Metroeconomica 52:3 (2001) 282–296

CLASSICAL ECONOMICS AND THE PROBLEM OF EXHAUSTIBLE RESOURCES

Heinz D. Kurz and Neri Salvadori
University of Graz, Austria, and University of Pisa, Italy

ABSTRACT

In this paper we discuss in terms of the simple model of exhaustible resources proposed by Bidard and Erreygers some of their propositions. The concept of 'real rate of profit' introduced by them is shown to be of no analytical use. It is stressed that the mathematical properties of the economic system under consideration are independent of the numéraire adopted. The classical treatment of exhaustible resources in terms of differential rent is shown to be correct under well-defined conditions. It is argued that it is complementary to, rather than incompatible with, the approach which emphasizes that in conditions of free competition the rate of profit obtained by conserving the resource equals that in production processes.

In section 1 we shall discuss the mathematical properties of the simple model proposed by Bidard and Erreygers (2001). We shall solve the model for a given real wage rate paid at the beginning of the uniform production period. In section 2 we shall question the usefulness of the concept of a 'real profit rate' suggested by Bidard and Erreygers and their view that the choice of numéraire can have an impact on the mathematical properties of the system under consideration. In sections 3 and 4 we assess some of the propositions put forward by Bidard and Erreygers. Section 3 deals with the fact that any economic model is bound to distort reality in some way and therefore can never be more than an attempt to 'approximate' important features of the latter. This is exemplified by means of the labour theory of value in classical economics, on the one hand, and by Ricardo's assimilation of the case of exhaustible resources to that of scarce land and thus its subsumption under the theory of differential rent, on the other. In certain well-specified circumstances royalties are replaced by rents, while in other circumstances neither rents nor royalties play any role. In section 4 we turn to the so-called Hotelling rule. It is stressed that in order for this rule to apply there must be no obstacles whatsoever to the uniformity of the rate of profit across conservation and production processes, and the available amounts of the resources must be bounded and known with certainty. Therefore Hotelling's rule cannot be considered so generally applicable as Bidard and Erreygers seem to suggest.

1. THE CORN–GUANO MODEL WITH A GIVEN REAL WAGE RATE

Bidard and Erreygers propose a simple model to investigate the elementary properties of an economy employing exhaustible resources, a model, they maintain, which 'constitutes an adaptation and the theoretical equivalent of the standard corn model for the classical theory of long-term prices' (p. 244). We find their concern with simplicity laudable. However, with Albert Einstein we insist that while a model should be as simple as possible, it must not be simpler than that. Indeed in our view the model suggested by Bidard and Erreygers, or rather their interpretation of it, neglects aspects of the problem under consideration that are important and can already be seen at the suggested low level of model complexity.

The two authors point out that the argument in their 'corn–guano model' could be formulated either in terms of a given real wage rate or in terms of what they call a given 'real rate of profit'. They then decide to develop fully only the second variant but stress that in the alternative case the 'dynamic behaviour of the system is completely similar'. In both models wages are paid at the beginning of the production period. Since, as will be made clear below, we doubt that the concept of 'real rate of profit' can be given a clear meaning and useful analytical role in the investigation under discussion, we shall start from a given real (i.e. corn) wage rate paid *ante factum*.

In accordance with the two authors we assume that there are two commodities, corn and guano, which can be produced or conserved by the processes depicted in table 1, where a_1 and a_2 are corn inputs per

Table 1

	Inputs			*Outputs*	
	Corn	*Guano*		*Corn*	*Guano*
(1)	a_1	1	→	1	—
(2)	a_2	0	→	1	—
(3)	—	1	→	—	1

unit of corn output inclusive of the corn wages paid to labourers ($0 < a_1 < a_2 < 1$). The quantity side of the model is not made explicit by Bidard and Erreygers; it is just assumed that from time 1 to time T processes (1) and (3) are operated, from time T to infinity process (2) is operated, and at time $T+1$ guano is exhausted and therefore processes (1) and (3) cannot be operated anymore. This assumption involves some sort of implicit theorizing and is invoked by us only in order to keep close to the procedure followed by Bidard and Erreygers. However, on the assumptions stated no difficulty appears to arise.[1]

The model has the following equations:

$$p_{t+1} = (1 + r_t)(a_1 p_t + z_t) \qquad 1 \leqslant t \leqslant T \tag{1.1}$$

$$p_{t+1} = (1 + r_t)a_2 p_t \qquad t \geqslant T \tag{1.2}$$

$$z_{t+1} = (1 + r_t)z_t \qquad 1 \leqslant t \leqslant T \tag{1.3}$$

where p is the price of corn, r the nominal rate of profit and z the price of guano at the time indicated by the corresponding subscript. The sequence of nominal rates of profit $\{r_t\}$ is assumed to be given. However, it is easily checked that the given sequence plays no role in determining the relative present value prices in the sense that, if the sequences $\{p_t\}$ and $\{z_t\}$ are a solution to system (1) for the given sequence $\{r_t\}$, then the sequences $\{q_t\}$ and $\{u_t\}$ such that

$$q_t = \prod_{\tau=0}^{t-1} \frac{1 + \sigma_\tau}{1 + r_\tau} p_t$$

$$u_t = \prod_{\tau=0}^{t-1} \frac{1 + \sigma_\tau}{1 + r_\tau} z_t$$

are also a solution to system (1) for a given sequence $\{\sigma_\tau\}$. This is so because r_t is the *nominal* rate of profit.

It is also easily checked that the above model can determine only the relative present value prices in the sense that, if the sequences $\{p_t\}$ and

[1] Things would be different in the case in which wages are paid *post factum*. In this case, in fact, if the process producing corn without guano is more expensive in terms of labour input but less expensive in terms of corn input than the process producing corn with guano, we cannot exclude that corn is produced first without guano, then with guano until guano is exhausted, then without guano once again. For an example of this type, see Kurz and Salvadori (1997, pp. 248–9).

$\{z_t\}$ are a solution to system (1), then the sequences $\{\eta p_t\}$ and $\{\eta z_t\}$ are also a solution, where η is a positive scalar. This means that there is room for a further equation fixing the numéraire. The numéraire is chosen by the observer and is not related to an objective property of the economic system, apart from the obvious fact that the numéraire must be specified in terms of valuable things (e.g. commodities, labour) that are a part of the economy that is being studied. As Sraffa emphasized in the context of a discussion of the particular numéraire suggested by him: 'Particular proportions, such as the Standard ones, may give transparency to a system and render visible what was hidden, *but they cannot alter its mathematical properties*' (Sraffa (1960, p. 23), emphasis added). We maintain that, whenever the choice of the numéraire seems to affect the objective properties of the economic system under consideration, then there is something wrong with the theory or model: the objective properties of the economic system must be totally independent of the numéraire adopted by the theorist. Hence the choice of a particular numéraire may be useful or not, but it cannot be right or wrong.

In order to fix the numéraire and to preserve the property that a change in the nominal rates of profit does not affect relative prices, the numéraire is to be set in terms of present value prices (at time θ); i.e. we could add, for example, the equation

$$\sum_{t=0}^{\infty}(h_t p_t + k_t z_t)\sum_{\tau=0}^{t-1}(1 + r_\tau)^{\theta - t} = 1 \tag{2}$$

where $\{h_t\}$ and $\{k_t\}$ are sequences of known non-negative magnitudes such that for some t either h_t or k_t, or both, are positive and $k_t = 0$ for all $t > T$.

In the following we will assume that $r_t = 0$, for each t. A change to another sequence of nominal rates of profit can be made at will, as indicated above. We shall also assume that $h_t = 0$ for each $t \neq T$, $h_T = 1$, $k_t = 0$ for each t, and $\theta = T$ in equation (2). Then system (1)–(2) is more simply stated as

$$p_{t+1} = a_1 p_t + z_t \qquad 1 \leqslant t \leqslant T \tag{3.1}$$

$$p_{t+1} = a_2 p_t \qquad t \geqslant T \tag{3.2}$$

$$z_{t+1} = z_t \qquad 1 \leqslant t \leqslant T \tag{3.3}$$

$$p_T = 1 \tag{3.4}$$

From equation (3.3) we see that

$$z_t = z_0 \qquad 1 \leqslant t \leqslant T \tag{4}$$

and then from difference equation (3.1), taking account of equation (3.4), we get

$$p_t = \frac{z_0}{1 - a_1} + \frac{1 - a_1 - z_0}{1 - a_1} a_1^{t-T} \qquad 0 \leqslant t \leqslant T + 1 \tag{5}$$

Then from difference equation (3.2), taking account of equation (3.4), we obtain

$$p_t = a_2^{t-T} \qquad t \geqslant T \tag{6.1}$$

Finally, taking account of the fact that equations (3.1) and (3.2) are both satisfied for $t = T$, we obtain

$$\frac{z_0}{1 - a_1} + \frac{1 - a_1 - z_0}{1 - a_1} a_1 = p_{T+1} = a_2$$

Hence

$$z_0 = a_2 - a_1$$

which, substituted in equations (4) and (5), completes the solution:

$$z_t = a_2 - a_1 \qquad 1 \leqslant t \leqslant T \tag{6.2}$$

$$p_t = \frac{a_2 - a_1}{1 - a_1} + \frac{1 - a_2}{1 - a_1} a_1^{t-T} \qquad 0 \leqslant t \leqslant T + 1 \tag{6.3}$$

What happens if there is a change of numéraire? Clearly, *relative* prices are unchanged. In fact, if $\{h_t\}$ and $\{k_t\}$ are sequences of known non-negative magnitudes (at least one of which is positive) such that the series $\sum_{t=T+1}^{\infty} h_t a_2^{t-T}$ is convergent, and if equation (3.4) is substituted by the equation

$$\sum_{t=0}^{T} (h_t p_t + k_t z_t) + \sum_{t=T+1}^{\infty} h_t p_t = 1$$

then the solution becomes

$$z_t = H(a_2 - a_1) \qquad 1 \leqslant t \leqslant T$$

$$p_t = H\left(\frac{a_2 - a_1}{1 - a_1} + \frac{1 - a_2}{1 - a_1} a_1^{t-T}\right) \qquad 0 \leqslant t \leqslant T + 1$$

$$p_t = H a_2^{t-T} \qquad t \geqslant T$$

where

$$H^{-1} = \frac{a_2 - a_1}{1 - a_1} \sum_{t=0}^{T} h_t + \frac{1 - a_2}{1 - a_1} \sum_{t=0}^{T} h_t a_1^{t-T} + \sum_{t=T+1}^{\infty} h_t a_2^{t-T} + (a_2 - a_1) \sum_{t=0}^{T} k_t.$$

The reader might also be interested in what happens if we use a sequence of $\{r_t\}$ different from $r_t = 0$, for each t. As an example, let us consider the case of a constant positive sequence:

$$r_t = r > 0 \qquad \text{for each } t$$

If, once again, $p_T = 1$, we get

$$z_t = (a_2 - a_1)(1 + r)^{t-T} \qquad 1 \leqslant t \leqslant T$$

$$p_t = \frac{(a_2 - a_1) + (1 - a_2) a_1^{t-T}}{1 - a_1} (1 + r)^{t-T} \qquad 0 \leqslant t \leqslant T + 1$$

$$p_t = [(1 + r) a_2]^{t-T} \qquad t \geqslant T$$

Obviously, in the special case in which

$$r = \frac{1 - a_2}{a_2}$$

the price of corn is constant from $t = T$ onward.

2. THE QUESTION OF THE 'REAL RATE OF PROFIT'

We have seen that, if the nominal rate of profit is zero at each t, then the price of guano is constant over time. We may now ask: is there also a sequence of nominal profit rates such that the price of corn is constant over time? Certainly there is.[2] One might also give a special name to this sequence, as is done by Bidard and Erreygers, and call it the sequence of 'real rates of profit'. But this is merely a name: the particular case referred to by it is neither more nor less important than any of the infinitely many possible representations of the genuine properties of the model under consideration. These properties concern in particular the following facts: (i) relative prices are independent of the numéraire; and (ii) relative prices corresponding to the same t are also independent of the sequence of nominal rates of profit. As a consequence, at any given point in time the amount of corn the proprietor of guano can obtain by selling one unit of guano (or the amount of corn a worker can obtain for a unit of labour; or the amount of corn the owner of one unit of corn can obtain by investing it either in corn production or in guano conservation; or the amount of guano the owner of one unit of guano can obtain by investing it either in corn production or in guano conservation) is independent of the numéraire and of the sequence of nominal rates of profit.

Starting from the two amounts just mentioned we can obtain what in the literature is known as the *own rate of return* of corn and guano, respectively. Consider an investor who possesses one unit of commodity j (corn or guano) and divides the investment at time t in two parts: d_1 in the production of corn and d_2 in the conservation of guano ($d_1 + d_2 = 1$ and $d_2 = 0$ if $t > T$). Hence, if $t \leqslant T$, at time t inputs are bought which at time $t+1$ yield $d_1\pi_{jt}/(a_1 p_t + z_t)$ units of corn and $d_2\pi_{jt}/z_t$ units of guano, where π_{jt} is the price of commodity j at time t. However, if $t > T$, then at time t an input of corn is obtained which at time $t+1$ yields $\pi_{jt}/a_2 p_t$ units of corn. If at time $t+1$ the investor wants to assess the yield of the investment in terms of commodity j, the answer is given by

$$\frac{d_1\pi_{jt}}{a_1 p_t + z_t}\frac{p_{t+1}}{\pi_{jt+1}} + \frac{d_2\pi_{jt}}{z_t}\frac{z_{t+1}}{\pi_{jt+1}} = (1 + r_t)\frac{\pi_{jt}}{\pi_{jt+1}} \qquad (t \leqslant T)$$

$$\frac{\pi_{jt}}{a_2 p_t}\frac{p_{t+1}}{\pi_{jt+1}} = (1 + r_t)\frac{\pi_{jt}}{\pi_{jt+1}} \qquad (t > T)$$

[2] The reader can easily check that the required sequence is

$$r_t = \frac{(1 - a_1)(1 - a_2)a_1^t}{(a_2 - a_1)a_1^T + (1 - a_2)a_1^{t+1}} \qquad 0 \leqslant t \leqslant T$$

$$r_t = \frac{1 - a_2}{a_2} \qquad t \geqslant T$$

Thus the own rate of return on an investment of commodity j is

$$\rho_{jt} = (1 + r_t)\frac{\pi_{jt}}{\pi_{jt+1}} - 1 = r_t\left(1 - \frac{\pi_{jt+1} - \pi_{jt}}{\pi_{jt+1}}\right) - \frac{\pi_{jt+1} - \pi_{jt}}{\pi_{jt+1}}$$

This rate is independent of the sequence of nominal rates of profit and of the numéraire adopted, as simple calculations show.[3] It is a *real* rate in the sense of Wicksell who defined interest in terms of parting with present goods 'in order in some way or other to obtain future goods *of the same kind*' ([1893] 1954, p. 107; emphasis added). The concept of a real or commodity rate of interest was also referred to by Sraffa in his criticism of Hayek's monetary overinvestment theory of the business cycle. Sraffa stressed that outside a long-period position of the economy (or, in Hayek's terminology, outside an 'equilibrium') 'there might be at any one moment as many "natural" rates of interest as there are commodities' (Sraffa (1932, p. 49)). He added with regard to an investor taking a loan:

> Loans are currently made in the present world in terms of every commodity for which there is a forward market. When a cotton spinner borrows a sum of money for three months and uses the proceeds to purchase spot, a quantity of raw cotton which he simultaneously sells three months forward, he is actually 'borrowing cotton' for that period. The rate of interest which he pays, per hundred bales of cotton, is the number of bales that can be purchased with the following sum of money: the interest on the money required to buy spot 100 bales, plus the excess (or minus the deficiency) of the spot over the forward prices of the 100 bales.— In [a long-period] equilibrium the spot and forward price coincide, for cotton as for any other commodity; and all the 'natural' or commodity rates are equal to one another, and to the money rate. (Sraffa (1932, p. 50))

It could be argued that for $t \geqslant T$ the price of corn is constant over time if the nominal rate of profit is $(1 - a_2)/a_2$, which is the rate of profit

---

[3] Let $\{r_t\}$, $\{\sigma_t\}$, $\{\pi_t\}$, and $\{s_t\}$ be sequences such that $\{r_t\}$ and $\{\sigma_t\}$ are non-negative, $\{\pi_t\}$ and $\{s_t\}$ are positive and

$$s_t = h\prod_{\tau=0}^{t-1}\frac{1+\sigma_\tau}{1+r_\tau}\pi_t\{r_t\}$$

Then

$$(1 + r_t)\frac{\pi_t}{\pi_{t+1}} = (1 + \sigma_t)\frac{s_t}{s_{t+1}}$$

that holds in the long period in an economy in which only the backstop technique is employed, and in this respect it can be considered the *long-period real* rate of profit (a well-defined concept). Hence one might be inclined to see whether the concept carries over to intertemporal analysis. However, any such inclination would immediately be frustrated: the fact that the price of corn is constant over time from time T onward if the nominal rate of profit coincides with the long-period *real* rate of profit is a consequence of the fact that there is only one reproducible commodity contemplated by the model. Suppose instead that there are two commodities, corn and iron. Then at time T the relative price of the two does not need to be the one prevailing in the long run even if they are produced from time T onward with the backstop processes. As a consequence, even if we take the long-period rate of profit as the nominal rate of profit from time T onward, prices will tend to the long-period prices only at infinity and will oscillate from time T onward. Using a modelling similar to that discussed in Kurz and Salvadori (2000), the following can be proved: to assume that the prices at time T are proportional to the long-period prices is equivalent to a very special assumption regarding the amounts of commodities and resources available at time 0. For lack of space we have to refrain from developing the argument in detail.

To conclude on the question of the *price* which Bidard and Erreygers take to be constant over time, we have to say that we were puzzled by sentences like:

> The *real profit* of an activity can be defined only by means of a standard. A measure of the real profitability of an activity is obtained when the value of the inputs at time t and the value of the outputs at time $t+1$ are translated into comparable units of 'what really counts'. (p. 246–7)

Unfortunately, the authors do not tell us 'what really counts', and why. Their references to Torrens and Fisher are not able to enlighten us. Until they can show conclusively that the concept of the 'real rate of profit' can be properly defined *and* can be put to productive analytical use, we see no reason to employ it.

3. ON THEORETICAL 'APPROXIMATIONS'

Bidard and Erreygers maintain: 'A post-Sraffian economist who feels disquiet about the labour theory of value and has imposed upon himself the intellectual requirement of working with a consistent theory of prices cannot be satisfied with the "approximation" of royalty by rent. A consistent theory of exhaustible resources is needed just as much as a consistent theory of prices' (p. 245). Economic theorists cannot do without bold assumptions whose role is to allow for approximations of the properties of the economic system under consideration. The labour theory of value was such an approximation or theoretical device to render transparent what otherwise would have remained impenetrable, given the analytical tools available at the time. It was a useful tool at a certain stage of the development of the analysis. As Ludwig Wittgenstein remarked, a particular theory may be compared to a ladder that is useful to reach a higher standpoint. However, once this standpoint is reached and a fuller view of the landscape is possible, the ladder may turn out to be an instrument that is inferior to some other device to reach the higher standpoint, and possibly beyond, and it will therefore be abandoned. This applies also to the labour theory of value: it was an instrument that provided useful services to the classical economists by introducing a constraint binding changes in the distributive variables, but once the problem of the relationship between income distribution and relative prices, given the system of production in use, had been fully solved, the labour theory of value had to be dispensed with because it did not provide a correct and fully satisfactory picture of that relationship.

Any economic model is bound to distort reality in some way. Otherwise it would be identical to the 'seamless whole' and thus useless in interpreting aspects of the latter. Sraffa was prepared to allow such 'distortions' in his own conceptualization of the production process. He was clear at an early stage of his work, which was to lead to his 1960 book, that the assumption of self-replacement of an economic system does not mimic reality. In the following note dated 25 March 1946, from his hitherto unpublished papers, he first points out a difference between a *physical real cost* approach to the problem of value and distribution, which he endorsed, and the labour theory of value:[4]

> The difference between the 'Physical real costs' and the Ricardo–Marxian theory of 'labour costs' is that the first does, and the latter does not, include in them the

[4] The papers are kept at Trinity College Library, Cambridge. The references follow the catalogue prepared by Jonathan Smith, archivist. We are grateful to Pierangelo Garegnani, Sraffa's literary executor, for permission to quote from the hitherto unpublished material.

natural resources that are used up in the course of production (such as coal, iron, exhaustion of land) — [Air, water etc. are not used up: as there is an unlimited supply, no subtraction can be made from ∞]. This is fundamental because it does away with 'human energy' and such metaphysical things.

He added:

> But how are we going to replace these natural things? There are 3 cases: a) they can be reproduced by labour (land properties, with manures etc.); b) they can be substituted by labour (coal by hydroelectric plant: or by spending in research and discovery of new sources and new methods of economising); c) they cannot be either reproduced nor substituted[5]—and in this case they cannot find a place in a theory of *continuous* production and consumption: they are dynamical facts, i.e. a stock that is being gradually exhausted and cannot be renewed, and must ultimately lead to destruction of the society. But this case does not satisfy our conditions of a society that just manages to keep continuously alive. (Sraffa's papers, D3/12/42: 33)

While a 'dynamic theory' would be needed to deal properly with exhaustible resources, Sraffa also reminded us of the intrinsic difficulties of elaborating such a theory. One of his notes reads:

> It is 'a fatal mistake' of some economists that they believe that, by introducing complicated dynamic assumptions, they get nearer to the true reality; in fact they get further removed for two reasons: a) that the system is much more statical than we believe, and its 'short periods' are very long, b) that the assumptions being too complicated it becomes impossible for the mind to grasp and dominate them—and thus it fails to realise the absurdity of the conclusions. (Sraffa: D3/12/11 (33))

In his book Sraffa mentioned exhaustible resources only in passing and on a par with land: 'Natural resources which are used in production, such as land and mineral deposits...' (Sraffa (1960, p. 74)). This is not the place for a full analysis of his view on the issue at hand. It must suffice to mention that he considered exhaustible resourses to be 'dynamical facts' which might be difficult to take account of in a 'theory of

[5] This is Sraffa's formulation, which we left as it is.

continuous production and consumption'. However, this did not make him abandon long-period analysis.

4. ON HOTELLING'S RULE[6]

Bidard and Erreygers write: 'Hotelling's rule is neither neoclassical nor classical; it is a necessary consequence of the notion of competitive solution: it is economic theory full stop' (p. 246). To be clear, Hotelling's rule (see Hotelling (1931)) is nothing but the application of the concept of a uniform rate of profit to all processes in the economy, whether these are conservation or production processes. But precisely because this is so, it applies only in certain circumstances and not in others. In particular, it presupposes that the following assumptions hold.

(i) The resource is available in homogeneous quality and in a quantity which at any moment of time is known with certainty.
(ii) The amount of the resource that can be extracted in a given period is only constrained by the amount of it left over from the preceding period.

If one of these assumptions is not met, then Hotelling's rule needs to be modified. The following case exemplifies this. It hardly needs to be stressed that assumption (i) is very bold indeed. In everyday experience new deposits of exhaustible resources are repeatedly discovered. The opposite extreme would consist in assuming the following.

(i*) For each exhausted deposit of the resource another one with the same characteristics is discovered and the cost of the search (in terms of labour and commodities) is always the same.

In this case, while each deposit would be exhaustible, the resource as such would not; and each deposit could in fact be treated as if it were a (reproducible) machine: the price of the new machine equals the cost of the search and the price of an old machine of age t equals the value of the deposit after t periods of utilization (see Kurz and Salvadori (1995, pp.

[6] In this section we shall only touch upon some aspects of the problem under consideration, a full treatment of which is beyond the scope of this short note. Such a treatment will be the object of a forthcoming paper.

359–60)). The price of the resource would be constant over time, as is commonly assumed in long-period analysis.[7]

Assumption (i*) may be compared to the assumption employed in much of the literature on exhaustible resources and also in the paper by Bidard and Erreygers: the assumption that there is a productive backstop technique, which is known from the beginning. Both assumptions have the effect, in Sraffa's words, of satisfying the conditions of a society that manages 'to keep continuously alive'. These assumptions are indeed devices to avoid the 'end of the world' scenario, on which there is nothing to be said.

The classical economists and especially Ricardo were concerned with a world in which none of the above assumptions (i), (i*) and (ii) was taken to apply. Smith and Ricardo typically saw mines of different 'fertility' being wrought simultaneously as the normal state of affairs. By fertility they meant the amount of the resource that can be extracted from the mine in a given period of time, e.g. a year, given the technique of extraction. This concept is analogous to their concept of fertility of land which refers to the amount of agricultural product that can be grown on a given plot of land of a given quality, using a given method of production. Hence, to extract a resource from a mine takes time, and a *capacity constraint* giving the upper limit of the resource that can be extracted per unit of time is the obvious assumption to make. The amount of a resource 'which can be removed' (Ricardo (1951, p. 68)) will generally fall short of the amount of the resource *in situ* at the beginning of an extraction period.[8] It is against this background that Ricardo maintained: 'If there were abundance of equally fertile mines, which any one might appropriate, they could yield no rent; the value of their produce would depend on the quantity of labour necessary to extract the metal from the mine and bring it to market' (Ricardo (1951, p. 85)). The absence of an abundance of equally fertile mines and the capacity constraint limiting the yearly output of any single mine in

[7] Adam Smith (1976) wrote about the discovery of new mines: 'In this search [for new mines] there seem to be no certain limits either to the possible success, or to the possible disappointment of human industry. In the course of a century or two, it is possible that new mines may be discovered more fertile than any that have ever yet been known; and it is just equally possible that the most fertile mine then known may be more barren than any that was wrought before the discovery of the mines of America' (WN, I,xi.m.21). Hence, the option the theorist has is either to model the uncertainty referred to by Smith or to make a simplifying assumption such as, for instance, assumption (i*) above.

[8] The assumption of a capacity constraint becomes clear, for example, when Ricardo (1951, p. 331) refers to the case of innovations in extracting coal: 'by new processes the quantity should be increased, the price would fall, and some mines would be abandoned'.

general necessitate the utilization of mines of different fertility in order to meet the effectual demand for the resource. In such circumstances, Ricardo emphasized, it is the 'relative fertility of mines [which] determines the portion of their produce, which shall be paid for rent of mines' (1951, p. 330). Smith and Ricardo were clear that the exhaustion of resources may constrain human productive activity. Thus Smith pointed out that 'useful fossils and minerals of the earth, &c. naturally grow dearer as the society advances in wealth and improvements' (WN I.xi.i.3; see also I.xi.d).

We may now ask: in which conditions is the classical approach to the problem of exhaustible resources in terms of the principle of differential rent *strictly* correct? From what has already been said it follows that this is a situation in which there are capacity constraints with regard to mines, and effectual demand is always such that it can never be met without operating also the backstop technology.[9] In this case prices are constant over time and the owners of mines receive a rent precisely as Ricardo maintained.

It need hardly be stressed that this observation does not imply a refutation of Hotelling's rule. The latter applies in certain conditions (possibly with some modifications), but not in all, and we employ it when appropriate (see, for example, Kurz and Salvadori (2000), and sections 1 and 2 above). However, the reader will by now, at the latest, be aware of the fact that that rule follows from a set of specific assumptions which define a particular theoretical 'world'. The analysis of such a theoretical world allows one to grasp some (but not all) aspects of the actual world exhibiting actual mines and oil or gas deposits. Obviously, to study different theoretical objects which allow one to grasp different aspects of actual processes of the exhaustion of resources is a perfectly sensible thing to do in order to increase one's understanding of the problem at hand. One might even consider the possibility of incorporating all these aspects in a single and more general model.

[9] Obviously no assumption of this type is found in Smith or Ricardo, but an institutional aspect referred to by Smith has the same implication: 'There are some [coal-mines], of which the produce is barely sufficient to pay the labour, and replace, together with its ordinary profits, the stock employed in working them. They afford some profit to the undertaker of the work, but no rent to the landlord. They can be wrought advantageously by nobody but the landlord, who being himself undertaker of the work, gets the ordinary profit of the capital which he employs in it. Many coal-mines in Scotland are wrought in this manner, and can be wrought in no other. The landlord will allow nobody else to work them without paying some rent, and nobody can afford to pay any' (WN, I.xi.c.13; quoted by Ricardo (1951, pp. 329–30)). Obviously, when these mines are exhausted they can no longer perform the same role as the backstop technology.

REFERENCES

Bidard Ch., Erreygers G. (2001): 'The corn-guano model', *Metroeconomica*, 53, 243–53 this issue.
Hotelling H. (1931): 'The economics of exhaustible resources', *Journal of Political Economy*, 39, pp. 137–75.
Kurz H. D., Salvadori N. (1995): *Theory of Production. A Long-period Analysis*, Cambridge University Press, Cambridge.
Kurz H. D., Salvadori N. (1997): 'Exhaustible resources in a dynamic input–output model with "classical" features', *Economic Systems Research*, 9 (3), pp. 235–51.
Kurz H. D., Salvadori N. (2000): 'Economic dynamics in a simple model with exhaustible resources and a given real wage rate', *Structural Change and Economic Dynamics*, 11, pp. 167–79.
Ricardo D. (1951): *On the Principles of Political Economy and Taxation*, 1st edn 1817; in *The Works and Correspondence of David Ricardo*, edited by Piero Sraffa with the collaboration of Maurice H. Dobb, Vol. I, Cambridge University Press, Cambridge.
Smith A. (1976): *An Inquiry into the Nature and Causes of the Wealth of Nations*, 1st edn 1776; in *The Glasgow Edition of the Works and Correspondence of Adam Smith*, edited by R. H. Campbell, A. S. Skinner, and W. B. Todd, Vol. II, Oxford University Press, Oxford. In the text quoted as WN, book number, chapter number, section number, paragraph number.
Sraffa P. (1932): 'Dr. Hayek on money and capital', *Economic Journal*, 42, pp. 42–53.
Sraffa P. (1960): *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.
Wicksell K. (1954): *Value, Capital and Rent* (first published in German in 1893), George Allen & Unwin, London.

Department of Economics
University of Graz
RESOWI-Zentrum, F4
A-8010 Graz
Austria
Email: heinz.kurz@kfunigraz.ac.at.