MAT.311UB (WS 2023/2024):

Numerische Mathematik 2

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References

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1. Numerical linear algebra

1.1. **Iterative solution of systems of linear equations.**

Λ1. *Linear system (LS) of equations.* Existence and uniqueness of solution of LS, condition number, a-priori and a-posteriori stability analysis.

Λ2. *Methods of solution of LS.* Direct solution methods, computational complexity, consistency and convergence, convergence rate and stopping criteria, classification of iterative methods.

Λ3. *Linear stationary iterative methods.* Jacobi, Gauss–Seidel (GS), successive over-relaxation (SOR) methods.

Λ4. *Non-stationary iterative methods.* Richardson, gradient, and steepest descent methods.

Λ5. *Projection methods.* Projection on conjugate directions, conjugate gradient (CG) method, preconditioned CG.

Λ6. *Krylov subspace methods.* Krylov subspaces, Arnoldi and Lanczos algorithms, generalized minimal residual (GMRES) method.

1.2. **Numerical solution of eigenvalue problems.**

Λ7. *Numerical solution of eigenvalue problem (EVP).* Matrix eigenvalue problem, stability and conditioning of EVP, power and QR-iteration methods.

Λ8. *Numerical solution of symmetric EVP.* Minimax property, Rayleigh quotient, tridiagonalization using Lanczos method, EV of tridiagonal matrix.

2. Solution of ordinary differential equations (ODE)

2.1. **Numerical solution of 1st order ODE.**

Λ9. *1st order ODE.* Cauchy initial value problem (IVP), existence, uniqueness, and stability of solution of Cauchy problem.

Λ10. *One-step numerical methods.* Explicit Euler method, truncation error and consistency, zero stability, convergence error, implicit Crank–Nicolson method

Λ11. *Runge–Kutta (RK) method.* Explicit and implicit RK methods, consistency and convergence, adaptive step-size.

Λ12. *Multi-step numerical methods.* Consistency, explicit and implicit Adams methods, root condition, Dahlquist theorems.

Λ13. *ODE system and stability.* System of equations, stiff systems, stability polynomial, absolute-, A- and zero- stability.

2.2. **Numerical solution of 2nd order ODE.**

Λ14. *Two-point boundary value problem (BVP).* Integral formula of solution, discretization by finite-difference method (FDM) and spectral collocation method.

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Λ15. *Variational problem (VP) in 1D.* Weak solution, Lagrange finite-element method (FEM), $\mathbb{P}_2$-stiffness and mass matrices, interpolation and approximation errors.

Λ16. *Advection-diffusion problem.* Variational solution in 1D, $\mathbb{P}_1$-FEM and FDM approximations, Péclet number and stability, stabilized FEM.