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THE CHAMPERNOWNE-KAHN-FORMULA: A NOTE ON THE VALUE OF INVESTED CAPITAL*

In their celebrated article 'The Value of Invested Capital' ([1953–54], reprinted in J. Robinson [1956]) D. G. Champernowne and R. F. Kahn discussed the effect of the rate of profits on the value of capital involved in the use of a given technique of production under equilibrium conditions within the framework of a model where time is regarded as a continuous variable.

This note is confined to a presentation of a simple way to derive the relationship between the value of a partly worn-out durable capital good and the price of the brand-new durable capital good within the framework of a von Neumann-Sraffa model where time is regarded as a discontinuous variable. In accordance with Champernowne and Kahn it is assumed that during its (physical) life of T years the fixed capital good is of constant efficiency and at the end of its life its scrap value is zero.[1]

The von Neumann-Sraffa point of view implies that the same fixed capital good should be treated as so many different products, each with its own price or book-value, as there are different ages of the fixed capital good. Let us denote the respective price per unit of the t years-old fixed capital good by P_t $(t = 0, 1, 2, \ldots, T-1)$.

The assumption of constant efficiency implies that the input–output structure of the industry using this particular fixed capital good remains constant over the latter's lifetime. This means that the input quantities of, say, n circulating capital goods (such as raw materials, energy inputs and the like) $a_1, a_2, \ldots, a_n$ periodically used up by the fixed capital good, the amount of direct labour per period, l needed to run the production processs, and the quantities of final outputs per period $b_1, b_2, \ldots, b_n$ remain the same irrespective of the age of the fixed capital good in use. Hence the price system of this industry will satisfy (cf. Sraffa [1960], p. 65).

$$
\begin{aligned}
&(1+r)P_0 + (1+r)(a_1p_1+\ldots+a_np_n) + wl = b_1p_1+\ldots+b_np_n + P_1\\
&(1+r)P_1 + (1+r)(a_1p_1+\ldots+a_np_n) + wl = b_1p_1+\ldots+b_np_n + P_2\\
(1)\quad &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&(1+r)P_{T-1} + (1+r)(a_1p_1+\ldots+a_np_n) + wl = b_1p_1+\ldots+b_np_n,
\end{aligned}
$$

where r is the rate of profits, w the wage rate, and p_i the price of the ith product $(i = 1, 2, \ldots, n)$.

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[1] It should be noted, however, that the method used is not confined to fixed capital goods of this peculiar kind but can be applied also to fixed capital goods of variable efficiency and positive scrap value; see Sraffa [1960], p. 66.

We can re-write (1) as

$$
(2)\quad
\begin{aligned}
P_0 - RP_1 &= Rc\\
P_1 - RP_2 &= Rc\\
&\dots\dots\dots\\
P_{T-2} - RP_{T-1} &= Rc\\
P_{T-1} &= Rc,
\end{aligned}
$$

where $R = (1+r)^{-1}$ and $c = \sum_{i=1}^{n} [b_i - (1+r)a_i]p_i - wl.$

Let us define

$$
\mathbf{Q} = \begin{bmatrix} 1 & -R & 0 & 0 & \dots & 0 \\ 0 & 1 & -R & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & \ddots & \vdots \\ \vdots & \vdots & & & \ddots & -R \\ 0 & 0 & \dots & \dots & \dots & 1 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_{T-1} \end{bmatrix} \text{ and } \mathbf{C} = c\mathbf{1} = c \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.
$$

Thus (2) can be written as

(3) $\mathbf{QP} = R\mathbf{C}$

Since $\mathbf{Q}$ is non-singular there exists the inverse matrix $\mathbf{Q}^{-1}$

$$
\mathbf{Q}^{-1} = \begin{bmatrix} 1 & R & R^2 & \dots & R^{T-1} \\ 0 & 1 & R & \dots & R^{T-2} \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & & \dots & R \\ 0 & 0 & & & 1 \end{bmatrix}
$$

Solving (3) for $\mathbf{P}$ gives us

(4) $\mathbf{P} = \mathbf{Q}^{-1}R\mathbf{C}$

If we now express the book-value of a t years old fixed capital good as a fraction of the price of the brand-new item we get from (4)

$$
(5)\quad \frac{P_t}{P_0} = \frac{(R^0 + R^1 + \dots + R^{T-t-1})Rc}{(R^0 + R^1 + \dots + R^{T-1})Rc}
$$

Numerator and denominator of this expression contain the same geometric series with different lengths. According to the summation formula for geometric series we have

$$
\frac{P_t}{P_0} = \frac{1 - R^{T-t}}{1 - R^T} = \frac{R^{-T} - R^{-t}}{R^{-T} - 1}
$$

and hence

$$
(6)\quad \frac{P_t}{P_0} = \frac{(1+r)^T - (1+r)^t}{(1+r)^T - 1},
$$

which is the Champernowne–Kahn formula. It shows that, given the price of the new fixed capital good, the value of the partly worn-out fixed capital good rises relative to the original price with a rise in the rate of profits.

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REFERENCES

CHAMPERNOWNE, D. G. and KAHN, R. F. (1953–54), 'The Value of Invested Capital', *Review of Economic Studies*, Vol. 21, pp. 107–11.

ROBINSON, J. (1956), *The Accumulation of Capital*, Macmillan.

SRAFFA, P. (1960), *Production of Commodities by Means of Commodities*, Cambridge University Press.