Goods and bads: Sundry observations on joint production, waste disposal, and renewable and exhaustible resources

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Abstract: The paper starts from the premise that nobody intends on purpose to pollute the environment, render animal species extinct, *etc*. And yet what nobody intends happens all the time. The present paper discusses to what extent the nonintended consequences of purposeful human activities are related to the fact that production is generally joint production, generating both goods and bads. After a brief introduction into the problems at hand with reference to major economists, the implications of the following assumptions are investigated within a simple analytical framework: (a) free disposal and the Rule of Free Goods; (b) costly disposal and the negativity of some price; (c) product-cum-process innovations that render it possible to transform bads into goods. The paper concludes with some observations on renewable and exhaustible resources.

Keywords: disposal activities; exhaustible resources; innovations; joint production; negative price; renewable resources; rule of free goods; unintended consequences; waste disposal.

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1 Introduction

The main problem the social sciences are struggling with was well expressed by Adam Ferguson who stated: "History is the result of human action, but not the execution of any human design" (Ferguson, 1793, p.205). Man attempts to achieve his or her goals in terms of purposeful action. However, man typically achieves not only these goals, but something else in addition, or he or she misses them and gets something different. If all actions of man had only and exclusively the intended effects, we would hardly be in need of a scientific discipline to study man's volitions and actions and their outcomes: it would be sufficient to ask the agents and then let their answers be confirmed by the facts. The unintended consequences of purposeful activity block the picture. Recognition of this fact and a discussion of its implications for the study of socio-economic phenomena play an important role in the philosophical tradition began by the Scottish Enlightenment. The theme was picked up by Adam Smith and developed into a doctrine known as the law of 'unintended social outcomes'. To Smith, a great range of social activities had the characteristic that men, in pursuing their own objectives, seem frequently to contribute to outcomes, which they did not intend or foresee. The doctrine is commonly referred to, in Smith's case, as the doctrine of the 'invisible hand', which states, using a well-known formulation taken from the *The Wealth of Nations*, that man is "led by an invisible hand to promote an end which was no part of his intention" (Smith, 1976a, *WN*, IV.ii.9).

While Smith noted that human action fuelled by motives ranging from self-love and love of distinction to selfishness and rapacity may also have effects that are detrimental to society at large, he was, on the whole, an optimist. He thought that the effects that are beneficial to society can be expected to somehow outweigh those that are detrimental to it. His optimism was premised on the belief that Providence 'neither forgot nor abandoned those who seem to have been left out in the partition' of the earth among the commonwealth of all living beings. The self-seeking agent can therefore more often than not be trusted, "without intending it, without knowing it, [to] advance the interest of the society, and afford means to the multiplication of the species" (Smith, 1976b, *TMS*, IV.i.10).

The optimism of Smith and many of his fellow economists has repeatedly been questioned in view of the facts and has openly been criticised in several quarters of the profession and neighbouring disciplines. In the 19th century, it was first and foremost the so-called 'Social Question' that brought his doctrine under attack from socialists and Marxists. They took issue with romanticist glorifications of reality in his works, such as, for example, his contention that the rich 'divide with the poor the produce of all their improvements' and that "In ease of body and peace of mind, all the different ranks of life are nearly upon a level, and the beggar, who suns himself by the side of the highway, possesses that security which kings are fighting for" (Smith, 1976b, *TMS*, IV.i.10). The pauperisation of ever wider strata of society, the physical and mental deprivation of the poor, their social degradation, child labour and more threw in doubt Smith's promise of the participation of all strata of society in the affluence of the new world.

More recently, the emphasis has shifted somewhat away from the concern with the difference between poor and rich in a given society in a given time to the concern with the difference between poor and rich over time. The degradation of our natural habitat is said to benefit the living generations at the cost of future generations. Our riches will be reflected in the poverty of those that are born onto a pillaged planet. Smith and, with him, generations of economists are accused of having ignored the intertemporal implications of unfettered industrialisation and economic growth: depletion of natural resources, environmental damage, pollution, the extinction of species and so forth.

This paper is obviously not the place to enter into a thorough investigation of the empirical, theoretical and doctrinal questions raised. I shall, rather, start from a simple observation, which will then lead me to discuss an important aspect of the issue at hand. The observation is: Nobody who can be considered sane in his or her mind deliberately decides to pollute the environment, render animal species extinct or degrade the quality of the environment in which we live. And yet what nobody wants or intends happens all the time and at an ever larger scale. These non-intended consequences of purposeful activities are typically dealt with in economics under the heading of 'externalities' or 'external effects'. The production and consumption activities of agents who operate within a common environment are not independent of one another, contrary to what is typically assumed in large parts of economic theory. There, these activities are taken to be connected only via the 'market'. This is clearly a simplification which, while perhaps heuristically of some value because it allows one to clarify the role of markets in ideal conditions, neglects the fact that economic processes such as production and consumption are often, if not always, directly interdependent. These processes are time-consuming and often interrelated and as such their (joint) outcomes are more or less uncertain. The directness of the interdependence of economic processes, their time-consuming nature and their apparently complex interaction with one another and with the environment in which they are taking place, make for a great deal of the uncertainty experienced by agents and economists alike.

In this short paper I shall focus on one aspect of the problems under consideration: production. In much of economic theory, the student encounters what may be called the 'dogma of single production'; that is, that there is only a single discernible and physically measurable output in each and every production process. While this may be acceptable as a first approximation, in general it is not so. Quite the contrary. Human productive activities typically generate several measurable results; multiple-products processes are ubiquitous, and *joint production* is the rule. Besides, there is no reason to assume that the processes known to producers at a given moment of time generate only 'goods', that is, products capable of satisfying human needs and wants. This is, however, the assumption implicit in the dogma of single production.

The technological knowledge at our disposal, and the actual productive capacities installed, often require that in order to meet given consumption levels, in addition to the goods wanted, also some 'bads' necessarily have to be produced. Bads are products nobody wants and which may even be harmful to humans if not disposed of safely by means of disposal processes. People want electricity. They get electricity plus nuclear waste. Closely related to the problem of bads is that of pollution and externalities, which can be considered a more indirect evidence of joint production. Hence, what we find in the modern industrialised world are complex *systems of production-cum-disposal*. By means of disposal activities, societies try to get rid of waste emerging from both production and consumption.

The composition of the paper is as follows. Section 2 provides a brief summary of early contributions in economics to an analysis of joint production. Section 3 discusses the assumption of free disposal, which can be traced back to Adam Smith; this is done in an exceedingly simple analytical framework. Section 4 then introduces the assumption of costly disposal in this framework and draws its implications, especially as to the emergence of the phenomenon of negative prices. Costly disposal of unwanted substances provides a strong incentive to find out ways to use these substances and transform them into vendible output. The technological dynamism inherent in joint production systems is illustrated in Section 5 in terms of an invention that allows the transformation of what hitherto has been considered a bad that had to be costly disposed of into a good for which a new market emerges on which it can profitably be sold. This is the case of a process-cum-product innovation. Section 6 generalises the case of costly disposal to an economy in which many products are produced by means of products and several of these products have to be disposed of by means of products. Section 7 concludes the paper by briefly turning to the problems of renewable and exhaustible resources.

2 Joint production in economics: a brief historical overview

Conventional economic analysis found in contemporary microeconomic textbooks typically focuses attention on single-product processes of production and single-product firms, giving only passing reference to the case of joint production. The inattentive reader might get the impression that, whereas single production is both empirically important and theoretically significant, joint production is neither and can safely be set aside without prejudicing the issue and blurring our understanding of real-world phenomena. However, this would be a misconception. Joint production is both empirically important and its presence can qualitatively alter some of the characteristic features of the economic system.[1] In fact, joint production is the general case, and single production, provided it exists at all, is an exception to the rule. This view has already been expressed by William Stanley Jevons as early as 1871 who stressed “that these cases of joint production, far from being ‘some peculiar cases’, form the general rule, to which it is difficult to point out any clear or important exception” (Jevons, 1965, p.198). This was Jevons’s response to John Stuart Mill who, some two decades earlier, had reckoned joint production among ‘some peculiar cases of value’ with regard to which the Ricardian (labour value-based) doctrine had yet to be completed. Mill defined the case he had in mind in the following way: “It sometimes happens that two different commodities have what may be termed a joint cost of production. … For example, coke and coal-gas are both produced from the same material, and by the same operation” (Mill, 1965, pp.582–583). This phenomenon is more widespread than Mill’s ‘sometimes’ indicates and there is no reason to restrict the case to two jointly produced products. There may be two or more and there is no reason to presume that both (or all) products will be useful to man, given the current state of technological knowledge. Some may be bads or ‘discommodities’. Some early authors were not only aware of the phenomenon under consideration but also tried to draw consequences from it for economic theory.[2]

Since husbandry, estate management, farming and the raising of animals abound with multiple-product processes of production, it should come as no surprise that cases of joint production are mentioned in the writings of the mercantilists, the physiocrats and the early classical economists. A few examples must suffice. Petty, for instance, with his objectivistic perspective on economic phenomena (see Kurz, 2003) in his *Political Anatomy of Ireland*, first published in 1672, provides detailed accounts of the breeding of cattle and sheep and the “increase of their Flesh, Skin and Tallow” (Petty, 1986, p.174). In another essay, he noted carefully which of the joint products Ireland exported in which quantities (Petty, 1986, pp.594–596). In still another pamphlet, he was concerned with the negative implications of production and consumption activities on the health of humans in densely populated areas such as towns. He observed that because of pollution, ‘the Country is more *healthful* than the City’ and illustrated his view with reference to ‘the Fumes, Steams and Stenches’ of London (Petty, 1986, p.393). In François Quesnay and other physiocrats, we encounter examples such as corn and straw, the catch of fish that differ in size and kind, and the mining of ores that differ in grade and kind. James Steuart in his *Inquiry into the Principles of Political Economy*, published in 1767, mentioned cases of joint production in agriculture in the context of a discussion of the spatial distribution of productive activities in concentric circles around a town, anticipating Johann Heinrich von Thünen’s later analysis in his famous ‘scheme of rings’ of specialisation in *The Isolated State*, first published in German in 1826.

Adam Smith moved beyond mere description to deal with joint production in analytical terms. In the context of a discussion of native North American societies, he put forward what may be considered the first expression of the ‘Rule of Free Goods’. In a society of hunters (and in most other societies as well), animals are the source of a multitude of useful items, or ‘use values’, including different kinds of meat, furs, hides, bones, and tendons, some or all of which can be used either directly or indirectly to satisfy various wants. Each animal represents a *compositum mixtum* of goods (and in addition possibly some bads), which accrue as joint products in the separation process. In Chapter XI of Book I, ‘Of the rent of land’, Smith writes:

> “The skins of the larger animals were the original materials of cloathing. Among nations of hunters and shepherds, therefore, whose food consists chiefly in the flesh of those animals, every man, by providing himself with food, provides himself with the materials of more cloathing than he can wear. If there was no foreign commerce, the greater part of them would be *thrown away as things of no value*. This was probably the case among the hunting nations of North America, before their country was discovered by the Europeans, with whom they now exchange their surplus peltry, for blankets, fire-arms, and brandy, which gives it some value.” (Smith, 1976a, *WN*, I.xi.c.4; emphasis added)

The passage demonstrates Smith’s clear perception of the existence of joint-product processes of production. It also shows his awareness of the possibility that with joint production, the proportions in which the products occur need not coincide with those in which they are wanted. Finally, in it we encounter, possibly for the first time in the history of economic thought, the Rule of Free Goods.

In the sequel to the passage quoted, Smith makes it very clear that circumstances will determine which of the joint products will be free goods and, furthermore, that the phenomenon of free goods is not limited to the ‘early and rude state’ of society. He illustrates this in terms of the following example:

> “In some parts of the highlands of Scotland the bark is the only part of the wood which, for want of roads and water-carriage, can be sent to the market. The timber is left to rot upon the ground.” (Smith, 1976a, *WN*, I.xi.c.5)

In short, whether some of the joint products will be in excess supply, and which, depends on the ‘progress of improvement’ of society (Smith, 1976a, WN, I.xi.m.3). There are also examples of joint production from agriculture, distilling, fishing and mining.

Smith illustrates his argument in terms of historical material collected from several countries. The emphasis is on long-run trends of relative prices of various joint products and the impact on them of tariffs, export and import restrictions and other regulations. The relevance of Smith’s analysis has, from time to time been confirmed by events, even in the present era. The conflict in 1990 between the USA and the European Union over corn gluten feed is an example. Europe banned this by-product of American whiskey production used in animal rearing, but welcomed the whiskey (see Kurz, 1992).[3]

Smith was clear that in competitive conditions, the total price of the jointly produced products must cover the total cost of production plus the ordinary rate of profits on the capital advanced. Apparently, he was of the opinion that the cost of production and thus the total price of the composite output could be ascertained independently of, and prior to, the way in which 'this price is to be divided upon the different parts' of the produce (Smith, 1976a, *WN*, I.xi.m.12) – a view also entertained by several later authors. However, this is typically not so. In a general framework of the analysis, it is clear that the prices of outputs must be determined through the same mechanism, and at the same time, as the prices of the inputs and thus the total price of the joint products.

Karl Marx was a most attentive student of the contemporary literature on the history of technology, technical inventions, machinery and industry. This is well-documented in his so-called 'excerpt books' composed between 1845 and 1851 containing excerpts from the vast literature on technology he consulted. He came across, and noted carefully, examples of joint production from almost all branches of industry, including mining, forestry, paper manufacturing, the chemical industries, mechanical engineering and elsewhere. He typically distinguished between a main product and one or several by-products, which he also dubbed 'accessory products'. He was fully aware that some of the products were non-marketable and had to be disposed of. In *Capital*, Volume III, he also spoke of 'excretions of production' and stressed that "the so-called waste plays an important role in almost every industry" (Marx, 1959, p.101).

Marx saw the generation of such waste on a large scale and the necessity to remove it as a major stimulus to technical invention. Costly disposal of significant amounts of such waste in combination with rising prices of raw materials provides a strong incentive to search for ways of recycling the waste. This will lead to 'improved machinery' that reduces the generation of waste and 'scientific progress, particularly of chemistry, which reveals the useful properties of such waste'. Marx illustrates this in terms of examples from the flax, cotton and silk industries (Marx, 1959, pp.101–102), and adds:

> "The most striking example of utilising waste is furnished by the chemical industry. It utilises not only its own waste, for which it finds new uses, but also that of many other industries. For instance, it converts the formerly almost useless gas-tar into aniline dyes, alizarin, and more recently, even into drugs." (Marx, 1959, p.102)

The technological development of modern industry is thus characterised by two trends: the prevention of the occurrence of waste and the recycling of it. The goal was, in Marx's words, "the reduction of excretions of production to a minimum, and the immediate utilisation to a maximum of all raw and auxiliary materials required in production" (Marx, 1959, p.103). This is seen to be systemically enforced by competition, which is taken to force upon the capitalist a "fanatical insistence … that nothing is lost or wasted" (Marx, 1959, p.83).

While Marx stressed the empirical importance of joint production and waste more than most other economists then and now, yet in his theory of value and distribution, these matters hardly played any role at all. This is perhaps due to his assumption, mentioned earlier, that the joint outputs of a process can typically be divided into a main product, whose price has to bear the brunt of cost of production, and some by-products.

However, as Smith's historical analysis has already indicated, the division between a main product and one or several by-products cannot be decided *a priori*. Also, as the chemical industries show, for example, there may be more than one product to contribute to covering costs. We should not presume, as Marx appears to have done, that systems in which joint production is ubiquitous, can be treated as if they were characterised by single-product industries throughout.

Johann Heinrich von Thünen's space model reflects the predominance of joint production in the primary sector of production. For example, the ring immediately around the town is specialised in, in addition to hard-to-transport and quickly perishable vegetables and fruits, the production of milk, which requires hay and straw for feed and litter. In this ring, von Thünen stresses, "grain will be grown only for the sake of its by-product of straw" (von Thünen, 1966, p.14; my translation). Obviously, the main product cannot always be ascertained *a priori*.

Manure is both a joint effect of human consumption and a joint product of animal breeding. For the inhabitants of villages and towns, the manure is a 'bad' that has to be disposed of. "The citizens want to get rid of the dung even if they do not receive anything in exchange for it, but have to pay for its removal" (von Thünen, 1966, p.209). Here, we have a clear statement of the possibility of a *negative* price: while with goods, money and product move in opposite direction, with bads, the direction is the same. Von Thünen also observes that for farmers, manure is a good that is indispensable to production.

Von Thünen notes perceptively that the quality of land is generally transformed by productive activity, that is, land is not a fixed immutable endowment, unaffected by the way in which it is utilised (von Thünen, 1966, p.57). Indeed, the productive employment of land is best treated within a joint-products framework where a given quality of land can be regarded as part of the annual intake of a process, and possibly qualitatively different land as part of the annual joint output, of which the more conspicuous part consists of marketable commodities that are the primary object of the process. Von Thünen, himself an estate owner, deals at length with devices that are designed to reestablish or improve the original fertility of the soil, such as crop rotation and three-fallowing (see especially von Thünen, 1966, Sections 14–17 and 21–3). The principle of crop rotation may schematically be described in the following way:

$$\text{Land of quality}_1 \oplus \text{Complementary inputs}_1 \oplus \text{Labour}_1 \rightarrow \text{Crop}_1 \oplus \text{Land of quality}_2$$

$$\text{Land of quality}_2 \oplus \text{Complementary inputs}_2 \oplus \text{Labour}_2 \rightarrow \text{Crop}_2 \oplus \text{Land of quality}_3$$

$$\text{Land of quality}_3 \oplus \text{Complementary inputs}_3 \oplus \text{Labour}_3 \rightarrow \text{Crop}_3 \oplus \text{Land of quality}_1$$

Here, land of quality 1 is used to produce crop 1 (for example, wheat) together with land of quality 2, which in turn is used to produce crop 2 (alfalfa) together with land of quality 3, which is then used to produce crop 3 (clover) and *uno actu* transform land back into its original quality 1. Clearly, if this qualitative reconversion of land by means of crop rotation does not work, land cannot be treated as a renewable resource, as in classical rent theory, but has to be treated as a depletable resource (see the discussion in Section 7 below).

Mountiford Longfield in his *Lectures on Political Economy*, published in 1834, interestingly reckoned transport activities among joint production processes. He expounded:

> "Thus the freight of goods outwards and homewards ought to pay the expense of the voyage in and out, with the wear and tear of the ship, usual profits, &c. If, then, any circumstance connected with the state of trade, usually creates a greater demand for freight homewards, it will have a proportional effect in diminishing the freight of the cargo outwards." (Longfield, 1971, p.246)

The notion of the transport system as a multiple-product industry became prominent a few decades later in a famous controversy about railway rates which involved, among others, F. Y. Edgeworth and A. C. Pigou. As Steedman (1984, p.13) pointed out, a single bus or train, which starts from Stop 0 and proceeds to Stops 1, 2,…, n produces $n(n + 1)/2$ different possible passenger journeys; if it then makes the return trip, a total of $n(n + 1)$ 'products' will have been produced.

John Stuart Mill has variously been credited with having been the first economist ever to deal with joint production in his *Principles of Political Economy*, first published in 1848. Clearly, this view cannot be sustained. Mill and Longfield are, however, interesting from the point of view of doctrinal history. Their analyses ring in a gradual shift away from the cost-of-production approach of the classical economists to the demand-and-supply approach of the marginalist authors. As Mill contended with regard to the determination of the prices of two jointly produced commodities – the familiar workhorse of many early contributions to the subject – "a principle is wanting to apportion the expenses of production between the two. Since cost of production here fails us, we must revert to a law of value anterior to cost of production, and more fundamental, the law of demand and supply" (Mill, 1965, pp.245–246). At first sight, Mill's view appears to be plausible, because if two commodities are produced by a single process, there will be only one cost of production equation in order to determine two unknowns. Such a system is mathematically underdetermined. The crucial assumption implicit in Mill's argument is, of course, that there are no technical alternatives to produce the two joint products in different (net) output proportions. If there were, then it cannot be excluded that two processes are profitably used side by side. With two equations, both prices can, in principle, be ascertained. Hence, the reason Mill gives in support of his rejection of the cost-of-production theory is not conclusive. As long as the system can be shown to gravitate towards a cost-minimising system of production in which the number of processes of production operated equals the number of commodities, the prices of which have to be determined, it would be premature to maintain that the classical theory of value 'fails us'. As the recent formalisations of the classical theory of value and distribution have shown, 'square' systems of production have a genuine significance (see Kurz and Salvadori, 1995).

Interestingly, Mill draws an analogy between joint production and international trade (see also Kurz and Salvadori, 1995, pp.149–150, 152). From an analytical point of view, a country's foreign trade (which will be taken to be balanced) can indeed be regarded as an infinite set of fictitious joint production processes, by means of which a number of commodity inputs, that is, *exports*, are turned into a number of commodity outputs, that is, *imports*.

William Stanley Jevons in his *Theory of Political Economy*, first published in 1871, claimed that "cases of joint production … form the general rule, to which it is difficult to point out any clear or important exception" (Jevons, 1965, p.198). He also insisted that there is no reason to think that all products that are jointly produced are goods: some can well be bads or discommodities, that is, "substances or things which possess the quality of causing inconvenience or harm" (Jevons, 1965, p.58). He reiterated Mill's criticism of the classical cost-of-production principle in terms of an attack on the labour theory of value, which in its crude form conceives of value as proportional to the labour needed (directly and indirectly) to produce a commodity. With reference to the case of one process and two commodities, he insisted that "it is impossible to divide up the labour and say that so much is expended on producing *X*, and so much on *Y*" (Jevons, 1965, p.20), where *X* and *Y* are, of course, the quantities of the two jointly produced commodities. Jevons's criticism suffers from the same defect as Mill's: it starts from the premise that there is only one process and not several ones. As soon as two processes can be shown to co-exist profitably, the prices of the two commodities as well as their labour values can be ascertained simultaneously in terms of the two production equations corresponding to the two processes.

Jevons stated very clearly the possibility of negative prices and stressed their empirical importance:

> "There cannot be the least doubt that people often labour, or pay money to other labourers, in order to get rid of things, and they would not do this unless such things were hurtful, that is, had the opposite quality to utility – disutility. … Reflection soon shows, in short, that no inconsiderable part of the values with which we deal in practical economics must be *negative values*." (Jevons, 1965, p.127)

He left things at that. In particular, he did not introduce (costly) disposal processes in the analysis. He took into account, however, pollution and negative externalities connected with certain forms of the disposal of bads. He argued, for example, that the waste products of chemical firms are often "fouling the rivers and injuring the neighbouring estates" (Jevons, 1965, p.202).

It is interesting to note that as late as 1906, the American economist Fisher (1965, p.120) in *The Nature of Capital and Income* criticised Jevons on the grounds that discommodities are never of great importance and need not receive special attention.

Alfred Marshall with his practical sense in his *Principles of Economics*, first published in 1890, noted carefully the overwhelming importance of the phenomenon of joint cost and developed an analytical scheme and graphical illustration, to deal with the determination of the prices of the joint products introducing the concept of 'derived supply curve' (see Marshall, 1977, p.322). In this case, as well as in others, he appears to have benefited from his thorough knowledge of the German economics literature, for his approach had been anticipated by Hans von Mangoldt in the early 1860s (see Kurz, 1986).

While earlier authors, safe a few exceptions including von Thünen, generally assumed that the output proportions of the joint products are rigidly fixed, Marshall abandoned this assumption. He did this both within a static and a dynamic context. As regards the former, he generalised the marginalist concept of subsitutability from factors of production or inputs, to products or outputs.[4] In the extreme, the proportion in which two joint products can be produced varies continuously in a given interval. A case in point is the amount of meat produced relative to that of the hide, which depends on the age at which the animal is slaughtered. With flexible proportions, supply of the two products can perhaps be adjusted to demand.

The dynamic variant, which is developed in some detail in *Industry and Trade*, first published in 1919, is perhaps more interesting because it emphasises, as Marx did a few decades earlier, the incentive to innovate in an environment in which industrial waste is produced on a substantive scale. This is said to induce both product and process innovations:

> "Something which was apparently almost valueless is suddenly made the foundation of an important product, either through a new technical discovery or through the rise of a new demand. ... Some of the by-products of petroleum are subtle and costly chemical compounds, for pharmaceutical and other uses; and they demand the thought and care of highly trained professional analysts. The same is true in regard to several by-products of the 'heavy' chemical industries. ... In all such industries, new products are frequently coming to the front; and a business, which has abundant capital and is controlled by men with scientific interests and large faculty for high enterprise, may constantly introduce into the world not only new methods, but also new things."

He added with special reference to the colour industry:

> "Nothing is hastily dropped; every intermediate product and every by-product is tried in various combinations, with the hope of getting some new result of value in some industrial process, or of getting at an old result by a shorter or more economical method. Such work as this can be effectively done only by a business that can afford to work long and patiently at each of a vast number of problems." (Marshall, 1970, pp.238–240)

These and other passages show that Marshall was particularly aware of the technological and organisational dynamism inherent in modern industry and the role played by joint production. Finding out the useful properties of substances and turn a bad, which has to be disposed of at great cost, into a good that can be sold profitably involves a most attractive option. It will stimulate research and development and spur the generation of product and process inventions many of which can be expected to become innovations. In this regard, Marshall is perhaps closest to Marx, who emphasised the endogeneity of scientific and technological revolutions characteristic of the capitalist mode of production.

The most important empirical account in the 19th century of progress made in the economic utilisation of waste and refuse and accessory products from manufactures was, however, written by a non-economist. *Waste Products and Undeveloped Substances* by the Victorian journalist Peter Lund Simmonds, published in 1873, is a fascinating testimony of the technological dynamism characteristic of modern society and bears witness to the importance of joint production (Simmonds, 1873). In several hundred pages, the author provides innumerable empirical examples of multiple-product processes of production and shows how whole sequences of product and process innovations can be understood as responses to the challenges posed by the generation of bads. Being continuously on the lookout for opportunities to reduce costs and increase profits will lead to the development of new industrial designs, new qualities of known products and entirely new products. The *leitmotiv* of Simmonds's study can be summarised as 'Dirt is only matter in the wrong place' and 'Where there's muck, there's brass!'. It is the task of ingenious people to find the 'right place' that yields 'brass'.

Strangely enough, early general equilibrium theorists from Léon Walras to Vilfredo Pareto had little to say about joint production. (An exception is Knut Wicksell who at least touched upon the problem.) It was only with John von Neumann, in a paper first given in 1930 at the Institute for Advanced Study in Princeton and then published in German in 1937 and translated into English in 1945, that joint production was given full attention (von Neumann, 1945). Von Neumann in fact assumed universal joint production coupled with the assumption of free disposal according to which any product in excess supply, whether good or bad, can miraculously be removed at zero cost. The problem of joint production is also dealt with in some detail in Piero Sraffa's reformulation of the classical approach to the theory of production, distribution and value (Sraffa, 1960).

The following four sections are more analytical in character. We begin with a brief discussion of the free disposal assumption and its implications, followed by three sections dealing with various aspects of costly disposal. In the first three sections, the argument will be kept as simple as possible. Their purpose is to illustrate some of the more important aspects related to the case under consideration. We assume, for example, that all income is directly related to the labour performed and thus set aside the profits of capital. These will, however, be brought back into the picture in Section 6.

3 Free disposal: an exceedingly simple framework

Let us illustrate free disposal in terms of the following elementary example. Assume that there is only a single constant returns to scale process of production in which a single factor of production, homogeneous labour, produces two products.[5] Let us normalise the elementary production process by assuming that one unit of labour is employed. In this case, we may tabulate the process in the following way:

$$1 \rightarrow y_1 \oplus y_2 \tag{P.I}$$

where y_1 denotes the amount of product 1 and y_1 the amount of product 2 generated by one unit of labour. Taking the income per unit of labour, ω, as the standard of value, the following production (alias price) equation must hold:

$$\omega = 1 = y_1 p_1 + y_2 p_2 \tag{I}$$

where p_1 is the price of one unit of product 1 in terms of labour and p_2 is the price of one unit of product 2. Free disposal with regard to a product implies that in addition to production process (P.I), there is another process with the same outputs and the same inputs except for the amount of the product under consideration. With regard to our simple example, free disposal involves that in addition to process (P.I) there are the following two processes:

$$1 \rightarrow y_1 \tag{P.II.1}$$

$$1 \rightarrow y_2 \tag{P.II.2}$$

In other words, *net* amounts of each of the two products can also be produced singly. Hence, with processes (P.I) – (P.II) all non-negative proportions in which the two products are wanted can exactly be met.

Alternatively, we may describe free disposal by explicitly introducing disposal processes with regard to the two products. These could be written in the following way:

$$\text{Any amount of product 1} \oplus \text{No further inputs} \rightarrow 0 \tag{FD.1}$$

$$\text{Any amount of product 2} \oplus \text{No further inputs} \rightarrow 0 \tag{FD.2}$$

This way of describing the premise of free disposal emphasises neatly that it is at odds with a fundamental principle of thermodynamics: the principle of the conservation of mass. In case one of the two products is not wanted at all, or not wanted in the amount produced even if its price were to vanish, then it fetches a zero price and the whole burden of cost will be borne by the other product. Assume, for example, that this applies to product 2, then the price of product 1 would be given by:

$$p_1 = \frac{1}{y_1}$$

We can illustrate the assumption of free disposal with regard to each of the two products in the following diagram (see Figure 1). In it the (net) output vector per unit of labour associated with process (P.I) is given by $\mathbf{y} = (y_1, y_2)$; the (net) output vectors per unit of labour related to the two processes involving free disposal, (P.II.1) and (P.II.2), are given by $\mathbf{y}_1 = (y_1, 0)$ and $\mathbf{y}_2 = (0, y_2)$, respectively. ABC gives the net output frontier when one unit of labour is employed: it is the locus of all net output quantities of the two products that can be produced with the three processes available. Obviously, in order to provide an output combination that is located between A and B, process (P.I) must be linearly combined with process (P.II.2), whereas between B and C, it must be combined with process (P.II.1). Hence, in A and between A and B, product 1 is a free good, whereas in C and between B and C, product 2 is a free good. Since the price vector, $\mathbf{p} = (p_1, p_2)$, may be represented as a line drawn normal to the net output frontier, we can neatly illustrate our results:

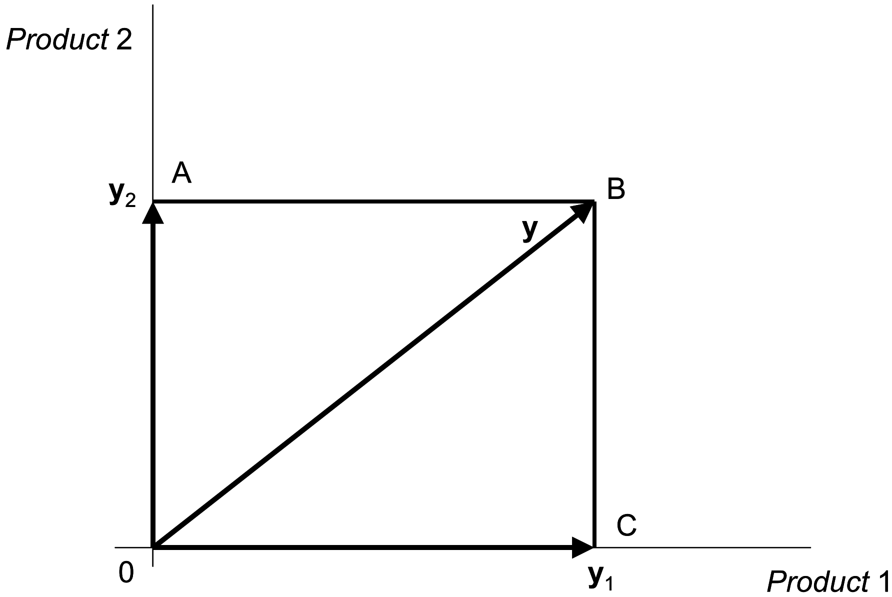
In A and between A and B: $\mathbf{p}_1 = \left(0, \frac{1}{y_2}\right)$

In C and between B and C: $\mathbf{p}_2 = \left(\frac{1}{y_1}, 0\right)$

In case the needs and wants (per unit of labour employed) of the society under consideration happen to request the two products exactly in the proportion in which they are produced, the prices are indeterminate: they can move together in an inverse manner within the intervals defined by price vectors $\mathbf{p}_1$ and $\mathbf{p}_2$.

While the denial of a fundamental principle of thermodynamics may be considered admissible, if there are disposals from a ‘small’ level of economic activity in a ‘large’ environment, things are different in the world in which we live. Therefore, the free disposal assumption cannot be sustained other than as a first approximation to the problem under consideration. Eventually, it has to give way to the assumption of costly disposal. In the ‘real world’, disposal is never really free. It may seem so because of the complete absence of property rights. Examples of this are smoke disposed of in the air and radioactive waste dumped in the ocean. Or it may seem so because of complex questions of externalities in the presence of only partially defined or insufficiently specified property rights. A case in point is the erection of waste disposal sites. Be that as it may, more or less costly disposal is the rule rather than the exception.

Figure 1 Free disposal



In the following, we deal with costy disposal on the bold assumption that the disposal of whichever substance has to be removed is both smooth and perfect. Hence, as in the case of free disposal, we assume that whatever it is that is being removed just disappears from the scene and thus does not impinge upon the remaining production and consumption processes. We illustrate the case of costly disposal first by means of a small adaption of our simple example above.

4 Costly disposal: a very simple analysis

Like the production process, the disposal processes are also taken to use only homogeneous labour as an active input, whereas the product to be disposed of may be considered a passive input. Let 1 unit of labour be required in order to get rid of z_1 units of product 1, then the disposal process corresponding to product 1 can be tabulated as:

$$1 \oplus z_1 \rightarrow 0 \qquad \text{(CD.1)}$$

Similarly, with regard to product 2, where one unit of labour can make z_2 units of product 2 disappear:

$$1 \oplus z_2 \rightarrow 0 \qquad \text{(CD.2)}$$

We may now write down the corresponding price equations. Obviously, the price equation of the production process is still given by Equation (I). In addition, we have the price equations associated with the two costly disposal processes:

$$1 + z_1 p_1 = 0 \qquad \text{(II.1)}$$

$$1 + z_2 p_2 = 0 \qquad \text{(II.2)}$$

In the case in which product 2 is not wanted at all, or not wanted in the quantity in which it occurs relative to product 1, a certain amount of product 2 has to be costly disposed of. In this case, the production process (P.1) and the disposal process (CD.2) will be operated simultaneously and define the following system of production-cum-disposal:

$$1 = y_1 p_1 + y_2 p_2 \tag{I}$$

$$1 + z_2 p_2 = 0 \tag{II.2}$$

These are two independent equations in two unknowns, p_1 and p_2. Solving (II.2) directly with respect to p_2, and inserting the result in Equation (I) gives:

$$p_2 = -\frac{1}{z_2} \tag{S.2}$$

$$p_1 = \frac{y_2 + z_2}{y_1 z_2} \tag{S.1}$$

The costly character of the disposal activity is first reflected in the *negativity* of the price of the product that is being disposed of: p_2 is the more negative, the smaller is the quantity of product 2 that can be removed by one unit of labour.

It is secondly reflected in a price of product 1, which is higher:

– the larger is the amount produced of product 2 (y_2)

– the smaller is the amount produced of product 1 (y_1)

– the smaller is the amount of product 2 (z_2) that can be disposed of.

We can now illustrate the argument again by means of a diagram. In Figure 2, $\mathbf{y} = (y_1, y_2)$ again gives the net output vector of the production process, whereas $\mathbf{z}_1 = (-z_1, 0)$ and $\mathbf{z}_2 = (0, -z_2)$ are the net output vectors per unit of labour employed by the two disposal processes, (CD.1) and (CD.2). The net output frontier of the system of production-cum-disposal per unit of labour is given by DEF. Following the line of reasoning in Section 3, we now get the following results:

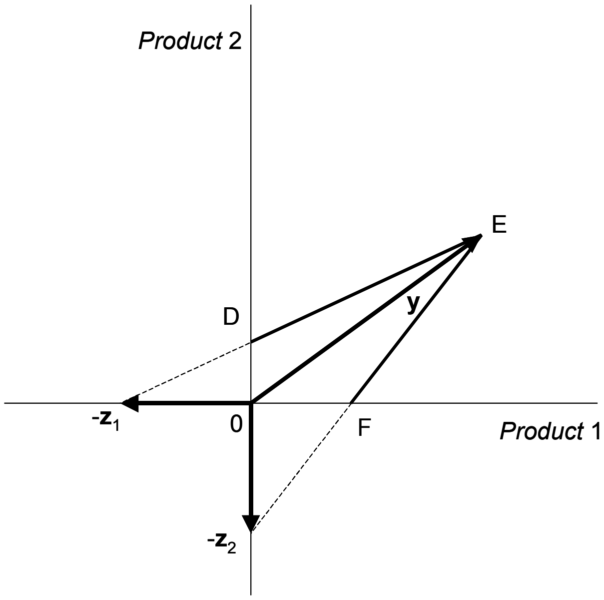
In D and between D and E: $\mathbf{p}_{D1} = \left(-\frac{1}{z_1}, \frac{y_1 + z_1}{y_2 z_1} \right)$

In F and between E and F: $\mathbf{p}_{D2} = \left(\frac{y_2 + z_2}{y_1 z_2}, -\frac{1}{z_2} \right)$

In case the needs and wants (per unit of labour employed) of the society under consideration happen to request the two products exactly in the proportion in which they are produced, the prices are again indeterminate: they can move together, but their movement is constrained by the interval defined by price vectors $\mathbf{p}_{D1}$ and $\mathbf{p}_{D2}$.

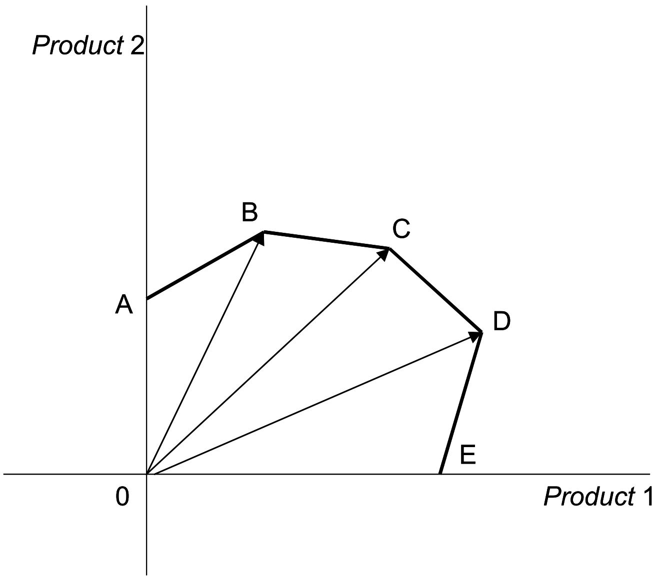
The talk of *negative* price is perhaps unfamiliar to some readers and may even strike them as repulsive. But the price of a product that must be disposed of is, of necessity, negative, if nobody is willing to take it for free. We may look upon the disposal activity also from another point of view. Accordingly, a disposal activity renders a service to the community, which will only be forthcoming if a positive price is paid for the service.

Figure 2 Costly disposal



Up until now, we have assumed that there is only a single production process and there are disposal processes for each of the products. Figure 3 illustrates the case in which there are three (non-dominated) production processes instead of only one. The net output frontier per unit of labour is now given by ABCDE. The reader will have no difficulty extending the argument to this case.

Figure 3 Production-cum-costly disposal



5 Transforming a bad into a good: a process-cum-product innovation

The above exceedingly simple framework may also be used to illustrate the case of an innovation that allows one to transform what hitherto has been a bad or discommodity, which had to be costly disposed of, into a good or commodity that is profitably marketed. Let the original situation be described again in terms of the co-existence of production process (P.I) and disposal process (CD.2). Then the corresponding price equations are (I) and (II.2) with solutions (S).

Assume now that a new process of production is being invented that allows one to produce a new product, product 3, obviously a good, by means of product 2 and labour. For simplicity, it is assumed that the new process is a single-product process. Here we have a combined *process* and *product invention* in front of us. The process (again normalised in such a way that one unit of labour is employed) can be tabulated as follows:

$$1 \oplus x_2 \rightarrow y_3 \qquad \text{(P.II)}$$

where x_2 is the amount of product 2 needed per unit of labour to produce the amount y_3 of product 3. Since the invention renders possible the conversion of at least a part of the amount generated of the former bad into a good, the invention can be expected to become an *innovation*. Hence, the process will actually be introduced and a new market – the market for product 3 – will emerge. In this case, the following price equation related to process (P.II) must be satisfied:

$$1 + x_2 p_2 = y_3 p_3 \qquad \text{(III)}$$

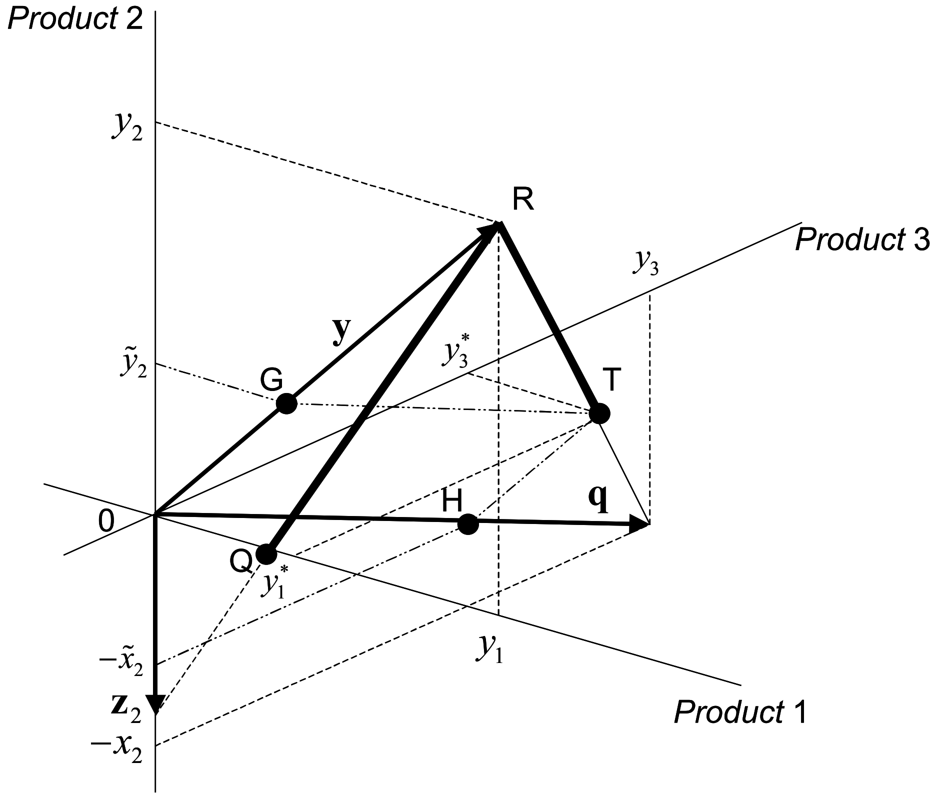
We can now illustrate the new situation after the introduction of process (P.II). In Figure 4 which, of necessity, has three dimensions instead of only two, the net output vector of process (P.II) is given by $\mathbf{q} = (0, -x_2, y_3)$, those of the other two processes by $\mathbf{y} = (y_1, y_2, 0)$ and $\mathbf{z}_2 = (0, -z_2, 0)$. In the case in which the entire amount of product 2 produced by process (P.I) is used as an input in process (P.II), the disposal process (CD.2) need not be activated at all and the net output per unit of labour is given by T. Process (P.I) is activated at level 0G and produces the amount $\tilde{y}_2$ of product 2, while process (P.II) is activated at level 0H and absorbs $\tilde{x}_2$ of product 2, where:

$$\tilde{y}_2 + \tilde{x}_2 = 0$$

The total amounts produced (net) per unit of labour employed of the two commodities are y_1^* and y_3^*. This compares rather favourably with the net output point Q in the original situation. In case only some of product 2 can be used, the disposal process (CD.2) has to be operated in order to get rid of the residual waste. The reader is invited to draw the implications with regard to the different possibilities involved and ascertain the prices of the three products in each situation.

One of the lessons the above example teaches us is that there is generally no *a priori* distinction possible between goods or commodities on the one hand and bads or discommodities on the other. Whether a product belongs to one class or the other depends not only on the needs and wants of society but also on the technical alternatives available to producers. What in one system of production-cum-disposal is a bad, might in another one be a good.

Figure 4 A product-cum-process innovation



As long as not the entire quantity generated of product 2 can be employed in the production of some good or other, there will be an incentive to 'reveal the useful properties' of the residual waste. Further, since the assumption of single production entertained in the above with regard to process (P.II) is difficult to sustain, assuming a multiple-product process instead gives us an idea of how an innovation, while solving a problem at a given time, becomes the source of one or several new problems. Imagine that instead of (P.II) we have:

$$1 \oplus x_2 \rightarrow y_3 \oplus y_4 \oplus y_5 \qquad \text{(P.III)}$$

While process (P.III) allows one to utilise product 2 in order to produce product 3, a good, it also generates two further substances – products 4 and 5. All would be well, at least from an anthropocentric point of view, if these products also turned out to be goods. However, one or even both of them could be bads, which will have to be disposed of. Hence, there is not only room for further inventions regarding production and disposal processes and thus products; such inventions might in fact be badly needed in order to be able to cope with the emergence of new harmful substances.

It can perhaps be said without too much of an exaggeration that in the presence of ubiquitous joint production and the generation of large amounts of waste, a solution to one pressing problem typically comes at the cost of the creation of one or several new problems. These may look insignificant at first, and in this case will typically be disregarded, but over time they may quickly gain in importance. This is but an expression of the law of unintended social outcomes. Without taking into consideration the technological dynamism inherent in systems that jointly produce goods and bads, it appears to be difficult to understand the recent economic history of modern industrial systems.

Up until now we have implicitly assumed that all income in the economy under consideration is distributed in proportion to the labour performed. In the following section, a slightly more general analysis of costly disposal will be put forward. We abandon the premise that there are no profits and assume instead that the wage rate per unit of labour is given (in terms of some standard). With free competition, there will be a uniform rate of profits on capital.

6 Costly disposal: a slightly more general analysis

We generalise the analysis somewhat in order to cover the case in which commodities are produced by means of commodities in a circular flow, which fits James Mill's famous dictum that man cannot create matter, man can only decompose and recompose it, change its form and move it. Production involves destruction, and the real cost of a commodity consists in the commodities destroyed in the course of its production, that is, means of production and means of subsistence in the support of producers. Production can now be described in terms of joint production processes such as $(\mathbf{a}_j, \mathbf{b}_j, l)$, $j = 1, 2, ..., m$. Process j transforms the vector of inputs, $\mathbf{a}_j$, by means of l units of labour during the (uniform) production period into the vector of outputs, $\mathbf{b}_j$. Collecting the information available about all processes of production in matrices **A** and **B** and labour input vector **l**, we may describe the technology comprising all processes by:

$$(\mathbf{A}, \mathbf{l}) \rightarrow \mathrm{B}$$

If there are n producible products and m processes of production, then matrices **A** and **B** are of dimension $m \times n$ and vector **l** is of dimension m.

Some of the products generated by means of the given technology may be bads, or discommodities, that nobody wants and which, if not removed, may be harmful to man (and other creatures). Hence, in any real system there will generally exist customs and mores, often materialised in legal terms, necessitating the disposal of such substances. Costly disposal can now be formalised in the following way. Assume that for each process producing a bad $(\mathbf{a}, \mathbf{b}, l)$, there is another process $(\mathbf{a}^*, \mathbf{b}^*, l^*)$ with the same output vector except that the bad is not produced and $\mathbf{a}^* = \mathbf{a} + \boldsymbol{\alpha}$ and $l^* = l + \lambda$. Obviously, $(\boldsymbol{\alpha}, \lambda)$ are the material and the labour inputs required to produce the goods in the production process under consideration and also to remove the quantity of the bad produced. (If there were to be free instead of costly disposal, we obviously would have $\boldsymbol{\alpha} = \mathbf{0}$ and $\lambda = 0$.) Process $(\mathbf{a}^*, \mathbf{b}^*, l^*)$ is thus constructed in such a way that it already combines the corresponding production process with the disposal process by means of which one can get rid of the amount of the bad produced. Hence the bad no longer appears at all: it is as if it had never been produced, because in the very moment in which it is about to make an appearance, it is removed from sight. Yet if one wants to render the generation and removal of the bad visible, the two processes – the production and the disposal process – must be disentangled. If this is done with respect to all bads and all products produced that are in excess supply even if their prices were to vanish, bads and overproduced products that have to be costly disposed of will generally have negative prices in cost-minimising techniques. Clearly, the price of the bad in our example above cannot be lower than $-(\alpha^T\mathbf{p} + \lambda w)$, where $\mathbf{p}$ is the price vector that is found together with the competitive rate of profits, r, as a solution of the following system of price equations:

$$\mathbf{B}^*\mathbf{p} = (1 + r)\mathbf{A}^*\mathbf{p} + w\mathbf{l}^*$$

Here, $\mathbf{B}^*$, $\mathbf{A}^*$ and $\mathbf{l}^*$ refer to those production and disposal processes actually chosen by cost-minimising producers, given the wage rate w in terms of a standard of value that consists of the n-dimensional, semi-positive vector of amounts of commodities, $\mathbf{b}$. Thus:

$$\mathbf{b}^T\mathbf{p} = 1.$$

Obviously, this vector will typically contain only positive quantities of some or all commodities, that is, products that fetch a positive price.[6]

We have heard that man cannot create matter, man can only decompose or recompose it or change its form and location. All production involves man's interaction with the world around him, his use of the directly or potentially useful resources he finds there. Therefore a discussion of the problems under consideration would be even less complete than it will be in this paper if the issues of renewable and exhaustible resources were not at least mentioned. This will be done in the concluding section.

7 Renewable and exhaustible resources

Renewable resources are resources capable of regenerating themselves provided the environment that nurtures them remains favourable. Examples are populations of birds or fish. The environment can be natural or artificial (that is, man-made). Agricultural, aquacultural, and zootechnical products are grown in an (at least partly) artificial environment. The size of the latter is enlarged or reduced in proportion to the harvest that is persistently to be obtained. The laws of the growth of such resources, nurtured in an artificial environment, can be described by a set of alternative processes, and the choice of technique of cost-minimising producers to meet needs and wants will also determine the size of the artificial environment utilised. The costs connected with the use of such resources are the costs incurred in raising them. Yet, the natural laws of the growth of animal populations living in a natural environment are not controlled by man. The costs connected with the use of such resources are typically the costs of their appropriation, for example, the costs of the catch in the case of fish that grows naturally. These costs will depend on the size of the natural environment under consideration, the total stock of fish in existence and its distribution in the environment.

It is beyond this paper to provide a proper analysis of renewable resources nurtured in a natural environment. A few observations on a particular case – the catch of fish – must suffice. The usual assumption in bioeconomic models and in the fishery literature is that the growth of the stock of fish is a non-monotonic function simply of the size of the stock itself. With u_t as the stock of fish in existence at time t, and setting aside human catch of fish, then at time $t + 1$ there will be u_{t+1} fish, where:

$$u_{t+1} = u_t + f(u_t)$$

$f(u_t)$ is taken to be negative both for very small and for very high values of u_t, whereas in the range for which $f(u_t) \geq 0$, $f(u_t)$ is first increasing and then decreasing. This is motivated in terms of the following considerations. If the population of fish is very small, then procreation is difficult, and if it gets very large, then the 'carrying capacity' of the environment will eventually be met. There is typically a threshold level of fish stock, u^*, which is essential for the survival of the species. If the population happens to fall below this threshold level, reproduction is less than natural mortality, and the species gradually dies out. Obviously, the extinction of the species is unavoidable if the catch of fish per year, c_t, is continually higher than the natural replenishment of the species, that is, if:

$$c_t > f(u_t)$$

For a discussion of a case in which there is a choice between raising fish aquaculturally and catching it in its natural environment, see Kurz and Salvadori (1995, Ch. 12, Section 3). Obviously, the lower the cost per unit of output with the best-practice aquacultural process and the higher the cost of the catch, the larger is the probability that fish living in its natural environment is disposed of freely, which, in the present context does not mean its catch (*i.e.*, 'destruction') but rather its 'conservation'. Yet if the conservation of fish happens to trigger a growth in stock, then unit costs of the catch will tend to fall and render the alternative process of generating fish more profitable. With a sufficient growth in stock and the corresponding fall in unit costs, there might eventually be a switch back from aquacultural to more conventional ways of providing fish to the market. This, however, involves a discontinuous shift in regime and may again endanger the wildly living species owing to excess harvesting.

In the case contemplated above, where $c_t > f(u_t)$, a renewable resource will be depleted. This is a borderline case between renewable and exhaustible resources. The latter are defined as resources that are available in given stocks that cannot be increased. These stocks can only be left as they are or they can be depleted. They are indeed depleted each time parts of them are removed for productive or consumptive purposes. A characteristic feature of exhaustible resources therefore is that their natural rate of growth is zero (or very close to zero). Typical examples of exhaustible resources are fossil fuels and oil.

Exhaustible resources pose a number of intricate problems to the economic theorist. Here is not the place to enter in some depth into these problems. Again, a few observations must suffice. Some exhaustible resources (coal, oil, gas) are used to produce energy that can safely be assumed to be an input in all production processes. This precludes their investigation in terms of a partial analysis. Instead, a general analysis is needed. While recycling is in some cases possible, for obvious reasons it is necessarily incomplete. Hence, deposits of such resources will be exploited and eventually fully exhausted. However, if for each exhausted deposit of the resource, another one with the same characteristics were to be discovered, and if the cost of the search (in terms of labour and means of production) were to be always the same, then the resource would not be exhaustible. In this case, each deposit could be treated as if it were a durable means of production, a machine, for example: the price of the new machine (deposit) equals the cost of search and the price of the old machine of age T equals the value of the deposit after T periods of utilisation. Since the 'resource' would not be exhaustible any more, its price could be constant over time.

If, however, the resource gets gradually exhausted, its price will have to rise over time. This is so because in competitive conditions, selling some of the resource from a given deposit and leaving the rest of it *in situ* must be equally profitable. Hence, the storing of the resource, which may be considered a 'conservation industry', cannot be operated if it does not pay a 'royalty' to the owner of the resource. The size of the royalty must be such that the general rate of profits is earned also in this activity. This, however, means that the price of the resource must increase from one period to the next, with the going rate of profits as the rate of its increase:

$$p_{t+1} = p_t(1 + r_t)$$

where p_{t+1} is the price per unit of the resource in period $t + 1$, p_t is the price of the resource in period t, and r_t is the rate of profits in that period. This is known as *Hotelling's rule*. For a discussion of some problems connected with the presence of exhaustible resources in a general framework of the analysis of classical derivation, see Kurz and Salvadori (2000) and the special issue of *Metroeconomica* (2001).

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Notes

1 On the empirical importance of joint production, see, for example, Steedman (1984). Steedman has also pointed out in several papers how the presence of even one multiple-product process of production in an otherwise single-product system can qualitatively affect the properties of the system; see, for example, Steedman (1982).

2 For a fuller account of the contributions of classical and early marginalist economists to an analysis of joint production, see Kurz (1986).

3 In this context, see Smith's discussion of sugar cultivation and the joint production of rum and molasses (Smith, 1976a, *WN*, I.xi.b.32) and von Thünen's observation: "The waste products from distilling can be most profitably used for stock feed" (von Thünen, 1966, p.275; my translation).

4 In this regard he was anticipated by von Thünen (see Kurz, 1986).

5 The construction contradicts a fundamental principle of thermodynamics: the principle of the conservation of mass. For the purpose of the following analysis, this is taken to be admissible in the interest of expository simplicity. We shall, however, swiftly come across cases where a denial of that principle cannot be accepted.

6 For a proper discussion of systems of production of this kind, see Kurz and Salvadori (1995).