Wicksell and the Problem of the “Missing” Equation

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Triggered by a stimulating paper by Bo Sandelin (1980), there has been a debate about Knut Wicksell’s theory of capital and distribution that is known as “Wicksell’s missing equation.” To this debate have contributed, among others, Sandelin (1980, 1982), Takashi Negishi (1982a; 1982b; 1985, chap. 9), Larry Samuelson (1982), and Tom Kompas (1992, chap. 4). The issue under consideration is whether or not Knut Wicksell had put forward a theory of capital and interest that is closed in the sense that the data, or independent variables, from which he started suffice to determine the unknowns, or dependent variables, especially the “natural” rates of wages, rents, and interest. The mentioned authors claim that there is one equation “missing” in Wicksell’s theory and that his formal system of equations is underdetermined. The question then is how to close the system in a way that is faithful to Wicksell. The authors under consideration differ in terms of the closures they suggest.

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In this paper I will argue that while the contributions under discussion are valuable because they help to clarify some of Wicksell's arguments and illustrations, their common premise is dubious: there is no equation missing in Wicksell's theory. The problem is rather that in the course of his endeavor to develop a coherent long-period supply-and-demand analysis of income distribution, Wicksell became increasingly aware of the fact that his attempt to establish the rate of interest as the "reward for waiting" was confronted with a serious, indeed insurmountable, problem: that of defining the "quantity of capital" independently of the rate of interest. He understood that with heterogeneous capital goods and deprived of Eugen von Böhm-Bawerk's device of the "average period of production" to aggregate them, the initial endowment of the economy of capital could only be given in value terms. Wicksell saw that this undermined the basic idea underlying neoclassical theory: that there is an analogy between the different factors of production—labor, land, and capital—and their rates of remuneration. Originally put forward by Johann Heinrich von Thünen, that idea had inspired several authors, including Léon Walras, Böhm-Bawerk, and Wicksell himself, to elaborate a theoretical edifice explaining the distribution of income in terms of a single principle: that of the (relative) scarcity of the factors of production. In the course of his work Wicksell became increasingly aware that the idea met with considerable difficulties. In particular, whereas the original factors of production—labor and land—can be measured in their own technical units, the capital endowment of the economy had to be given in terms of a sum of value. His lack of enthusiasm for this option—the only one at his disposal, if the supply-and-demand approach to the long-period theory of income distribution was to be adhered to—therefore reflects a fundamental difficulty of the theory. While this is explicitly or implicitly confirmed by those authors who suggest some alternative closures of the system, the inattentive reader might (wrongly) get the impression that these closures allow one to overcome that theoretical difficulty. In this article I will attempt to draw the attention back to the central problem of Wicksell's theory and to put the discussion about the missing equation in the perspective of his overall intellectual program.

I will show that Wicksell approached the two problems of capital theory, the remuneration of capital on the one hand and its accumulation on the other, in separate logical stages. In a first stage he took the capital endowment of the economy as given; that is, he treated it as a datum or
an independent variable, and determined the natural rate of interest in terms of the relative scarcity of capital. It is in this part of the analysis that he deemed it possible to postulate functional relations of known properties. In the corresponding system of equations the "quantity of capital" is treated as an *exogenous* magnitude. In a second stage he dealt with the formation of capital, focusing attention on the factors affecting its accumulation over time. In this part of the analysis the capital stock at any moment of time is taken to be a dependent or *endogenous* variable. However, because of the complexity and variability of the factors affecting saving and investment behavior, Wicksell was convinced that this part of the analysis was not amenable to a treatment in terms of functional relations, at least not for the time being. He therefore confined himself to what are essentially qualitative considerations. For obvious reasons, if one were to stick closely to Wicksell's own approach, then the debate about his missing equation ought to relate only to the first stage, that is, that part of the analysis which in his view allows for a mathematical treatment.

The composition of this article is as follows. Section 1 provides a brief summary account of the alternative closures suggested by some of the authors who maintain that there is an equation missing in Wicksell's formal analysis. Section 2 argues that a common element of these contributions is that they interpret Wicksell's argument and his "static" method of analysis as referring to a stationary economy *strictu sensu*. Section 3 then documents that Wicksell showed little interest in strictly stationary conditions, because the principal object of his inquiry was a growing economy in which capital accumulates. The static method he adopted was explicitly designed to study, however imperfectly, the distribution of income in such a growing economy. Yet, since neither Wicksell nor his contemporaries distinguished carefully between "static method" and "stationary state," the two were often confounded and Wicksell's analysis was misunderstood as referring to strictly stationary conditions. Section 4 discusses briefly Wicksell's consecutive attempts to develop a theory of income distribution by generalizing the principle of supply and demand from the singularly special case of a "non-capitalistic" production with homogeneous labor and homogeneous land to the general case of a "capitalistic" production with heterogeneous capital goods. Section 5 contains some concluding remarks.
1. Alternative Closures

Bo Sandelin (1980, 29) introduces the discussion of Wicksell’s theory of capital and interest as follows:

It is a well-known fact that one equation is “missing” in Wicksell’s various formalizations of his capital theory; this implies that one central magnitude has to be determined exogenously. After some vacillation Wicksell chooses the value of capital as an exogenous variable of his system. (emphasis added)

Hence, strictly speaking there is no equation missing in Wicksell’s theory of capital and interest.¹ Wicksell attempted to determine the amounts of the different commodities produced, the distribution of income, and relative prices in terms of the following sets of data: (1) the preferences of consumers and (2) the technical alternatives from which cost-minimizing producers can choose. To these he added (3) the given endowments of the economy of the original factors of production—labor and land—and, as Sandelin rightly points out, the economy’s endowment of capital, conceived as a given value of capital.

More precisely, Wicksell took the quantity of capital as a given in terms of a unit of consumption that also serves as the numeraire to express wages and prices. This value magnitude is given from outside the system: it is an exogenous variable designed to represent the amount of social capital in existence in the economy at a given moment in time. The productive powers of the economy under consideration are taken to be defined in terms of data (2) and (3). In Wicksell’s theory of the rate of interest the size of (the value of) capital is thus not determined endogenously. It is not reckoned among the unknowns of the problem under consideration, but among its data.²

¹. This is confirmed by Larry Samuelson (1982, 301), who begins his comment on Sandelin as follows: “It is well known that Wicksell’s capital theory is one equation short of being determinate . . . and that Wicksell addresses this problem by assuming the value of capital to be given exogenously.”

². From a logical point of view there appears to be no principal difference within the framework of Wicksell’s supply-and-demand approach to the theory of income distribution between giving the amount of capital and giving the amounts of labor and land. Hence, with the same right with which the closing of his system in terms of a given capital endowment is questioned, one might question taking the supplies of labor and land as given. However, as we shall see, whereas giving the amounts of labor and land (in their own technical units) has a clear meaning, this is not so with regard to value capital.
Sandelin is very clear about this fact. Yet Wicksell’s “vacillation” prompts Sandelin to contemplate alternative ways of closing the system. Rather than assuming an exogenously given value of social capital, one might, he suggests, follow Friedrich August von Hayek and start with “a given structure of real capital”; or Luigi Pasinetti and treat the rate of interest as an independent variable. Yet there is still another alternative to “close” the system:

In this article we shall consider a third possibility: basing the discussion on the wine-storage problem, we shall derive one additional equation which describes the condition for an optimal amount of labor, as seen from the entrepreneur’s point of view; in other words, we shall introduce the “missing equation.” (1980, 29)

In substance, Sandelin’s proposal amounts to replacing constant returns to scale by variable returns in Wicksell’s wine-storage problem. In this case, and assuming a partial framework, characterized by a given world market price of wine as a known function of its age, the profit-maximizing (representative) firm chooses its optimum size and thus the optimum amount of labor to be employed. The value of capital is then endogenously determined by appropriately discounting forward the wage payments invested in the storage of wine. Sandelin stresses that this “closure” of the system involves a significant departure from Wicksell’s original approach. He writes that “the marginal productivity of social capital in the Wicksellian sense now becomes a somewhat

3. It is not clear how a “given structure of real capital” should be compatible with a long-period equilibrium of the economy, characterized by a uniform rate of interest—Wicksell’s “natural” rate. As is well known, Walras in the Elements ([1874] 1954) assumed that the economy’s endowment of capital is given in terms of quantities of physically specified capital goods, that is, a given structure of physical capital. However, in the fourth edition of his magnum opus Walras had to admit that, contrary to his previous view, with an arbitrarily given vector of capital goods proper there is no reason to presume that the requirement of a uniform “rate of net income”—Walras’s expression for the rate of interest—is met; see, for example, Kurz and Salvadori 1995, 439–41. Hayek, in The Pure Theory of Capital, to which Sandelin refers, effected a break with long-period marginalist theory and claimed to be no longer concerned with the determination of the rate of interest (see Hayek 1941, 41). Otherwise he would have had to allow, as Wicksell knew very well (see section 4 below), that the “structure of real capital” cannot be taken as given, but rather had to be treated as an endogenous variable.

4. Taking with Pasinetti the rate of interest as given involves, of course, a fundamental change in the theory. In fact, Pasinetti, a critic of neoclassical theory, advocates the “classical” approach to the problem of value and distribution as it was revived by Piero Sraffa (1960). It should be stressed already at this point that in a classical framework taking the rate of interest as given does not imply stationary or steady-state conditions.
obscure concept” and that “one cannot follow Wicksell in deriving the marginal productivity of social capital. . . . This means that the Wicksellian marginal productivity of social capital becomes a doubtful notion” (1980, 29–30).

To avoid a possible misunderstanding, it should be noted that it was Wicksell himself who, upon his discovery of the famous so-called Wicksell effect (see, for example, Wicksell [1893] 1954, 137–38), drew the conclusion that the theory of interest, as it had been put forward by Thünen, could not be sustained (other than in a partial context). In fact, Wicksell was no follower of that theory, but rather advocated what he considered to be the more general and, as he hoped, logically coherent supply-and-demand theory of income distribution. One could, however, say, as I have done already in the above, that the alternative closures suggested draw attention to a fundamental difficulty of Wicksell’s own construction.

In a comment on Sandelin’s paper, Negishi (1982b, 310) confirms that there is an equation missing in Wicksell’s theory of capital; in a related paper dealing with Böhm-Bawerk’s famous “Three Grounds” he rightly stresses that closing the system via a given amount of capital in value terms deprives the analysis of much of its explanatory power (1982a, 164; see also Negishi 1985, chap. 9). In addition, he argues that the three closures mentioned by Sandelin do not exhaust the set of alternatives and suggests himself two further variants. Instead of changing the production function in the case of the example of the wine-storage problem, Negishi (1982b) introduces explicitly the saving behavior of the capitalist, which derives from intertemporal utility maximization. This involves considering “the value of capital as an endogenous variable” (1982b, 310). In his other contribution (Negishi 1982a) he develops an overlapping generations model with a stationary population, where each agent lives for two periods, the first being the working period, the second the retirement period. In the former the income of the agent exceeds his consumption, that is, he saves, whereas in the latter things are the other way round, that is, he dissaves all the capital previously built up. Negishi demonstrates that even assuming away time preference, the rate of interest may be positive due to the individual agent’s concern with better provision for wants in the second than in the first period and the superiority of more roundabout processes of production. Hence, Wicksell is said to have been right in his criticism
of Böhm-Bawerk that time preference was not all that important in the theory of interest.\footnote{The two causes given relate, of course, to the first and the third of Böhm-Bawerk's three grounds. We shall come back to them in section 4 below, where we will deal with Wicksell's interpretation of their interplay.}

Thus it seems that there are a number of ways to interpret and eventually solve the problem of the missing equation. Depending on the alternative adopted one arrives at a different system. This in turn seems to imply that Wicksell's analysis is characterized by a certain openness and arbitrariness which contradicts the otherwise praised clarity and definiteness of his reasoning. We may then ask: Why did Wicksell close the system as he did? However, before we turn to that question we shall first ask whether the alternative closures suggested share a common element and whether that common element reveals why Wicksell, given his intellectual program, did not adopt any of the solutions proposed.

2. The Common Element in the Suggested Interpretations

A closer look at the proposed alternative closures of Wicksell's capital theory shows that they all presuppose stationary economic conditions in the strict sense of the concept of stationarity, that is, time invariant data (1)-(3), listed above. This involves, in particular, a constant working population, a constant technical knowledge, and, most important in the present context, an unchanging endowment of capital. That is, in the conceptualizations suggested there is neither capital accumulation nor capital decumulation over time: gross savings (which are taken to equal gross investments) are just sufficient to replace periodically the produced means of production used up in the production process. In these interpretations Wicksell's "static" point of view is taken to imply a concern with economic systems characterized by a capital stock that does not change over time. Starting from such a presupposition involves, of course, looking for a state of the economic system such that the forces working in the direction of a growing capital stock are exactly balanced by the forces working in the opposite direction. To determine such a state then amounts to determining endogenously the capital equipment of the economy, both as regards its overall size (in terms of the numeraire) and its composition, that is, that capital stock which, together with the other data (preferences, technical alternatives,
and endowments of the primary factors of production, labor and land), accounts for strictly stationary conditions.

The treatment of the capital endowment (of the stationary economy) as a dependent rather than an independent variable is indeed the common characteristic feature of the interpretations under consideration.\(^6\) Interestingly, prior to the debate about Wicksell's missing equation, Guy Arvidsson (1956) had maintained that strictly stationary conditions and thus the constancy of social capital are perfectly compatible with a positive level of the rate of interest, provided each individual at the end of his life is inclined to bequeath the same amount of wealth or capital that he inherited. And Jack Hirshleifer (1967) had argued that with intertemporal utility maximization stationary conditions obtain if the rate of interest equals the rate of time preference. With the corresponding quantity of capital in an otherwise Wicksellian framework there are no motives to any further accumulation (or decumulation) of capital. Yet, did Wicksell's "static" method involve strictly stationary conditions, which is the explicit or implicit view held in much of the interpretative literature on Wicksell's theory of capital and interest?

This is answered in the positive by Tom Kompas (1992) in an interesting attempt to come to grips with the complexities of Wicksell's approach to the problem of capital and interest and to that of capital accumulation.\(^7\) In Kompas's view, "Wicksell, except in provisional terms, with substantial qualification, does not take the value of aggregate capital as given to solve for a stationary equilibrium" (132). By "stationary equilibrium" Kompas means a situation in which net savings are nil, which in turn is taken to imply the equality between the rate of interest and the (collective) rate of time preference (114). According to Kompas, Wicksell intended to close the system in terms of a savings function and intertemporal preferences, but essentially "for analytical simplicity" (11) took the value of capital as a datum. Wicksell and Walras are said

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6. As has been indicated in footnote 4, while stationary (or steady-state) conditions imply the endogenous determination of the value of capital, the contrary is not true. Thus, with given levels of gross output, given technical alternatives of production, and a given real wage rate (or, alternatively, a given rate of profits), as in Sraffa 1960, the value(s) of social capital compatible with one (or several equiprofitable) cost-minimizing system(s) of production is (are) determined, but the system(s) under consideration need not be in a stationary or a steady state; see, for example, Kurz and Salvadori 1995.

7. I am grateful to one of the referees for referring me to Kompas's book, which had escaped my attention.
to have “set out clear and consistent theories of long-run equilibrium for a stationary economy” (vii).

3. Wicksell’s “Static” Method vs. the Stationary State

As is well known, notwithstanding important differences between different authors, the marginalist economists from William Stanley Jevons to Alfred Marshall, from Léon Walras to Gustav Cassel, and from Eugen von Böhm-Bawerk to Knut Wicksell all attempted to explain the shares of wages, profits, and rent in terms of a single principle: that of the relative scarcities of the respective factors of production, labor, capital, and land. Whereas the classical authors, especially David Ricardo, applied that principle only in order to explain the rent of land, the marginalist authors were convinced of the universal applicability of that principle to all factors and their remunerations alike.

Wicksell was deeply impressed by the Böhm-Bawerkian version of neoclassical theory, that is, what Paul A. Samuelson (1987, 908) called the “marginal-productivity-of-time paradigm.” In *Value, Capital, and Rent*, originally published in German in 1893, Wicksell ([1893] 1954, 20) defined his own contribution essentially as an attempt to provide an “exact, mathematical treatment of the theory of capital interest” in a general equilibrium framework based on Böhm-Bawerk’s temporal approach. In Wicksell’s view the theory of capital and interest had to tackle two main problems: (1) it had to explain the *origin and level of interest*, that is, identify the factors that give rise to a positive rate of interest; and (2) it had to explain the *origin and formation of capital* (cf. 21–22). The former problem belonged to the theory of interest proper, whereas the latter belonged to the theory of capital accumulation and economic growth. The former problem, Wicksell surmised, may be dealt with using the “static” method, whereas a satisfactory treatment of the latter necessitated a “dynamical” analysis.

According to Lionel Robbins (1930, 195), the ambiguity of the concept of “static” method was responsible “for some of the most important doctrinal confusions of the past.” Indeed, as Robbins emphasized, that method of analysis was not distinguished with sufficient clarity from the concept of “stationary equilibrium,” so that people were easily misguided to confound the two. While Robbins did not explicitly deal with Wicksell’s contributions, his general assessment applies also to them. In
fact, following Böhm-Bawerk's lead, both in *Value, Capital, and Rent* and in the *Lectures on Political Economy* Wicksell ([1901] 1934, 7) approached the first of the two problems mentioned "mainly from the static point of view, i.e. we shall assume, in principle, a society which retains unchanged from year to year the same population, the same area of territory and the same amount of capital, and remains on the same level of technical achievement." In another place he stated that, "for the moment, . . . we shall content ourselves with what has been called the static aspect of the problem of equilibrium, i.e. the conditions necessary for the maintenance, or the periodic renewal, of a stationary state of economic relations" ([1901] 1934, 105). These and similar specifications of the data and thus the framework in terms of which he sought to determine the rate of interest look indeed as if stationary conditions *strictu sensu* are implied. However, this impression is quickly dispelled by numerous other passages. Thus, in his early contribution to capital theory he referred to the "fundamental—and simplest—hypothesis" of a "stationary economy in which capital and the other economic factors can be thought of as an approximately unalterable sum" ([1893] 1954, 22; second emphasis added), a formulation that is echoed in his mature work (cf. Wicksell [1901] 1934, 184, 193). He left no doubt, however, that this is a simplifying device in order to come to grips, as a first approximation, with the problem of distribution. The real economy of his time, Wicksell kept stressing (cf. in particular part 3 of volume 1 of the *Lectures*), was an economy in motion in which capital accumulated. The static method is therefore not meant to do away with this fact: "the accumulation of capital is itself, even under stationary conditions, a necessary element in the problem of production and exchange" ([1901] 1934, 203). The clearest expression of Wicksell's method of analysis, and of the problematic character of the terminology used, is perhaps the following:

We shall assume *stationary conditions* as the foundations of our observations. This will not prevent us from considering changes in the quantities concerned, provided that we do not take into account the actual transition stage, which is a much more complicated problem, but assume that these changes have already become final, so that "static equilibrium" (a stationary state) is again restored. ([1901] 1934, 152)

To study the problem of income distribution in a "dynamic" framework, Wicksell surmised, was not yet possible: "the laws of capital
formation have been too little studied for a treatment of the subject in its entirety to be of much real use” (203).

To summarize, Wicksell opted for a treatment of the two main problems of the theory of capital and interest in two consecutive steps. First, the determination of the rate of interest, the wage rate, and the rent rate should be approached in a static framework, in which the amounts of the respective factors of production—capital, labor, and land—are taken to be in given supply; he in fact even assumed vertical supply functions (cf. [1901] 1934, 105). The rates of remuneration determined in this way would reflect the relative scarcities of the factors of production. In a second step he would then proceed to the discussion of the impact of changes in one or several of the data, in particular the amounts of the productive factors, on the distribution of income. Part 3 of volume 1 of the Lectures, “Capital Accumulation,” documents well his comparative static analysis of the problem of income distribution when capital accumulates. This approach in two steps is clearly expressed in the following passage:

Both logically and for purposes of exposition it would seem right to begin by examining the effects of a given supply of capital already accumulated, and then to inquire the causes which influence, and eventually alter, this supply.” (155; first emphasis added)

Therefore, it would be wrong to think that in dealing with the problem of income distribution Wicksell assumed stationary conditions strictu sensu. He did not.8

8. Larry Samuelson maintained that Wicksell intended “to close the model via a theory of savings behavior” (1982, 301; see also pp. 302–3 and 306). It can hardly be doubted that Wicksell would have liked to be possessed of a theory of savings of sufficient generality that could also be formalized. This would have allowed him to tackle both the problem of the remuneration of capital and that of its accumulation in terms of a single mathematical theory of the production and distribution of wealth. Alas, in his view such a theory of savings was not available. This is why he felt obliged to adopt his two-stage procedure. This was clearly a second-best solution, but the only one at his disposal. Kompas (1992) has put forward a careful study of Wicksell’s analysis that contains several interesting observations. However, in a fundamental respect I think his interpretation is wrong. He is well aware of Wicksell’s agnosticism as regards the theory of savings. Nevertheless he feels entitled to do what Wicksell thought could not be done: “To close the system, add an expression representing savings behaviour,” and, in addition, impose a “zero net savings” condition (1992, 94) in order to obtain strictly stationary conditions. Kompas follows his own suggestion and introduces savings functions as they are to be found in the more recent neoclassical literature (see, for example,
There is additional evidence that Wicksell did not intend to study the problem of distribution in terms of a strictly stationary state of the economy. To see this we have to turn to his criticism of Walras and his successors. These are said to hold a theory of interest which contains both formal and material defects and which is seriously incomplete. Walras' formula for interest, as may easily be seen . . . [,] reduces itself, on the assumption of stationary conditions, simply to the equation \( F(i) = 0 \), in which \( F(i) \) is the amount of annual savings conceived as a function of the rate of interest \( i \). In other words, it expresses the truism that, in the stationary state, the inducement to new savings must have ceased; but it affords no answer to the question why a given amount of existing social capital gives rise to a certain rate of interest, neither higher nor lower. ([1901] 1934, 171; the second emphasis is Wicksell's)

Hence Walras's theory is accused of being “seriously incomplete” because it determines the rate of interest only for the strictly stationary state, in which there is no incentive to net savings or dissavings, and fails to determine it for an arbitrarily “given amount of existing social

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In this way, the capital endowment of the economy becomes an endogenous variable. This interpretation is difficult to sustain. First, Kompas takes Wicksell's "static" method to involve a concern with a "stationary equilibrium," which it does not (see, again, Robbins's clarification). Second, had Wicksell seen the possibility of closing the system in terms of a general savings function, he could be expected to have done so. Therefore, in this regard Kompas is forced to read into Wicksell what he cannot find there verbatim. Yet, had Wicksell believed it possible to close the system in terms of a general savings function, then there would have been no need for him to tinker with a stationary equilibrium: he could have provided a theory cast from a single die, dealing both with the production, distribution, and accumulation of wealth. In short, he could have provided a full-fledged dynamic theory. It is not clear why he should not have done so in case he could. Third, Kompas stresses that determining endogenously the capital endowment by assuming a stationary state involves fixing the rate of interest at the exogenously given level of the (collective) rate of time preference. This amounts to treating the rate of interest, which Wicksell was keen to determine, as a datum or independent variable. Hence, what in Wicksell was an unknown, in the interpretation under consideration has become a known magnitude. (If the rate of time preference were itself to be considered an endogenous variable that tends to follow any upward or downward trend of the actual rate of interest, as Wicksell in places appears to assume, then Kompas's suggestion would involve closing the system by directly fixing the rate of interest instead of the value of capital.)
capital.”9 Böhm-Bawerk on the other hand is credited with having attempted to provide precisely the missing piece of analysis.10

We may conclude that Wicksell was not really interested in the stationary state of the economy strictu sensu. The actual trend of the economy he experienced exhibited capital accumulation and economic growth and there were no indications that the stationary state was around the corner. Economic theory had to study this state of affairs and not the purely hypothetical one in which net savings were nil. The static point of view adopted by him was designed to throw some light on the actual, growing economy in terms of a comparative static analysis of consecutive states of the economy characterized, inter alia, by different “quantities of capital” in existence. Wicksell was aware that this was not fully satisfactory, because defining any such state by a given and unchanging social capital implied that the net social product consisted only of consumption goods, whereas in an economy in which capital accumulates it consists also of investment goods. However, he was of the opinion that with a slowly growing economy the error involved was perhaps negligible. The static method, he concluded, was the best at hand and allowed one to investigate, albeit imperfectly, the implications of changes in factor endowments and technical knowledge on the distribution of the product.

9. Because Wicksell was not concerned with a stationary equilibrium but with an economy in which capital accumulates, there is nothing “perplexing” (cf. Kompas 1992, 102) about the fact that in the Lectures he would still treat fixed capital items as “rent goods.” Since in a growing economy there is no presumption that long-lived fixed capital is ever fully adjusted to the other data of the system, which are themselves subject to permanent change, it should come as no surprise that “the adjustment to equilibrium is explicitly ignored” with regard to fixed capital (cf. Kompas 1992, 102).

10. Böhm-Bawerk is criticized, however, for his attempt to solve the problem of the existence of interest independently of that of the level of the rate of interest (cf. Wicksell [1901] 1934, 171). In fact, Böhm-Bawerk thought that the former problem could be settled without any reference to the endowments of “capital,” labor, and land of the economy, whereas the latter required taking these quantities as known. In this context it should be mentioned that by taking these endowments as given, Böhm-Bawerk did not imply strictly stationary conditions. In this regard his method of analysis does not differ from that employed by Wicksell. In the excursuses to his Positive Theory of Capital he vehemently denied Ladislaus von Bortkiewicz’s accusation that his analysis was based on such an assumption: “Of course, the concept of ‘static’ or ‘stationary’ cannot be given such an unusual and contradictory meaning as Bortkiewicz once did in his polemic zeal. . . . Bortkiewicz recognizes a society only as ‘stationary’ when it neither makes actual progress, nor has ever made it in the past. But it is apparent that such a limitation of the concept of ‘stationary’ is not only arbitrary and very unusual, but it also lacks its right of existence” (Böhm-Bawerk [1889] 1959, 3:216 n. 39).
In particular, it was taken to allow one to determine, at least "approximately," one of the key variables of the economy: the general rate of interest.

After having expounded the method of analysis employed by Wicksell we may now briefly summarize the content of his theory in his two main contributions to the theory of capital and interest. This then leads us back to the problem of the closure of his system and thus to the question of whether there is an equation missing.


As is well known, Böhm-Bawerk ([1889] 1959) had put forward "Three Grounds" for interest: (1) "different circumstances of want and provision" in the present and in the future; (2) the "under-estimation of the future," that is, a positive time preference; and (3) the "technical superiority of present over future goods," that is, the superiority of more "roundabout" processes of production.

Wicksell shared Böhm-Bawerk's basic theoretical vision and was convinced that the latter's analysis contained the key to solving the two main problems of the theory of capital and interest. Yet, in Wicksell's view Böhm-Bawerk had not fully grasped the proper status of each of the three grounds and their interaction. Already in Value, Capital, and Rent Wicksell ([1893] 1954, 21–22) set out his own understanding of the proper division of labor among the three grounds in tackling the two problems; essentially the same view is found in the Lectures ([1901] 1934, 154–56). For given endowments of the factors of production, including capital, the third ground is said to allow one to determine the rate of interest, \( i \), as the "marginal product of waiting." This provides a preliminary answer to the first main problem of capital theory. In an economy that, according to Wicksell, was still far away from being saturated with capital, the resulting "natural" rate of interest may be expected to be larger than the (average) rate of time preference in society, \( \rho \), contemplated by the second ground. With \( i > \rho \), a sufficient condition for positive net capital formation is met. This leads immediately to the first ground, which supposes a growing income per capita

11. Similar views are in Wicksell 1928. For the argument below that follows, see also Hansson 1993; Boianovsky 1998; and Kurz 1998.
and which now turns out to be merely a consequence of \( i \) exceeding \( \rho \). Finally, the greater the difference between \( i \) and \( \rho \), the greater, ceteris paribus, the pace at which capital accumulates and the economy grows. Setting aside technical progress and population growth, as capital accumulates, its relative scarcity decreases, which will be reflected in a falling rate of interest. Other things equal, this implies a gradual deceleration in the formation of new capital. As Wicksell ([1901] 1934, 209) stressed, "Under such conditions, we should therefore expect a continual accumulation of capital—though at a diminishing rate—and, at the same time, a continual fall in the rate of interest." This is taken to provide some elements of a preliminary answer to the second main problem.

As regards the concept of the quantity of capital in given supply, Wicksell had been aware since the beginning of his investigation that this required him to define a measure of the capital endowment of the economy, which consists of heterogeneous capital goods, that is independent of the rate of interest and relative prices. The same problem had already bothered Böhm-Bawerk, who, as is well known, had attempted to replace a vector of physically heterogeneous capital goods with a scalar: the "average period of production." According to this concept, time could serve as the sought measure of capital.

**Value, Capital, and Rent**

When Wicksell came across that concept, or, as he preferred to call it, the "average period of investment," he was at first enthusiastic about its potentialities. In *Value, Capital, and Rent* he expressed the view that Böhm-Bawerk's concept "will presumably prove extremely fruitful" (Wicksell [1893] 1954, 22). His own formalization of the theory was indeed designed to demonstrate this:

12. The parallel to the simple neoclassical growth model of Robert Solow, with a given rate of population growth \( \lambda \), a given proportional savings function with \( s \) as the (marginal and average) propensity to save, a linear homogeneous Cobb-Douglas production function, and setting aside technical progress, is obvious. The rate of growth outside the steady state, \( g \), is given by

\[
g = s F_K + \alpha \lambda,
\]

where \( F_K \) is the marginal product of capital, which equals the rate of interest, and \( \alpha \) is the partial elasticity of production with regard to labor. (There is no time preference in Solow's model.) As capital accumulates relative to labor, the marginal product of capital will fall and so will the rate of growth.

Since . . . the relatively definite and very simple concept of the lengthening of the process of production [i.e., of the "average period of production"] replaces the older, vague, and multiform idea of productivity of capital, the theory of capital-interest can be treated in as exact a fashion as the theory of ground-rent before. (116–17)

He stressed that a “definite” solution of the problem of distribution required taking the amount of capital as a given magnitude. “We can,” he maintained, “determine without difficulty the position of equilibrium finally attained, with the help of our equations set forth above—but only if we assume that the present capital is a known magnitude” (156).14

However, already in his early contribution there are passages indicating that the concept was perhaps not as powerful as Wicksell would have liked it to be. First, there is a remarkable contrast in the passage just quoted between “relatively definite” and “exact.” Yet there is more direct evidence available. Abandoning Böhm-Bawerk’s assumption of natural services as free goods, Wicksell got two “average periods of investment”—one related to labor, the other to land. There is only a singularly special case, which he qualified as “a first approximation” (147), in which the two kinds of capital, or “average periods,” can be aggregated independently of relative prices and income distribution: this is the case in which all commodities (final products) exhibit the same proportions of labor to land at every single stage of their production, that is, the dated quantities of labor and of land show the same profiles in the production of all commodities.15 Wicksell also saw that Böhm-Bawerk’s concept was unable to deal properly with fixed capital and decided to evade the problem by treating durable instruments of production as “rent-goods” (99).16 Yet, it is not clear whether or not by the time of the publication of Value, Capital, and Rent he had been fully aware of the fact that the concept of the “average period” breaks down even in the case with a single primary factor and circulating capital only, the workhorse of much of Böhm-Bawerk’s argument, if compound instead of single interest is

14. It deserves to be mentioned that not only in the Lectures but already in Value, Capital, and Rent Wicksell expressed the quantity of capital as a value sum. However, as Garegnani (1960, 127–30) observed, on the assumption that the concept of the “average period” was valid, there was no need to do so in the earlier work.

15. The parallel of this case to the case of a uniform “organic composition of capital” across all industries in Marx is close at hand.

used in the calculations. While he saw that compound interest was necessitated by the assumption of free competition, he seemed to think that using simple interest involved an admissible simplification and no "essential alteration" (126). As we know, this presumption cannot be sustained (see, for example, Kurz and Salvadori 1995, 436–37).

Wicksell’s original expectation as to the potentialities of the “average period of investment” was frustrated. The demand-and-supply theory of the rate of interest was confronted with the problem that the average period of investment did not provide a measure of the quantity of capital in given supply that was independent of the rate of interest. Was there a way out of the impasse?

*Lectures on Political Economy*

The idea that heterogeneous capital goods could be aggregated independently of (relative) prices and the rate of interest had turned out to be illusory. Wicksell ([1901] 1934, 145) drew the consequences and admitted that “all these requisites [i.e., produced means of production] have only one quality in common, namely that they represent certain quantities of exchange value, so that collectively they may be regarded as a single sum of value.” Yet, the need to express the available quantity of capital in the economy as a sum of *value* in terms of some *numeraire* destroyed the alleged analogy between the three kinds of income—wages, rents, and profits—and the corresponding factors of production—labor, land, and capital:

This analogy between interest, on the one hand, and wages and rent, on the other, is incomplete. . . . Whereas labour and land are measured each in terms of its own *technical* unit (e.g. working days or months, acre per annum), capital, on the other hand, . . . is reckoned . . . as a sum of *exchange value*. . . . In other words, each particular capital-good is measured by a unit extraneous to itself. However good the practical reasons for this may be, it is a *theoretical anomaly which disturbs the correspondence which would otherwise exist between*

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17. In his paper “Kapitalzins und Arbeitslohn,” published in 1892, Wicksell showed some awareness that the value of capital depends on the way of calculating interest (see Wicksell 1892, 846; see also Wicksell [1893] 1954, 123 n, 143 n). I am grateful to one of the referees for having drawn my attention to these passages.

18. Apparently, by the time of the Lectures Wicksell was aware of the problem; see, for example, Wicksell [1901] 1934, 205.
One might contemplate, with Walras, the possibility of treating each kind of capital good as a separate factor, which would remedy the "defect," Wicksell surmised. He added: "But, in that case, productive capital would have to be distributed into as many categories as there are kinds of tools, machinery, and materials, etc., and a unified treatment of the rôle of capital in production would be impossible." However, in competitive equilibrium the rate of interest "is the same on all capital," that is, the interest obtained is proportional to the values of the different capital goods ([1901] 1934, 149). Concerned with a "unified treatment of the rôle of capital," Wicksell had no alternative but to assume the capital endowment of the economy as given in value terms. This involved a "theoretical anomaly"—but deprived of the "average period" there was no other way open to him, if the demand-and-supply approach to the theory of income distribution in a long-period framework was to be preserved.

It was also no longer possible to describe the production of single commodities in terms of the average period of investment. Wicksell therefore decided to consider "the total amount of a commodity produced as a function (homogeneous and linear) of all the quantities of labour and land employed (i.e. annually consumed) both current and saved up" (203). He thus postulated a production function for commodity $j$, which in our notation can be written as

$$y_j = f_j(l_{1j}, l_{2j}, l_{3j}, \ldots ; b_{1j}, b_{2j}, b_{3j}, \ldots) \quad (j = 1, 2, \ldots, n),$$

where $l_{1j}$ and $b_{1j}$ indicate current services of labor and land, $l_{2j}$ and $b_{2j}$ indicate services in the previous period, and so forth. He reiterated his earlier view that capital is not an original factor of production, but a derived one: it is nothing but "a single coherent mass of saved-up labour and saved-up land" ([1901] 1934, 150). Accumulated labor and land are taken to "have been able to assume forms denied to them in their crude state, by which they attain a much greater efficiency for a number of productive purposes." Capitalistic processes of production are roundabout, and it is the time element of production that is important: the increase in efficiency is a "necessary condition of interest" (150). The upshot of Wicksell's mature theory of capital and interest is summarized in the following statement:
Capital is saved-up labour and saved-up land. Interest is the difference between the marginal productivity of saved-up labour and land and of current labour and land. (154)

With the wage per unit of (homogeneous) labor and the rent per acre of (homogeneous) land paid at the end of the elementary production period (month or year), in static equilibrium the values of the marginal products of the dated quantities of labor and land are equal to the wage rate and rent rate, \( w \) and \( q \), properly discounted forward (see pp. 156, 204); that is,

\[
\frac{\partial y_j}{\partial l_{kj}} p_j = w(1 + i)^{k-1} \quad (j = 1, 2, \ldots, n; k = 1, 2, \ldots)
\]

\[
\frac{\partial y_j}{\partial b_{kj}} p_j = q(1 + i)^{k-1} \quad (j = 1, 2, \ldots, n; k = 1, 2, \ldots),
\]

with \( p_j \) as the price of commodity \( j \) and \( i \) as the rate of interest. All value magnitudes are expressed in terms of a common numeraire consisting of one or several consumption goods.

In equilibrium the total quantity demanded of each factor equals the total quantity supplied of that factor. The formulation of this condition causes no problem with regard to labor and land, which can be measured in terms of their own technical units. With given quantities of labor, \( L \), and land, \( B \), whose supplies are taken to be given and independent of the respective rates of remuneration, in equilibrium we have (see p. 204): 20

\[
L = \sum_{j=1}^{n} \sum_{k=1}^{\infty} l_{kj}
\]

\[
B = \sum_{j=1}^{n} \sum_{k=1}^{\infty} b_{kj}.
\]

19. As Wicksell ([1919] 1934, 228) pointed out in his criticism of Cassel's theory of general equilibrium, there is no presumption that all factors in given supply can be fully employed and fetch a positive income. Wicksell was in fact one of the first authors to indicate that general equilibrium should be characterized in terms of inequalities rather than equations. However, in the bit of his own analysis we are concerned with here he proceeded as if all factors could be fully employed. With a sufficient degree of substitutability in production, which he assumed, this is indeed the case.

20. In Wicksell's formalization the time index (\( k \) in the formulas) does not go to infinity, but to a given finite period. Since that period cannot be known independently of the solution of the system of equations, the above formulation appears to be more correct.
The supply-equals-demand condition is more difficult with regard to capital. This is due to the fact that it may be difficult—if not impossible—to define this concept of social capital with absolute precision, as a definite quantity. In reality, it is rather a complex of quantities. (165)

However, Wicksell insisted that both the question of the existence and that of the actual level of the rate of interest cannot be answered "without referring to the market for capital" (171). Finally, in order to be consistent with the concept of a full competitive equilibrium, characterized by a uniform rate of interest, the "amount of capital" (204) available in the economy at the beginning of the production period, $K$, can be given in value terms only, representing a certain quantity of the numeraire. In equilibrium that sum of value must be equal to the value of capital employed (204), which consists of "labour power capital" and "land power capital":

$$K = w \sum_{j=1}^{n} \sum_{k=2}^{\infty} l_{kj} (1 + i)^{k-1} + q \sum_{j=1}^{n} \sum_{k=2}^{\infty} b_{kj} (1 + i)^{k-1}.$$ 

It is this latter equation that caused Wicksell a lot of headaches.21 In fact, it is not clear what sort of constraint is this that forces the right-hand side of the equation to be equal to a given amount of a consumption good, for example, corn. What does it mean here for $(w; r; l_{11}, l_{21}, \ldots ; b_{11}, b_{21}, \ldots ; \ldots ; l_{1n}, l_{2n}, \ldots ; b_{1n}, b_{2n}, \ldots)$ to be constrained in this way?

Yet, the logic of his supply-and-demand approach to the theory of income distribution forced Wicksell to invoke such an equation.22 He emphasized with reference to his model with two industries that if these values are summed and are put equal to a certain given quantity—the total exchange value of the capital employed in the two industries together, expressed in terms of the first commodity, we shall

21. Sandelin (1980, 38) rightly stresses that Wicksell "did not accept measuring the productive capital in value units without objections. But for practical reasons, and to study the rate of interest, he found no better solution."

22. It should come as no surprise that the same theoretical necessity is felt in the wine-storage example. There Wicksell assumed that "the capital of the community is just sufficient for a storage period of $t$ years—$t$ being assumed to be known." He then calculated the demand for value capital and concluded "if the social capital is exactly equal to this there will be equilibrium" ([1901] 1934, 179).
then obtain the necessary [additional] relation, and the problem will at last be completely determinate. (204–5; first and third emphases added)

In this conceptualization the physical composition of social capital $K$ in terms of the $l_{kj}$ and $b_{kj}$ is a part of the equilibrium solution to the problem of value and distribution rather than one of its data. “In equilibrium,” Wicksell emphasized, “the composition of the sum total of capital is thus definitely fixed” (204).

Hence, if there was a “vacillation” on Wicksell’s part, it concerned not so much the type of closure of the system—it certainly had to be closed in terms of a given amount of capital—as the fact that he was forced to retreat to the concept of a value measurement of capital. The meaning of a constraint on production and distribution specified in these terms was unclear and deprived the theory of its definiteness. Indeed, it questioned the usefulness of the entire theory.23 Wicksell did not draw this radical conclusion, but contented himself with the supposition that giving the capital endowment in value terms provided a sufficiently good approximation of the amount of what he called “real capital” (165). Apparently, he was inclined to interpret the difficulty under consideration broadly in the light of his earlier observation that “in such questions we can never achieve more than approximately valid conclusions” (184). The weakness of this supposition is close at hand. As Pierangelo Garegnani (1990, 38) stressed, “What such a justification of a value measurement ignores is the fact that, in order to speak of one of the magnitudes as a workable approximation to another, we should first be able to define the second magnitude exactly: and the

23. Ian Steedman has pointed out to me that the “Wicksell closure problem” is just a variant of the problem of the wages fund. Consider a two-period Wicksell model, with wages paid ex post. If $Y = Y(L_0, L_1)$, then $\partial Y/\partial L_0 = w$ and $\partial Y/\partial L_1 = (1 + i)w$. With a linear homogeneous production function, we have

$$Y = wL_0 + (1 + i)wL_1$$

or

$$Y = wL + iK,$$

with $L = L_0 + L_1$ and $K = wL_1$. Since in this model there are no advances on the first stage, $wL_1$ is the “wages fund” of the second stage. What is the meaning of taking $K$ as given? What kind of constraint is this that requires $wL_1$ to be equal to a given amount of corn per period? As is well known, Wicksell was critical of the wages fund doctrine. His reluctance to accept the closure of his theory of distribution in terms of a given value of capital may have had as its deeper reason the fact that he saw that the critique leveled at the wages fund theory applied also to his closure.
‘real capital’ magnitude is precisely what, in most relevant cases, cannot be defined” (see also Sandelin 1990). Today we know that Wicksell’s hope was futile: modern capital theory has shown that the value magnitude of capital can vary in any direction and to almost any degree as distribution changes, even though “real capital,” that is, the vector of capital goods, is unaltered (see, for example, Garegnani 1990; and Mas-Colell 1986).

5. Conclusion

In this essay I have argued that there is no equation “missing” in Knut Wicksell’s theory of capital and interest. The uneasiness with which Wicksell in the Lectures introduced the given amount of capital as a value sum, which, in equilibrium, is taken to be equal to the value of capital in demand by cost-minimizing producers, rather reflects his awareness of the difficulties of the theory of distribution he had elaborated, starting from Böhm-Bawerk’s conceptualization. The promise that the “average period of production” would allow one to consistently aggregate heterogeneous capital goods had turned out to be illusory, because that concept could not be defined independently of the rate of interest, that is, the unknown of the problem under consideration. There was only the option of defining the capital endowment of the economy in value terms, the meaning of which, however, was dubious. Wicksell’s belief, and it was just a belief, that value capital could be considered as approximating “real capital” is untenable in general.

In the contributions to the debate about Wicksell’s missing equation the problem of defining the capital endowment of the economy independently of the rate of interest is avoided. The emphasis is on stationary (or steady) states strictu sensu, in which both the size and the composition of the social capital are determined endogenously. This involves a significant departure from Wicksell’s analysis. As we have seen, Wicksell showed little interest for the singularly special case of the stationary (or steady) state. He was rather concerned with an economic system in motion in which capital accumulates and income per capita grows. He sought to approach the two main problems of capital theory, that is, the problem of the origin and level of interest and that of the origin and formation of capital, in separate logical stages. In a first stage, which according to Wicksell allowed postulating functional relations of known properties, the amount of capital was treated as an independent variable:
its relative scarcity was taken to hold the key to the determination of the rate of interest. This was effected in terms of a system of simultaneous equations. In a second stage he then discussed the formation of new capital. In his view this problem had not yet been studied carefully enough to be put into mathematics. “Unfortunately, such a theory [of savings and investment] has not been worked out, and the phenomena which it should explain depend on a number of motives—partly selfish, partly altruistic, but in any case very complex” (Wicksell [1901] 1934, 207–8).

Hence, while the alternative closures suggested provide useful insights into some of Wicksell’s considerations, they are not able to remedy the deficiency of his supply-and-demand theory of distribution.

References


