Endogenous Growth

Ever since the inception of systematic economic analysis at the time of the classical economists from William Petty to Adam Smith and David Ricardo, the problem of economic growth—its sources, forms and effects—was high on the agenda of economists. In the early authors, economic growth, or rather economic development, was considered as generated from within the socio-economic system as a consequence of the purposeful activities of humans. These activities generated a number of effects, some intended, others not. History was therefore seen as the ‘result of human action, but not [of] human design’ (Ferguson, 1793, p. 205). Adam Smith placed special emphasis on the role of an ever-deeper division of labour on labour productivity. Capital accumulation, by increasing the extent of markets, was seen to be a prime mover of economic development and productivity growth. An aspect of the division of labour was that research and development became the ‘sole trade and occupation of a particular class of citizens’, engaging ‘philosophers or men of speculation, whose trade it is, not to do anything, but to observe every thing: and who, upon that account, are often capable of combining together the powers of the most distant and dissimilar objects’ (Smith, 1776 [1766], p. 21). The writings of the classical economists foreshadow the concepts of induced and embodied technical progress, learning by doing, and learning by using. Labour did not constrain growth because it was considered to be created endogenously and attuned to the needs of capital accumulation.

Karl Marx, too, considered the development of the productive powers of society and of economic growth as the outcome of the forces at work in a capitalist economy. Capital accumulation and innovation are seen as the necessary result of competitive conditions in which the single capitalist can survive only if his business expands and is continually modernised: ‘Accumulate, accumulate! This is Moses and the prophets’ (Marx, 1976 [1867]). Labour-saving technical progress feeds an industrial reserve army of the unemployed which constrains the aspirations of the working class.

The idea of an economic system growing exclusively because some exogenous factors make it grow is of a much later date. We encounter it, for example, in Alfred Marshall and in Gustav Cassel. It was, however, only with the neoclassical models of Robert Solow and Trevor Swan that it gained prominence. In Solow’s (1956) model, the steady-state growth rate is given from outside; it equals the (exogenous) rate of growth of the working population plus the (exogenous) rate of labour-saving technical progress.

The idea of endogenous growth figured prominently in post-Keynesian growth theory championed by Nicholas Kaldor and Joan Robinson, with the emphasis on the role of effective demand in the long run. However, it became particularly prominent from the 1980s onward even in mainstream economics due to a growing disenchantment with and critical assessment of the theoretical features and empirical implications of the Solovian model. Key properties of the ‘new’ or ‘endogenous’ growth models are:

- the steady-state rate of growth is determined by the behaviour of agents;
- innovation occurs in response to profitability;
- the emphasis is on explaining intensive growth, that is, growth of per capita income;
- capital accumulation and innovation generate externalities; and
- the endogenous mechanisms at work prevent the returns to capital from falling.

A few remarks on the most prominent new growth models are apposite (see Romer et al., 1994; Barro and Sala-i-Martin, 1995; Jones, 1998; and Kurz and Salvadori, 2003).

One class of models sets aside all non-accumulable factors of production (labour and land) and assumes that all inputs are ‘capital’ of some kind. The simplest version of this class is the so-called ‘AK model’, or ‘linear model’, which assumes that there is a linear relationship between total output, $Y$, and a single factor capital, $K$, both consisting of the same commodity:

$$Y = AK,$$  \hspace{1cm} (1)

where $1/A$ is the amount of that commodity required to produce one unit of itself. The rate of return on capital $r$ is given by:

$$r + \delta = \frac{K}{K} = A,$$ \hspace{1cm} (2)

where $\delta$ is the exogenously given rate of depreciation. This model is essentially a variant of Ricardo’s ‘corn model’ in which corn is produced by means of corn, land is a free good and $\delta = 0$. There are a large variety of models of this type in the literature. In the two-sector version in Rebelo (1991) it is assumed that the capital good sector produces the capital good by means of itself and nothing else. It is also assumed that there is only one method of production to produce the capital good. Therefore, the rate of profit is determined by technology alone.

Then the saving-investment mechanism jointly with the assumption of a uniform rate of growth determines a relationship between the growth rate, $g$, and the rate of profit, $r$. Rebelo obtains either:

$$g = \frac{A - \delta - \rho}{\alpha} = \frac{r - \rho}{\alpha},$$ \hspace{1cm} (3)

or:

$$g = (A - \delta) \alpha = sr.$$ \hspace{1cm} (4)

Equation (3) is obtained when savings are determined on the assumption that there is an immobile representative agent maximising the following intertemporal utility function:

$$\int_0^\infty e^{-\rho t} \frac{1}{1 - \sigma} [c(t)^{1-\sigma} - 1]dt,$$
subject to constraint (1), where $\rho$ is the given discount rate or rate of time preference, $1/\sigma$ is the elasticity of substitution between present and future consumption (1 $\neq \sigma > 0$), and $Y = \beta(t) + K$ (where $\beta(t)$ is consumption at time $t$ and $K$ is real investment). Equation (4) is obtained when the average propensity to save $\alpha$ is given. Hence, in this model the rate of profit is determined by technology alone and the saving-investment mechanism determines the growth rate. Obviously, the lower $\rho$ and $\sigma$ (the higher $\beta$), the higher is the steady-state rate of growth. With a constant population, $g$ gives of course the rate of growth of per capita income.

King and Rebelo (1990) assumed that there are two kinds of capital: real capital and human capital. There are two lines of production, one which produces a consumption good that serves also as a physical capital good, whereas the other produces human capital. The production functions relating to the two kinds of capital are assumed to be homogeneous of degree one and strictly concave. There are no diminishing returns to (composite) capital for the reason that there is no non-accumulable factor such as simple or unskilled labour. The rate of profit is uniquely determined by the technology and the maximisation of profits. The growth rate of the system is then endogenously determined by the saving–investment equation. The greater the propensities to accumulate human and physical capital, the greater the growth rate.

The linear models do not really contain any new insights into the growth process. They were anticipated in a two-sectoral framework by Robert Torrens and by Karl Marx in his schemes of extended reproduction. The most sophisticated linear model of endogenous growth, taking into consideration fixed capital and joint production and allowing for a choice of technique, was elaborated by John von Neumann (1937-1945). A macro version of the model was first put forward by E.D. Domar.

Another class of models preserves the dualism of accumulable and non-accumulable factors but restricts the impact of an accumulation of the former on their returns by a modification of the aggregate production function. Jones and Manuelli (1990), for example, allow for both labour and capital and even assume a convex technology. However, a convex technology requires only that the marginal product of capital is a decreasing function of its stock, not that its vanishes as the amount of capital per worker tends towards infinity. Jones and Manuelli assume that:

$$m(k) \approx bk, \text{ each } k \geq 0,$$

where $m(k)$ is the per capita production function and $b$ is a positive constant. The special case contemplated is:

$$m(k) = f(k) + bk,$$  \hspace{1cm} (5)

where $f(k)$ is the conventional Solovian per capita production function. As capital accumulates and the capital–labour ratio rises, the marginal product of capital will fall, approaching asymptotically $b$, its lower boundary. With a given propensity to save, $\alpha$, and assuming capital to be everlasting, the steady-state growth rate $g$ is endogenously determined: $g = \alpha b$. Assuming, on the contrary, intertemporal utility maximisation, the rate of growth is positive provided that the technical parameter $b$ is larger than the rate of time preference $\rho$. In the case in which it is larger, the steady-state rate of growth is given by equation (3) with $\rho = b$.

The models dealt with up until now can hardly be said to contain any original novelties or to provide new insights into the process of economic development and growth. More interesting in this respect are the large and growing class of models contemplating various factors counteracting any diminishing tendency of returns to capital and generating a growth in per capita income. Here the focus will be on the following two subcategories: human capital formation and knowledge accumulation. In both kinds of models positive external effects play an important part, they offset any fall in the marginal product of capital and are the source of a growing income per person.

Models of the first subclass attempt to formalise the role of human capital formation in the process of growth. Elaborating on some ideas of Uzawa (1965), Lucas (1988) assumed that agents have a choice between two ways of spending their (non-leisure) time: to contribute to current production or to accumulate human capital. Lucas’s conceptualisation of the process by means of which human capital is built up is the following:

$$h = \varphi(1 - u),$$

where $u$ is a positive constant.

With the accumulation of human capital there is said to be associated an externality: the more human capital society as a whole has accumulated, the more productive each single member will be. This is reflected in the following production function:

$$Y = AK(uN)^{1-\beta}(w^{**})^{\gamma},$$

(7)

where the labour input consists of the number of workers, $N$, times the fraction of time spent working, $u$, times $h$ which gives the labour input in efficiency units. Finally, there is the term $w^{**}$ designed to represent the externality. The single agent takes $h^*$ as a parameter in his/her optimising by choice of $c$ and $u$. However, for society as a whole the accumulation of human capital increases output both directly and through the externality. Here we are confronted with a variant of a public good problem. The individual optimising agent faces constant returns to scale in production: the sum of the partial elasticities of production of the factors he/she can control, that is, the individual’s physical and human capital, is unity. Yet for society as a whole the partial elasticity of production of human capital is not $1 - \beta$, but $1 - \beta + \gamma$. As is well known, whenever there is a public good problem there is room for economic policy designed to correct an inefficient private supply of the good.

It can be shown (see Kruz and Salvadori, 1998) that if the above-mentioned externality is not present ($\gamma$ in equation (7) equals zero) and therefore returns to scale are constant, endogenous growth in Lucas’s model is obtained in essentially the same way as in the linear models: the rate of profit is determined by technology and profit maximisation alone; and for the predetermined level of $r$ the saving–investment mechanism determines $g$. Yet, as Lucas himself pointed out, the endogenous growth is positive independently of the fact that there is the above-mentioned externality. Therefore, while complicating the picture, increasing returns do not add substantially to it: growth is
endogenous even if returns to scale are constant. If returns to scale are not constant then neither the competitive technique nor the associated rate of profit is determined by technical alternatives and profit maximisation alone. Nevertheless, these two factors still determine, in steady states, a relationship between the rate of profit and the rate of growth. This relationship, together with the relationship between the same rates obtained from the saving-investment mechanism, determines both variables.

Models of the second subclass attempt to portray technological change as generated endogenously. The proximate starting-point of this kind of models was Arrow’s (1962) paper on ‘learning by doing’. Romer (1986) focuses on the role of a single variable called ‘knowledge’ or ‘information’ and assumes that the information contained in inventions has the property of being available to anyone to use make of it at the same time. In other words, information is considered essentially a non-rival good. Yet, it need not be totally non-excludable, that is, through some institutional arrangements (for example, patent rights) it can be monopolised at least temporarily. It is around the two different aspects of publicness – non-rivalry and non-excludability – that the argument revolves. Discoveries are made in research and development (R&D) departments of firms. This requires that resources be withheld from producing current output. The basic idea of Romer’s model is that there is a trade-off between consumption today and knowledge that can be used to produce more consumption tomorrow. He formalises this idea in terms of a ‘research technology’ that produces ‘knowledge’ from forgone consumption. It is boldly assumed that knowledge is cardinal measurable.

Romer stipulates a research technology that is concave and homogeneous of degree one,

\[ \dot{k}_i = G(\dot{I}_i, k_i), \]  

(8)

where \( \dot{k}_i = \frac{dk_i}{dt}, \dot{I}_i \) is an amount of forgone consumption in research in firm \( i \) and \( k_i \) is the firm’s current stock of knowledge. The production function of the consumption good relative to firm \( i \) is

\[ Y_i = f(k_i, X_i), \]  

(9)

where \( X \) is the accumulated stock of knowledge in the economy as a whole and \( x_i \) is the vector of all inputs different from knowledge. The function is taken to be homogeneous of degree one in \( x_i \) and homogeneous of a degree greater than one in \( k_i \) and \( X \). Romer assumes that factors other than knowledge are in fixed supply. Spillovers from private R&D activities increase the public stock of knowledge \( X \). Again, a positive externality is taken to be responsible for per capita income growth. And again there is room for economic policy in order to overcome a socially suboptimal generation of new knowledge.

Assuming, contrary to Romer, that the above production function (9) is homogeneous of degree one in \( x_i \) and \( X \) involves a constant marginal product of capital: the diminishing returns to \( k_i \) are exactly offset by the external improvements in technology associated with capital accumulation. In this case it can be shown that, similar to the models previously dealt with, the rate of profit is determined by technology and profit maximisation alone, provided, as is assumed by Romer, that the ratio \( XK \) equals the given (1) number of firms. Once again, endogenous growth does not depend on an assumption about increasing returns with regard to accumulable factors, whereas a growing per capita income does. Such an assumption would only render the analysis a good deal more complicated. In particular, a steady-state equilibrium does not exist, and in order for an equilibrium to exist the marginal product of capital must be bounded from above. This is affected by Romer in terms of an ad hoc assumption regarding equation (8).

Finally, there are so-called ‘neo-Schumpeterian’ models of growth which take into account the fact that self-seeking behaviour which leads to technical and organisational innovation and economic growth typically generates both positive and negative externalities; see in particular Aghion and Howitt (1998). These models revolve around Joseph Schumpeter’s concept of ‘creative destruction’. A firm that innovates successfully manages to obtain a temporary monopoly position and can reap extra profits. Wages growing in line with labour productivity at the same time render capital stocks embodying older vintages of technical knowledge obsolete. This poses, among other things, once again the question of what is the socially optimal rate of ‘technical innovation’.

In conventional theory, whenever increasing returns that are (dominantly) internal to the firm, externalities, public goods (or bads), incomplete and asymmetric information and so on are involved, there is a problem of market failure. Since the literature on ‘new’ or ‘endogenous’ growth revolves around precisely these phenomena, the question of public policy, institutional arrangements and mechanism design are close at hand. While capital accumulation is still at the centre of the analysis, these wider issues, which figured prominently in the classical authors, have been brought back into the picture.

**References**


