Gauge invariant spectra of SU(2) theories with BEH effect

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University of Graz

Outline

- Motivation: GI formulation of Higgs theories.
- FMS mechanism.
- Examples of lattice spectra with FMS predictions.
- Composite massless vector state investigation in SU(2)+adjoint.
- Composite fermion bound states lattice spectroscopy.

Motivation: Gauge invariant formulation of Higgs theories

Why a gauge invariant formulation?

[1] Maas and Sondenheimer, "Gauge-invariant description of the Higgs resonance and its phenomenological implications".

Why a gauge invariant formulation?

BSM theories with Higgs construction

- A Gauge Invariant (GI) formulation could provide a different spectrum than the one obtained with usual Perturbation Theory (PT) in a fixed gauge, given certain conditions.
- GUT setup: need to find a massless vector state which is GI.

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Insight on mass hierarchy

- Fermions have to be rethinked with a different operator structure.
- GI treatment of Higgs mass renormalization has only logarithmic divergences.[1]
- FMS provide potentially observable phenomenological consequences, can bring further insight in areas of tension of SM, or create new ones where there are none.

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Standard Higgs approach

Gauge Higgs theory prototype

$$\mathcal{L} = -\frac{1}{2}\mathrm{tr}(W_{\mu\nu}W^{\mu\nu}) + \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi + \lambda(\phi^\dagger\phi - v^2)^2$$

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- One performs 'spontaneous symmetry breaking'.
- In a suitable fixed gauge, the split $\phi(x) = vn + \varphi(x)$ is possible.
- In this gauge $\langle\phi\rangle = vn$.

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- This construction is gauge dependent[2].

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Main motivation

- No spontaneous breaking of local symmetries exists.[3]
- Wilson GI formulation has $\langle \phi \rangle = 0$, also there exist gauges in which $\langle \phi \rangle = 0$.[4]
- Perturbative results are gauge dependent, potentially nonphysical.
- No phase boundary for BEH effect with fund. Higgs $\rightarrow$ symmetry breaking is a gauge fixing artifact.[5]

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- Why PT works so well for the Standard Model?
- Is PT always reliable then?
- Answers lie in a gauge-invariant formulation of the theory.

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Gauge invariant formulation of SM observables

- Elementary fields are treated as observable in PT, even if not gauge invariant.
- The real physical objects must be described with gauge invariant composite operators[6].

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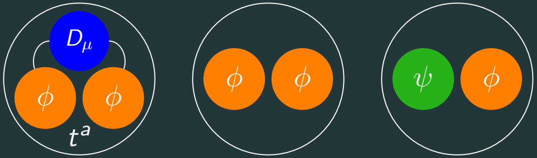
- Elementary Higgs $\phi(x)$ $\rightarrow$ Physical Higgs $(\phi^\dagger\phi)(x)$
- Elementary Fermion $\psi(x) \rightarrow$ Physical fermion $(\phi^\dagger\psi)(x)$
- Elementary Vector $W^a_\mu(x) \rightarrow$ Physical vector $(t^a\phi^\dagger D^\mu\phi)(x)$



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FMS Mechanism

- Correlators of GI operators give access to physical spectrum → explanation for ordinary PT spectrum validity.

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- Expand the correlator, $H(x) = \sqrt{2}\,\mathrm{Re}(n^\dagger \phi)$

$$\langle O^\dagger_{0^+}(x) O_{0^+}(y) \rangle = 4v^2 \,\langle H(x)^\dagger H(y) \rangle + \mathcal{O}(v)\,.$$

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- Mapping of on-shell properties. Off-shell contributions from non leading terms[7].

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Vector channel and custodial charge

- GI vector operator: Custodial Triplet $O^a_{1^-,\mu}(x) = tr(t^a \phi^\dagger D_\mu \phi)(x)$

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- Expansion the correlator,

$$\langle O^a_{1^-,\mu}(x) O^a_{1^-,\mu}(y)\rangle = -\frac{(gv)^2}{4} c^{ab} c^{ac} \langle W^b_\mu(x) W^c_\mu(y)\rangle + \mathcal{O}(v)\,.$$

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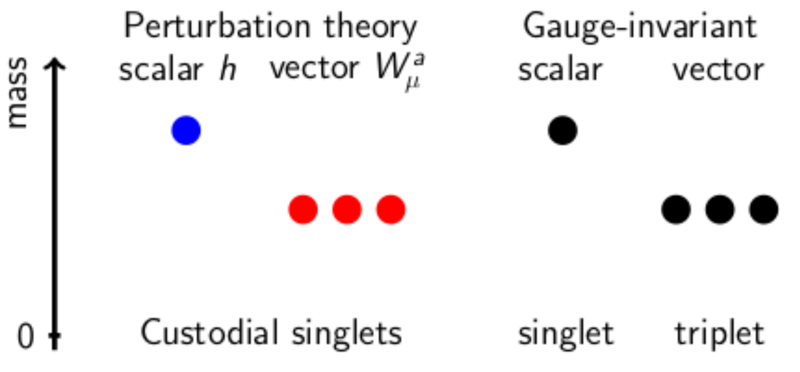
- The gauge triplet of the A^a_μ is mapped to the GI custodial triplet O^a_μ.
- Weak sector of SM $\rightarrow$ Custodial group $SU(2)$ = Gauge group $SU(2)$.
- Only true quantum numbers: J^{PC} and custodial charge.

FMS Mechanism for weak sector of SM

- GI spectrum of the weak sector corresponds to perturbation theory.



FMS for toy SU(3) GUT

- A SU(3) theory with a fundamental scalar has been investigated on lattice.

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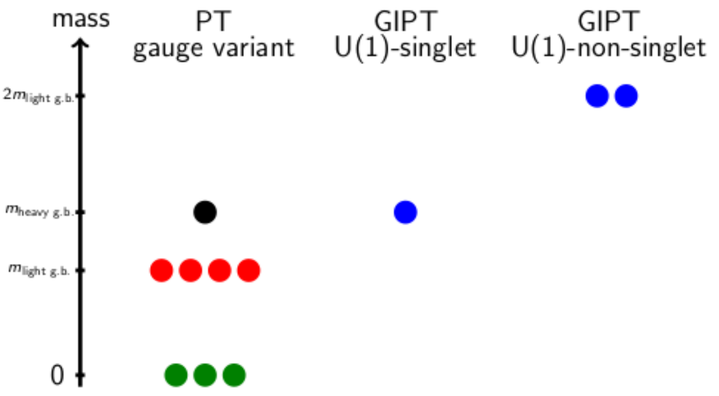

- Lattice spectrum support FMS predictions.[8]

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New challenges for the FMS mechanism

Massless composite GI vector state in Higgs theories

- Composite GI operators have been analyzed on the lattice, also for BSM theories.
- There is always a mass gap.

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GI fermions on the lattice

- The fermion sector of the SM, as GI bound states, has never been analyzed nonperturbatively.
- Parity violation is not accessible on the lattice[9].

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Massless vector state in the SU(2) adjoint Higgs theory[a]

[a] Afferrante, Maas, and Törek, "Composite massless vector boson".

SU(2) Gauge theory coupled with an adjoint Scalar

$SU(2)$+Adjoint Higgs theory

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mathrm{tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi) \right] - V(\Phi)\,.$$

[10] Georgi and Glashow, "Unity of All Elementary Particle Forces".

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- $\Phi(x)$ is the scalar field in the adjoint representation.
- Potential:

$$V = -\mu^2 \,\mathrm{tr}\,\Phi^2 + \frac{\lambda}{2}(\mathrm{tr}\,\Phi^2)^2\,.$$

- We look for potentials that allow a BEH effect.

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- We look for potentials that allow a BEH effect.
- It is a toy GUT model[10] with a low energy QED (massless vector).

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Brout-Englert-Higgs Effect - Perturbation theory approach

- The only relevant breaking pattern which leads to a potential with a minimum is $SU(2) \to U(1)$.
- Split scalar field in vev and fluctuations:

$$\Phi(x) = \langle\Phi\rangle + \phi(x) \equiv w\Phi_0 + \phi(x)\,.$$

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- Φ_0 is the direction of the vev: $\Phi_0^a \Phi_0^a = 1$, can always be chosen diagonal.
- Perturbative tree level mass matrix for the gauge fields:

$$(M_A^2)^{ab} = -2(gw)^2 \,\mathrm{tr}\Big([T^a, \Phi_0][T^b, \Phi_0]\Big)\,.$$

- Φ_0 direction: massless gauge boson,
- Remainder coset: 2 massive gauge bosons $m_A = gw$.

FMS mechanism for the SU(2) adjoint Higgs

GI operator for the vector channel:

$$
\begin{aligned}
O^{\mu}_{1^-} &= \frac{\partial_\nu}{\partial^2}\,\mathrm{tr}[\Phi F^{\mu\nu}] \\
&= -\,w\,\mathrm{tr}\Big[\Phi_0(\delta^{\mu}_{\nu} - \partial^{\mu}\partial_{\nu}/\partial^2)A^{\nu}\Big](x) + \mathcal{O}(A,\phi) \\
&= -\,w\,\mathrm{tr}\big[\Phi_0 A^{\mu}_{\perp}\big](x) + \mathcal{O}(A,\phi)\,.
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With $\Phi_0^a = \delta_{a3}$

$$
\langle O^{\mu}_{1^-}(x)O_{1^-,\mu}(y)\rangle = \frac{w^2}{4}\,\langle A^{3\mu}_{\perp}(x)A^{3}_{\perp,\mu}(y)\rangle + O(w^0)
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We expect a massless composite vector bound state[11].

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Spectrum for the SU(2) adjoint Higgs vector channel

Perturbation theory spectrum

- One massless gauge boson.

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- One massless vector boson (from first order expansion).

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Spectrum for the SU(2) adjoint Higgs vector channel

Perturbation theory spectrum

- One massless gauge boson.
- Two massive vector states with mass $m_A = gw$.

FMS mechanism

- One massless vector boson (from first order expansion).
- One next level state with mass $2m_A$ (further expansion in the coupling, constituent picture)[12].

[12] Maas, Sondenheimer, and Törek, "On the observable spectrum of theories with a Brout-Englert-Higgs effect".

Scan of the phase diagram

- Lattice implementation of the theory has been performed.
- Previous lattice results point to phase transition: QCD-like vs. BEH[13].

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- We can use the vev and the plaquette as order parameters.
- We fixed $\beta = 4, \lambda = 1$ and we varied κ over $[0.1, 0.8]$.
- Lattice sizes: $8^4, 16^4, 24^4, 32^4$.

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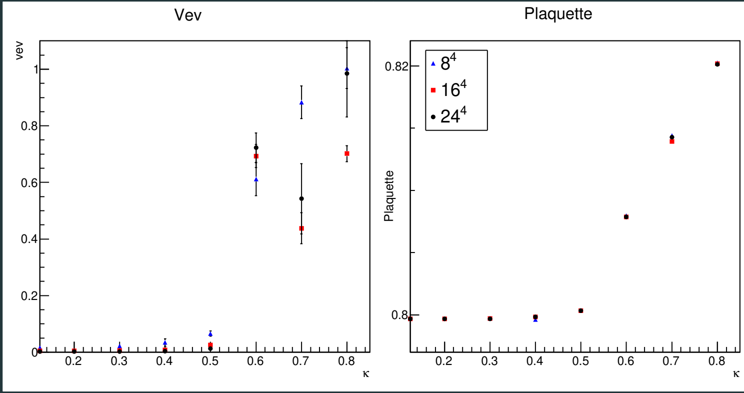
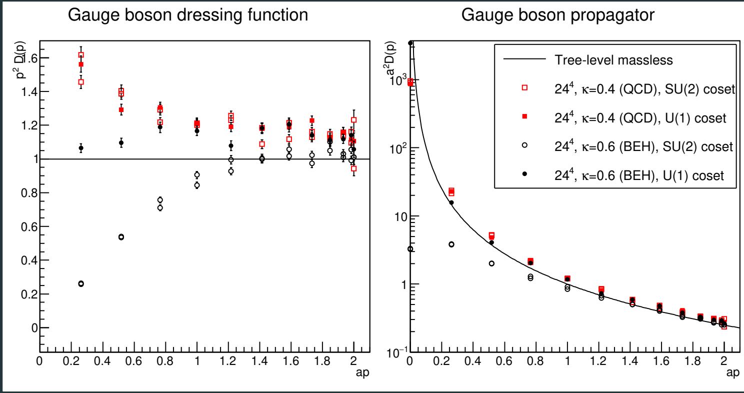


- The vev and plaquette variable show a discontinuity around $\kappa \sim 0.5$ (with β, λ fixed).
- We expect to be in the BEH phase for $\kappa > 0.5$.

Gauge boson propagator



- Phase diagram and FMS RHS result, using gauge propagator in a fixed gauge, at $\kappa = 0.4$ and $\kappa = 0.6$.
- The split of the two cosets at $\kappa = 0.6$ is a strong hint of the BEH phase.

Operator for lattice spectroscopy

- Check LHS of FMS (lattice realization of $O^{\mu}_{1^-}$)[14]:

$$B^i(x) = \frac{1}{\sqrt{2\,\mathrm{Tr}(\Phi^2)}}\,\mathrm{Im}\,\mathrm{Tr}\Big(\Phi(\vec{x},t)U^{jk}(\vec{x},t)\Big)\,.$$

- Correlator showed no signal in rest mass frame.

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- Correlator showed no signal in rest mass frame.
- We give the operator a non-zero momentum via

$$B^j(\vec{p},t) = \frac{1}{\sqrt{V_{\vec{x}}}}\,\mathrm{Re}\sum_{\vec{x}} B^j(\vec{x},t)e^{i\vec{p}\cdot\vec{x}}\,.$$

- We choose as momentum the smallest one in the z direction

$$\vec{p}_z = \left(0, 0, \frac{2\pi}{N_z}\right)\,.$$

[14] Lee and Shigemitsu, “Spectrum Calculations in the Lattice Georgi-glashow Model”.

Transverse and Longitudinal Correlator

Split of the correlator in the transverse and the longitudinal part

$$C_\perp(t) = \frac{1}{N_t} \sum_{t'=0}^{N_t-1} \sum_{j=1}^{2} \langle B^j(\vec{p}_z, t') B^j(\vec{p}_z, t+t') \rangle \ ,$$

$$C_\parallel(t) = \frac{1}{N_t} \sum_{t'=0}^{N_t-1} \langle B^3(\vec{p}_z, t') B^3(\vec{p}_z, t+t') \rangle \ .$$

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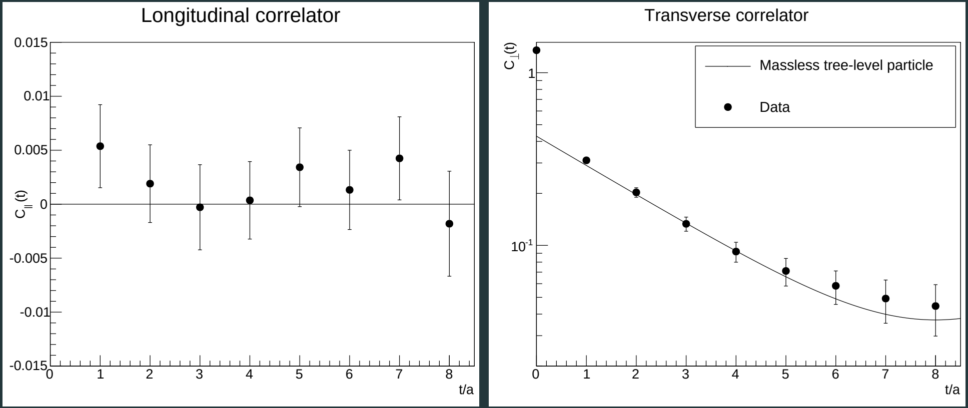
We expect the two correlators to behave as

$$C_{\perp}(t) \propto \exp(-E(P_z)t) \, ,$$

$$C_{\parallel}(t) \propto \delta(t=0) \, .$$

Transverse and Longitudinal Correlator - results



- No signal from the longitudinal correlator.
- Transverse correlator compatible with massless ansatz.

Expanded Basis

- We enlarge the basis by adding two more operators:

- $B_{1^-}^{\Phi,i}(x) = 2\,\mathrm{tr}(\phi^2)\,B_{1^-}^{i}(x)$
- $B_{1^-}^{2,i}(x) = \left(\sum_j B_{1^-}^{j}(x) B_{1^-}^{j}(x)\right) B_{1^-}^{i}(x)$

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- To further increase the basis, APE smearing has been performed, up to 5 times.
- Full basis for variational analysis consisted of 20 operators.

Massless state investigation

- For a lattice state with nonzero momentum we expect

$$\cosh(aE) = \cosh(am) + \sum_i (1 - \cos(ap_i))$$

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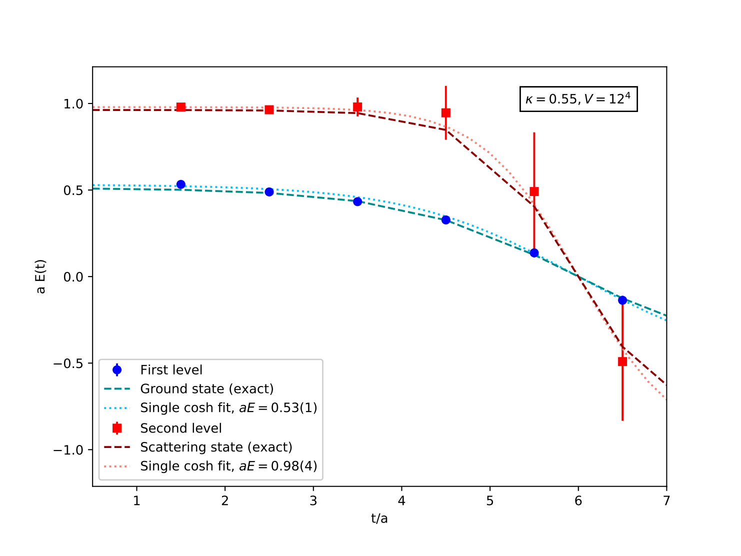
$$\cosh(aE) = \cosh(am) + \sum_i (1 - \cos(ap_i))$$

- Energies extracted from

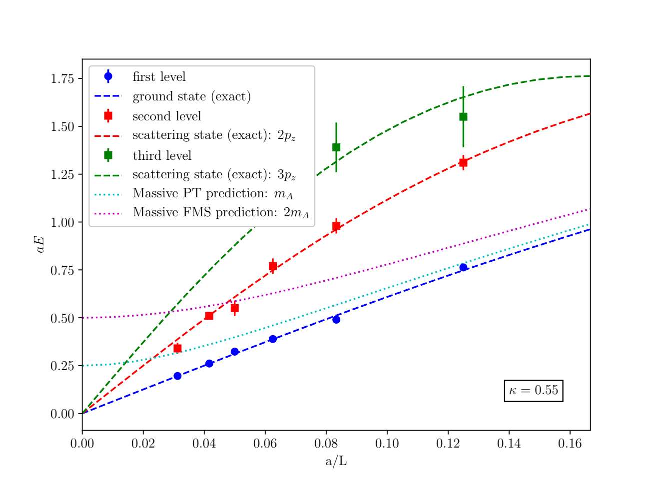
$$E_{eff}^{(k)}(t + 0.5) = \log\left(\frac{\lambda_\perp^{(k)}(t)}{\lambda_\perp^{(k)}(t+1)}\right)$$

- Fitting procedure for energy extraction, with the expected cosh behaviour for a massless or a massive state.

Spectroscopy fit examples



Spectroscopic results



Lattice results confirm the massless hypothesis.

Gauge invariant fermion spectrum[a]

[a] Afferrante et al., "Testing the mechanism of lepton compositness".

FMS Mechanism for fermions in SM

- Left handed fermions in SM are not GI $\rightarrow$ they can be treated with the FMS mechanism.
- We can employ fermionic GI bound states $\Psi(x) = X^\dagger(x)\psi(x)$, but never proven, $X = (\tilde{\phi}\ \phi)$.

$$\langle \Psi(x)\overline{\Psi}(y)\rangle = \frac{v^2}{2}\langle \psi(x)\overline{\psi}(y)\rangle + O(\varphi)$$

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- The flavor weak doublet is mapped to a custodial doublet.
- If the FMS construction holds, the mass of the bound state should be the same as the elementary one.
- The main goal of this work is to analyze this hypothesis on the lattice.

FMS for fermions

- Lattice chiral fermions formulation is a longstanding problem $\rightarrow$ Vectorial fermions.

FMS for fermions

- Lattice chiral fermions formulation is a longstanding problem → Vectorial fermions.
- Toy model of SM Weyl fermions with vectors of Dirac fermions.
- Vectorial fermion $\psi = (\psi_1, \psi_2)$ which is gauged → $L = (\nu_L, l_L)$.
- Vectorial fermion $\chi = (\chi_1, \chi_2)$ which is ungauged → (ν_R, l_R).

Vectorial fermion action

$$S = \int \mathrm{d}^4x \Big[-\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) + \overline{\psi} \left(\mathrm{i}\not{D} - m\right) \psi + \overline{\chi}_{\bar{k}} \left(\mathrm{i}\not{\partial} - m\right) \chi_{\bar{k}} - y\left(\overline{\psi}\tilde{\phi}\chi_1 + \overline{\chi}_1 \tilde{\phi}^\dagger \psi\right) - y\left(\overline{\psi}\phi\chi_2 + \overline{\chi}_2 \phi^\dagger \psi\right) - V(\phi^\dagger \phi) \Big] .$$

Elementary mass spectrum

- Apply the Higgs mechanism $\phi = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \varphi$.

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$$M = \begin{pmatrix} m & 0 & \frac{v}{\sqrt{2}}y & 0 \\ 0 & m & 0 & \frac{v}{\sqrt{2}}y \\ \frac{v}{\sqrt{2}}y & 0 & m & 0 \\ 0 & \frac{v}{\sqrt{2}}y & 0 & m \end{pmatrix} .$$

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$$M^{\pm} = m \pm \frac{yv}{\sqrt{2}}.$$

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- The ψ and χ doublets are degenerate at a tree level.

NLO masses eigenstates

- We can add the NLO correction to TL masses

$$
\begin{aligned}
m_\psi^{(1)} &= m(1 + c_{\rm y} y^2 + c_{\rm W}\alpha_{\rm W}),\\
m_\chi^{(1)} &= m(1 + c_{\rm y} y^2),
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- NLO eigenmasses

$$
\begin{aligned}
M^\pm &= \frac{m_\psi^{(1)} + m_\chi^{(1)}}{2} \pm \frac{1}{2}\sqrt{\left(m_\psi^{(1)} - m_\chi^{(1)}\right)^2 + 2y^2 v^2}\\
&= m\left(1 + c_y y^2 + \frac{c_{\mathrm{W}}}{2}\alpha_{\mathrm{W}}\right) \pm \frac{1}{2}\sqrt{c_{\mathrm{W}}^2 \alpha_{\mathrm{W}}^2 m^2 + 2v^2 y^2}.
\end{aligned}
$$

- Behaviour with respect to y is not linear.
- Small y: larger split of $m_\psi - m_\chi$.

Lattice setup

- Fermion propagator obtained by inversion of the Dirac operator, quenched setting

$$\begin{pmatrix}\overline{\psi} & \overline{\chi}\end{pmatrix} D \begin{pmatrix}\psi \\ \chi\end{pmatrix} = \begin{pmatrix}\overline{\psi} & \overline{\chi}\end{pmatrix} \begin{pmatrix} D^{\overline{\psi}\psi} & D^{\overline{\psi}\chi} \\ D^{\overline{\chi}\psi} & D^{\overline{\chi}\chi}\end{pmatrix} \begin{pmatrix}\psi \\ \chi\end{pmatrix} ,$$

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- Standard Wilson-Dirac operator, first diagonal block:

$$D^{\overline{\psi}\psi}(x|y)_{ij} = \mathbb{1}\delta_{ij} - \kappa_F \sum_{\mu=\pm 1}^{\pm 4} (\mathbb{1} - \gamma_\mu)\, U_\mu(x)_{ij}\, \delta_{x+\hat{\mu},y} ,$$

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- Here $\kappa_F = \frac{1}{2(m+4)}$.
- The second diagonal block is the free Wilson-Dirac operator for χ.

- Non-diagonal blocks are the Yukawa terms

$$
\begin{aligned}
D^{\bar{\psi}\chi}_{if'}(x|y) &= \delta_{xy}\mathbb{1}\left(YX_{i1}\delta_{1f} + YX^{\dagger}_{i2}\delta_{2f}\right), \\
D^{\bar{\chi}\psi}_{f'i}(x|y) &= D^{\bar{\psi}\chi\dagger}_{if'}(x|y).
\end{aligned}
$$

[15] Jegerlehner, "Krylov space solvers for shifted linear systems".

[16] Maas and Mufti, "Two- and three-point functions in Landau gauge Yang-Mills-Higgs theory".

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- Two parameters: $\kappa_F = 0.11, 0.12$ and the Yukawa coupling $Y = 0.01, 0.05, 0.1$.
- Lattice sizes $8^4, 12^4, 16^4, 20^4, 24^4$.

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[16] Maas and Mufti, "Two- and three-point functions in Landau gauge Yang-Mills-Higgs theory".

Observables

- Interesting GI observables are

$$O_1 = \phi^\dagger \psi \overset{\text{FMS}}{\propto} \psi_2 + \cdots \quad , \quad O_2 = \tilde{\phi}^\dagger \psi \overset{\text{FMS}}{\propto} \psi_1 + \cdots \quad , \quad \chi_1 \quad , \quad \chi_2 \, .$$

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- Correlators GI matrix

$$M_{\mathrm{GI}}(x|y) = \begin{pmatrix} X^\dagger(x)(D^{-1})_{\bar{\psi}\psi}(x|y)X(y) & (D^{-1})_{\bar{\psi}\chi}(x|y)X(y) \\ X^\dagger(x)(D^{-1})_{\bar{\chi}\psi}(x|y) & (D^{-1})_{\bar{\chi}\chi}(x|y) \end{pmatrix} .$$

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- We add to the base the two gauge variant component ψ_1, ψ_2, which are evaluated on a smaller subset of gauge fixed configurations.

ψ and χ masses

- Propagators of ψ(with GF) and χ show a good plateau $\rightarrow$ eigenmasses.
- Propagator of χ $\rightarrow$ First mass eigenvalue M^-.
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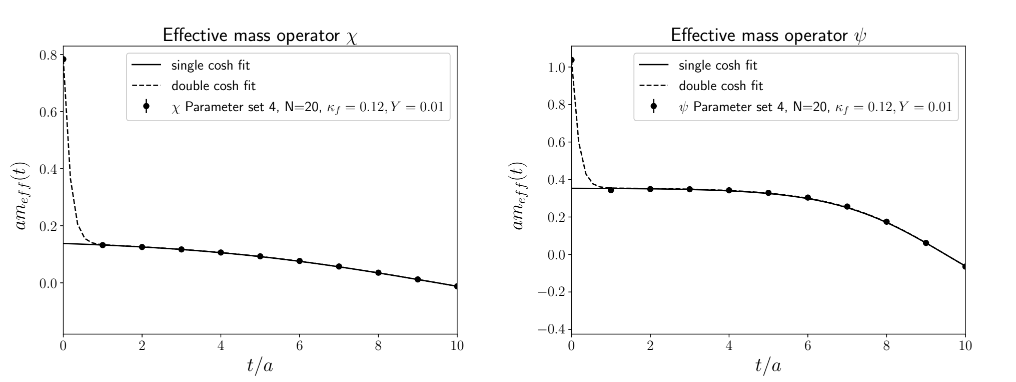
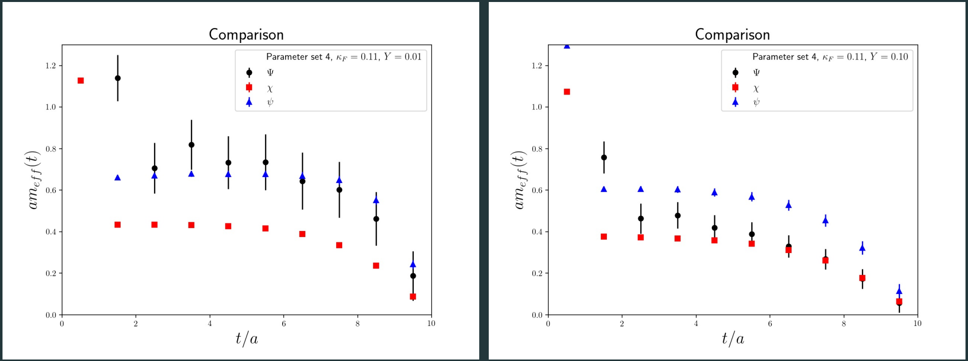
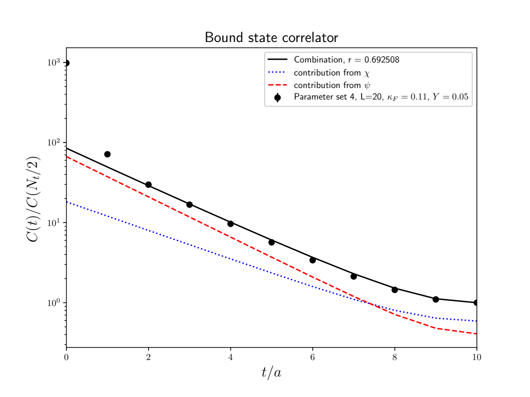


Comparison of bound state

- The bound state mixing depends on the Yukawa coupling.
- Small Yukawa: compatibility with ψ.
- Large Yukawa: compatibility with χ.



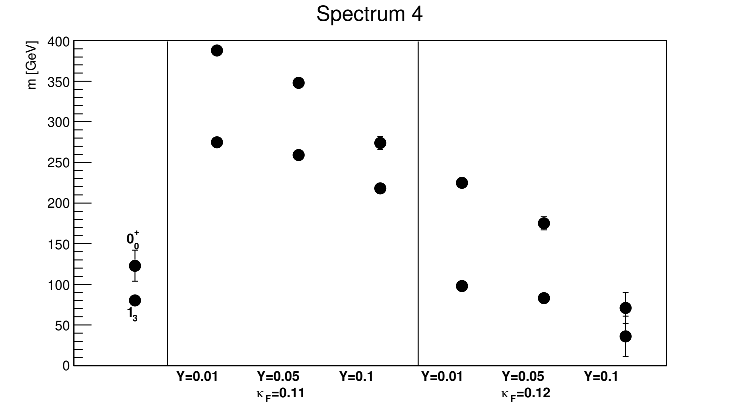
Bound state mixing



- Intermediate Yukawa: combination of ψ and χ gives a good fit

$$\frac{C(t)}{C(N_t/2)} = \frac{1}{1+r}\left[\cosh\left(M^-(t-N_t/2)\right) + r\cosh\left(M^+(t-N_t/2)\right)\right]$$

Spectroscopic results



- Mass dependence on Yukawa coupling independently of Dirac mass parameter.
- Two stable states at larger Yukawa. Infinite volume extrapolate results.

Conclusions

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[17] Fernbach et al., "Constraining the Higgs valence contribution in the proton".

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- A GI composite photon in a SU(2) GUT type theory with an adjoint Higgs has been studied in a lattice simulation.
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Outlook:

- Lattice: exploration of $SU(3)$ + fundamental and adjoint $\rightarrow$ better understanding of BSM Higgs theories.
- Pheno: Valence Higgs contributions[17] can be explored with the HL-LHC and the newly proposed linear lepton colliders $\rightarrow$ flavor and $g-2$ anomalies.

[17] Fernbach et al., "Constraining the Higgs valence contribution in the proton".

Thanks for the attention!

Backup slides

Lattice action for SU(2)+adjoint

A multihit Montecarlo has been implemented, with action

$$S[\Phi, U] = S_W[U] + \sum_x 2\,\mathrm{tr}(\Phi(x)\Phi(x)) + \lambda(2\,\mathrm{tr}(\Phi(x)\Phi(x)) - 1)^2 - 2\kappa \sum_\mu \mathrm{tr}\Big(\Phi(x)U_\mu(x)\Phi(x+\hat\mu)U_\mu^\dagger(x)\Big)$$

3 parameters: β, κ, λ. Center symmetry Z_2.

Explicitating the generators of the algebra

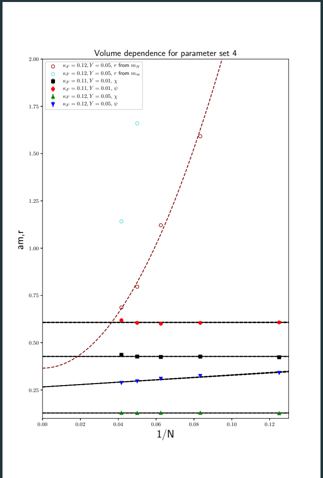
$$S[\phi, U] = S_W[U] + \sum_{x,a} \Phi^a(x)\Phi^a(x) + \lambda(\Phi^a(x)\Phi^a(x) - 1)^2 - 2\kappa \sum_{\mu,a,b} \Phi^a(x)V_\mu^{ab}(x)\Phi^b(x+\hat\mu)$$

where

$$V_\mu^{ab}(x) = \mathrm{tr}\Big(T^a U_\mu(x) T^b U_\mu^\dagger(x)\Big)\,.$$

Infinite volume behaviour

Masses behaves exponentially with respect to infinite volume limit.



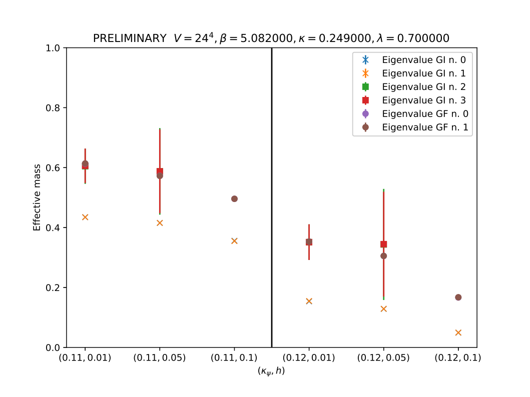
Full results

We examined 5 different parameters set for the bosonic sector.

#	β	κ	λ	a^{-1} [GeV]	$m_{0_0^+}$ [GeV]	α_W(200 Gev)	v(200 GeV) $= \frac{m_{1_3^-}}{\sqrt{\pi\alpha_W}}$ [GeV]
1	2.7984	0.2954	1.328	384	118(9)	0.544	39
2	2.7984	0.2978	1.317	326	129(12)	0.495	64
3	3.9	0.2679	1	509	116(19)	0.140	121
4	5.082	0.249	0.7	636	123(19)	0.170	110
5	5.082	0.2552	0.7	427	131(5)	0.0794	161

#	κ_F	Y	aM^-	aM^+	r
1	0.12	0.01	$0.421^{+0.001}_{-0.008}$	0.817(3)	1.9
1	0.12	0.05	0.407(6)	0.77(3)	0.4
1	0.12	0.1	0.353(9)	0.54(1)	0.2
1	0.11	0.01	0.137(1)	0.58(1)	1.4
1	0.11	0.05	0.111(1)	0.45(1)	0.2
1	0.11	0.1	0.044(5)	0.21(1)	0.1
2	0.12	0.01	0.422(3)	0.810(4)	1.4
2	0.12	0.05	0.406(3)	0.75(2)	0.6
2	0.12	0.1	0.352(2)	0.62(3)	0.4
2	0.11	0.01	0.136(1)	0.583(4)	1.4
2	0.11	0.05	0.103(1)	0.49(2)	0.3
2	0.11	0.1	0.032(2)	0.17(1)	0.2
3	0.12	0.01	$0.422^{+0.001}_{-0.006}$	0.674(3)	6.5
3	0.12	0.05	0.407(5)	0.645(2)	0.3
3	0.12	0.1	0.357(3)	0.574(4)	0.09
3	0.11	0.01	0.136(1)	0.426(5)	3.0
3	0.11	0.05	$0.112^{+0.004}_{-0.002}$	0.385(2)	0.8
3	0.11	0.1	0.043(1)	0.24(1)	0.2
4	0.12	0.01	0.422(1)	0.604(2)	11.5
4	0.12	0.05	0.402(2)	0.54(1)	0.6
4	0.12	0.10	0.331(7)	0.43(1)	0.1
4	0.11	0.01	0.136(3)	0.346(2)	2.4
4	0.11	0.05	0.098(1)	0.27(2)	1.0
4	0.11	0.10	0.036(9)	0.09(1)	0.4
5	0.12	0.01	0.422(5)	0.599(2)	7.1
5	0.12	0.05	0.39(1)	0.51(1)	1.0
5	0.12	0.1	0.305(5)	0.35(1)	0.4
5	0.11	0.01	0.126(4)	0.347(6)	7.1
5	0.11	0.05	0.086(2)	0.22(1)	1.0
5	0.11	0.1	0.03(2)	$0.1^{+0.09}_{-0.05}$	0.1

Variational analysis



- Results point a compatibility between the second GI eigenvalues and the elementary mass.
- Statistical noise is very high in this extrapolation.