Limit Orders and Knightian Uncertainty

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Motivation and Result

not all uncertainty in the world is objective

one way to deal with such uncertainty is subjective expected utility (SEU)

Ellsberg experiments provide examples of behavior that cannot be explained by SEU

many alternative preference models have since been developed,

mostly motivated by Ellsberg experiments,

with finance (so far) the most successful area of application

↪ this paper: ambiguity aversion cannot lead to different behavior than SEU in finance when people have access to limit orders

Examples of Implicitly considered Ambiguity Aversion Models

this is true for all models of ambiguity aversion that

- evaluate risk (objective uncertainty) with expected utility
- satisfy a monotonicity axiom

these include most models:

Choquet expected utility - Schmeidler (1989, ECMA)
maxmin expected utility - Gilboa and Schmeidler (1989, JME)
variational and multiplier preferences - Hansen and Sargent (2001, AER)
and Maccheroni, Marinacci, and Rustichini (2006, ECMA)
incomplete preference model of Bewley - (2002, DEF)
smooth ambiguity model - Klibanoff, Marinacci, and Mukerji (2005, ECMA)
confidence function preferences - Chateauneuf and Faro (2009, JME)
uncertainty aversion preferences - Cerreia-Vioglio, Maccheroni, Marinacci, and Montrucchio (2011, JET)
and many others

Ambiguity Aversion in Finance

ambiguity aversion can help to explain quantitative empirical puzzles in finance:

- home-bias investment behavior; e.g., Epstein and Miao (2003, JEDC) and Uppal and Wang (2003, JFin)
- ambiguity premium to explain equity premium puzzle; e.g., Klibanoff, Marinacci, and Mukerji (2005, ECMA), Ju and Miao (2012, ECMA) and Collard, Mukerji, Sheppard, and Tallon (2018, QE), Brenner and Izhakian (2018, JFinE), Izhakian (2020, JET)
- excess volatility puzzle; using e.g., the model of Epstein and Wang (1994, ECMA), Illeditsch (2011, JFin)

the driving force behind this: ambiguity averse investors tend to take more conservative positions than subjective expected utility investors

as first identified by Dow and Werlang (1992, ECMA)

$\hookrightarrow$ this paper: ambiguity aversion cannot lead to different behavior than SEU in finance when people have access to limit orders

Ellsberg '61

urn A: 100 balls, an uncertain number of which is • and the rest •

urn B: 49 of 100 balls are •

Choose

bet on red •: you win (€1000) if a red ball is drawn from urn A

bet on blue •: you win (€1000) if a blue ball is drawn from urn A

bet on gray •: you win (€1000) if a gray ball is drawn from urn B

Analysis experiment

There are also two states: the drawn ball from urn A is either ● or ●

Probabilities of winning

state / bet	●	●	●
●	1	0	$\frac{49}{100}$
●	0	1	$\frac{49}{100}$

there is no (pure strategy) dominance relationship

but, I could throw a fair coin and after heads choose ● and after tails choose ●

in both states the overall probability of winning of this "mixed strategy" is $\frac{1}{2}$

the "mixed strategy" strictly dominates strategy ● (Raiffa, 1961)

this is a general argument (Kuzmics, 2017, GEB)

goes back to Pearce's (1984) Lemma, back to Wald's complete class theorem (1947, 1950)

Pearce and Wald

Pearce (1984): a strategy in a two-player game is undominated (in the space of mixed strategies) if and only if it is a best response to a belief about the opponent's strategy

translated to decision theory (player 1 = decision maker; player 2 = nature):

close to Wald (1947): an act is undominated (in the space of mixed acts) if and only if it is Bayesian (optimal given some belief about the state) under the same Bernoulli utility function

implication: an act that is optimal for an ambiguity averse DM with monotone preferences but not optimal for a SEU DM

- must be dominated by a mixed act
- must be undominated in the space of pure acts

$\hookrightarrow$ this paper: when trading can be done through limit orders there are no such acts

A simple investment problem

there is one asset

that can have a value of $+1$ or -1 tomorrow

you cannot attach probabilities to these two states

you can buy or sell this asset at price $p \in (-1, 1)$ or do nothing

Subjective expected utility

suppose you have a belief about the likelihood of the two states

say μ is the probability of the state being $+1$

suppose you are risk neutral

then you should buy if

$$\mu(+1) + (1 - \mu)(-1) > p$$

and sell if

$$\mu(+1) + (1 - \mu)(-1) < p$$

for "almost all" prices you should either buy or sell, never do nothing

Dow Werlang (1992)

what if you have Gilboa und Schmeidler (1989) maxmin expected utility?

then there is a range of prices for which you would prefer to do nothing

this is a possible explanation for why people do not participate in the stock market

Limit Orders

suppose you have to submit a limit order before the price is known

is your demand function also the limit order you would like to submit?

to answer that we need to model your view of prices

suppose, for the motivating example, that you know the distribution (or have a firm belief) over possible prices when you submit your limit order

Objective distribution of prices - example

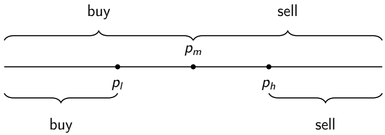
suppose the DM "knows" that the price p follows some atomless full support distribution F on $[-1, 1]$

a (generalized) limit order is a measurable function from the set of possible prices $[-1, 1]$ to the action set $\{+1, -1, 0\}$ (for buy, sell, do nothing)

the payoff function:

$$v(p, a, x) = a(x - p)$$

Objective distribution of prices - example



additional payoff of threshold limit order over Dow Werlang limit order is

$$\frac{1}{2}\left(F(p_h) - F(p_l)\right)\left(\mathbb{E}\left[p \mid p > p_m\right] - \mathbb{E}\left[p \mid p < p_m\right]\right)$$

in both states $x \in \{-1, +1\}$ [where p_m is the conditional median price]

doing nothing is dominated by "buy cheap, sell dear"

dominance is in terms of pure strategies

recall: almost all models of ambiguity aversion satisfy a monotonicity (dominance) axiom

Relaxed Assumptions in General Model

prices may be informative

DM may have fundamental uncertainty about prices and final values

DM may have (almost) any continuous and increasing Bernoulli utility function

DM can buy (sell) any amount in an interval (that includes 0)

Uncertainty Model - State Space

the state space is a compact metrizable space Y of probabilistic models

for each model $y \in Y$ there is a joint density over prices and final values, $h : \mathbb{R} \times \mathbb{R} \times Y \to \mathbb{R}$ continuous in Y such that

$$\int \int h(p, x, y) \; \mathrm{d}p \; \mathrm{d}x = 1 \text{ for all } y \in Y.$$

Uncertainty Model - Actions and Bernoulli Utility

DM invests an amount in $[b, t]$ with $b < 0 < t$.

payoffs from net-gains from investing are given by an increasing and continuous $u : \mathbb{R} \to \mathbb{R}$

to guarantee that expected utilities are defined and finite, we assume that there is an integrable function $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ satisfying

$$|u(tx - tp)|h(p, x, y) + |u(bx - bp)|h(p, x, y) \leq d(p, x) \text{ for all } (p, x) \in \mathbb{R}^2 \text{ and all } y \in Y.$$

restriction most severe for risk-neutral DM (among all risk-averse DMs)

even then tails can be quite heavy; includes for instance bivariate t-distributions of any degrees of freedom greater than $2 + \epsilon$

Limit Orders

a limit order is a measurable function from $\mathbb{R}$ into the set $A = [b, t]$ [equivalence classes]

the set of limit orders L endowed with the topology of convergence in measure and the corresponding Borel σ-algebra

L is embedded in the space of mixed limit orders $\Delta(L)$ (via point masses)

Payoffs for Limit Orders and Dominance Notions

payoff function $v : \mathbb{R} \times A \times \mathbb{R} \to \mathbb{R}$ given by $v(p, a, x) = u\big(a(x - p)\big)$

the expected payoff from $\mu \in \Delta(L)$ if the true model is y is

$$V(\mu, y) = \int\int\int v\big(p, l(p), x\big) h(x, p, y)\, \mathrm{d}p\, \mathrm{d}x\, \mathrm{d}\mu(l).$$

$\mu' \in \Delta(L)$ strictly dominates $\mu \in \Delta(L)$ if $V(\mu', y) > V(\mu, y)$ for all $y \in Y$

a (mixed) limit order that is not strictly dominated by another mixed limit order is undominated

a (mixed) limit order that is not strictly dominated by a nonrandomized limit order is deterministically undominated

Results - Dominance

a version of the Pearce/Wald result

A limit order λ is undominated if and only if there exists a probability distribution $\beta \in \Delta(Y)$ such that λ maximizes $\int V(\cdot, y)\, \mathrm{d}\beta(y)$.

proof uses Battigalli, Cerreia-Vioglio, Maccheroni, and Marinacci (2016, ECMA)

key result

A limit order λ is undominated if and only if it is deterministically undominated.

Main Result

main result

Any optimal choice of a DM with monotone (not necessarily complete or transitive) preferences that is compatible with the Bernoulli utility function u is Bayesian (a choice that an SEU DM with same u could also have chosen).

Additional Results

For those AA models that can be represented with sets of beliefs, there is a rationalizing belief (of the SEU DM) in the closed convex hull of the set of beliefs.

The access to limit orders is sufficient for the result.

Undominated limit orders may be complicated functions, but can be implemented through a combination of stop-loss limit orders.

Existence of undominated limit orders is guaranteed.

Market participation: an AA or SEU DM can abstain from participating in the stock market for some prices only if there is a belief under which at these prices the price equals the expected final value of the asset.

Take away

One

A non-SEU person's demand function and her limit order do not necessarily coincide.

Two

When limit orders are available ambiguity aversion cannot explain more behavior than SEU can.