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Natural Sciences

Dark Matter in Neutron Stars

Bachelor Thesis

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Abstract

It is estimated that most of the mass of the universe appears in the form of dark matter. Because neutron stars could contain a mixture of dark and ordinary matter, their extreme conditions make them interesting cosmic laboratories for studying the nature of dark matter particles. This thesis studies observable properties of neutron stars like the mass, the radius, or the second Love number of the star and how those observables are influenced by dark matter inside the star. For ordinary matter, the equation of state developed in [5] is used, and for dark matter, the equation of state from [4] is used. Using those two models for ordinary and dark matter, the two-fluid Tolman-Oppenheimer-Volkoff equations are solved using a 4th-order Runge Kutta algorithm with adaptive step size. For different sets of parameters, the mass-radius relation and plots of the second Love number over the Compactness are presented and investigated further. At the end, the results are compared to experimental data.

Contents

1 Introduction to Compact stars and Dark matter

This thesis aims to understand dark matter in neutron stars, connecting two seemingly separate fields, namely particle physics and general relativity. This chapter is a short introduction to compact objects (for example neutron stars), dark matter in general and the possibility of dark matter in compact objects.

1.1 Compact stars

The term compact star or compact object is used collectively for stars that have a much higher density than ordinary stars. Examples of compact stars are white dwarfs and neutron stars. The latter will be the main focus of this thesis.
White dwarfs, neutron stars and black holes are the final stages of the evolution of most ordinary stars. In ordinary stars, the gravitational pull trying to contract the star is counteracted by the thermal pressure. The thermal energy stems from fusion reactions in the core of the star. Once all the elements for which fusion is an exothermic reaction are used, the star explodes in a supernova. What is left of the star is either a white dwarf, a neutron stars or a black hole depending on the previous mass of the star.
The name neutron star stems from the fact that this star consists mostly of neutrons. The gravitational pressure causes the protons and electrons to merge into neutrons. The neutron star counteracts the gravitational pull with the degeneracy pressure. This pressure is the result of the fact that neutrons are fermions and obey the Pauli principle. They cannot all occupy the same quantum state and this results in a repulsive interaction between the neutrons. (For further details see [6] and [3])

1.2 Dark matter

Many astronomical phenomena cannot be explained within the current standard model. This led physicists to believe that there is a new form of matter called dark matter (DM). Examples are the flatness of the universe or the fact that ordinary matter (OM) is subject to greater gravitational forces than the visible mass would exert. This type of matter does not interact with photons and therefore cannot be directly observed, hence the name dark matter. It is believed that the majority of the universe´s mass appears in the form of DM. The exact nature of DM is still subject to much speculation. There are different methods to detect DM using particle accelerators or scattering experiments, but this thesis studies the properties of DM by investigating the effect the presence of DM has on neutron stars.
In recent years gravitational waves caused by mergers of two neutron stars orbiting each other have been detected. The gravitational wave signals of such merger events depend on the deformation of the binary neutron star system in the inspiral stage. Therefore the tidal deformability of the stars can be extracted from the gravitational wave signals. Because the tidal deformability

changes with the amount of DM present in the stars, it is possible to gain insight into the properties of DM from these gravitational wave signals.
(For further details see [2] and [3])

2 Mathematical theory

This chapter summarizes the mathematical formulas needed for this thesis.

2.1 Tolmann-Oppenheimer-Volkoff Equation

There are two forces acting on a neutron star, the gravitational pull and the degeneracy pressure. The Tolmann-Oppenheimer-Volkoff (TOV) equation (1) describes the equilibrium of these two forces for a spherically symmetric, non-rotating and electrically neutral object. [6] For a detailed introduction and derivation see [3]

$$\frac{dp}{dr} = -\frac{G\epsilon(r)m(r)}{c^2r^2}(1+\frac{p(r)}{\epsilon(r)})(1+\frac{4\pi r^3 p(r)}{m(r)c^2})(1-\frac{2Gm(r)}{c^2 r})^{-1} \tag{1}$$

Here r is the radial distance from the center of the star, p(r) is the pressure at radius r, $\epsilon(r)$ is the energy density, m(r) is the integrated mass and c is the speed of light.
The dimensionless pressure p', energy density ϵ', mass m' and radius r' are defined by $p = p'\frac{m_F^4 c^8}{(\hbar c)^3}$, $\epsilon = \epsilon'\frac{m_F^4 c^8}{(\hbar c)^3}$, $m = m'\frac{m_p^3}{m_F^2}$ and $r = r'\frac{l_p m_p}{m_F^2}$ where m_F is the mass of the fermion (neutron mass), m_p and l_p are the Planck mass and Planck length and $\hbar$ is Planck's constant. Using these dimensionless quantities one can rewrite the TOV equation to obtain (2).

$$\frac{dp'}{dr'} = -\frac{(\epsilon' + p')(m' + 4\pi r'^3 p')}{r'(r' - 2m)} \tag{2}$$

To solve for the mass and the radius of the star a second equation is needed. The change of the integrated mass is given by $dm = 4\pi\rho r^2 dr$. Using $\rho = \frac{\epsilon}{c^2}$ and substituting the dimensionless quantities one arrives at the following equation:

$$\frac{dm'}{dr'} = 4\pi r'^2 \epsilon' \tag{3}$$

The total mass of the star is $M = m(r = R)$ where R is the radius of the star.

Equations (2) and (3) form a system of coupled differential equations. Given an equation of state $\epsilon'(p')$ and an initial pressure p'_0 this system of equations can be solved numerically. In this thesis a Runge Kutta 4th Order algorithm with adaptive step size is used for this purpose. (Note that because the system is two dimensional a second initial value for the mass is required. This initial mass is not independent from the initial pressure and is given by $m_0' = \frac{4}{3}\pi r_0'^3 \epsilon'(p_0')$ where r_0 is a small radius element and the starting point of the Runge Kutta method.) The algorithm terminates the Runge Kutta method once the pressure is as close to zero as desired. A solution could look like Figure 1

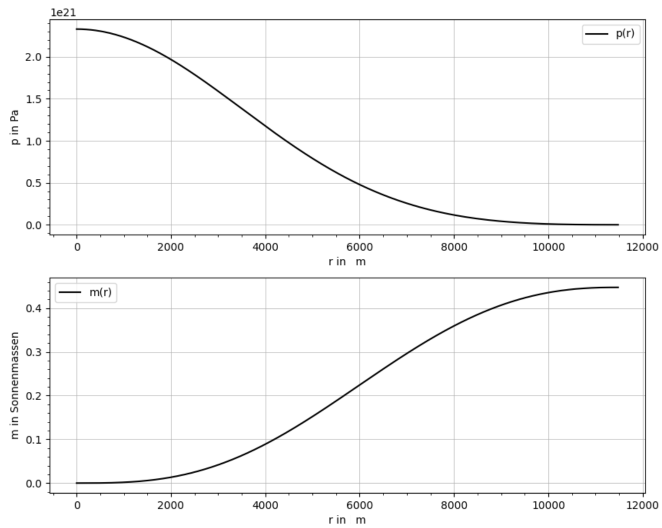

Figure 1: Solution of the TOV equations for a given initial pressure. (m in solar masses and p in Pa)

The radius of the neutron star R is the point where the pressure reaches zero and we end up in vacuum.

2.2 Dark Matter

Dark matter in neutron stars can be treated in exactly the same way as ordinary matter. Because the two different types of matter influence each other the neutron star is described by a system of four coupled differential equations. (4)

$$\begin{aligned}\frac{dp'_{OM}}{dr'} &= -(p'_{OM} + \epsilon'_{OM})\frac{d\nu}{dr'} \\ \frac{dm'_{OM}}{dr'} &= 4\pi\epsilon'_{OM}r'^2 \\ \frac{dp'_{DM}}{dr'} &= -(p'_{DM} + \epsilon'_{DM})\frac{d\nu}{dr'} \\ \frac{dm'_{DM}}{dr'} &= 4\pi\epsilon'_{DM}r'^2 \\ \frac{d\nu}{dr'} &= \frac{(m'_{OM} + m'_{DM}) + 4\pi r'^3(p'_{OM} + p'_{DM}))}{r'(r' - 2(m'_{OM} + m'_{DM}))}\end{aligned} \tag{4}$$

Instead of using the initial pressures of ordinary and dark matter as the two independent initial values, in this thesis the initial pressure of ordinary matter $p'_{OM,0}$ and the ratio $r = \frac{p'_{DM,0}}{p'_{OM,0}}$ are used as initial conditions. Given $p'_{OM,0}$ and r as well as two equations of state for ordinary and dark matter $\epsilon'_{OM}(p'_{OM})$ and $\epsilon'_{DM}(p'_{DM})$ one can calculate the four initial values needed to solve the four dimensional system of equations: $p'_{OM,0}$, $p'_{DM,0}$, $m'_{OM,0} = \frac{4}{3}\pi r_0'^3\epsilon'_{OM}(p'_{OM}))$ and $m'_{DM,0} = \frac{4}{3}\pi r_0'^3\epsilon'_{DM}(p'_{DM})$ A solution for the system of equations (4) could look like Figure 2

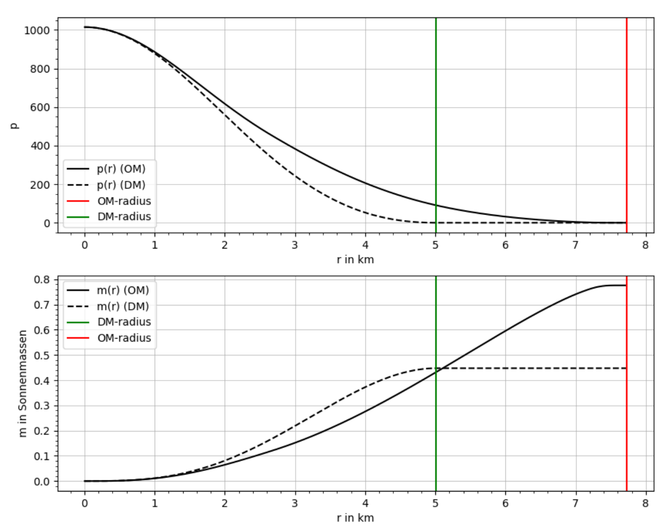

Figure 2: Solution of the two fluid TOV equations for a given initial pressure and ratio. (m in solar masses and p in MeV/fm^3)

2.3 Equation of state

The equation of state (EoS) $\epsilon(p)$ of the matter inside the star is needed to solve the TOV equations (4).

Polytropic equation of state

Different models can be used to describe a compact star. If the matter inside the star is treated like an ideal Fermi gas, one arrives at polytropic equations of state. (5) (for a detailed derivation see [6])

$$p(\epsilon) = K\epsilon^{\frac{n+1}{n}} \tag{5}$$

K is an arbitrary constant and n is called the polytropic index. Different n represent different physical scenarios. For example the non-relativistic Fermi gas corresponds to $n = 1.5$ while the relativistic Fermi gas corresponds to $n = 3$. (See Figure 3)

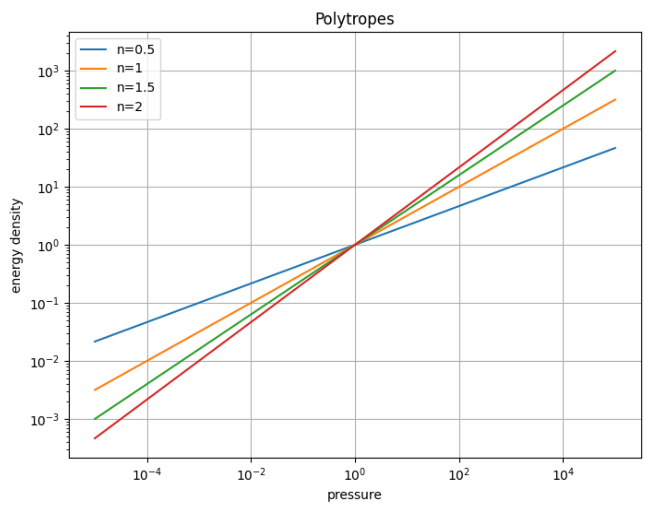

Figure 3: Polytropes for different polytropic indices.

Models using Quantum chromodynamics (QCD)

For ordinary matter, the equation of state from [5] is used in this thesis. (see Figure 4) While a full nonperturbative calculation of the EoS is not possible due to the so-called Sign Problem of lattice QCD, they managed to predict the EoS using chiral effective field theory (EFT) for low pressures and perturbative QCD (pQCD) for high pressures. There is still an unknown region left between those limiting cases that can't be described by either of those two theories.

Different interpolations using piece-wise polytropic functions are used for this region which yield many different equations of state. An EoS has to support all astronomical observations and therefore, the number of possible EoS can be constrained. First of all, the speed of sound given by $c^2_{s,OM} = \frac{dp'_{OM}}{d\epsilon'_{OM}}$ cannot exceed the speed of light. Other constraints come, for example, from the existence of neutron stars with mass $2M_\odot$, or most recently from gravitational wave signals caused by neutron star mergers. For more details see [5]

For dark matter, the equation of state from [4] is used. In their approach they studied a QCD-like theory to avoid the sign problem. In this modified version the gauge group of QCD SU(3) is replaced by the Lie group G_2. To determine the equation of state they looked at available data from lattice simulations. For more details see [4]

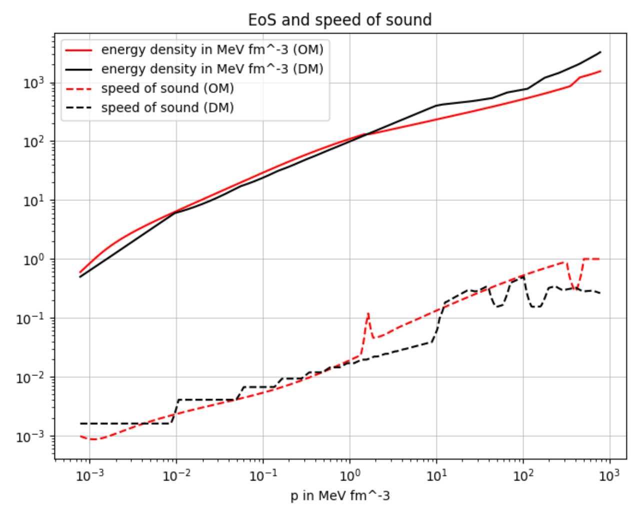

Figure 4: Plot of the equation of state

2.4 Tidal Deformability

The tidal deformability Λ is a dimensionless parameter describing how a neutron star is deformed if two neutron stars orbit each other. (It can be determined experimentally by measuring the gravitational wave singal of a neutron star merger.)
The tidal deformability Λ can be calculated using equation (6).

$$\frac{2}{3}\frac{R^5}{G}k_2 = \Lambda\frac{M^5 G^4}{c^{10}} \tag{6}$$

k_2 is called the second love number and can be calculated using equation (7) where $C = \frac{M}{R}$ is called the compactness and y is obtained by solving the differential equation (8) with the initial value $y(r' = 0) = 2$.

$$
\begin{aligned}
k_2 = \frac{8C^5}{5}&(1-2C)(2+2C(y-1)-y) \cdot \{2C(6-3y+3C(5y-8))+ \\
&4C^3(13-11y+C(3y-2)+2C^2(1+y))+ \\
&3(1-2C)^2(2-y+2C(y-1))ln(1-2C)\}^{-1}
\end{aligned} \tag{7}
$$

$$
r\frac{dy(r')}{dr'} + y(r')^2 + y(r')F(r') + r'^2 Q(r) = 0 \tag{8}
$$

$F(r')$ and $Q(r')$ are given by (9) and (10) respectively.

$$
F(r') = \frac{r - 4\pi r'^3((\epsilon'_{OM} + \epsilon'_{DM}) - (p'_{OM} + p'_{DM}))}{r' - 2m'(r')} \tag{9}
$$

$$
\begin{aligned}
Q(r') = &\frac{4\pi r'(5(\epsilon'_{OM} + \epsilon'_{DM}) + 9(p'_{OM} + p'_{DM}) + \frac{\epsilon'_{OM} + p_{OM'}}{c^2_{s,OM}} + \frac{\epsilon'_{DM} + p_{DM'}}{c^2_{s,DM}} - \frac{6}{4\pi r'^2})}{r' - 2m'(r')} \\
&- 4(\frac{m' + 4\pi r'^3(p'_{OM} + p'_{DM})}{r'^2(1 - \frac{2m'}{r'})})
\end{aligned} \tag{10}
$$

where $m'(r') = m'_{OM}(r') + m'_{DM}(r')$, $c^2_{s,OM} = \frac{dp'_{OM}}{d\epsilon'_{OM}}$ and $c^2_{s,DM} = \frac{dp'_{DM}}{d\epsilon'_{DM}}$

2.5 Observations from gravitational wave signals

The gravitational wave signal of the last moments before two inspiraling neutron stars merge is called the "chirp signal". Two parameters can be easily determined from a chirp signal - the chirp mass M_C and a combination of the tidal deformabilities of the inspiraling objects $\tilde{\Lambda}$. The chirp mass M_C is a combination of the individual masses of the two stars and is given by (11).

$$
M_C = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1 + m_2)^{\frac{1}{5}}} \tag{11}
$$

In 2017 the Laser Interferometer Gravitational-Wave Observatory (LIGO) managed to measure a gravitational wave signal from a neutron star inspiral for the first time. LIGO was able to determine the chirp mass of the two stars, the result was $M_C = 1.188^{+0.004}_{-0.002}$ $M_\odot$. Additioinally, LIGO constrained the parameter $\tilde{\Lambda}$ given by (12) to $\tilde{\Lambda} < 800$ and the tidal deformabilities of the individual stars to $\Lambda < 800$. [1]

$$
\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5} \tag{12}
$$

3 Calculation & Results

In this chapter, the procedure for analyzing a given equation of state is explained, and the results for different EoS are shown.

3.1 Mass-Radius relation and stability analysis

Let us first consider a star without any dark matter present. To analyze an EoS one solves the TOV equations for a range of different central pressures. Every central pressure leads to a distinct solution like the one in Figure 1. The mass-radius relation (MRR) is a plot of all the different possible masses and radii and could look like Figure 5. The central pressure increases along the curve, starting from the right at large radii. The MRR can be split up into two parts, the first one between points D and E describing neutron stars, and the second one to the right of point A describing white dwarfs.

Stability analysis

Not all solutions of the TOV equations are stable and the unstable solutions are represented by a dashed line in Figure 5. The procedure to determine the stable solutions works as follows: (for more details see [7]) All of the eigenfrequencies $\omega_0, \omega_1, \omega_2, ...$ resulting from the Sturm-Liouville eigenvalue equation have to be real. To check whether those frequencies (or modes) are real, one starts at a solution for low pressures (somewhere in the white dwarf branch), which is known to be stable. The pressure is increased until a point where $\frac{dM}{dp_0} = 0$ is reached. This occurs at the maximum point A. If at this point $\frac{dR}{dp_0}$ is positive, an odd mode squared changes sign, if $\frac{dR}{dp_0}$ is negative then an even mode squared changes sign. At point A the radius is decreasing, therefore an even mode squared gets negative and the solutions become unstable. At point B the radius is increasing, and therefore an odd mode squared changes sign. Now both, an even and an odd mode squared are negative, and the solution is still unstable. At point C, the radius is increasing and thus the odd mode squared changes sign and becomes positive again. Finally, at point D, the radius is decreasing and thus the even mode squared becomes positive again. Now all frequencies are real, and the solution is stable until the point E is reached.

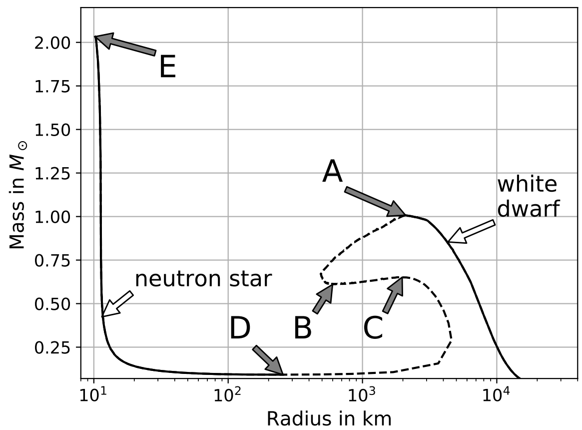

Figure 5: A possible mass-radius relation

Dark matter

A star containing both ordinary matter and dark matter is described by an EoS for ordinary matter $\epsilon'_{OM}(p'_{OM})$, and an EoS for dark matter $\epsilon'_{DM}(p'_{DM})$. The initial values for the TOV equations (4) are the central pressure of ordinary matter $p_{OM,0}$ and the ratio r as described in section 2.2. Let's first consider r fixed and solve the TOV equations for many different central pressures $p_{OM,0}$. Every central pressure leads to a solution like Figure 2 and thus to the masses and radii of the OM part and the DM part: R_{OM}, R_{DM}, $M_{OM} = m_{OM}(R_{OM})$, $M_{DM} = m_{DM}(R_{DM})$. To determine whether a solution is stable, one has to look at the two mass-radius relations $M_{OM}(R_{OM})$ and $M_{DM}(R_{DM})$. If the solution is in the stable regions in both mass-radius relations, then it is overall stable.
Many different options for the overall MRR are possible, this thesis will focus mainly on plotting the total mass $M_{tot} = M_{OM} + M_{DM}$ vs the radius of the ordinary matter part R_{OM}. Another option would be to plot the mass of the total star M_{tot} vs the radius of the total star $R_{max} = \max(R_{OM}, R_{DM})$. The reason why the former MRR is chosen is that those two quantities M_{tot} and R_{OM} can be directly measured. Current radius measurements rely on the observation of photons and since dark matter does not interact with photons only R_{OM} can be directly measured. In contrast the radius of the DM part can theoretically be obtained by observing a dark halo around the neutron star, which appears if R_{DM} exceeds R_{OM}. However up to this point no experiment observing this phenomenon has been performed. The total mass of the star can be obtained through the rotational frequency of two rotating neutron stars.
Up to this point, we have considered the ratio r fixed, if now additionally, r is varied, this gives rise to a family of curves.

k_2 vs C plots

Other interesting quantities for neutron stars are the tidal deformability Λ and the second Love number k_2 because they can be experimentally constrained by analyzing gravitational wave signals of merger events. (This thesis will mainly focus on the second Love number, but this is arbitrary since both quantities are proportional to each other, and one can be calculated out of the other using equation (6))
For many different central pressures and a fixed ratio r, additionally to solving the TOV equations (4), one solves equation (8) and calculates k_2 using equation (7). From the solution of the TOV equations the compactness $C = \frac{M}{R}$ can be calculated, where M is the total dimensionless mass of the star, and $R = max(R_{OM}, R_{DM})$ is the maximum dimensionless radius of the star. Now, the second Love number k_2 can be plotted against the compactness C of the star. Again, if r is varied, this results in a family of curves.

3.2 Polytropes

At first, the results for compact stars, which contain only ordinary matter described by a polytropic EoS (see section 2.3), are shown. Figure 6 is the mass-radius relation for different polytropic indices and in Figure 7 the second Love number is plotted against the compactness $C = \frac{M}{R}$ of the star. The $n = 0$ polytrope (which is very close to the $n = 0.01$ polytrope in Figure 6) is the causal limit, this means that there are no solutions allowed on the left of this curve in the mass-radius relation.

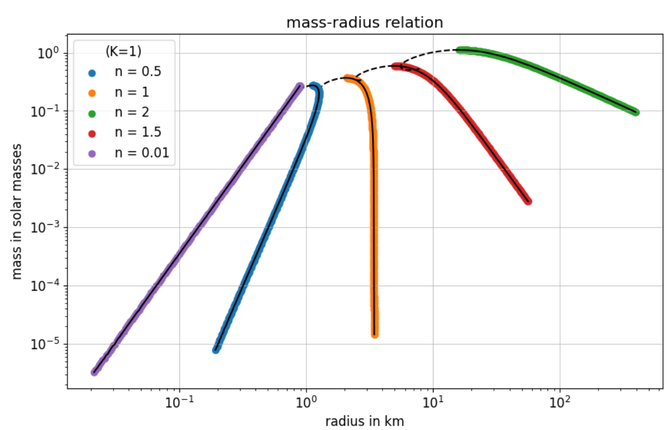

Figure 6: Mass radius relation for the polytropic EoS

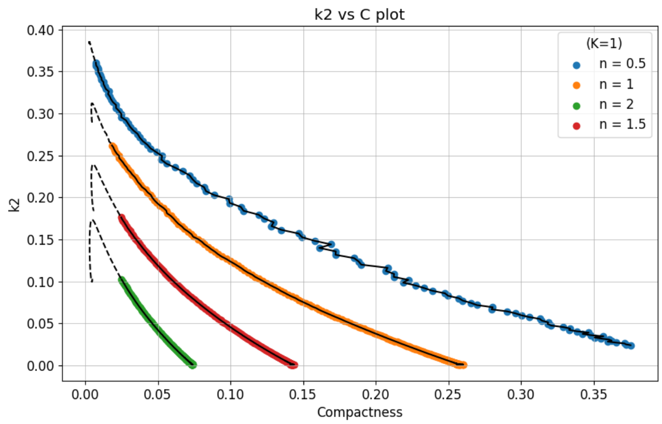

Figure 7: Second Love number vs Compactness plot for polytropic EoS

3.3 Mixed stars

In this section the results for stars containing both ordinary and dark matter are shown and discussed. The models for the equations of states obtained from QCD are explained in section 2.3. As explained in section 3.1, every value of r yields a distinct mass-radius relation. Additionally, the mass of the dark matter particle M_{DM} can also be varied, which leads to a two dimensional parameter space. The mass-radius relations and the k_2 vs C plots for different parameters can be seen in Figure 8 and Figure 7, respectively. The unstable parts of the curve are represented by dashed lines and only the neutron star branch is considered.

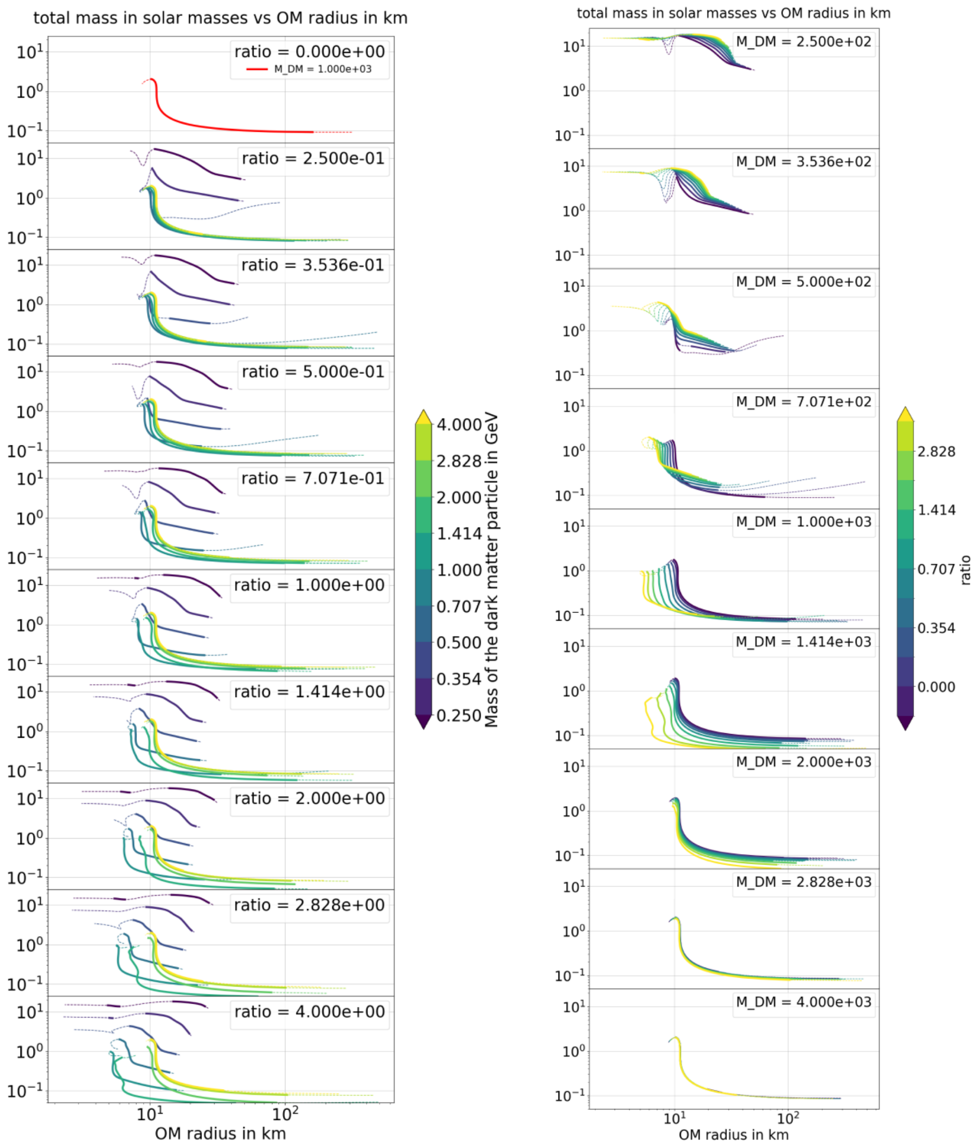

Figure 8: Mass radius relation for different combinations of parameters;
Left figure: In each subplot the ratio is fixed and the mass of the dark matter particle M_{DM} / GeV is varied. The ratio of $r = 0$ corresponds to a pure OM star and a ratio of $r = -1$ corresponds to a pure DM star.
Right figure: In each subplot the mass of the dark matter particle M_{DM} / MeV is fixed and the ratio is varied.

Observations from the mass-radius relation

First, we look at the left panel, where in each subplot the ratio r is fixed. For a fixed ratio it can be observed that the total mass of the star at first decreases with increasing mass of the dark matter particle M_{DM} until it reaches a minimum value of around $M_{DM} = 1$ GeV, then the mass of the star starts to increase again with increasing M_{DM}. The same thing happens with the radius of the star. For a fixed ratio r, at first, the radius of the star decreases with increasing M_{DM}, until it reaches its minimum around $M_{DM} = 1$ GeV, then the radius starts to increase again. These tendencies can be observed for all ratios.
Now we look at the right panel, where in each subplot, the mass of the dark matter particle M_{DM} is fixed. At first, it can be observed that the mass-radius relation only really depends on the ratio if M_{DM} is less than 2 GeV. For masses greater than 2 GeV, the curve looks the same for different ratios. This is due to the fact that the dark matter starts to decouple from the ordinary matter once the mass of the dark matter particle is too heavy compared to M_N. (The mass of a neutron is $M_N = 0.9396$ GeV) If M_{DM} is smaller than the mass of the neutron, the mass of the total star increases as the ratio increases. Once M_{DM} surpasses the neutron mass, the opposite case can be observed: As the ratio is increased, both the radius and the mass of the star decrease.
The highest total mass is reached for $M_{DM} = 0.25$ GeV and $r = 4$ and is $M_{tot} = 18.23$ $M_\odot$. For this set of parameters, the star is made of 99 % dark matter. The smallest total mass is reached for the parameters $M_{DM} = 1.414$ GeV and $r = 2.828$, and ist $M_{tot} = 0.048$ $M_\odot$. In this case the star is made of approximately 75 % dark matter.

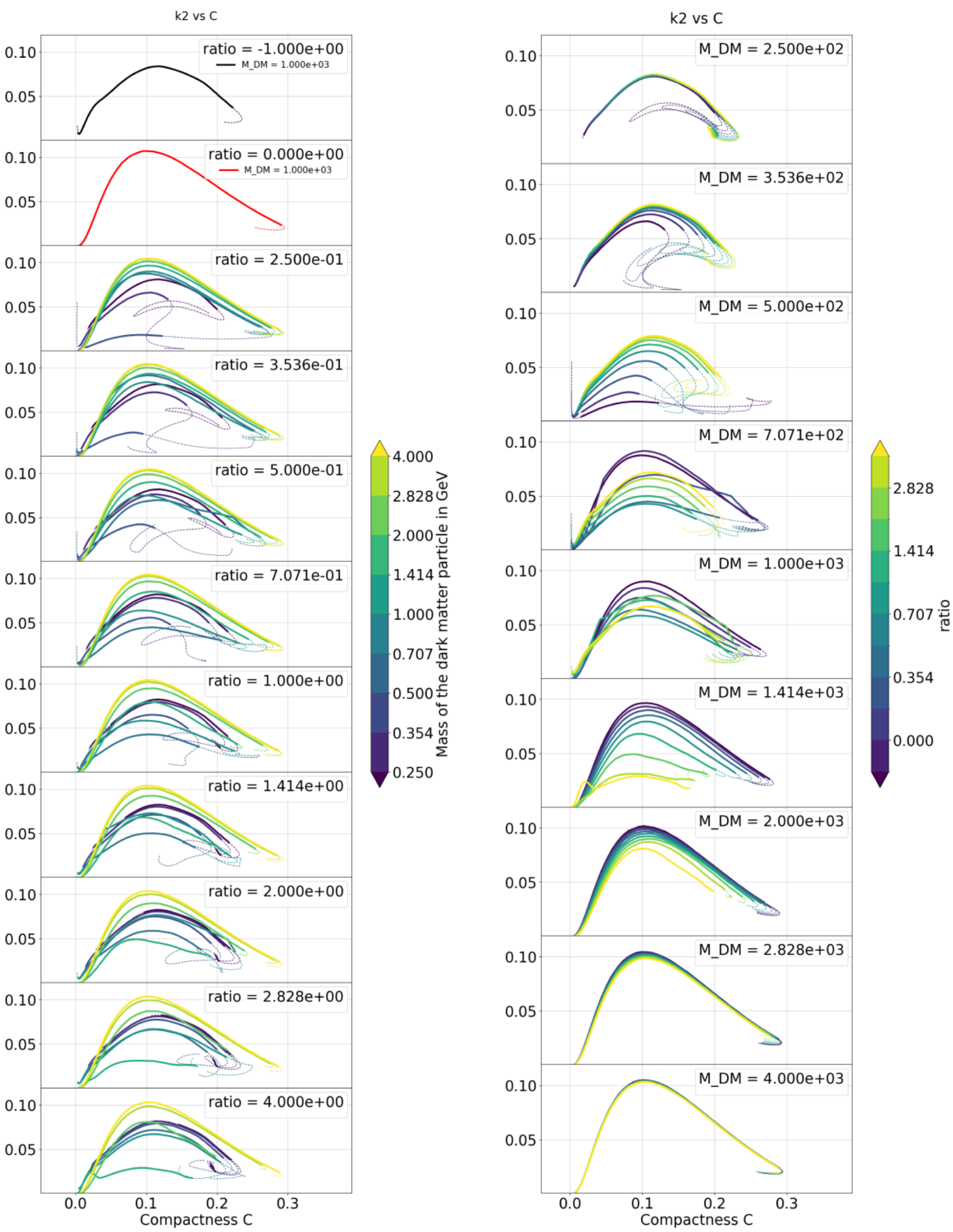

Figure 9: k_2 vs C plots for different combinations of parameters;
Left figure: In each subplot the ratio is fixed and the mass of the dark matter particle M_{DM} / GeV is varied. The ratio of $r = 1$ corresponds to a pure OM star and a ratio of $r = -1$ corresponds to a pure DM star.
Right figure: In each subplot the mass of the dark matter particle M_{DM} / MeV is fixed and the ratio is varied.

Observations from the k2 vs C plots

First, we look at the left panel, where in each subplot the ratio r is fixed. In each subplot, it can be observed that as M_{DM} is increased, k_2 first decreases, then reaches a minimum and increases again. The maximum value for k_2 always occurs for the highest value of M_{DM}. The mass M_{DM} at which the minimum value for k_2 occurs, seems to change slightly with r: As we go through the different subplots from top to bottom (increasing r), the mass that leads to the minimum, increases.

Now we look at the right panel, where in each subplot, the mass of the dark matter particle M_{DM} is fixed. The same effect, as in the mass-radius relation occurs, that if M_{DM} is too large compared to the mass of the neutron, the curve does not depend on the ratio anymore. If M_{DM} is clearly below the mass of the neutron then k_2 increases with increasing ratio, and if M_{DM} is clearly higher than the neutron mass then k_2 decreases with increasing ratio.

The highest value for k_2 is reached for $M_{DM} = 4$ GeV and $r = 0.25$ and is $k_2 = 0.1045$.

Another interesting feature in the k_2 vs C plots is the appearance of kinks (sudden changes in the slope of the curve) in some of the curves. To better understand why those kinks appear, different mass-radius relations are plotted for fixed parameters $r = \sqrt{2}$ and $M_{DM} = 1$ GeV in Figure 10. For low central pressures (right side of the curve), the radius of the MRR is defined by the radius of the DM star. As the pressure is increased along the curve, a kink appears. From this point forward, the radius of the star is the radius of the OM star. Because of the kink in the M_{tot} vs R_{max} curve and because $C = \frac{M_{tot}}{R_{max}}$, there also appears a kink in the k_2 vs C plot.

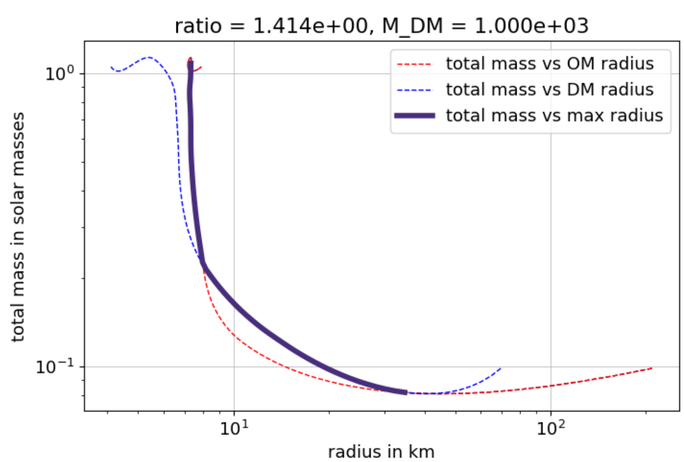

Figure 10: Different mass-radius relations for fixed r and M_{DM}

Limiting cases

For this discussion, it is best to look at the MRR $M_{tot}(R_{max})$ instead of $M_{tot}(R_{OM})$. (see Figure 14 in Appendix A) For high masses of the dark matter particle M_{DM} (compared to the neutron mass $M_N = 0.9396$ GeV), the MRR and the k_2 vs C plots approach the case of a pure OM star, and for low M_{DM} they approach a pure DM star. This can easily be seen if one looks at the right panel of Figure 14 and Figure 9 and compares those plots to the pure OM and pure DM cases at the top of the left panel.
For high ratios r, the MRR and the k_2 vs C plots approach the case of a pure DM star, and for low ratios r they approach a pure OM star. This is most apparent if one looks at the subplot in Figure 14 and Figure 9 with the mass of the dark matter particle fixed at $M_{DM} = 1$ GeV and compares the limiting cases to the pure OM and pure DM cases at the top of the left panel.

Because r and M_{DM} form a two-dimensional parameter space the results can be displayed in a three-dimensional plot. From every MRR in Figure 8, the maximum value is chosen, and the total mass and the OM radius are plotted as a function of the two parameters. (see Figure 11 left and middle panel) Analogously, for the k_2 vs C plots (Figure 9), the maximum value of every curve is chosen and plotted as a function of the parameters. (see Figure 11 right panel)

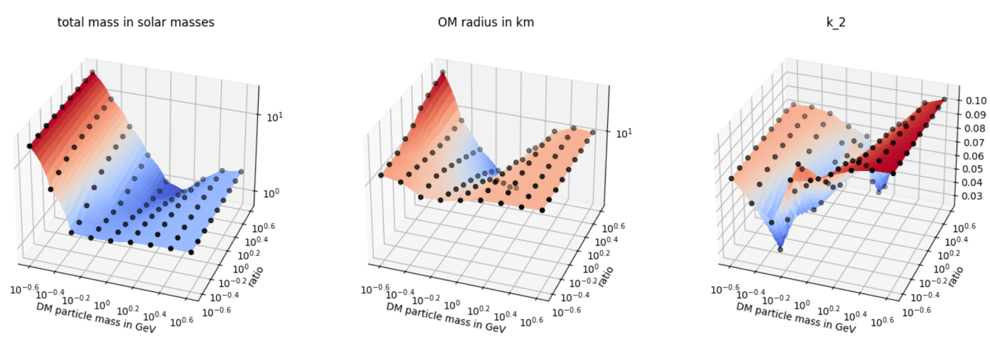

Figure 11: 3D plots

Observations from the 3D plots

First of all, the ways in which M_{tot},R_{OM} and k_2 change as the parameters vary, which are described in the discussion of the MRR and k_2 vs C plots above, can be directly seen in the 3D plots. One more highly interesting feature appears in the 3D plots: For all three plots, there seems to be one special straight line in the parameter space for which M_{tot}, R_{OM}, and k_2 have their minimal values.

Comparison to experimental data

Lastly, the results can be compared to the experimental data measured by LIGO. (see section 2.5) They constrained the chirp mass to $M_C = 1.188^{+0.004}_{-0.002}$ M$_\odot$. To see if the results are consistent with this measurement a Λ-Λ plot is created. (see Figure 12) To do this, the results for different sets of parameters (fixed r and M_{DM}) are searched for pairs of stars, such that their chirp mass is within the limits given by LIGO. If such a pair is found, the tidal deformabilities of the two stars are plotted against each other. It can be seen that for ratios smaller than 1 the results fall well within the 90 % or 50 % credibility lines determined by LIGO.

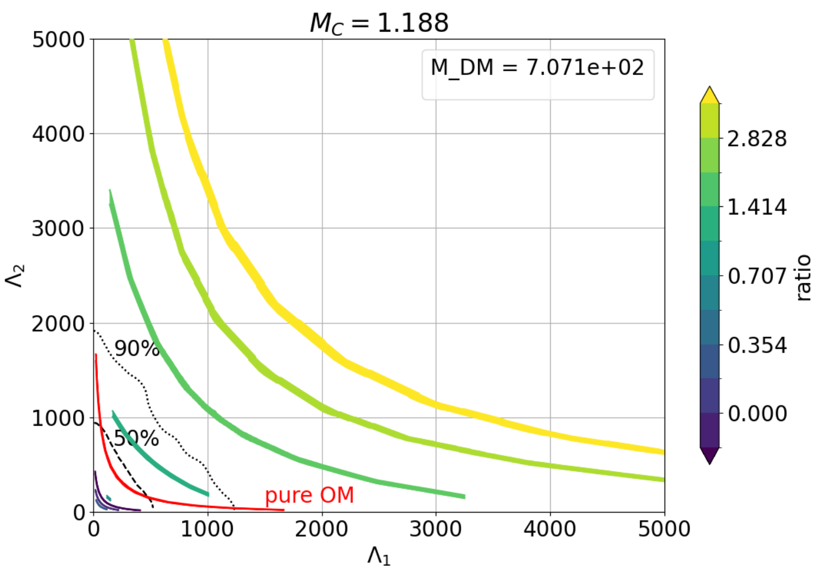

Figure 12: Λ-Λ plots for $M_{DM} = 0.7071$ GeV and different ratios as well as a pure OM star. Each point in this plot represents two neutron stars with a chirp mass within $M_C = 1.188^{+0.004}_{-0.002}$ M$_\odot$. The dotted and dashed lines are the 90% and 50% credibility limit determined by LIGO.

LIGO also constrained the dimensionless tidal deformability of the individual neutron stars to $\Lambda < 800$. To compare our results to this measurement, the dimensionless tidal deformability is plotted against the total mass of the star. (see Figure 13) It can be seen that for most sets of parameters, the results support stars with a tidal deformability below 800.

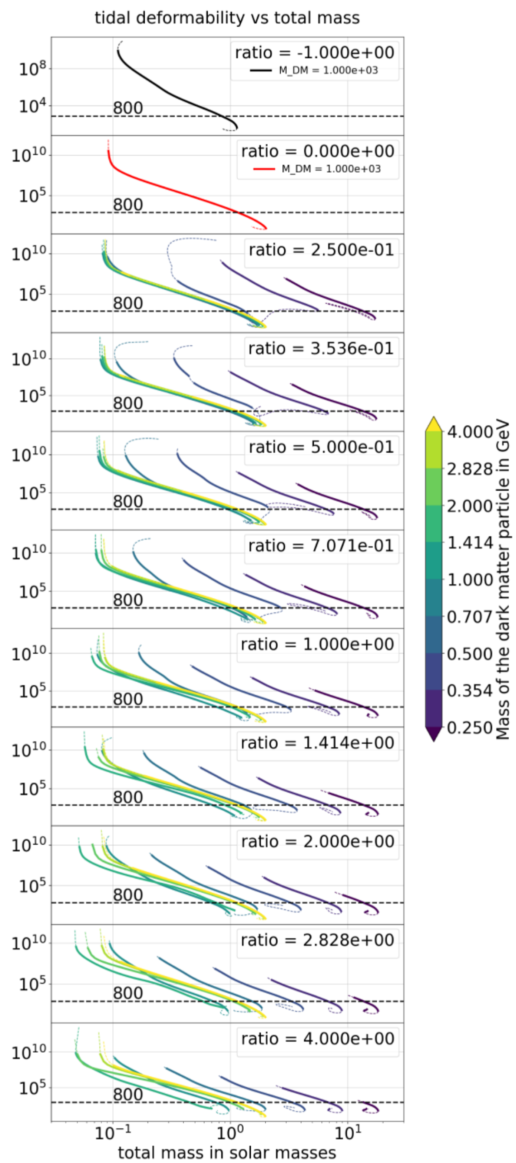

Figure 13: the dimensionless tidal deformability plotted against the total mass in solar masses for different combinations of parameters;
In each subplot the ratio is fixed and the mass of the dark matter particle M_{DM} / GeV is varied. The ratio of $r = 1$ corresponds to a pure OM star and a ratio of $r = -1$ corresponds to a pure DM star.
The dashed line is the limit for the dimensionless tidal deformability determined by LIGO.

4 Summary and Outlook

In this work, the influence of dark matter on neutron stars has been studied. For this purpose, the two-fluid TOV equations were solved using the OM model from [5] and the DM model from [4]. For different masses of the dark matter particle and different ratios, the mass-radius relation and k_2 vs C plot were computed. Because the parameter space is two dimensional the results were also displayed in a three dimensional plot. Finally, the results were compared to the experimental data from a neutron star merger measured by LIGO.
By analyzing those plots, many tendencies on how the different observables of a neutron star depend on the mass of the dark matter particle and the ratio were observed and discussed. The most interesting observation is that there appears to be a special straight line in the parameter space where a valley appears in the 3d plots for several different observables for example M_{tot}, R_{OM} and k_2.
Finally, it was found that there are neutron stars containing dark matter that lie within the experimental constraints given by LIGO. [1]
In the future, the parameter study could be expanded. One could incorporate a broader range of ratios and masses of the DM particle or use a higher density of parameter points. Another avenue of investigation is to further constrain and improve the equations of state.

A Further Plots

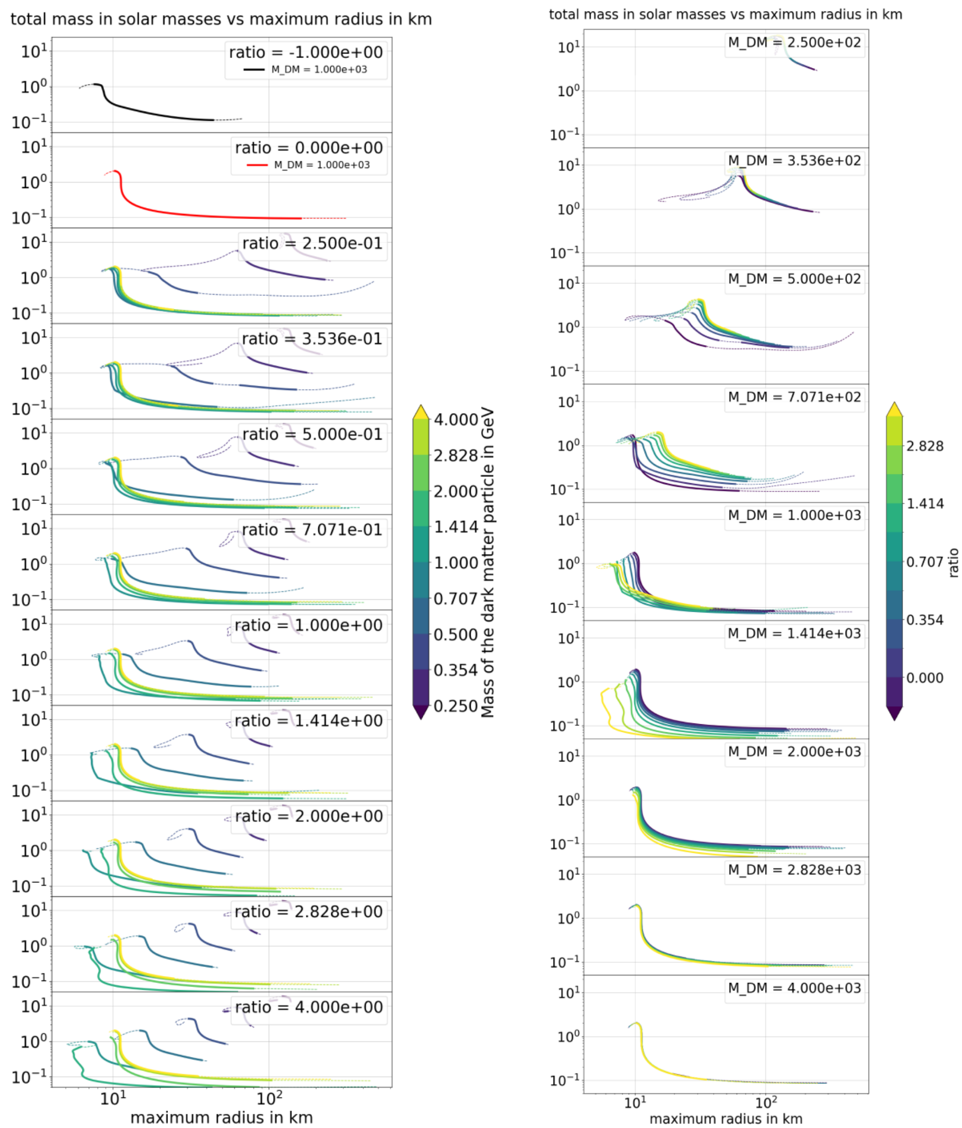

Figure 14: Mass radius relation for different combinations of parameters;
Left figure: In each subplot the ratio is fixed and the mass of the dark matter particle M_{DM} / MeV is varied. The ratio of $r = 0$ corresponds to a pure OM star and a ratio of $r = -1$ corresponds to a pure DM star.
Right figure: In each subplot the mass of the dark matter particle M_{DM} / MeV is fixed and the ratio is varied.

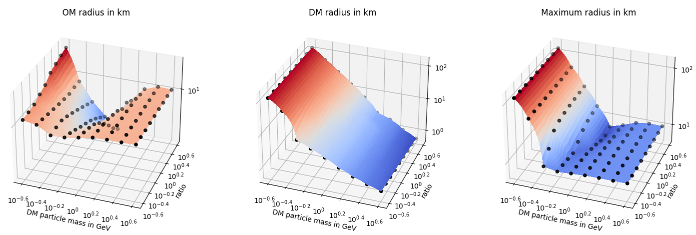

Figure 15: From every MRR in Figure 8, the maximum value is chosen, and the different radii are plotted as a function of the two parameters.

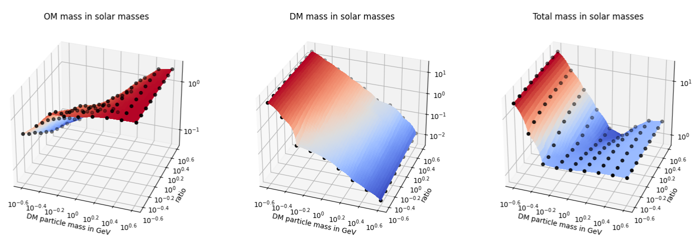

Figure 16: From every MRR in Figure 8, the maximum value is chosen, and the different masses are plotted as a function of the two parameters.

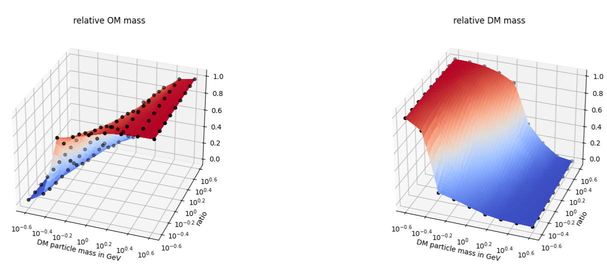

Figure 17: From every MRR in Figure 8, the maximum value is chosen, and the OM mass/total mass and the DM mass/total mass are plotted.

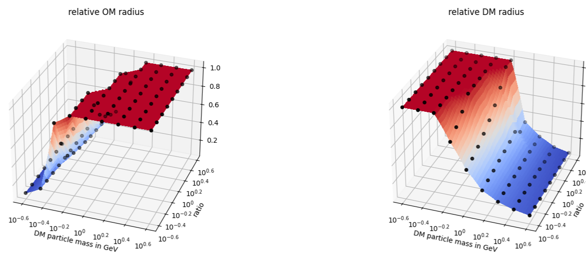

Figure 18: From every MRR in Figure 8, the maximum value is chosen, and the OM radius/maximum radius and the DM radius/maximum radius are plotted.

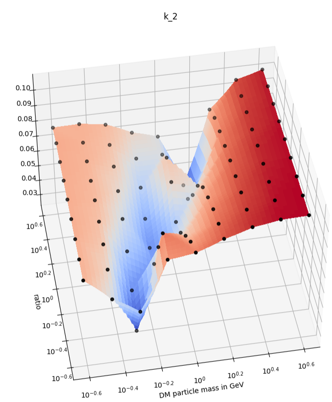

Figure 19: From every k_2 vs C plot in Figure 9, the maximum value is chosen, and the k_2 value is plotted. This is the same plot as in Figure 11 (right panel) but from a different perspective to highlight the line in the parameter space where the valley occurs.

B Python Code

The following is the Runge Kutta 4-th order algorithm with adaptive stepsize, which has been used in this thesis to solve the two fluid TOV equations.

```python
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Mo, Mar 6 15:08:17 2023

```

```python
@author: Kevin Radl
"""

from cmath import log
from shutil import which
import numpy as np
import matplotlib.pyplot as plt
import scipy.constants as const
import math as m
import scipy.interpolate
from scipy.misc import derivative

A = np.array([[0,0,0,0] , [0.5,0,0,0] , [0,0.5,0,0], [0,0,1,0]])
B = np.array([1/6 , 1/3 , 1/3 , 1/6])
C = np.array([0 ,0.5 , 0.5 , 1])
d=len(B)

def RK4_calc(func,cur_y,cur_time,eps,termination_factor): #function to calculate the next Runge Kutta step
    k = np.zeros((len(y0),d))
    for j in range(d):
        sum_1 = np.zeros(len(y0))
        for l in range(d):
            sum_1 += A[j,l] * k[:,l]
        u = cur_time + eps * C[j]
        v = cur_y + eps*sum_1

        k[:,j] = func(cur_time + eps * C[j], cur_y + eps*sum_1)

    sum_2 = np.zeros(len(y0))
    for j in range(d):
        sum_2 += B[j] * k[:,j]
    y_new = cur_y + eps * sum_2
    return y_new


def RK4(func,t0,y0,t_max,eps,A,B,C,delta=1e-10):            # main function for the Runge Kutta method with adaptive stepsize
    print("Starting RK4...")
    print(f"y0 = {y0}")

    termination_factor = 1e-8


```

```python
    # time steps and y-values for those time steps are going to be stored
    # in the following two arrays
    t_array = np.array([t0])
    y_values = np.array([[y0[0]],[y0[1]],[y0[2]],[y0[3]],[y0[4]]])

    i = 0
    OM_terminated = False
    DM_terminated = False


    while not OM_terminated or not DM_terminated:
        cur_time = t_array[i]
        cur_y = y_values[:,i]
        print(cur_time)
        print(cur_y)



        #Termination Condition:
        if cur_y[0] < y0[0] * termination_factor and not OM_terminated:
            print(f"OM terminated at {cur_time}")
            OM_terminated = True
            OM_termination_radius = cur_time

        if cur_y[2] < y0[2] * termination_factor and not DM_terminated:
            print(f"cur_y[2] = {cur_y[2]}")
            print(f"DM terminated at {cur_time}")
            DM_terminated = True
            DM_termination_radius = cur_time


        err = delta +1
        while delta < err:
            dt = eps
            y_1 = RK4_calc(func,cur_y,cur_time,eps/2,termination_factor)

            #Termination Condition:
            if cur_y[0] < y0[0] * termination_factor and not OM_terminated:
                print(f"OM terminated at {cur_time}")
                OM_terminated = True
```

```python
                OM_termination_radius = cur_time
            if cur_y[2] < y0[2] * termination_factor and not DM_terminated:
                print(f"cur_y[2] = {cur_y[2]}")
                print(f"DM terminated at {cur_time}")
                DM_terminated = True
                DM_termination_radius = cur_time

            y_1 = RK4_calc(func,y_1,cur_time+eps/2,eps/2,termination_factor)

            #Termination Condition:
            if cur_y[0] < y0[0] * termination_factor and not OM_terminated:
                print(f"OM terminated at {cur_time}")
                OM_terminated = True
                OM_termination_radius = cur_time
            if cur_y[2] < y0[2] * termination_factor and not DM_terminated:
                print(f"cur_y[2] = {cur_y[2]}")
                print(f"DM terminated at {cur_time}")
                DM_terminated = True
                DM_termination_radius = cur_time

            y_2 = RK4_calc(func,cur_y,cur_time,eps,termination_factor)

            #Termination Condition:
            if cur_y[0] < y0[0] * termination_factor and not OM_terminated:
                print(f"OM terminated at {cur_time}")
                OM_terminated = True
                OM_termination_radius = cur_time
            if cur_y[2] < y0[2] * termination_factor and not DM_terminated:
                print(f"cur_y[2] = {cur_y[2]}")
                print(f"DM terminated at {cur_time}")
                DM_terminated = True
                DM_termination_radius = cur_time

            err = np.linalg.norm(y_2[0:3] - y_1[0:3])/15
            rho = delta/(err)
            eps = min(0.9*eps*rho**(1/5),2*eps)

            if cur_time > 1000:      #just in case...
                return t_array[:(i-1)],y_values[:,:i-1],(0,0)
```

```python

        if OM_terminated:   # If one of the stars is terminated, the pressure is set to zero (vacuum)
            y_1[0] = 0

        if DM_terminated:
            y_1[2]=0


        y_new = y_1
        y_values = np.column_stack((y_values, y_new))
        t_new = t_array[i] + dt
        t_array = np.append(t_array, t_new)
        i += 1
    print(f"Termination Radii: (OM,DM) = {OM_termination_radius} and {DM_termination_radius}")
    return t_array[:(i-1)],y_values[:,:i-1],(OM_termination_radius,DM_termination_radius)


def read_data(filename):            #function to import the equations of state
    infile = open(filename)
    energy_density = []
    pressure = []
    lines = infile.readlines()
    for line in lines:
        line = line.replace('\n', '')
        line = line.replace('\t', ' ')
        line_list = line.split(" ")
        line_list = [value for value in line_list if value != '']
        #print(line_list)
        energy_density.append(float(line_list[0]))
        pressure.append(float(line_list[1]))
    return np.array(energy_density), np.array(pressure)
```

```python

if __name__ == "__main__":
    print('Start')

    #Defining Constants:

    #using SI units:
    G = const.gravitational_constant       #Gravitational constant
    c = const.speed_of_light           #speed of light
    M = 1.9891 * 1e30                    #mass of the sun in kg
    m_p = 2.176434 * 1e-8
    m_f = 1.674927 * 1e-27               #mass of the fermion (neutron)
    l_p = 1.616 *1e-35   #m

    hbarc = 197.3  #MeV fm
    E_N = 1000#939.565   #Mev/c^2
    p_0 = E_N**4 / hbarc**3
    m_0 = m_p**3/m_f**2
    r_0 = l_p*m_p**2/m_f**2



    #-----------------------------------------------------------------------------

    #Import OM EoS and create a function:
    energy_density, pressure = read_data("EoS_new.dat")
    print(energy_density,pressure)

    energy_density_dimless = energy_density / p_0
    pressure_dimless = pressure / p_0
    EoS = scipy.interpolate.interp1d(pressure_dimless,energy_density_dimless, fill_value='extrapolate')


    #-----------------------------------------------------------------------------

    #Import DM EoS and create a function:
    pressure_DM,energy_density_DM = read_data("EoS_G2_new.dat")
    print(energy_density_DM,pressure_DM)


```

```
    EoS_DM = scipy.interpolate.interp1d(pressure_DM,energy_density_DM, fill_value='extrapolate')


    #---------------------------------------------------------------------------


    #implementation of the derivative using interpolation and scipy derivative
    p = np.logspace(-18,3,10000)
    eps_DM = np.zeros(len(p))
    for i,cur_p in enumerate(p):
        eps_DM[i] = EoS_DM(cur_p)

    EoS_DM_2 = scipy.interpolate.interp1d(eps_DM,p,fill_value='extrapolate')

    def Q(r,p_OM,p_DM,m_OM,m_DM):
        c_s_OM = derivative(EoS, p_OM, dx=1e-18)**(-1)
        c_s_DM = derivative(EoS_DM, p_DM, dx=1e-18)**(-1)
        if c_s_OM == 0:
            OM_term = 0
        else:
            OM_term = (EoS(p_OM) + p_OM)/c_s_OM
        if c_s_DM == 0:
            DM_term = 0
        else:
            DM_term = (EoS_DM(p_DM) + p_DM)/c_s_DM
        aa = (4*np.pi*r*(   5*(EoS(p_OM)+EoS_DM(p_DM)) + 9*(p_OM + p_DM) +  OM_term + DM_term - 6/(4*np.pi*r**2)))
        bb = (r-2*m_OM-2*m_DM)
        cc = 4*((m_OM + m_DM+4*np.pi*r**3*(p_OM + p_DM))/(r**2*(1-2*(m_OM+m_DM)/r)))**2
        return aa/bb - cc



    def F(r,p_OM,p_DM,m_OM,m_DM):
        return (r - 4*np.pi*r**3*((EoS_DM(p_DM) + EoS(p_OM))-(p_DM+p_OM)))/(r-2*(m_OM+m_DM))


    #Define differential equation
    #y[1] = m_OM    y[0] = p_OM     y[3] = m_DM     y[2] = p_DM
    def func(t,y):
```

```python
    dvdr = (y[1] + y[3] + 4 * np.pi*t**3*(y[0] + y[2])) / (t*(t-2*(y[1] + y[3])))
    return np.array( [-(y[0] + EoS(y[0]))*dvdr    ,    4 * np.pi * t**2 * EoS(y[0])   ,   -(y[2] + EoS_DM(y[2]))*dvdr   ,   4 * np.pi * t**2 * EoS_DM(y[2])   ,   -1/t*(y[4]**2 +y[4]*F(t,y[0],y[2],y[1],y[3]) + t**2*Q(t,y[0],y[2],y[1],y[3]))])


small_initial_radius = 1e-3
N = 100  # Number of data points per set off parameters.
path = r"/Users/kevin/Desktop/Studium/Bachelorarbeit/Finale Berechnungen/New_EoS_OM_DM_r=1_data/"
filename_start = "final_calculations_testfile"


with open(path+"README"+".txt","w") as f:
    print('initial_pressure_OM dimensionless initial_pressure_DM dimensionless   radius_OM in km radius_DM in km mass_OM in sun masses   mass_DM in sun masses   compactness second_love_num tidal deformability',file = f)





#-------------------------------------------------------------------------------
#creating the List of parameters which should be calculated

#starting parameters: r=1 M=1 GeV
parameters = []      #(r,M) M in GeV
power = 0
ratios = []
for i in range(-power,power+1):
    ratios.append(2**i)

#carteasian product
for i1 in ratios:
    for i2 in ratios:
        parameters.append((i1,i2))

def delete_parameters(parameters,cur_power):        # delete already calculated parameters
    delete_list=[]
```

```python
    for par in parameters:
        if not(par[0]==2**cur_power or par[1]==2**cur_power or par[0]==2**(-cur_power) or par[1]==2**(-cur_power)):
            delete_list.append(par)
    for par in delete_list:
        parameters.remove(par)
    return parameters

#parameters = delete_parameters(parameters,2)
print(ratios)
parameters = [(1,0.5),(1,1),(1,2)]
print(parameters)


#----------------------------------------------------------------------------


for ratio,M_DM in parameters:      # Main loop over the different parameters
    filename = path + filename_start + f"_r_{ratio}" + f"_M_DM_{M_DM}" +".txt"

    hbarc = 197.3  #MeV fm
    E_N = 1000 * M_DM#939.565   #Mev/c^2
    m_f = E_N / (5.609586167219 * 10**29)
    p_0 = E_N**4 / hbarc**3
    m_0 = m_p**3/m_f**2
    r_0 = l_p*m_p**2/m_f**2

    energy_density_dimless = energy_density / p_0
    pressure_dimless = pressure / p_0
    EoS = scipy.interpolate.interp1d(pressure_dimless, energy_density_dimless,fill_value='extrapolate')


    for i,initial_pressure_OM in enumerate(np.logspace(-7,-1,N)):       #loop over different central pressures and calculating all relevant quantities

        initial_pressure_DM = initial_pressure_OM * ratio
        initial_mass_OM = 4*np.pi/3 * small_initial_radius**3 * EoS(initial_pressure_OM)
```

```python
            initial_mass_DM = 4*np.pi/3 * small_initial_radius**3 * EoS_DM(initial_pressure_DM)

            y0 = np.array([initial_pressure_OM,initial_mass_OM, initial_pressure_DM,initial_mass_DM,2])

            r,y,r_termination = RK4(func,small_initial_radius,y0, small_initial_radius*1e6,small_initial_radius*5,A,B,C)
                            #y[1] = m_OM     y[0] = p_OM      y[3] = m_DM      y[2] = p_DM     y[4]=y_tidaldef

            y_tidaldef=y[4,-1]
            C_tidaldef = (y[1,-1] + y[3,-1]) / max(r_termination[0],r_termination[1])
            second_love_number = 8*C_tidaldef**5 / 5 * (1-2*C_tidaldef)**2 * (2+2*C_tidaldef*(y_tidaldef-1)-y_tidaldef)* (2*C_tidaldef*(6-3*y_tidaldef+3*C_tidaldef*(5*y_tidaldef-8)) + 4*C_tidaldef**3*(13-11*y_tidaldef + C_tidaldef*(3*y_tidaldef-2) + 2*C_tidaldef**2*(1+y_tidaldef)) + 3*(1-2*C_tidaldef)**2 * (2-y_tidaldef+2*C_tidaldef*(y_tidaldef-1))*np.log(1-2*C_tidaldef))**(-1)
            gamma_tidaldef = 2/3 * (y[1,-1] + y[3,-1])**(-5) * max(r_termination[0],r_termination[1])**(5)  * second_love_number #*(G**(-5)* c**(10))

            print(f"OM-Mass = {y[1,-1]}, DM-Mass = {y[3,-1]} and Radius = {r_termination}")
            print(f"OM-Mass in sun masses = {y[1,-1]*m_0 / M}, DM-Mass in sun masses= {y[3,-1]*m_0 / M} , OM-Radius in km = {r_termination[0] * r_0 /1000} and DM-Radius in km = {r_termination[1] * r_0 /1000}")

            with open(filename,"a") as f:
                print(f"{initial_pressure_OM} {initial_pressure_DM}   {r_termination[0]* r_0 /1000}   {r_termination[1]* r_0 /1000}   {y[1,-1]*m_0 / M}   {y[3,-1]*m_0 / M}   {C_tidaldef}    {second_love_number}   {gamma_tidaldef}",file=f)

```

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[3] Norman K. Glendenning. *Compact Stars*. Springer, 2000.

[4] Ouraman Hajizadeh **and** Axel Maas. "Constructing a neutron star from the lattice in G2-QCD". **in***The European Physical Journal A*: 53.10 (**october** 2017). DOI: `10.1140/epja/i2017-12398-x`. URL: `https://doi.org/10.1140%2Fepja%2Fi2017-12398-x`.

[5] Aleksi Kurkela **andothers**. "CONSTRAINING NEUTRON STAR MATTER WITH QUANTUM CHROMODYNAMICS". **in***The Astrophysical Journal*: 789.2 (**june** 2014), **page** 127. DOI: `10.1088/0004-637x/789/2/127`. URL: `https://doi.org/10.1088%2F0004-637x%2F789%2F2%2F127`.

[6] Irina Sagert **andothers**. "Compact stars for undergraduates". **in***European Journal of Physics*: 27.3 (**april** 2006), **pages** 577–610. ISSN: 1361-6404. DOI: `10.1088/0143-0807/27/3/012`. URL: `http://dx.doi.org/10.1088/0143-0807/27/3/012`.

[7] Saul A. Teukolsky Stuart L. Shapiro. *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects*. John Wiley Sons, 1983.