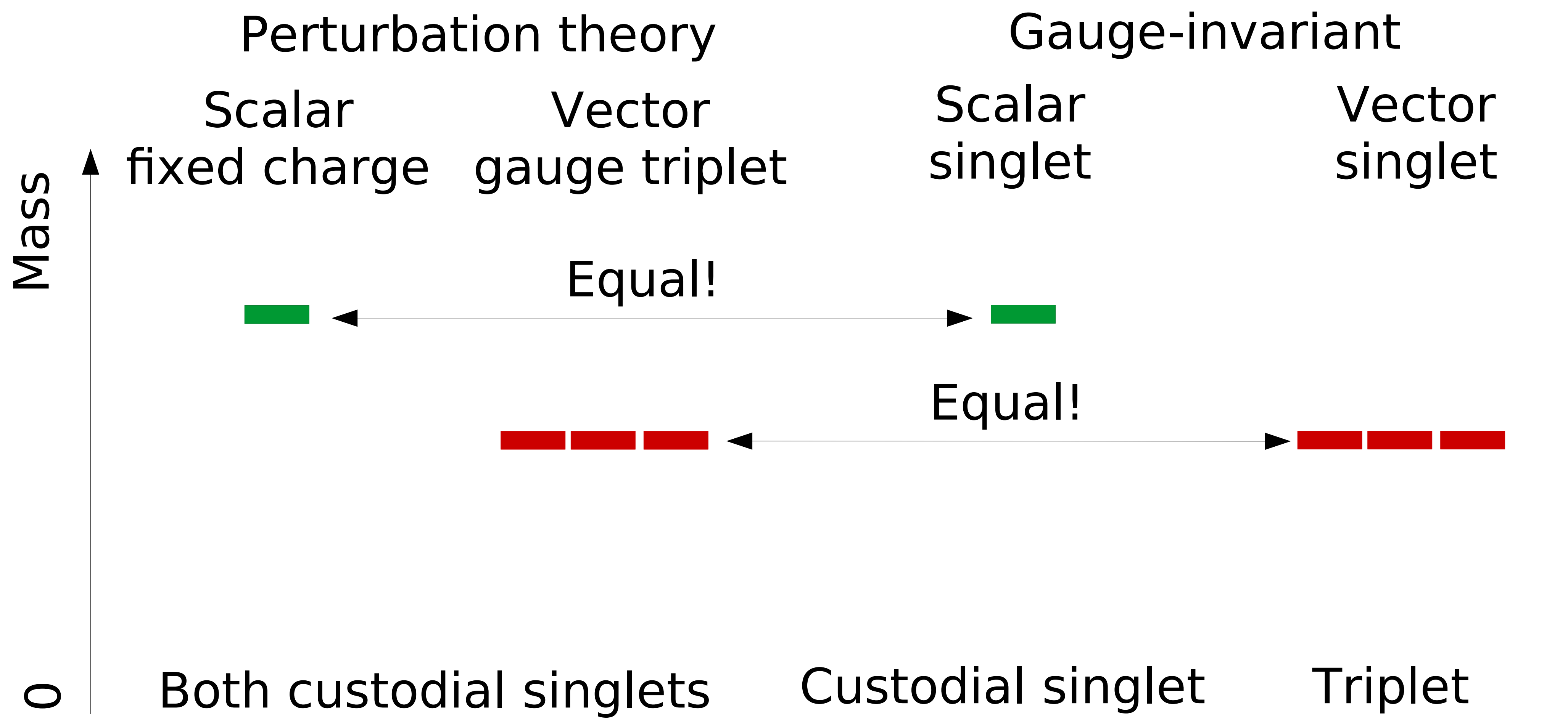

Why?

A microscopic origin
-
Fröhlich-Morchio-Strocchi mechanism

How to make predictions

[Fröhlich et al.'80,'81,
Maas & Törek'16,'18,
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  - Bound state structure – non-perturbative methods?
  - But coupling is still weak and there is a BEH
  - Perform double expansion [Fröhlich et al.'80, Maas’12]
    - Vacuum expectation value (FMS mechanism)
    - Standard expansion in couplings
    - Together: Augmented perturbation theory

Augmented perturbation theory

[Fröhlich et al.’80,’81
Maas'12,’17]

1) Formulate gauge-invariant operator

Augmented perturbation theory

[Fröhlich et al.'80,'81
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0^+ singlet: $\langle (h^+ h)(x)(h^+ h)(y)\rangle$

Higgs field

Augmented perturbation theory

[Fröhlich et al.’80,’81
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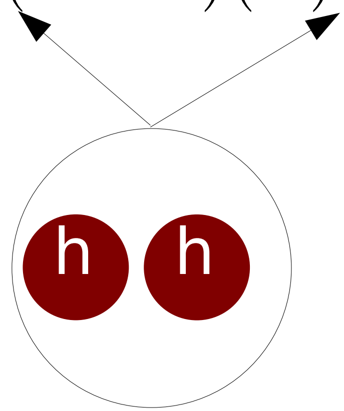

Augmented perturbation theory

[Fröhlich et al.’80,’81
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Trivial two-particle state

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Standard Perturbation Theory

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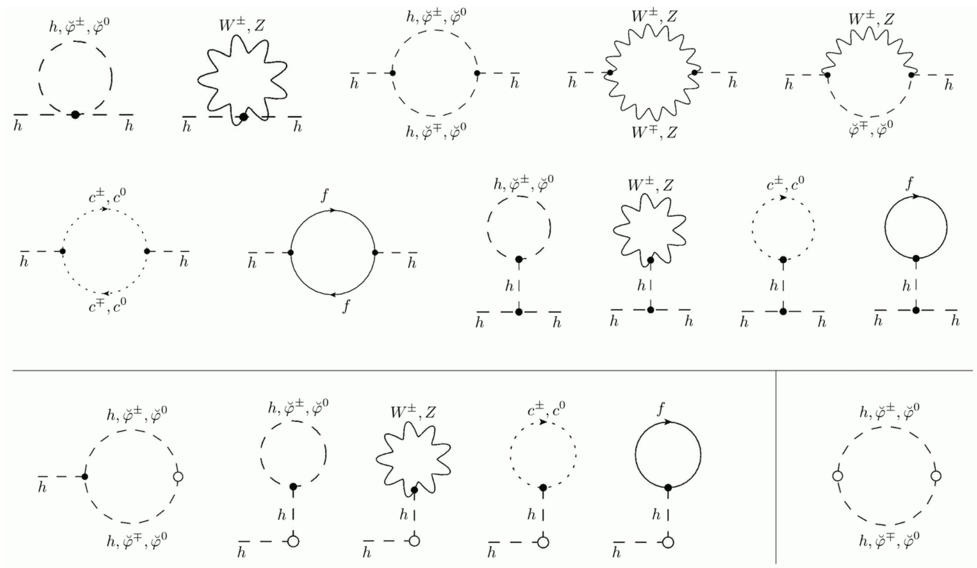

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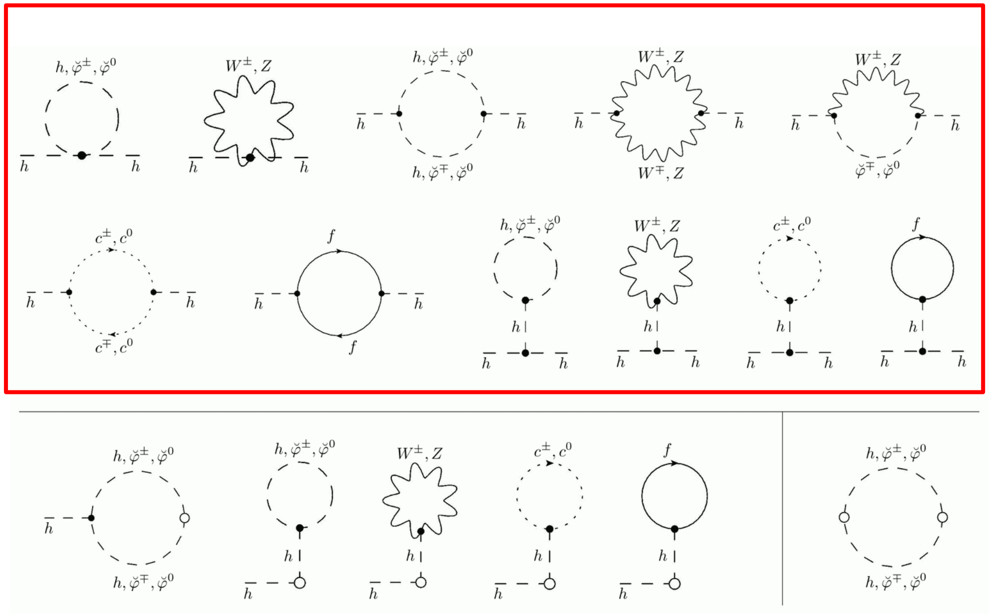

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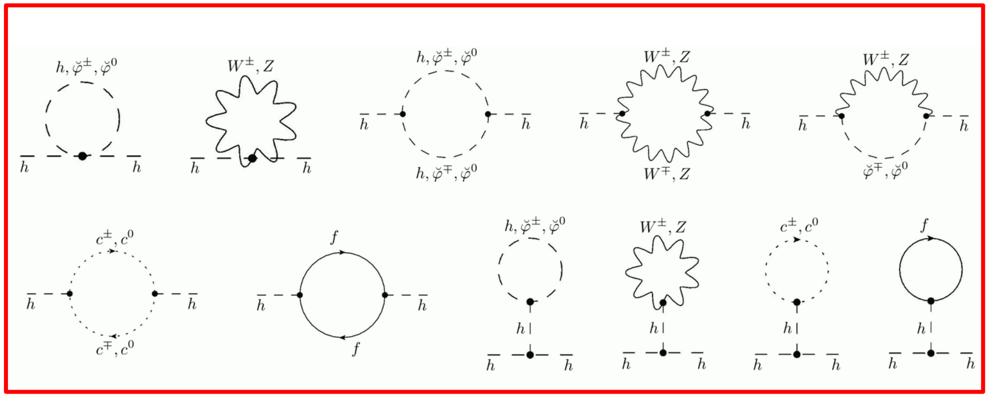

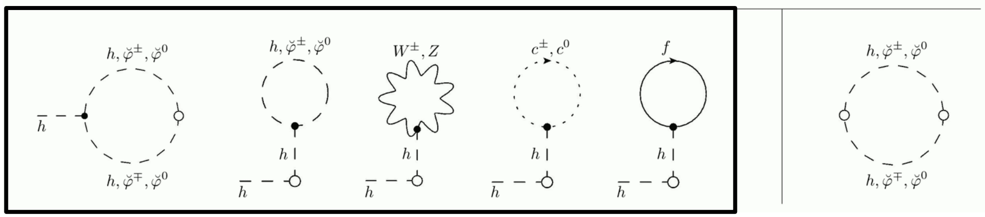

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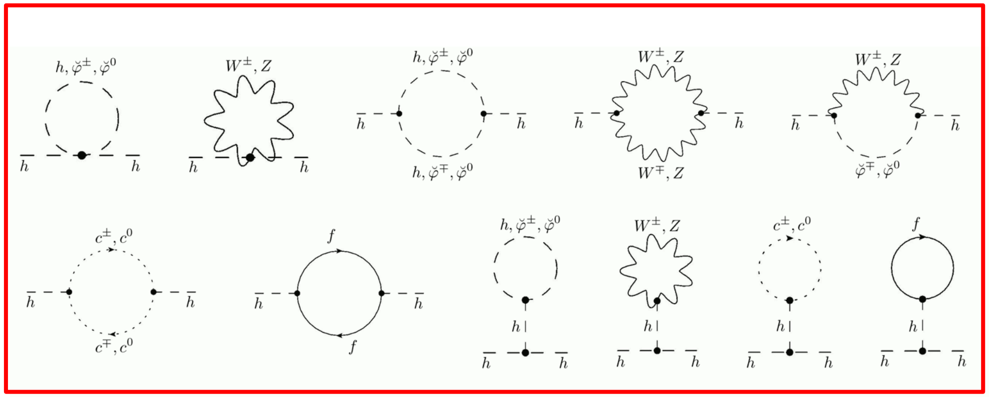

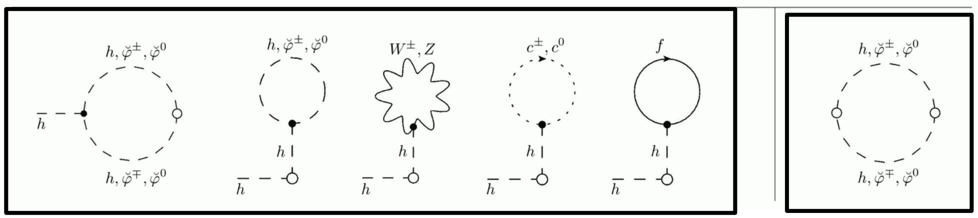

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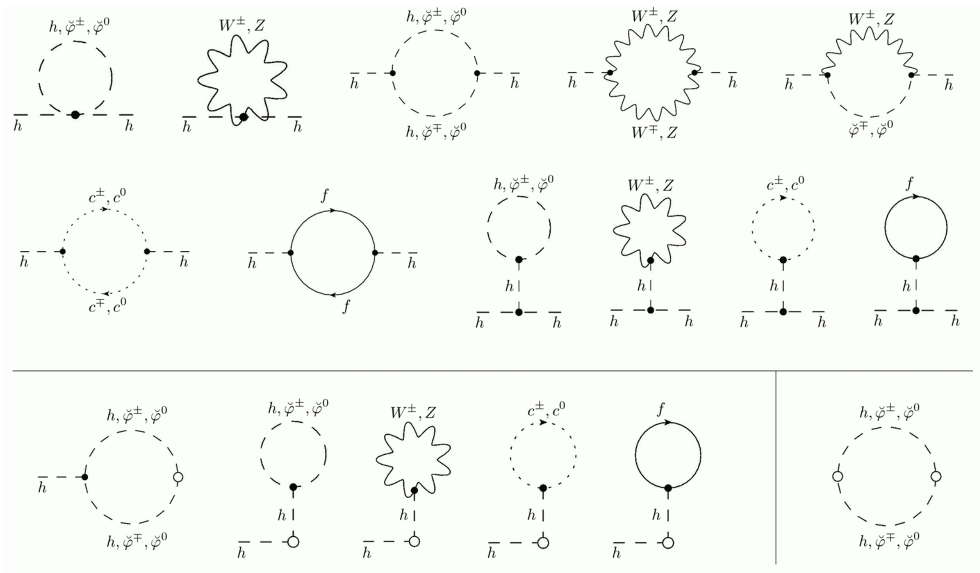

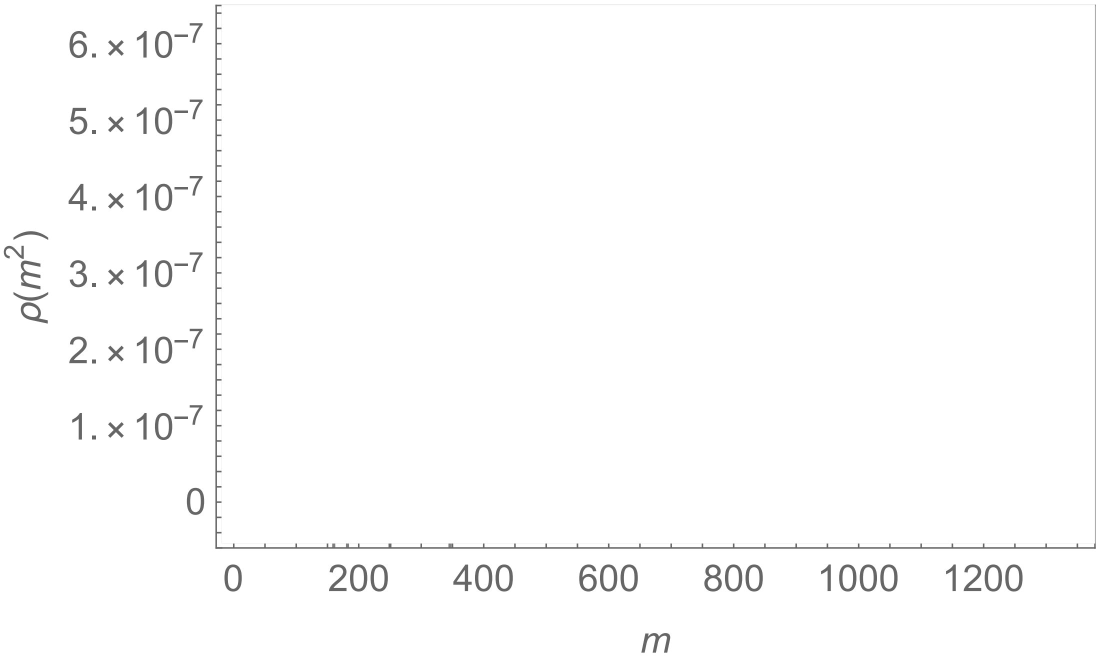

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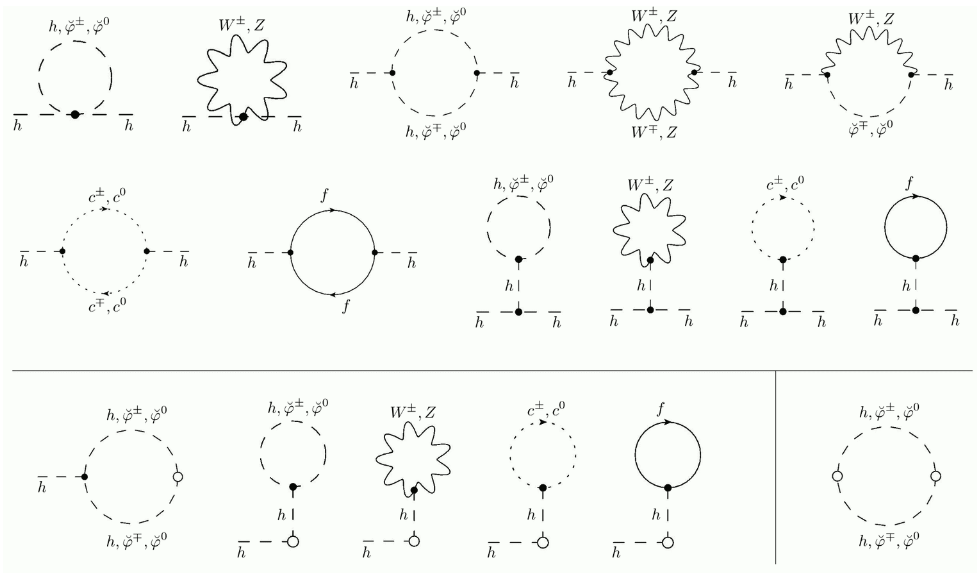

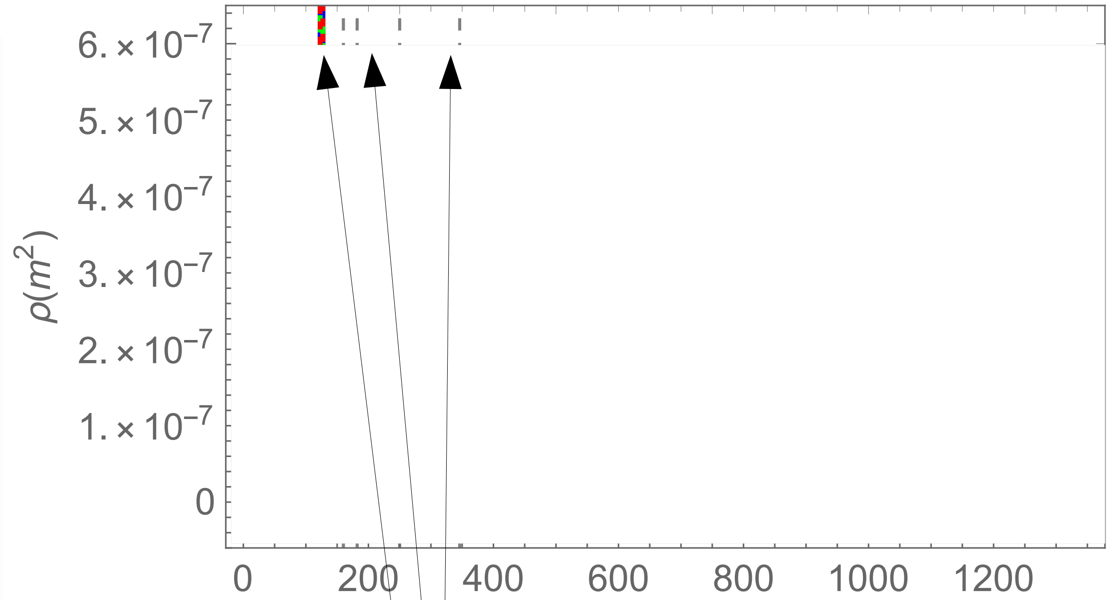

Physical thresholds

Consequences: The Higgs

[Maas'12,'17
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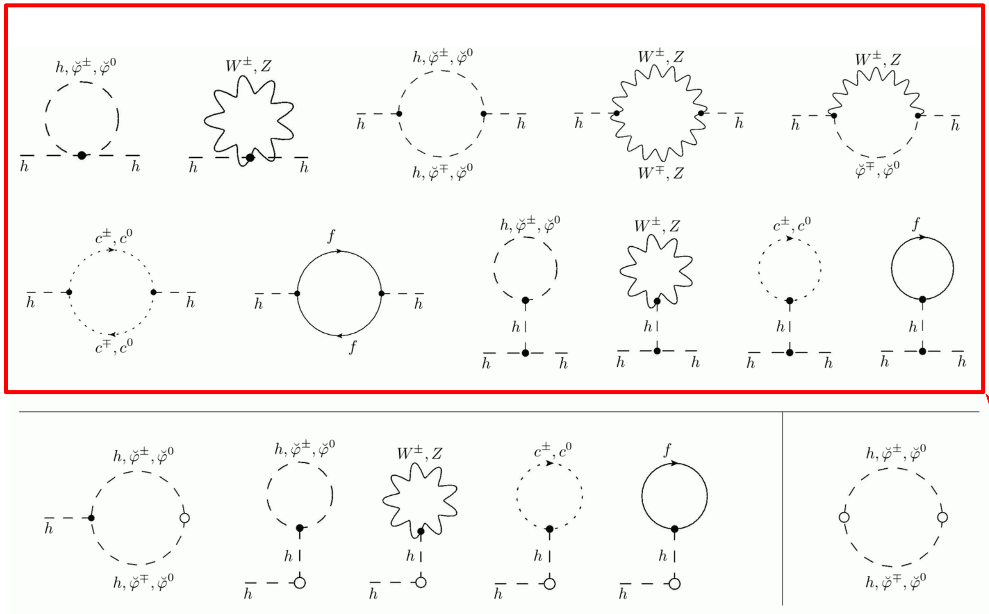

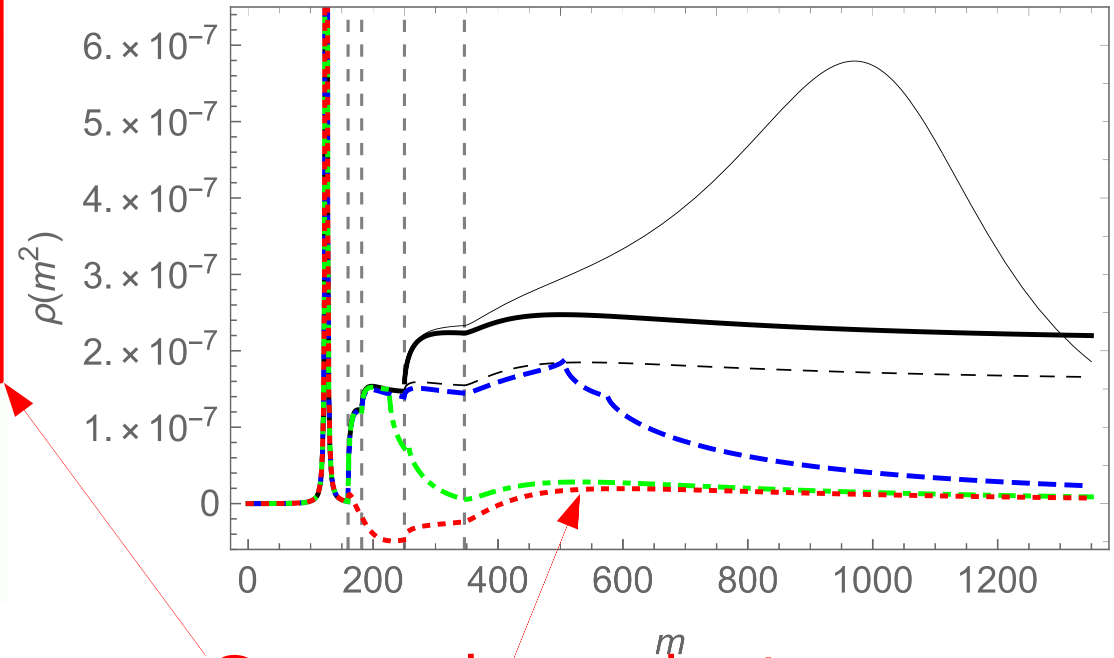

Gauge-dependent

Consequences: The Higgs

[Maas'12,'17
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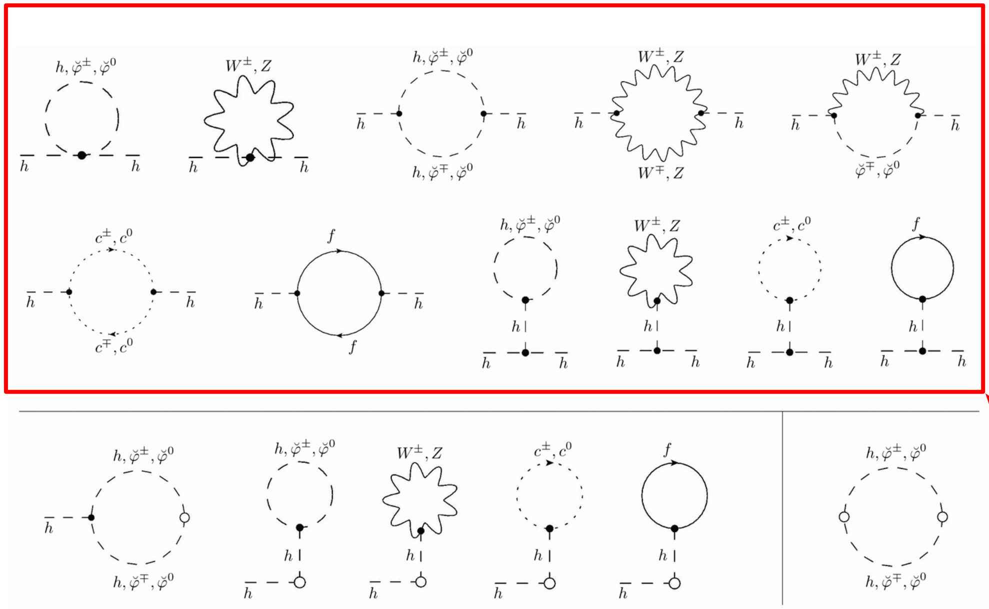

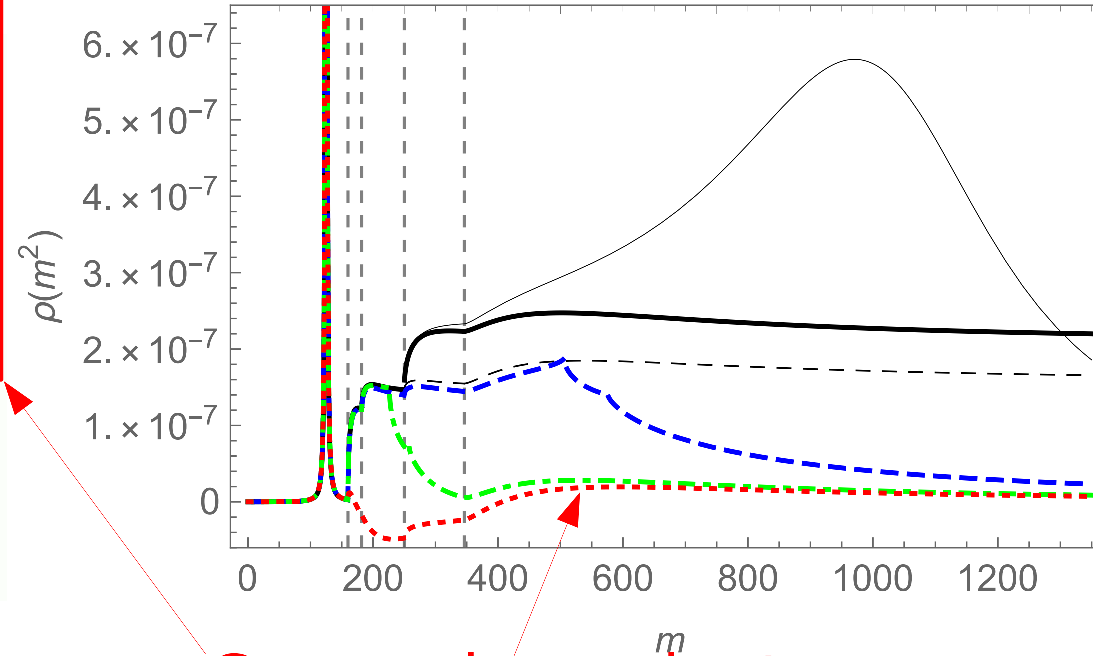

Gauge-dependent
Unphysical features:
Positivity violation
Additional thresholds

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[Maas'12,'17
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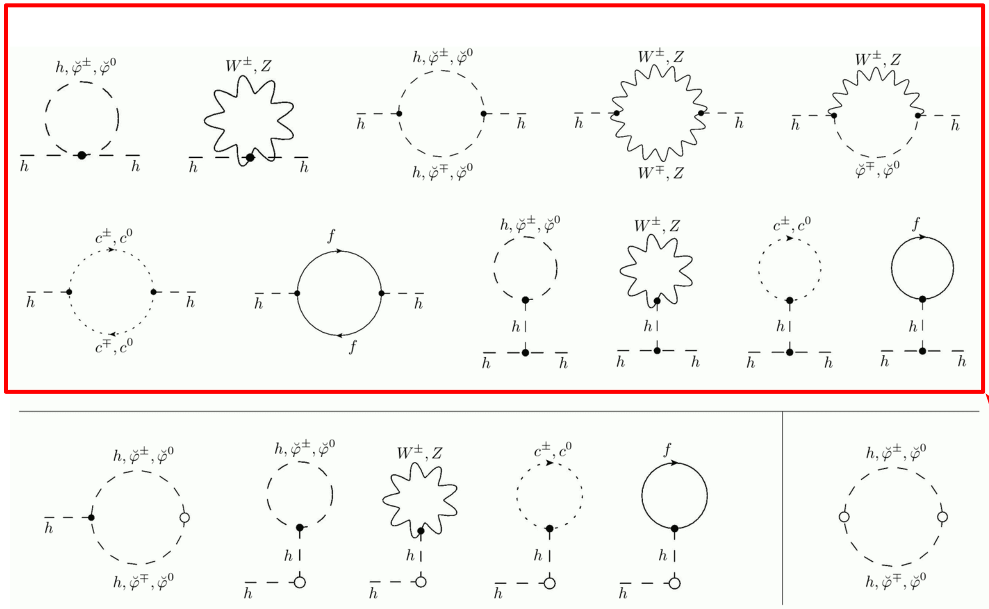

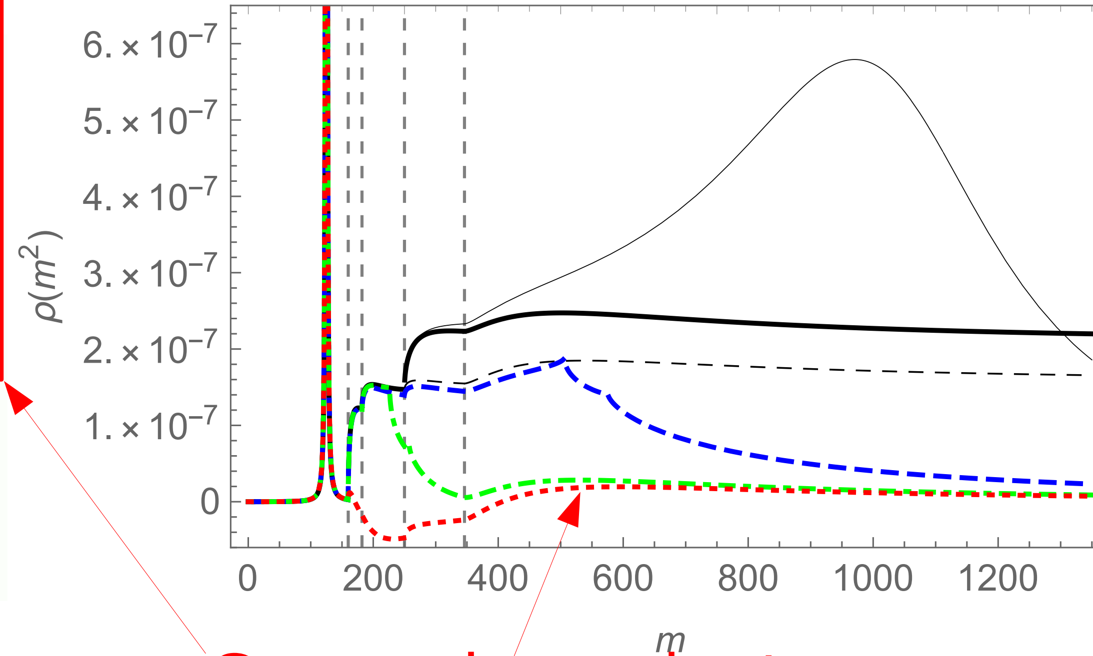

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Unphysical features:
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Not a consequence
of instability: Occurs even
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Higgs in a toy theory

Consequences: The Higgs

[Maas'12,'17
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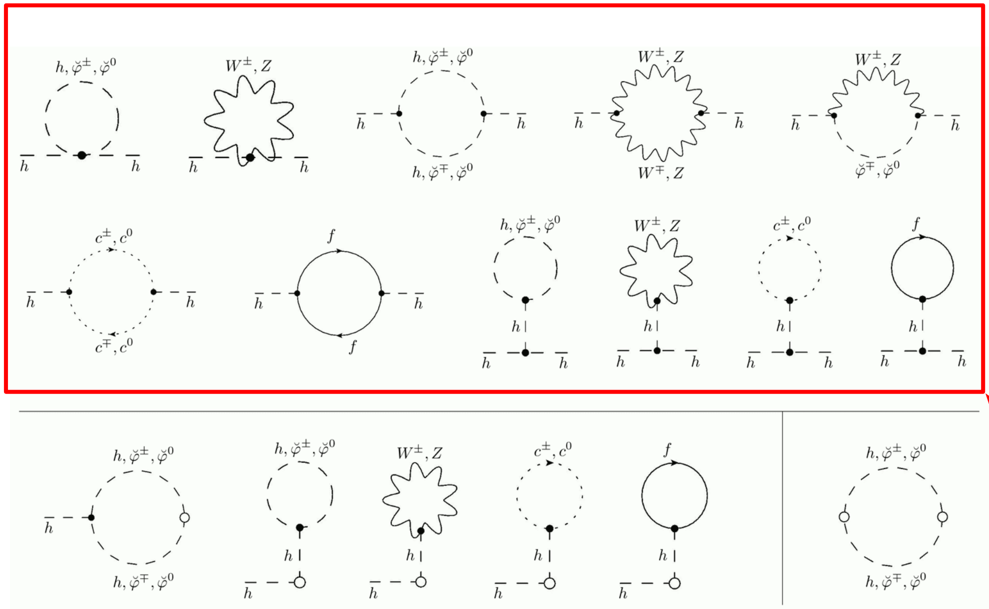

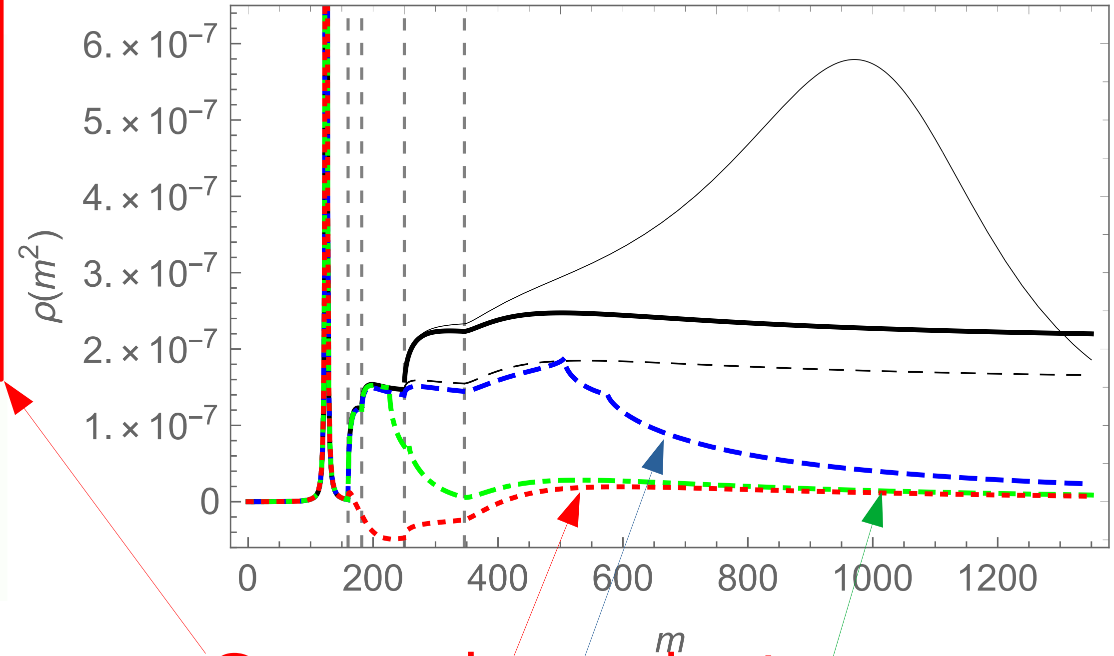

Gauge-dependent

Other gauge choices

Consequences: The Higgs

[Maas'12,'17
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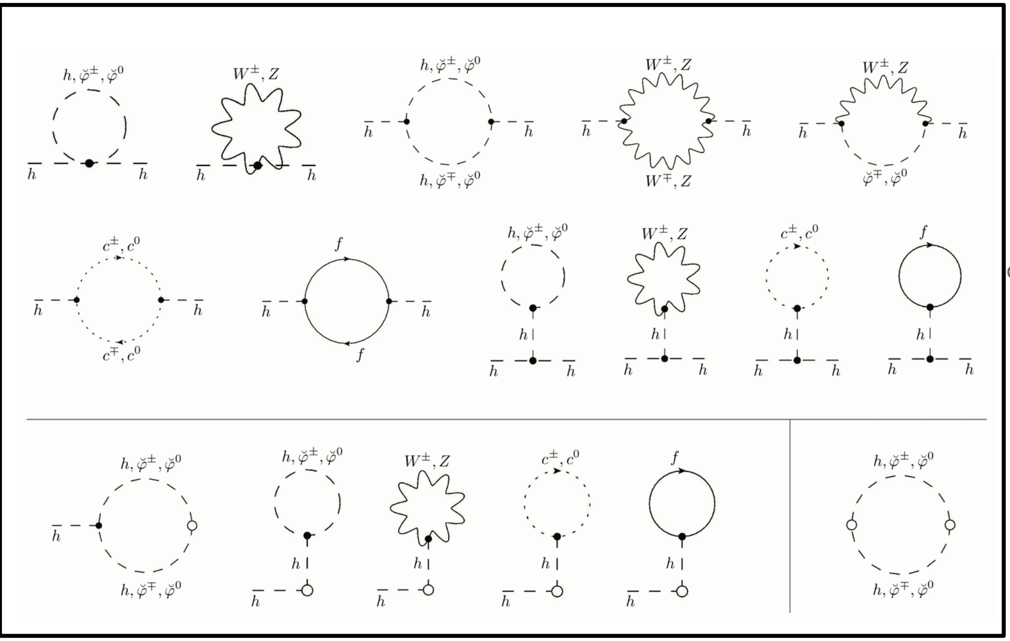

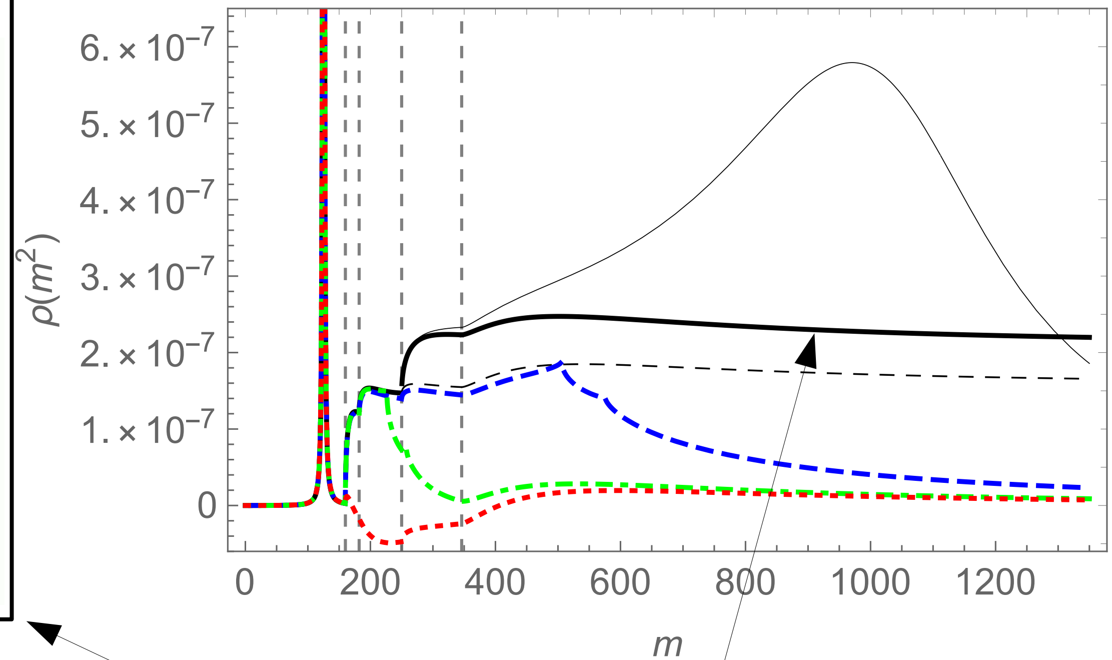

Physical – same for all gauge choices

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What about the vector?

[Fröhlich et al.’80,’81
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Exactly one gauge boson for every physical state

Phenomenological Implications
-
Can we measure this?

Bound states as extended objects

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- Two possibilities to measure extension

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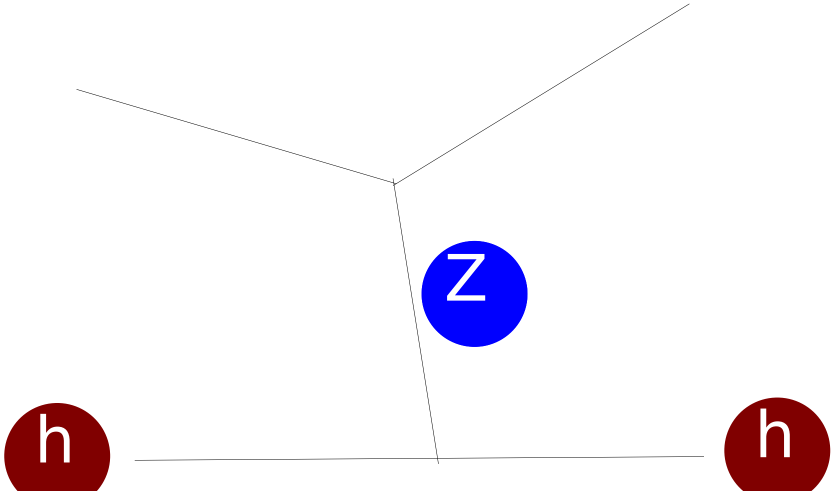
  - Form factor
    - Difficult
      - Higgs and Z need to be both produced in the same process

[Maas, Raubitzek, Törek’18]



Bound states as extended objects

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    - Standard vector boson scattering process at low energies
    - Use this one

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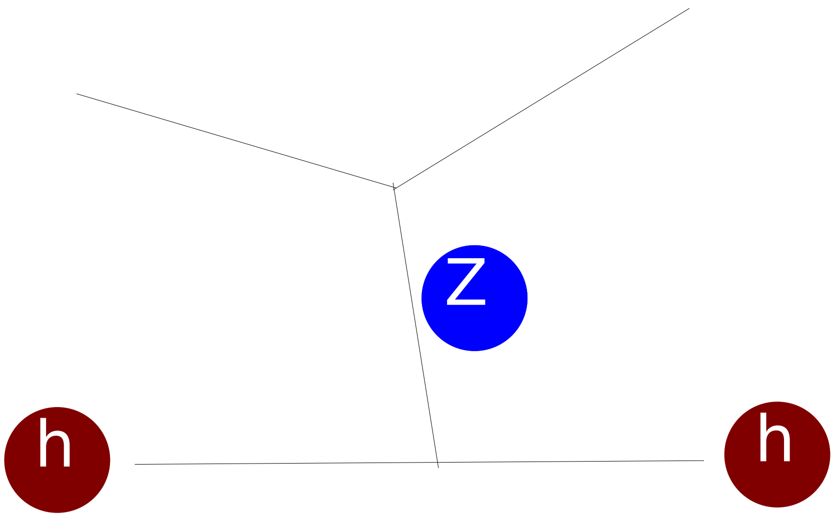

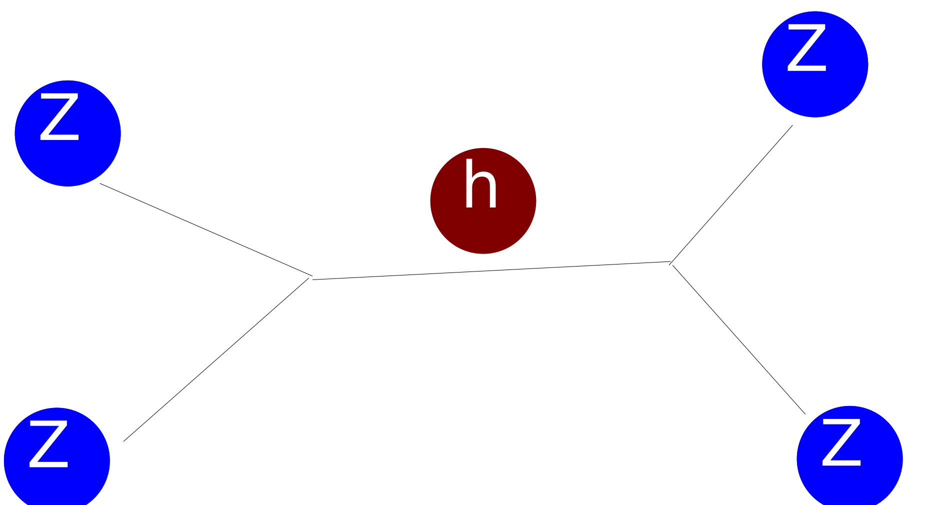

[Jenny, Maas, Riederer’22]

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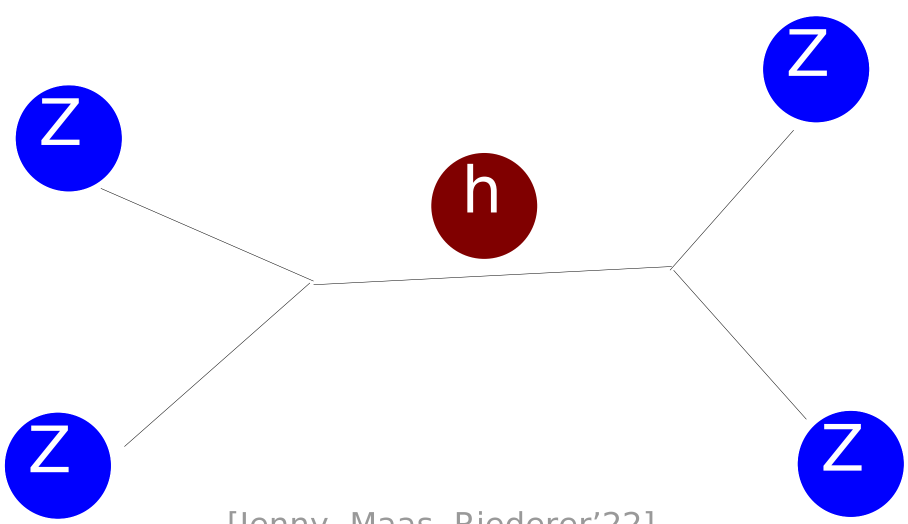

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Radius from elastic scattering in VBS

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Cross section

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Matrix element

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Legendre polynom

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$$M(s,\Omega) = 16\pi \sum_J (2J+1) f_J(s) P_J(\cos\theta)$$

$$f_J(s) = e^{i\delta_J(s)} \sin(\delta_J(s))$$

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Phase shift

Scattering length~"size"

Radius from elastic scattering in VBS

- Elastic region: $160/180\, GeV \leqslant \sqrt{s} \leqslant 250\, GeV$
  - *s* is the CMS energy in the initial/final ZZ/WW system
- Requires a partial wave analysis

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Phase shift

→ Lattice Lüscher analysis

Impact of a finite size of the Higgs

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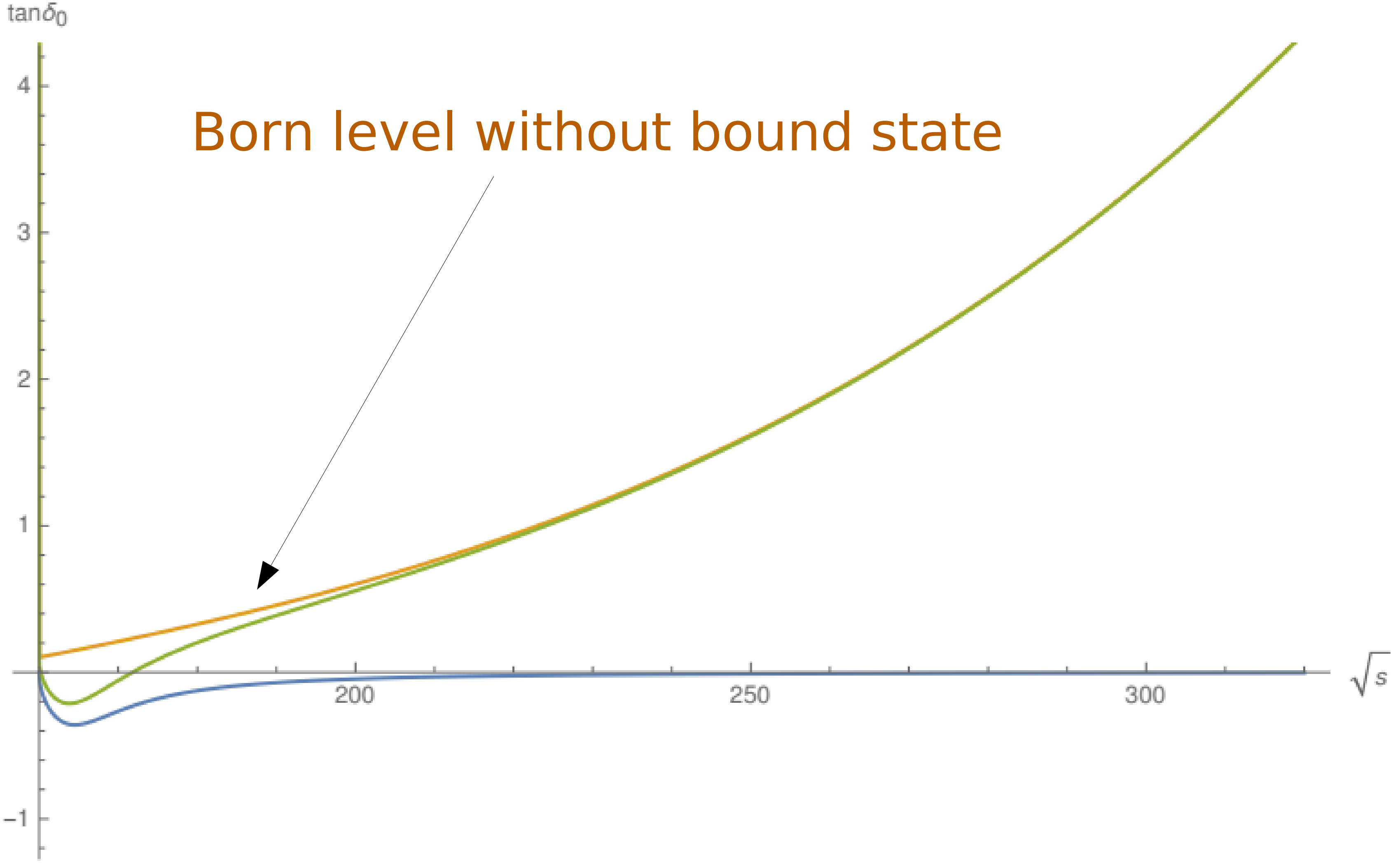
Impact of a finite size of the Higgs



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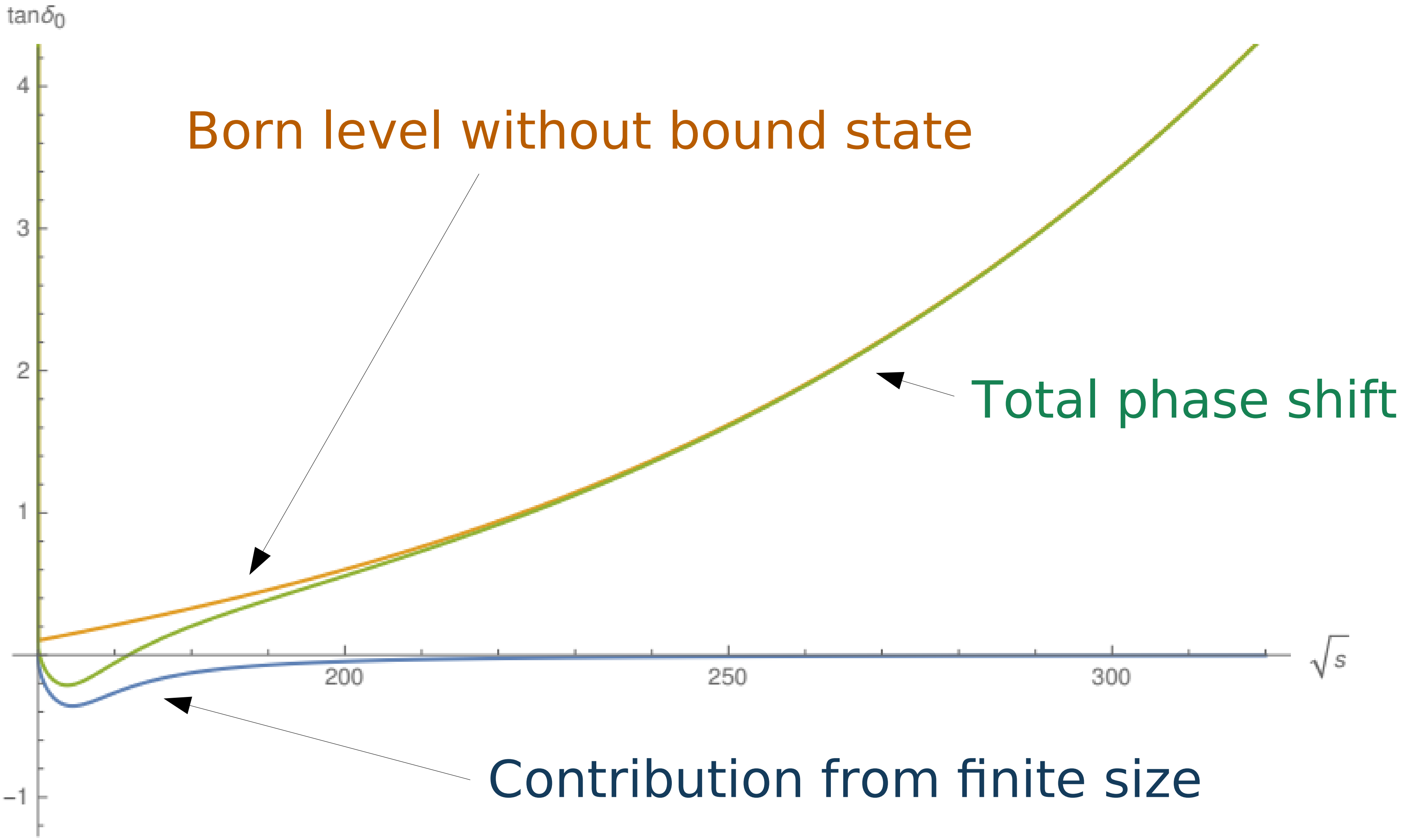

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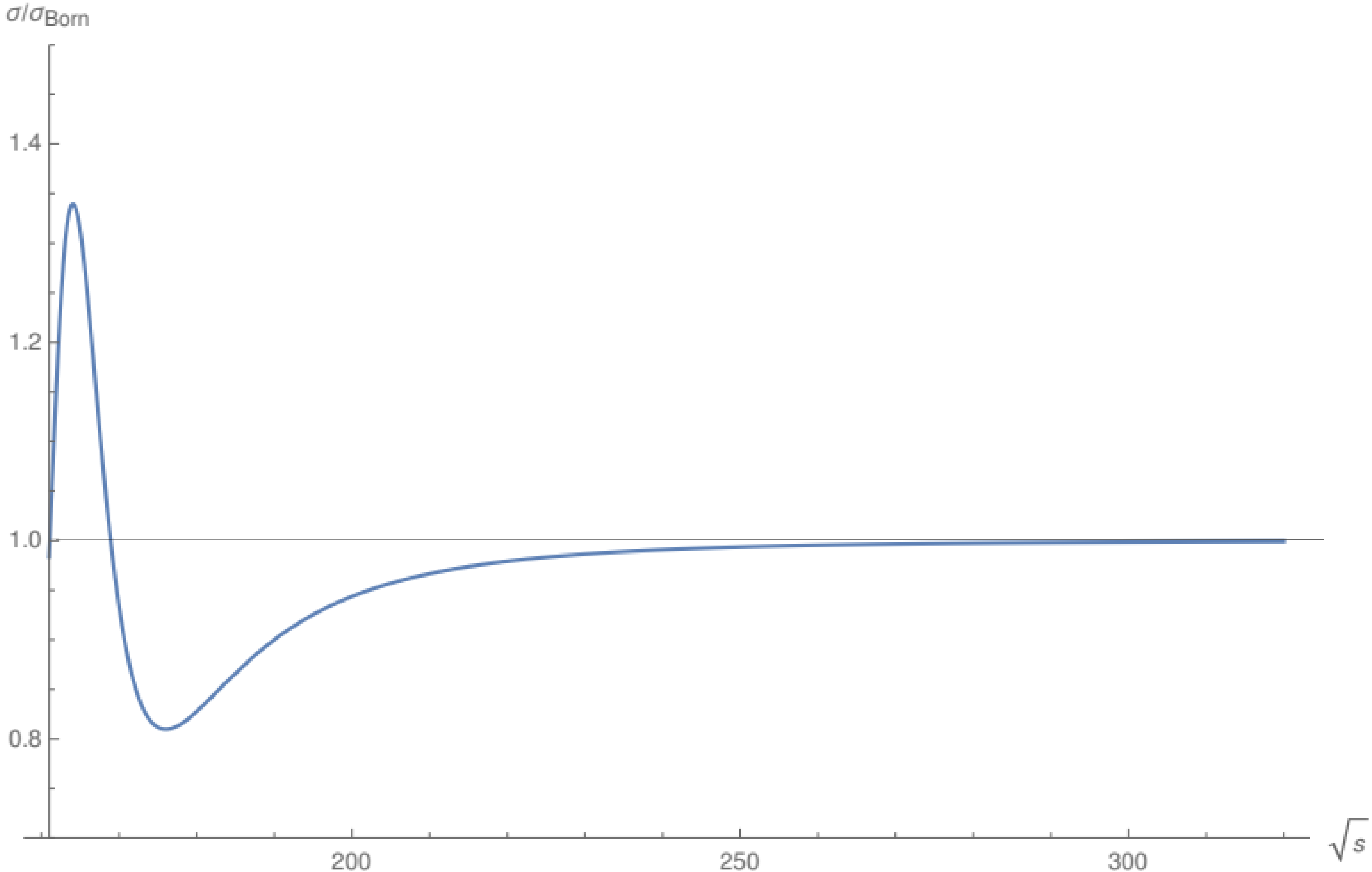

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Impact on the radius of the Higgs

- Reduced SM: Only W/Z and the Higgs
  - Parameters slightly different
    - Higgs 145 GeV and weak coupling larger

[Jenny, Maas, Riederer'22]

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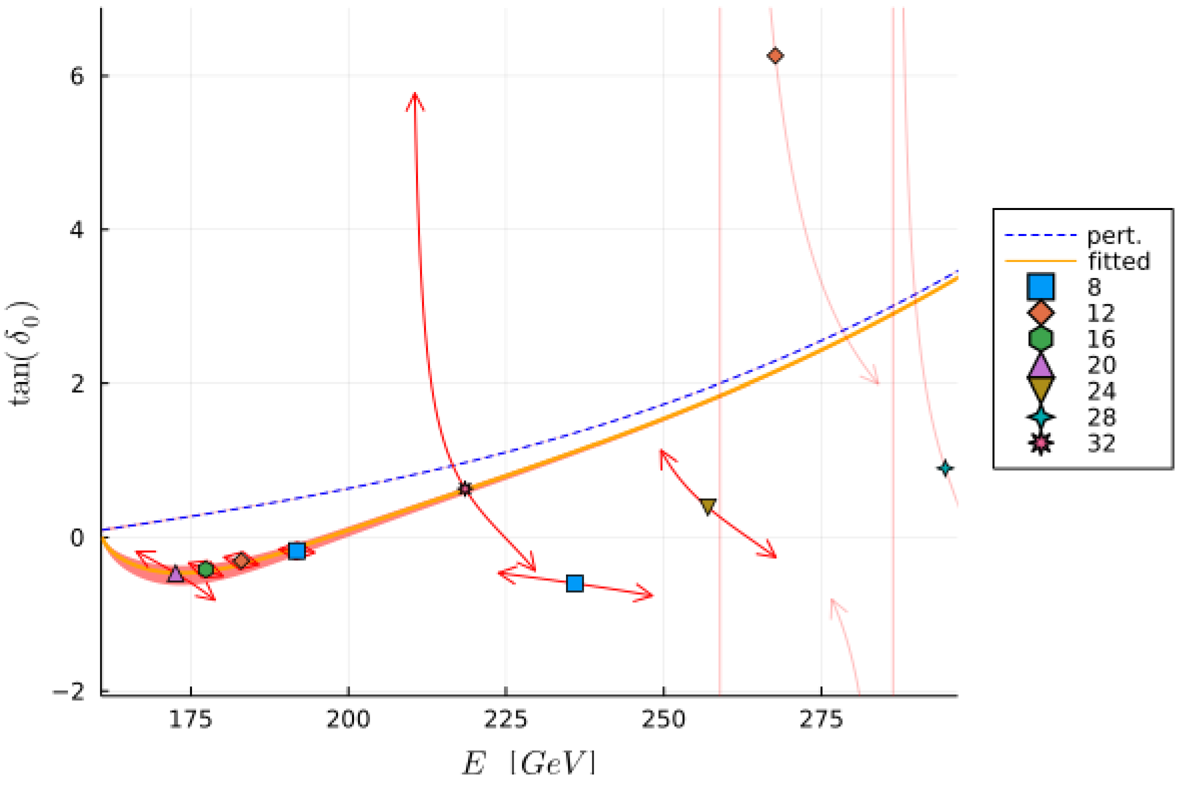

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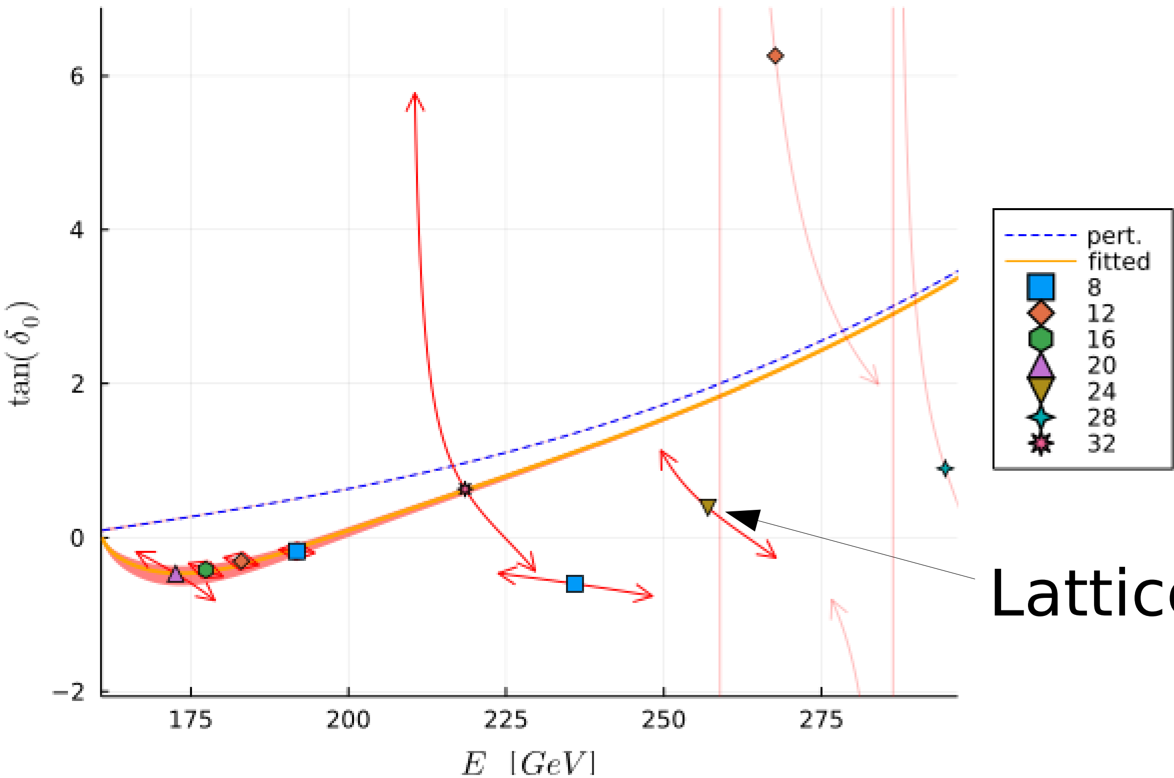

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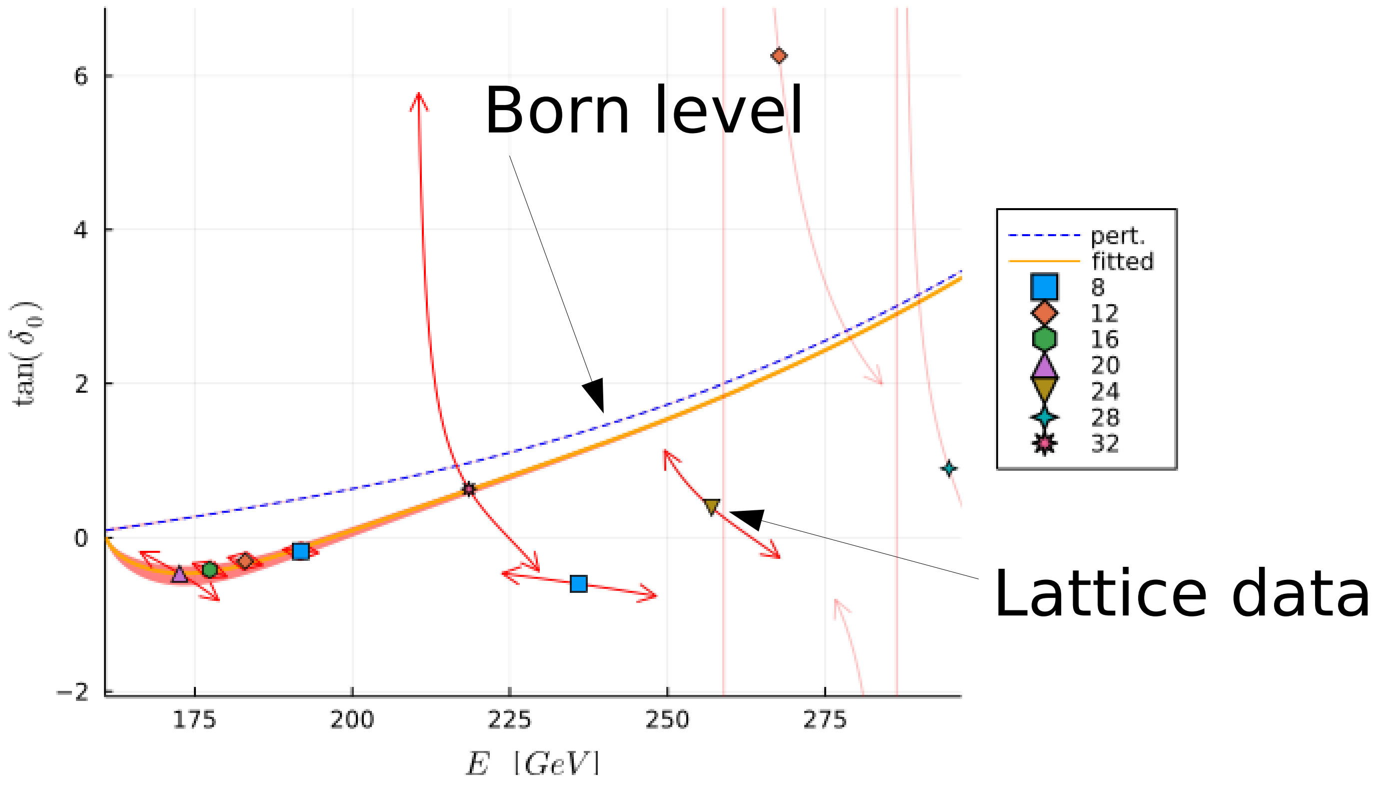

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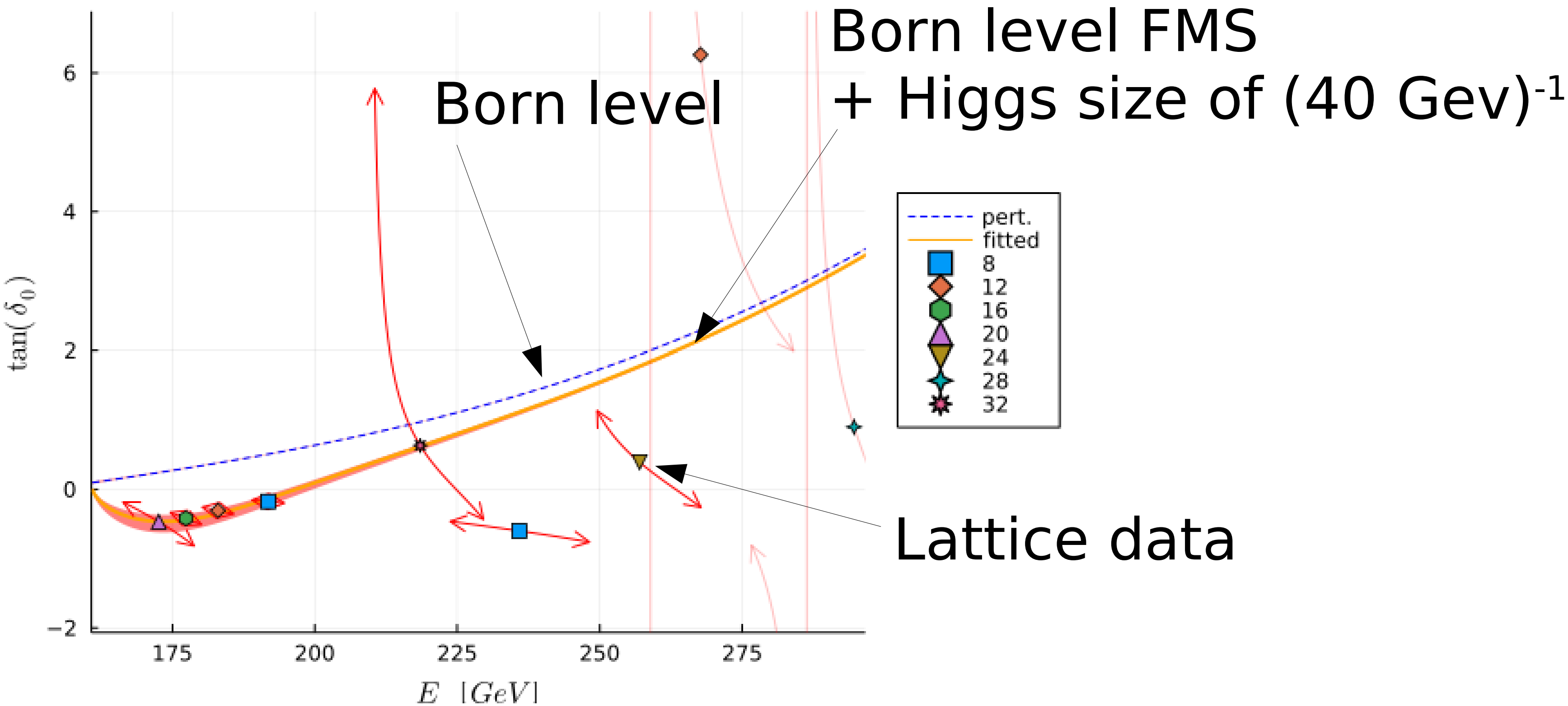

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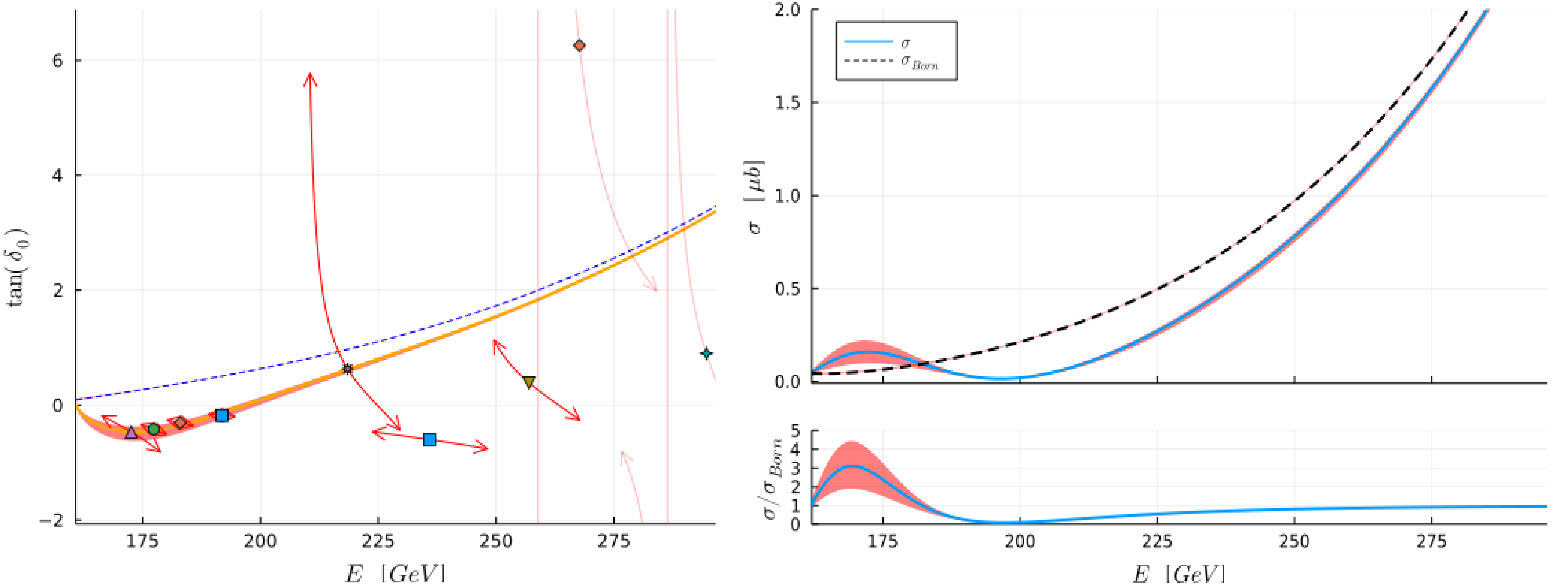

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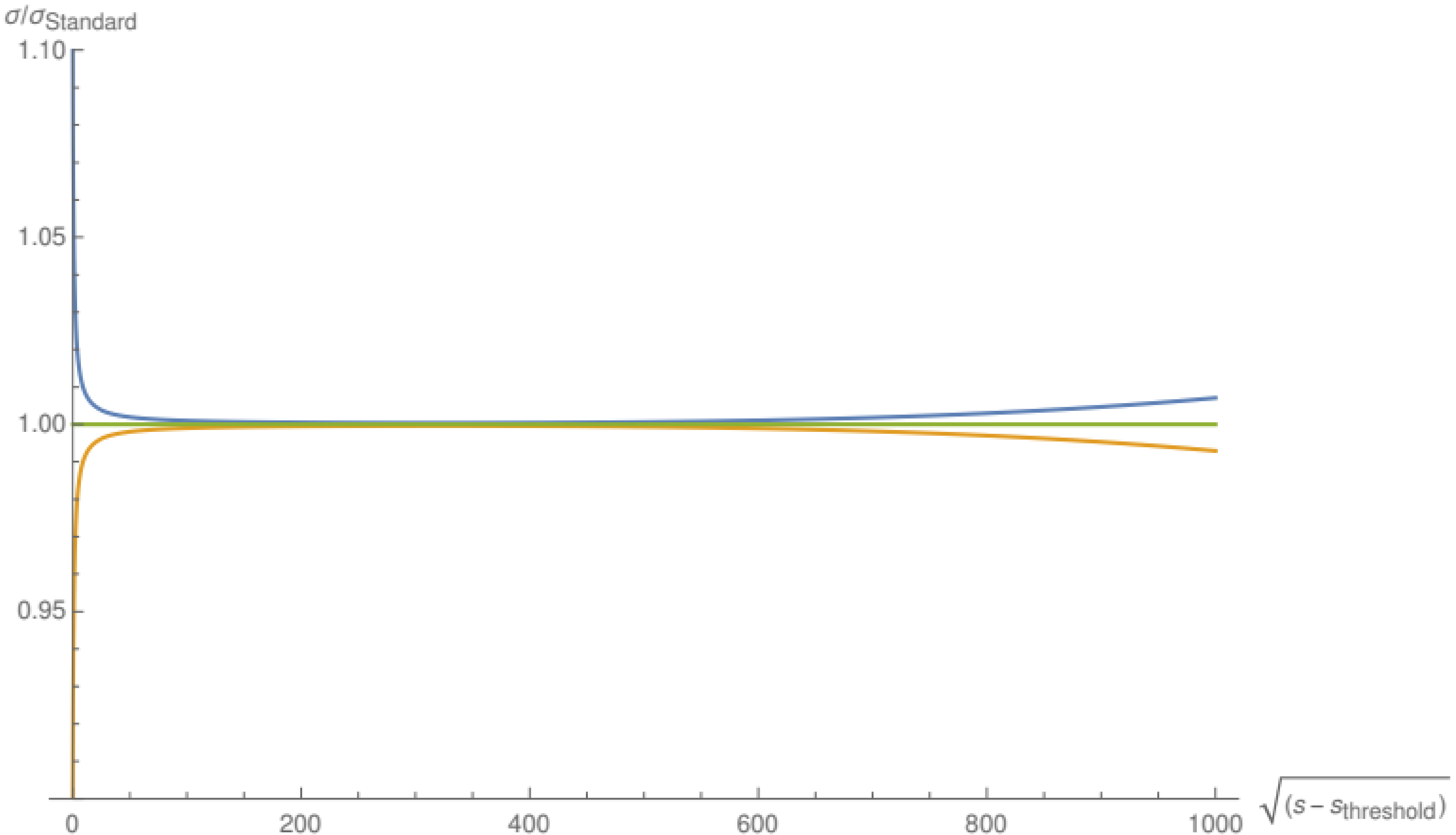
Generic behavior

Has been done for several observables

Generic behavior: DIS-like



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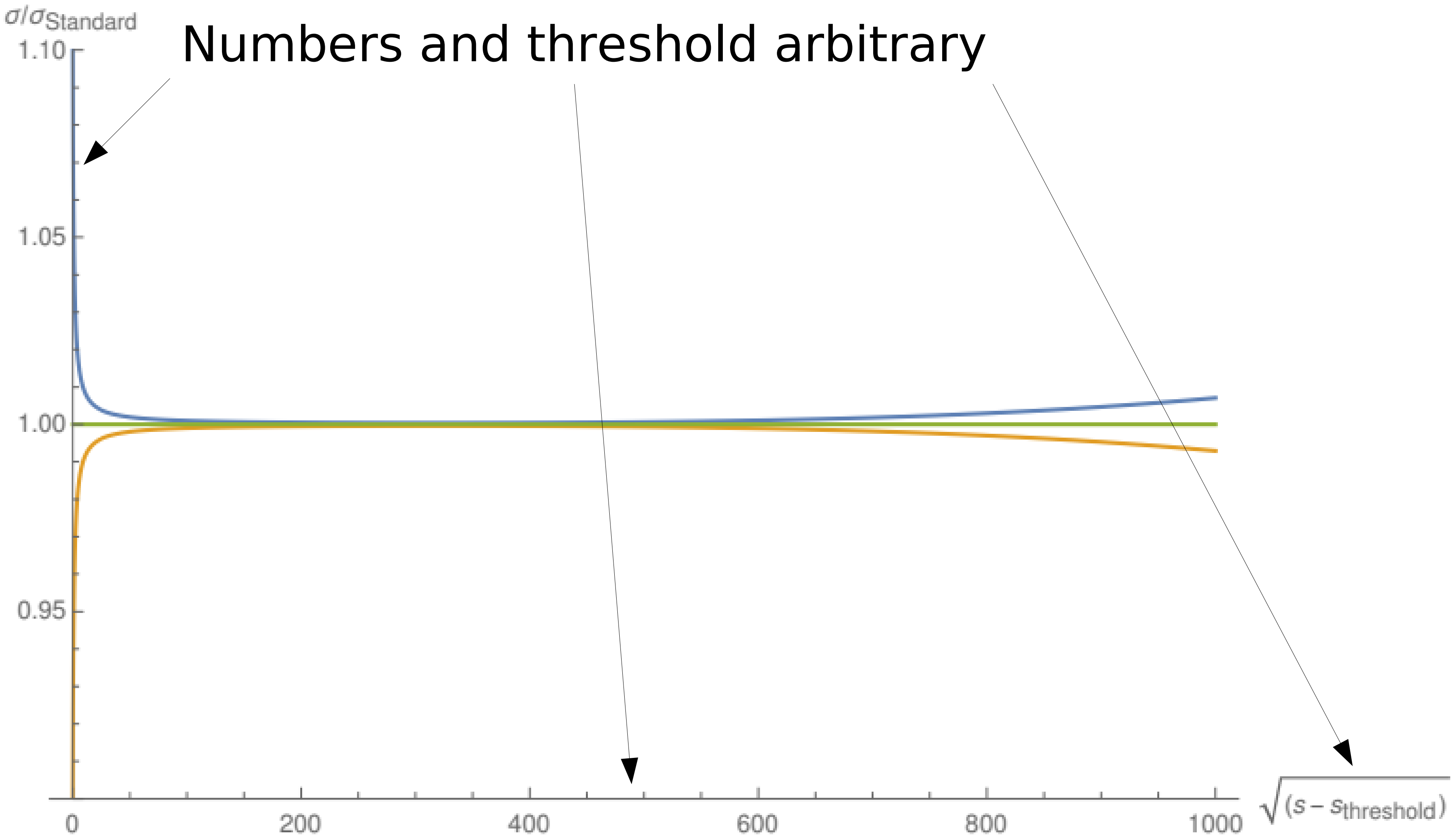

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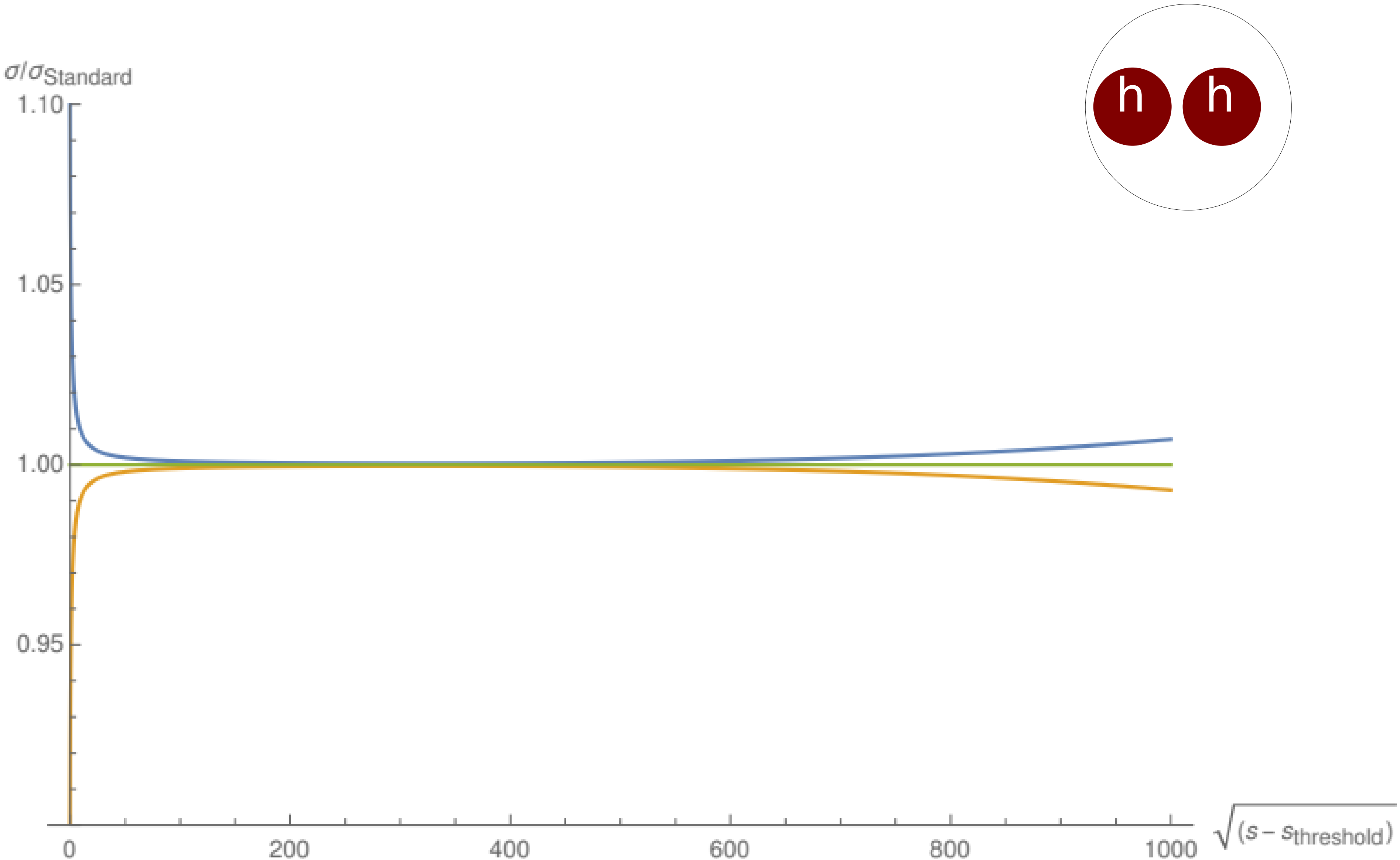

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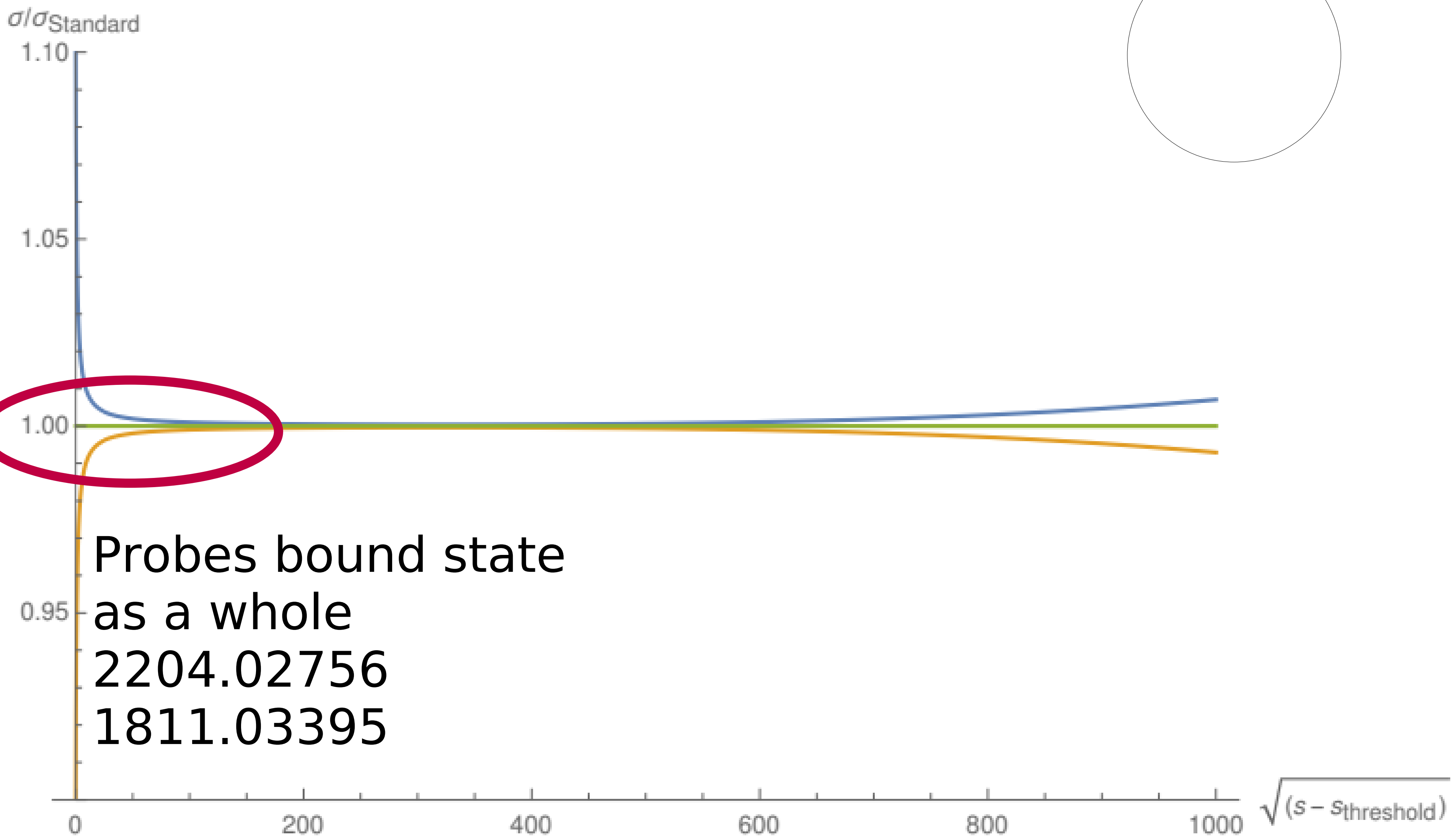
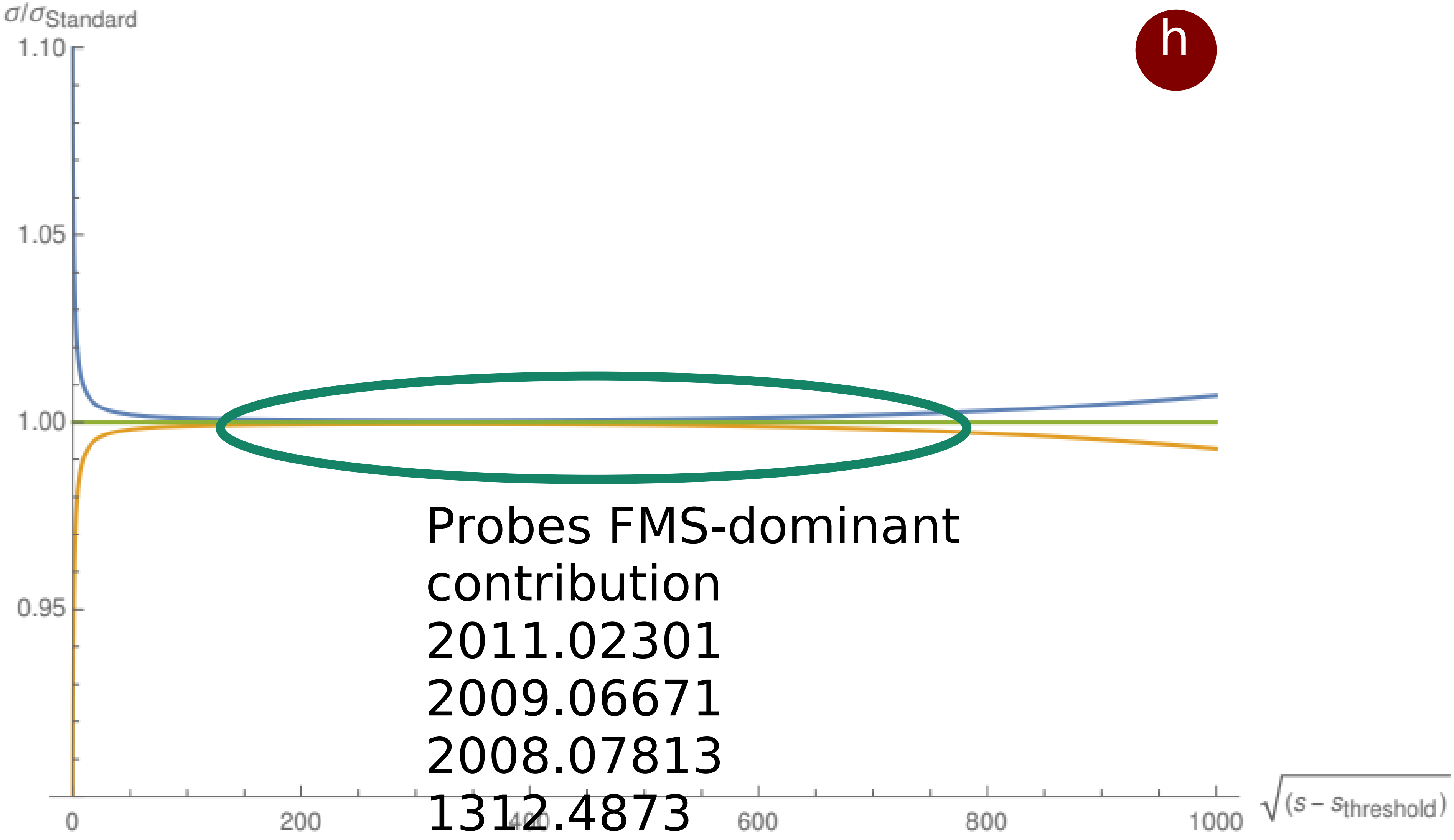


Probes bound state
as a whole
2204.02756
1811.03395

Has been done for several observables

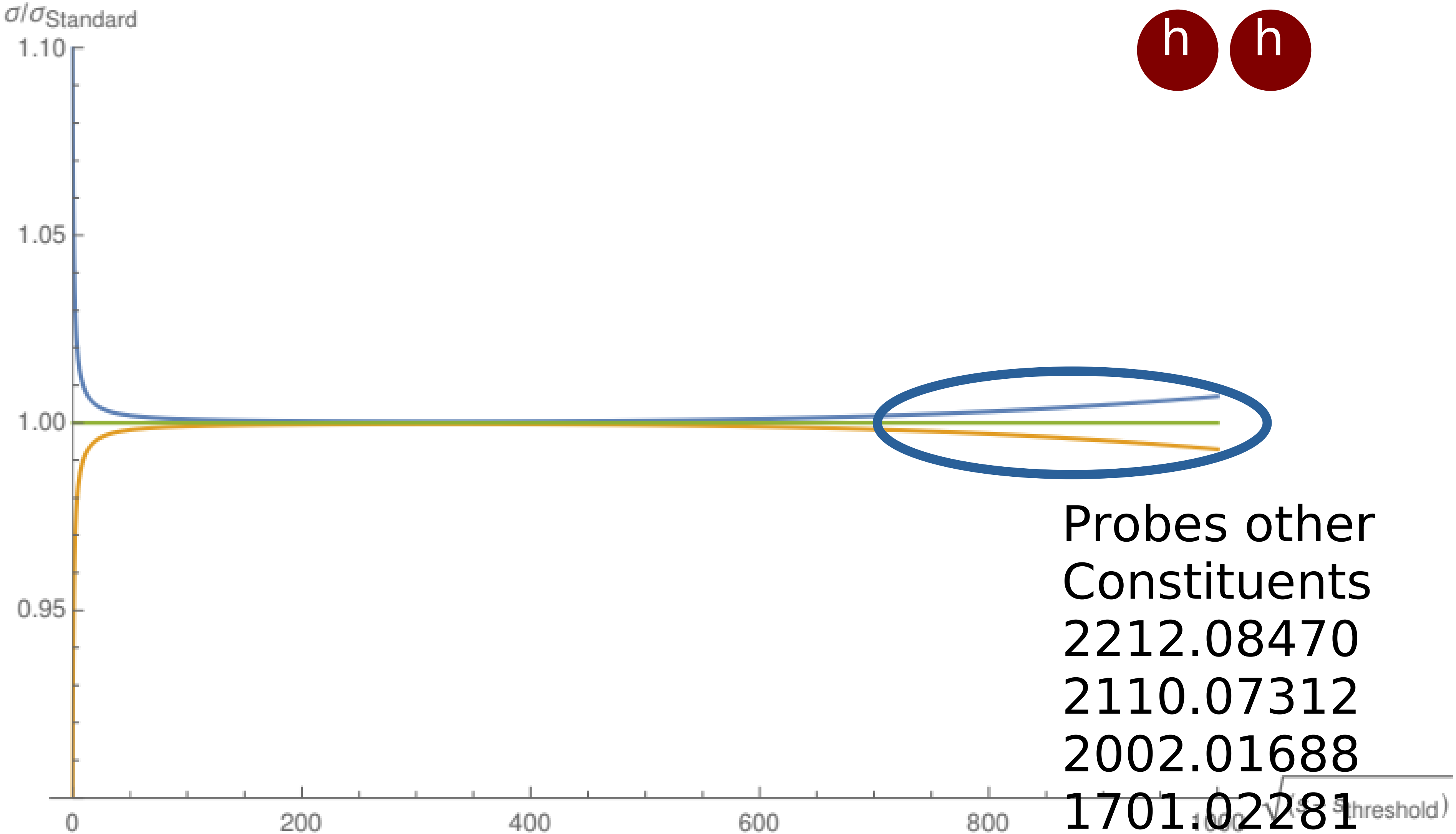
Generic behavior: DIS-like



Probes FMS-dominant contribution
2011.02301
2009.06671
2008.07813
1312.4873

Has been done for several observables

Generic behavior: DIS-like



Probes other Constituents
2212.08470
2110.07312
2002.01688
1701.02281

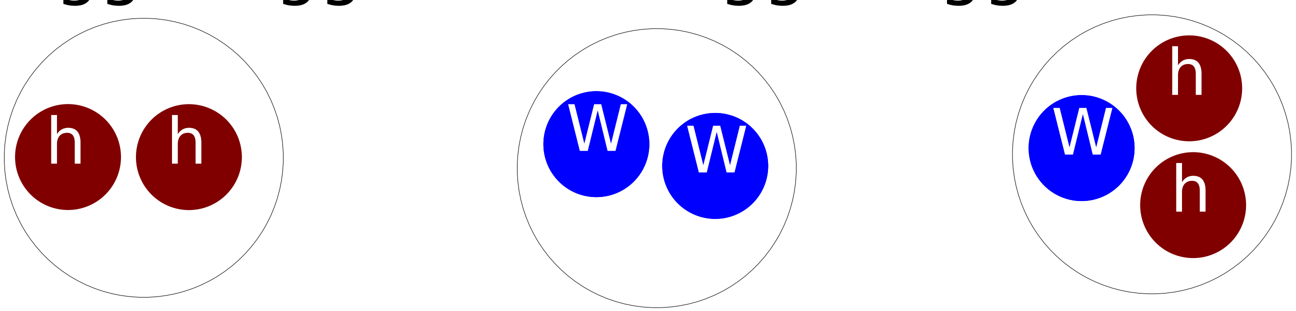
Has been done for several observables

Phenomenological Implications
-
Adding matter

Physical states

[Fröhlich et al.'80,
Banks et al.'79]

- Need physical, gauge-invariant particles
  - **Cannot** be the elementary particles
  - Non-Abelian nature is relevant
- Need more than one particle: Composite particles
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.



- Has nothing to do with weak coupling
  - Think QED (hydrogen atom!)
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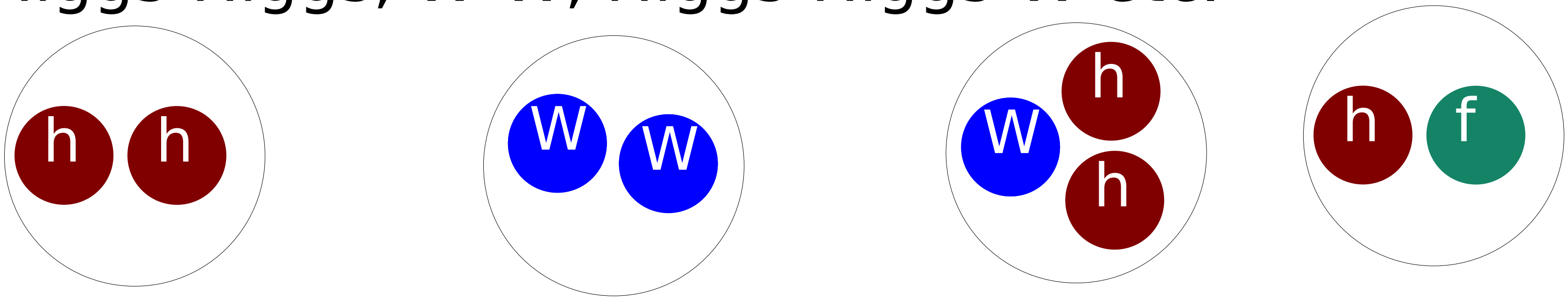

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Flavor

[Fröhlich et al.'80,
Egger, Maas, Sondenheimer'17]

- Flavor has two components
  - Global SU(3) generation
  - Local SU(2) weak gauge (up/down distinction)

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- Extends non-trivially to hadrons

Flavor on the lattice

[Afferrante,Maas,Sondenheimer,Török'20]

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  - Compressed mass scales
  - One generation
  - Degenerate leptons and neutrinos
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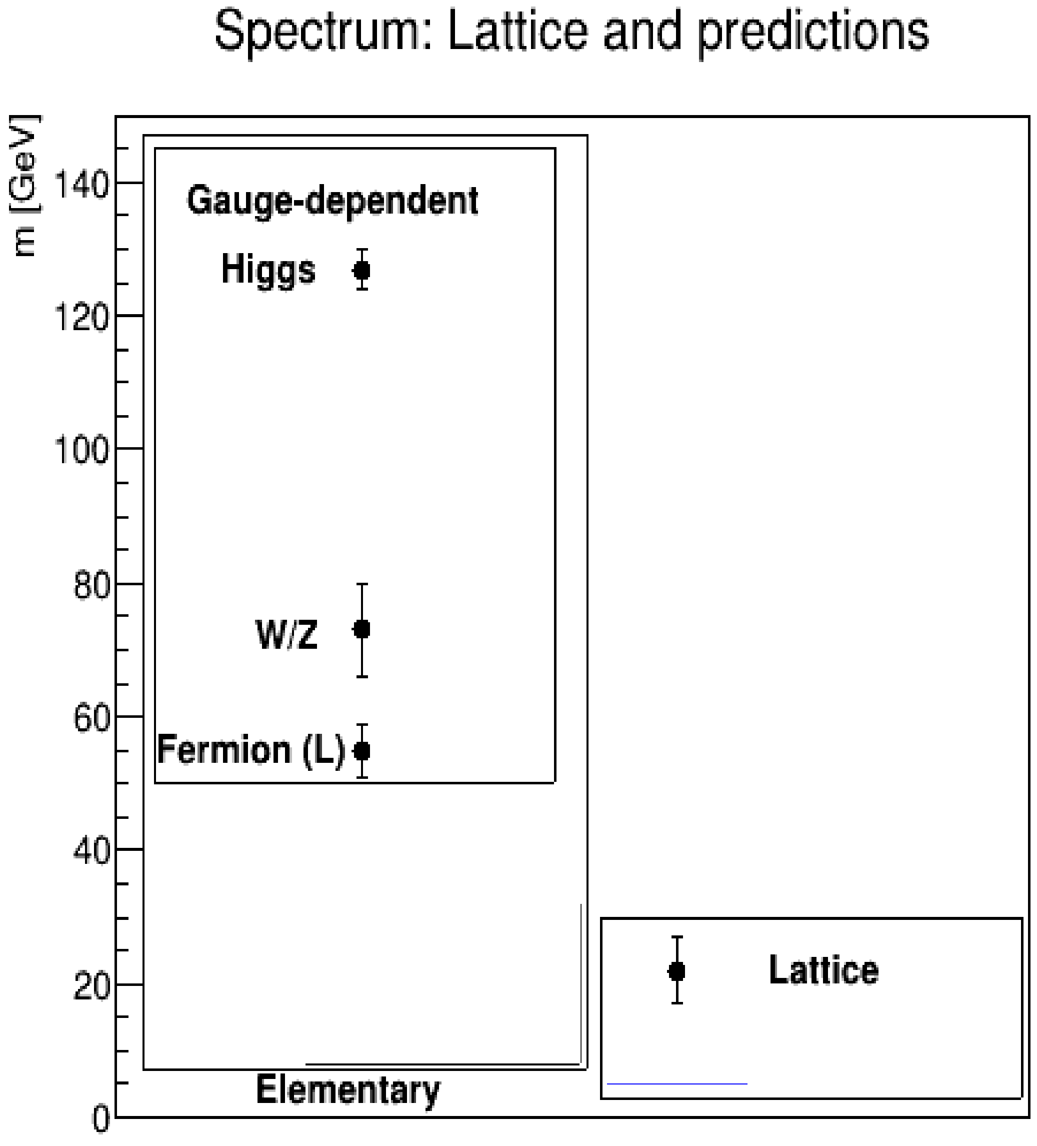

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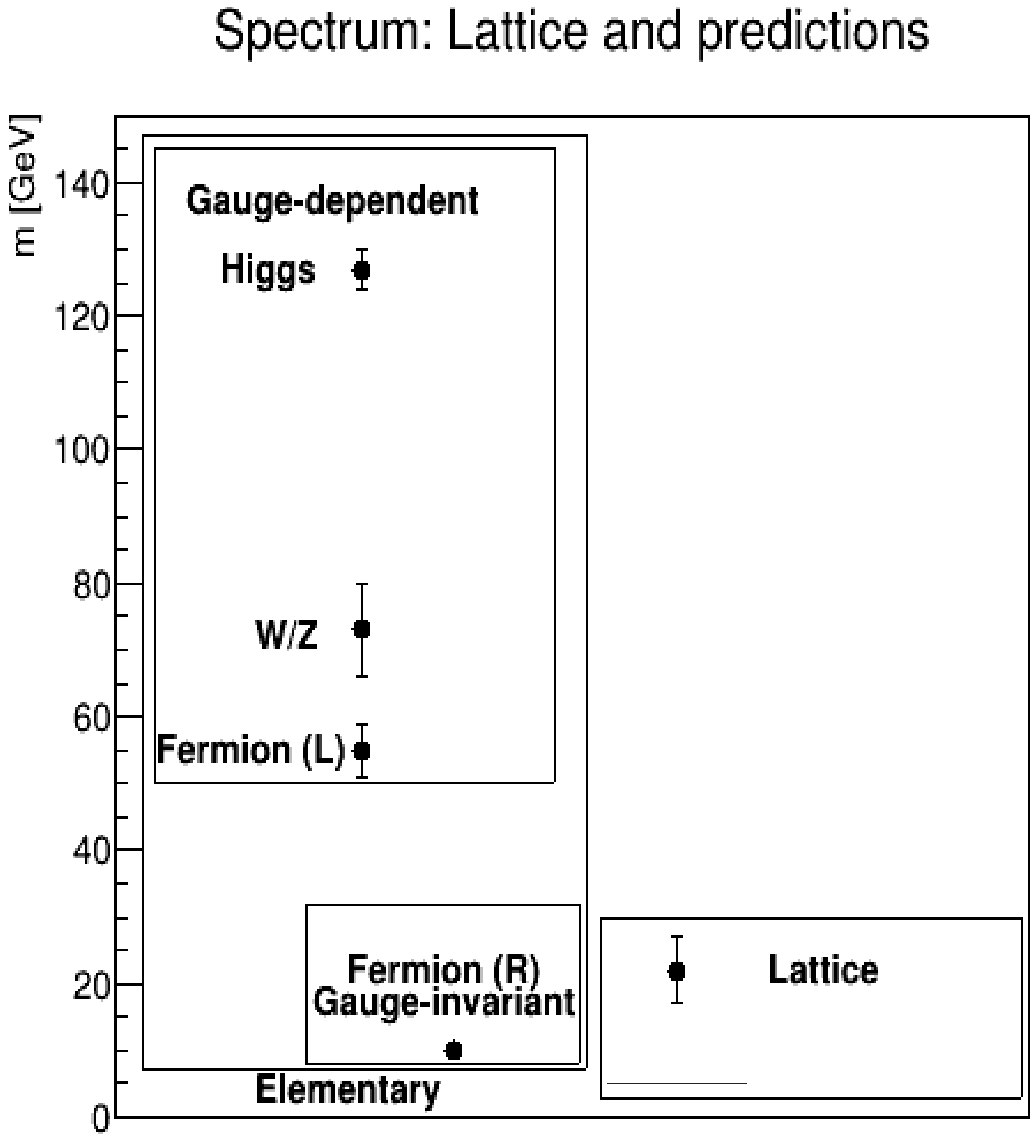

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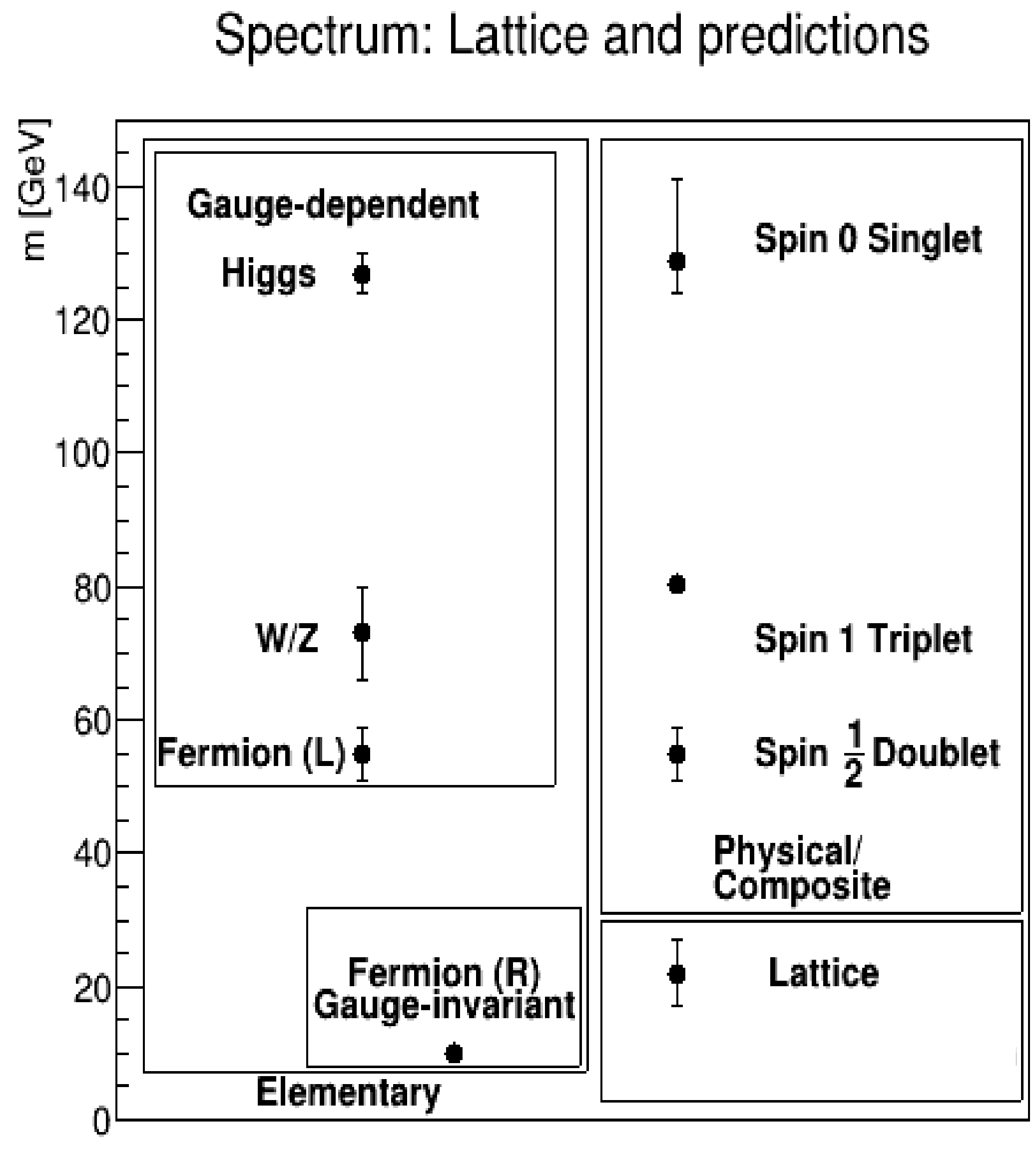

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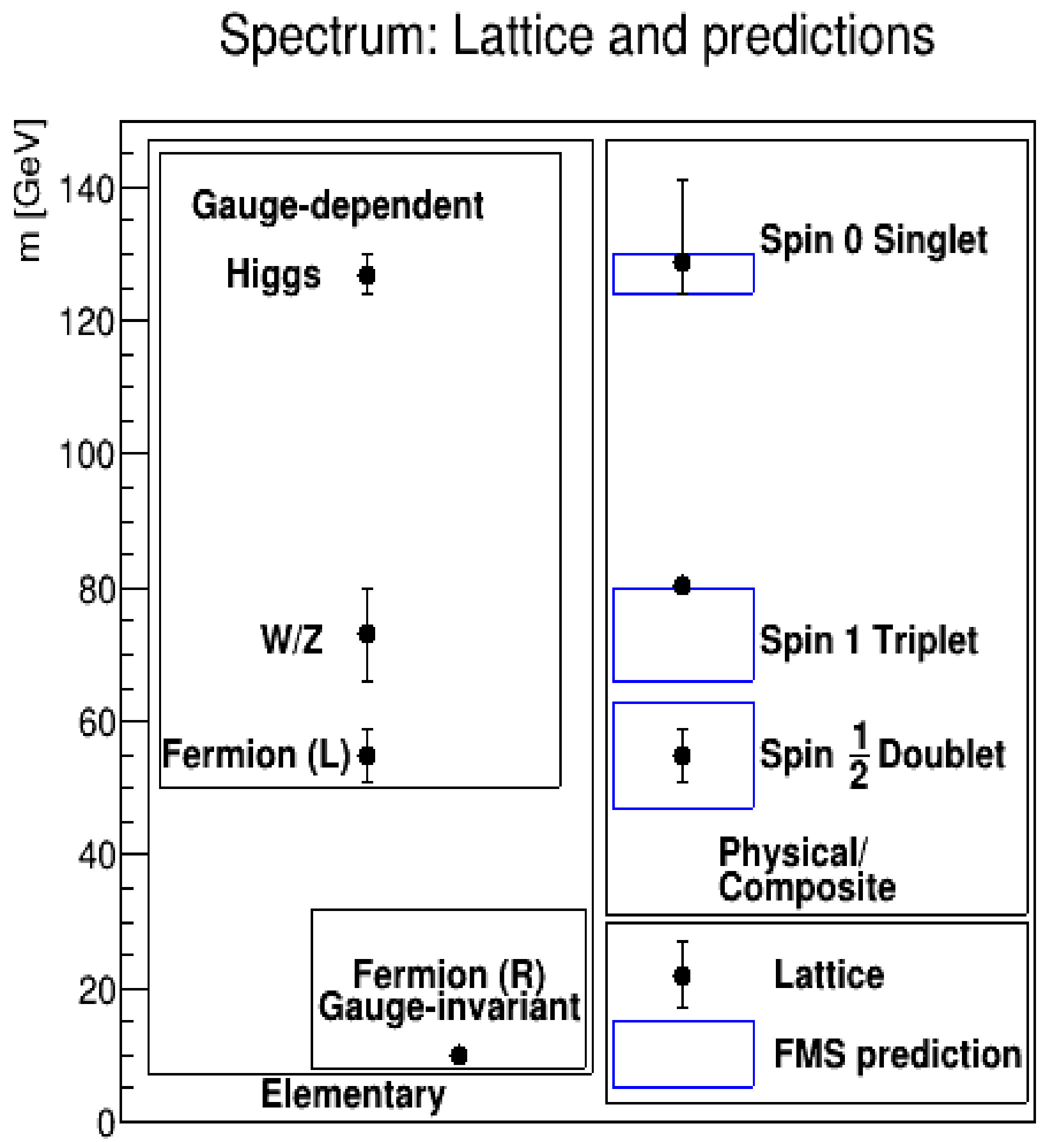

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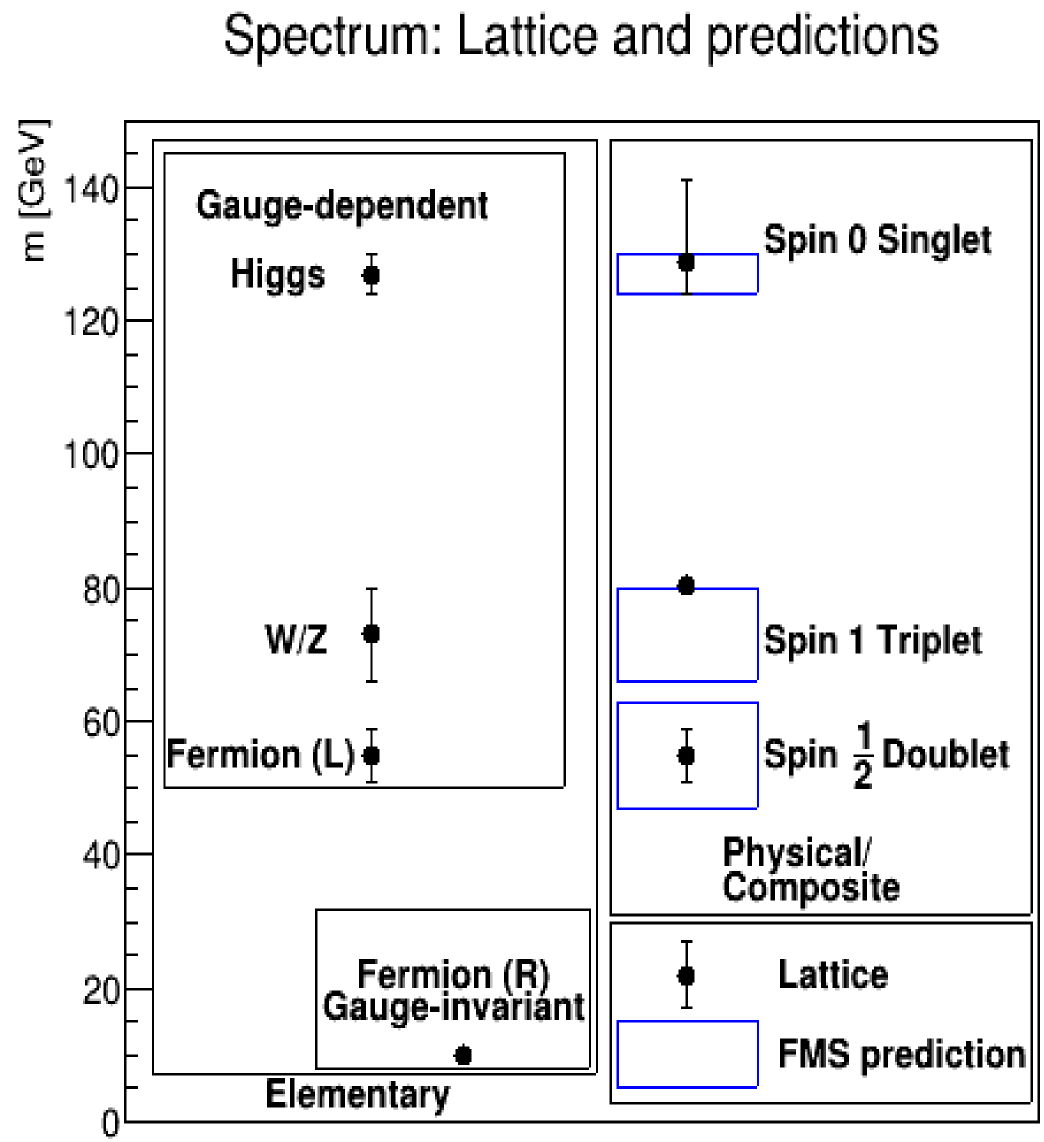
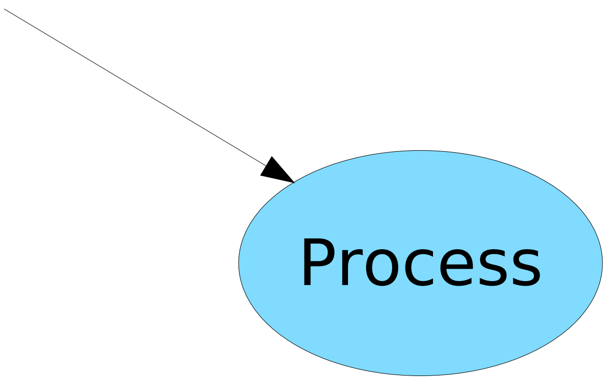
- Supports FMS prediction – grant for unquenching '24-'28



Scattering

[Maas et al.'17
Maas & Reiner '22
Maas, Plätzer et al.' unpublished]

Incoming (asymptotic) particle
Standard LSZ: Elementary particle



$$\left\langle f\left(p\right)...\right\rangle$$

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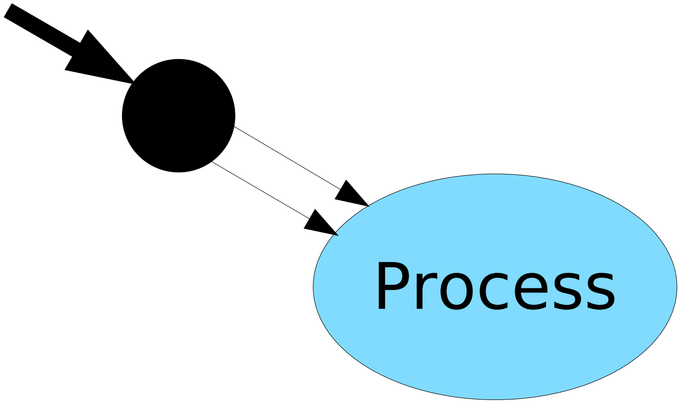

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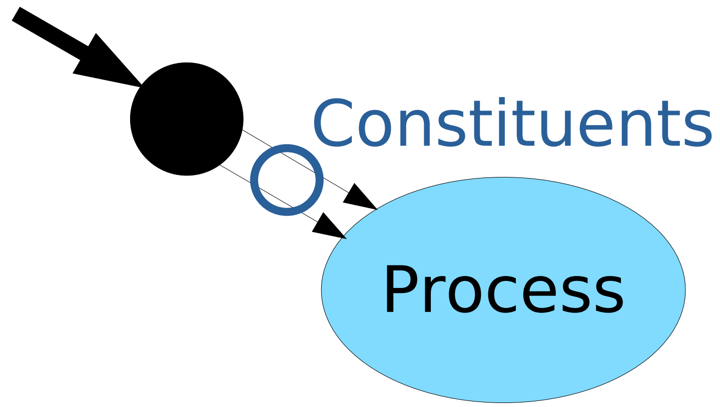

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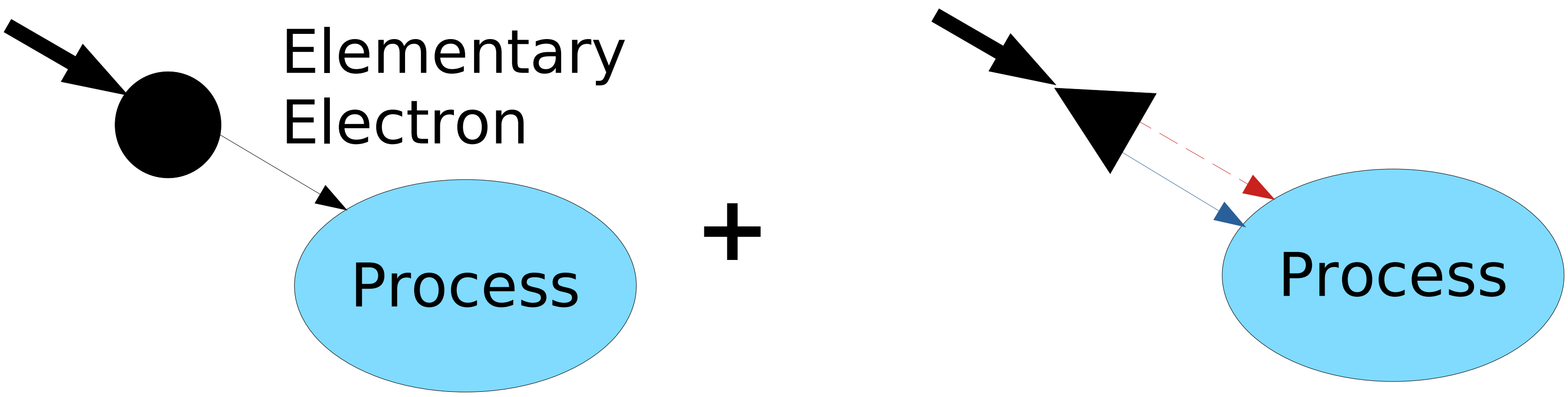

$$v\langle f(p)\ldots\rangle + \int dq\, \Gamma(P,q)\, D_f(p-q)\, D_h(q) \langle h(q) f(P-q)\ldots\rangle$$

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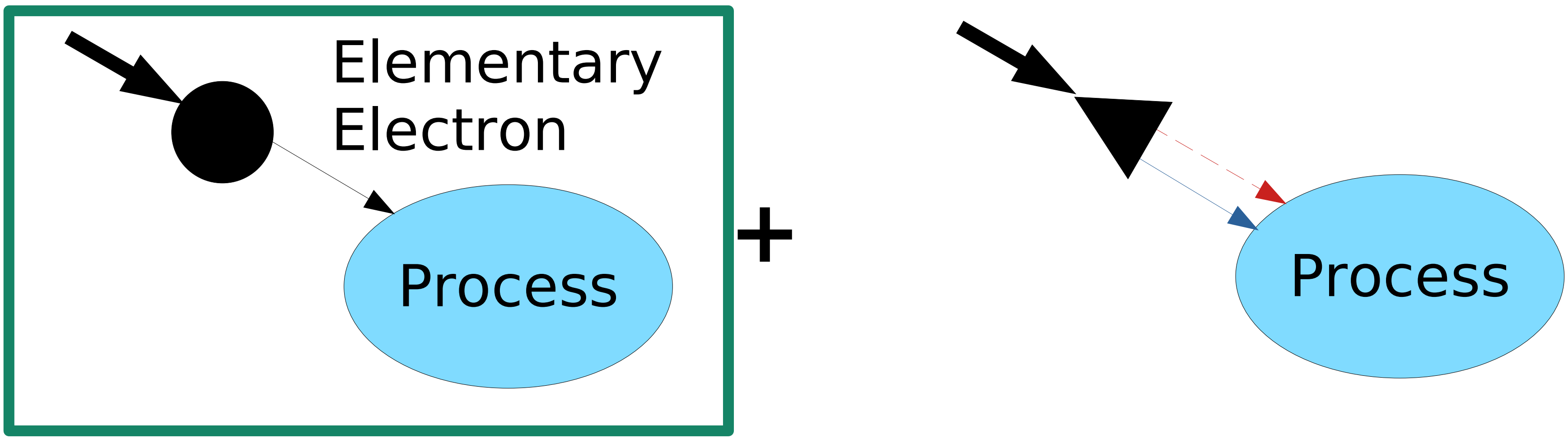

Standard perturbation theory

$$\boxed{v\langle f(p)\ldots\rangle}+\int dq\,\Gamma(P,q)\,D_f(p-q)\,D_h(q)\langle h(q)f(P-q)\ldots\rangle$$

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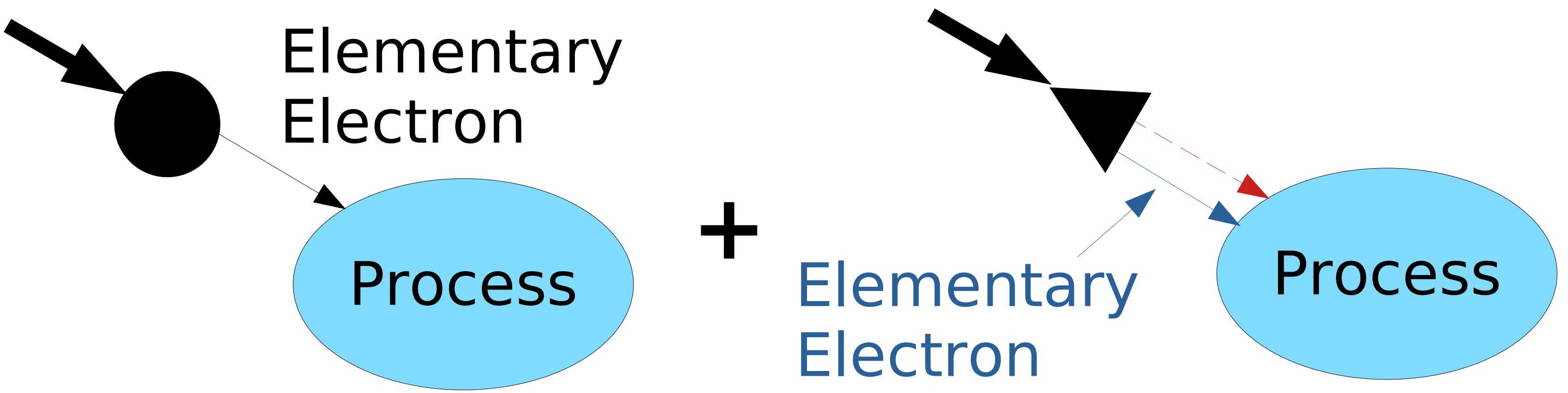

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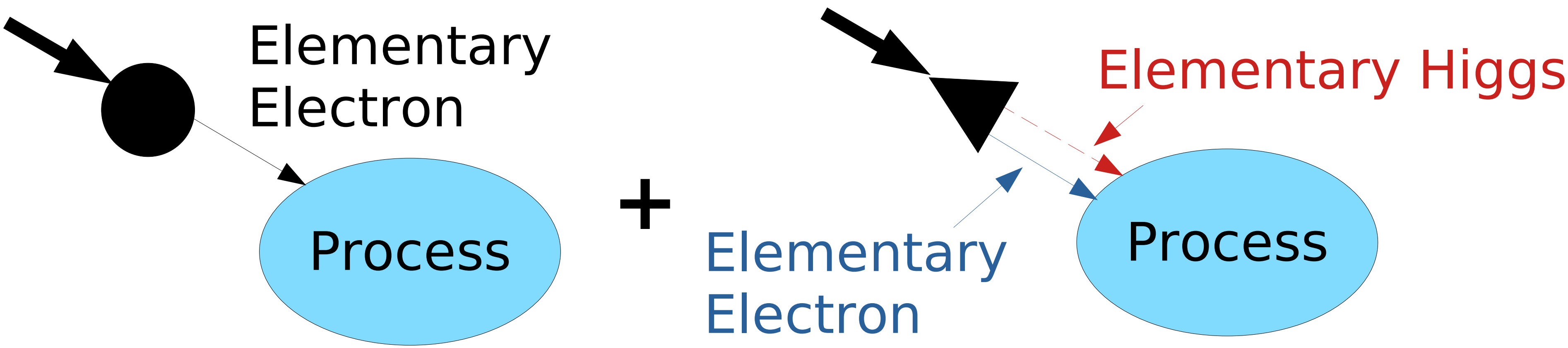

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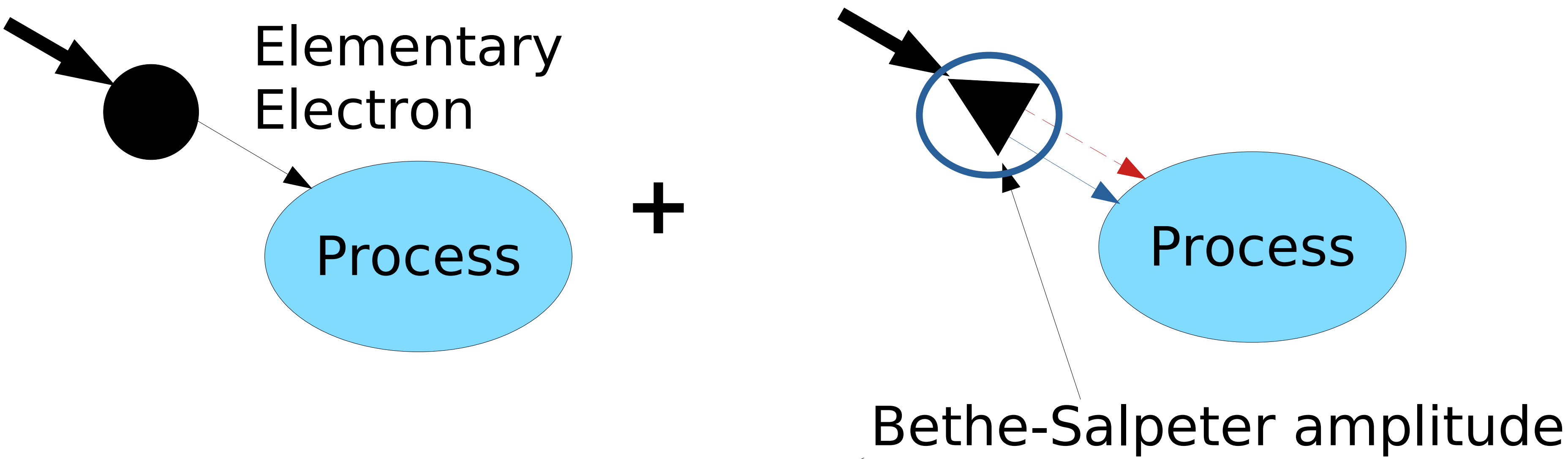

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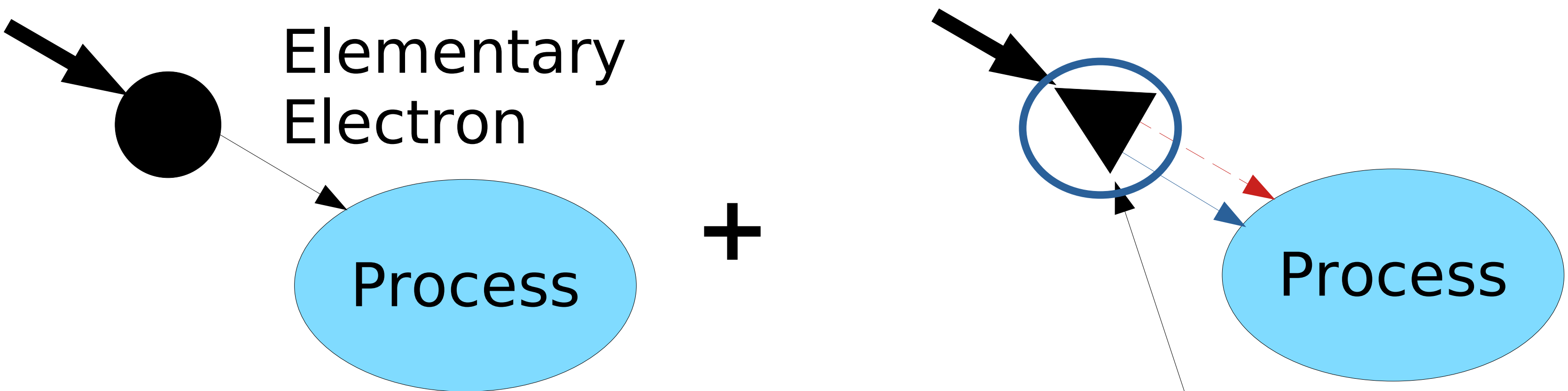

Bethe-Salpeter amplitude:
Calculabe in augmented PT

$$v\langle f(p)\ldots\rangle+\int dq\,\Gamma(P,q)\,D_f(p-q)\,D_h(q)\langle h(q)f(P-q)\ldots\rangle$$

Bethe-Salpeter Amplitude

[Maas, Plätzer et al. unpublished
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Bethe-Salpeter Amplitude

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Calculable itself in augmented perturbation theory

Bethe-Salpeter Amplitude

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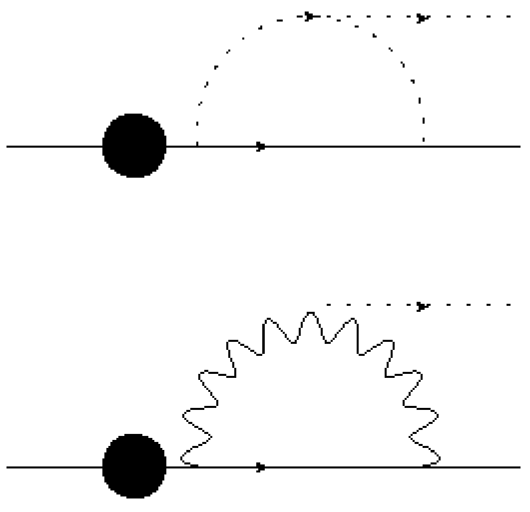

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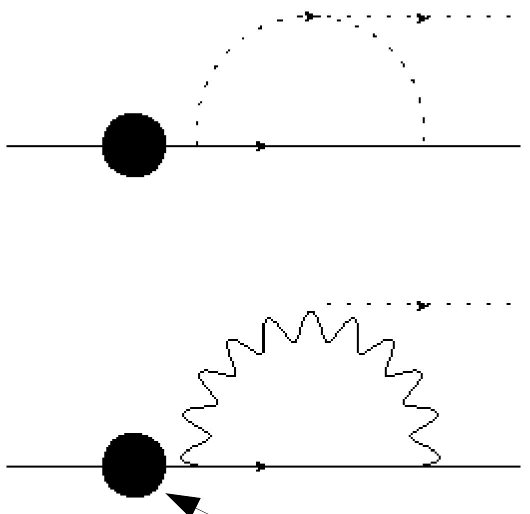

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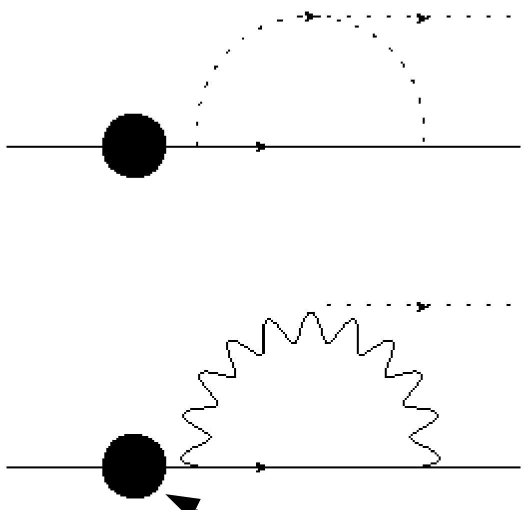

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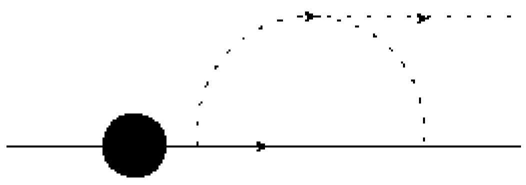

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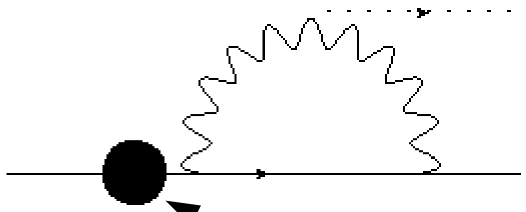

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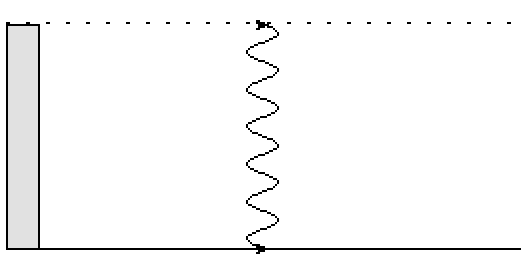

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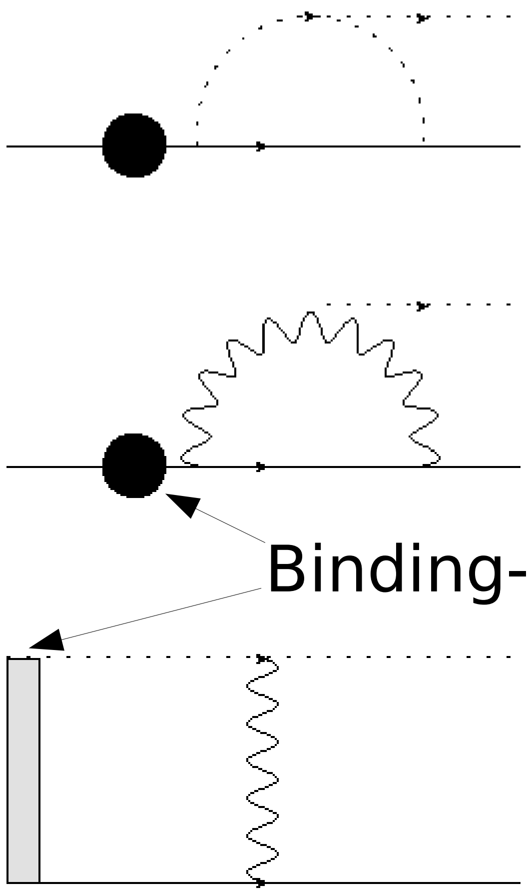

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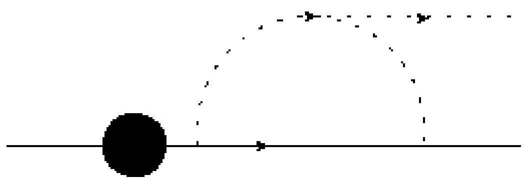

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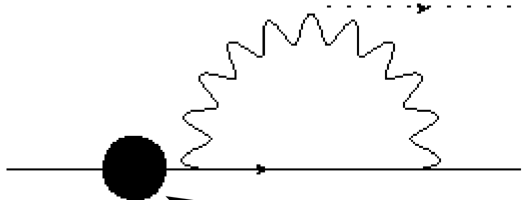

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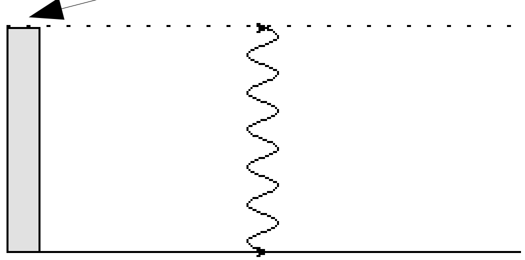

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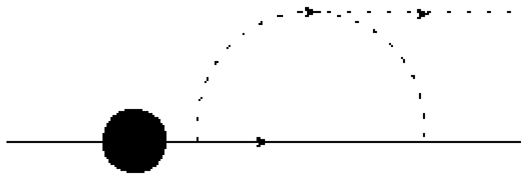

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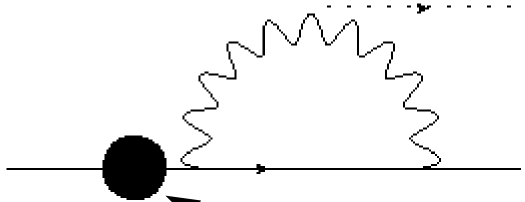

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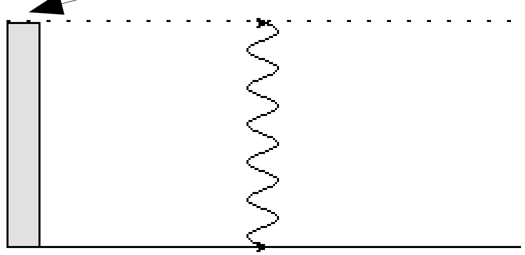

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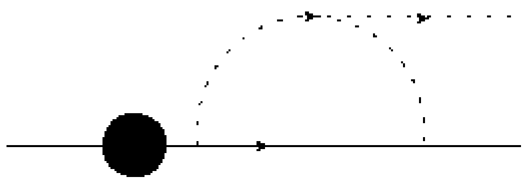

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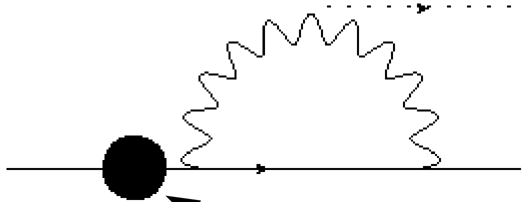

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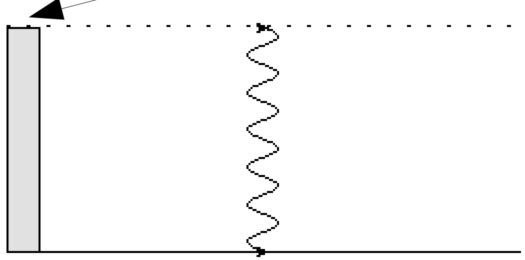

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Neither are Yukawa suppressed

Impact on processes

[Maas, Plätzer et al. unpublished
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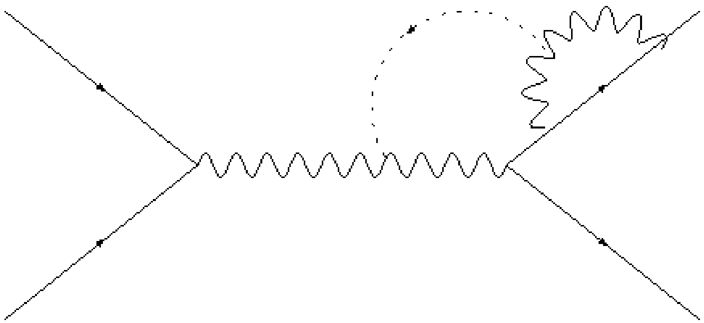
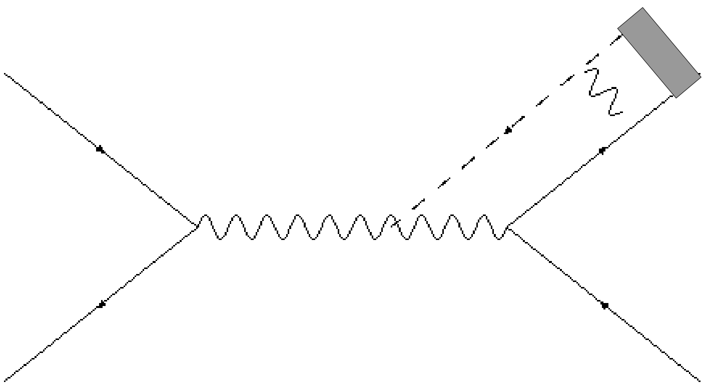
Process f̄f → f̄f: 2/1-loop (in g_{weak}) suppressed contribution





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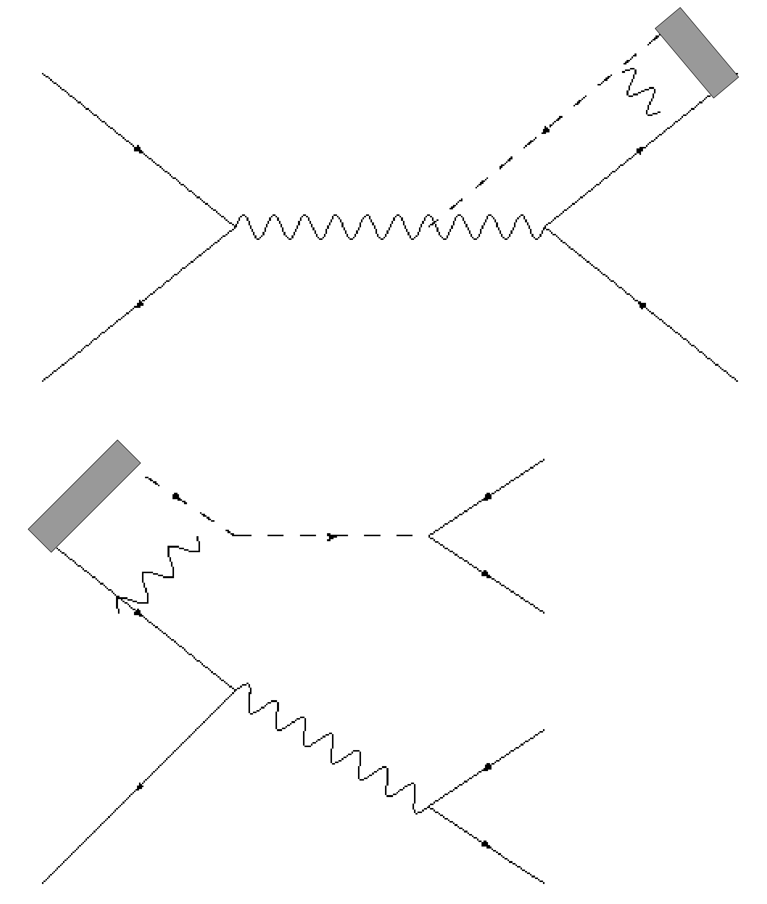

Process ff → ffff 1-loop ($g_{weak}y$) suppressed contribution

Resummation effects at $s \gg M_Z$ [Ciafaloni et al. '00]

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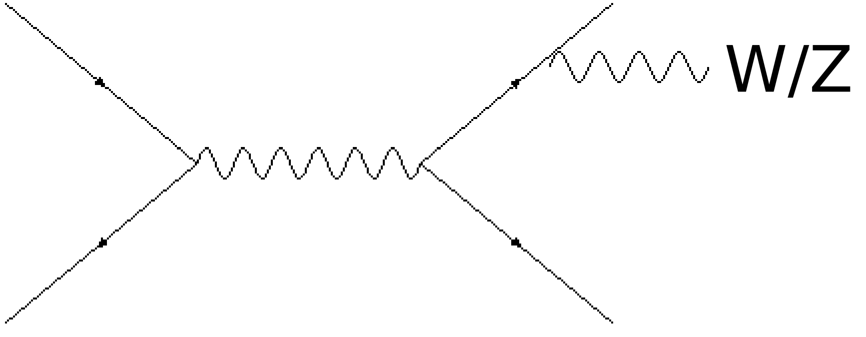
Standard perturbation theory



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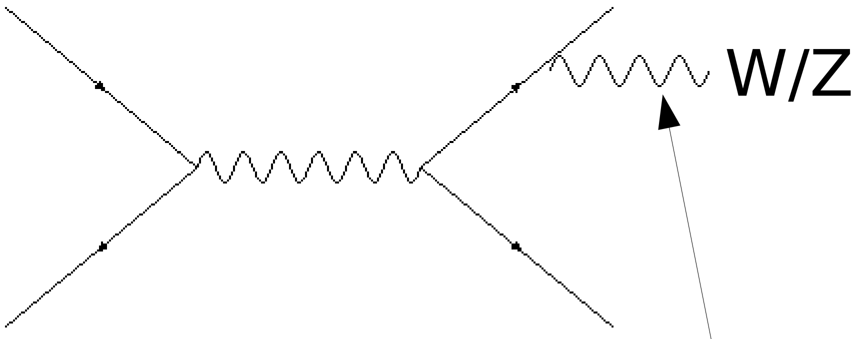

Resumming real emission

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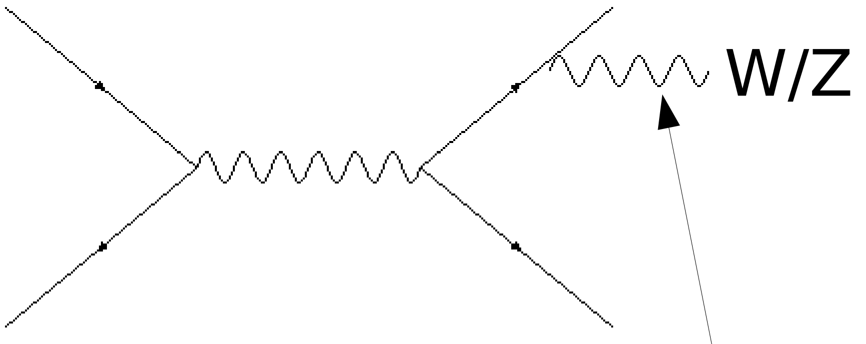

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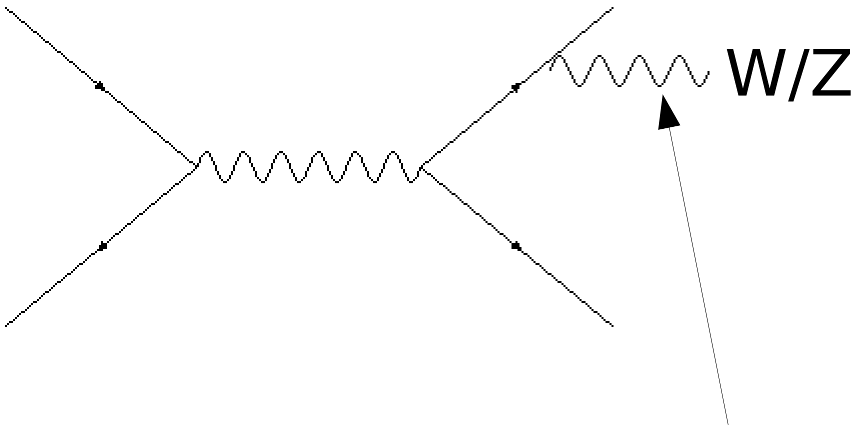

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at 1 TeV of the same order as strong corrections

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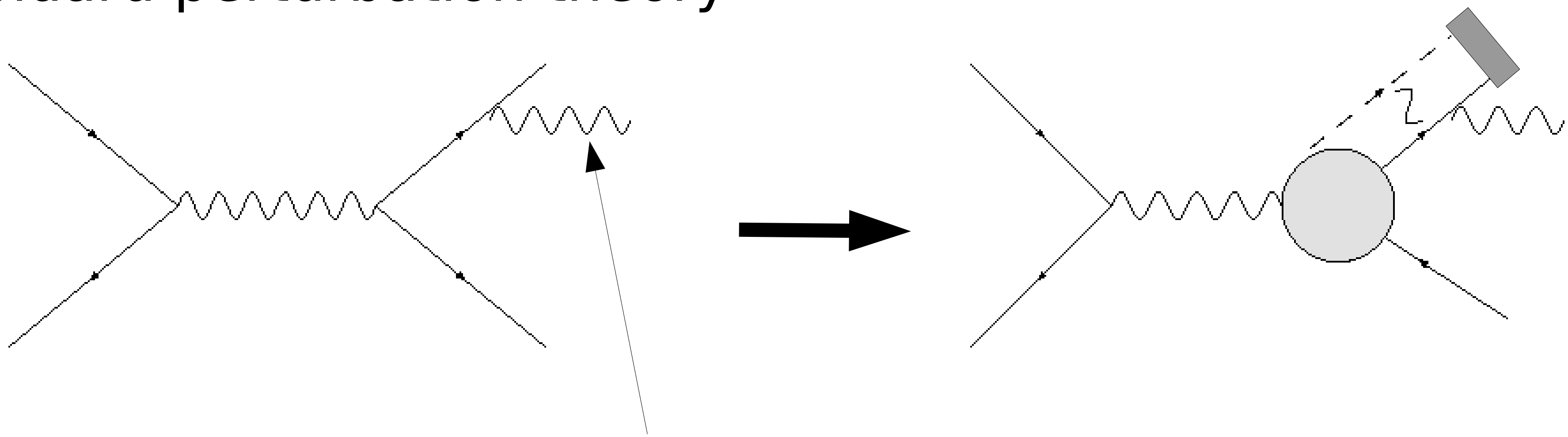
Standard perturbation theory

Augmented by correct asymptotic state



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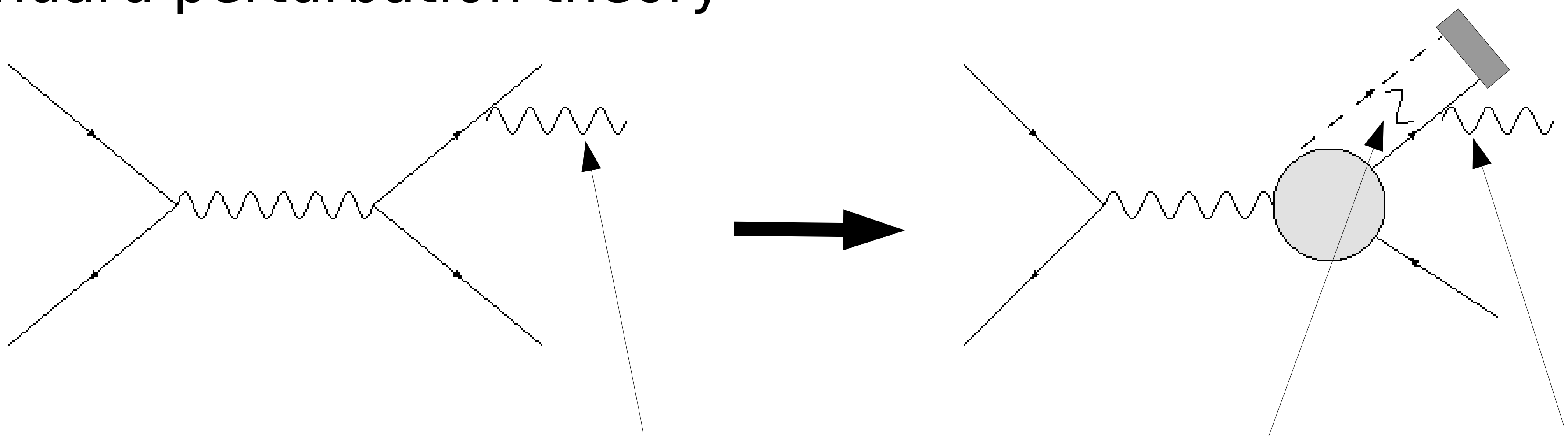

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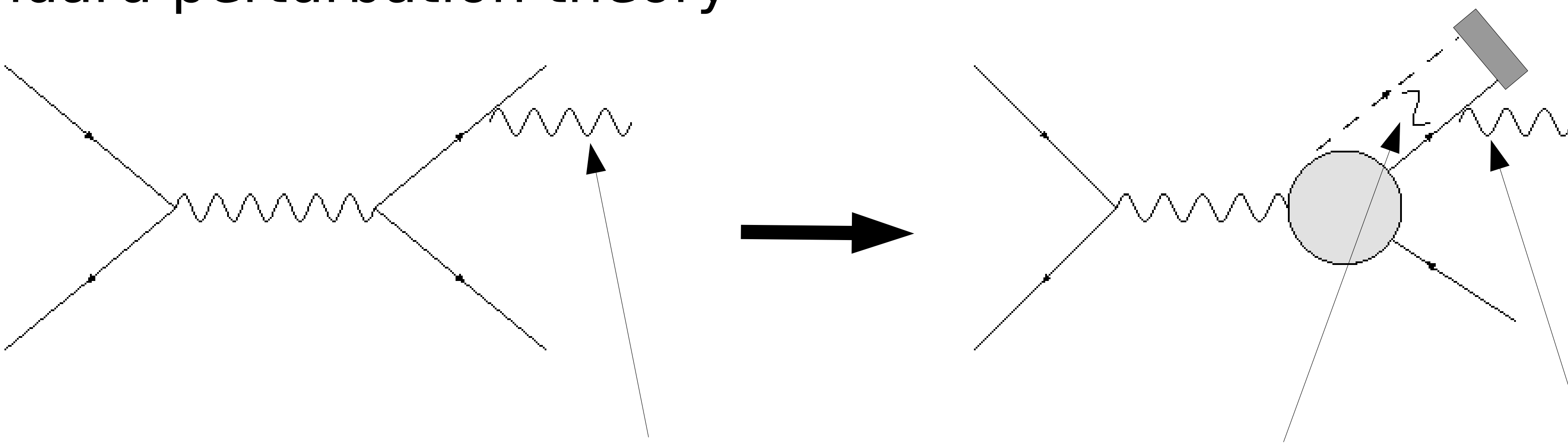

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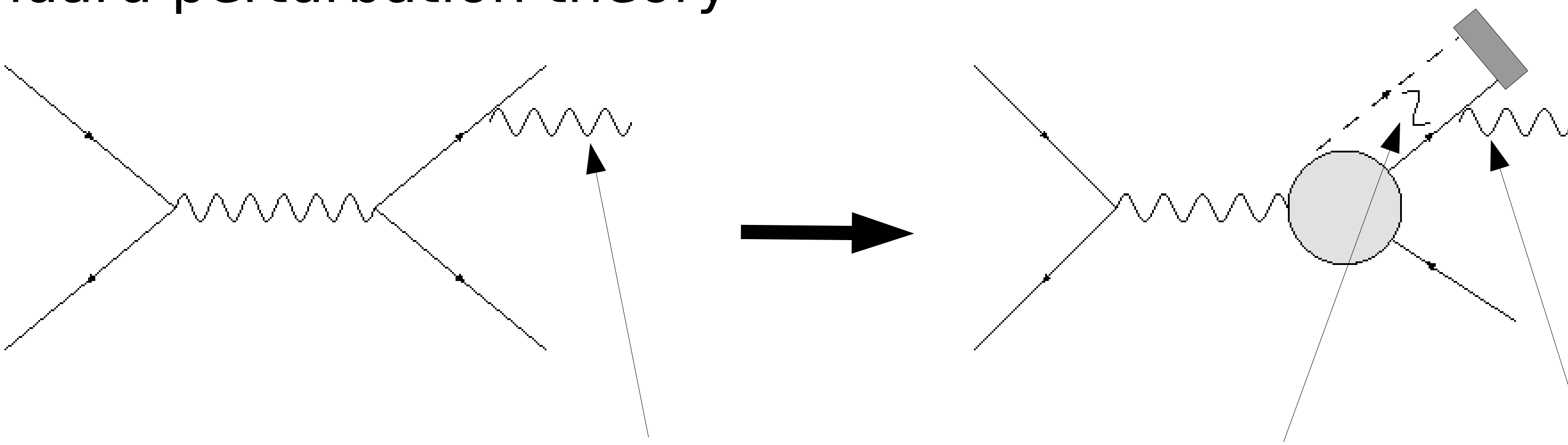

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New physics
-
Qualitative changes

Beyond the standard model

[Maas'15
Maas, Sondenheimer, Törek'17]

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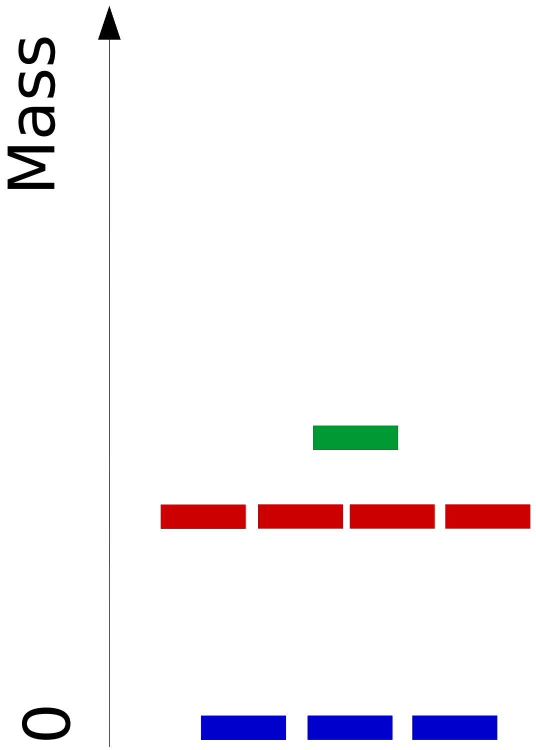
- Global U(1) custodial (flavor) symmetry
  - Acts as (right-)transformation on the scalar field only

$$W_\mu^a \rightarrow W_\mu^a \qquad h \rightarrow \exp(ia) h$$

Spectrum

Gauge-dependent

Vector



‘SU(3)→ SU(2)’

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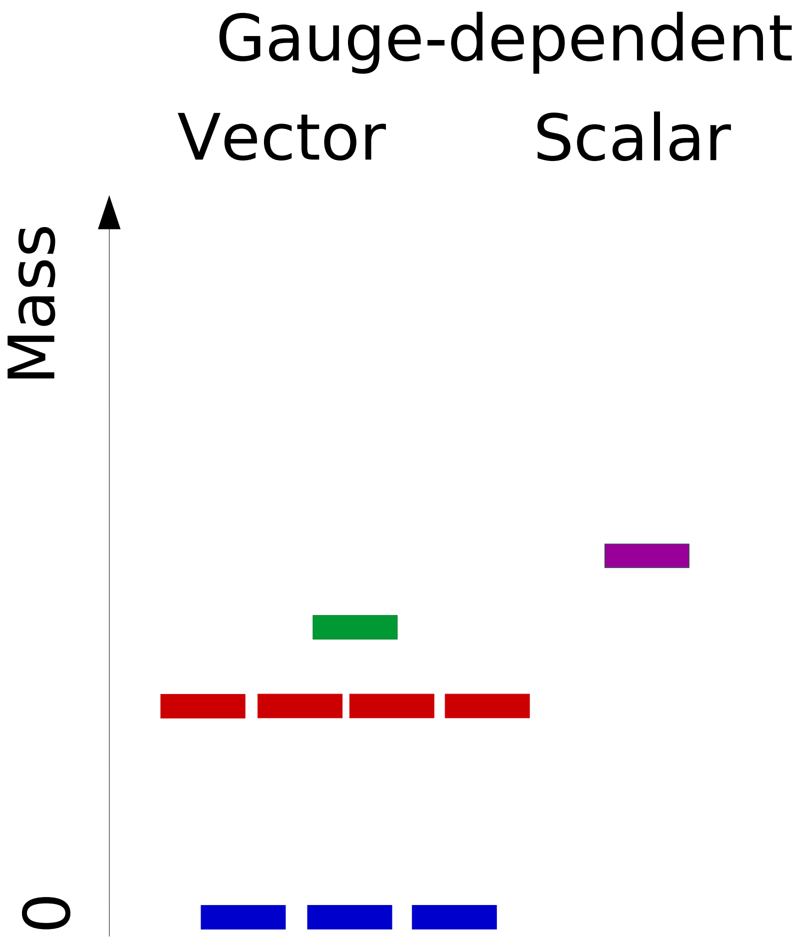

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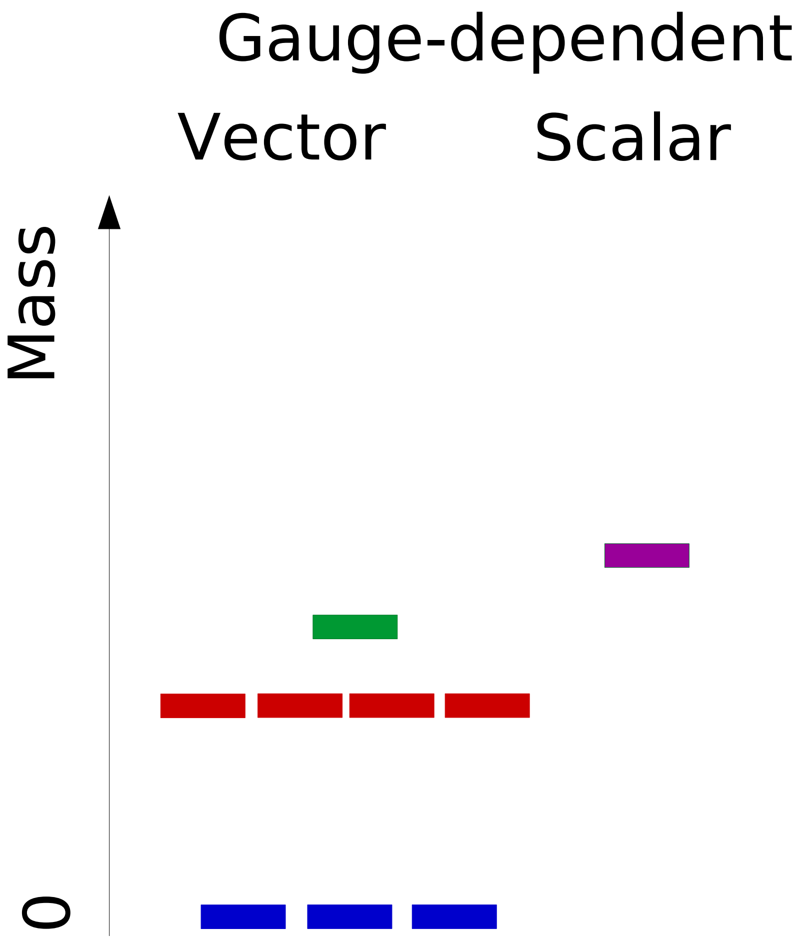

Confirmed in gauge-fixed
lattice calculations [Maas et al.’16]

Spectrum

[Maas & Török'16,'18
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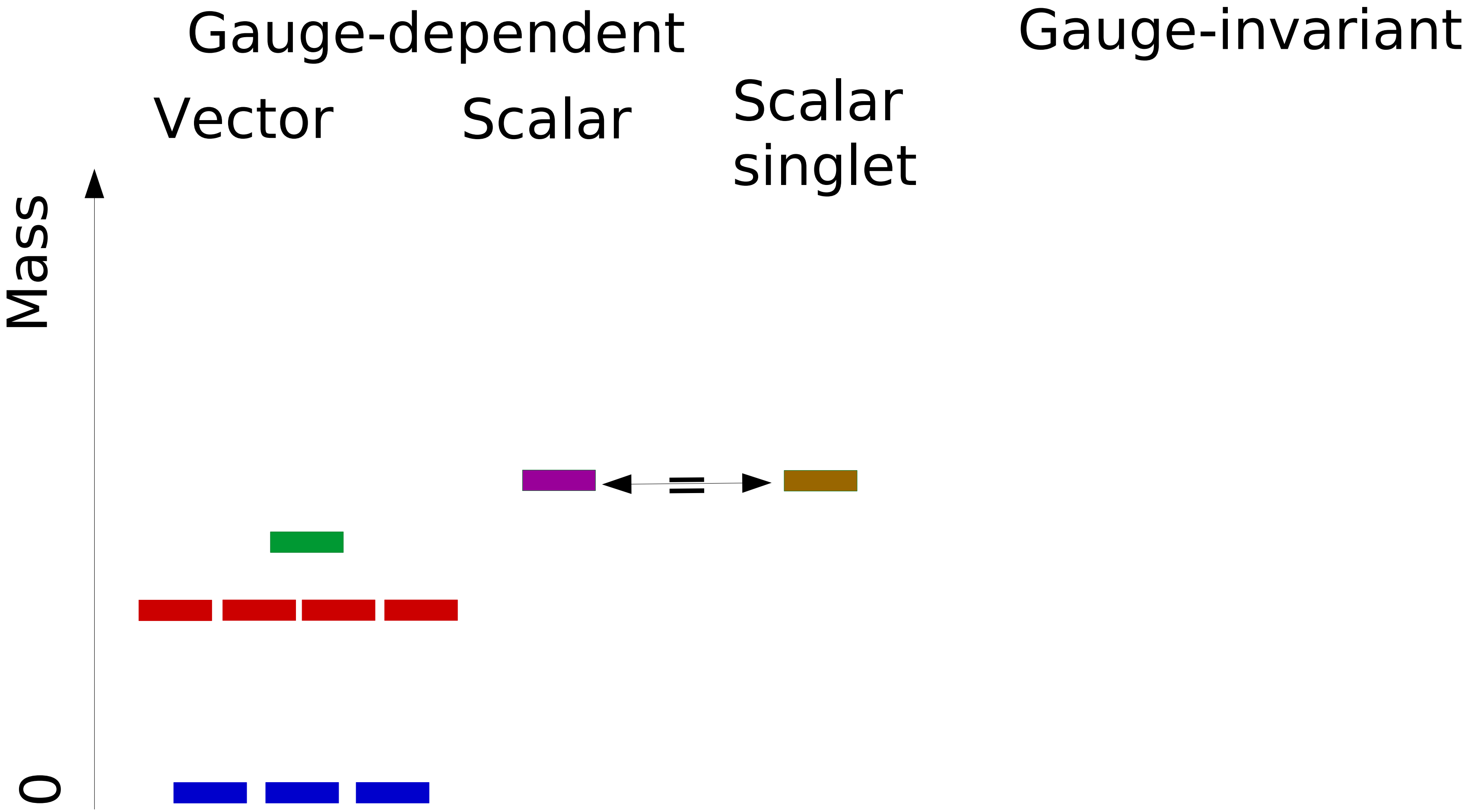

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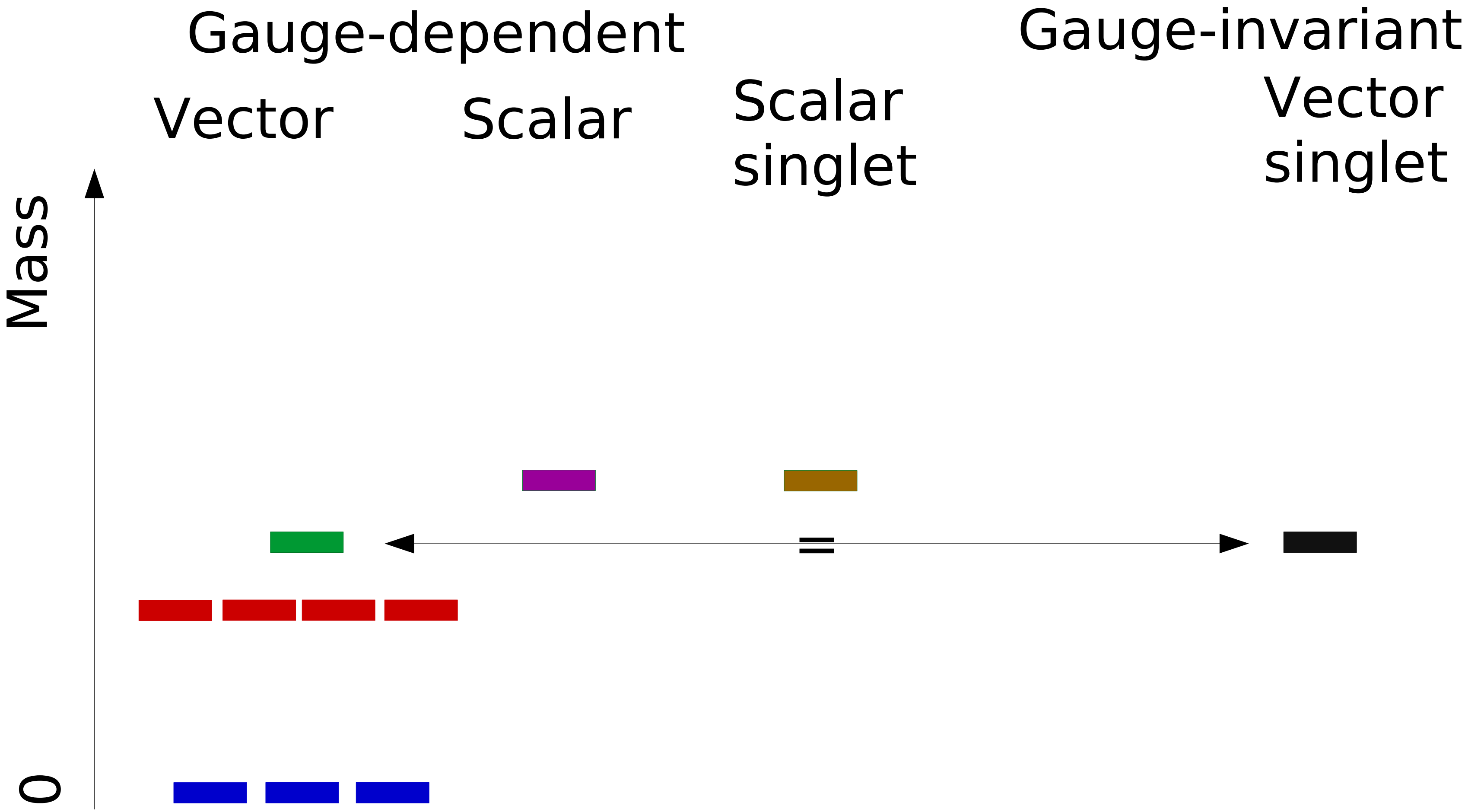

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Matrix from group structure

c^{ab} projects out only one field

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1^- singlet: $\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle$

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$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle = v^2 c^{ab} \langle W_\mu^a(x) W^b(y)^\mu \rangle + \dots$$
$$= v^2 \langle W_\mu^8 W_\mu^8 \rangle + \dots$$

Matrix from group structure

c^{ab} projects out only one field

Only one state remains in the spectrum at mass of gauge boson 8 (heavy singlet)

A different kind of states

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  - Because Higgs is bifundamental
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- Now: States without elementary analouge
  - Gauge-invariant states from 3 Higgs fields
  - Baryon analogue – U(1) acts as baryon number
  - Lightest must exist and be absolutely stable

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Spectrum

[Maas & Törek'16,'18
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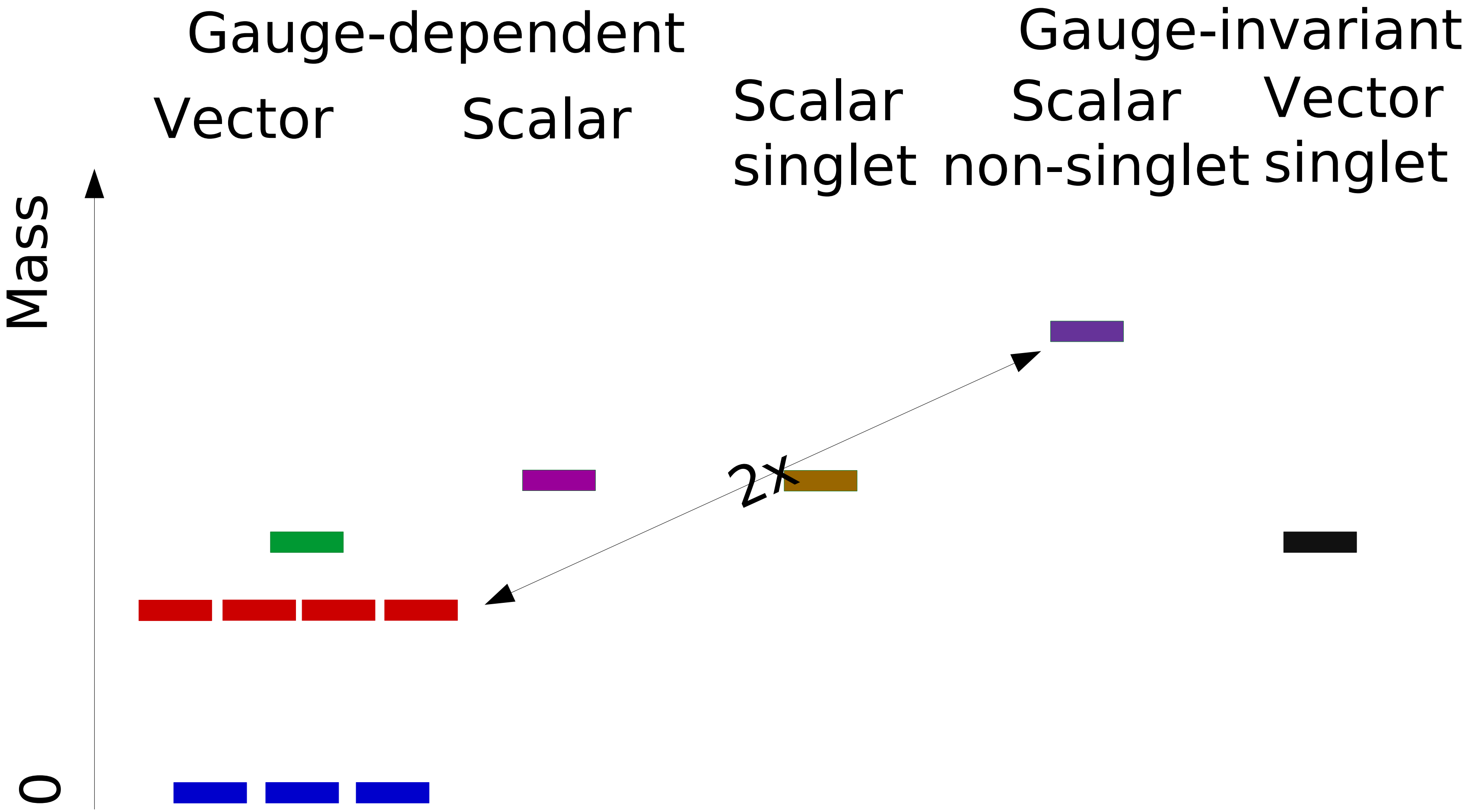

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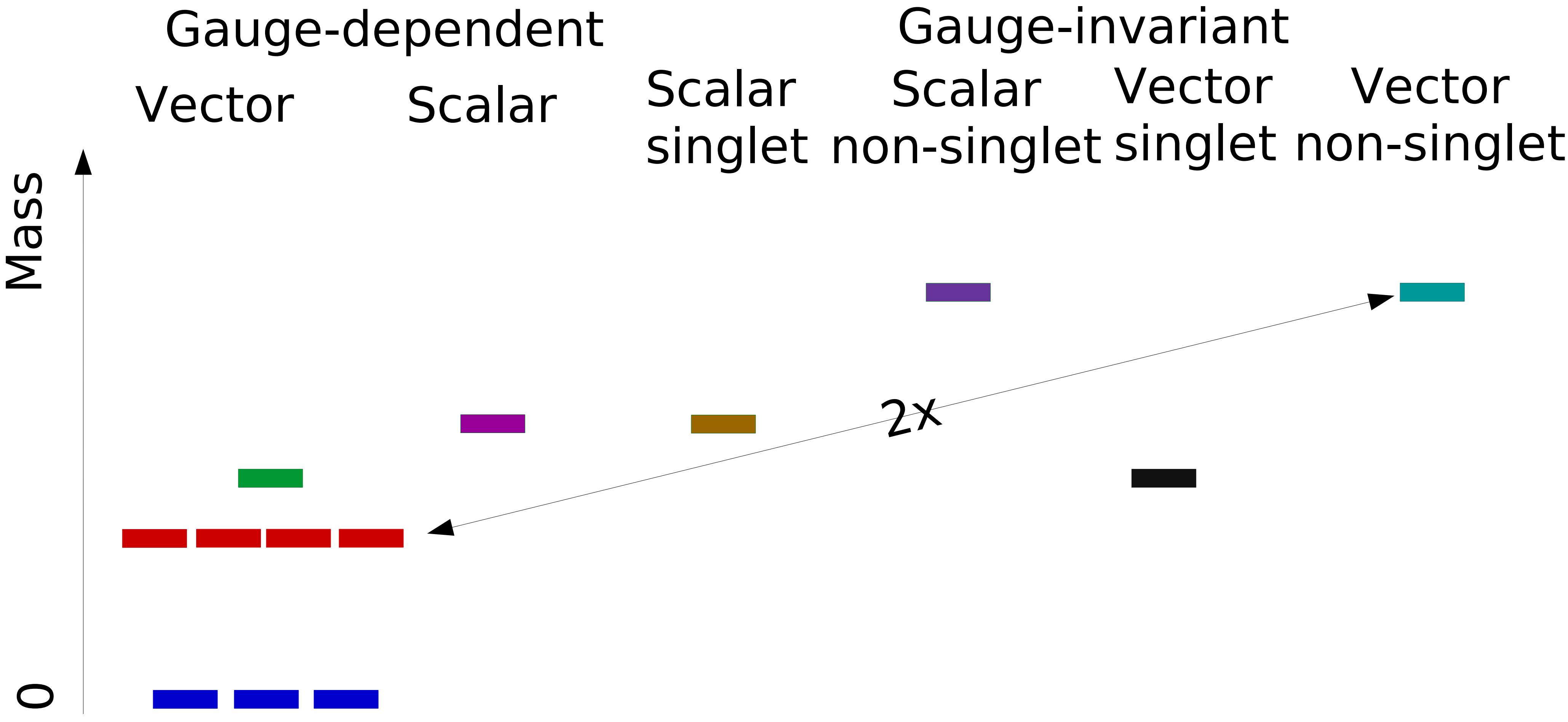

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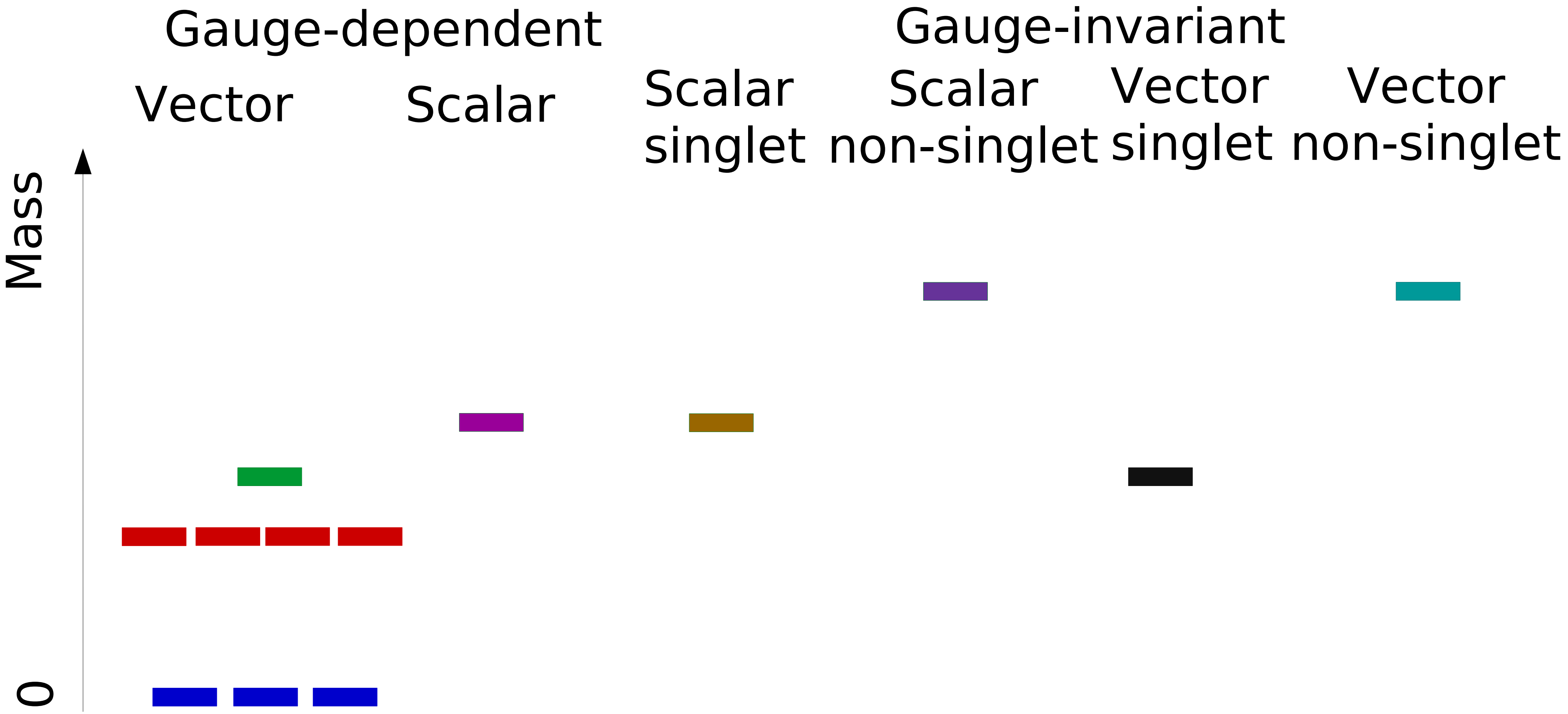

- Qualitatively different spectrum
- No mass gap!

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- What is the lightest state?
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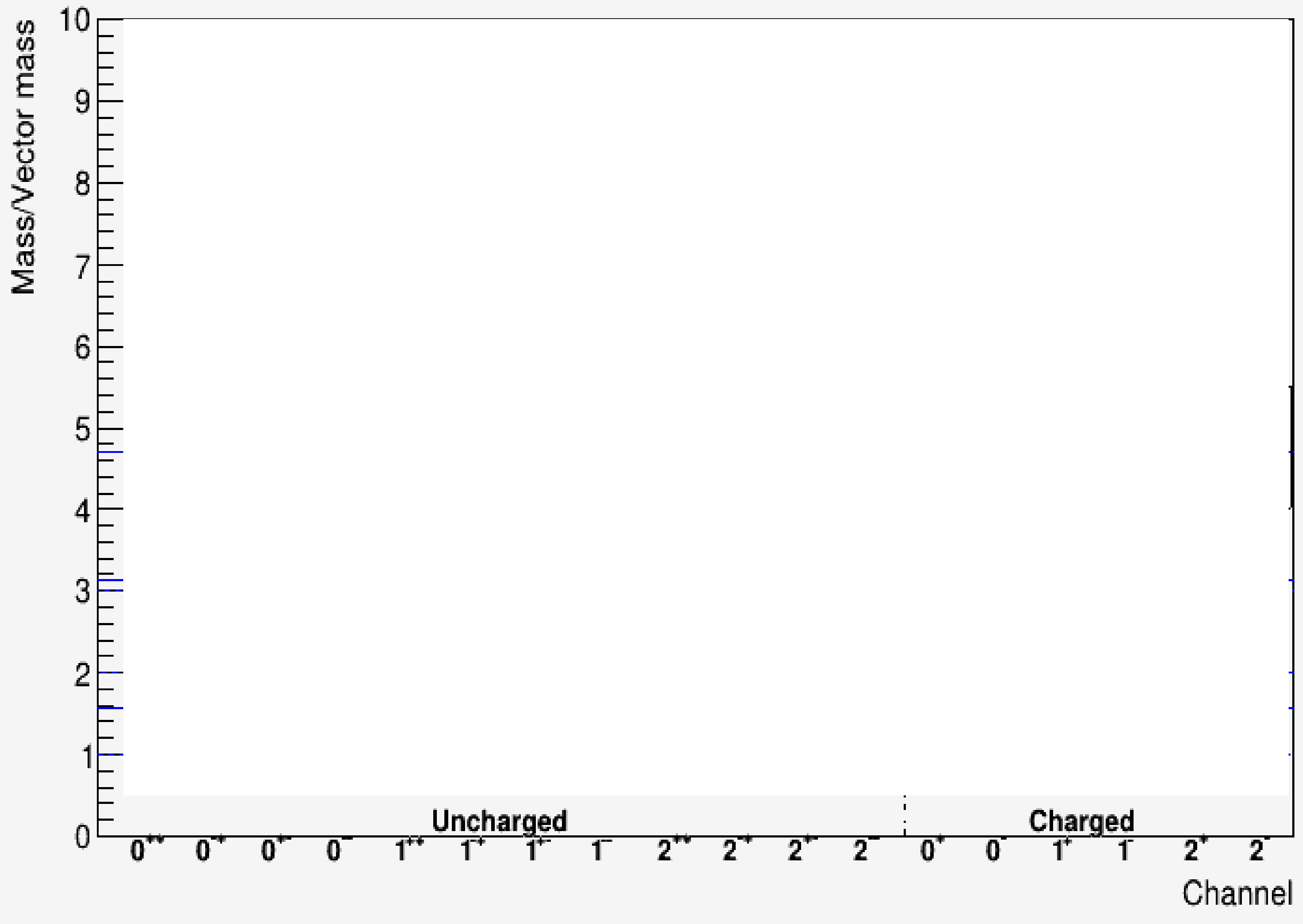
  - Lattice calulations
  - All channels: J<3
  - Aim: Ground state for each channel
    - Characterization through scattering states

Typical spectrum

PRELIMINARY

[Dobson et al.’25]



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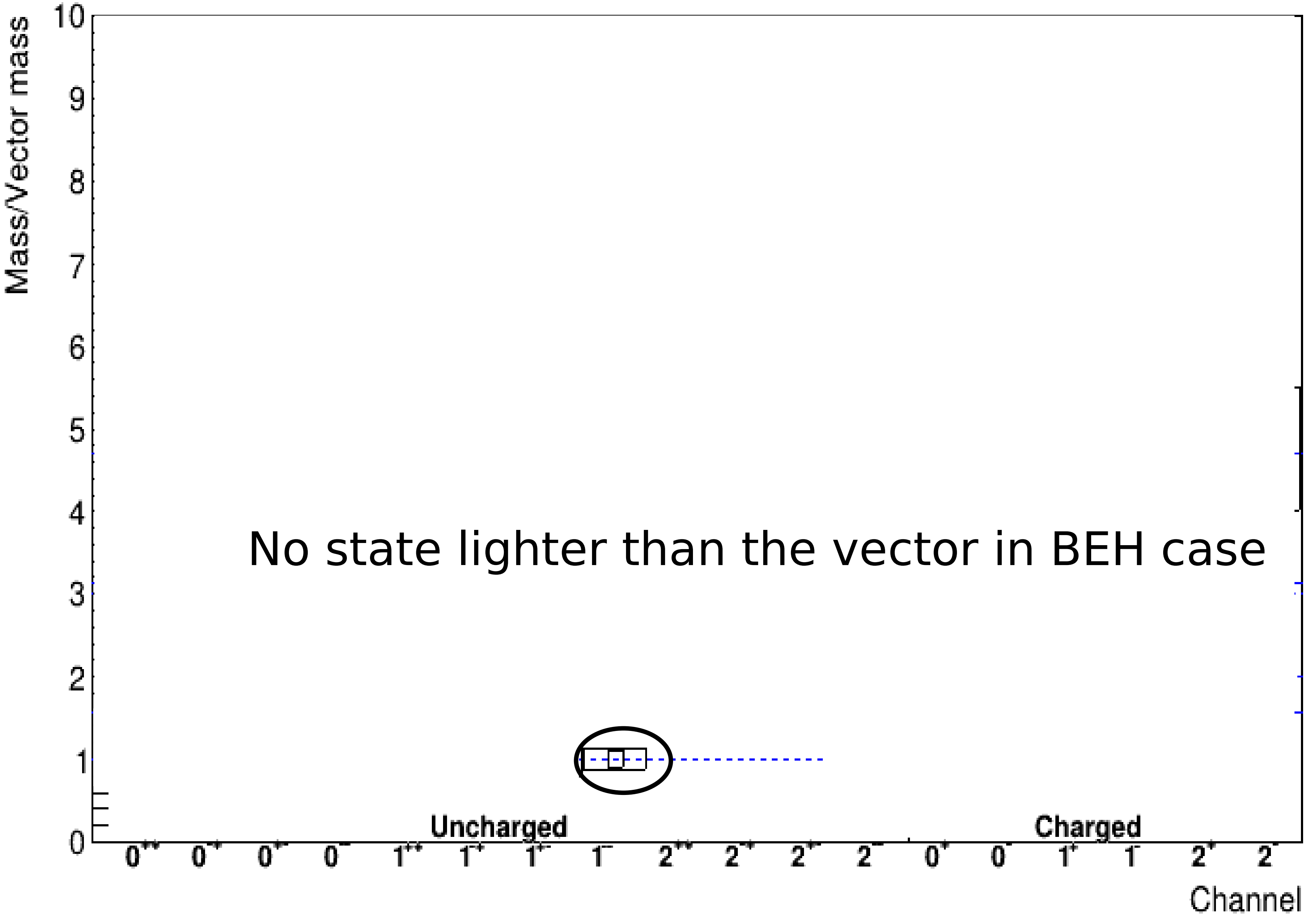

No state lighter than the vector in BEH case

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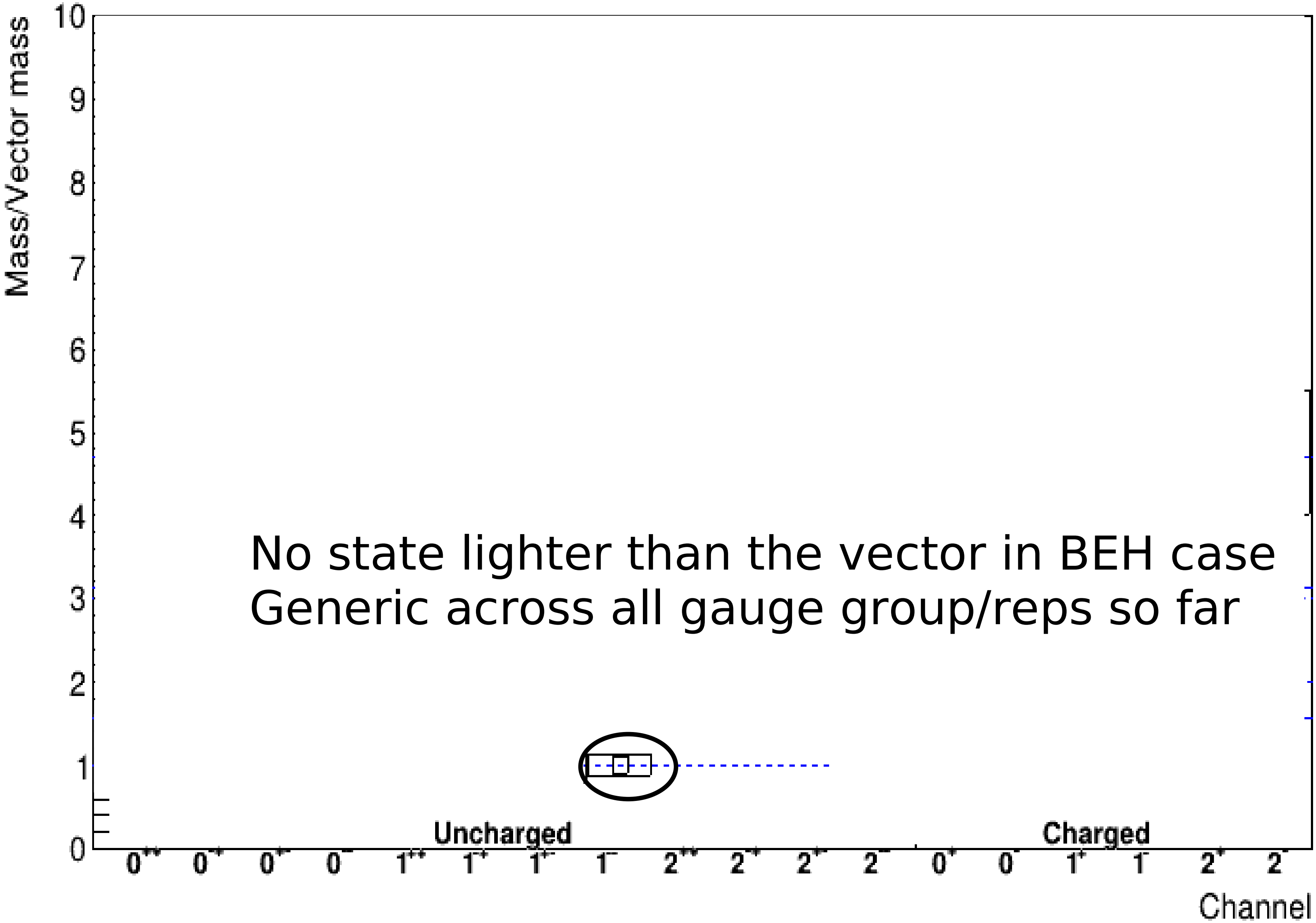

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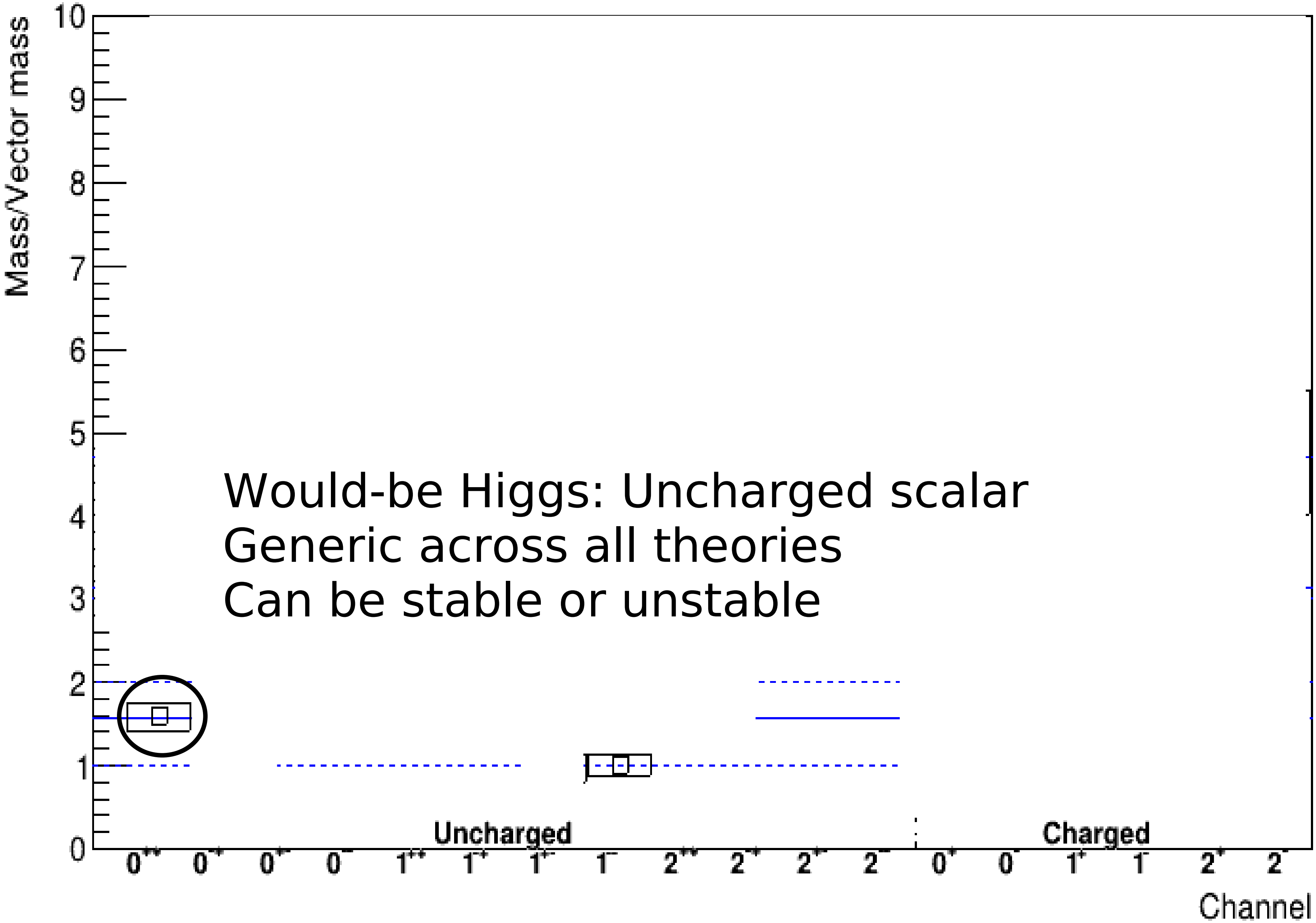

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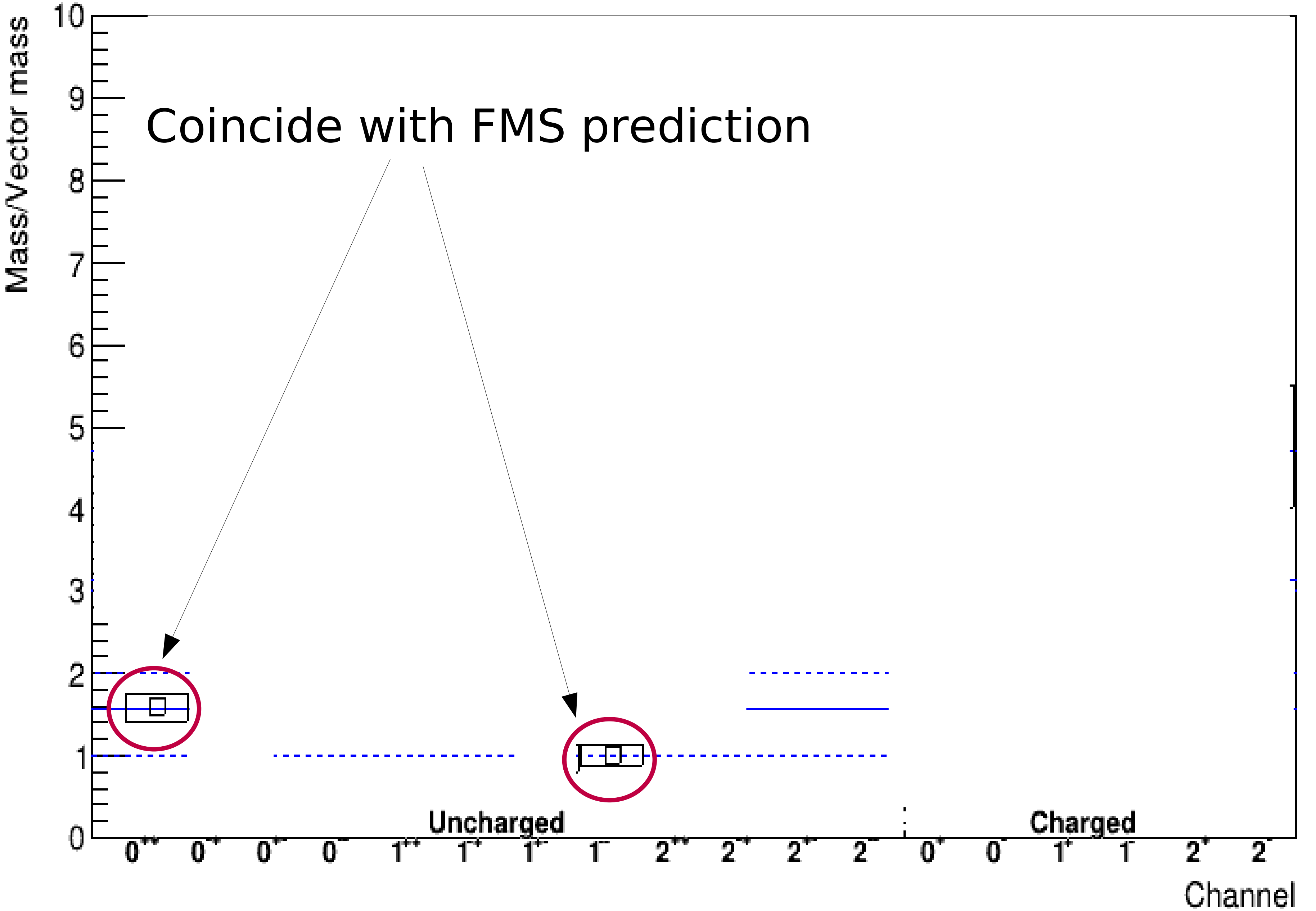

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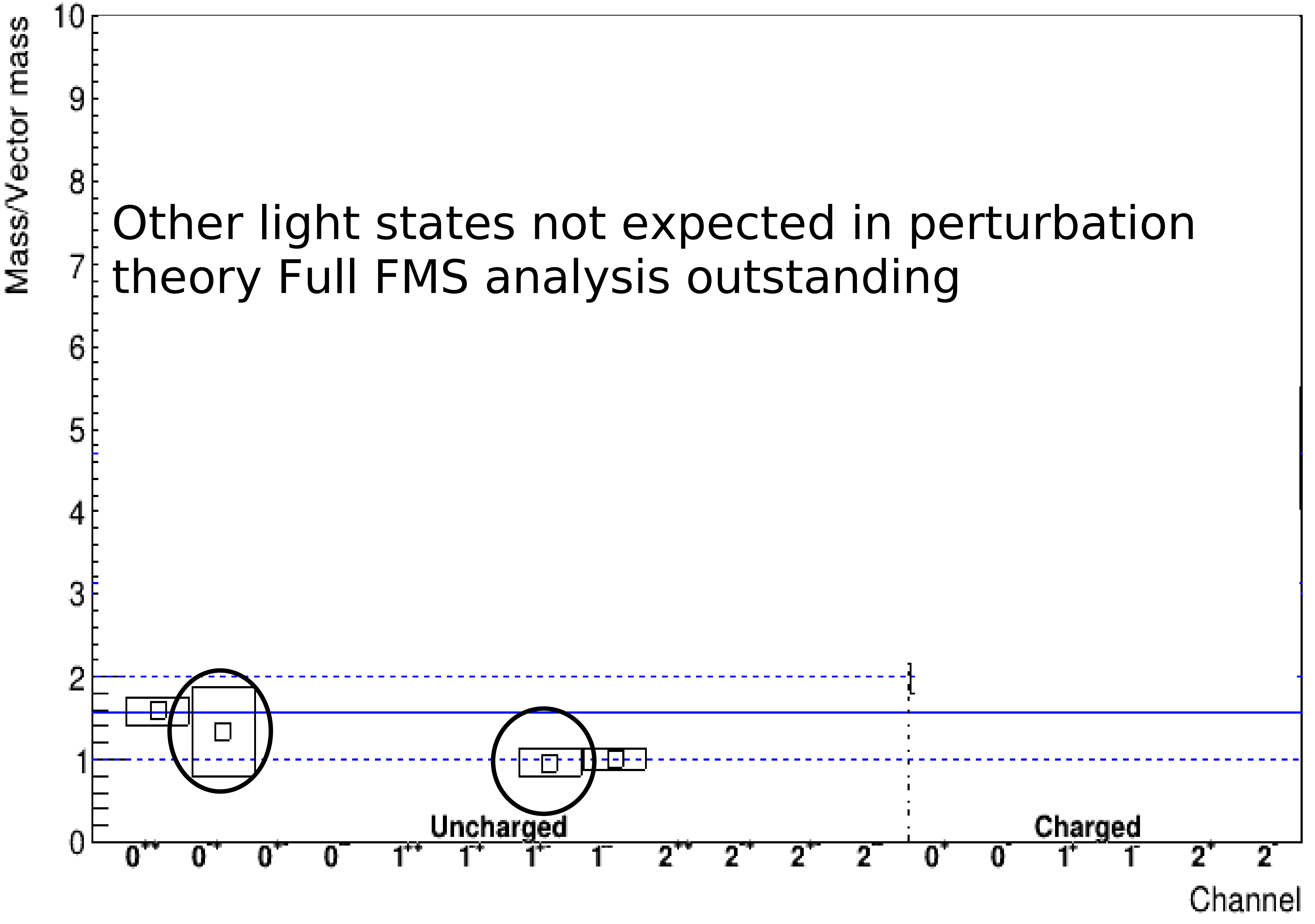

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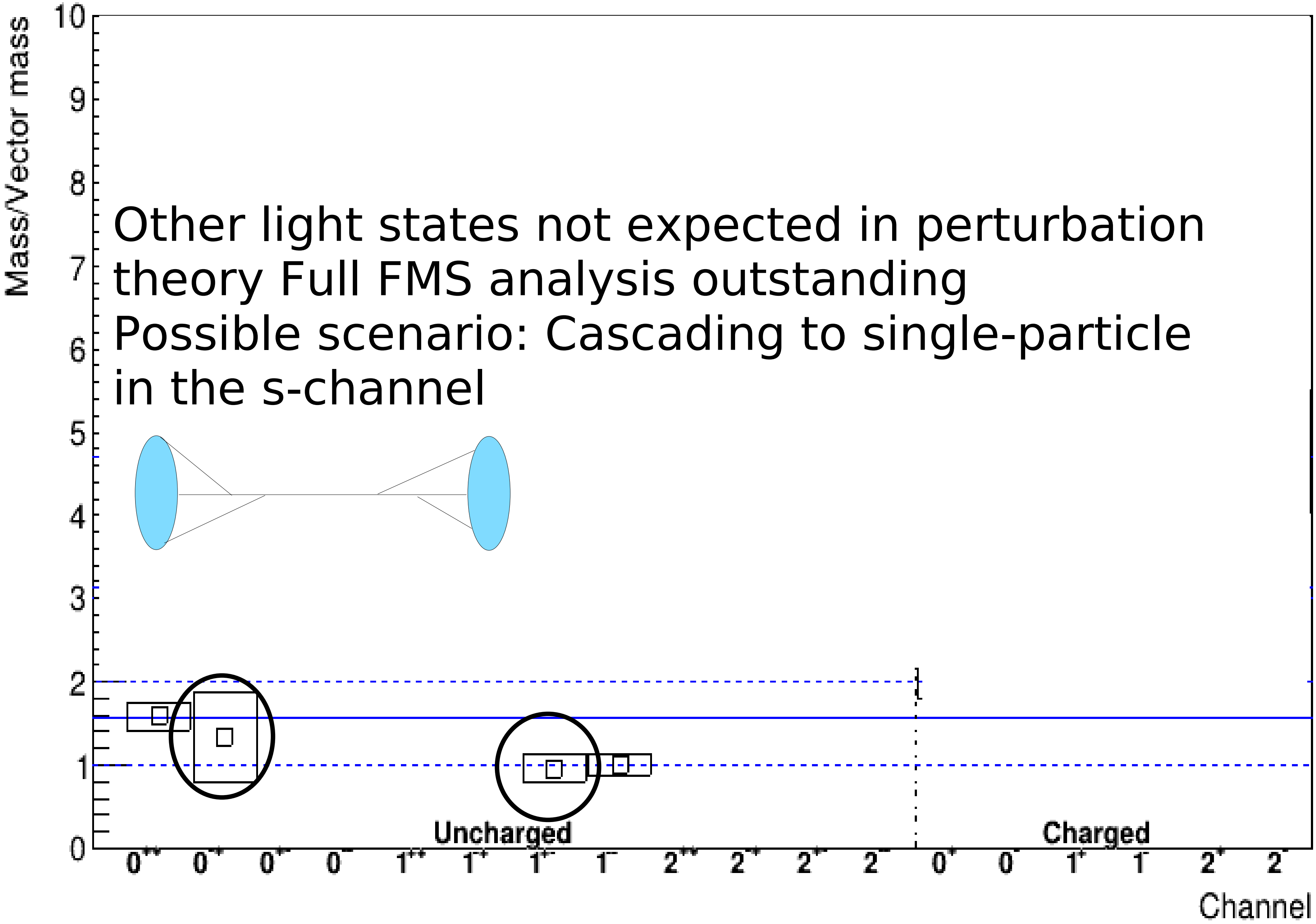

Other light states not expected in perturbation theory Full FMS analysis outstanding
Possible scenario: Cascading to single-particle in the s-channel

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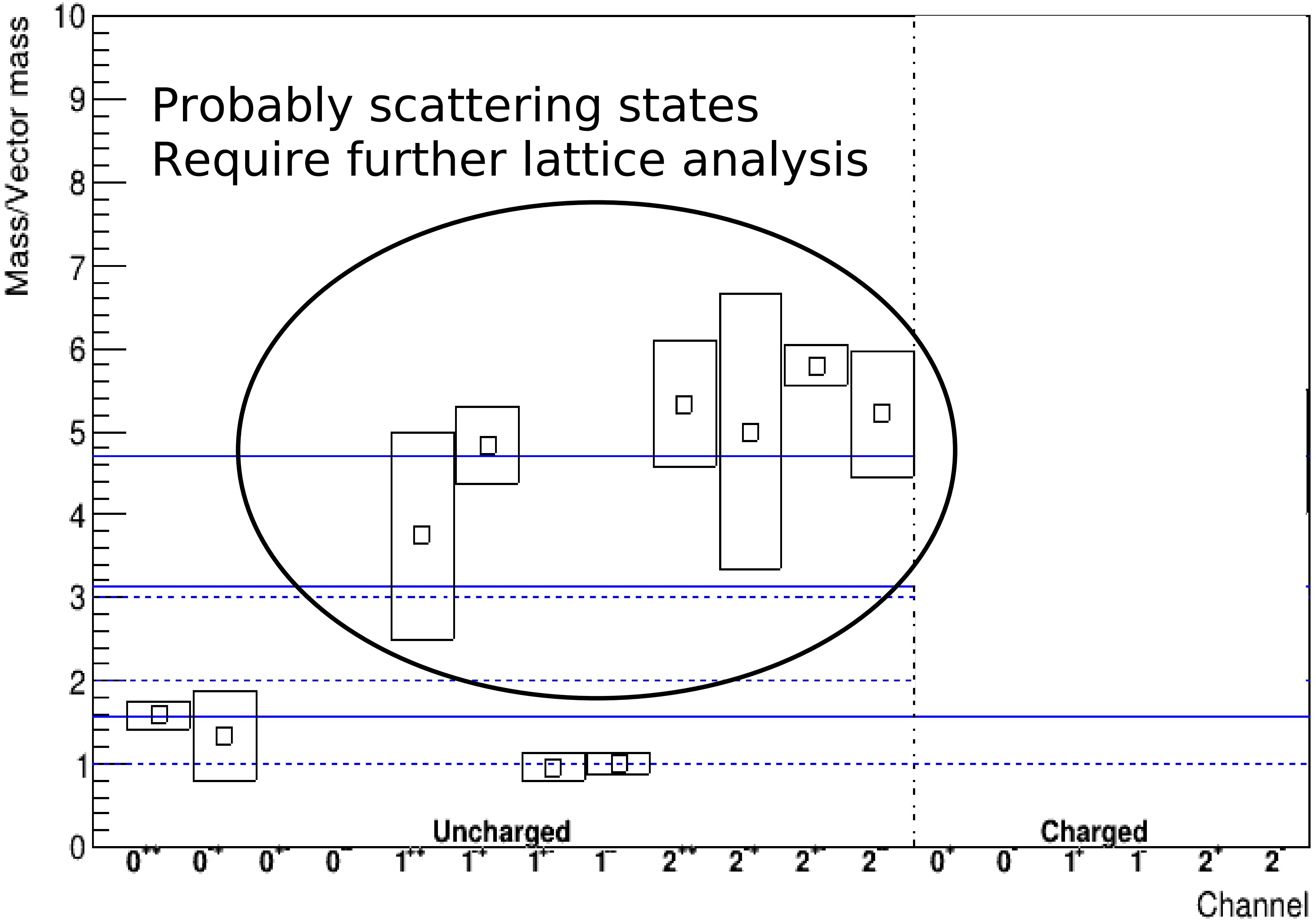

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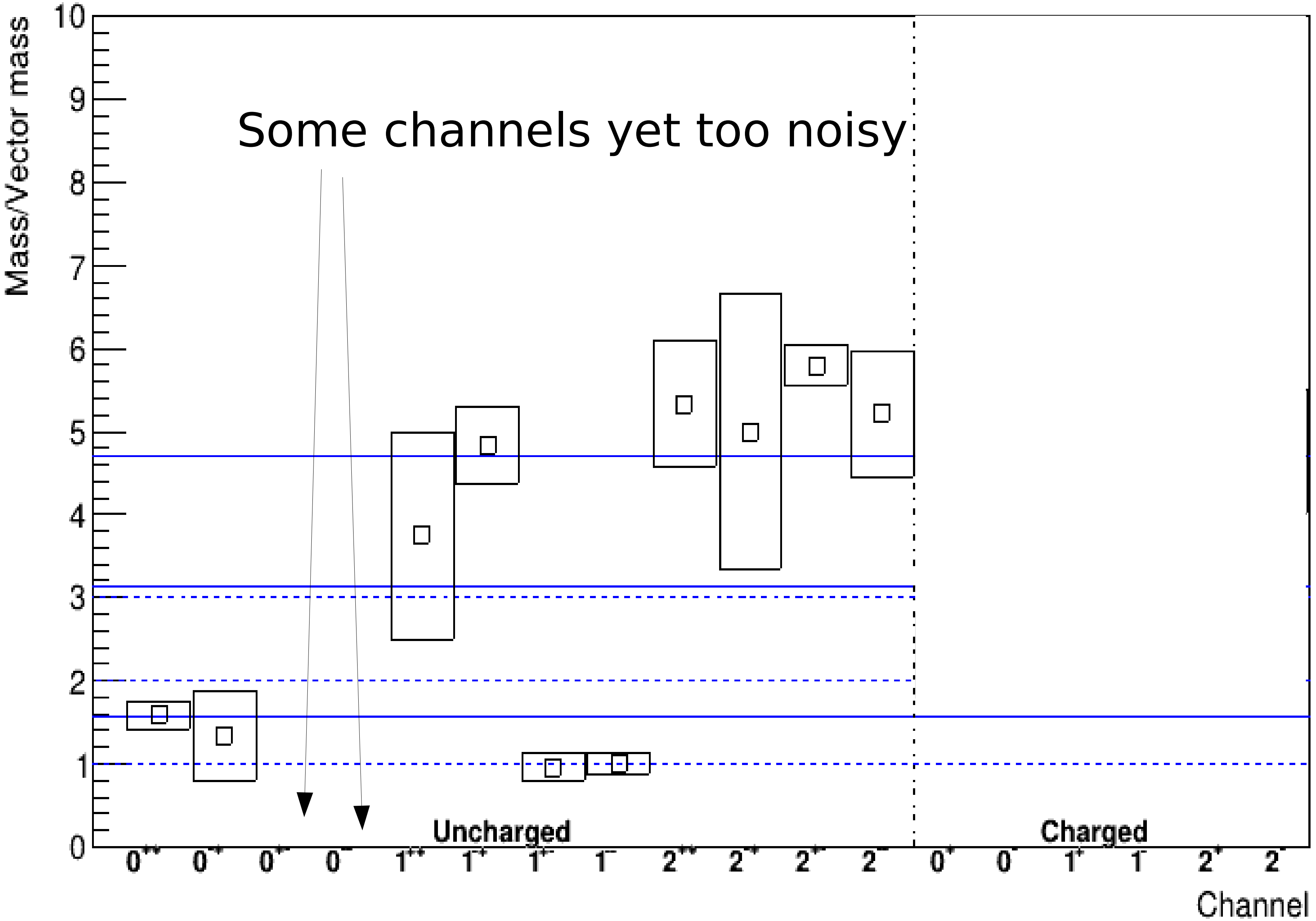

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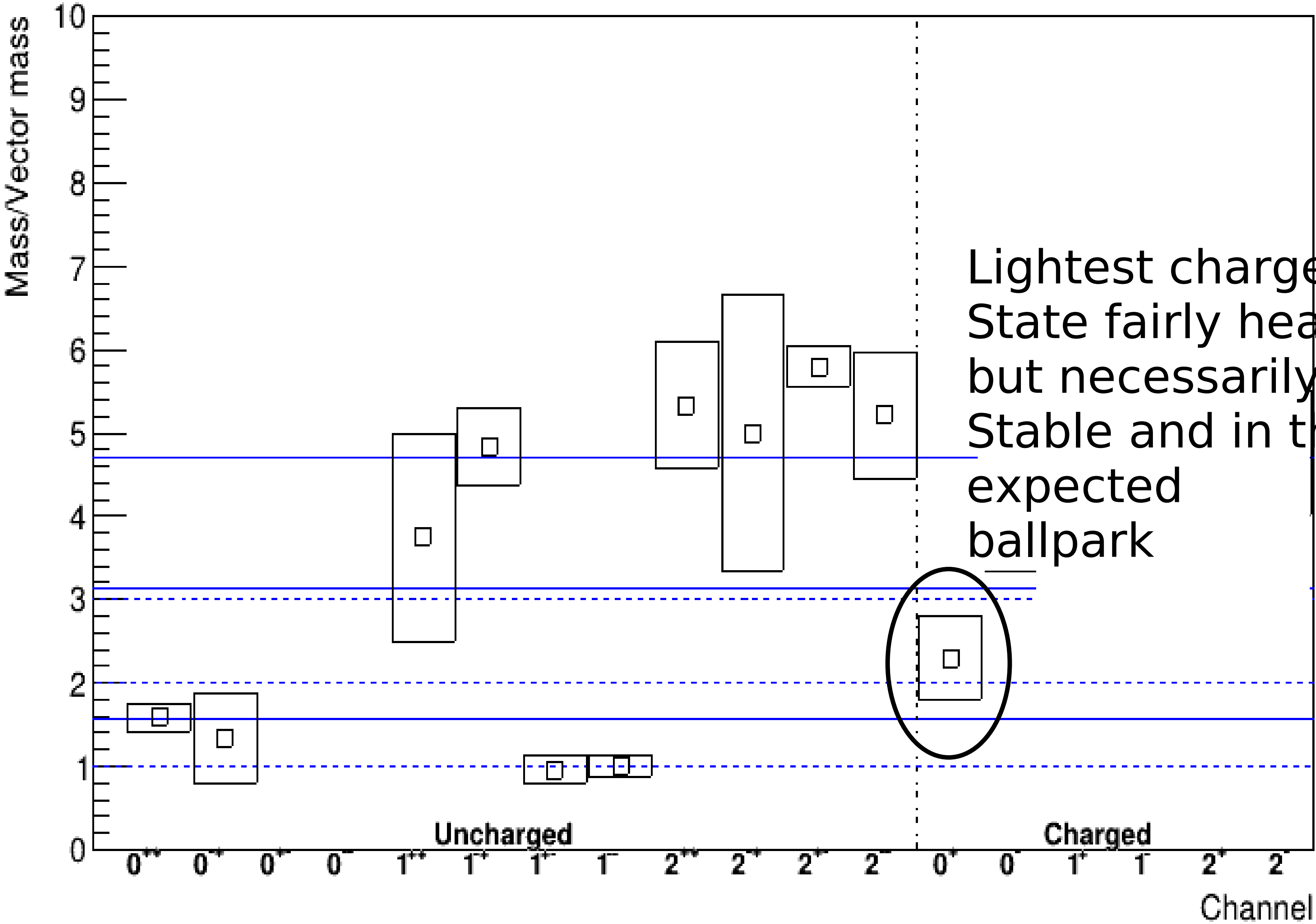

Lightest charged
State fairly heavy,
but necessarily
Stable and in the
expected
ballpark

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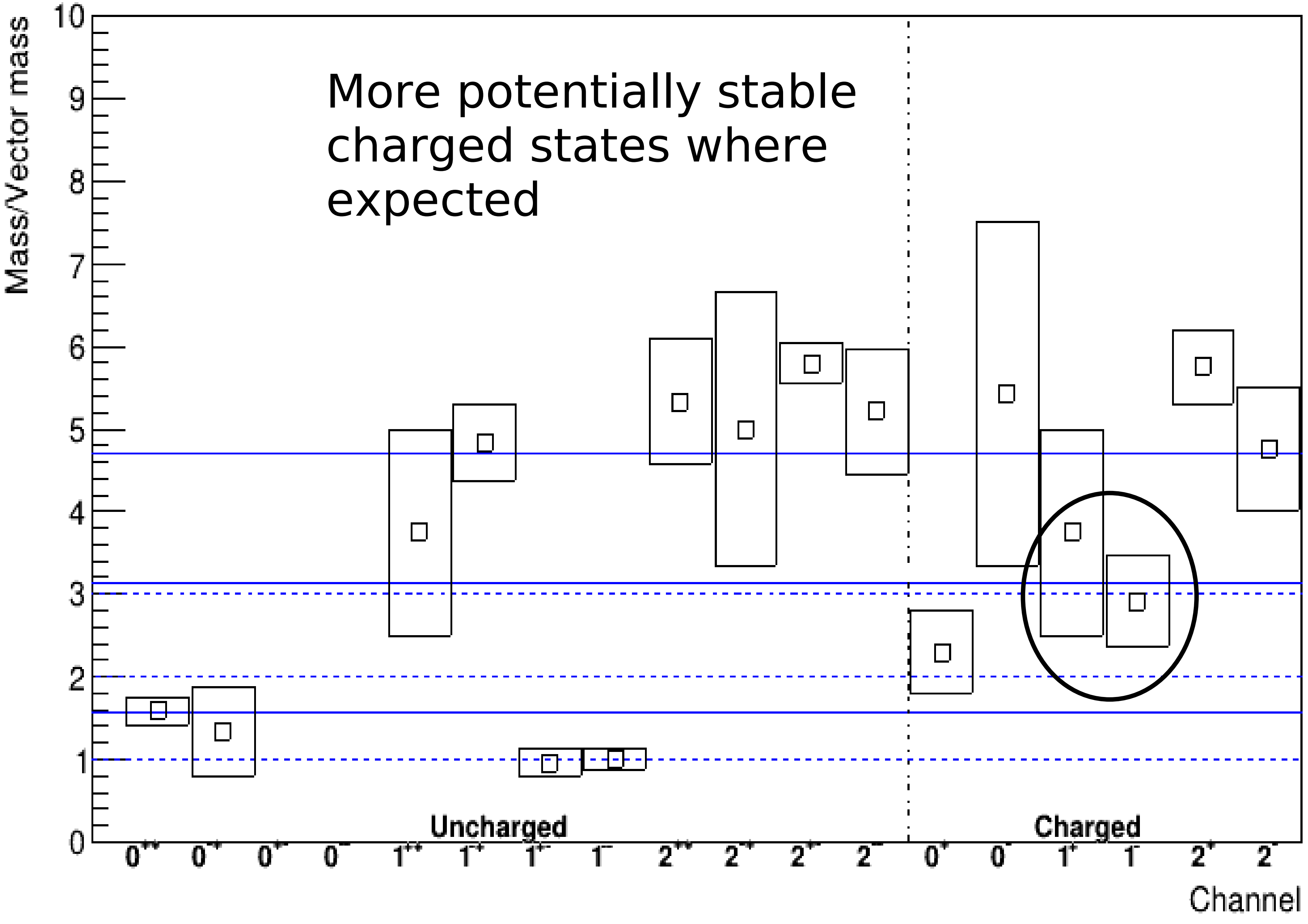

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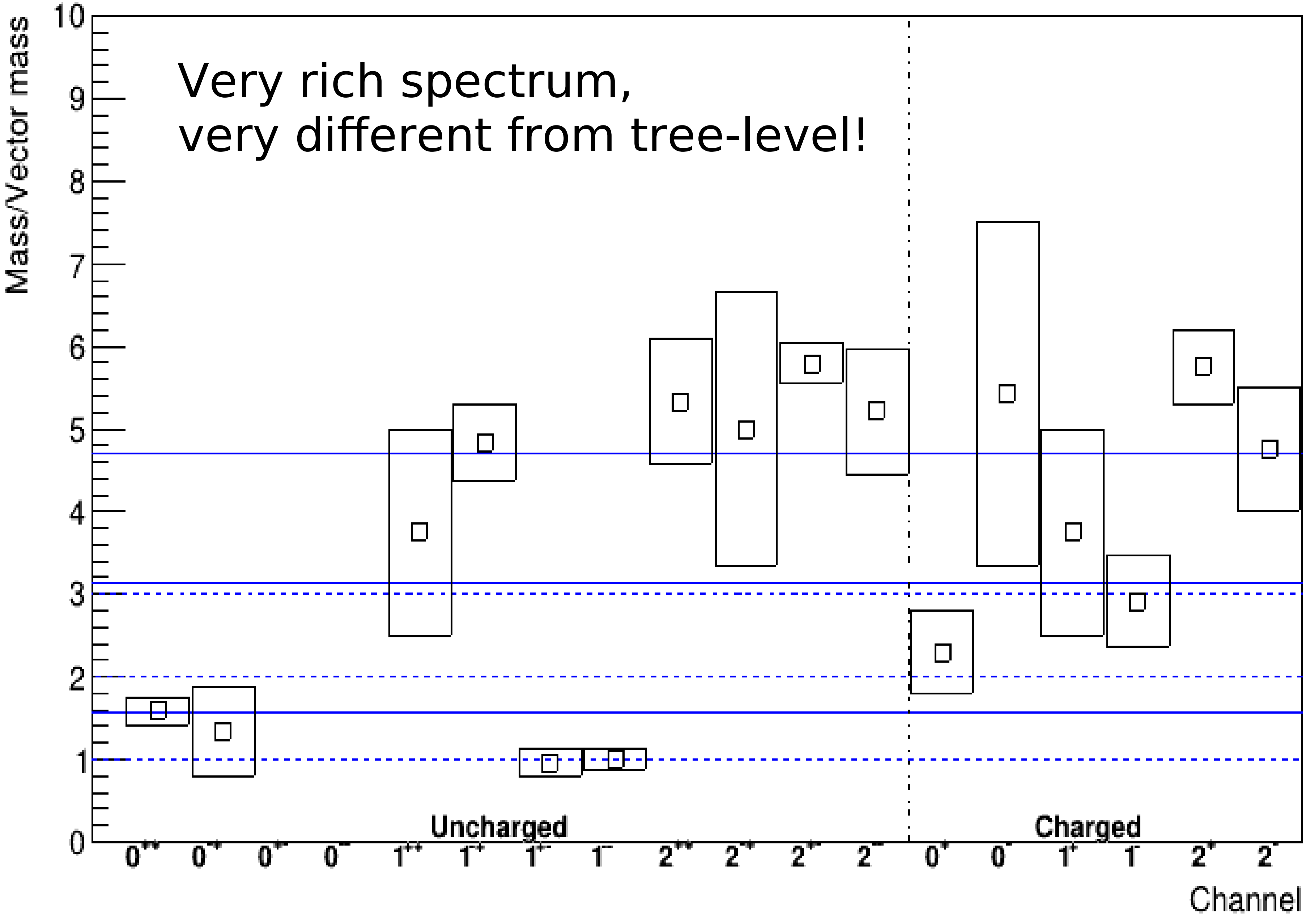

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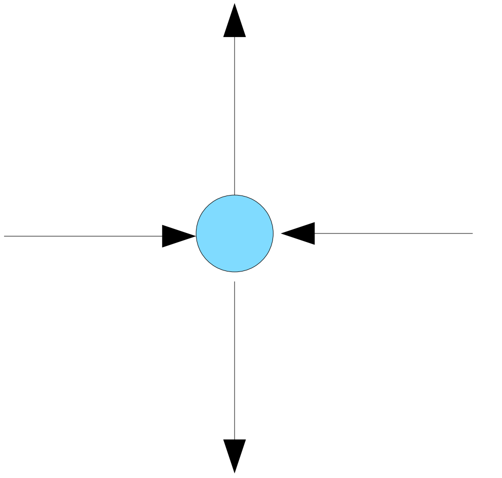

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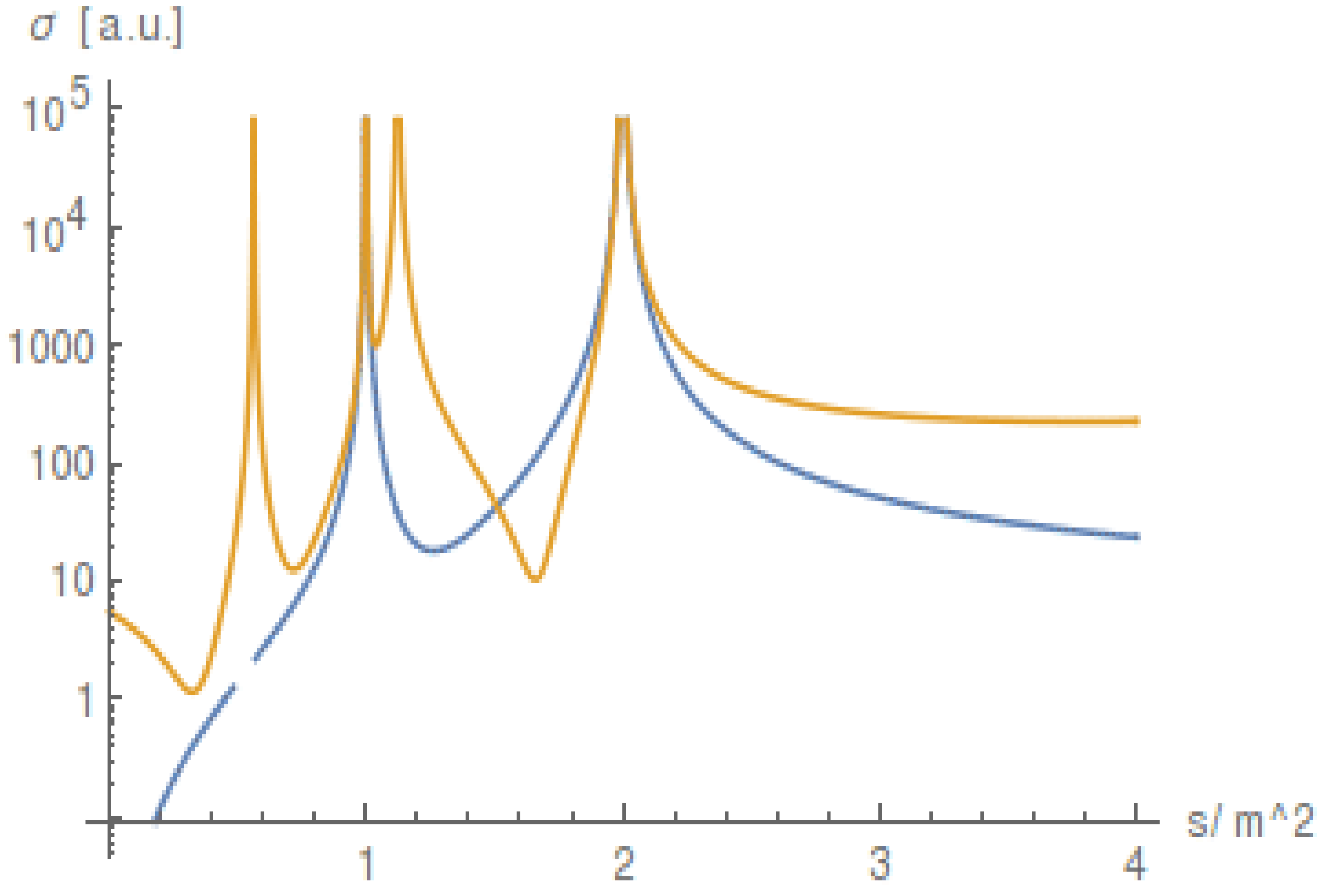

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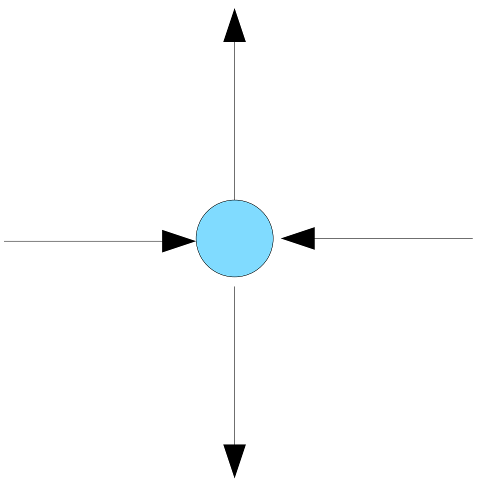

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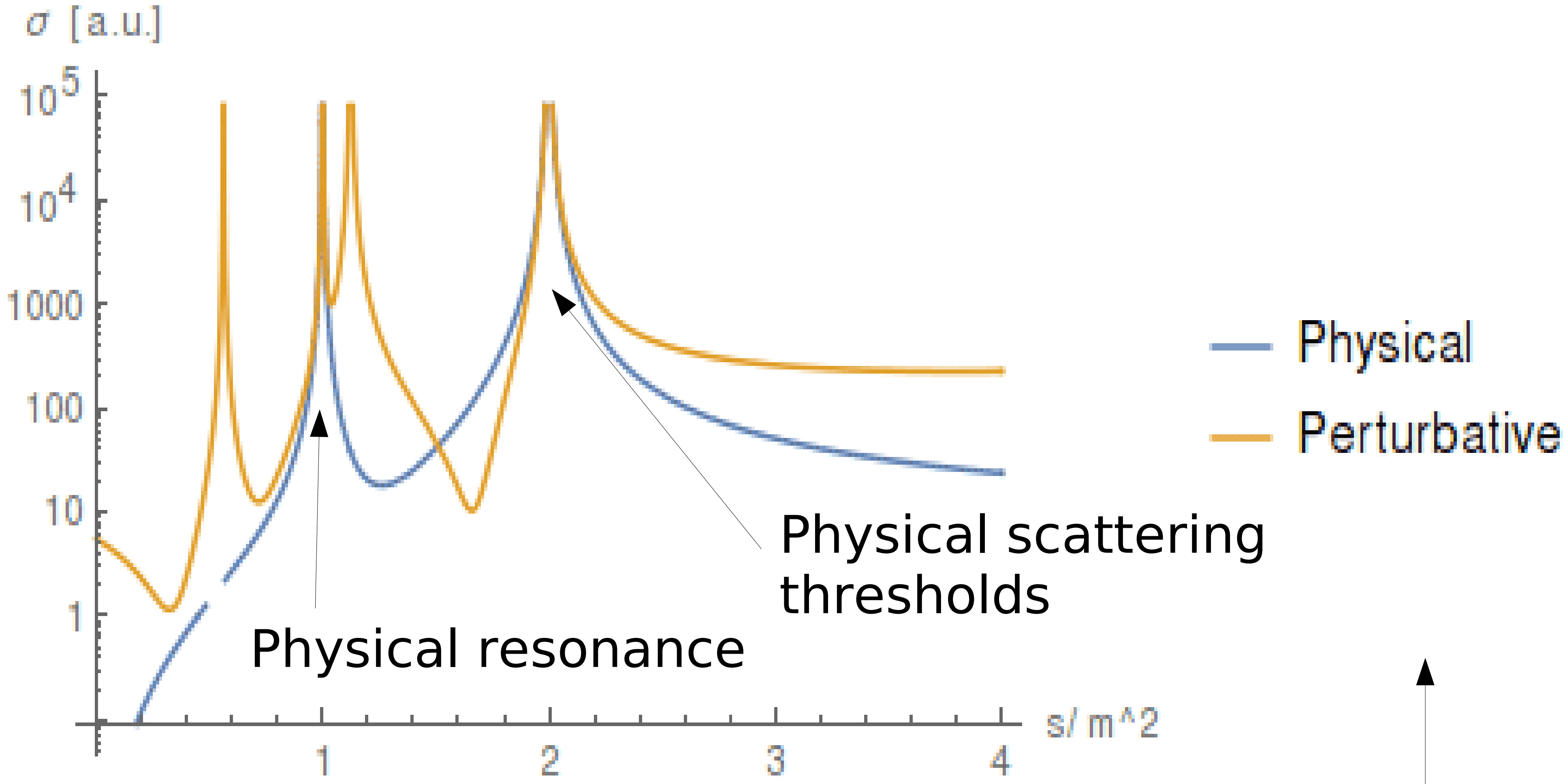

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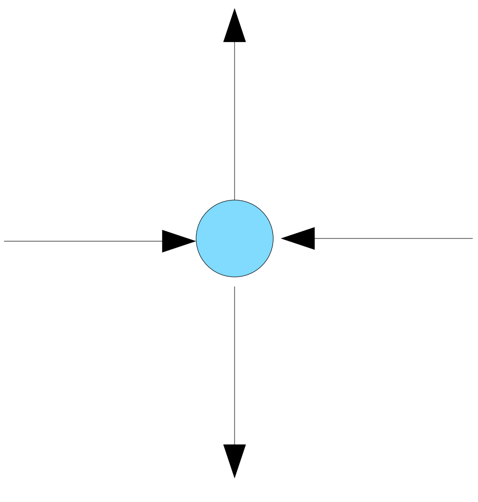

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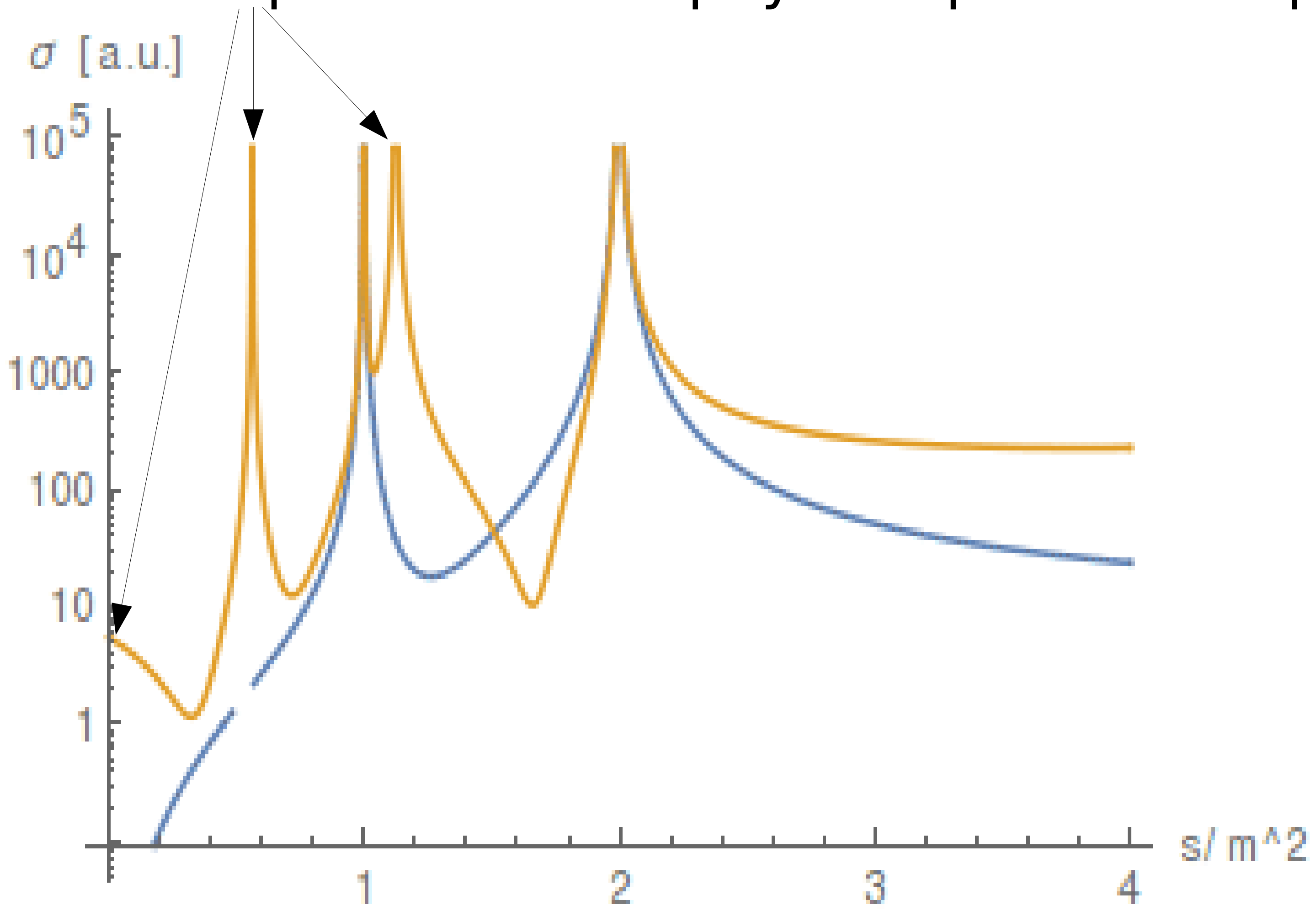
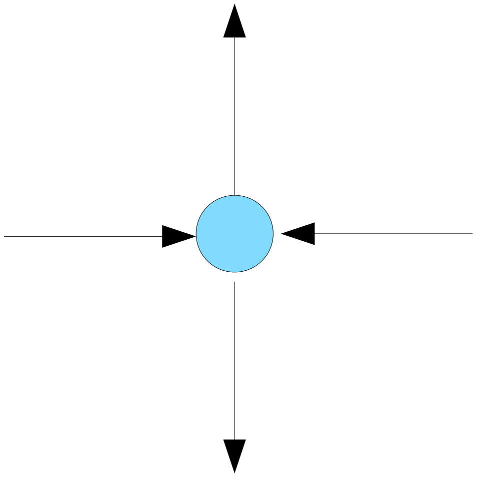
Ghost peaks from unphysical particles in perturbation theory





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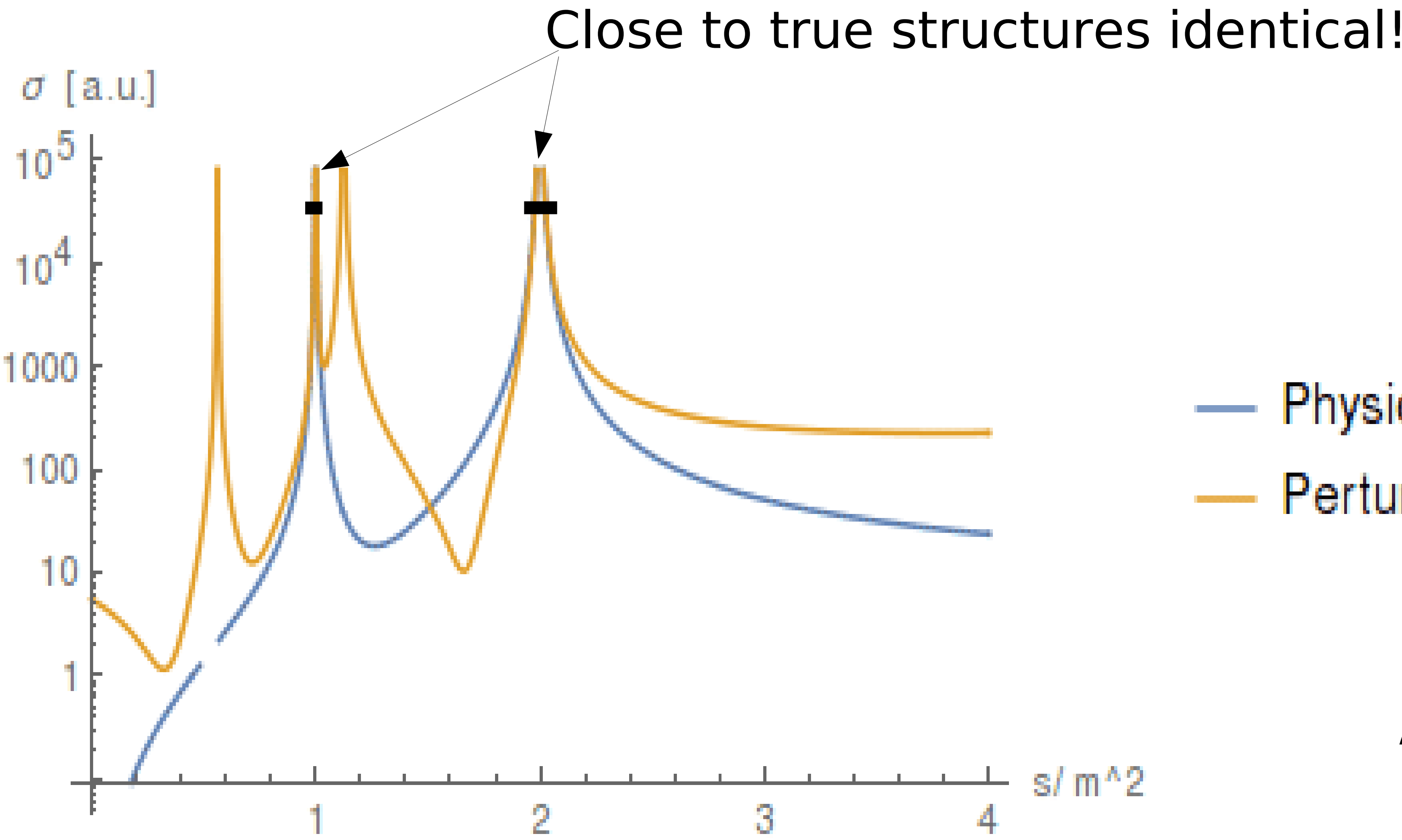

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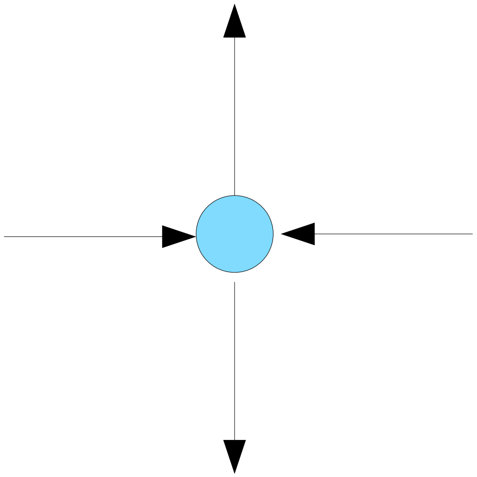

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  - Hints for such states seen in CDT [Maas, Plätzer, Pressler'25]

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Review: 1712.04721 Update: 2305.01960

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Review: 1712.04721 Update: 2305.01960