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Abstract: Discusses the problem of exhaustible resources in a dynamic input-output model with `classical' features in terms of the price and quantity and distribution sides. Definition of exhaustible resources; Main assumptions in characterizing the backstop technology and cost minimizing backstop processes; Algorithmic analysis in finding the solution in backstop processes.

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EXHAUSTIBLE RESOURCES IN A DYNAMIC INPUT-OUTPUT MODEL WITH `CLASSICAL' FEATURES

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ABSTRACT

This paper discusses the problem of exhaustible resources in a dynamic input-output model with `classical' features. Both the quantity side and the price and distribution side are discussed. The argument is developed using the simplifying assumptions that there is no technical progress and that no new deposits of the exhaustible resources are discovered. To avoid the `end-of-world' scenario, it is assumed that there is a `backstop technology': this implies that exhaustible resources are useful but not necessary in the production and reproduction of commodities. A necessary and sufficient condition for the existence of paths of prices, quantities produced, and stocks of resources converging to the ultralong-period position is determined, provided that the backstop technology exhibits some appropriate properties. In addition, an algorithm to determine these paths is suggested. A numerical example illustrates the findings.

KEYWORDS: Dynamic input-output model, exhaustible resources, cost minimization, backstop technology

1. Introduction

Much of conventional economic analysis is based on the twin assumptions of the free gifts of nature and the free disposal of wastes. Therefore, the environment is envisaged to be simultaneously a `horn of plenty' and a `bottomless sink'. Seen from this perspective, the environment does not really matter: production can be conceptualized as a process the inputs of which are only labour and produced means of production, and the outputs of which are commodities. This view is difficult to sustain. In this paper, we will partially abandon the first of the twin assumptions, in order to accommodate `exhaustible' natural resources. We shall, however, retain the second assumption, which amounts to adopting the (in)famous `free disposal axiom'.

Exhaustible resources are natural resources that are either not naturally regenerated at all or regenerated on a time-scale that is irrelevant to human exploitation. Typical cases are ores such as coal, oil and metals: setting aside the possibility of recycling (which, for obvious reasons, can only be incomplete in any case), any removal reduces their stocks. In dealing with exhaustible resources, it is first necessary to explain why the owners of such resources do not sell the whole amount that they own at current prices in order to invest the proceeds in an industry where the normal rate of profit can be obtained. In other words, the storage of natural resources, which may be considered as a `conservation industry', cannot be operated if it does not yield a `royalty' to the owners of the resources. Under competitive conditions, the size of this royalty must be such that the going rate of profit is earned in the business. As a consequence, a natural resource has to be revalued over time, i.e. its price has to increase over time, and the prices of all commodities the production of which involves natural resources directly or indirectly will have to change over time. This necessitates studying the changes in prices and quantities over time. The appropriate analytical framework for this task appears to be a dynamic input-output (IO) framework.

The dynamic IO model elaborated in this paper has `classical' features. To understand what is meant by this, it is perhaps useful to refer briefly to some classical authors and authors working in the classical tradition. A characteristic feature of this tradition is the method employed: the analysis is carried out in terms of comparisons of `Long-period' positions of the economic system, characterized--conditions of free competition--by a uniform rate of profit across industries.(n1) With regard to the content of the theory, it starts from (1) given levels of production of the different commodities; (2) a given set of methods of production from which cost-minimizing producers can choose; and (3) a given rate of profit (or a given real wage rate). On the basis of these given conditions, the theory determines (1) the share of income other than the given income and (2) relative prices. It is a characteristic feature of this approach that the prices at the beginning of the (uniform) production period are equal to the prices at the end of this period, i.e. prices are static. With exhaustible resources, the stationarity of the prices of exhaustible resources cannot be preserved. This appears to imply that the long-period method does not carry over to an analysis of exhaustible resources.

While this is essentially true, the classical authors and their followers were apparently convinced that the long-period method is the best available and that its abandonment does not lead us any further, but presents the danger of depriving the analysis of a solid base to study an economic system confronted with the depletion of (some of) its stocks of natural resources. Although this position is not always made explicit, it seems to underlie much of the analysis carried out in the classical tradition. A brief survey of some important contributions to the classical approach confirms this impression. Although Ricardo devoted a separate chapter to the problem of exhaustible resources, entitled `On the rent of mines', the length of the chapter (barely three pages) and its content show that he did not really feel the need to elaborate on an approach that is different from the approach applied to the problem of inexhaustible resources dealt with in the preceding chapter `On rent' (Ricardo, 1951, Chapters 2 and 3). This conviction is reiterated by Karl Marx, who stressed that "Mining rent proper is determined in the same way as agricultural rent" (Marx, 1959, p. 775). Sraffa mentioned exhaustible resources only in passing: "Natural resources which are used in production, such as land and mineral deposits..." (Sraffa, 1960, p. 74).

It was only in the aftermath of the revival of the classical approach initiated by the publication of Sraffa's book that attempts were made to tackle the problem of exhaustible resources. A start was made by Parrinello (1983), who questioned the appropriateness of the long-period method in the case under consideration. A more detailed discussion was provided by Schefold (1989,Part II, Chapter 19b). One of Schefold's concerns was with showing that the theory of exhaustible resources "is of practical relevance only under exceptional circumstances, so that the classicists may be excused for having ignored it by subsuming the incomes of mine owners to the general theory of rent" (Schefold, 1989, p. 228). Schefold provided four reasons in support of this defence of the approach of the classical authors and Sraffa:

(1) the uncertainty concerning the future course of prices;

(2) the unpredictability of the impact of technical progress on the process of extraction;

(3) the relatively slow change of the royalty in any one mine;

(4) the great importance of cost differentials between mines and, thus, differential rents in explaining the residual income of mine owners.(n2)

Nevertheless, we shall attempt in this paper to deal with exhaustible resources in a framework which gives up some of the salient features of the classical long-period approach. In particular, this concerns the stationarity of prices and the real wage rate, and the stationarity of the proportions in which the commodities are produced. The purpose of the model elaborated in this paper is to provide a necessary and sufficient condition for the existence of the path that the economy can take from time zero to infinity. We shall attempt to modify the original approach, but only to the extent to which it is indispensable, since the questions asked could not otherwise be answered. The argument will be developed on the premise that, in each period, a uniform rate of profit and a uniform wage rate are obtained in each production activity. With regard to the determination of income distribution, for the reasons clarified in the theory of capital (cf. Kurz & Salvadori, 1995, Chapter 14), we cannot adopt the neoclassical idea that the rate of profit and the real wage rate are determined by the demand for and the supply of capital and labour respectively. In keeping with the classical approach to the theory of value and distribution, we shall instead assume that all but one of the distributive variables is given from outside the IO system, whereas the remaining distributive variables are determined endogenously. (The royalties are determined via the changes in the prices of resources over time.) Thus, the model preserves important `classical' features.

The paths of prices, quantities produced and resources available determined by the model would be followed if there were perfect foresight. But what if there is not? Suppose that the actual path of the price of a resource happened to be above the mentioned path. If producers were totally `myopic', then a situation would be reached where some amount of that resource would still be available, but nobody would want it. Of course, the assumption of totaly myopic behaviour is as questionable as that of perfect foresight. A truly dynamic model would have to take into account the reactions of those producers who recognize the fact that there could be unwanted resources remaining, from the point in time onwards in which they do so. Similarly, the actual path of the price of a resource might be below the mentioned path. In the case of totally myopic behaviour, the resource would be exhausted before its price had time to approach the level at which nobody wanted it. Again, a proper dynamic analysis would have to take into account the reactions of those who (first) recognize this fact.

Before we enter into a discussion of our subject matter, the reader's attention should be drawn to the following proviso. In all cases in which prices are bound to change over time and agents can be assumed to be aware of this, the question of the `expectations' of agents, and the formation of these expectations, cannot, in principle, be avoided. This raises formidable problems for economic theorizing. The assumption of perfect foresight, i.e. perfect information on the part of agents on all future states of the world, as entertained in the theory of intertemporal equilibrium, simply evades the issue. In contrast, introducing expectations involves the danger of depriving the analysis of clear-cut results: depending on the particular hypothesis about the formation of the expectations and how they affect the actions of agents, `anything goes' appears to be the unavoidable conclusion. Here, we abstain from entering into a discussion of the problems associated with the indisputable uncertainty that clouds the future. Hence, the following analysis can be only a preliminary step towards a more satisfactory investigation of the subject.

Another crucial element of the analysis is that the technology is assumed to be such that exhaustible resources are useful, but not necessary in the production and reproduction of commodities. This can be interpreted in the following way. The exhaustible resources can be used to produce `energy', which is taken to be an input in many (if not all) production processes. However, there is a `backstop technology', based on the use of some non-depletable resource (such as solar energy or geothermal energy). Setting aside technical progress and the discovery of new deposits of natural resources, the economy may be expected to converge to a system of production in which only solar energy will be used.

The present formulation derives from Salvadori (1987) and Kurz and Salvadori (1995). The main difference with respect to the work of Salvadori (1987) is that, in that paper, the price vector at time zero is assumed to be given, whereas we have here (as in Kurz and Salvadori (1995,pp. 357-368)) that the vector of commodities available at time zero is given. In Kurz and Salvadori (1995), we thought that the problem to be solved was the determination of a finite time path of the exhaustion of the resources. Indeed, we assumed that it is always possible to fix a point in time sufficiently distant from today (or the present period) such that all resources which it is convenient to deplete are actually depleted, and we found a sufficient condition to determine the finite time path of the exhaustion of the resources, on the assumption that the exhaustion is completed before the given point in time. We failed to recognize the following: (1) at the end of that time span, some resources can still be unexhausted and positively priced; and, as a consequence of this, (2) the solution to the problem can only be an intermediate result, because the proof that the exercise has been performed for a sufficiently long time span is not trivial; and, finally, (3) the problem has actually a necessary and sufficient condition which can be proved in a much simpler way. In order to make the present paper self-contained, we have borrowed some of the concepts introduced in the relevant discussion in our previous book. Thus, the reader may consider part of this paper as a simplification and generalization of that discussion.

The structure of the paper is as follows. Section 2 states the main assumptions on which the subsequent argument is based, gives some basic definitions and describes the dynamic IO model. Section 3 contains the preliminary results which constitute the solution to the problem that we considered in our previous work. Section 4 presents the complete analysis. In Section 5, a simple algorithm is suggested to find the solution in terms of a numerical example. Section 6 contains some concluding remarks.

2. Main Assumptions, Basic Definitions and the Model

There are s deposits of substances which can enter into the production of commodities, but cannot be produced; the amounts of these substances which have been utilized cannot be reutilized (as, for example, land whose powers were considered `indestructible' by Ricardo). These substances will be called `exhaustible resources'. Each deposit contains one resource and, for each resource, there is one deposit. (Therefore, s is also the number of resources: different deposits of the same resource are dealt with as deposits of different exhaustible resources, and different grades of the same resource are also dealt with as different exhaustible resources.) Let y_t be the vector of prices of exhaustible resources available at time t is an element of N_0, and let z_t be the vector of the amounts of exhaustible resources available at time t is an element of N_0.

Each resource is available in homogeneous quality and in a quantity which, at time 0, is known with certainty. Hence, discoveries of new deposits (or resources) are excluded. It hardly needs to be stressed that the assumption according to which the amounts of resources are known with certainty is very strong. In everyday experience, new deposits are continuously discovered. The opposite extreme would consist of assuming that, for each exhausted deposit (resource), another deposit with the same characteristics is discovered and that the cost of the search (in terms of labour and commodities) is always the same. In this case, the resources would not be exhaustible and each deposit could actually be treated as if it were a machine: the price of the new machine equals the cost of the search (including profits at the going rate), and the price of an old machine of age t equals the value of the deposit after t periods of utilization.(n3) Because the resources would no longer be exhaustible, their prices would be constant over time, as is commonly assumed in long-period analysis. The difficulty results from the fact `reality' is between the two extremes. An analysis of the `realistic' case, however, is beyond the scope of this paper.

A finite number n of different commodities are produced in the economy and a finite number m (> n) of processes are known to produce them. Let p_t be the vector of prices of commodities available at time t is an element of N_0 and let x_t be the intensity of operation of processes at time t is an element of N. A process or method of production is defined by a quadruplet (a, b, c, l), where a is an element of R^n is a commodity input vector, b is an element of R^n is the output vector, c is an element of R^s is the exhaustible resources input vector and l is the labour input (a scalar); of course, a >/= 0, b >/= 0, c >/= 0, l >/= O. It is important to note that the inputs referred to in vector c are inputs of the resources as they are provided by nature; for example, extracted oil is not contained in c, but is in b if (a, b, c, l) is an extraction process, or is in a if (a, b, c, l) is a process that uses it, unless the extraction costs equal zero. The m existing processes are defined by quadruplets

(a_j, b_j, c_j, l_j), j = 1, ..., m

Then, define the matrices A, B, C and vector 1 as follows:(n4)

[Multiple line equation(s) cannot be represented in ASCII text]

The commodities required annually for consumption are defined by a vector d, which, for simplicity, is assumed to be given and constant over time, i.e. independent of prices and quantities, including the quantities of the exhaustible resources left over. If innovations are set aside and all exhaustible resources are private property, so that, for each time t is an element of N_0, a uniform (among sectors) nominal rate of profit r_t holds, then the following inequalities and equations are to be satisfied for each t is an element of N_0 (w_t is the nominal wage rate at time t is an element of N):

(1a) Bp_{t+1} </= $(1 + r_t) (Ap_t + Cy_t) + w_{t+1}$ 1

(1b) $x^{T, \text{ sub } t+1} Bp_{t+1} = x^{T, \text{ sub } t+1} [(1+r_t) Ap_t + Cy_t) + w_{t+1}$ 1]

(1c) $y_{t+1} </= (1+r_t)y_t$

(1d) $z^{T,\ sub\ t+1}\ y_{t+1} = (1 + r_t)\ z^{T,\ sub\ t+1}\ y_t$

(1e) $x^{T,\ sub\ t=1}\ B >/= x^{T,\ sub\ t+2}\ A + d^T$

(1f) $x^{T,\ sub\ t+1}\ Bp_{t+1} = (x^{T,\ sub\ t+2}\ A + d^T)p_{t+1}$

(1g) $z^{T,\ sub\ t} >/= x^{T,\ sub\ t+1}\ C + z^{T,\ sub\ t+1}$

(1h) $z^{T,\ sub\ t}\ y_t = (x^{T,\ sub\ t+1}\ C + z^{T,\ sub\ t+1})\ y_t$

(1i) $w_{t+1} > 0,\ p_t >/= 0,\ y_t >/= 0,\ z_t >/= 0,\ x_{t+1} >/= 0$

Inequality (1a) means that nobody can obtain extra profits by producing commodities available at time t + 1. Equation (1b) implies, because of inequalities (1a) and (1i), that commodities available at time t + 1 will only be produced if the ruling rate of profit is obtained. Inequality (1c) means that nobody can obtain extra profits by storing exhaustible resources from time t to time t + 1. Equation (1d) implies, because of inequalities (1c) and (1i) that exhaustible resources will be stored from time t to time t + 1 only if the ruling rate of profit will be obtained by this storage activity. Inequality (1e) implies that the amounts of commodities produced are not smaller than the amounts of commodities required, and equation (1f) implies that if an amount is larger, then the price of that commodity is zero. Inequality (1a) implies that the amounts of exhaustible resources available at time t are not smaller than the amounts of exhaustible resources available at time t+1 plus the amounts of exhaustible resources utilized to produce commodities available at time t + 1, and equation (1h) implies that if an amount is larger, then the price of that exhaustible resource is zero. The meaning of inequalities (ii) is obvious.

In the following, it will be assumed that the nominal wage rate at time 1 serves as the numeraire. We have

(1j) $w_1 = 1$

Model (1) is not yet complete, because some initial conditions are needed. A first obvious initial condition is that the amounts of exhaustible resources available at time 0 are given, i.e.

(1k) $z_0 = z$

A second initial condition, which is perhaps less obvious, is that the amounts of commodities available at time 0 are given. This can be stated as

(1l) $v^T >/= x^{T,\ sub\ 1}\ A + d^T$

(1m) $v^T\ p_0 = (x^{T,\ sub\ l}A + d^T)\ p_0$

where v is a given vector that is larger than d.

Using the model presented in this paper, it is not possible to determine the dynamics of the nominal profit rate and the nominal wage rate. The simplest assumption to be adopted is, of course, that both variables are constant over time, i.e.

(1n) $r_t = r_0$ (= r)

(1p) $w_{t+1} = w_1$ (=1)

The constancy of the nominal profit rate and of the nominal wage rate should not be interpreted to mean that they need to be constant but, rather, that the dynamics of these variables is external to the model. It would be possible, for instance, to assume that the rate of profit is changing over time, with the whole sequence of profit rates taken as being given. Similarly, the ratio between the nominal wage rate at time t (t= 2, 3, ...) and the nominal wage rate at time 1 can be given, whereas the nominal wage rate at time 1 is to be determined in the model (or set as the numeraire). For instance, if the nominal wage rate is assumed to increase at a uniform growth rate b, then (with a constant nominal rate of profit) inequalities (1a) and (1b) would be

$$Bp_{t+1} </= (1 + r) (Ap_t + Cy_t) + w_1 (1 + b)^t 1$$

$$x^{T, \text{ sub } t+1} Bp_{t+1} = x^{T, \text{ sub } t+1} [(1+r) (Ap_t + Cy_t) + w_1 (1+b)^t 1]$$

whereas the equation fixing the numeraire could again be w_1 1. A somewhat more interesting model could assume that the real wage rate rather than the nominal wage rate is constant. In fact, if prices are changing over time and the nominal wage rate is not, then the real wage, i.e. the amounts of commodities which a worker can afford, is also changing, but this would eliminate the linearity of the system of equations and inequalities investigated. In fact, with a constant nominal rate of profit and a constant real wage rate, inequalities (1a) and (1b) would be

$$Bp_{t+1} </= (1 + r) (Ap_t + Cy_t) + wld^T p_{t+1}$$

$$x^{T, \text{ sub } t+1} Bp_{t+1} = x^{T, \text{ sub } t+1} [(1 + r) (Ap_t + Cy_t) + wld^T p_{t+1}]$$

where r is the nominal rate of profit and w is the number of bundles d which constitutes the real wage rate (on the assumption that the real wage rate is defined in terms of the annual consumption vector of society). In this case, it is not possible to fix the numeraire by setting w = 1, because w is not a price; and to set w = 1 is, of course, an alternative to take r as given from outside the model. In this case, one of the following numeraires could be adopted, for example:

$$d^T p_1 = 1$$

$$wd^T p_1 = 1$$

Because ours is only a first step towards a more complete analysis, we consider the premise of a constant nominal wage rate, i.e. the adoption of equation (1p), to be good enough. Obviously, it is implicitly assumed that changes in the real wage rate do not endanger the capacity of workers to work and, thus, the viability of the economic system at large.

Model (1) is more simply stated as

(2a) $Bp_{t+1} \leq (1+r)(Ap_t + Cy_t) + 1$

(2b) $x^{T,\ \text{sub } t+1} Bp_{t+1} = x^{T,\ \text{sub } t+1} [(1 + r)(Ap_t + Cy_t) + 1]$

(2c) $y_{t+1} \leq (1+r)y_t$

(2d) $z^{T,\ \text{sub } t+1} y_{t+1} = (1 + r)z^{T,\ \text{sub } t+1} y_t$

(2e) $v^T \geq x^{T,\ \text{sub } 1} A + d^T$

(2f) $v^T p_0 = (x^{T,\ \text{sub } 1} A + d^T) p_0$

(2g) $x^{T,\ \text{sub } t+1} B \geq x^{T,\ \text{sub } t+2} A + d^T$

(2h) $x^{T,\ \text{sub } t+1} Bp_{t+1} = (x^{T,\ \text{sub } t+2} A + d^T) p_{t+1}$

(2i) $z^{T,\ \text{sub } t} \geq x^{T,\ \text{sub } t+1} C + z^{T,\ \text{sub } t+1}$

(2j) $z^{T,\ \text{sub } t} y_t = (x^{T,\ \text{sub } t+1} C + z^{T,\ \text{sub } t+1}) y_t$

(2k) $z_0 = z$

(2l) $p_t \geq 0,\ y_t \geq 0,\ z_t \geq 0,\ x_{t+1} \geq 0$

Each of the exhaustible resources is assumed to provide directly or indirectly(n5) services which can, however, also be produced by solar energy, the source of which does not risk exhaustion in any relevant time-frame. In particular, the commodities required annually for consumption, defined by the vector d, can be produced without using the exhaustible resources. In other words, there are vectors x^* and p^* which solve the system

(3a) $x^T (B - A) \geq d^T$

(3b) $x^T (B - A)p = d^T p$

(3c) $Bp \leq (1 + r) Ap + 1$

(3d) $x^T Bp = x^T [(1 + r) Ap + 1]$

(3e) $x \geq 0,\ p \geq 0$

where the processes (A, B, 0, 1) are obtained from (A, B, C, 1) by deleting all processes that directly use natural resources. That is, the process ($e^{T,\ \text{sub } i}$ A, $e^{T,\ \text{sub } i}$ B, $e^{T,\ \text{sub } i}$ C, $e^{T,\ \text{sub } i}$ 1) is in the set of processes (A, B, 0, 1) if and only if $e^{T,\ \text{sub } i} C = 0^T$. The processes (A, B, 0, 1) constitute the `backstop technology'. In the following discussion, we will refer to the processes operated at the intensity vector x (obtained by augmenting the vector $x^*$ with zeros) as the `cost-minimizing backstop processes', and we will denote these processes by the quadruplet (A, B, 0, 1).

The assumption that there is a backstop technology is necessary to avoid the `end-of-the-world' scenario, on which nothing needs to be said.(n6) This is the case because we excluded discoveries of new deposits (or resources) and innovations. Seen from this perspective, the assumption that the commodities required annually for consumption can be produced without using the exhaustible resources may be considered as a sort of simple corrective device to counterbalance the bold premises that underlie our analysis. The following assumptions characterize the backstop technology and the cost-minimizing backstop processes.

Assumption 1. The backstop technology is such that it converges to the processes (A, B, 0, 1). In other words, the backstop processes (A, B, 0, 1) are such that the system made up by inequalities (2a), (2b), (2e), (2f), (2g) and (2h), and the first and fourth parts of inequalities (21), with A = A, B = B, C = 0 and 1 = 1, is such that, for each of its solutions (if there is one), there is a natural number Theta^* such that, for each t >/= Theta^*, only the processes (A, B, 0, 1) are operated.

Assumption 2. The number of cost-minimizing backstop processes is exactly n (the number of commodities); the matrix B is invertible; the matrix B^{-1} A is non-negative; and the eigenvalue of maximum modulus of the matrix $B^{-1} A = A^*$ is lower than $(1 + r)^{-1}$.

Assumption 2 certainly holds if there is no joint production and if the rate of profit r is such that, for each commodity, no more than one process producing it can be operated in the long run. In fact, in this case, we can order the processes (A, B, 0, i) in such a way that B is diagonal, with the elements on the main diagonal all positive; finally, the property of the eigenvalue of maximum modulus of the matrix A^* means that the backstop technology can support the given rate of profit r.

3. A Preliminary Result

Assume that system (2) has a solution. Call the set of processes operated at time t in such a solution the position at time t. Because the number of processes is finite, the number of possible positions is also finite. Hence, at least one position is replicated for an infinite number of times. Because the amounts of exhaustible resources available at time 0 are finite, and because the vector of the amounts of resources utilized in a position employing exhaustible resources is bounded from below (because demand is fixed), any position which is replicated an infinite number of times cannot use exhaustible resources. Hence, we can divide the period from time 0 to infinity into two periods: a finite period from time 0 to time Tau' and an infinite period from time Tau' + 1 to infinity, on the condition that, in the second period, only the backstop processes (A, B, 0, 1) are operated. (This does not mean that, in the second period, exhaustible resources are already fully exhausted, or are not positively priced: they are simply not used in production.) Moreover, if Assumption 1 holds, then we can divide the period from time Tau' + 1 to infinity into two subperiods: a finite subperiod from time Tau' + 1 to time Tau" and an infinite subperiod from time Tau" + 1 to infinity, on the condition that, in the second subperiod, only the cost-minimizing backstop processes (A, B, 0, 1) are operated.

The argument just developed suggests the following problem, the study of which is a preliminary step towards an analysis of system (2). Let Theta be a positive natural number and let us investigate the following system:

(4a) $Bp_{t+1} \leq (1+r)(Ap_t + Cy_t) + 1,\ 0 \leq t \leq \text{Theta} - 1$

(4b) $x^{T,\ \text{sub } t+1}\ Bp_{t+1} = x^{T,\ \text{sub } t+1}\ (Ap_t + Cy_t) + 1],\ 0 \leq t \leq \text{Theta} - 1$

(4c) $y_{t+1} \leq (1 + r)y_t,\ 0 \leq t \leq \text{Theta} - 1]$

(4d) $z^{T,\ \text{sub } t+1}\ y_{t+1} = (1 + r)\ z^{T,\ \text{sub } t+1}\ y_t,\ 0 \leq t \leq \text{Theta} - 1$

(4e) $v^T \geq x^{T,\ \text{sub } 1}\ A + d^T$

(4f) $v^T\ p_0 = (x^{T,\ \text{sub } 1}\ A + d^T)\ p_0$

(4g) $x^{T,\ \text{sub } t}\ B \geq x^{T,\ \text{sub } t+1}\ A + d^T,\ 1 \leq t \leq \text{Theta} - 1$

(4h) $x^{T,\ \text{sub } t}\ Bp_t = (x^{T,\ \text{sub } t+1}\ A + d^T)p_t,\ 1 \leq t \leq \text{Theta} - 1$

(4i) $x^{T,\ \text{sub Theta}}\ B >/ x^T\ A + d^T$

(4j) $x^{T,\ \text{sub Theta}}\ Bp_{\text{Theta}} = (x^T\ A + d^T)\ p_{\text{Theta}}$

(4k) $z^T \geq x^{T,\ \text{sub } 1}\ C + z^{T,\ \text{sub } 1}$

(4l) $z^T\ y_0 = (x^{T,\ \text{sub } 1}\ C + z^{T,\ \text{sub } 1})\ y_0$

(4m) $z^{T,\ \text{sub } t} \geq x^{T,\ \text{sub } t+1}\ C + z^{T,\ \text{sub } t+1},\ 1 \leq t \leq \text{Theta} - 1$

(4n) $z^{T,\ \text{sub } t}\ y_t = (x^{T,\ \text{sub } t+1}\ C + z^{T,\ \text{sub } t+1})\ y_t,\ 1 \leq t \leq \text{Theta} - 1$

(4p) $p_t \geq 0,\ y_t \geq 0,\ 0 \leq t \leq \text{Theta}$

(4q) $z_t \geq 0,\ x_t \geq 0,\ 1 \leq t \leq \text{Theta}$

This system (4) can be considered as consisting of the first Theta steps of system (2), on the assumption that $x_{\text{Theta} + 1} = x$, i.e. on the assumption that, at time Theta + 1, the cost-minimizing backstop processes are operated and they are operated at the cost-minimizing backstop intensities.

System (4) can easily be transformed into a linear programming problem. This is immediately seen when current prices at time t are transformed into present or discounted prices, i.e. prices at time 0 of commodities or resources available at time t.(n7) We have

$p_t = (1 + r)^{-t} p_t$

$y_t = (1 + r)^{-t} y_t$

The use of discounted prices instead of actual prices is very common in intertemporal analysis. This use allows, for instance, the description of a process by means of netput vectors instead of pairs of input vectors and output vectors. (The description in terms of netput vectors is not feasible in a long-period analysis, because the actual prices at the beginning of the production period and those at the end of the production period are equal, so the discounted prices at the beginning of the production period are (1 + r) times the discounted prices at the end of the production period.) In terms of discounted prices, system (4) can be written as

(5a) $Bp_{t+1} \leq Ap_t + Cy_t + (1+r)[\text{sup } (1+t)\ 1, \ 0 \leq t \leq \Theta - 1$

(5b) $x^{T,\text{ sub } t+1} Bp_{t+1} = x^{T,\text{ sub } t+1} [Ap_t + Cy_t + (1 + r)^{-(1+t)\ 1}\ 0 \leq t \leq \Theta - 1$

(5c) $y_{t+1} \leq y_t, \ 0 \leq t \leq \Theta - 1$

(5d) $z^{T,\text{ sub } t+1} y_{t+1} = z^{T,\text{ sub } t+1} y_t, \ 0 \leq t \leq \Theta - 1$

(5e) $v^T \geq x^{T,\text{ sub } 1} A + d^T$

(5f) $v^T p_0 = (x^{T,\text{ sub } 1} A + d^T) p_0$

(5g) $x^{T,\text{ sub } t} B \geq x^{T,\text{ sub } t+1} A + d^T, \ 1 \leq t \leq \Theta - 1$

(5h) $x^{T,\text{ sub } t} Bp_t = (x^{T,\text{ sub } t+1} A + d^T) p_t, \ 1 \leq t \leq \Theta - 1$

(5i) $x^{T,\text{ sub Theta}} B \geq x^{-T} A + d^T$

(5j) $x^{T,\text{ sub Theta}} Bp_{\text{Theta}} = (x^{-T} A + d^T) p_{\text{Theta}}$

(5k) $z^{-T} \geq x^{T,\text{ sub } 1} C + z^{T,\text{ sub } 1}$

(5l) $z^T y_0 = (x^{T,\text{ sub } 1} C + z^{T,\text{ sub } 1}) y_0$

(5m) $z^{T,\text{ sub } t} \geq x^{T,\text{ sub } t+1} C + z^{T,\text{ sub } t+1}\ 1 \leq t \leq \Theta - 1$

(5n) $z^{T,\text{ sub } t} y_t = (x^{T,\text{ sub } t+1} C + z^{T,\text{ sub } t+1}) y_t, \ 1 \leq t \leq \Theta - 1$

(5p) $p_t \geq 0, \ y_t \geq 0, \ 0 \leq t \leq \Theta$

(5q) $z_{t[\ \geq 0,\ x[\text{sub } t} \geq 0, \ 0 \leq t \leq \Theta$

Because of the equilibrium theorem of linear programming, system (5) is equivalent to each of the following two linear programming problems, which are dual to each other. First, we will call primal the problem

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subject to

(6a) $v^{T} \geq x^{T,\ \text{sub}\ 1} A + d^{T}$

(6b) $x^{T,\ \text{sub}\ t} B \geq x^{T,\ \text{sub}\ t+1} A + d^{T}$, $1 \leq t \leq \Theta - 1$

(6c) $x^{T,\ \text{sub}\ \Theta} B \geq x^{-T} A + d^{T}$

(6d) $x^{T} \geq x^{T,\ \text{sub}\ 1} C + z^{T,\ \text{sub}\ 1}$

(6e) $z^{T,\ \text{sub}\ t} \geq x^{T,\ \text{sub}\ t+1} C + z^{T,\ \text{sub}\ t+1}$ $1 \leq t \leq \Theta - 1]$

(6f) z_t 0, $x_t \geq 0$, $1 \leq t \leq \Theta$

Next, we have will call dual the problem

[Multiple line equation(s) cannot be represented in ASCII text]

subject to

$Bp_{t+1} \leq Ap_t + Cy_t + (1 + r)$[sup -(1 + t)1, $0 \leq t \leq \Theta - 1$

$y_{t+1} \leq y_t$, $0 \leq t \leq \Theta - 1$

$p_t \geq 0$, $y_t \geq 0$, $0 \leq t \leq \Theta$

Note that the dual can also be stated without reference to the discounted prices,

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subject to

$Bp_{t+1} \leq (1+r)(Ap_t + Cy_t) + 1$, $0 \leq t \leq \Theta - 1$

$y_{t+1} \leq (1 + r) y_t$, $0 \leq t \leq \Theta - 1$

$p_t \geq 0$, $y_t \geq 0$, $0 \leq t \leq \Theta$

Henceforth, we will refer to this problem as the `dual'. The following proposition gives an `if and only if' condition of the existence of a solution to system (4).

Proposition 1. System (4) has a solution for Theta = Theta' if and only if the following assumption holds.

Assumption 3. There are two finite sequences x_t and z_t (t= 1, 2, ..., Theta') such that system (6) holds for Theta = Theta'.

Proof. Because the two finite sequences p_t = 0 and y_t = 0 (t = 0, 1, ..., Theta') are feasible solutions to the dual, both the primal and the dual have optimal solutions if and only if there is a feasible solution to the primal, because of the duality theorem of linear programming.

4. Main Results

Proposition 1 is a generalization and a simplification of the results presented in Kurz and Salvadori (1995,pp. 358-365). In Kurz and Salvadori (1995), we thought that a result such as that of Proposition 1 was enough; that what was needed in addition was merely to find a large enough Theta such that a solution to system (4) consisted only of the first Theta steps of a solution to system (2); and that this could be checked simply by an analysis of time Theta + 1. This view is incorrect. However, we are now in a position to provide a necessary and sufficient condition for a solution to system (2), on the assumption that the backstop technology is such that it converges to the cost-minimizing backstop processes, which exhibit some appropriate properties (Assumptions 1 and 2).

The following proposition informs about the solutions to system (4) for different Theta values.

Proposition 2. If system (4) has a solution for Theta = Theta', then it has a solution for Theta = Theta", with each Theta" >/= Theta' .

Proof. If the two finite sequences x'_t and z'_t (t = 1, 2, ..., Theta') satisfy system (6) for Theta = Theta', then the two finite sequences x''_t and z''_t (t= 1, 2, ..., Theta") with $x''_t = x'_t$ and $z''_t = z'_t$ for t = 1, 2, ..., Theta', and x''_t = x and $z''_t = z'_{Theta'}$ for t = Theta + 1, Theta' + 2, ..., Theta", satisfy system (6) for Theta = Theta".

Let us now assume that there is a natural number Theta' such that Assumption 3 holds. Then, because of Proposition 2, for each Theta >/= Theta', the minimum value of the primal exists; we will call it h_{Theta}. Moreover, for each Theta >/= Theta', four infinite sequences $\{x_{t\ Theta}\}$, $\{z_{t\ Theta}\}$, $\{p_{t\ Theta}\}$ and $\{y_{t\ Theta}\}$ are defined, where, for t </= Theta, $x_{t\ Theta'}$ and $z_{t\ Theta}$ equal the corresponding elements of the optimal solution of the primal, and $p_{t\ Theta}$ and $y_{t\ Theta}$ equal the corresponding elements of the optimal solution of the dual and, for t >/= Theta, we have

(7a) $x_{t\ Theta} = x$

(7b) $z_{t\ Theta} = z_{Theta\ Theta}$

(7c) $p_{t\ Theta} = p^* + (1 + r)^{t\ -\ Theta} (A^*)^{t\ -\ Theta} (p_{Theta\ Theta\ -\ p[sup\ *})$

(7d) $y_{t\ Theta} (1 + r)^{t\ -\ Theta}\ y_{Theta\ Theta}$

where $A^* = B^{-1} A$ has the properties mentioned in Assumption 2.(n8) It is immediately checked that the four sequences satisfy (2b)-(21). (Note that $p_{t \Theta}$ in equation (7c) is the solution of the difference equation

$$Bp_{t+1} = 1 + (1 + r) Ap_t$$

of the initial point $p_{\Theta \Theta}$.) Finally, if the sequences $\{p_{t \Theta}\}$ and $\{y_{t \Theta}\}$ do not satify the inequality (2a) for t = t' (necessarily, t' >/= Theta), then the first t' elements of the sequences $\{x_{t \Theta}\}$ and $\{z_{t \Theta}\}$ cannot be an optimal solution of the primal above for Theta = Theta" > t'. In other words, we have

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This argument suggests the following.

Proposition 3. If Assumptions 1 and 2 hold and there is a natural number Theta' such that Assumption 3 holds, then system (2) has a solution if and only if there is a natural number Tau >/= Theta such that

(8) [Multiple line equation(s) cannot be represented in ASCII text]

Proof. It is easily checked that

[Multiple line equation(s) cannot be represented in ASCII text]

and that, if Theta' </= Θ^* < Θ^{**}, then

[Multiple line equation(s) cannot be represented in ASCII text]

In other words, the sequence

$$\{h_{\Theta} + 1/r(1 + r)^{\Theta} x^T 1\}$$

is not increasing; therefore, we have

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Hence, if there is a Tau >/= Theta' such that equation (8) is satisfied, then, for each Theta" >/= Tau, we have that $x_{t \Theta''} = x_{t \mathrm{Tau}}$ and $z_{t \Theta''}$ = z[sub t Tau], and as a consequence, $p_{t \Theta''} = p_{t \mathrm{Tau}}$ and $y_{t \Theta''} = y_{t \mathrm{Tau}}$. This is enough to assert that the sequences $\{x_{t \mathrm{Tau}}\}$, $\{z_{t \mathrm{Tau}}\}$, $\{p_{t \mathrm{Tau}}\}$ and $\{y_{t \mathrm{Tau}}\}$ satisfy system (2). However, if system (2) has a solution, then the argument developed at the very beginning of Section 3 applies and there is a natural number Tau" such that, from time Tau" + 1 to infinity, only the cost-minimizing backstop processes (A, B, 0, 1) are operated, i.e. for t > Tau", we have that x_t = x. This implies that equation (8) holds with Tau = Tau" + 1.

5. A Simple Algorithm

The analysis developed so far is quite abstract. A simpler algorithm to find a solution to system (2) (provided it exists) is the following. First, set Theta so large that system (6) has a solution. If such a Theta value does not exist, then the conditions mentioned in Proposition 3 cannot be met. Next, determine the sequences {$x_{t\ \text{Theta}}$} and {z[sub t Theta}, the first Theta + 1 elements of which are the corresponding elements of the determined optimal solution to the primal above; the other elements are determined as in equations (7a) and (7b). Then determine an optimal solution to the dual above and, consequently, the sequences {$p_{t\ \text{Theta}}$} and {$y_{t\ \text{Theta}}$}, the first Theta + 1 elements of which are the corresponding elements of the determined optimal solution to the dual above; the other elements are determined as in equations (7c) and (7d). If the sequences {$p_{t\ \text{Theta}}$} and {$y_{t\ \text{Theta}}$} satisfy inequality (2a) for each t, then a solution to system (2) is found. If this is not the case, then perform the same exercise for a larger Theta value. We cannot be sure that this algorithm will actually converge.

Let us explore this algorithm with the help of a simple numerical example. The example was used in Kurz and Salvadori (1995,pp. 366-368) to show that, in a general framework, such as the one presently adopted, it is possible to switch from a situation in which resources are not used to one in which they are, and then to switch back again to a situation in which they are not used (being depleted). Here, we will use that example to illustrate the simple algorithm introduced in this section.

Let energy be the only `basic' commodity in an economy the technology of which has only single-product processes of production. By a `basic' commodity, we mean a commodity that enters directly or indirectly into the production of all commodities (cf. Sraffa, 1960, p. 7). The amount of energy consumed plus the amount of energy required to produce all the other (non-basic) commodities is assumed to be equal to unity per year. In other words, the economy requires one unit of energy every year (to satisfy `final demand') plus the energy required to produce energy itself. Energy can be produced by using oil--an exhaustible resource obtained at zero extraction cost--or by using solar energy--a nonexhaustible resource. The available processes to produce energy are described in Table 1. To rule out the problem of a choice of technique as regards the consumed non-basic products, it will be assumed that there is only one process available for each of them. (There is no need to specify the processes that produce non-basic commodities.) The amount of oil available at time 0 equals 3/4 units; the rate of profit is taken to be unity.

We may now distinguish between several cases, according to the amount of energy available at time 0. We will consider two cases only. In the first case, we assume that the amount of energy v existing at time 0 is v= 20001/18000. In the second case, v = 10/9. To simplify the notation, let x_t be the amount of energy produced at time t, by using oil, and let q_t be the amount of energy produced at time t without using oil. Moreover, let p_t be the price of energy available at time t, let y_t be the price of oil available at time t and let z_t be the amount of oil available at time t. The vector x mentioned in the previous analysis has a zero entry that corresponds to the process using oil and q = 10/9 that corresponds to the process not using oil. The vector p^* consists of one element: $p^* = 25/2$.

5.1. Case (1): v = 20001 / 18000

If Theta = 3, then the algorithm determines x_{13} = 0 for t is not equal to 3; and x_{33} = 1/18, q_{13} = 667/600, q_{23} = 67/60, q_{33} = 19/18, q_{t3} = 10/9 for t >/= 4; y_{t3} = 0 for each t >/= 0; p_{t3} = 25/2 + (1625/2)$(1/5)^t$ for each t >/= 0; z_{t3} = 3/4 for each 0 </= t </= 2; z_{t3} = 13/18 for each t >/= 3. It is easily checked that inequality (2a) is satisfied only if

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i.e. only if t </= 2. In contrast, if Theta >/= 5, then the algorithm determines $x_{t\ Theta}$ = 0 for 1 </= t </= 3; $x_{4\ Theta}$ = 73/162, $x_{5\ Theta}$ = 85/81, $x_{t\ Theta}$ = 0 for t >/= 6; $q_{1\ Theta}$ = 667/600, $q_{2\ Theta = 67/60,\ q[sub\ 3\ Theta}$ = 7/6, $q_{4\ Theta}$ = 62/81, $q_{t\ Theta}$ = 1/81, $q_{t\ Theta}$ = 10/9 for t >/= 6; $y_{t\ Theta}$ = (43/120)2^t for each t >/= 0; $p_{t\ Theta}$ = 25/2 + (14375/6)$(1/5)^t$ for each t >/= 0; $z_{t\ Theta}$ = 3/4 for each 0 </= t </= 3; $z_{4\ Theta}$ = 85/162, $z_{t\ Theta}$ = 0 for each t >/= 5. It is easily checked that expression (2a) is satisfied only if

(9) 2/5 [25/2 + 14375/6$(1/5)^t$] + 43/120 2^t + 1 >/= 25/2 + 14375/6 $(1/5)^{t+1}$

which is satisfied for each natural number t (as an equation for t = 3 or t= 4).(n9)

5.2. Case (2): v = 10/9

Let Theta >/= 2. Then, the algorithm determines $x_{t\ Theta}$ = 0 for each t; $q_{t\ Theta}$ = 10/9 for each t; $y_{t\ Theta}$ = y_0 2^t for each t: $p_{t\ Theta}$ = 25/2 - (25/2)$(1/5)^t$ for each t; $z_{t\ Theta}$ = 3/4 for each t. It is easily checked that inequality (2a) is satisfied only if

2/5 [25/2 - 25/2$(1/5)^t$ + y_0 2^t + 1 >/= 25/2 - 25/2 $(1/5)^{t + 1}$

which is satisfied for each t if y_0 >/= 9. This is the (unique) solution to system (2), which is determined with the algorithm introduced here--but it is not the unique solution to system (2). Indeed, it is easily checked that, if p_0 and y_0 are two non-negative real numbers such that, for each t is an element of N_0 we have

(p_0 - 25/2) $(1/5)^{t + 1}$ + y_0 2^t >/= 13/2

then x_t = 0, q_t = 10/9, y_t = y_0 2^t, p_t = 25/2 + (p_0 - 25/2)$(1/5)^t$, z_t = 3/4 is a solution to system (2).

6. Concluding Remarks

In this paper, a dynamic IO model has been elaborated which is able to deal in a simple way with exhaustible resources, such as ores of coal, oil or metal. The problem is dealt with in a general rather than a partial framework. The model is derived from modern formulations of the 'classical' approach to the theory of value and distribution, which takes all but one of the distributive variables as given from outside the system of production. However, while classical analyses generally assume static prices and static values of the endogenous distributive variables (the real wage rate and the rent rates), with exhaustible resources the stationarity postulate cannot be preserved. In our model, both the prices of commodities and the real wage rate can change over time, with the levels of the rate of profit and of the nominal wage rate being constant over time, and the level of the rate of profit being given from outside the system (the wage rate at time 1 serves as the numeraire).

The model sets aside technical progress in extracting or using a resource. To avoid the 'end-of-the-world' scenario, it is assumed that there is a backstop technology (using solar energy). A necessary and sufficient condition for the existence of paths of prices, produced quantities, and stocks of resources converging to the ultralong-period position is determined, provided that the backstop technology exhibits some appropriate properties. In addition, an algorithm to determine these paths is suggested. It goes without saying that many more steps will have to follow before one arrives at a moderately satisfactory theory of exhaustible resources.

Notes

(n1.) For a detailed exposition of the classical long-period method and its application to a variety of economic problems, see Kurz and Salvadori (1995).

(n2.) See also the contributions by Gibson (1984), Roncaglia (1983, 1985), Pegoretti (1986, 1990) and Quadrio Curzio (1983, 1986).

(3.) For a general analysis of fixed capital, see Kurz and Salvadori (1995). Chapter 7 is devoted to the case in which no more than one machine is used in each process, while Chapter 9 deals with the case in which machines may be used jointly.

(n4.) Transposition of a vector or a matrix is denoted by superscript T.

(n5.) Assume, for instance, that electric energy can be produced from oil which is extracted from the ground. The unextracted oil is the resource, whereas the extracted oil is a commodity produced by means of that resource. Then, we say that the resource produces electric energy indirectly.

(n6.) Although the 'end-of-the-world' has repeatedly been forecast (see, for example, the gloomy perspectives entertained by the Club of Rome), it has not yet become a part of the 'plan of Providence', to use a term employed by Adam Smith [1759] (1976, TMS, III. 5.7).

(n7.) We should stress that we did not grasp this point from the beginning. In previous versions of what is given as Proposition 1, we used a complicated compact format of system (4), which allowed us to use linear complementarily to obtain a sufficient condition definitely more restrictive than that of Proposition 1 (see Kurz & Salvadori, 1995, pp. 364-365). We later had the opportunity to read a manuscript by Diedrich (1994; see also Diedrich, 1995, 1996), which used an ingenious but complicated transformation of our compact format of system (4) in a pair of dual linear programmes. This transformation leads to the results presented here as Proposition I as they are now, and we used this transformation in lectures and in the presentation of the present paper at the IO conference in New Delhi. Afterwards, Giuseppe Freni remarked to one of us that the

complexities connected with these transformations can be avoided by adopting the concept of discounted prices. This is what we do in the following. We would like to take the opportunity to thank Martin Diedrich for having given us the opportunity to read his unpublished manuscript, and Giuseppe Freni for his valuable remark.

(n8.) *Note that, if these properties do not hold, then p_t does not need to approach p^* as t approaches infinity. These properties, which are not restrictive in the single production case, have the specific aim of ensuring that p_t approaches p^*.*

(n9.) *The reader can easily check the expression (9) can be stated as $(43/120)2^t$ >/= 13/2 - $(2875/6)(1/5)^t$ and (43/120) 2^t > 13/2 for each t >/= 5.*

Table 1. Processes to produce energy

	Inputs			
Energy	Oil	Labour		Eneryg
1/10	--	10	arrow right	1
1/5	1/2	1	arrow right	1

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