Trade Equilibrium Amongst Growing Economies

Some Extensions

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1. Introduction

In the 1970s Ian Steedman published several papers, some together with Stan Metcalfe, on the implications of Piero Sraffa's findings in his 1960 book (Sraffa, 1960) for the Heckscher-Ohlin-Samuelson theory of international trade (see Steedman, 1979a). This first and foremost critical activity had as by-products also some positive contributions to the problem under consideration which were then arranged in a neat and clear-cut essay titled *Trade Amongst Growing Economies* (see Steedman, 1979b). The book has ten chapters: the first eight of them are devoted to the small open economy, whereas the last two chapters deal with international trade equilibrium. The analysis is laid out in a simple format. The work horse employed to good effect is a model with only two capital goods and one consumption good. It is admirable how many interesting aspects of the problem at hand Steedman succeeds to highlight within this simple framework. The presence of two capital goods is, of course, crucial in order to put into sharp relief the difference between the analysis elaborated and the conventional neoclassical one which assumes that there is only a single capital good. As Steedman's

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contributions have made clear beyond doubt, neoclassical trade theory faces insurmountable difficulties when there is more than one capital good.[1]

In the last two chapters Steedman (1979b) provided an analysis of international trade equilibrium; he adopted a procedure that was very popular at the time and was inspired by Sraffa (1960). It consisted of first analysing the properties of a single technique, and of introducing the problem of a choice of technique only subsequently. While this two-step procedure is appropriate in many cases, in some it is not. One such case is the problem of land and of the rent of land, another one is the problem of international trade equilibrium. In these cases (as well as in a few others) the two-step procedure may render obscure the role of quantities in determining prices. In the case of extensive rent it is well known that marginal land depends on the amounts available of the different qualities of land and on the quantities of the various agricultural commodities that are to be produced. The presentation in terms of two steps, one devoted to the analysis of a single technique and one to the problem of the choice of technique, may somewhat remove from sight the role of quantities. Similarly in the case of international equilibrium in which the 'sizes' of countries matter. For instance, with only two countries, one being very large compared with the other one, the large country must obviously produce all commodities. As a consequence, the international prices are determined by the technology of the large country alone, whereas the small country will adopt an extreme form of specialisation by producing only a single commodity (except in freak cases in which it may produce more than one commodity) and will obtain all the other commodities via trade. In a less extreme case as to the relative size of the two countries, the large country produces all commodities except one, whereas the small country produces two commodities. (In any case there will be a commodity produced by both countries in order to satisfy the requirements for use.) And so on.

In this paper we construct a general model in order to analyse international trade equilibrium. The main difference with regard to the models in Steedman (1979b) is that we use inequalities instead of equalities. This amounts to determining international prices jointly with the processes that are operated in the two countries. It also allows us to extend the analysis beyond the case of single production and deal with joint production and fixed capital. As

[1] In later contributions Steedman showed that the HOS model is difficult to sustain even with zero profits (or interest) and even with only a single produced input, but more than one primary input. Within the conventional long-period setting of the HOS doctrine difficulties arise as soon as there are at least three inputs, not all of them being primary ones.

Steedman stressed in several of his contributions, there is reason to presume that these cases are empirically very important and therefore ought to be taken into account in economic analysis.

2. All commodities are tradeable

There are n commodities and s countries. Each country has a specific technology defined by the triplet $(\boldsymbol{A}_j, \boldsymbol{B}_j, \boldsymbol{l}_j)$ where $j = 1,2,...,s$, $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ are $m_j \times n$ matrices and $\boldsymbol{l}_j$ is a $m_j \times 1$ vector; $\boldsymbol{A}_j$ is the material input matrix, $\boldsymbol{l}_j$ is the labour input vector, and $\boldsymbol{B}_j$ is the output matrix. Following Steedman we assume that each country has its own uniform rate of profit r_j, whereas the growth rate g is common to all countries:

> While all commodities will be assumed to be internationally mobile ... money capital will be assumed not to flow [among] countries, so that no tendency toward an international uniformity of profit rates will be assumed. (Such an assumption is not, of course entirely realistic under the condition of modern capitalism but it is justified here on the grounds that it would not be sensible to relax it without entering into a full analysis of the inter-relationships between international trade and international investment, which is not possible in the present work.) (Steedman, 1979b, p. 109)
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> [T]he ... analysis of growing, trading economies will, unfortunately, have to be carried out under [the] assumption [that the constant rate of steady growth is] uniform not only for those output and labour quantities relating to a given country but also as [among] countries. (If the ... countries were to grow at different rates then, with complete or partial specialization by [such] countries, international prices would change through time, so that capitalists' expectations would have to be analysed explicitly – doubtless under assumptions no less implausible than the assumption of equal, steady growth rates.) (Ibid., p. 110)

Immediately after the first passage quoted Steedman adds that 'Labour will also be assumed not to move [among] countries.' We adopt the same assumption. In addition we set aside all monetary aspects of an international economy in terms of some bold assumption. Since the purchasing power of the wage rates received by workers in different countries (in terms of the local currency) depend crucially on the exchange rates, which, in their turn, depend on monetary variables, the most simple assumption consists in assuming that the exchange rates are such that the wage rate in country j equals $\alpha_j = w_j / e_j$, where w_j is the money wage rate in country j and e_j is the exchange rate of country j with respect to that currency that serves as the numéraire (see below). All α_j's are considered as given and determined by the monetary system.

It goes without saying that we adopt also the usual assumptions made in pure trade theory, such as the absence of transport costs.

For each country *j* the nonnegative vector of prices (in terms of/divided by the exchange rate) $\boldsymbol{p}_j \geqq \boldsymbol{0}$ must be such that no process pays extraprofits and therefore:

$$\left[\boldsymbol{B}_j - (1+r_j)\boldsymbol{A}_j\right]\boldsymbol{p}_j \leqq \alpha_j \boldsymbol{l}_j$$

In each country there is an effectual demand $\boldsymbol{d}_j \geq \boldsymbol{0}$ which needs to be satisfied. As a consequence, the nonnegative process intensity vector $\boldsymbol{x}_j \geqq \boldsymbol{0}$, the nonnegative import vector $\boldsymbol{z}_j \geqq \boldsymbol{0}$ and the nonnegative export vector $\boldsymbol{y}_j \geqq \boldsymbol{0}$ must satisfy the inequality

$$\boldsymbol{x}_j^T\left[\boldsymbol{B}_j - (1+g)\boldsymbol{A}_j\right] + \boldsymbol{z}_j^T \geqq \boldsymbol{d}_j^T + \boldsymbol{y}_j^T$$

where g is the growth rate, $\boldsymbol{x}_j$ is a $m_j \times 1$ vector and $\boldsymbol{d}_j$, $\boldsymbol{x}_j$, and $\boldsymbol{y}_j$ are $n \times 1$ vectors. Obviously, if a process is not able to obtain the uniform (country-specific) rate of profit, it will not be operated, and if a commodity is produced in excess supply its price will be zero. That is,

$$\boldsymbol{x}_j^T\left[\boldsymbol{B}_j - (1+r_j)\boldsymbol{A}_j\right]\boldsymbol{p}_j = \alpha_j \boldsymbol{x}_j^T \boldsymbol{l}_j$$

$$\boldsymbol{x}_j^T\left[\boldsymbol{B}_j - (1+g)\boldsymbol{A}_j\right]\boldsymbol{p}_j + \boldsymbol{z}_j^T\boldsymbol{p}_j = \boldsymbol{d}_j^T\boldsymbol{p}_j + \boldsymbol{y}_j^T\boldsymbol{p}_j$$

Let $\boldsymbol{p} \geqq \boldsymbol{0}$ be the international price vector in terms of the currency that serves as the monetary numéraire. (This means that the exchange rate of that currency equals unity; we will in addition consider the labour supplied by workers in that country as numeraire, so that also the wage rate in that country equals unity). In an open economy in which all commodities are tradeable, domestic prices equal international prices. We express this in the following way:

$$\begin{gathered}\boldsymbol{p}_j \leqq \boldsymbol{p}\\ \boldsymbol{z}_j^T\boldsymbol{p}_j = \boldsymbol{z}_j^T\boldsymbol{p}\\ \boldsymbol{p}_j \geqq \boldsymbol{p}\\ \boldsymbol{y}_j^T\boldsymbol{p}_j = \boldsymbol{y}_j^T\boldsymbol{p}\end{gathered}$$

The meaning of the equations and inequalities is close at hand: domestic prices cannot exceed international prices, and commodities whose domestic prices are lower than international price cannot be imported; international prices cannot exceed domestic prices and commodities whose domestic prices are larger than international prices cannot be exported.

Finally, international trade equilibrium requires

$$\begin{gathered}\sum \boldsymbol{y}_j \geqq \sum \boldsymbol{z}_j \\ \sum \boldsymbol{y}_j^T \boldsymbol{p} = \sum \boldsymbol{z}_j^T \boldsymbol{p}\end{gathered}$$

That is, the sum total of the imported amounts of the various commodities cannot exceed the sum total of the exported amounts of them, and the price of a product that is exported in excess equals zero. (Obviously, only 'free goods' can be exported in excess).

It is easily checked that the system of inequalities and equations considered has a solution. Let us show this for the case in which $s = 2$. The reader can easily check that this assumption only simplifies the notation.

Let

$$\boldsymbol{M}(r_1,r_2)=\begin{bmatrix}\boldsymbol{B}_1-(1+r_1)\boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_2-(1+r_2)\boldsymbol{A}_2 & \boldsymbol{0}\\ \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{I}\\ \boldsymbol{0} & \boldsymbol{I} & -\boldsymbol{I}\\ -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{I}\\ \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{I}\end{bmatrix},\ \boldsymbol{q}=\begin{bmatrix}\boldsymbol{p}_1\\ \boldsymbol{p}_2\\ \boldsymbol{p}\end{bmatrix},\ \boldsymbol{l}=\begin{bmatrix}\alpha_1\boldsymbol{l}_1\\ \alpha_2\boldsymbol{l}_2\\ \boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix},\boldsymbol{d}=\begin{bmatrix}\boldsymbol{d}_1\\ \boldsymbol{d}_2\\ \boldsymbol{d}\end{bmatrix},\ \boldsymbol{x}=\begin{bmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\\ \boldsymbol{z}_1\\ \boldsymbol{z}_2\\ \boldsymbol{y}_1\\ \boldsymbol{y}_2\end{bmatrix}.$$

Then the above equations and inequalities can be stated as

$$\begin{gathered}\boldsymbol{M}(r_1,r_2)\boldsymbol{q} \leqq \boldsymbol{l}\\ \boldsymbol{x}^T\boldsymbol{M}(r_1,r_2)\boldsymbol{q}=\boldsymbol{x}^T\boldsymbol{l}\\ \boldsymbol{x}^T\boldsymbol{M}(g,g)\geqq \boldsymbol{d}^T\\ \boldsymbol{x}^T\boldsymbol{M}(g,g)\boldsymbol{q}=\boldsymbol{d}^T\boldsymbol{q}\\ \boldsymbol{q}\geqq\boldsymbol{0},\quad \boldsymbol{x}\geqq\boldsymbol{0}\end{gathered}$$

Systems of this type are well known and can easily be solved using linear complementarity when $r_j \geq g$ and there is a nonnegative vector $\boldsymbol{v}$ such that $\boldsymbol{v}^T\boldsymbol{M}(r_1,r_2) > \boldsymbol{0}^T$, which is certainly the case if there are vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ such that $\boldsymbol{w}_j^T\left[\boldsymbol{B}_j-\left(1+r_j\right)\boldsymbol{A}_j\right]>\boldsymbol{0}^T$. These two conditions are readily interpreted in terms of the viability of the technologies of the two countries.[2] This means that if the autarkic technologies are viable, then there is an international equilibrium. However, it can also be shown that even if one or, in the extreme, even both autarkic technologies are not viable, but the world economy is viable, then an international equilibrium exists. This is an aspect of the gains from trade: the two countries could not survive in autarky, but they can if they engage in trade and (partial) specialisation.

Let $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ be nonnegative vectors such that

$$\boldsymbol{h}_1^T\left[\boldsymbol{B}_1-\left(1+r_1\right)\boldsymbol{A}_1\right]+\boldsymbol{h}_2^T\left[\boldsymbol{B}_2-\left(1+r_2\right)\boldsymbol{A}_2\right]>\boldsymbol{0}^T \tag{1}$$

Then we say that the world economy is viable at rates of profits r_1 and r_2. Let $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ be nonnegative vectors such that

$$\begin{aligned}\boldsymbol{h}_1^T\left[\boldsymbol{B}_1-\left(1+r_1\right)\boldsymbol{A}_1\right]&=\boldsymbol{u}_1^T-\boldsymbol{v}_1^T\\ \boldsymbol{h}_2^T\left[\boldsymbol{B}_2-\left(1+r_2\right)\boldsymbol{A}_2\right]&=\boldsymbol{u}_2^T-\boldsymbol{v}_2^T\end{aligned}$$

Obviously, there are two positive real numbers ε_1 and ε_2 such that

$$\boldsymbol{u}_1-\boldsymbol{v}_1+\boldsymbol{u}_2-\boldsymbol{v}_2-\left(\varepsilon_1+\varepsilon_2\right)\boldsymbol{e}>\boldsymbol{0},$$

where $\boldsymbol{e}$ is the sum vector. Hence

$$\begin{bmatrix}\boldsymbol{h}_1\\ \boldsymbol{h}_2\\ \boldsymbol{u}_1-\varepsilon_1\boldsymbol{e}\\ \boldsymbol{u}_2-\varepsilon_2\boldsymbol{e}\\ \boldsymbol{v}_1\\ \boldsymbol{v}_2\end{bmatrix}^T\begin{bmatrix}\boldsymbol{B}_1-\left(1+r_1\right)\boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{B}_2-\left(1+r_2\right)\boldsymbol{A}_2 & \boldsymbol{0}\\ \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{I}\\ \boldsymbol{0} & \boldsymbol{I} & -\boldsymbol{I}\\ -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{I}\\ \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{I}\end{bmatrix}>\boldsymbol{0}^T.$$

This proves also that a world of open economies exhibits higher levels of the maximum rates of profit compared with a world with closed economies.

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2 See Kurz and Salvadori (1995, pp. 231-4, 244-5, 245-6 (exercise 8.2)).

3. Only some commodities are tradeable

In this section we introduce the important distinction between tradeable and nontradeable commodities. We assume that the first n_t commodities are tradeables, whereas the other ones $n_{nt} = n - n_t$ are not. How must the model be adapted in order to accomodate this fact? Obviously, the vectors $\boldsymbol{p}$, $\boldsymbol{x}_j$, and $\boldsymbol{y}_j$ are now of dimension $n_t \times 1$. Let us introduce matrix $\hat{\boldsymbol{I}} = [\boldsymbol{I} \quad \boldsymbol{0}]$, where $\boldsymbol{I}$ is the n_t identity matrix and $\boldsymbol{0}$ is the $n_t \times n_{nt}$ zero matrix. It is immediately recognized that the problem can be stated as above by considering instead of matrix $\boldsymbol{M}(r_1, r_2)$ the matrix

$$\hat{\boldsymbol{M}}(r_1, r_2) = \begin{bmatrix} \boldsymbol{B}_1 - (1+r_1)\boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_2 - (1+r_2)\boldsymbol{A}_2 & \boldsymbol{0} \\ \hat{\boldsymbol{I}} & \boldsymbol{0} & -\boldsymbol{I} \\ \boldsymbol{0} & \hat{\boldsymbol{I}} & -\boldsymbol{I} \\ -\hat{\boldsymbol{I}} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & -\hat{\boldsymbol{I}} & \boldsymbol{I} \end{bmatrix}.$$

However, in this case the existence of nonnegative vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ such that inequality (1) is satisfied is not sufficient to ensure the existence of an international equilibrium.

4. Conclusion

In this paper we generalised the discussion of international trade equilibrium in Steedman (1979b) in order to cover many countries and many products, allowing for joint production. The main difference with Steedman's analysis is that we use inequalities instead of equalities and thus deal with the problem of the choice of technique right from the beginning. It is shown that under certain conditions an equilibrium exists and what its properties are. An aspect of the gains from trade is that an international equilibrium may exist even though the technologies available to the various countries in a hypothetical state of autarky are not viable. Trade and specialisation typically involve larger maximum rates of profits of the countries involved and thus corroborate Ricardo's view that foreign trade and improved machinery may have similar effects: 'If … by the extension of foreign trade, or by

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improvements in machinery, the food and necessaries of the labourer can be brought to the market at a reduced price, profits will rise.' (Ricardo, *Works*, Vol. I, p. 132)

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