Public Economics, Social Choice and Normative Economics

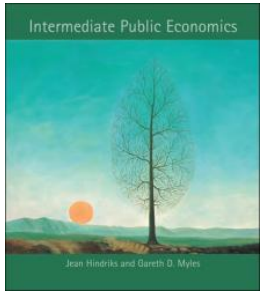
WS 2016/17

Christian Klamler

General Information

- Time and location: see UniGrazOnline for details
  - course ends before Christmas
- Course material
  - available on my webpage (access via usual uni-graz login)
  - christian.klamler@uni-graz.at
- Office hours
  - Mon 13.30 – 15.00
- Grading: 3 exams or short tests
  - 4 Nov (max 15 points)
  - 18 Nov (max 40 points), HS 15.04; 08.00am
  - 16 Dec (max 45 points), HS 15.04; 08.00am
  - in-class participation
  - **more than 50 points necessary for positive grade!**

LV-Info - Literatur



Hindriks, J. and G.D. Myles: Intermediate Public Economics, MIT Press, Cambridge, 2006

The Theory of Implementation of Social Choice Rules*

Roberto Serrano[†]

Department of Economics, Box B
Brown University
Providence, RI 02912, U.S.A.

Working Paper No. 2003-19
September 2003

Abstract. Suppose that the goals of a society can be summarized in a social choice rule, i.e., a mapping from relevant underlying parameters to final outcomes. Typically, the underlying parameters (e.g., individual preferences) are private information to the agents in society. The implementation problem is then formulated: under what circumstances can one design a mechanism so that the private information is truthfully elicited and the social optimum ends up being implemented? In designing such a mechanism, appropriate incentives will have to be given to the agents so that they do not wish to misrepresent their information. The theory of implementation or mechanism design formalizes this "social engineering" problem and provides answers to the question just posed. I survey the theory of implementation in this article, emphasizing the results based on two behavioral assumptions for the agents (dominant strategies and Nash equilibrium). Examples discussed include voting, and the allocation of private and public goods under complete and incomplete information.

Journal of Economic Literature Classification: C72, D78, D82.

Keywords: Implementation theory, mechanism design, asymmetric information, decentralization, game theory, dominance, Nash equilibrium, monotonicity.

Serrano, R.: "The Theory of Implementation of Social Choice Rules", 2003 (https://www.sss.ias.edu/files/papers/econpaper33.pdf)

PERSPECTIVES ON MECHANISM DESIGN IN ECONOMIC THEORY

Prize Lecture, December 8, 2007

by

Roger B. Myerson[1]

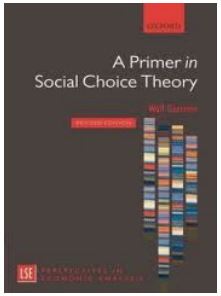
Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, USA.

1. AN HISTORICAL PERSPECTIVE

Economics began with Xenophon's "Oeconomicus" (c 360 BCE), in which Socrates interviews a model citizen who has two primary concerns. He goes out to his farm in the country to monitor and motivate his workers there. Then he goes back to the city, where his participation in various political institutions is essential for maintaining his rights to own this farm. Such concerns about agents' incentives and political institutions are also central in economic theory today. But they were not always.

Two centuries ago, economics developed as an analytical social science by focusing on production and allocation of material goods, developing methodologies of national-income accounting and price theory. Questions about resource allocation seemed particularly amenable to mathematical analysis, because flows of goods and money are measurable and should satisfy flow-balance equations and no-arbitrage conditions. From this perspective, the classical economic problem was that people's ability to satisfy their desires is constrained by limited resources. The classical economic result was that unrestricted free trade can achieve allocative efficiency, in the sense that reallocating the available resources cannot improve everyone's welfare.

Myerson, R.B.: "Perspectives on Mechanism Design in Economic Theory", American Economic Review, 98, 586-603, 2008.



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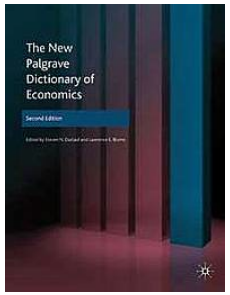

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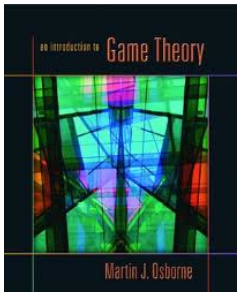

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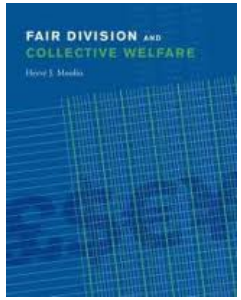

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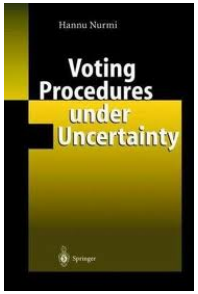

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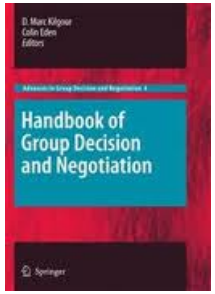

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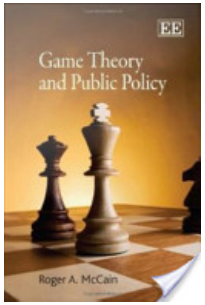

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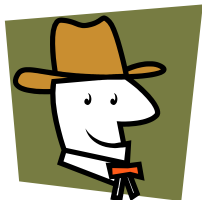
Public Econ, Social Choice and Normative Econ

- Content
  - Social Choice
    - Preferences
    - Aggregation Procedures
    - Impossibility Results
    - Inequality and Poverty Measures
  - Public Economics
    - Public Goods
    - Mechanism Design
    - Local Goods
    - Externalities
  - Fair Division

Public Econ, Social Choice and Normative Econ

- Public Economics
  - the part of Economics concerned with government policies
    - efficiency and equity
  - aggregation of individual preferences important to determine what government should do
    - what is "best" for society?
    - how can such social choices be made?
  - finding "good" public policies needs normative judgements
    - what "should" be done
    - which allocations are considered "fair"
  - how can we give the right incentives to tell the truth
    - government needs information to determine optimal policies
    - mechanism design

Social Choice Theory



Where will they have dinner?

Social Choice Theory - Introduction

- look for collective decisions
  - what possibilities in decision finding do we have?
  - in the sense, that
    - we are all "happy" with the result
    - small changes in the starting situation do not lead to large changes in the result
    - there will be no paradoxical results

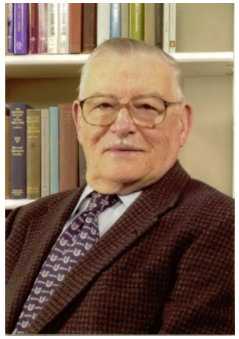
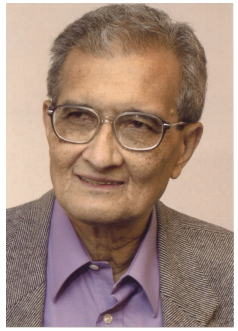
- collective decisions are to be made in many situations
  - elections, committees, groups of experts, family decisions, internet search-engines, Oscar celebration, Song-Contest, jurys in sports, etc.

Historical Aspects

- Historical Aspects
  - First statements of collective decisions more than 2000 years ago
  - first substantial research results in the 18th century
    - criticism of plurality rule
    - suggested alternatives
  - Marquis de Condorcet (1743 – 1794)
    - Académie Royale des Sciences
  - Jean-Charles de Borda (1733-1799)

Historical Aspects

- modern social choice theory developed around 1950
  - major contributions (among others) by following Nobel prize winners
    - Kenneth Arrow (Stanford) – 1972
    - James Buchanan (George Mason) - 1986
    - Amartya Sen (Harvard) - 1998
    - Eric Maskin (Princeton) – 2007
    - Lloyd Shapely (UCLA) – 2012

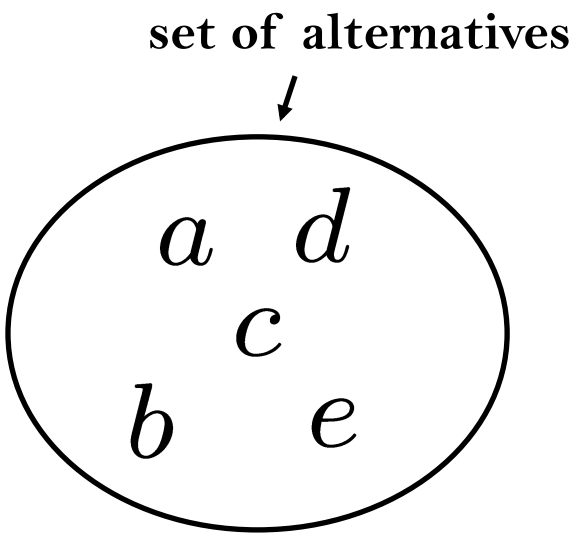




Social Choice Theory

- **Social Choice Theory**
  - need to transform individual preferences over a set of alternatives into a group preference/decision
  - which collective decision rules do you know?
  - does it make a difference in
    - homogeneous groups
    - heterogeneous groups
      - exactly here we need a convincing, acceptable and plausible system

Social Choice Theory

- **formal approach**
  - X = {a,b,c,...,x,y,z} ... finite set of alternatives
  - N = {P1,P2,...} ... finite set of individuals



- announce best alternative
- rank alternatives
- divide alternatives into groups
- evaluate each alternative individually

political parties, job candidates, dishes, etc.

Model

- so we start with individual preferences
  - usually a binary relation R over some finite set X of alternatives is defined as follows:

$$R \subseteq X \times X$$

  - $(x, y) \in R_i$ will denote x at least as good as y in i's terms.
  - equivalently written as xR_iy

Definition

A binary relation R on X is

- complete if $\forall x, y \in X$, either xRy or yRx
- reflexive if $\forall x \in X$, xRx
- transitive if $\forall x, y, z \in X$, xRy and yRz implies xRz
- quasi-transitive if $\forall x, y, z \in X$, xPy and yPz implies xPz
- acyclic if $\forall x, y, z_1, ..., z_l \in X$, xPz_1, z_1Pz_2, ..., z_lPy implies xRy

Model

Example

What properties do the following relations satisfy, where $X = \{x, y, z\}$?

1. $R = \{(x,x),(y,y),(z,z),(x,y),(y,x),(x,z),(z,x),(y,z),(z,y)\}$
2. $R = \{(x,y),(y,x),(z,x)\}$
3. $R = \{(x,x),(y,y),(z,z),(x,y),(x,z),(z,x),(y,z)\}$
4. $R = \{(x,x),(y,y),(z,z),(x,y),(y,x),(z,x),(y,z),(z,y)\}$
5. $R = \{(y,x),(x,z),(z,y)\}$
6. $R = \{\}$
7. $R = \{(x,x),(y,y),(z,z),(x,y),(y,x),(z,x),(z,y)\}$
8. $R = \{(x,x),(y,y),(z,z),(y,x),(x,z),(z,x),(y,z),(z,y)\}$

Model

- usually we start with a complete and transitive binary relation R over some finite set X of alternatives

- question:
  - what is a collective decision?

set of preference profiles

$$f : \mathcal{R}^n \Rightarrow \mathcal{R}$$

ind. preference profile, p

$$\begin{array}{ccccccc} a & c & e & d & b & d & a \\ b & a & c & e & c & c & d \\ c & e & a & a & a & b & e \\ d & d & b & b & e & e & c \\ e & b & d & c & d & a & b \end{array}$$

social choice rule

group decision

group ranking

election of one (or a set of) alternatives

Welfare Economics

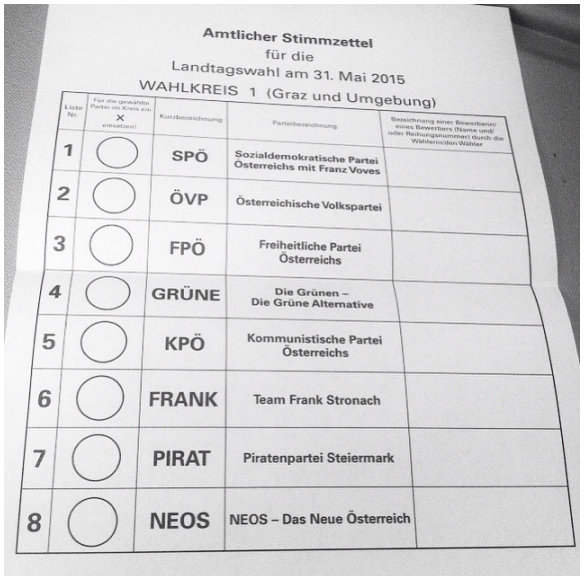
- **Welfare Economics**
  - concerned with the evaluation of resource allocations by markets and policy makers
  - major tool to evaluate allocations?
    - first fundamental theorem of welfare economics
    - PARETO criterion

P_1	P_2	P_3
a	b	e
b	d	a
c	f	f
d	a	b
e	c	c
f	e	d

  - what would be the Pareto ranking?
  - does it seem applicable in real world policy decisions?

How are we usually voting?

- **Plurality Rule**
  - each individual can vote for exactly ONE alternative
    - formal definition?
  - results of plurality rule used differently
    - proportional representation
    - majority representation



Partei	Zahl	Prozent	% +/-	Sitze†
SPÖ	189.763	29,29	▼8,97 %	15
ÖVP	184.300	28,45	▼8,74 %	14
FPÖ	173.332	26,76	▲16,1 %	14
Grüne	43.272	6,68	▲1,13 %	3
KPÖ	27.339	4,22	▼0,19 %	2
Stronach	11.292	1,74	(neu)	0
Piraten	1.406	0,22	(neu)	0
NEOS	17.078	2,64	(neu)	0

formal definition of plurality rule?

Definition (Plurality Rule)

f is called plurality rule if $\forall p \in \mathcal{R}^n$ and all $x, y \in X$, $xf(p)y$ if and only if $|\{i \in N : xR_iz, \forall z \in X\}| \geq |\{i \in N : yR_iz, \forall z \in X\}|$.

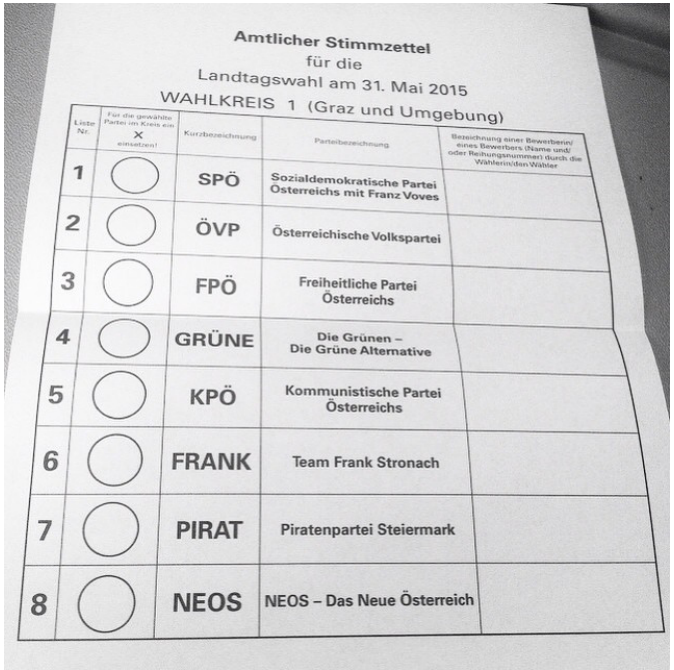
Plurality rule

- What's going on?

best alternative

worst alternative

a	a
b	e
c	d
d	c
e	b



Amtlicher Stimmzettel
für die
Landtagswahl am 31. Mai 2015
WAHLKREIS 1 (Graz und Umgebung)

Liste Nr.	Für die gewählte Partei im Kreis ein X einsetzen!	Kurzbezeichnung	Parteibezeichnung	Bezeichnung einer Bewerberin/ eines Bewerbers (Name und/ oder Reihungsnummer) durch die Wählerin/den Wähler
1		SPÖ	Sozialdemokratische Partei Österreichs mit Franz Voves	
2		ÖVP	Österreichische Volkspartei	
3		FPÖ	Freiheitliche Partei Österreichs	
4		GRÜNE	Die Grünen – Die Grüne Alternative	
5		KPÖ	Kommunistische Partei Österreichs	
6		FRANK	Team Frank Stronach	
7		PIRAT	Piratenpartei Steiermark	
8		NEOS	NEOS – Das Neue Österreich	

Lot of preference information ignored!

Plurality Rule

- US election 2000
  - voting system based on plurality rule and an "electoral college"
  - Gore had <u>more votes</u>
    - **50,999,897** vs 50,456,002
  - Bush <u>more electors</u>
    - **271** vs 266

were all preferences equally plausible?

Nader	*Nader*
Gore	*Bush*
Bush	*Gore*

→ strategic issues important

Plurality Rule

- plurality rule can also lead to other paradoxical situations
  - example: 6 voter with preferences over 3 alternatives

	2	1	2	1
worst	a	a	b	c
↓	c	b	c	b
best	b	c	a	a

→

group
a
b
c

  - what if all switch their preferences completely?

	2	1	2	1
best	b	c	a	a
↓	c	b	c	b
worst	a	a	b	c

→ 

Condorcet's Method

- Condorcet saw the following problem with plurality rule
  - missing democratic legitimacy in many voting situations

3	2	2
a	b	c
b	c	b
c	a	a

$\Rightarrow$

$b \succ a$

$b \succ c$

$c \succ a$

$\Rightarrow$

$group$
b
c
a

- **pairwise (simple) majority rule** (Condorcet)
  - pairwise vote over every pair of alternatives
  - elect the alternative winning over EVERY other alternative
  - formal definition?

Definition (Simple Majority Rule)

f is called simple majority rule if $\forall p \in \mathcal{R}^n$ and all $x, y \in X$, $xf(p)y$ if and only if $|\{i \in N : xR_iy\}| \geq |\{i \in N : yR_ix\}|$.

Pairwise Majority Rule

- problematic situations possible

1	1	1
a	b	c
b	c	a
c	a	b

$\Rightarrow$

$$a \succ b$$
$$b \succ c$$
$$c \succ a$$

$\Rightarrow$

$group$
?
?
?

  - violation of transitivity
  - **Condorcet Cycle**
  - what can be done against this?

Pairwise Majority Rule

- sequential majority rule
  - first a against b; then winner against c

1	1	1
a	b	c
b	c	a
c	a	b

$\Rightarrow$

$$a \succ b$$
$$c \succ a$$

$\Rightarrow$

$group$
c
a
b

  - if you are the first individual?
    - would you try to change something in the order?

1	1	1
a	b	c
b	c	a
c	a	b

$\Rightarrow$

Sequential Majority Rule

- could we elect an objectively bad alternative with this rule?
- first a against b; then winner against c; then winner against d

1	1	1
a	b	c
d	a	b
c	d	a
b	c	d

- is this an "acceptable" choice?

Borda Rule

- **Borda Rule**
  - assign points to each alternative according to the rank in the individual preference order
  - scoring procedure based on scores
    - points pre-determined

	1	1	1
3 points →	a	d	a
2 points →	d	b	d
1 point →	c	c	b
0 points →	b	a	c

	points
a	6
b	3
c	2
d	7

group
d
a
b
c

Borda Rule

	1	1	1
3 points →	a	d	a
2 points →	d	b	d
1 point →	c	c	b
0 points →	b	a	c

	points
a	6
b	3
c	2
d	7

group
d
a
b
c

Definition (Borda rule)

Let $b_i(x) = |\{y \in X : xP_iy\}|$ and $b(x) = \sum_{i \in N} b_i$. Then f is called Borda rule if $\forall p \in \mathcal{R}^n$ and all $x, y \in X$, $xf(p)y$ if and only if $b(x) \geq b(y)$.

- variant of the Borda rule used at Song-Contest
  - (12,10,8,7,6,5,4,3,2,1,0,...,0)
  - other variants (scoring rules) possible
  - is there a relationship between Condorcet's and Borda's rules?

Scoring Rules

- assign 1 point to the best two (three, etc.) alternatives
- does it make a difference?

3	2	2	2	1
a	d	b	b	b
c	c	a	c	d
d	a	d	d	a
b	b	c	a	c

	$Borda$
a	16
b	15
c	14
d	15

- Borda-Rule
  - ? wins
- vote for best 2
  - ? wins
- plurality rule
  - ? wins
- vote for best 3
  - ? wins

preferences did not change – group decision did!

Plurality Runoff

- **Plurality Runoff**
  - Use plurality rule. If no alternative receives absolute majority, let there be a contest between top two candidates.
  - presidential election

34	35	31
a	b	$\cancel{c}$
$\cancel{c}$	$\cancel{c}$	b
b	a	a

1. round

group
b
a
$\cancel{c}$

2. round

$$b \succ a$$

Plurality Runoff

- **Single Transferable Vote**
  - Use plurality rule. If no alternative receives absolute majority, eliminate alternative with fewest plurality votes. Repeat until one alternative receives absolute majority.
  - in both, plurality runoff and STV, the attempt is to provide legitimacy of choice by winning an "absolute majority" at least in some round

Official Ballot
Municipal Elections

INSTRUCTIONS TO VOTERS	Candidates for City Council District One (Three to be elected.)		*Only one vote per candidate* Only one vote per column
Mark Your Choices by Filling in the Numbered Boxes Only	Douglas Campbell	Dem.	1 2 3 4 5 6 7 8 9
Fill in the number one 1 box next to your first choice; fill in the number two 2 box next to your second choice; fill in the number three 3 box next to your third choice, and so on. You may fill in as many choices as you please. Fill in no more than one box per candidate. Fill in no more than one box per column.	Martha Dains	Rep.	1 2 3 4 5 6 7 8 9
	Terry Graybeal	Reform	1 2 3 4 5 6 7 8 9
	Robert Gomez	Dem.	1 2 3 4 5 6 7 8 9
	Cynthia Daniels	Indep.	1 2 3 4 5 6 7 8 9
	Robert Higgins	Rep.	1 2 3 4 5 6 7 8 9
	Write In		1 2 3 4 5 6 7 8 9
	Write In		1 2 3 4 5 6 7 8 9
	Write In		1 2 3 4 5 6 7 8 9

Plurality Runoff

3	2	1	1	1
a	c	b	e	d
b	b	e	b	b
c	d	d	c	c
d	e	c	d	e
e	a	a	a	a

- who wins with plurality rule?
  - *a*
  - **BUT**: clear majority prefers ALL other alternatives to a
- who wins with plurality runoff?
  - *c*
  - but also problems

Plurality Runoff

3	2	1	1	1
a	c	b	e	d
b	b	e	b	b
c	d	d	c	c
d	e	c	d	e
e	a	a	a	a

- strategic problems if too many "similar" alternatives
  - what if *b,d,e* have similar political orientation?
  - presidential elections in France 2002
    - Chirac – Le Pen

Plurality Runoff

- Plurality Runoff has problematic properties
  - what happens if a candidate receives additional support?

34	35	31
a	b	c
c	c	b
b	a	a

more support for b

30	4	35	31
a	b	b	c
c	a	c	b
b	c	a	a

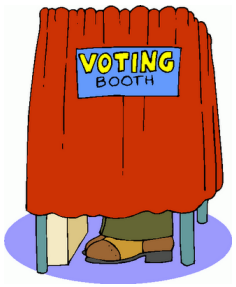
violation of **monotonicity**!

problem of many <u>sequential</u> election procedures!

Voting Rules

- experienced many different voting rules
  - common feature: **ordinal** information
    - at most the individual ranking of alternatives is used as information

- still different rules might lead to different results
  - might lead to indirect influence of results via choice of voting rule
  - i.e. i might be able – given the preference profile – to change the result of the collective decision



Voting Rules - Example

Plurality Rule?

Borda Rule?

2	3	1	4	3	2
a	a	a	d	c	b
b	b	c	c	b	c
c	d	b	b	d	d
d	c	d	a	a	a

Majority Rule?

Plurality Runoff?

Other Collective Decision Rules

- many other collective decision rules possible
  - adaptations or combinations of previous rules
  - rules not based on individual preference rankings, e.g.
- **Majority Judgment**
  - divide alternatives into acceptance classes
  - choose alternative with highest median value

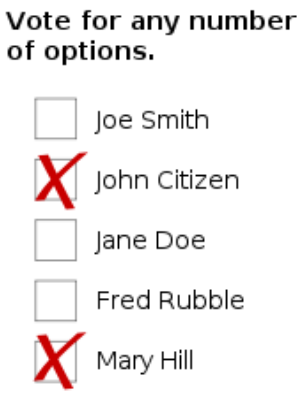
Other Collective Decision Rules

- **Approval Voting**
  - each individual divides set of alternatives into acceptable and inacceptable alternatives
  - rank alternatives according to number of individuals that find it acceptable

2	2	1	1	1	1
a	c	b	b	c	b
b	a	c	c	b	a
c	b	a	a	a	c

AV-outcome:

$a \succ b \succ a$

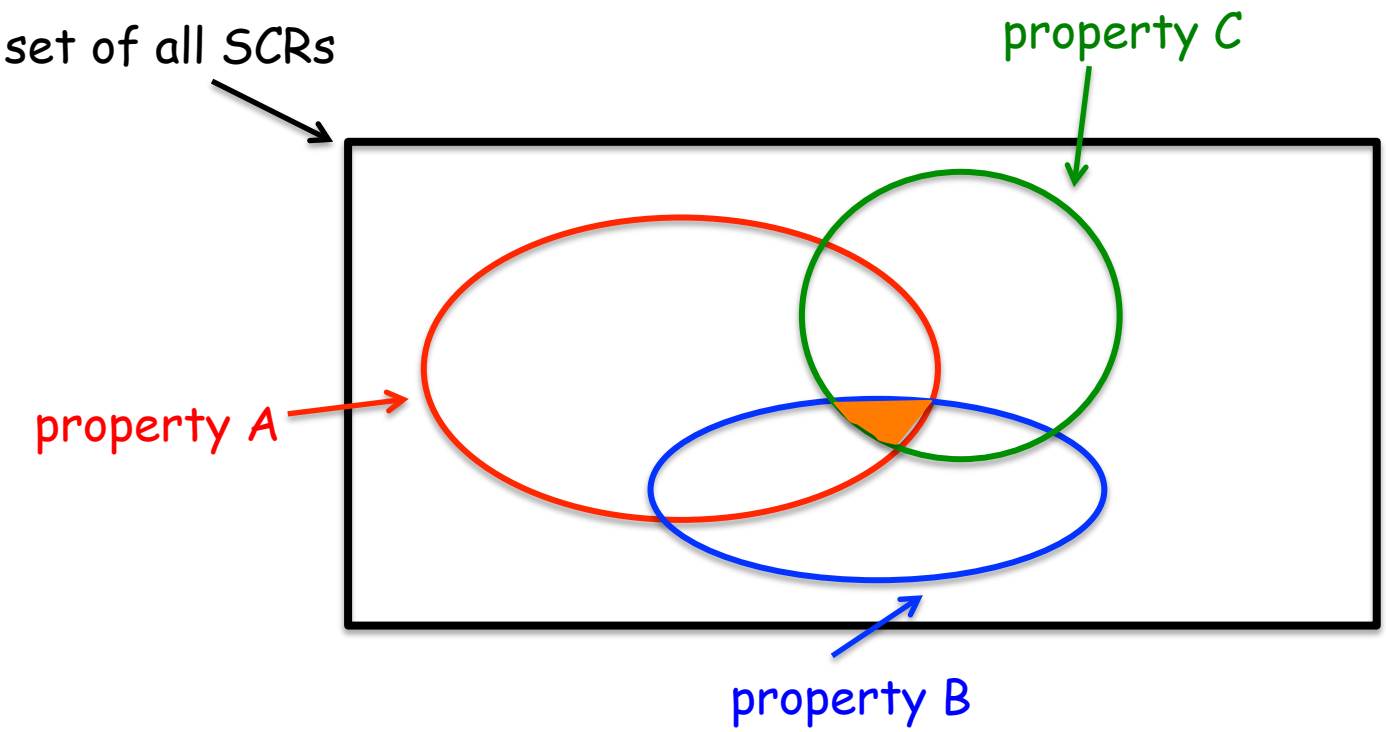


Other Collective Decision Rules

- **Range Voting**
  - evaluate alternative along a scale
  - rank alternatives according to the sum of evaluations
  - main feature: cardinal preference information
    - expresses the subjective intensity between alternatives
    - BUT: am I really able to say that pizza is 3 times as good as Sushi?
      - scientific community sceptical that this is doable
  - use:
    - Dancing Stars, Miss Austria
    - ski jumping, Amazon Ranking, etc.

Which Rule is the Best?

- which rule should we use?
- Social Choice Theory uses an axiomatic approach
  - what are (reasonable) properties of collective decision rules?



Which Rule is the Best?

- **Unrestricted domain**
  - all individual preferences should be allowed

a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

Definition (Unrestricted Domain)

The domain of f includes all logically possible n-tuples of individual weak orders over X.

Which Rule is the Best?

- **Weak Pareto principle** (unanimity)
  - if everybody prefers *a* over *b* then this should also hold for the collective decision

a	c
b	a
c	b

$\Rightarrow$ $a \succ b$

Definition (Weak Pareto)

For all $p \in \mathcal{R}^n$ and all $x, y \in X$; $\forall i \in N$, xP_iy implies xPy.

Which Rule is the Best?

- **Independence of Irrelevant Alternatives**
  - if between *a* and *b* there is no change in the ranking of the individuals, then there should also be no change in the group decision between *a* and *b*

1	1	1
a	d	a
d	b	d
c	c	b
b	a	c

if

$$d \succ a$$

then

1	1	1
a	d	a
d	b	b
c	c	c
b	a	d

$$d \succ a$$

Definition (Independence of Irrelevant Alternatives)

For all $p, p' \in \mathcal{R}^n$ and all $x, y \in X$; $\forall i \in N$, $xR_iy \Leftrightarrow xR'_iy$ implies $xRy \Leftrightarrow xR'y$.

Arrow's Impossibility Theorem

- Which rules remain, if
  - there are at least 2 individuals and 3 alternatives
  - a transitive group ranking (**social welfare function**) is required
  - the above 3 properties have to be satisfied

Arrow's Impossibility Theorem (1951)

- which rules violate which of the properties?
- many other properties used to determine "good" voting rules
  - monotonicity, etc.

Single Peaked Preferences

- domain restrictions
  - to avoid paradoxical situations
  - single-peaked preferences
  - sometimes natural restrictions on the domain
    - preferences based on distance from an optimal point on line
    - left-right orientation in politics

R_1	R_2	R_3	R'_1	R'_2	R'_3
a	c	b	a	c	c
b	a	c	b	a	b
c	b	a	c	b	a

  - single-peaked preferences are sufficient for a Condorcet winner to occur

Sen's Liberal Paradox

- so far not considered any aspects of choices among alternatives that lie in one's private domain
  - "If you prefer to have pink walls rather than white, the society should permit you to have this even if a majority of the community would like to see your walls white." (Sen, 1970)

Let $f : \mathcal{R}^n \to \mathcal{A}$ be a social decision function and consider the following property:

Definition (Minimal Liberalism)

There exist at least 2 individuals s.t. each of them is decisive over at least one pair of alternatives, i.e. if i is decisive over (x, y), then $xP_iy \Rightarrow xPy$.

Theorem (Sen, 1970)

There exists no social decision function satisfying UD, WP and ML.

Sen's Liberal Paradox

Proof.

Let $X = \{x, y, z\}$ and $i, j \in N$ be such that $\bar{D}_i(x, y)$ and $\bar{D}_j(x, z)$. The preferences are considered as follows:

R_i	R_j	$rest(k \neq i, j)$
x	y	yP_kz
y	z	
z	x	

- xPy because of ML of i
- yPz because of WP
- zPx because of ML of j
- Leads to a cycle!

$\square$

Strategic Aspects

My scheme is only intended for honest men! [Borda]

Voters adopt a principle of voting which makes it more of a game of skill than a real test of the wishes of the electors. [Dodgson]

Politicians are continually poking and pushing the world to get the results they want. The reason they do this is they believe (and rightly so) that they can change outcomes by their efforts. It is often the case that voting need not have turned out the way it did. [Riker]

Strategic Aspects

- can outcome be changed in one's favor by voting strategically?

1	1	1
a	d	a
d	b	d
c	c	b
b	a	c

	points
a	6
b	3
c	2
d	7

group
d
a
b
c

1	1	1
a	d	a
d	b	b
c	c	c
b	a	d

	points
a	6
b	4
c	3
d	5

group
a
d
b
c

- manipulation as problem
  - but a lot of information necessary to individuals
  - a problem for many rules

Strategic Aspects

- use as property: non-manipulability (strategy-proofness)
  - a rule is non-manipulable, if no single individual can get a better outcome by changing her preference

Definition (Manipulability)

Social choice function $f : \mathcal{R}^n \to X$ is manipulable by i at profile p via R_i' if $f(p')P_i f(p)$.

Theorem (Gibbard-Satterthwaite)

Let $|N| \geq 2$ and $|X| \geq 3$. If f is non-manipulable and satisfies WP, it is a dictatorship.

  - how can we proof this?