

# Subleading effects for future lepton colliders

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Supported by  
Paul Urban Scholarship Foundation

September 28th, 2022

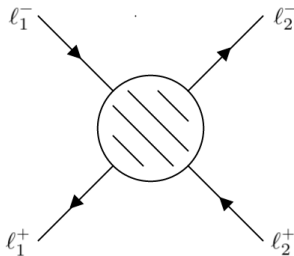
## Goal

NLO precision calculation of a leptonic scattering process

$$l_1^- l_1^+ \rightarrow l_2^- l_2^+$$

## Motivation

- ▶ Future lepton colliders opens new energy-regimes
- ▶ Experimentally easily accessible process
- ▶ Able to polarize initial particles  $\rightarrow$  effects?



## Problem

- ▶ Elementary particles not gauge-invariant
- ▶ Fundamental requirement in order to represent measurable quantities
- ▶ Strong agreement between standard perturbation and experiments

## Question

Why does the usual approach work so well?

## Solution

1. Start with a gauge-invariant (GI) description of particles, combine fermions with Higgs field  $h$

$$\ell^\pm \rightarrow L^\pm = h\ell^\pm$$

2. Fröhlich-Morchio-Strocchi (FMS) mechanism relates quantities of GI particles to perturbation theory<sup>123</sup>

$$L^\pm \xrightarrow{\text{FMS}} \ell^\pm$$

3. LO contribution equal to regular calculations  
 $\langle L^\pm \dots \rangle = \langle \ell^\pm \dots \rangle + \text{Higher orders}$

<sup>1</sup>Fröhlich, Morchio, and Strocchi 1980

<sup>2</sup>For a review see Maas 2019

<sup>3</sup>For the lepton composite-operator see Afferrante et al. 2021

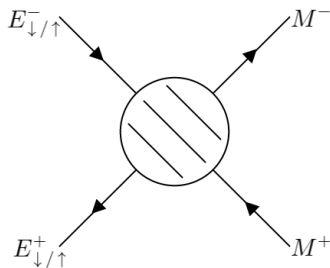
## Our initial process

1. Use four-point function of composite electron-positron scattering to muon-antimuon<sup>4</sup>  
 $\langle E^- E^+ M^- M^+ \rangle$
2. FMS expansion in elementary  $e$ ,  $\mu$  and  $h$   
 $\langle E^- E^+ M^- M^+ \rangle = \langle e^- e^+ \mu^- \mu^+ \rangle + \langle h e^- e^+ \mu^- \mu^+ \rangle + \dots$
3. Leads to additional contributions in the (differential) cross section
4. Effects of GI approach are compatible with data so far, but...

<sup>4</sup>Egger, Maas, and Sondenheimer 2017

## Central questions

- ▶ Are the additional contributions relevant in high-energy collisions?
- ▶ Can better precision in low-energy results resolve these contributions?
- ▶ How are different initial helicity configurations affected?



## Approach

- ▶ Calculate (differential) cross section from standard matrix elements  $\mathcal{M}_S$  up to NLO for initial states

$$E^-E^+, E_{\uparrow}^-E_{\uparrow}^+, E_{\uparrow}^-E_{\downarrow}^+ \dots \rightarrow M^-M^+$$

- ▶ Include contributions from the GI approach

$$d\sigma_{FMS} \propto |\mathcal{M}_S + \mathcal{M}_{FMS}|^2$$

- ▶ Compare results between both approaches

$$\frac{d\sigma_{FMS}}{d\sigma} \propto 1 + \frac{2\text{Re}(\mathcal{M}_S\mathcal{M}_{FMS}^*)}{|\mathcal{M}_S|^2} + \frac{|\mathcal{M}_{FMS}|^2}{|\mathcal{M}_S|^2}$$

## Results

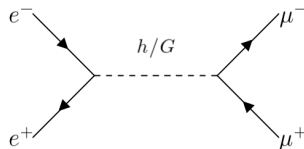
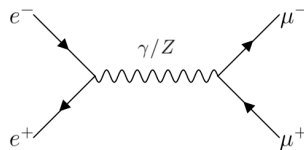
- ▶ Four contributions to the tree-level cross sections

$$d\sigma \propto |\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_h + \mathcal{M}_G|^2$$

- ▶ No effects from the FMS approach

$$d\sigma_{FMS} = d\sigma$$

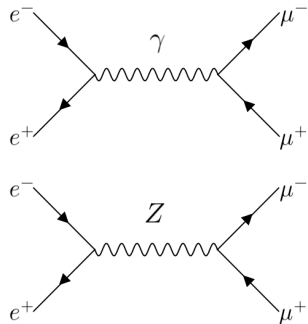
- ▶  $\gamma$  and  $Z$  main contributions





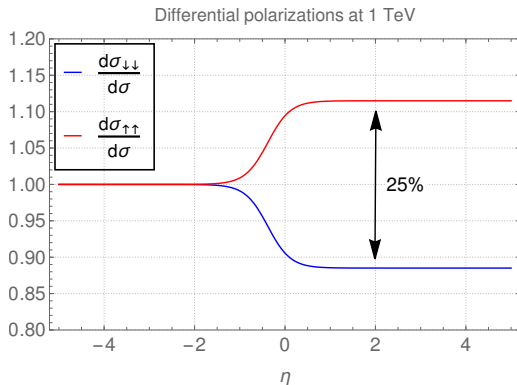
## Results

- ▶ External masses can be neglected
- ▶ For massless external particles only helicity configurations  $e^-_\downarrow e^-_+$  and  $e^-_+ e^-_+$  survive
- ▶ Scalar effects vanish as Yukawa-coupling  $\propto m_{e/\mu}$
- ▶ Compare different initial helicity cross sections  $d\sigma_{\downarrow\downarrow}$  and  $d\sigma_{\uparrow\uparrow}$



## Highlights

- ▶ Polarization effects remain visible at high energies
- ▶ Helicity most relevant for forward-scattering



## First remarks

- ▶ FMS terms couple for massless external states only to fields with negative helicity  $L_{\downarrow}^{\pm} = h l_{\downarrow}^{\pm}, L_{\uparrow}^{\pm} = l_{\uparrow}^{\pm}$

- ▶ Leads to matrix elements with up to four additional Higgs contributions  $\rightarrow$  16 possible sets of matrix elements

$$\langle E_{\downarrow}^{-} E_{\downarrow}^{+} M_{\downarrow}^{-} M_{\downarrow}^{+} \rangle$$

$$\rightarrow \langle e_{\downarrow}^{-} e_{\downarrow}^{+} \mu_{\downarrow}^{-} \mu_{\downarrow}^{+} \rangle + \langle h e_{\downarrow}^{-} e_{\downarrow}^{+} \mu_{\downarrow}^{-} \mu_{\downarrow}^{+} \rangle + \dots + \langle h e_{\downarrow}^{-} h e_{\downarrow}^{+} h \mu_{\downarrow}^{-} h \mu_{\downarrow}^{+} \rangle$$

- ▶ No difference to standard perturbation in process

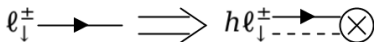
$$E_{\uparrow}^{-} E_{\uparrow}^{+} \rightarrow M_{\uparrow}^{-} M_{\uparrow}^{+}$$

- ▶ Strongest amplification possibly for experimental setup of

$$E_{\downarrow}^{-} E_{\downarrow}^{-} \rightarrow M^{-} M^{+}$$

## Feynman diagrams

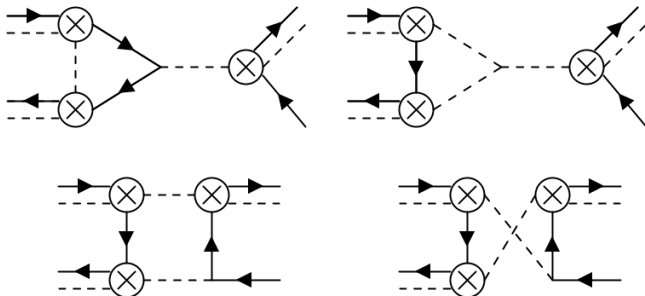
- ▶ Same topologies for loop diagrams for one Higgs insertion in FMS contributions
- ▶ Composite state contributes as special interaction vertex  
→ allows for diagrams with Yukawa term in external states
- ▶ Diagrams more restricted for more Higgs insertions
- ▶ How do these two aspects interplay?



# One-loop level

## Three Higgs insertions

►  $he_{\downarrow}^{-}he_{\downarrow}^{+} \rightarrow h\mu_{\downarrow}^{-}\mu_{\uparrow}^{+}$



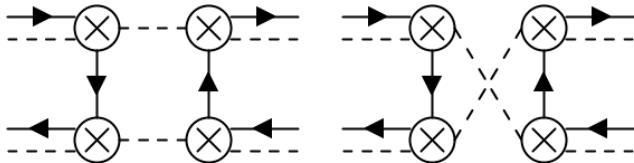
► Only triangle and box diagrams

$$e_{\downarrow}^{\pm} \longrightarrow \Longrightarrow h e_{\downarrow}^{\pm} \longrightarrow \otimes$$

# One-loop level

## Four Higgs insertions

►  $he_{\downarrow}^{-}he_{\downarrow}^{+} \rightarrow h\mu_{\downarrow}^{-}h\mu_{\downarrow}^{+}$



► Only box diagrams

$$e_{\downarrow}^{\pm} \longrightarrow \Longrightarrow h e_{\downarrow}^{\pm} \text{---} \otimes$$

## Overview

- ▶ Generation of helicity one-loop diagrams with QGraf<sup>5</sup> for different helicities
- ▶ Strong difference in position of Higgs insertion

$$h_1 e^- h_2 e^+ \rightarrow h_3 \mu^- h_4 \mu^+$$

Helicity	No FMS	$h_1$	$h_1 h_2$	$h_1 h_3$	$h_1 h_2 h_3$	$h_1 h_2 h_3 h_4$
↓↓↓↓	197	181	21	224	21	9
↓↓↓↑	206	238	21	27	9	0
↓↓↑↑	196	181	22	0	0	0
↑↓↑↑	266	31	0	13	0	0
↑↑↑↓	265	31	0	0	0	0
↑↑↑↑	197	0	0	0	0	0

<sup>5</sup>Nogueira 2022

## Next steps

- ▶ Deal with WF-renormalization and external composite states
- ▶ Calculation of  $d\sigma$  for different initial and final leptons
- ▶ Include external masses
  - Spin treatment for chirality  $\neq$  helicity
  - Increased number of contributions







- ▶ Gauge-invariant description of the SM particles leads to composite-operator formalism

$$l^\pm \rightarrow L^\pm = hl^\pm$$

- ▶ Yields further matrix amplitudes and therefore adds to the differential cross section

$$d\sigma_{FMS} \propto |\mathcal{M}_S + \mathcal{M}_{FMS}|^2$$

- ▶ Effects on leptons could be relevant for future high-energy and precision measurements in FCC-ee, CLIC, ILC...

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-  Afferrante, Vincenzo et al. (2021). “Testing the mechanism of lepton compositeness”. In: *SciPost Physics* 10.3. DOI: [10.21468/scipostphys.10.3.062](https://doi.org/10.21468/scipostphys.10.3.062). URL: <https://doi.org/10.21468%2Fscipostphys.10.3.062>.
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-  Shtabovenko, Vladyslav, Rolf Mertig, and Frederik Orellana (2020). “FeynCalc 9.3: New features and improvements”. In: *Computer Physics Communications* 256, p. 107478.