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Input–Output Analysis from a Wider Perspective: a Comparison of the Early Works of Leontief and Sraffa

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ABSTRACT *This paper compares Leontief's 1928 PhD thesis and Sraffa's work in Cambridge in 1927–1928 as reconstructed from Sraffa's unpublished papers. Both authors showed that relative prices and the interest rate can be determined exclusively in terms of the observable amounts of commodities produced and used up during a year – without any reference to demand and supply. While Sraffa continued to elaborate a comprehensive objectivist, surplus-based alternative to the marginalist theory, Leontief's interest shifted towards applying the new tool of input-output to practical problems. However, his assumption of given value added coefficients in his price equations cannot be sustained.*

KEY WORDS: Circular flow, classical theory, input-output analysis, Wassily Leontief, Piero Sraffa

1. Introduction

'Input–output analysis is a practical extension of the classical theory of general interdependence which views the whole economy of a region, a country and even of the entire world as a single system and sets out to describe and to interpret its operation in terms of directly observable basic structural relationships' (Leontief, 1987, p. 860).

According to this statement, input–output analysis is based exclusively on magnitudes that are directly observable and that can be measured, using the ordinary instruments for measurement in economics. This objectivist concern is already present in Leontief's Berlin PhD thesis, his 1928 paper 'Die Wirtschaft als Kreislauf' (Leontief, 1928, an abridged English translation of which was published as 'The economy as a circular

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flow', Leontief, 1991*),[1] and permeates his entire work. In the 1928 essay, the objectivist approach to economic phenomena is counterposed with the then dominant Marshallian analysis and its stress on subjective factors. Marshall, it should be recalled, had tried to patch over what was a major breach with the objectivist tradition of the English classical economists, especially David Ricardo and Robert Torrens. He did this in terms of completely re-defining the received concept of 'real cost'. While originally this was meant to capture the materials (means of production and means of sustenance in the support of workers) used up in the production of a commodity, or the amount of labour actually bestowed upon it, Marshall used it to refer to 'the exertions of all the different kinds of labour that are directly and indirectly involved in making it; together with the abstinences or rather the waitings required for saving the capital used in making it' (Marshall, [1890] 1970, p. 282). Apparently, Leontief was not impressed by Marshall's re-definition and he was also not convinced by Marshall's insistence that prices could be determined only with reference to the forces of 'demand' and 'supply', conceived of as functional relationships between the price of a commodity and the amount demanded or supplied, as is famously exemplified by Marshall's analogy with the two blades of a pair of scissors. Indeed, one of the main messages of Leontief's 1928 paper was that relative prices can be determined exclusively in terms of the observable amounts of commodities that are respectively produced and used up during a year – without any reference to demand and supply.

This was an important finding of Leontief's maiden paper. Unfortunately, he did not pursue much further the line of thought upon which it was based. Concerned with applying the new tool of input–output to practical problems made him put on one side, and eventually lose sight of, certain properties of the economic system. This applied first and foremost to the problem of value and distribution and the role the physico-economic scheme of production played with regard to it. While at the centre of interest in his 1928 essay, this problem disappeared from the scene, or rather was eventually replaced by *given* 'value added' coefficients in Leontief's price equations (see Leontief, 1941). The difficulty with this approach is that the magnitudes of value added per unit of output in the different industries cannot generally be determined prior to, and independently of, the system of prices. In this conceptualization the constraint binding changes in the distributive variables shaped by the system of production in use, and the dependence of relative prices on income distribution – facts stressed by Leontief in his 1928 paper – are removed from the scene.

Value and distribution were already set aside as part and parcel of the analytical problem at hand in Leontief's seminal 1936 paper on 'Quantitative input–output relations in the economic system of the United States' (Leontief, 1936*). Leontief introduced his study in the following terms: 'The statistical study presented in the following pages may be best defined as an attempt to construct, on the basis of the available statistical materials, a *Tableau Economique* of the United States for the year 1919' (Leontief, 1936*, p. 105).

Giving his study the name of *Tableau Economique* is indeed appropriate. As is well known, François Quesnay's original Tableau contains a summary account of national production, distribution and consumption during a given year, in the mid 18th century, for France. Most importantly for our purpose, it takes the distribution of the surplus product, or *produit net*, and the corresponding prices to be given and known. However, it is clear that the given distribution of the net product exclusively in terms of rents to landlords[2] and the given prices must satisfy the constraints imposed by the given system of

production, which distinguishes only between two production sectors of the economy, primary production (agriculture, mining, forestry etc), or *la classe productive*, and manufacturing, or *la classe stérile*. In his 1936 paper, Leontief follows Quesnay closely in that he also takes distribution and prices to be given and reflected in the available national accounting system. He is actually forced to do so, because there is no statistical description of the production process of the economy during a year in purely material terms. The available 'description of the flow of commodities and services as it enters the given enterprise (or household) through one end and leaves it by the other' (Leontief, 1936*, p. 106) is typically in money terms. In this perspective, the wider theoretical concerns of Leontief's 1928 paper play no role any more. The concern with applying the tool of input–output to practical problems in the way with which we are familiar is effected at the cost of narrowing its analytical scope.

Clearly, the analytical potentialities and practical usefulness of an approach that starts from a description of the production process of the economy as a whole in material terms – a 'circular flow' – go beyond conventional input–output analysis. To see this we may start from Paul Samuelson's (1991*) commentary on the abridged English version of Leontief's 1928 paper, because in it Samuelson places Leontief's contribution in a wider theoretical context. Samuelson observed that a historian of science 'would expect that somewhere a Leontief and a Sraffa would be independently discovering at about the same time the theory of input–output.' In a footnote he added that simultaneously John von Neumann perfected his model of a two-person zero-sum game which led him towards his famous model of balanced economic growth (von Neumann, 1945 [1937]). Samuelson commented: 'good things in science often come in three [sic].' He continued with regard to Leontief's essay: 'A new embryo contains the future organism, but not even the most discerning eye can see in the constellation of early cells the beautiful baby that is to come.' He went on:

> In embryo, when all mammals begin superficially alike, the semblance is much greater. Therefore, the Sraffian paradigms that remained forever innocent of empirical investigation started out more closely resembling the 1928 Leontief formulations than did the ultimate 1960 classic, *The Production of Commodities by Commodities.* Oddly, the 1928 non-mathematical Italian began more with algebra and formal mathematics than did the *wunderkind* of mathematical economics. (Samuelson, 1991*, pp. 177–178)[3,4]

Samuelson concluded: 'The present article [Leontief's 1928 paper] contains no matrix, much less a determinant. It is, so to speak, primarily taxonomic and topological. The pioneer is carving our [sic] a new language, prior to composing a scientific poem in that language.' (Samuelson, 1991*, p. 178)

We agree with Samuelson that the contributions of Leontief, Sraffa and von Neumann share important common features.[5] These concern both the method and content of the analysis. At the same time there are notable differences. In this paper we focus attention on the early contributions of Leontief and Sraffa. We are especially interested in finding out why 'in embryo' the semblance of the two is much greater than in maturity. In other words, we reflect upon the development of input–output analysis from Leontief's 1928 essay to his 1936 and later contributions and compare them with Piero Sraffa's early work on the theory of production. Since the opening of Sraffa's papers at Trinity

College Library in Cambridge, UK, we are able to study in detail Sraffa's independent, but parallel attempt at elaborating an economic approach that proceeds exclusively in terms of magnitudes that can be observed and measured. Also, Sraffa saw his analysis as rooted in the contributions of the classical economists from William Petty to David Ricardo and he too equated his scheme with the *Tableau Economique*. However, while in the years 1927–1928 Leontief and Sraffa may be said to have been independently pursuing similar lines of thought, they soon afterwards, apparently again without knowing of each other's work, parted company, with Leontief turning to the practical application of a stripped-down version of the new instrument, and Sraffa relentlessly seeking to solve the intricate problems the approach posed in the course of its elaboration.

The composition of this paper is as follows. In the next section we provide a summary account of crucial features of the classical economists' approach to the problem of value and distribution as it emerges from Piero Sraffa's studies in the late 1920s. The third section then turns briefly to Leontief's 1928 paper and shows how closely it is related to the classical approach to the theory of production, distribution and value. The fourth section provides a summary account of Sraffa's work in the period 1927–1931. In it, Sraffa managed to solve several of the problems that the approach was confronted with, some of which had been mentioned but not fully answered by Leontief in his maiden paper. In the short fifth section it will be argued that, due to his premature abandonment of his original line of research, Leontief later proceeded essentially in an eclectical way. In particular, when confronted with the time-honoured problem of value and distribution he developed a concept of prices and distribution that sits uncomfortably with the rest of his scheme. The sixth section draws some conclusions.

This paper and the paper by Kurz and Salvadori (2000) can be seen as complementary to each other. The latter is concerned with the prehistory of input–output analysis and therefore covers essentially only authors whose contributions predate the publication of Leontief's seminal papers; Sraffa (1960) is therefore only mentioned in passing. The present paper compares instead Leontief's 1928 essay with the hitherto unpublished manuscripts and working notes of Piero Sraffa that were composed at around the same time. Interestingly, both Leontief and Sraffa were disenchanted with the marginalist doctrine as it had been handed down by Alfred Marshall. They despised the subjectivist character of the explanation of value and distribution given and explicitly sought to elaborate an objectivist alternative to it. Both authors saw their own work as firmly rooted in the contributions of the physiocrats and the English classical political economists. There is no evidence known to us that the two authors knew of each other's similar endeavours and there is every reason to think that no such evidence exists. We are thus confronted with the fact that two major economists of the 20th century developed, independently of each other, similar approaches to the problem of production. These were meant to revolutionize the subject in the traditional sense of the word as overcoming the ruling doctrine of the time by returning to an earlier one which, as Sraffa (1960, p. v) was to write, 'has been submerged and forgotten since the advent of the "marginal" method.' This return necessitated shedding the weaknesses of the doctrine's earlier formulations, which were at least partly responsible for its premature abandonment and replacement by the marginalist one, and elaborating on its strengths. The question then is how far Leontief and Sraffa got in this regard in the late 1920s and early 1930s, that is, whether they succeeded in providing a coherent approach to the problems under consideration that was faithful to their objectivist outlook on economic phenomena.

2. Circular Flow and Physical Real Costs

Input–output analysis has its roots in the classical economists from William Petty via Richard Cantillon and the Physiocrats to Robert Torrens and David Ricardo and to authors working in the classical tradition, such as Karl Marx, Vladimir K. Dmitriev, Ladislaus von Bortkiewicz and Georg von Charasoff. Since we have elsewhere dealt with the prehistory of input–output analysis (see Kurz and Salvadori, 2000), we may directly turn to some characteristic features of the early classical economists' view of the economic system and the analytical challenges it poses.

The arguably most important and closely intertwined features of the starting point of the classical approach to the theory of production, distribution and value are the following:

(1) Production consists essentially in a *transformation of matter and energy into other forms of matter and energy*; this process is subject to the laws of science (especially physics, chemistry and biology).
(2) Production involves destruction, and the real cost of a commodity consists first and foremost in the commodities necessarily destroyed in the course of its production. This leads to the concept of *physical real cost*.
(3) For the reason just given there is no such thing as production carried out by unassisted labour: *it is impossible to produce something out of nothing*.
(4) Production is essentially a *circular flow*: commodities are produced by (means of) commodities.
(5) Production typically generates a *social surplus*. The surplus refers to those quantities of the different commodities that are left over after the necessary means of production are used up and the means of subsistence in the support of labourers have been deducted from the gross outputs produced during a year.

Features (1) and (3) are well expressed by James Mill's famous dictum that man cannot create matter, man can only decompose and recompose it, change its form and move it (Mill, [1821] 1826, p. 107). They also help us to explain the *objectivist* nature of the analysis of the classical authors or, as William Petty put it in 1690, its 'physician's outlook', the upshot of which is to express oneself only 'in Terms of *Number*, *Weight* or *Measure* ... and to consider only such Causes, as have visible foundations in Nature' (Petty, 1986, p. 244). Given this starting point, economics had to be elaborated in full recognition of the laws of science. Without too much of an exaggeration one can indeed say that in their analyses the classical economists tried to respect what nowadays are known as the laws of thermodynamics.

The perhaps clearest expression of the physical real cost approach, feature (2), has been put forward by James Mill in his *Elements of Political Economy*, first published in 1821. Mill insisted that, in the last instance, '*The agents of production are the commodities themselves* .... They are the food of the labourer, the tools and the machinery with which he works, and the raw materials which he works upon' (Mill, [1821] 1826, p. 165, emphasis added).

This feature also invites one to draw an analogy between a product that obtains as the result of the productive consumption or 'destruction' of necessary quantities of means of production and means of subsistence, on the one hand, and a chemical reaction conceived

of as a balance of the weights of inputs and outputs.[6] In both cases the balance expresses conservation of matter and energy. Sraffa traced the objectivist or natural science point of view back to William Petty and the Physiocrats. In the Physiocrats, he pointed out, 'il valore sia una quantità intrinseca degli oggetti, quasi una qualità fisica o chimica (let value be an intrinsic quantity of objects, a sort of physical or chemical property)' (D3/12/12: 7).[7] And with regard to Adam Smith's doctrine of 'natural value' he emphasized that the Scotsman was concerned with 'that physical, truly natural relation between commodities' (D3/12/11: 83). He also used the term 'physical value' of products and insisted that it '*is* equal to what has been consumed' (D3/12/1: 5; see also D3/12/10: 54).

The idea expressed in feature (4) can be traced back to William Petty and Richard Cantillon and was most effectively expressed by François Quesnay ([1759] 1972) in the *Tableau Economique*. The different parts of an economy are typically interdependent – they form a connected system of production – and therefore cannot generally be analysed independently of one another. A *general analysis* is needed; a partial analysis will typically not do. Such an analysis of general economic interdependence was in fact for the first time provided by Quesnay. Both Leontief and Sraffa pay tribute to him because of this fact.

Feature (5) raises a number of important issues and is the source of major conceptual and analytical problems that constituted (and still constitute) formidable stumbling blocks to economists. First, in systems characterized by the conservation of matter (and energy) the question is, in what sense is it possible to have a surplus? Second, once this question is satisfactorily answered, how is this surplus distributed amongst different claimants, and what are the implications of different distributions with respect (a) to the properties of the given system of production in use and (b) the forces at work that transform the system over time? The former problem leads directly to the classical analysis of the relationship between income distribution and relative prices, the latter to the analysis of the relationship between income distribution on the one hand and capital accumulation and economic growth and development on the other.

It is this rich picture of the economic system elaborated by the classical authors and those working in their tradition that allows one to adopt a wider perspective from which one may assess input–output analysis conceived as 'a practical extension of the classical theory of general interdependence'. Before doing this we show that there are some remarkable parallels between Leontief's 1928 essay and Sraffa's work in the late 1920s. We begin with a brief summary account of some of the ideas contained in Leontief's essay.

3. Input–Output Analysis, Mark I: Leontief's Essay of 1928

In his essay 'Die Wirtschaft als Kreislauf' Leontief put forward a two-sectoral input–output system. Throughout his investigation he assumed single production and constant returns to scale; scarce natural resources are mentioned only in passing. In much of the analysis it is also assumed that the system of production (and consumption) is indecomposable. Much of his analysis focuses on the case of a stationary system characterized by constant technical coefficients. He tabulated the 'relations of production' in the following way (Leontief, 1991 [1928], p. 194):

$$aA + bB \rightarrow A \tag{1a}$$

$$(1 - a)A + (1 - b)B \rightarrow B \tag{1b}$$

where A and B give the total quantities produced of two, possibly composite, commodities, and a and b [$(1 - a)$ and $(1 - b)$] give the shares of those commodities used up as means of production and means of subsistence in the first (second) sector.

Leontief premised his analysis on the conviction that economics should start from 'the ground of what is objectively given' (Leontief, 1928, p. 583); economic concepts are said to be meaningless and potentially misleading unless they refer to magnitudes that can be observed and measured. He adopted explicitly a 'naturalistic' or 'material' perspective (p. 622 [p. 211]). The starting point of the marginalist approach, *homo oeconomicus*, he considered inappropriate because it is said to give too much room to imagination and too little to facts (pp. 619–620). Economic analysis should rather focus on the concept of circular flow which expresses one of the fundamental objective features of economic life. A careful investigation of its 'technological' aspects is said to be an indispensable prerequisite to any kind of serious economic reasoning.

Leontief then distinguished between 'cost goods' and 'revenue goods'; the latter satisfy final demand. The concept of revenue good indicates that the economy is taken to produce a surplus over and above what is consumed productively. He suggested (p. 585) that the process of production should be described in terms of three sets of 'technical coefficients': (i) 'cost coefficients', that is, the proportion in which two cost goods participate in the production of a good; (ii) 'productivity coefficients', that is, the total quantity produced of a good in relation to the total quantity used up of one of its inputs; and (iii) 'distribution coefficients', that is, the proportion of the total output of a certain good allotted to a particular group of property income receivers.

Leontief stressed that because of the circular character of production 'a complete elimination of a factor of production from the given system is in principle impossible.' He added: 'Of course, the size of the "capital factor" can be reduced to any chosen level by referring back to even earlier periods of production' (p. 622 [p. 211]). The reference is to what became known as the method of reduction to dated quantities of labour (see Sraffa, 1960, chapter VI). This reduction, Leontief stressed, has nothing to do with a historical regress (p. 596, fn. 6 [p. 192 fn]).

Most important for the purpose of this paper, Leontief left no doubt that the physical scheme of production contemplated also held the key – or, to be more precise, one of the (two) keys – to an investigation of exchange relationships. The latter, he stressed, had to fulfil some 'general conditions' imposed by 'the framework of a circular flow' (p. 598 [p. 193]). It deserves to be emphasized that the corresponding concept of value is explicitly qualified as one that refers to the 'exchange relation *deduced from all the relationships ... analysed so far*' (p. 598 [p. 193]; emphasis added). Leontief thus expressed clearly the fact that the exchange ratios under consideration flow directly from the interdependent structure of the system of production in use.

A part of the (surplus) product of each sector is taken to be appropriated by a so-called ownership group: 'In the general circular flow scheme, income from ownership is of course considered alongside other cost items without the slightest direct reference to how it originates (the phenomenon of ownership). It is the task of the theory of interest to investigate these fundamental relationships' (p. 600 [p. 196]). Leontief's argument resulted in setting up price equations that reflect not only the socio-technical conditions of production, but also the rule that fixes the distribution of the surplus product. This rule is the second key to a determination of relative prices. Only if both the system of production and the sharing out of the surplus between different claimants in terms

of wages, profits (or interest) and rents is known, can relative prices be determined. Two 'keys' are required in order to solve the problem of value and distribution.

There is compelling evidence that Leontief was very clear about this. Counting unknowns and equations, he found that without fixing the distribution of the surplus the number of variables exceeds the number of equations by one. This led him to the idea of investigating the impact of hypothetical variations of one of the unknowns on the levels of the others: 'One may vary at will the exchange proportions and consequently the distribution relationships of the goods without affecting the circular flow of the economy in any way' (pp. 598–599 [p. 194]). In other words, the same physical input–output schema can accommodate different price systems reflecting different distributions of income. He related this finding to the classical economists who are explicitly said to have advocated a 'surplus theory' of value and distribution (p. 619 [p. 209]). Hence the exchange ratios of goods reflect not only 'natural', that is, essentially technological, factors, but also 'social causes'. For example, assuming free competition, as the classical economists did in much of their analysis, the surplus is distributed in terms of a uniform rate of return on capital across all industries of the economy. With this specification, the general rate of profit together with relative prices can be determined in terms of the system of production in use and given real wages. 'But this is the "law of value" of the so-called objective value theory' (p. 601 [p. 196]), Leontief insisted.

These are remarkable propositions. They show that the young Leontief was possessed of a deep understanding of the classical economists' approach to the problem of value and distribution. Comparing his view of the classical economists with the one expressed in the received interpretations of a Marshall, Edwin Cannan or Jacob Hollander testifies to Leontief's astounding originality and profundity. There is of course reason to think that Leontief's perspective on the classical authors was at least partly shaped by the supervisor of his PhD thesis in Berlin, the eminent economic theorist and statistician Ladislaus von Bortkiewicz; see, in particular, von Bortkiewicz (1906–1907 and 1907). As we shall see in the following section, Leontief had independently arrived at a view that is very similar to the one Sraffa elaborated at around the same time. In particular, Leontief had clearly understood that the classical approach provided a coherent explanation of value exclusively in material terms and could entirely do without any reference to labour values. It constituted an analysis that was fundamentally different from the marginalist one and avoided the concepts of demand and supply functions which had no objective contents and to which nothing corresponded in the real world.

A few years later Leontief presented his input–output approach as a development of 'general equilibrium theory' which, at the time, could be expected to be identified with the Walras–Pareto–Cassel theory.[8] See, in this context, the interpretation provided by Gilibert (1981, 1991). Leontief's alleged 'conversion' to neoclassicism remains somewhat of a riddle, not least because scrutiny shows that it is more apparent than real.

We now turn to the work Sraffa carried out in the period 1927–1931. As his unpublished papers document in great detail, there were several stumbling blocks he had to remove on the way to developing a coherent theory of value and distribution exclusively in objectivist terms (quantities of materials, labour etc). These included, *inter alia*, the problems of how to deal with (i) durable instruments of production, (ii) scarce natural resources, such as land, and (iii) joint production. Here we set aside these problems,

which played an important role in Sraffa's work in the period under consideration, and focus attention instead on the more basic ones he tackled at the beginning of this period. This limitation allows us to concentrate on what is common to Sraffa and Leontief in the late 1920s.

4. Sraffa's Early Work on the Theory of Production, Distribution and Value

Physical concepts were widely discussed by economists in the late 19th and at the beginning of the 20th century. Economists thus responded to developments in the sciences, especially John Dalton's elaboration of the 'atomic theory' in chemistry. This theory was based on two laws: (1) the Law of the conservation of mass; and (2) the Law of definite proportions. The latter stated that in a given chemical compound, the elements are always combined in the same proportion by mass. The first Law met largely with approval amongst economists which, however, did not necessarily mean much. More surprisingly, even the second Law appealed to some economists. It was discussed, for example, by Pantaleoni (1894, pp. 99 *et seq*.), whose work Sraffa had meticulously studied while still in Italy. When he was lecturing at the University of Perugia, Sraffa in 1925 criticised the adoption of this law in economics. He came back to his criticism whilst preparing his lectures on advanced theory of value he was supposed to give in Cambridge from Michaelmas term 1927–1928, but which he then postponed for a year. He insisted that the Law does not carry over from chemistry to economics essentially for two reasons. First, workers can subsist in different ways and yet produce the same kind of commodity. (Similarly, they can subsist in the same way and yet produce different kinds of commodities.) Secondly, one and the same commodity can be produced using different methods of production which request the productive consumption of different means of production.

The laws of science could thus not simply be adopted by economists, rather they had to be adapted judiciously to the economic field and its specific nature. The fundamental questions Sraffa faced were: was an objectivist or material-based approach to the theory of production, distribution and value at all possible? Could property incomes such as profits and rents be explained exclusively in terms of magnitudes that can be observed and measured, or did one have to have recourse to subjectivist concepts such as utility and disutility, as conventional (marginalist) theory maintained? These questions Sraffa began to answer from November 1927 onward in terms of the elaboration of several sets of 'equations', each of which was designed to address, and solve, a particular problem. He followed a growing order of complexity of the problems studied.

4.1. First Equations: Production without a Surplus

Sraffa quite naturally first analysed an economy that produces just enough, neither more, nor less, to recover the necessary means of production used up in the process of production and the necessary means of subsistence in the support of workers – a situation reflected in what he called his 'first equations'. He emphasized that this amounts to considering workers' remuneration 'as amounts of fuel for production' (D3/12/7: 138) and identified the situation as the realm of *pure necessities*, or 'natural economy'.[9] In this case the concept of physical real costs applied in an unadulterated way. The means of subsistence in the support of workers are an indispensable part of physical real costs, because

only their (recurrent) consumption 'enables' workers to perform their function. The periodic destruction of such commodities is a necessary condition the economic system has to meet in order to realize a 'self-replacing state', but it is not also sufficient. The system must be able to restore periodically the initial distribution of resources in order for the (re)productive process to continue unhampered. With a division of labour and in the absence of a central coordination of economic activities, this coordination must be achieved in terms of interdependent markets. Commodities must be exchanged for one another at the end of the uniform period of production. But which exchange ratios guarantee the repetition of the process? Sraffa showed that the sought ratios, or what in his interpretation Ricardo had called 'absolute' values, were uniquely determined by the socio-technical conditions of production and could be ascertained by solving a set of linear homogeneous production equations. Whether actual markets led up to the same solution was a different question.

We may illustrate Sraffa's reasoning in terms of a document dated winter 1927, which contemplates the 'no surplus' case of an economy composed of three industries:

$$\left.\begin{aligned} A &= a_1 + b_1 + c_1 \\ B &= a_2 + b_2 + c_2 \\ C &= a_3 + b_3 + c_3 \end{aligned}\right\} \quad \text{where} \quad \begin{aligned} A &= \Sigma a \\ B &= \Sigma b \\ C &= \Sigma c \end{aligned}$$

> These are homogeneous linear equations. They have infinite sets of solutions, but the solutions of each set are proportional. These proportions are univoche {unique}.
>
> These proportions we call ratios of Absolute values. They are purely numerical relations between the things A, B ... They are not necessarily the ratios, in which exchange will actually take place in any community in which the quantities of things respectively used in production (i.e. consumed) and produced satisfy those equations: such actual ratios of exchange are also conditioned by such things as legal institutions, etc, which vary in different organisations of society and which are 'arbitrary', i.e. irrelevant, from our present point of view (D3/12/5: 2).

The above equations may be read in different, yet mutually compatible ways. First, they can simply be read as a tabulation of the productive operations carried out, with A, B and C as the gross outputs of the three industries and a_i, b_i and c_i as the amounts of the three inputs consumed in industry i ($i = 1, 2, 3$). A second way of reading them becomes perhaps clearest when we draw a parallel between Sraffa's equations and the representation of chemical reactions, since Lavoisier in the late 18th century, as algebraic equations, with the names of the substances expressing the equality of constituents and compounds. Obviously, an expression of the type '$2H_2O = 2H_2 + O_2$' may even be regarded as a proper algebraic equation when interpreted as follows: the mass of two molecules of water is equal to the mass of two molecules of hydrogen plus the mass of one molecule of oxygen. In this interpretation H_2O is not just a symbol for water, but has a quantitative aspect: it is the mass of a molecule of water. Similarly, an expression of the type '$A = a_1 + b_1 + c_1$' could be interpreted both as a tabulation of a production process (and in this case it is not an algebraic equation) and as an algebraic equation stating: the value of the output of the first industry, A, equals the sum of the values of its inputs, a_1, b_1, and c_1.

Yet in order to determine the values of the three kinds of commodities, the above equations are of little use. They had to be replaced by equations of the following type:

$$Ap_1 = a_1p_1 + b_1p_2 + c_1p_3$$
$$Bp_2 = a_2p_1 + b_2p_2 + c_2p_3$$
$$Cp_3 = a_3p_1 + b_3p_2 + c_3p_3$$

where p_i is the 'absolute value', or price, of one unit of commodity i ($i = 1, 2, 3$). This Sraffa did in early 1928. He was clear that only two of the equations could be independent of one another. Hence, setting the price of one of the commodities (or of a bundle of commodities) equal to unity allows one to determine the prices of the remaining two commodities in terms of it.

Before we proceed, the following observations are apposite. First, there is a close parallel between Leontief's claim that the exchange ratios of commodities can be 'deduced from all the relationships' describing the conditions of production in the economy and Sraffa's above demonstration. Indeed, relative prices (or 'absolute values') can be ascertained exclusively in terms of the physical input–output quantities given: there is no need to have recourse to demand and supply schedules and the like. As Sraffa stated in a related note: 'It is clear at once that these technical relations of production leave no room to play with: the values are rigidly fixed, and neither preferences nor ... {the punctuation mark is Sraffa's} can have any influence unless they change these relations. – It must be noted that they do not represent only the cost of production: they equally show the use, or disposal, of each product' (D3/12/2: 31). Secondly, Sraffa emphasized that a system of such algebraic equations is non-contradictory *only* in the case in which there is no surplus (see, for example, D3/12/6: 16 and D3/12/2: 32–35). The with-surplus case Sraffa tackled almost in parallel with the no-surplus case in the winter of 1927–1928.

4.2. Second Equations: Production with a Surplus and Given Real (Commodity) Wages

Systems with a surplus are the object of what Sraffa called his 'second equations'. More precisely, the latter deal with a system of production that produces a surplus over and above the necessary physical real costs, including the means of sustenance of workers, and in which the surplus is distributed according to a uniform rate of interest on the value of capital invested. Sraffa, thus, at first retained the earlier assumption of given real wages in commodity terms in each of the different industries. Therefore, both in his first and second equations Sraffa felt no need to invoke, and then define, the concept of 'labour': all that mattered were physical real costs, that is, amounts of commodities used up in the course of production. In his 1960 book Sraffa aptly spoke of 'the methods of production and productive consumption' (Sraffa, 1960, p. 3).

There is, however, a crucial difference between the no-surplus and the with-surplus case: 'When we have got surplus, natural economy stops' (D3/12/11: 42) and social and institutional factors become important. Technically this is reflected in the fact that 'the equations become contradictory' (D3/12/6: 16). Materially, 'the "absolute values" have no more the appeal to commonsense of restoring the initial position – which is required if production has to go on' (D3/12/6: 10). Indeed, in the with-surplus economy, a whole range of exchange ratios is, in principle, compatible with the condition of

self-replacement (see D3/12/6: 9). Sraffa stressed that 'within those limits value will be indeterminate.' And: 'It is therefore necessary to introduce some new assumption, which in substance will amount to determine ... according to which criterion the surplus is distributed between the different industries' (D3/12/6: 16). With free competition, and focusing attention on the case of only circulating capital, the surplus is distributed in terms of a uniform rate of interest on the value of the 'capital' advanced in the different industries. Obviously, with heterogeneous material inputs (means of production and means of subsistence) the value of capital cannot be ascertained independently of, but only *simultaneously* with, the prices of commodities.

By mid 1928 Sraffa had managed, with the help of his colleague and friend Frank Ramsey, to establish that a solution existed and what it was (see Kurz and Salvadori, 2001, pp. 262–264). The system he and Ramsey discussed on 26 June 1928 was the following one:

$$v_a A = (v_a a_1 + v_b b_1 + c_1)r$$
$$v_b B = (v_a a_2 + v_b b_2 + c_2)r$$
$$C = (v_a a_3 + v_b b_3 + c_3)r$$

(see D3/12/2: 29). Here v_i is the value (or price) of one unit of commodity i ($i = a, b$), with the third commodity serving as standard of value, and r is the interest factor ($=$ 1 + interest rate); the meaning of the other magnitudes should be evident. Ramsey transformed the linear homogeneous system into its canonical form and set the determinant equal to zero in order to ascertain the non-trivial solutions.

An investigation of the with-surplus system and given commodity wages yielded the following result. The rate of interest and relative prices are determined exclusively in terms of the given physical input–output scheme. All that matters are the amounts of the different commodities produced and the amounts of them used up, where obviously we now have $A \geq \Sigma_i a_i$, $B \geq \Sigma_i b_i$, and $C \geq \Sigma_i c_i$, and where at least with regard to one of the commodities the strict inequality sign applies. Hence in this respect there is a close parallel between the no-surplus and the with-surplus cum given-commodity-wages case. In a draft of parts of the preface of his book, probably written in the 1950s, Sraffa maintained that this point of view 'implies replacing the notion that "commodities are produced by factors of production" with the other one that "*commodities are produced by commodities*"', which in turn amounted to 'replacing the idea that the process of production has a beginning and an end with that that it is a circular one – an idea first introduced by the Tableau économique' (D3/12/7: 2; emphasis added). And when in the second half of the 1950s Sraffa began to put together his book, for a while he thought of giving it the title 'Outline of an Economic System, or, The Production of Commodities by Commodities' (see D3/12/80: 2). Thus, Sraffa for a while actually tinkered with the idea of giving his book the title Samuelson attributed to it. With regard to his first and second equations this would indeed have been appropriate. (Things are somewhat different with regard to his 'third equations'; see further down.)

Sraffa constructed simple numerical examples in order to illustrate his finding, including the following one with two industries:

$$17V = (6V + 10)r$$
$$23 = (5V + 4)r$$

(D3/12/8: 26). Here V is the value of one unit of the first product in terms of the second and r is again the interest factor. Sraffa calculated that r will be approximately equal to 1.582 and V equal to 2.108.[10] Sraffa also represented graphically the relationships between V and r given by the two equations and identified the solution of the system as the intersection of the two curves. (See also D3/12/7: 60, 96-100, and 173 together with the corresponding diagram in 57.) Figure 1 illustrates the case under consideration.

Next, Sraffa turned to the vexed problem that had already bothered a great deal David Ricardo, and after him many other authors including Karl Marx and Knut Wicksell: how does the rate of interest, and do relative prices, change consequent upon a change in wages, given the system of production? Answering this question implied disclosing the mathematical properties of a given system of production as regards the distributional alternatives it allows for and the corresponding price vectors supporting these alternatives.

4.3. Third Equations and Ricardo's 'Proportional Wages'

The study of the impact of a hypothetical variation in wages on the rate of interest and relative prices, given the system of production, Sraffa carried out in terms of what he called his 'third equations'. Following Ricardo's lead (see *Works*, Vol. I, p. 50), he did this first in terms of a redistribution of the surplus product away from profits and towards wages in proportion to the original vector of the surplus product. This assumption allowed him to conceive of the redistribution of the surplus in straightforward physical terms and yet advocate a share concept of surplus wages that is independent of relative prices. He demonstrated that an increase in wages implied a decrease in the rate of interest and in general a change in relative prices. However, for obvious reasons, he was not satisfied with the idea of variable surplus wages with a constant commodity composition. If workers could spend some of their wages on beef, it makes no sense to assume that they consume only bread.

He saw that Ricardo had also allowed for a participation of workers in the surplus product and was especially fascinated by the way Ricardo had done this analytically in terms of what Sraffa called 'proportional wages', which had allowed him to

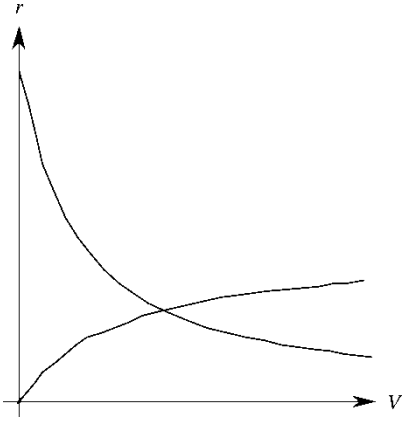

Figure 1. The relationship between V and r

circumnavigate the problem just mentioned. He credited Ricardo with the proposal that a *share* concept of wages was appropriate in the new situation. More specifically, Ricardo had insisted that what could be taken as a given magnitude in the theory of value and distribution is 'the proportion of the annual labour of the country . . . devoted to the support of the labourers' (Ricardo, *Works*, Vol. I, p. 49; see also pp. 274–275 and 420). As Sraffa swiftly noted, Ricardo's labour-based share concept was subsequently adopted by Marx in terms of a given 'rate of surplus value', S/V, that is, the ratio between the portion of the net (labour) value added that goes to capital owners, or surplus value, S, and the portion that goes to workers, or variable capital, V. While Sraffa also adopted a share concept, unlike Ricardo (and Marx) he defined wages as a proportion of the national income evaluated in terms of normal prices.

The germs of Sraffa's work in this regard can be traced back to the latter part of the first period of his constructive work (1927–1931), but it was only as a consequence of his work on the Ricardo edition in the 1930s that he understood more clearly the new conceptualization of real wages as proportional wages Ricardo had adopted in the *Principles* (see also Sraffa, 1951, p. lii). In particular, Sraffa became aware of the fact that Ricardo's argument was not meant to be limited to the case of a given economy at a given time but was designed to cover in at least one important respect also the development of the economy *over time*. More specifically, Ricardo's demonstration of the inverse relationship between the rate of profits and wages was seen to encompass the case in which the productivity of labour changes. It was on the basis of the new wage concept (and on the premise that the social capital consisted only of, or could be reduced to, wages) that Ricardo had felt he could assert what may be called his 'fundamental proposition on distribution': that the *rate* of profits depends on *proportional wages*, and on nothing else (see Kurz, 2006).

There was only one further step to be taken in order to arrive at the price equations one encounters in Sraffa (1960, chap. III). While Ricardo (and Marx) had consistently assumed wages to be paid *ante factum*, that is, at the beginning of the (uniform) production period, and thus as belonging to the capital advanced in each industry, Sraffa, after some deliberation, decided to treat wages as paid *post factum*, that is, at the end of the production period and thus out of the product. He admitted that this was a hard choice because of the undeniable ever-present element of subsistence in wages, but compared with the alternatives at hand it was the least unsatisfactory one. In familiar matrix notation, we get the following system of price equations:

$$\mathbf{p} = (1 + r)\mathbf{A}\mathbf{p} + w\mathbf{l}$$

where $\mathbf{p}$ is the n-dimensional vector of prices, $\mathbf{A}$ is the matrix of technical coefficients regarding means of production, $\mathbf{l}$ is the vector of direct labour coefficients, r is now the rate of profits or rate of interest (and no longer the interest factor), and w is the wage. In order to be able to interpret w as the share of wages in national income, some normalizations are needed (see Sraffa, 1960, chap. III). Most important, it is only now that Sraffa needed a concept of the quantity of 'labour' on the basis of which wages are calculated. In order to render heterogeneous labour commensurable, Sraffa followed the classical economists and had recourse to given wage differentials: quantities of different kinds of labour are aggregated via the relative wage rates of those different kinds of labour (see Kurz and Salvadori, 1995, chap. 11).

What is important to note is the following. Once a standard of value (or *numéraire*) is fixed and the share of wages (or, alternatively, the rate of interest) is given, prices and the rate of interest (or, alternatively, the share of wages) can be determined. However, as soon as prices are ascertained so is the value of the net product of the system, the value of the capital employed in each industry and in the economy as a whole, etc. In other words, the 'value added' is known. This value added depends on prices and therefore on income distribution.

5. Input–Output Analysis, Mark II: Leontief's Post-1928 Contributions

While Leontief conceived of his early contribution as firmly rooted in the classical tradition, he called his input–output method developed in the 1930s and 1940s 'an adaptation of the neo-classical theory of general equilibrium to the empirical study of the quantitative interdependence between interrelated economic activities' (Leontief, 1966, p. 134). Scrutiny shows, however, that in his input–output analysis he preserved the classical concept of circular flow and did not, as is maintained by some interpreters, adopt the Walras–Cassel view of production.[11] In the second edition of *The Structure of American Economy*, published in 1951, he even explicitly rejected the view of production as a one-way avenue that leads from the services of the 'original' factors of production: land, labour and capital – the 'venerable trinity' – to final goods (Leontief, 1951, p. 112). Unlike the theories of Walras and Cassel, in Leontief there are no given initial endowments of these factors. This is why we said earlier that the change is more apparent than real.[12]

In order to understand the real difference between his later and his earlier approach we have to turn to the way in which Leontief within the new input–output framework determined prices.[13] Obviously, attention has to focus on the so-called 'open' model. Leontief proposed a set of 'value-added price equations'. The price each productive sector is assumed to receive per unit of output equals the total outlays incurred in the course of its production. These outlays comprise the payments for material inputs purchased from the same or other productive sectors plus the 'value added'. Surprisingly, the latter is assumed to be *given* from the outside. Assuming a closed economy without a government, the latter represents the sum total of payments to the owners of productive factors: wages, rents, interest and profits. The main problem with this approach is that the magnitudes of value added per unit of output in the different sectors cannot generally be determined prior to and independently of the system of prices. Another way of putting it is that in this formulation two things are lost sight of: the constraint binding changes in the distributive variables, and the dependence of relative prices on income distribution – facts well understood and for perfectly good reasons, it seems, stressed by Leontief in his 1928 paper.[14]

6. Concluding Remarks

The paper compares the analytical structure of the approach Leontief adopted in his maiden paper published in 1928 with that of Sraffa in his early work on his equations of production in 1927 and 1928. It is argued that the two approaches are very similar and explain relative prices and the distribution of the social surplus essentially in the same way. Both focus attention on physical real costs of production and thus base their argument on 'what is objectively given' (Leontief) in an economy. This approach,

whose roots both trace back to the classical economists, is counterposed to the demand and supply approach of the marginalist authors, especially Alfred Marshall. The important lessons the two authors drew from their analyses is that information about the quantities of commodities produced and actually used up, or the physico-economic system of production of the system as a whole, plus information about the rule according to which the surplus is distributed is sufficient in order to determine relative prices: no other data are needed or could have a place in the argument.

Whereas the starting point of the two authors was very similar, the paths they followed afterwards diverged. While Sraffa with untiring effort generalized the argument to the cases of fixed capital, scarce natural resources, joint production, and a choice of technique, Leontief applied the new tool of input–output to practical problems. This made him set aside, and eventually lose sight of, certain important analytical findings contained in his 1928 essay. In particular, Leontief henceforth no longer took into account the fact that the 'value added' in each industry is endogenously determined in terms of the system of production in use and the rule fixing the sharing out of the product between the different claimants, typically workers and capital owners. Leontief instead assumed *given* 'value added' coefficients in his 'value-added price equations' (see Leontief, 1941). The difficulty with this procedure is that the magnitudes of value added per unit of output in the different industries can only be known when distribution and prices are known. Therefore, one might say that Leontief's price determination in terms of given value-added coefficients involves a regression compared with his earlier analysis.

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Notes

[1]Papers by Leontief whose years of publication carry an asterisk (*) are reprinted in Kurz *et al.* (1998).

[2]Quesnay also mentions interest on fixed capital, but interest plays at most a secondary role in the Tableau.

[3]The title of Sraffa's book in Samuelson's quote is not fully accurate: the correct title is *Production of Commodities by Means of Commodities*. However, for reasons that will become clear below, Sraffa for a while contemplated the possibility of adopting the title given by Samuelson.

[4]Of the three scholars mentioned, in our view only von Neumann deserves to be called a 'wunderkind' of mathematical economics.

[5]We have commented on this fact in several of our own contributions; see, in particular, Kurz and Salvadori (1995, chap. 13; 2000; 2004).

[6]As we shall see below, Sraffa at first wrote down systems of equations in which apparently heterogeneous things were added up and equated with one another (see on this aspect Garegnani, 2004).

[7]References to Sraffa's papers which are kept at Trinity College Library, Cambridge, follow the catalogue prepared by Jonathan Smith, archivist. Unless otherwise stated, all emphases are in the original, where words or passages Sraffa underlined are italicised by us. Sraffa frequently abbreviated 'and' by '+'; we shall use the word instead of the symbol. Since in his texts Sraffa used both round and square brackets,

all additions by us will be bracketed by { and }. We are grateful to Jonathan Smith and the staff of Trinity College Library for continuous assistance while working on the Sraffa papers.

[8]In *The Structure of American Economy*, first published in 1941, Leontief characterized the volume as an 'attempt to apply the economic theory of general equilibrium' to an empirical study of economic interrelations. Interestingly, he added that a better term than economic equilibrium would be 'general interdependence' (Leontief, 1951, p. 3). Léon Walras is mentioned only when Leontief introduced the important technical assumption of fixed coefficients underlying his empirical analysis (Leontief, 1951, pp. 37 and 201; the first page reference is not included in the index of the book). However, since apart from Walras only John Maynard Keynes, Vilfredo Pareto, François Quesnay and the German historicist Gustav Schmoller are mentioned, several readers appear to have implied that the reference to general equilibrium theory pointed in the direction of the analyses of Walras and Pareto. Things are not left in the open in other works of Leontief, in which a connotation with neoclassical theory is clearly spelled out; see, for example, the reference to 'neo-classical theory of general equilibrium' in Leontief (1966, p. 134).

[9]The same assumption of given 'necessities of life' paid out of the capital advances at the beginning of the uniform period of production also underlies von Neumann's model (1945, p. 2). For a discussion of the latter and a comparison with the classical approach to value and distribution on the one hand and intertemporal equilibrium theory of Arrow–Debreu on the other, see Kurz and Salvadori (2004).

[10]Actually V is closer to 2.107.

[11]For a characterisation of the Walras–Cassel point of view, see, for example, Kurz and Salvadori (1995, chapter 13, subsection 7.1).

[12]For a comparison of Leontief's approach and that of Walras, and the different traditions to which the two belong, see Kurz and Salvadori (2000, pp. 173–174); see also Gilibert (1981, 1991).

[13]Since the general reader of this journal can be expected to know very well Leontief's closed and open models we refrain from providing a formal discussion of them.

[14]See also Kurz and Salvadori (2000, pp. 173–174) and especially Steedman (2000).

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