

Strongly Interacting Dark Matter from $Sp(4)$ Gauge Theory

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mostly based on [\[2202.05191\]](#)



NAWI Graz
Natural Sciences



Der Wissenschaftsfonds.

Quark Confinement and the Hadron Spectrum
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Dark Matter

- Nature of Dark Matter (DM) unclear
- Only gravitational effects observed
- Hypothesis: Particle Dark Matter
 - At least one additional DM particle to SM
 - Coupling to the SM extremely weak
 - Stable over tens of billions of years

From WIMPs to SIMPs

(Strongly Interacting Massive Particles)

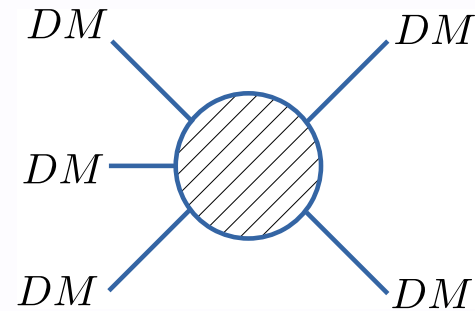
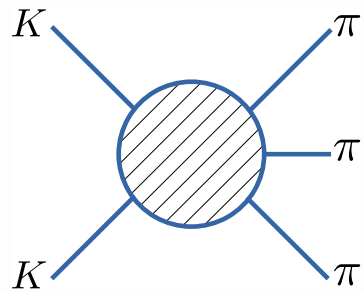
- WIMPs: DM as thermal relic from early universe
- Decouple below certain temperature \rightarrow freeze out
- Density distribution of DM constraints theories
- Constraint given by DM depletion process

WIMPs: $2\text{DM} \rightarrow 2\text{SM} \Rightarrow m_D \approx \text{TeV}$

SIMPs: $3\text{DM} \rightarrow 2\text{DM} \Rightarrow m_D \approx \mathcal{O}(100)\text{MeV}$ [1]

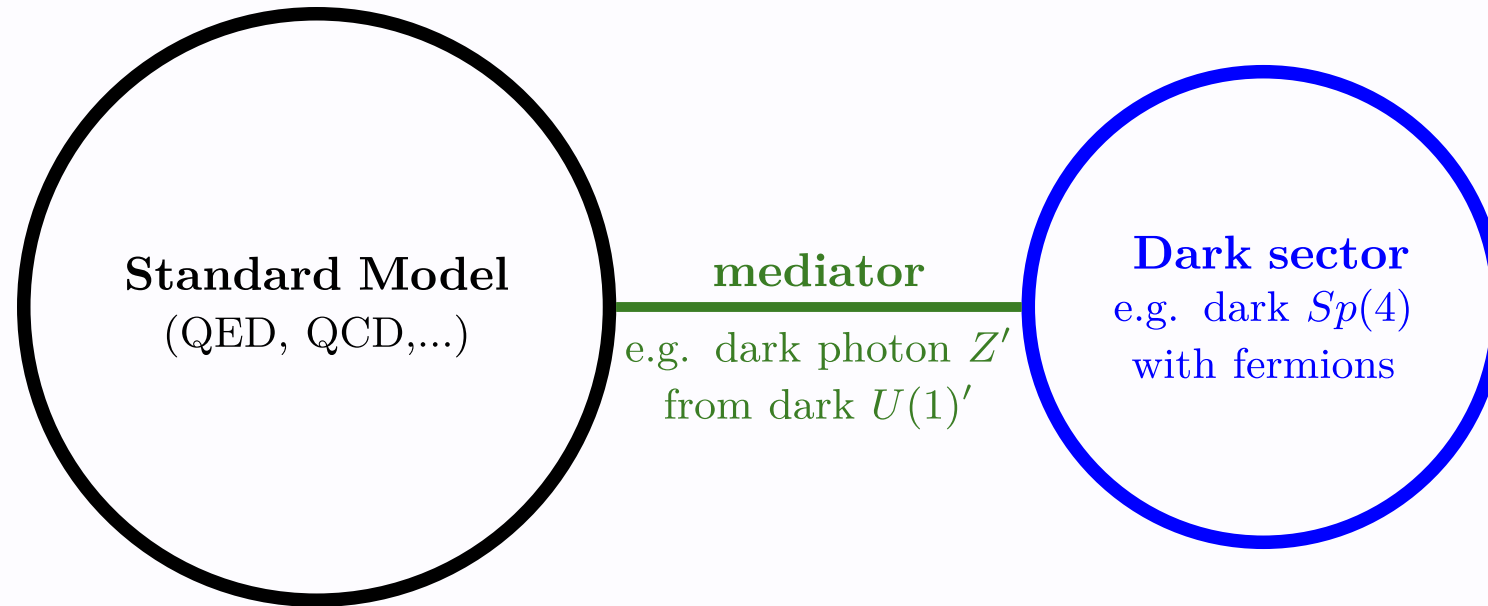
3 \rightarrow 2 occurs in chiral effective theories!

- Spontaneous chiral symmetry breaking
 \Rightarrow relatively light (pseudo-)Goldstone states
 ≥ 5 Goldstones: effective 5-point-interaction
- In QCD this describes the $2K \rightarrow 3\pi$ decay.



Idea [1]: Non-Abelian gauge theory with $3 \rightarrow 2$
Goldstones as Dark Matter candidates + mediator

Models of SIMP Dark Matter



- Strong, confining dark sector \Rightarrow dark hadrons
- **Dark fermions do not carry any SM charge**
- Small coupling to the SM via Z' - γ -mixing
- DM has self-interactions

A minimal SIMP model

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{Sp(4)} + \mathcal{L}_{\text{mediator}}$$

- $Sp(4)$ with $N_f = 2$ has exactly 5 Goldstones
- Dark hadrons DM candidates \rightarrow non-perturbative
- Low energy effective theory (EFT) needed
- **Combine the methods with lattice field theory**
 - Derive low energy EFT for dark sector + mediator
 - Low energy constants (LECs) from lattice
 - Use EFT for astro/collider/direct detection pheno

Lagrangian of $Sp(4)_c$ with fermions

$$\mathcal{L}_{Sp(4)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{f=u,d} \bar{\psi}_f (i \not{D} + m_f) \psi_f$$

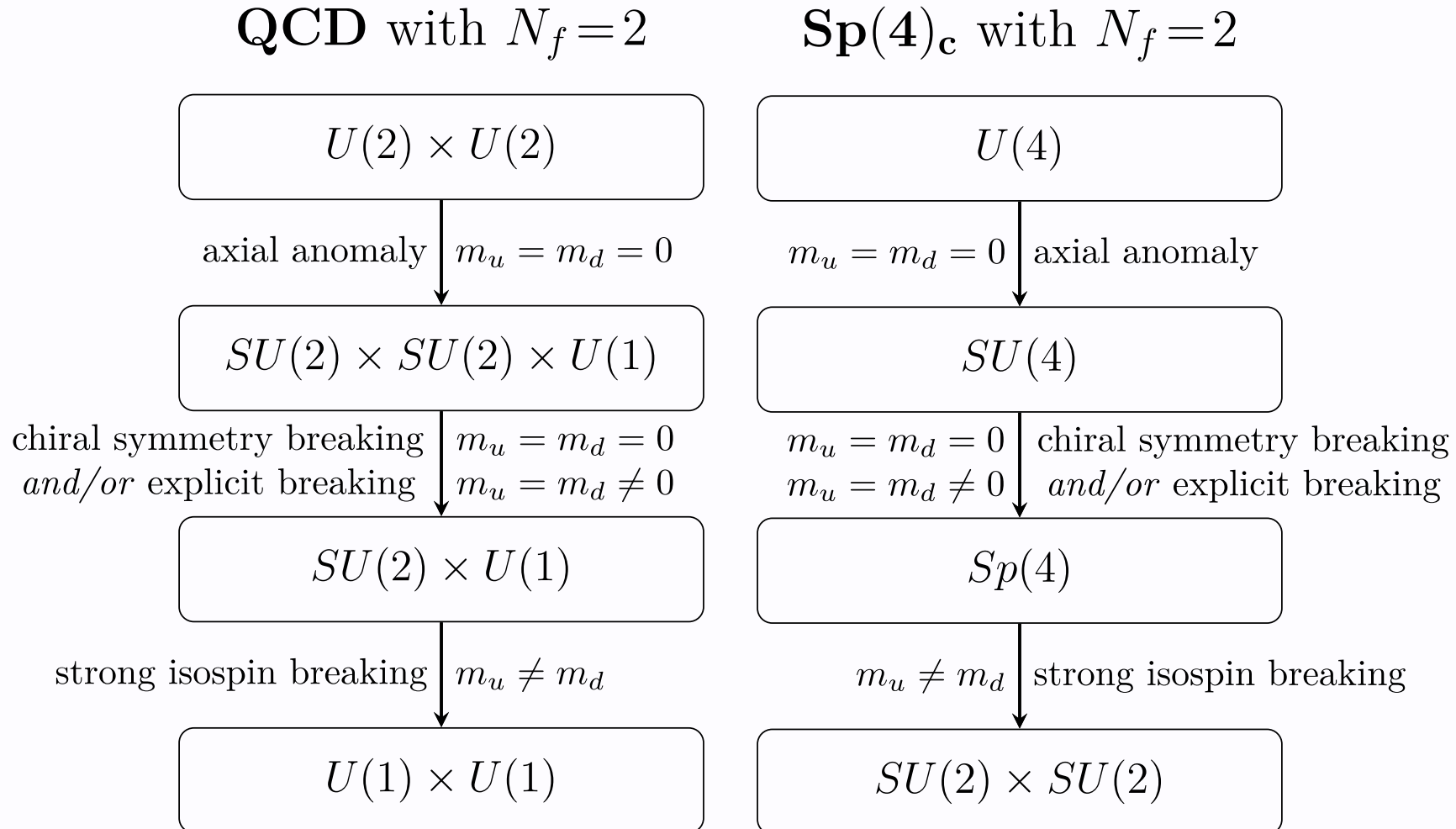
- Higher symmetry than QCD-like theories

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -SC u_R^* \\ -SC d_R^* \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \\ \tilde{u}_R \\ \tilde{d}_R \end{pmatrix} \quad \begin{array}{l} C \dots \text{charge conj.} \\ S \dots \text{colour matrix} \end{array}$$

$$\mathcal{L}_{Sp(4)} = i \bar{\Psi} \not{D} \Psi - \frac{1}{2} (\Psi^T S C M \Psi + h.c.) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- generators τ_a in fundamental repr. : $S \tau_a S = -\tau_a^T$

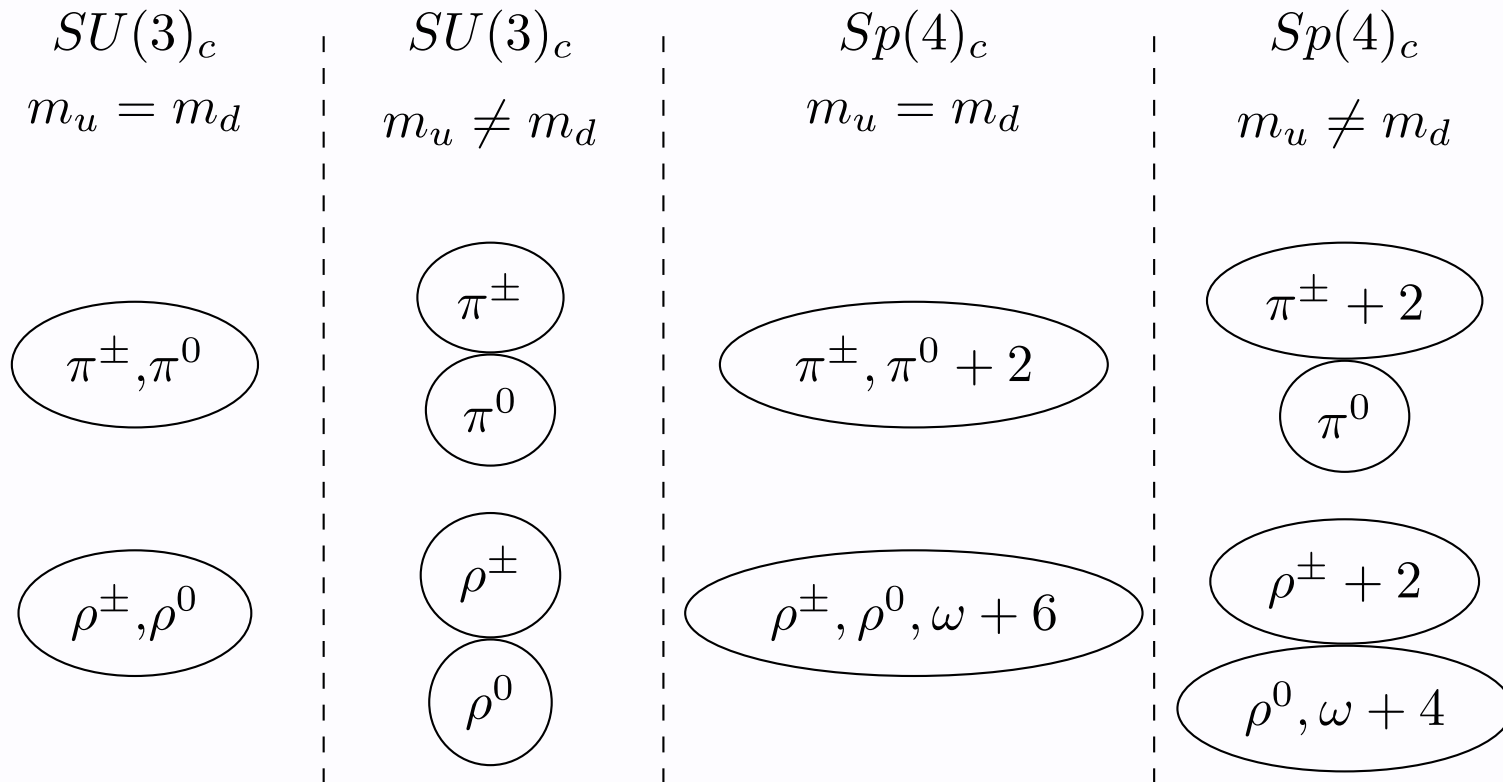
Constructing EFTs: Symmetries



see e.g. Kogut et. al. [[hep-ph/0001171](#)], von Smekal [[1205.4205](#)]

Symmetries of dark hadrons (without mediator)

- Global symmetries are enlarged compared to QCD
- New quark-quark and antiquark-antiquark states



Dark photon Z' mediator

- will break global symmetry even further
- Symmetry breaking for degenerate masses:
 - a) $Sp(4) \rightarrow SU(2) \times U(1)$
 - b) $Sp(4) \rightarrow U(1) \times U(1)$
- 5 Goldstones split into $3 + 2$ (a) or $1 + 2 + 2$ (b) **[1]**
- A singlet vector meson always exists **[2]**

Particle stability

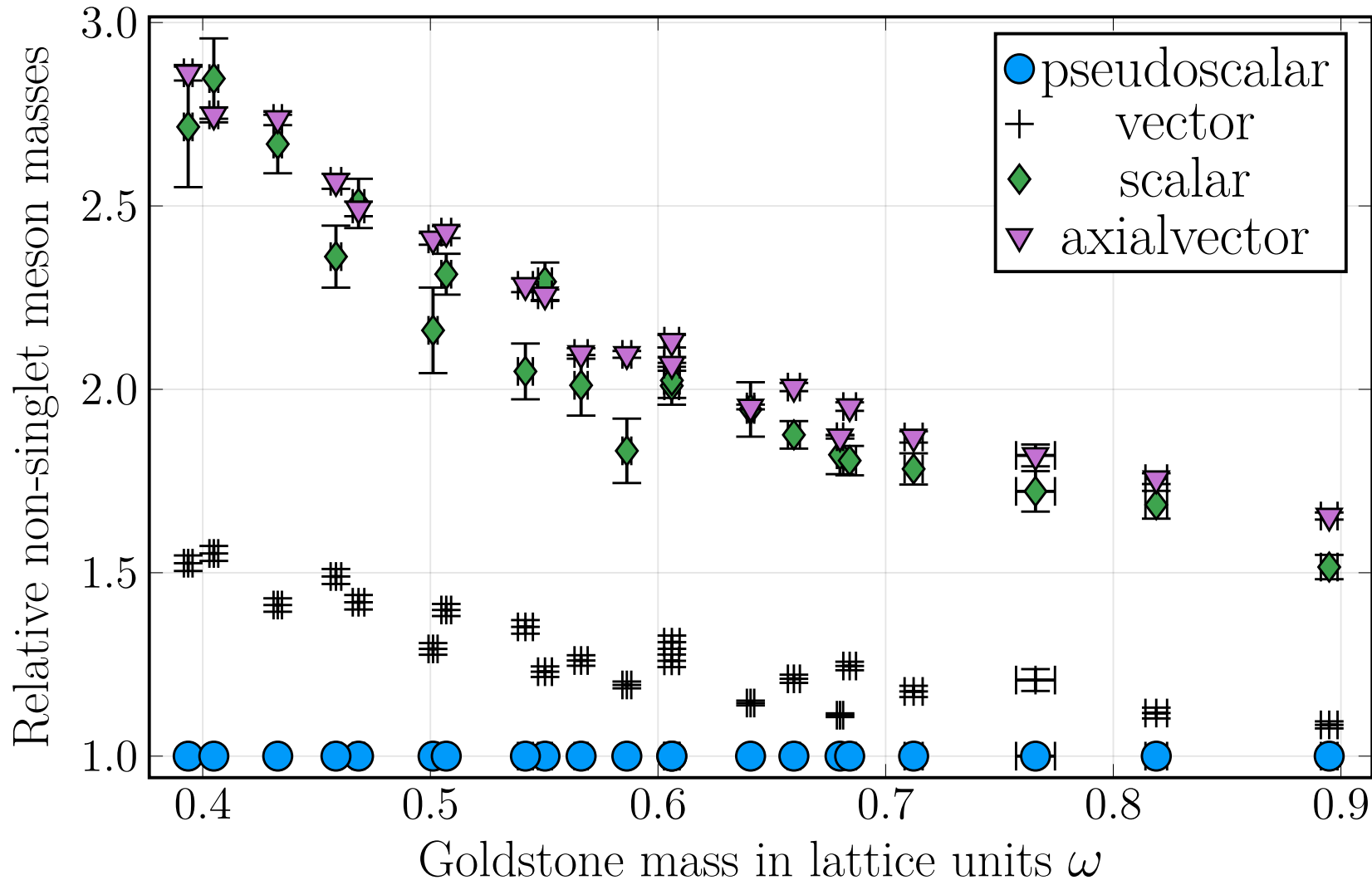
- Only multiplets are protected by symmetry
- Singlets can decay
 - $m_u = m_d$: Charge assignments without π singlets
 - $m_u \neq m_d$: Even without a Z' the π^0 is a singlet
- **For a viable DM candidate the decay of flavour singlet Goldstones needs to be suppressed**

Lattice investigations

- HiRep code [1] (Wilson action) used for study of
 - non-singlet hadronic ground state masses
 - decay constants of Goldstones π and vectors ρ
- results on Glueball spectrum available [2]
⇒ heavier than mesonic states
- same setup useful for other symplectic groups
lattice results for $SU(2)$ available [3]
- No fermionic bound states!

Hadron Spectrum from the Lattice, $m_u = m_d$

$Sp(4)$ with degenerate fermions - data from 1909.12662

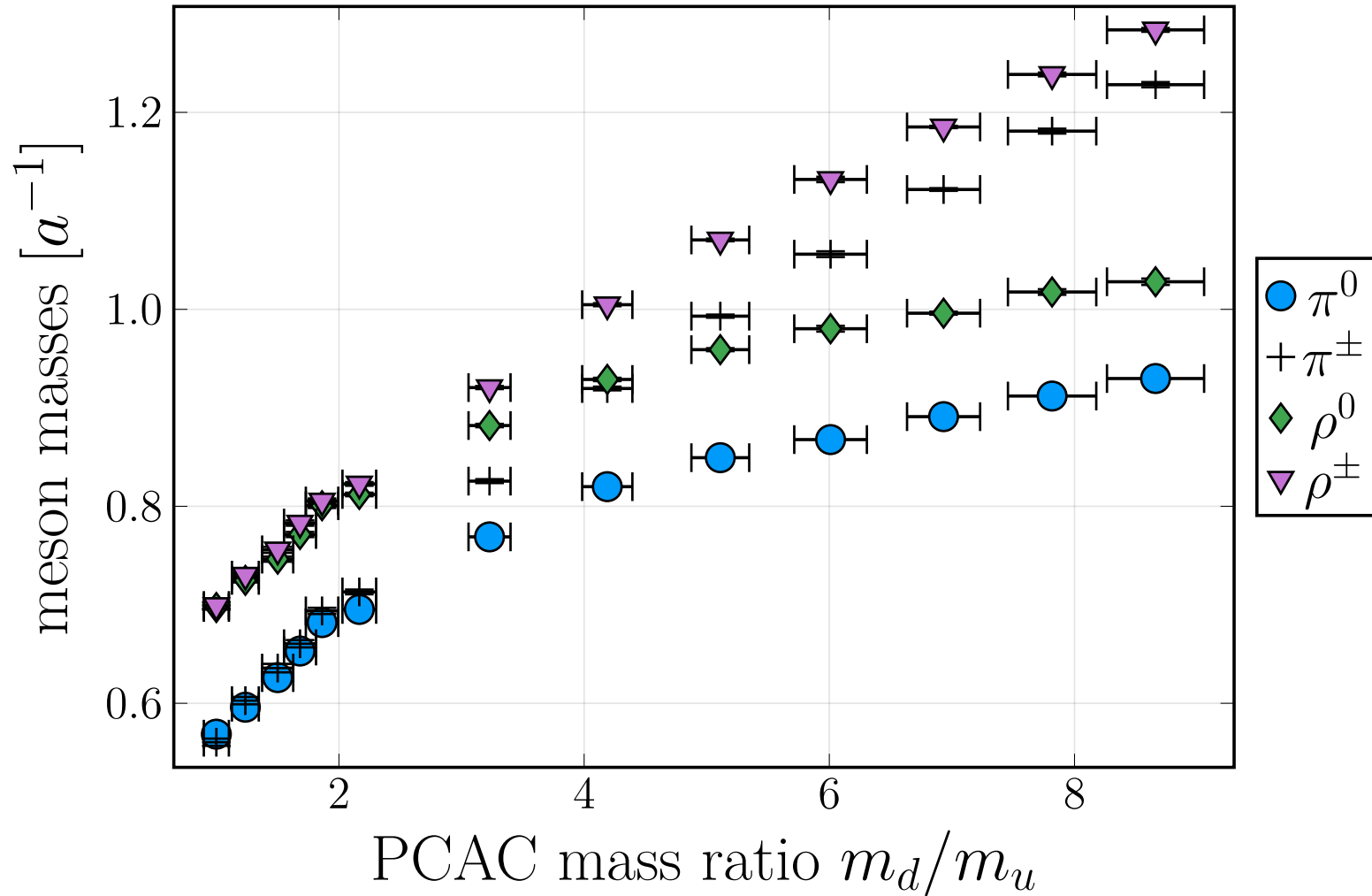


Hadron Spectrum from the Lattice, $m_u = m_d$

- (Non-singlet) scalar and axialvectors are heavy
- Glueballs are even heavier [1]
- **Goldstones + vectors** lightest nonsinglet states [2]
(for degenerate fermions)

Hadron Spectrum from the Lattice, $m_u \neq m_d$

$$\beta=6.9 \quad \left(\frac{m(\rho)}{m(\pi)}\right)_{deg} = 1.24(1)$$



Non-degenerate fermions: $Sp(4)_c$ with $N_f = 2$

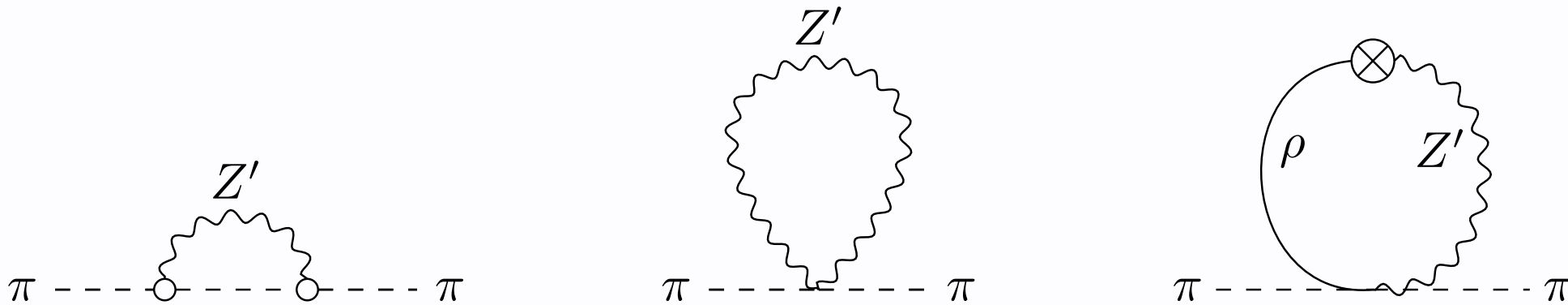
- Singlet Goldstone π^0 is the lightest state
- Unflavoured vectors lighter than flavoured ones

$$m(\rho^0) < m(\rho^\pm)$$

- Change in mass hierarchy at large splitting
- System resembles heavy-light quarks
- EFT for almost degenerate π 's constructed **[1]**
 - m_π and f_π are the low energy constants

Further Low Energy Constants from the lattice: Z'

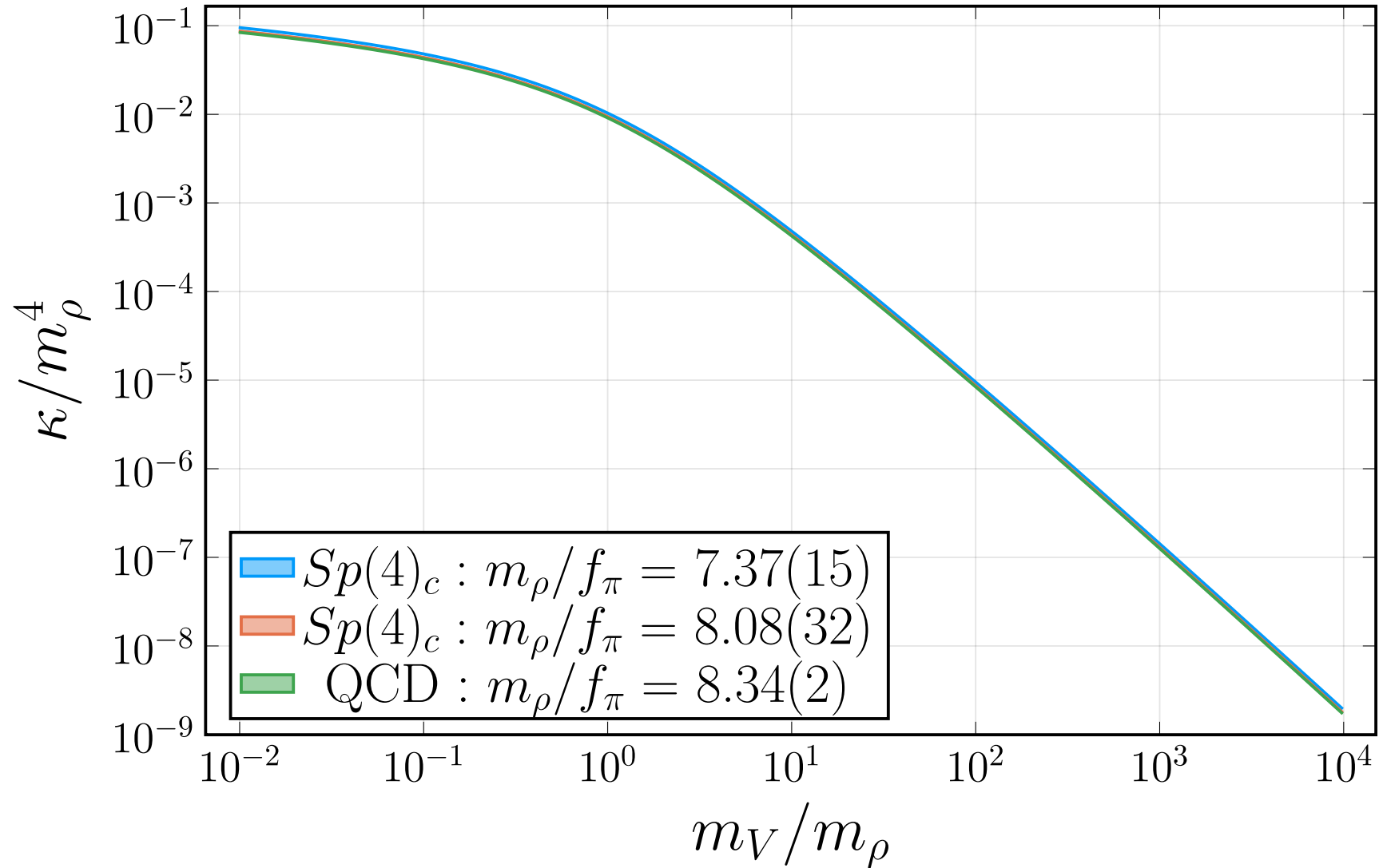
- Inclusion of Z' leads to different Goldstone masses
- Mass difference parametrized by one quantity κ



$$\Delta m_\pi = \frac{2\kappa e_D^2}{f_\pi^2}$$

$$\kappa \approx \frac{3}{4\pi^2} \frac{f_\pi^2 m_\rho^4}{m_{Z'}^2 - m_\rho^2} \log \left(\frac{m_{Z'}^2}{m_\rho^2} \right)$$

$U(1)'$ breaking parameter κ against dark photon mass m_V



- Similar to QCD (and possibly other gauge theories) 18

Conclusion

- SIMP Dark Matter is an interesting candidate model
- EFTs and meson multiplets constructed for
 - $N_f = 2$ with $m_u = m_d$ and $m_u \neq m_d$
 - Dark photon Z' mediator
- Hadron mass hierarchies and LECs from the lattice

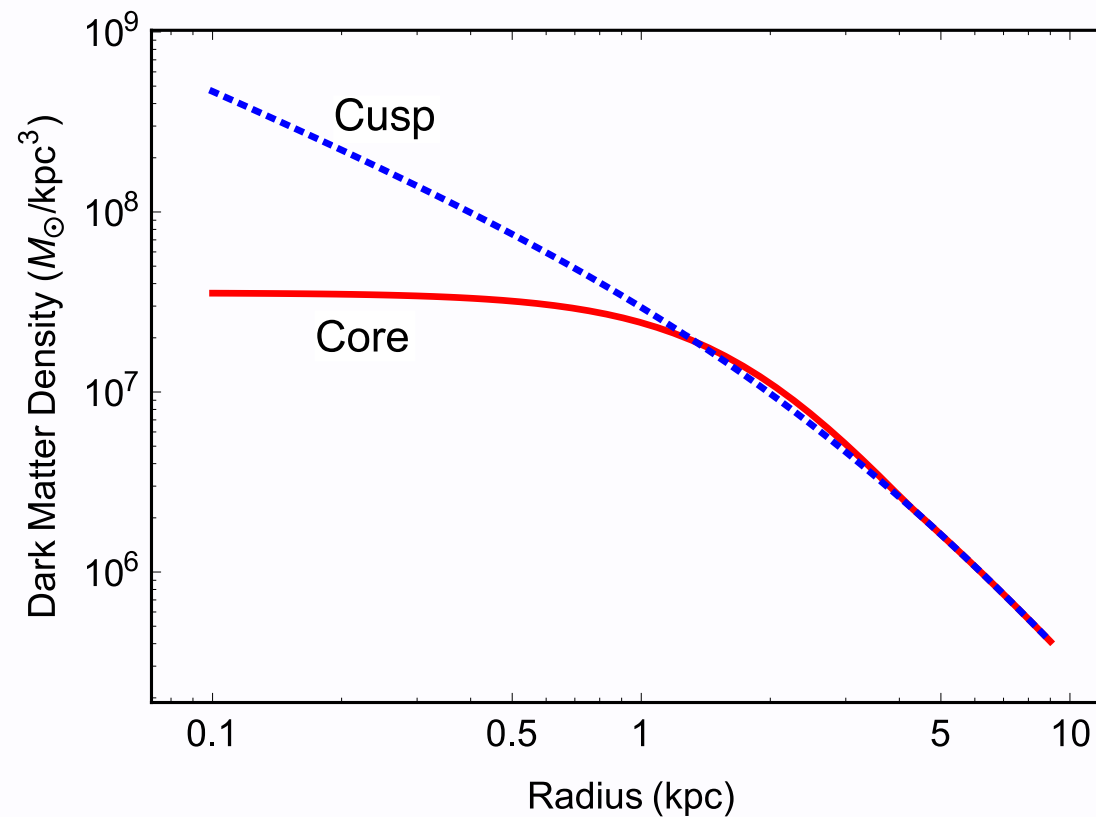
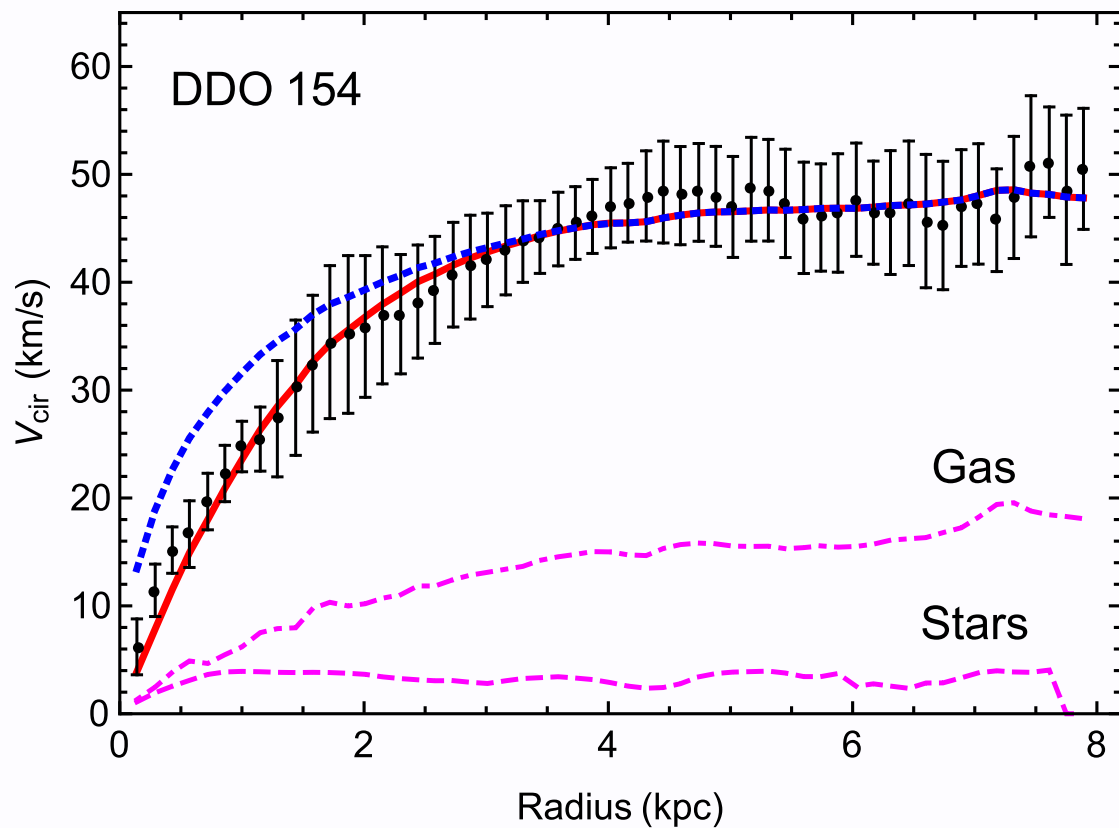
Next steps

- singlet mesons, $2\pi \rightarrow 2\pi$ scattering, collider searches/constraints

Thank you!

Backup slides

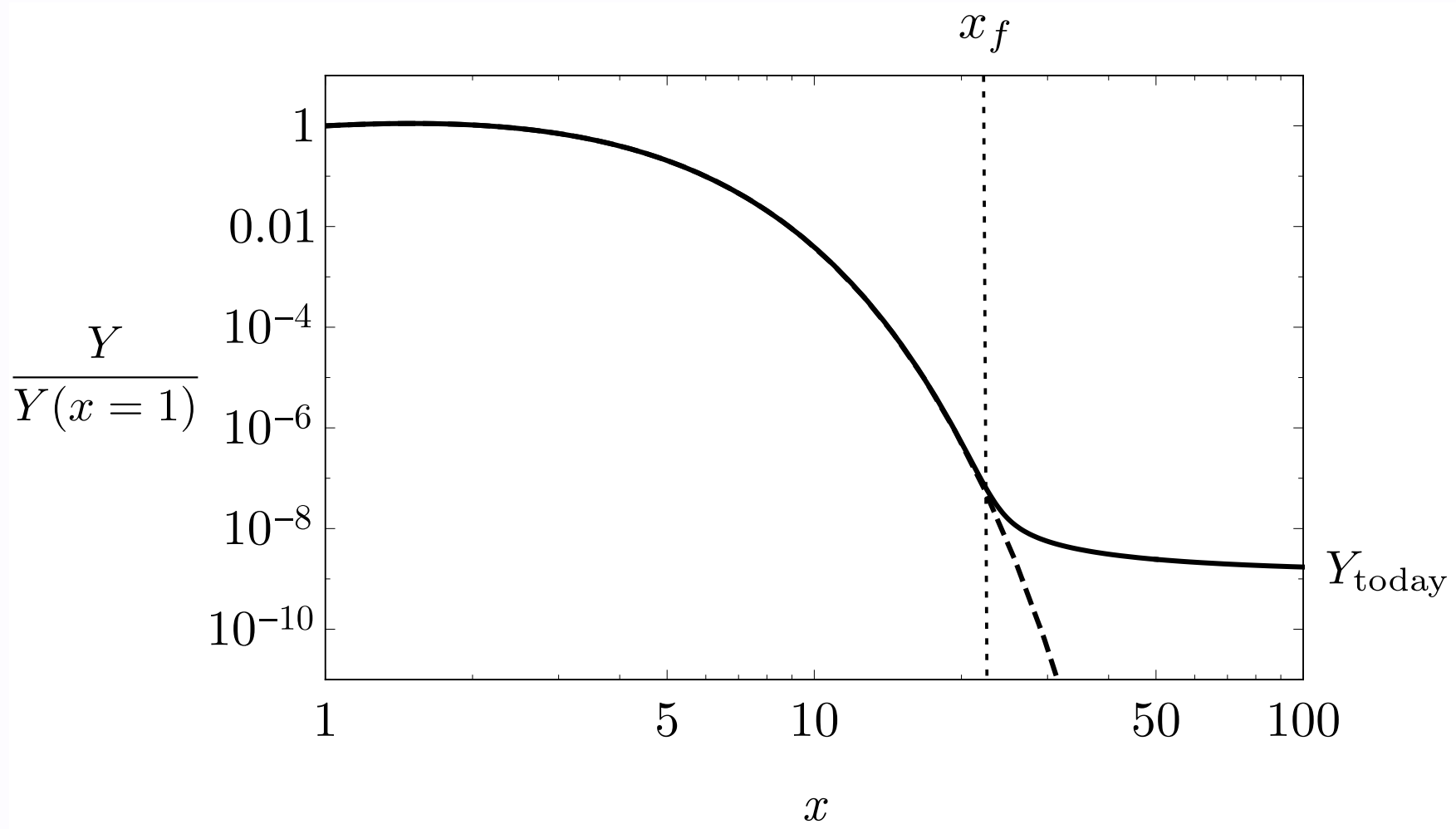
Core-vs-cusp problem



Thermal Dark Matter - Freeze out

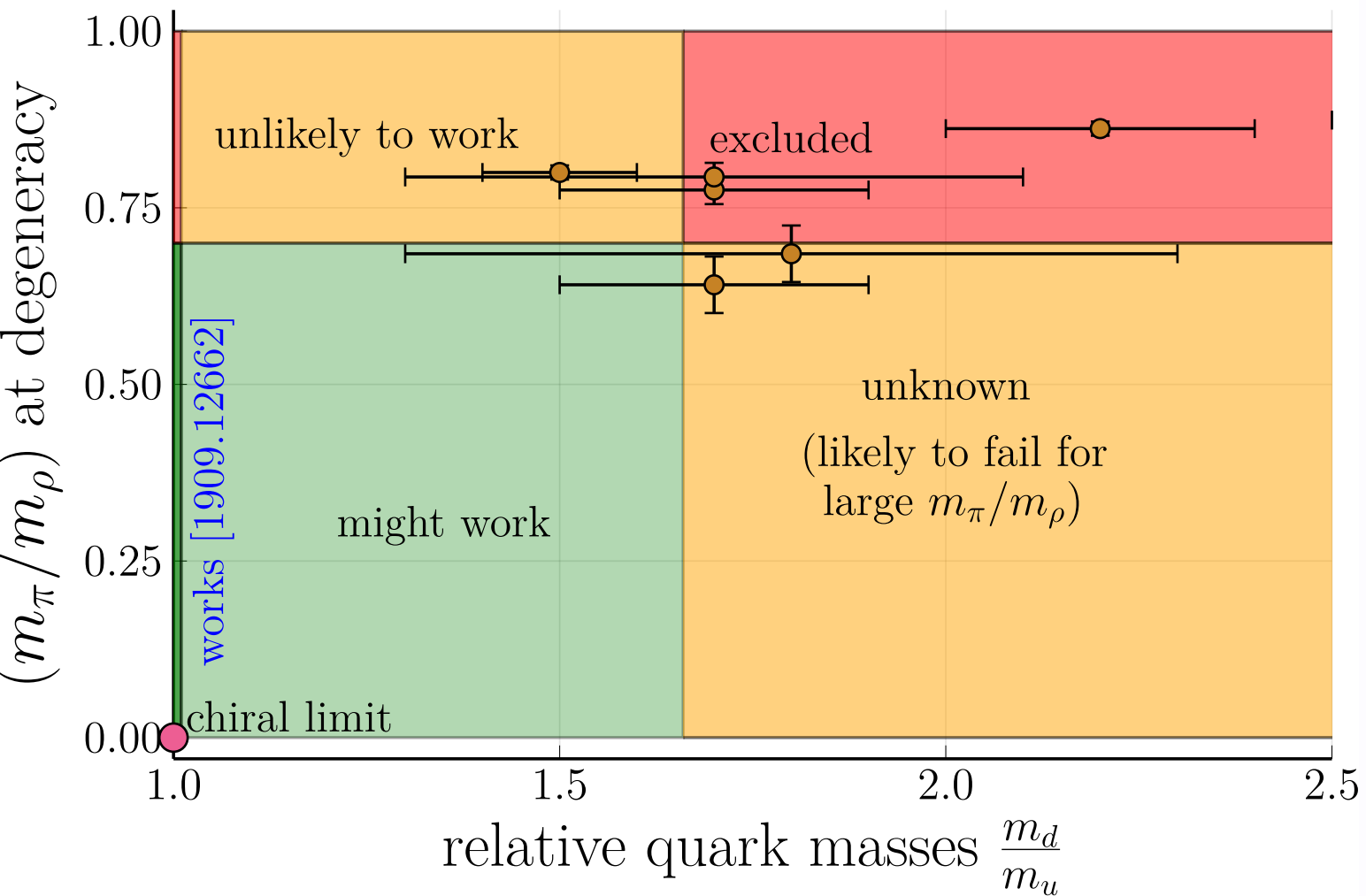
- Early universe: Thermal and chemical equilibrium
- DM depletion process, e.g. $\chi\chi_{DM} \leftrightarrow \psi\psi_{SM}$
 - Universe cools: $\psi\psi \rightarrow \chi\chi$ kinematically forbidden
 - *Dark Matter number decreases* through $\chi\chi \rightarrow \psi\psi$
 - Universe expands: χ annihilation stops
- DM eventually thermally decoupled: **freeze out**

Freeze-out process



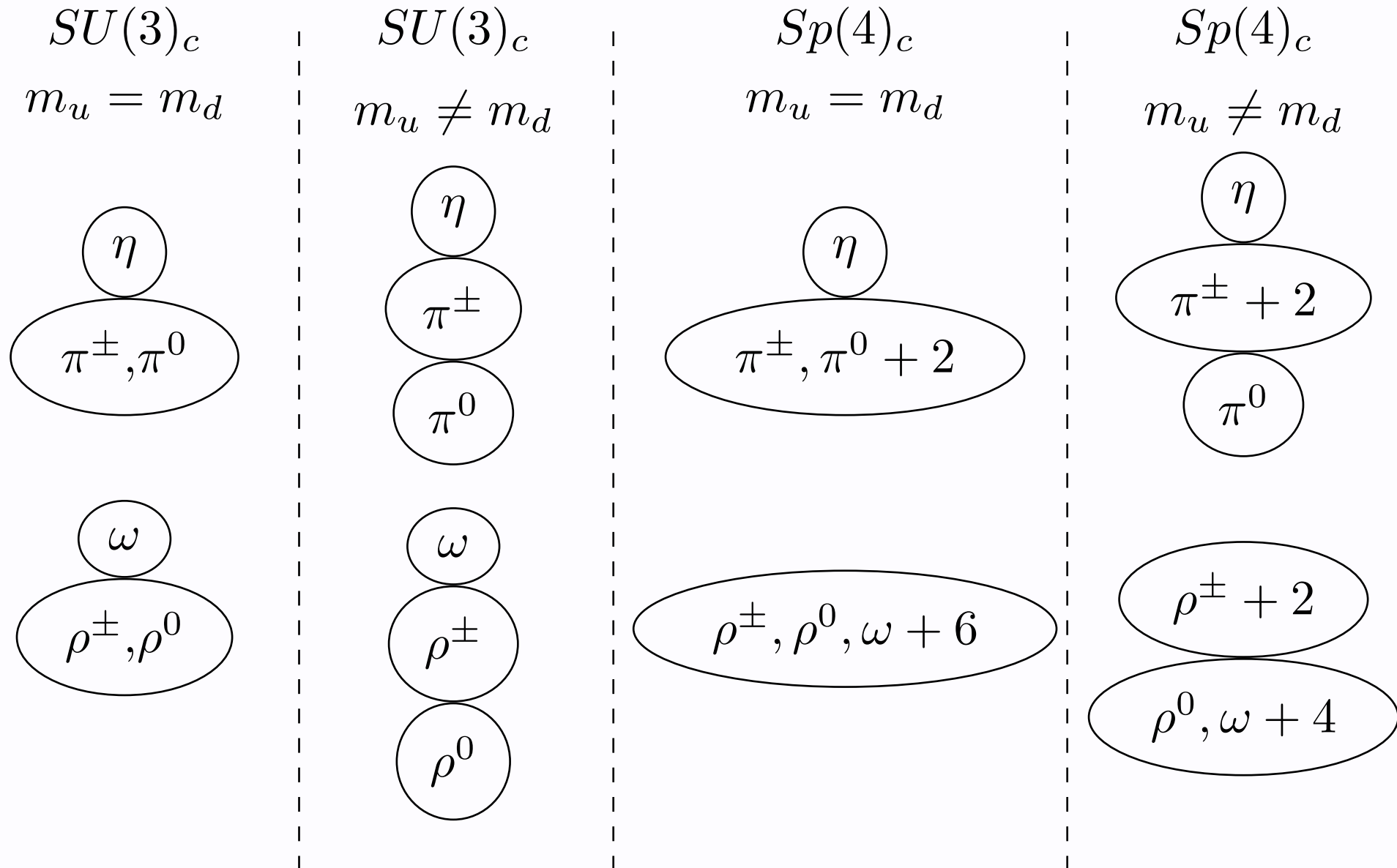
- Y ... DM number density, $x = m_{DM}/T$

rough sketch of the validity of LO χPT in Δm_q



- data points: breakdown of LO χPT in Δm on the lattice
- $\frac{m_d}{m_u} = 1 + \Delta m$
- $\frac{m_\pi}{m_\rho}$ fixed at degeneracy

Pseudoscalar (PS) and vector (V) multiplets



Meson Spectroscopy

- Obtain 2-point function of Operator $\mathcal{O}(x)$

$$\mathcal{O}_{\pi^+} = \bar{u}\gamma_5 d$$

$$\mathcal{O}_{\pi^-} = \bar{d}\gamma_5 u$$

$$\mathcal{O}_{\pi^0} = (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) / \sqrt{2}$$

$$\mathcal{O}_{\eta} = (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) / \sqrt{2}$$

- For scalars replace $\gamma_5 \rightarrow \mathbb{I}$, for vectors $\gamma_5 \rightarrow \gamma_i$

$$\langle \mathcal{O}(\vec{x}, t) \bar{\mathcal{O}}(0) \rangle = \sum_n \langle 0 | \hat{\mathcal{O}} | n \rangle \langle n | \hat{\mathcal{O}}^\dagger | 0 \rangle e^{-tE_n}$$

Diagrammatical representation

$$O_{\pi^\pm} = - \begin{array}{c} \text{d} \\ \text{---} \\ \text{n} \quad \text{m} \\ \text{---} \\ \text{u} \end{array}$$

$$2O_{\pi^0} = - \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \quad \text{m} \\ \text{---} \\ \text{u} \end{array} - \begin{array}{c} \text{d} \\ \text{---} \\ \text{n} \quad \text{m} \\ \text{---} \\ \text{d} \end{array} - \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array} - \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array} + \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{u} \begin{array}{c} \text{u} \\ \text{---} \\ \text{m} \end{array} + \begin{array}{c} \text{d} \\ \text{---} \\ \text{n} \end{array} \text{d} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array}$$

$$2O_\eta = - \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \quad \text{m} \\ \text{---} \\ \text{u} \end{array} - \begin{array}{c} \text{d} \\ \text{---} \\ \text{n} \quad \text{m} \\ \text{---} \\ \text{d} \end{array} + \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array} + \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array} + \begin{array}{c} \text{u} \\ \text{---} \\ \text{n} \end{array} \text{u} \text{u} \begin{array}{c} \text{u} \\ \text{---} \\ \text{m} \end{array} + \begin{array}{c} \text{d} \\ \text{---} \\ \text{n} \end{array} \text{d} \text{d} \begin{array}{c} \text{d} \\ \text{---} \\ \text{m} \end{array}$$

- Quark lines "come back" \Rightarrow **disconnected** diagrams
- Cancellation for π^0 in symmetric limit

Always present for singlet mesons!

Goldstone bosons and parity

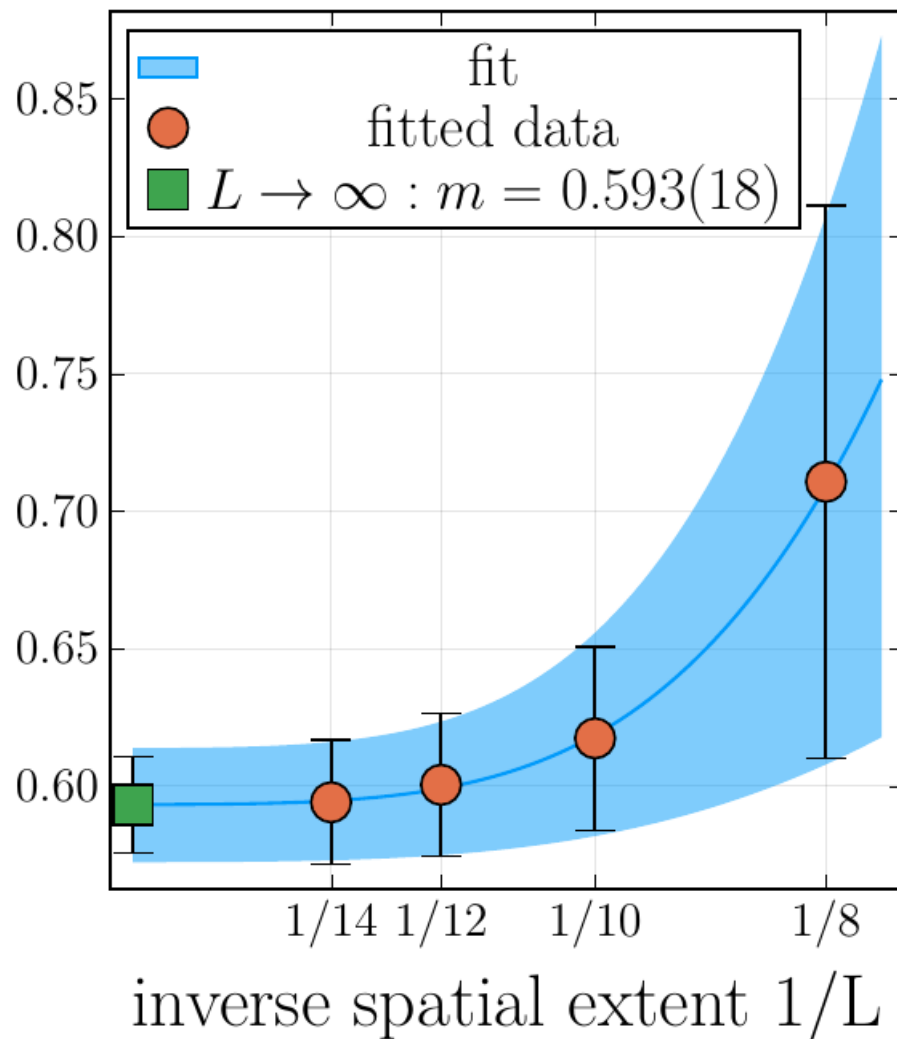
$$P : \psi(x, t) \rightarrow \gamma_0 \psi(-x, t)$$

$$D : \psi(x, t) \rightarrow \pm i \gamma_0 \psi(-x, t)$$

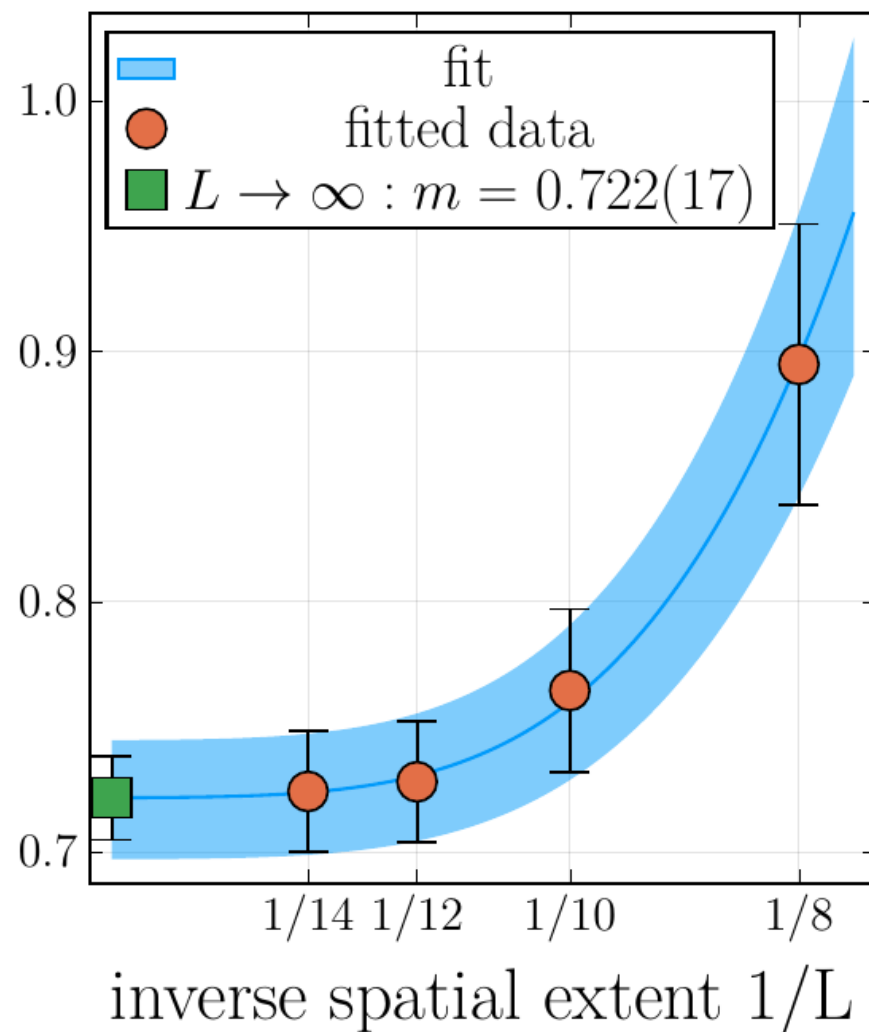
name	operator	J^P	J^D
π^-	$\bar{u} \gamma_5 d$	0^-	0^-
π^+	$\bar{d} \gamma_5 u$	0^-	0^-
π^0	$\bar{u} \gamma_5 u - \bar{d} \gamma_5 d$	0^-	0^-
π_{qq}	$u^T S C \gamma_5 d$	0^+	0^-
$\pi_{\bar{q}\bar{q}}$	$\bar{u} S C \gamma_5 \bar{d}^T$	0^+	0^-

Finite Volume: Meson masses

π^0 mass [a] at $\beta = 6.9$

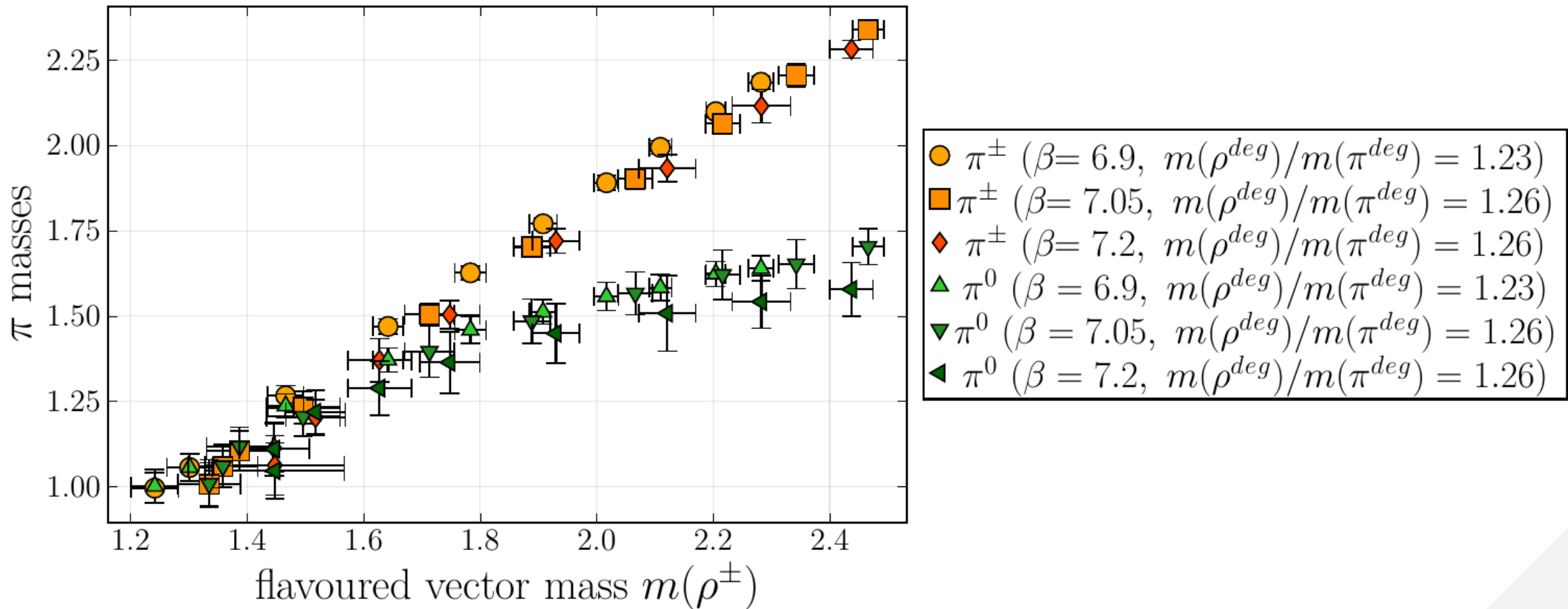


ρ^0 mass [a] at $\beta = 6.9$

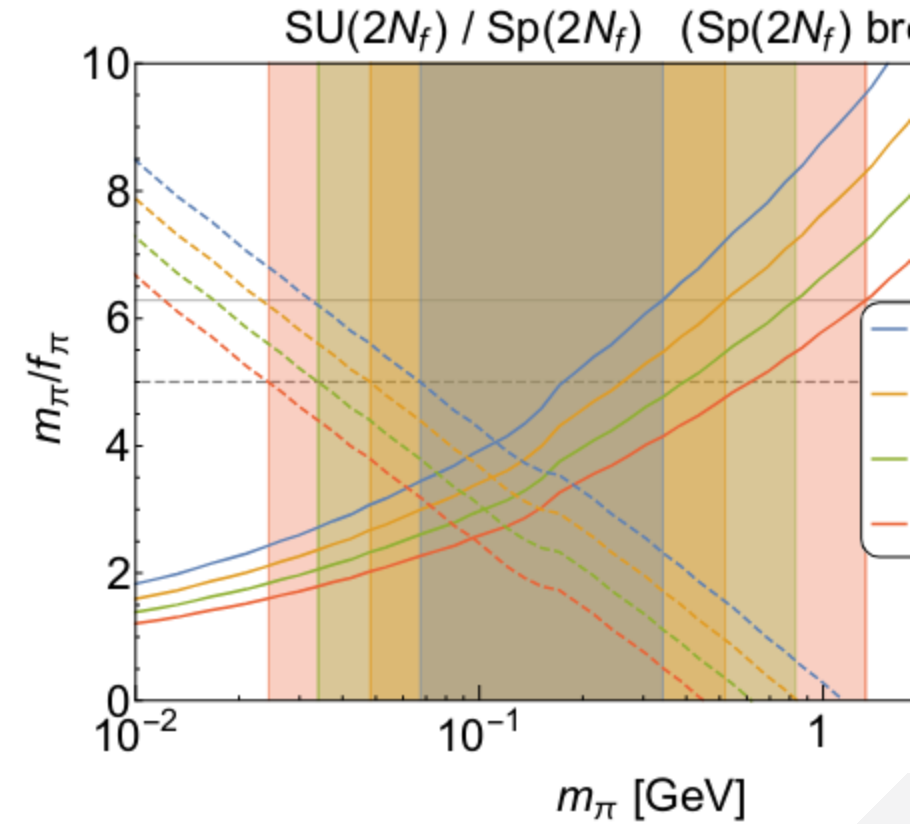
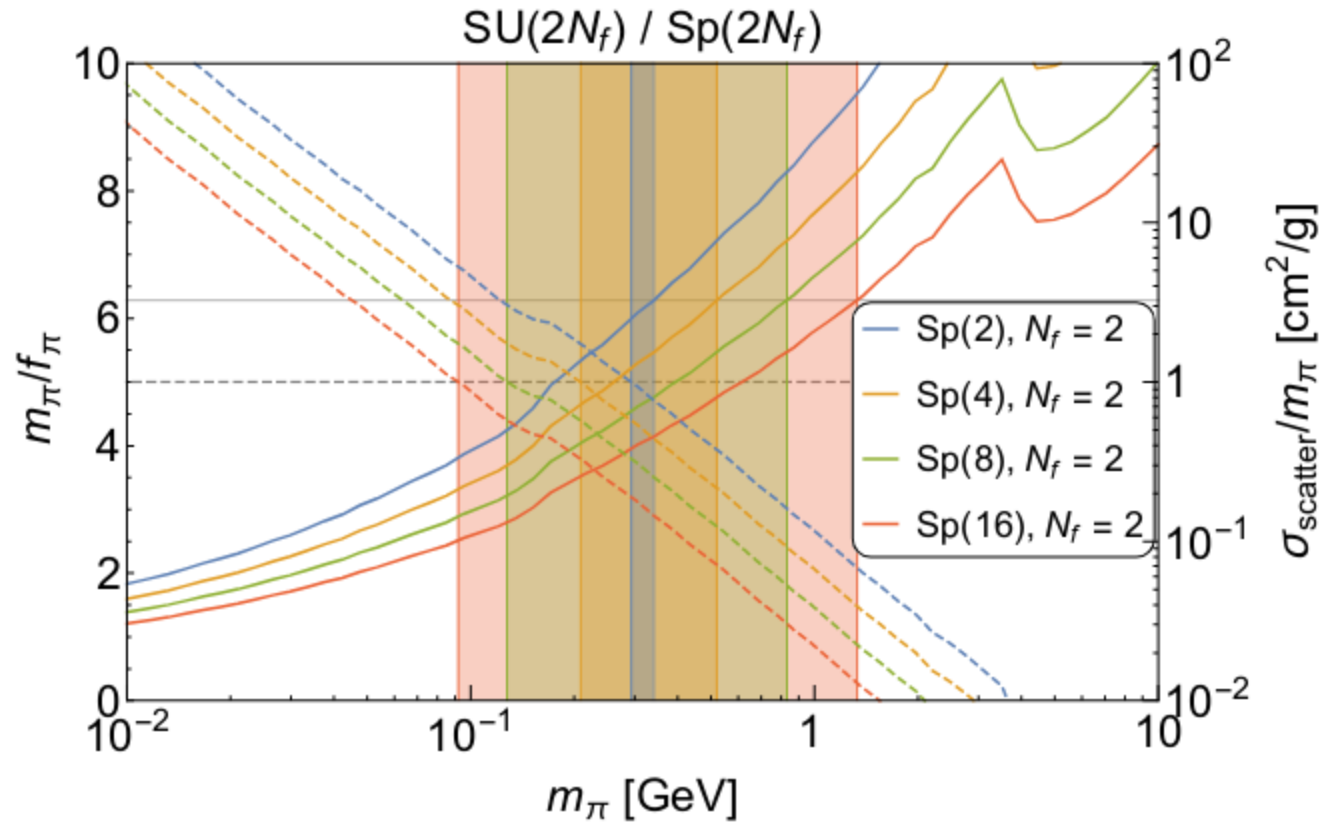


Spacing effects: Goldstone masses

Different $\beta = 6.9, 7.05, 7.2$ [$m(\pi^{deg})$]



Effects of varying N_c and $Sp(2N_f)$ breaking



taken from [1411.3727]