

# The Dynamic Leontief Model and the Theory of Endogenous Growth

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**ABSTRACT** *This paper shows that the dynamic Leontief model can be interpreted as a linear model of endogenous growth. The long-term rate of growth is determined within the economic system—either as the outcome of the saving and investment behaviour of agents or as the outcome of some planner’s maximization of some objective function.*

**KEYWORDS:** *Dynamic Leontief model; endogenous growth; saving; investment*

## 1. Introduction

After a decade or so of negligence, recent years have seen a remarkable revival in growth theory. The state of the art of so-called ‘new’ growth theory is summarized in Barro & Sala-i-Martin (1995) and Aghion & Howitt (1998). Because, in the models that are in high esteem in this literature and in contradistinction to the growth model of Solow (1956), the steady-state rate of growth is endogenously determined, these models are also known as models of ‘endogenous’ growth. Immediately after the publication of the papers by Romer (1986) and Lucas (1988), which triggered an avalanche of growth literature, there was considerable excitement in the profession about the original novelties contained in this literature. However, it slowly gave way to a more sober assessment of the achievements of ‘new’ growth theory. It was pointed out that many, if not all, of the ideas put forward had been known for a long time and that there were older growth theories prior to the ‘new’ ones which fulfilled the criterion of ‘endogeneity’ advocated in contemporary growth economics. This criterion requires that long-run growth is determined ‘within the model’ rather than by some exogenously growing variables (Barro & Sala-i-Martin, 1995, p. 38). It was stressed, for example, that both the theories of capital accumulation and economic growth of the classical economists, Karl Marx’s theory of extended reproduction and John von Neumann’s famous growth model all fulfil that criterion (see, for instance, Kurz & Salvadori, 1997, 1998, 1999).<sup>1</sup>

In this paper, we shall show that Wassily Leontief’s dynamic input–output model can also be interpreted as belonging to the theory of endogenous growth.

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In fact, in the interpretation given, the model satisfies the defining characteristic of that theory: the long-run growth rate is determined within the system—either as the outcome of the saving and investment behaviour of agents or as the outcome of some planner's or policy maker's maximization of some objective function.

The composition of the paper is the following. In Section 2 we shall briefly recall the features of Leontief's dynamic input–output model. Section 3 presents a particular version of that model, which is then used to analyse the implications of different given objective functions that are to be maximized. A characteristic feature of the model developed is that the process describing the consumption of workers and the (re)production of labour is taken to be a part of the Leontief matrix, thus preserving an element of the closed model. This assumption is introduced in order to prepare the ground for Section 4, in which the dynamic Leontief model will be compared to the recent linear models of endogenous growth. It will be shown that, properly specified, the dynamic Leontief model can indeed be considered an endogenous growth model. Section 5 contains some concluding remarks.

## 2. The Dynamic Leontief Model: a Summary Account

Dynamic input–output analysis derives from the static model through consideration of lags or rates of change over time of sectoral interdependencies (see Leontief, 1953, ch. 3). The attention focuses on structural relations between stocks of durable instruments of productions and flows of material inputs and flows of outputs. In static input–output models, the final demand vector comprises not only consumption goods, but also investment goods; that is, additions to the stocks of fixed capital items such as buildings, machinery, tools, etc. In dynamic input–output models, investment demand cannot be taken as given from outside, but must be explained within the model. The approach chosen is the following: the additions to the stocks of durable capital goods are technologically required, given the technique in use, in order to allow for an expansion of productive capacity that matches the expansion in the level of output effectively demanded. A simple dynamic model has the following form

$$(\mathbf{I} - \mathbf{A})\mathbf{x}_t - \mathbf{B}(\mathbf{x}_{t+1} - \mathbf{x}_t) = \mathbf{y}_t$$

where  $\mathbf{I}$  is the  $n \times n$  identity matrix,  $\mathbf{A}$  is the usual material input matrix (inclusive of wear and tear of fixed capital goods, or rather depreciation),  $\mathbf{B}$  is the square matrix of fixed capital coefficients,  $\mathbf{x}$  is the vector of total outputs and  $\mathbf{y}$  is the vector of final deliveries, excluding fixed capital investment;  $t$  refers to the time period. It deserves to be stressed that, in this approach, time is treated as a discrete variable. The coefficient  $b_{ij}$  defines the stock of products of industry  $j$  required per unit of capacity output of industry  $i$  and is thus a stock–flow ratio.

The time path of all the  $n$  components of final demand,  $\mathbf{y}_t, t = 1, 2, \dots$ , as well as the levels of all outputs at the initial point of time,  $\bar{\mathbf{x}}$ , are assumed to be given. Hence, we have a system of  $n$  difference equations. A major shortcoming of the simple model presented here is that, with an arbitrarily given time path of final demand and some given initial endowments, it cannot be guaranteed that the solution to the model will always have non-negative output levels. Closely connected with this is the model's inability to deal with situations in which one or several industries do not fully utilize their productive capacity, but exhibit excess capacities. As Leontief stressed: 'While bringing to the fore the crucial role that a complete set of capital coefficients has to play—in addition to a complete set of current input

coefficients—in the detailed description of the structural framework of a given economy, such a set of difference equations is too rigid a tool to be used to describe and project the actual process of economic development and change’ (Leontief, 1987, p. 863).

This is certainly a valid observation. In order to be useful in applied economics, the model needs to be adapted quite a bit.<sup>2</sup> However, this does not mean that there are no sensible uses to which the dynamic Leontief model can be put. In this paper we will show that it serves well the purpose of illustrating in a multisector framework the potential for endogenous growth. To do this we shall use a simplified form of the dynamic Leontief model, setting aside all fixed capital items and thus matrix **B**. There will be only circulating capital goods in the system; the matrix of material inputs will be given by **A**. Notice that **A** does not contain depreciation quotas of durable instruments of production, simply because there are no such instruments. It would not be difficult, however, also to take into consideration fixed capital. Yet, in order to solve the problem of depreciation, this would necessitate the formulation of a price model. Without such a model and a formalization of the problem of the choice of the optimal patterns of utilization and optimal lifetimes of fixed capital items, the problem of depreciation could not be solved consistently. Simply introducing some *ad hoc* rule such as ‘depreciation by evaporation’ or ‘depreciation by radioactive decay’ is not good enough (see, for instance, Kurz & Salvadori, 1995, chs 7 and 9).<sup>3</sup>

The main purpose of this paper is to relate the simplified version of the dynamic input–output model developed in the next section to some of the ‘new’ growth models.

### 3. Endogenous Growth and the Dynamic Leontief Model

Let us take a dynamic Leontief model of the type

$$\mathbf{x}_t \geq \mathbf{A}\mathbf{x}_{t+1} + \alpha_t \mathbf{d} \tag{1}$$

where **d** is a vector of consumed commodities and  $\alpha$  is a scalar. **A** is a Leontief matrix, but contrary to the usual formulation of the open Leontief model it includes a sector (a row) portraying the consumption of workers which, at the same time, is assumed to portray the production of labour by means of commodities and labour; and, contrary to the usual formulation of the closed Leontief model, it does not include consumption by capital owners. In the following, the consumption vector **d** will therefore be interpreted as referring only to the consumption of members of the society other than workers. In Section 4 we shall assume that these people are the capital owners whose income consists of profits (or interest) and who will spend their income partly on consumption goods (in proportion to vector **d**, i.e. all commodities are considered perfect complements) and will partly save and invest it. It should be pointed out that this approach is similar to that of the classical economists from Adam Smith to David Ricardo, who broadly considered labour as generated within the system and adjusted to the needs of capital accumulation. The most simple conceptualization possible, which is the one adopted here, is that labour is made available in whichever amount is needed at a given unit cost, which is equal to the given real wage rate.

In this conceptualization, labour is accordingly considered a producible factor of production. This assumption is invoked because, as we shall see in Section 4, in the ‘new’ growth literature something similar is done: there it is suggested that

'labour' can be treated as, or rather replaced by, a factor called 'human capital'; that is, an input in the production process that can be produced and accumulated. The removal from the picture of all factors that are non-accumulable is indeed one of the 'devices' by means of which, in this literature, the way is paved for unhampered perpetual growth.

Our model is not determined until the  $\alpha$ s are determined and we know which of the weak inequalities are satisfied as equations. A common way to determine the model is to assume that a planner or policy maker fixes an objective function for each period  $t$

$$f(\alpha_t, t)$$

so that it is possible to maximize

$$\sum_{t=0}^{\infty} f(\alpha_t, t)$$

under the constraint (1), where  $\mathbf{x}_t$  is non-negative for each  $t$ , and  $\mathbf{x}_0 \leq \bar{\mathbf{x}}$ , where  $\bar{\mathbf{x}} (> \mathbf{0})$  is the vector of the stocks of goods available at the beginning of the time considered.<sup>4</sup>

Let us first consider two simple examples. These are but steps towards a third example, which prepares the ground for a comparison with some of the 'new' growth models.

*Example 1.* In the first example it is assumed that the policy maker focuses attention exclusively on consumption at time  $\theta$ . This would obviously be a policy that could be characterized in terms of the famous dictum 'après moi le déluge'; the purpose of this example is purely illustrative. In this case

$$\sum_{t=0}^{\infty} f(\alpha_t, t) = \alpha_\theta$$

and, therefore, the problem to be solved consists of

$$\max \alpha_\theta \tag{2}$$

subject to

$$\mathbf{x}_t \geq \mathbf{A}\mathbf{x}_{t+1} + \alpha_t \mathbf{d}$$

$$\mathbf{x}_t \geq \mathbf{0}$$

$$\alpha_t \geq 0$$

$$\mathbf{x}_0 \leq \bar{\mathbf{x}}$$

It is easily checked that  $\alpha_\theta$  can be determined as the solution to the problem

$$\max \alpha_\theta$$

subject to

$$\alpha_\theta \mathbf{A}^\theta \mathbf{d} \leq \bar{\mathbf{x}}$$

$$\alpha_\theta \geq 0$$

Hence

$$\alpha_0 = \left( \max_i \frac{\mathbf{e}_i^T \mathbf{A}^\theta \mathbf{d}}{\mathbf{e}_i^T \bar{\mathbf{x}}} \right)^{-1}$$

where  $\mathbf{e}_i$  is the  $i$ th unit vector, i.e.  $\mathbf{e}_i^T = (0, \dots, 0, 1, 0, \dots, 0)$ .

Then, the solution to problem (2) is completed by

$$\begin{aligned} \mathbf{x}_t &= \alpha_0 \mathbf{A}^{\theta-t} \mathbf{d} & 0 \leq t \leq \theta \\ \alpha_t &= 0 & t \neq \theta \\ \mathbf{x}_t &= \mathbf{0} & t > \theta \end{aligned}$$

*Example 2.* The second example is also of the kind described, but slightly less bizarre: now the policy maker's attention focuses on consumption at two different times and attributes different weights to the two consumption levels:

$$\sum_{t=0}^{\infty} f(\alpha_t, t) = \psi \alpha_0 + (1 - \psi) \alpha_\tau$$

Hence the problem amounts to

$$\max \psi \alpha_0 + (1 - \psi) \alpha_\tau \tag{3}$$

subject to

$$\begin{aligned} \mathbf{x}_t &\geq \mathbf{A} \mathbf{x}_{t+1} + \alpha_t \mathbf{d} \\ \mathbf{x}_t &\geq \mathbf{0} \\ \alpha_t &\geq 0 \\ \mathbf{x}_0 &\leq \bar{\mathbf{x}} \end{aligned}$$

It is easily checked that  $\alpha_0$  and  $\alpha_\tau$  can be determined as the solution to the problem

$$\max \psi \alpha_0 + (1 - \psi) \alpha_\tau$$

subject to

$$\begin{aligned} \alpha_0 \mathbf{A}^\theta \mathbf{d} + \alpha_\tau \mathbf{A}^\tau \mathbf{d} &\leq \bar{\mathbf{x}} \\ \alpha_0 &\geq 0 \\ \alpha_\tau &\geq 0 \end{aligned}$$

Then, the solution to problem (3) is completed by

$$\begin{aligned} \alpha_t &= 0 & \theta \neq t \neq \tau \\ \mathbf{x}_t &= \alpha_0 \mathbf{A}^{\theta-t} \mathbf{d} + \alpha_\tau \mathbf{A}^{\tau-t} \mathbf{d} & 0 \leq t \leq \theta \\ \mathbf{x}_t &= \alpha_\tau \mathbf{A}^{\tau-t} \mathbf{d} & \theta < t \leq \tau \\ \mathbf{x}_t &= \mathbf{0} & t > \tau \end{aligned}$$

*Example 3.* In order to facilitate a comparison with the ‘new’ growth theory, let us now consider the case in which

$$\sum_{t=0}^{\infty} f(\alpha_t, t) := \sum_{t=0}^{\infty} (1 + \rho)^{-t} (1 - \sigma)^{-1} (\alpha_t^{1-\sigma} - 1)$$

where  $\rho$  can be interpreted as a rate of time preference at which consumption at future dates is discounted, and  $1/\sigma$  ( $1 \neq \sigma > 0$ ) can be interpreted as the elasticity of substitution between present and future consumption. Hence, the problem amounts to

$$\max \sum_{t=0}^{\infty} (1 + \rho)^{-t} (1 - \sigma)^{-1} (\alpha_t^{1-\sigma} - 1) \tag{4}$$

subject to

$$\begin{aligned} \mathbf{x}_t &\geq \mathbf{A}\mathbf{x}_{t+1} + \alpha_t \mathbf{d} \\ \mathbf{x}_t &\geq \mathbf{0} \\ \alpha_t &\geq 0 \\ \mathbf{x}_0 &\leq \bar{\mathbf{x}} \end{aligned}$$

It is easily checked that the  $\alpha$ s can be determined as the solution to the problem

$$\max \sum_{t=0}^{\infty} (1 + \rho)^{-t} (1 - \sigma)^{-1} (\alpha_t^{1-\sigma} - 1) \tag{5}$$

subject to

$$\begin{aligned} \sum_{t=0}^{\infty} \alpha_t \mathbf{A}^t \mathbf{d} &\leq \bar{\mathbf{x}} \\ \alpha_t &\geq 0 \end{aligned}$$

The Kuhn–Tucker–Lagrange conditions amount to

$$\alpha_t = [(1 + \rho)^t \mathbf{z}^T \mathbf{A}^t \mathbf{d}]^{-1/\sigma} \tag{6a}$$

$$\sum_{t=0}^{\infty} \alpha_t \mathbf{A}^t \mathbf{d} \leq \bar{\mathbf{x}} \tag{6b}$$

$$\mathbf{z} \geq \mathbf{0} \tag{6c}$$

$$\sum_{t=0}^{\infty} \alpha_t \mathbf{z}^T \mathbf{A}^t \mathbf{d} = \mathbf{z}^T \bar{\mathbf{x}} \tag{6d}$$

where  $\mathbf{z}$  is a vector of Lagrangians. Once this problem is solved, the solution to problem (4) is completed by

$$\mathbf{x}_t = \sum_{\tau=t}^{\infty} \alpha_{\tau} \mathbf{A}^{\tau-t} \mathbf{d}$$

Problem (5) does not have an easy solution. However, if we are interested in the steady-state solution only, then the problem is relatively simple. In this case  $\bar{\mathbf{x}}$  cannot be arbitrary, but must be chosen in such a way that

$$\alpha_t = \alpha_0(1 + g)^t$$

where  $g$  is a constant to be determined. Since equations (6a) and (6c) hold, and since, on the assumption that matrix  $\mathbf{A}$  has  $n$  distinct eigenvalues,

$$\mathbf{A}^t = \mathbf{T}^{-1} \mathbf{L}^t \mathbf{T}$$

where  $\mathbf{T}$  is the matrix of the left eigenvectors of matrix  $\mathbf{A}$ ,  $\mathbf{L}$  is the diagonal matrix with the eigenvalues of matrix  $\mathbf{A}$  on the main diagonal ( $\mathbf{TA} = \mathbf{LT}$ ),

$$\mathbf{z}^T = \beta \mathbf{q}^T$$

where  $\mathbf{q}^T$  is the Perron–Frobenius left eigenvector of matrix  $\mathbf{A}$  normalized in some way (we will use the normalization  $\mathbf{q}^T \mathbf{d} = 1$ ). Hence we have

$$\alpha_t = [(1 + \rho)^t \lambda^t \beta \mathbf{q}^T \mathbf{d}]^{-1/\sigma} = \beta^{-1/\sigma} [(1 + \rho)\lambda]^{-t/\sigma}$$

$$\sum_{t=0}^{\infty} \beta^{-1/\sigma} [(1 + \rho)\lambda]^{-t/\sigma} \mathbf{A}^t \mathbf{d} = \beta^{-1/\sigma} \{\mathbf{I} - [(1 + \rho)\lambda]^{-1/\sigma} \mathbf{A}\}^{-1} \mathbf{d} = \bar{\mathbf{x}}$$

$$\mathbf{x}_t = \beta^{-1/\sigma} [(1 + \rho)\lambda]^{-t/\sigma} \{\mathbf{I} - [(1 + \rho)\lambda]^{-1/\sigma} \mathbf{A}\}^{-1} \mathbf{d} = [(1 + \rho)\lambda]^{-t/\sigma} \bar{\mathbf{x}}$$

Note that matrix  $\mathbf{I} - [(1 + \rho)\lambda]^{-1/\sigma} \mathbf{A}$  is invertible with a positive inverse if and only if

$$(1 + g) = [(1 + \rho)\lambda]^{-1/\sigma} < \lambda^{-1} \tag{7}$$

that is,

$$\rho > \lambda^{\sigma(1 + \sigma)} - 1$$

Then we assume that inequality (7) holds from the beginning. Inequality (7) means that the actual growth factor  $[(1 + \rho)\lambda]^{-1/\sigma}$  is smaller than the maximum one,  $\lambda^{-1}$ . This requires that, for a given  $\sigma$  and a given  $\lambda$ , the rate of time preference  $\rho$  is sufficiently large, which will be assumed.

#### 4. A Comparison with the ‘New’ Growth Literature

The last few years have seen the publication of a bewildering variety of ‘new’ growth models. Despite all their differences, these models share some common features, the most important of which is, of course, that the steady-state rate of growth is determined endogenously. This is indeed the salient feature of this kind of model *vis-à-vis* the Solow model. In the latter, the actual rate of growth depends on the saving rate and thus agents’ behaviour, whereas the steady-state rate of growth does not. There is no endogenous growth in the very long run in the Solow model. Seen from this perspective, the main novelty of the ‘new’ growth models consists of also rendering the steady-state rate of growth an endogenous variable.

The reason why, in the Solow model, the long-term rate of growth is exogenous rather than endogenous is the presence of a non-accumulable factor, labour, which is responsible for a diminishing marginal product of capital as capital accumulates relative to labour (see Kurz & Salvadori, 1998). In order to have perpetual growth over and above the growth of the labour force (on the assumption that the latter is always fully employed), the marginal product of capital must not fall to zero (or to some lower boundary, given by a minimum level of the rate of profit at which accumulation ceases). Hence, there are essentially three ways open to ‘new’ growth theory: provide arguments that guarantee either that the curve giving the marginal

product of capital does not fall, but is a line parallel to the abscissa, or falls, but its fall is bounded from below at a level larger than zero (or larger than the minimum rate of profit) or, instead of falling, rises. As can be shown, the first route was chosen in the so-called 'linear' or 'AK models' (Rebelo, 1991, and King & Rebelo, 1990); the second in the model by Jones & Manuelli which assumes a convex technology with returns to capital sufficiently bounded from below (Jones & Manuelli, 1990); and the third by models based on the formation of human capital and the externalities associated with it (see, in particular, Lucas, 1988) or by models formalizing research and development and the endogenous generation of technical progress (see, in particular, Romer, 1986).

Since the dynamic Leontief model presupposes a given linear technology that does not change over time, the natural counterpart of it in the 'new' growth literature is the linear or AK models. We shall therefore focus attention on the latter.

It is a characteristic feature of the linear growth models that they set aside all factors of production that are non-accumulable. In its simplest version it is assumed that there is a linear relationship between total output,  $Y$ , and a broad measure of the accumulable factor capital,  $K$ , both consisting of the same commodity:

$$Y = AK \quad (8)$$

where  $1/A$  is the amount of that commodity required to produce one unit of itself. Capital is assumed to encompass both physical and human capital.<sup>5</sup> In this model, time is assumed to be continuous. Because of the linear form of the aggregate production function, the marginal product of capital, which equals the instantaneous rate of profit,  $\hat{r}$ , is given by

$$\hat{r} + \delta = \frac{Y}{K} = A \quad (9)$$

where  $\delta$  is the exogenously given rate of depreciation.

The continuous time framework prevents an immediate comparison of the AK model with the above dynamic Leontief model. In particular, one might be inclined to think that focusing attention on circulating capital only, as in the present dynamic Leontief model, amounts to assuming that, in the AK model,  $\delta$  would have to be set equal to unity. However, whereas with discrete time, the assumption  $\delta = 1$  would indeed mean that all capital is consumed in one unit of time, in a continuous time framework that assumption would mean that capital is consumed at the same instant of time at which produced commodities leave the production process. Yet with no time elapsing between inputs and outputs, there would simply be no capital at all: with continuous time, the premise  $\delta = 1$  would remove all capital from the picture and not only fixed capital. Moreover, in order to allow for the possibility that a capital good is consumed in a finite amount of time, we would have to introduce an infinite number of commodities for each capital good, each of these infinitely many commodities representing the capital good at the appropriate (continuous) vintage. With continuous time, then, the idea that a capital good depreciates in the sense that a part of it evaporates is not only the simplest one available to capture the idea of capital, but also the only one which, as far as we know, avoids the necessity of having recourse to an infinite number of capital goods. (This does not mean, of course, that it is fully satisfactory.)

Now, as equation (9) shows, it is a remarkable fact that, in the AK model, the

rate of profit is determined by technology alone. Then, the saving–investment mechanism, jointly with the assumption of a uniform rate of growth (that is, a steady-state equilibrium), determines a relationship between the instantaneous rate of growth,  $\hat{g}$ , and the instantaneous rate of profit,  $\hat{r}$ . Rebelo (1991, p. 504) obtains<sup>6</sup>

$$\hat{g} = \frac{A - \delta - \hat{\rho}}{\sigma} = \frac{\hat{r} - \hat{\rho}}{\sigma} \tag{10}$$

where  $\hat{\rho}$  is the instantaneous rate of time preference. Hence the growth factor (defined with respect to one unit of time) equals

$$e^{(\hat{r} - \hat{\rho})/\sigma}$$

Equation (10) is obtained when savings are determined on the assumption that there is an immortal ‘representative agent’ who is concerned with maximizing an intertemporal utility function,  $u = u(c(t))$ , over an infinite horizon. Choosing the path that maximizes consumption involves maximizing the integral of instantaneous utility,

$$\int_0^{\infty} e^{-\hat{\rho}t} u(c(t)) dt$$

In the case under consideration this integral is maximized subject to constraint (8), where  $Y = c(t) + \dot{K}$  and

$$u(c(t)) = \frac{c(t)^{1-\sigma}}{1-\sigma}$$

We may now compare this model with the above dynamic Leontief model. With labour as an endogenously produced factor of production whose costs of production are given and constant in terms of given amounts of wage goods and labour, and assuming that there is free competition in the economic system, the rate of return on capital  $r$  will tend to be uniform across all sectors. In this case, the normal prices ruling in the above dynamic Leontief model are given by the following equation

$$\mathbf{p}^T = (1 + r)\mathbf{p}^T \mathbf{A}$$

that is,

$$\lambda \mathbf{p}^T = \mathbf{p}^T \mathbf{A}$$

where

$$\lambda = \frac{1}{1 + r}$$

Scalar  $\lambda$  is the Perron–Frobenius eigenvalue of matrix  $\mathbf{A}$ . The profit factor is  $\lambda^{-1}$  and is thus equal to the maximum growth factor compatible with the given conditions of production (and productive consumption of the workers). From equation (7) we have

$$1 + g = \left( \frac{1 + r}{1 + \rho} \right)^{1/\sigma}$$

Taking into account that

$$\hat{r} = \log(1 + r)$$

$$\hat{\rho} = \log(1 + \rho)$$

we have that

$$1 + g = e^{(\hat{r} - \hat{\rho})/\sigma}$$

## 5. Concluding Remarks

The paper has looked at a special version of the dynamic Leontief model from the perspective of so-called ‘new’ growth theory, whose characteristic feature, in contradistinction to the Solovian model, is that the long-term rate of growth is determined within the model. It has been shown that a dynamic Leontief model, which fulfils the requirement of endogeneity of the steady-state growth rate, exhibits a close family resemblance with the linear variants of ‘new’ growth theory, such as the AK model. In both kinds of models the endogeneity of the growth rate is due to the fact that there are no primary factors in given (or exogenously growing) supply that could constrain economic expansion. Natural resources are set aside and it is suggested that there is a technology producing a surrogate for what the classical economists (and Solow) called ‘labour’. In the ‘new’ growth literature, that factor has merely been given new names and enters the stage either as ‘human capital’ or ‘information’ or ‘knowledge’. If there is such a technology, and if it fulfils the usual properties attributed to production processes, then the rate of profit is either technologically given or, if there is a choice of technique, it results from the cost-minimizing behaviour of producers. For a given and constant rate of profit, saving behaviour determines the rate of growth.

## Notes

1. For an interpretation of the dynamic Leontief model as a model of a quasi-stationary economy: that is, one that grows at a constant rate of growth, preserving its sectoral composition, see Meyer & Schumann (1977).
2. See, for example, the adaptation of the model to the study of the employment effects of the diffusion of new technologies in Leontief & Duchin (1986) and Kalmbach & Kurz (1992).
3. In input–output analysis, one is occasionally confronted with the view that the presence of ‘capital owners’ and thus profits is tied to the presence of fixed capital. Hence, without fixed capital there can be no profits. We do not share this view, but side rather with the classical economists, from Adam Smith to Ricardo and authors working in their tradition, such as von Newmann and, in at least some of his analysis, Sraffa. These authors conceived of both circulating capital goods (raw materials etc) and the means of subsistence in the support of workers (i.e. wages) as advances to be made available by capitalists at the beginning of the production period.
4. Jochen Schumann has pointed out to us that the introduction of an objective function into the input–output model is extraneous to Leontief’s way of thinking. In this context he quoted the following saying of Leontief’s: ‘The objective function is the most subjective part of the model!’. In the present context, the introduction of such a function serves only the purpose to relate the dynamic model to the new growth literature.
5. In King & Rebelo (1990) two types of capital are distinguished: physical and human capital, and it is assumed that both kinds of capital goods as well as the consumption good, which is taken to be identical with the capital good, are produced by means of both kinds of capital goods.
6. In the case in which the average propensity to save  $s$  is given from outside, Rebelo (1991, p. 506) obtains

$$\hat{g} = s(A - \delta) = s\hat{r}$$

This is formally identical to the famous ‘Cambridge equation’ of the post-Keynesian theory of growth and distribution, advocated by Kaldor, Robinson and Pasinetti.

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