



Interpretation of quantum mechanics

Silvester Sabathiel

Ao.Univ.-Prof. Dr.Ulrich Hohenester

Karl-Franzens Universität Graz

Institut für Physik

Graz, November 2014

Contents

1	Introduction	1
2	The EPR Paradox	3
2.1	Correct- and completeness of theories	3
2.2	Classical and quantum entanglement	3
2.3	The Bell's theorem	5
3	Decoherence and its mathematical tools	8
3.1	The density operator	9
3.1.1	The density operator for a pure state	9
3.1.2	The density operator for a mixed state	10
3.1.3	The reduced density matrix	11
3.2	The measurement problem	12
3.2.1	The preferred-basis problem	13
3.2.2	The problem of outcomes	14
4	Decoherence in detail	14
4.1	The Von Neumann-equation	14
4.2	The Krauss representation	15
4.3	The Master equation	15
4.3.1	The Markov approximation	16
4.3.2	Lindblad equation	17
4.4	Example of Decoherence	18
5	Information perspective of quantum physics	20
5.1	Missing Information - Entropy	20
5.2	Quantum discord	22
6	Alternative Interpretations	24
6.1	The orthodox and the Copenhagen interpretation	24
6.2	The relative-state interpretation	25
6.3	Bohmian mechanics	26

1 Introduction

Quantum theory has been well established since its launch at the beginning of the 20th century. Including not only describing processes and phenomena that have occurred at that time (wave-particle duality, quantized observables, superposition of states...), which have not been understood with the classical models of physics. It also helped to understand the origin of the classical models. Newton's classical equation of motion has been proven to arise as a limiting case from quantum physics by the well known Ehrenfest theorem, and electrodynamics, as a limit of relativistic quantum mechanics. Such that two models that have been considered as two separate elementary models have been generalized by an even more elementary model - quantum physics. Yet the predictions of the quantum model have been so successful that up to now all experimental data have been described properly by quantum mechanics.

So why should the model of quantum physics or at least its interpretation be unsatisfying or lacking? To answer this question one has to know what the properties of a satisfying theory are. The goal of physics or any other natural science is to find the most simple model to describe observations in a wide range of natural processes, ideally all processes. The problem about the conventional model of quantum physics is that it is divided in two elementary processes, whereby none of them can describe the whole range of perceptions separately. One submodel is describing deterministic processes by the linear Schrödinger equation.

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle, \quad |\psi(t_0)\rangle = \psi_0 \quad (1)$$

With this equation, consisting of the initial state ψ_0 and the Hamiltonian H of the system one can calculate the state at any arbitrary time. Since the Schrödinger equation is linear, any linear combination of different solutions presents a solution too. The statistical interpretation connects the Schrödinger equation with a physical observable: the probability density $\rho(t)$, e.g. for the position-space wave function the probability density is given by $\rho(x, t) = |\psi(x, t)|^2$. So $|\psi(x, t)|^2 dx$ describes the probability to find a particle in the spatial interval $(x, x + dx)$.

Beside this we have the non-deterministic, irreversible so called '*collapse*'. In the measurement process the superposition of eigenstates reduces to a single eigenstate of the measurement apparatus.

$$|\Psi\rangle \xrightarrow{\text{Measurement}} |\psi_n\rangle \quad (2)$$

With this prescription quantum mechanics does not predict the outcome of a single

event, but only the distribution of outcomes for a large number of identical tests. In classical models probabilistic approaches are often related to predictions of outcomes of practically indistinguishable tests, but not fundamental identical tests (like the outcome of throwing a dice). This leads to the question whether the prepared identical quantum systems with different outcomes are really completely identical or, if we take a closer look at them, we would find something that distinguishes them in a certain variable. This fundamental issue will be examined in the first chapters about the EPR-paradox and Bell's inequality.

The big puzzle about the collapse is how a nondeterministic process can emerge from a deterministic process, as described by the Schrödinger equation. Furthermore it raises the question why we can't see superpositions in our daily life despite the fact that the classical world is based on elementary "processes" which in principle do allow superpositions.

One early launched and still widely accepted approach is the Copenhagen interpretation, which solves this problem by introducing measurement as the key process for the collapse. In this interpretation measurement is defined as the interaction of a quantum system with a classical measuring device. The state of the measuring device after measurement can only correspond to the state that has interacted with a single definite state of the quantum system, since it cannot be in a superposition itself. Thus it declares this state at the moment of measurement. On the one hand this defines the classical world as the world of definite outcomes without superpositions of multiple states. On the other hand it does not declare the boundary between these two worlds - what behaves classical and what quantum like?

One obvious boundary between classical and quantum systems would be to distinguish by classifying them by macro- and microscopic systems. But as seen in many experiments, especially in the past 20 years, quantum effects are not limited to microscopic scales.

A much more satisfying concept is the concept of decoherence. The following chapters will show that decoherence helps us getting closer to understand the process of collapse. It shows how superpositions vanish in a deterministic way, such that the system is left with multiple but definite states.

2 The EPR Paradox

2.1 Correct- and completeness of theories

The EPR-Paradox was published by Einstein, Podolsky and Rosen in 1935 [1] to emphasize and to stimulate the discussions about the problems of the fundamental basis on which the framework of quantum physics rests.

In this paper they critically examine the definition of a satisfactory theory. There are two properties that declare such a theory.

1. **Correctness:** 'is judged by the degree of agreement between the conclusion of the theory and human experience'
2. **Completeness:** 'there is an element in the theory corresponding to each element in reality'

So far there is no doubt about the sufficiently high correctness of the theory. But Einstein, Rosen and Podolsky questioned the completeness of the theory by illustrative examples which is summarized by their following conclusion:

'In the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality.'

So for the momentum p and position x of a particle, the position can only be obtained by measurement which alters the state such that it destroys the knowledge about the momentum p . More important: The conclusion of quantum mechanics is that if p is known x has no physical reality.

Since the wave function is assumed to contain a complete description of the physical reality and quantities like the momentum and position are each measurable in reality for a certain way of measurement, it is concluded that quantum theory is an incomplete theory.

One possibility to make the quantum theory complete is to introduce hidden variables. To illustrate the idea of the hidden variables we consider an illustrative example of entanglement.

2.2 Classical and quantum entanglement

Entanglement describes the physical phenomenon in which systems that also can be locally separated from each other, are correlated in such a way that from the

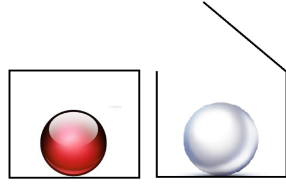


Figure 1: Classical entanglement

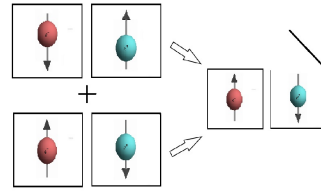


Figure 2: Quantum entanglement

information of the property from one object a conclusion about the property from an other object can be drawn. This phenomenon is not limited to quantum physics, but also occurs in classical physics as one can see at a simple example.

We consider two classical objects, such as balls, one red and one white. Both are put in a closed box and mixed, so we don't know in which box the red or the white object is. As soon as we open one of the boxes, we not only know the color of the opened box, but also the one of the closed box. Thus we can conclude from the property of one object the property of another. The objects, or the properties of the objects, are entangled.

The same conclusion also applies to quantum objects. Instead of the property color (red and white) one could consider the property spin (up and down) for two distinguishable particles. So for the initial state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ there are two possibilities to measure. Either particle 1 has spin 'up' and particle 2 has spin 'down' which corresponds to state $1 \rightarrow |\uparrow\downarrow\rangle$ or vice versa state $2 \rightarrow |\downarrow\uparrow\rangle$. After the measurement of one of the spins the principle is the same as for the classical example. Measuring the spin of one particle determines the spin of the other particle.

The controversial part of this is, that there are two possibilities how it comes to this result. Either it was determined from the beginning which particle has which spin, or the particle decides just at the moment of measurement whether it has spin up or down. Quantum physics and its principle equation, the Schrödinger equation claims the second case. This is becoming obvious in the von Neumann measurement scheme which will be discussed in chapter 6: before the measurement the particles are in a super position of both states until the measurement is taken and the system is in only one state (collapse). There is no possibility to predict the state of the particle when it is measured even though one knows all variables of the system. Einstein, Podolsky and Rosen had problems with this view of nature. They claimed that there must be hidden variables, which the equations of quantum physics do not include, but if they were known one could predict the exact measurement.

2.3 The Bell's theorem

The EPR-paradox leads to the question whether there is an experiment to prove the existence of hidden variables or not. In 1964 the northern Irish physicist John S. Bell answered this question with his paper 'On the Einstein-Podolsky-Rosen-paradox'. [2] John Bell considered an experiment in which a pair of spin one-half particles in the singlet spin state

$$\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2) \quad (3)$$

is measured. Quantum mechanics says that one cannot predict which combination $\langle\uparrow\downarrow|$ or $\langle\downarrow\uparrow|$ you get until you take the measurement. The realists' point of view is that this inability of prediction is caused by ignorance of a missing variable that has to be taken into consideration. In Bell's thought experiment he allowed to measure different components of the spin. Particle 1 is measured in the direction of the normalized vector \vec{a} and particle 2 in direction of normalized vector \vec{b} . For spin up we get the value $+1$ for and for spin down the value -1 (both in units of $\frac{\hbar}{2}$). Let the expectation value of the product of the measured spins in dependence of the two direction \vec{a} and \vec{b} be called $P(\vec{a}, \vec{b})$. If we chose $\vec{a} = \vec{b}$ we will get $P(\vec{a}, \vec{b}) = P(\vec{a}, \vec{a}) = -1$, since it is a single state where the two particles always have two opposite spins ± 1 thus the product always becomes -1 . The quantum mechanical expectation value in general is corresponding to

$$P(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} \quad (4)$$

The proof for this is found in the appendix.

To avoid both of the problems, incompleteness and nonlocality, according to the EPR-Paradox there has to be so called hidden variables such that the result of the measurement of the spin is completely determined by its complete state. We now take the idea of hidden variables into consideration for an experiment in which an electron-positron pair is created in the interaction of a photon with a nucleus or any other neutral boson.[4] For the measured spins of particle A and particle B we get

$$A(\vec{a}, \lambda) = \pm 1 \quad B(\vec{b}, \lambda) = \pm 1$$

In this case we already assumed that the result of the measured spins are dependent on a hidden variable λ . The variables are assumed to be *local*, which means that if the measurement setup is constructed in an appropriate way, the created particles cannot interact anymore if the distance between them is large enough. Thus each of the particles 'carries' its fixed variable which determines the outcome of the future

measurement.

If the spins of the two particles are measured in the same direction $\Rightarrow \vec{a} = \vec{b}$ we get a perfect anti-correlation between these two functions due to total spin conservation:

$$A(\vec{a}, \lambda) = -B(\vec{a}, \lambda) \quad (5)$$

The expectation value of the product of the measured spins is

$$P(\vec{a}, \vec{b}) = \int \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda. \quad (6)$$

We introduced $\rho(\lambda)$ as the probability density for the hidden variable, for which we don't have any further assumptions except that it is normalized.

Since equation (5) applies, we can write down the expression completely in terms of the function for particle A.

$$P(\vec{a}, \vec{b}) = - \int \rho(\lambda) A(\vec{a}, \lambda) A(\vec{b}, \lambda) d\lambda$$

Now we subtract this expression by the expectation value for a third arbitrary direction.

$$P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) = - \int \rho(\lambda) [A(\vec{a}, \lambda) A(\vec{b}, \lambda) - A(\vec{a}, \lambda) A(\vec{c}, \lambda)] d\lambda$$

And with $[A(b, \lambda)]^2 = 1$ follows

$$P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c}) = - \int \rho(\lambda) [1 - A(\vec{b}, \lambda) A(\vec{c}, \lambda)] A(\vec{a}, \lambda) A(\vec{b}, \lambda) d\lambda.$$

Since $-1 \leq [A(\vec{a}, \lambda) A(\vec{b}, \lambda)] \leq +1$ and $\rho(\lambda) [1 - A(\vec{b}, \lambda) A(\vec{c}, \lambda)] \geq 0$ we get

$$|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq \int \rho(\lambda) [1 - A(\vec{b}, \lambda) A(\vec{c}, \lambda)] d\lambda,$$

which is the same as

$$\boxed{|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq 1 + P(\vec{b}, \vec{c})} \quad \text{Bell inequality} \quad (7)$$

But this is incompatible with equation (4), based on the predictions of quantum mechanics. To show that the result of Bell's inequality is not satisfied for every arbitrary directions of measurement one can find one example which does not satisfy it.

For this example we chose \vec{c} such that it forms an angle of 45° with \vec{a} and \vec{b} and insert the general results of quantum mechanics, $P(\vec{a}, \vec{b}) = 0$, $P(\vec{a}, \vec{c}) = P(\vec{b}, \vec{c}) = -\vec{a} \cdot \vec{c} = -\vec{b} \cdot \vec{c} = -0.707$ into Bell inequality.

Thus one can directly see the contradiction with Bell's inequality.

$$\Rightarrow 0.707 \not\leq 1 - 0.707 = 0.293$$

Bell assumed a theory with local hidden variables and showed that there is a contradiction with a general result of quantum mechanics.

So the final conclusion of Bell's inequality is that if quantum physics has hidden variables, they must be non-local.

It is important to emphasize that a non-local interaction would allow deterministic theories, such as the De Broglie-Bohm theory is, (see chapt. 6). The problem is that such non-locality might violate the principles of special relativity. Whereby an important issue is whether locality in special relativity addresses to finite velocity of *signal propagation* or *causal propagation*, since a non-local quantum theory would not necessarily violate signal locality.[10] So far the results of the above mentioned experiment agreed with the results of the quantum theory (in particular with equation (4)), such that one has to give up at least the idea of local hidden variables in the theory of quantum mechanics. Nevertheless there is even criticism about that conclusion arguing that the experiments have crucial problems, that are often referred to as 'loopholes' for the Bell inequality.

3 Decoherence and its mathematical tools

In quantum physics there are some a priori introduced postulates that have to be assumed to build up the theory. The main equation, the Schrödinger equation, explains the dynamics of an isolated system. The equation can have many or infinitely many solutions $|\phi_i\rangle$ and since it is linear any superpositions of these solutions gives a valid solution as well.

$$|\psi\rangle = \sum_i c_i |\phi_i\rangle$$

The Schrödinger equation does not describe a system in the moment of measurement, since this implies interaction with another system such that it is no isolated system anymore. There are two processes which the Schrödinger equation for the isolated system does not explain, which leads to the failure of describing the measurement.

1. The process of the transition from a state including superpositions of eigenstates to a state consisting of only definite eigenstates without superposition.
2. The so called "collapse" from a range of eigenstates to one single definite eigenstate.

The first process seems to lead to a contradiction. The main property that characterizes and distinguishes the two models is that there is superposition in quantum physics but not in classical physics. In other words, we can't see superpositions in everyday life. As an illustrative picture of that paradox, Erwin Schrödinger came up with the gedankenexperiment of 'Schrödinger's cat', in which a cat is put into a box and its' fate is dependend on whether a single radioactive atom is decayed or not. If the nuclei of the atom decays the cat is killed by a linked mechanism. As long as the state of the whole system is not measured, the state of the nucleus is in a superposition of $|\text{decayed}\rangle$ and $|\text{not} - \text{decayed}\rangle$ According to quantum physics the cat thus would be in a superposition of the two states $|\text{dead}\rangle$ and $|\text{alive}\rangle$, since the two systems are entangled. Though this behaviour of a classical object, such as the cat is, does not fit to our experience. This chapter will help us to understand and get closer to the solution for this problem.

The idea of decoherence, which we will introduce in this chapter, is treating one of the two processes namely the vanishing superpositions (process (1)) but does not explain the collapse (2). The main idea of decoherence is to include the environment of a system. As we will see, the environment will lead to a state with definite eigenfunc-

tions of the system of interest without superpositions. In the following section we will introduce the mathematical tool of the density operator. It will help us to work with a composite system, consisting of two subsystems, e.g. the system of interest and the environment. And further to express and characterize the superpositions of states in mathematical terms.

3.1 The density operator

3.1.1 The density operator for a pure state

For an arbitrary state $|\psi\rangle$ we define the density operator:

$$\boxed{\rho(t) = |\psi(t)\rangle \langle \psi(t)|} \quad \text{Density operator} \quad (8)$$

Mathematical properties

There are three important mathematical properties that follow from this definition.

- $\rho(t) = \rho(t)^\dagger$ is positive hermitian operator
- $\rho(t) = \rho(t)^2$ is a idempotent operator
- $Tr(\rho(t)) = 1$

Physical properties

The physical relevant and measurable property is the expectation value of an observable. We can extract this from the density operator.

Therefore we start from the definition of the expectation value of the operator A to the corresponding observable with eigenvalue A_i in quantum mechanics:

$$\begin{aligned} \langle A \rangle &= \langle \psi | A | \psi \rangle = \sum_n \langle \psi | A | n \rangle \langle n | \psi \rangle = \sum_n \langle n | \underbrace{|\psi\rangle \langle \psi|}_{\rho(t)} A | n \rangle \\ &\Rightarrow \langle A \rangle = Tr(\rho(t)) \end{aligned} \quad (9)$$

Another physical meaning of the density operator gets obvious if we take a look at the elements of the density operator.

With $|\psi\rangle = \sum_n c_n |n\rangle$ we get

$$\rho = |\psi\rangle \langle \psi| = \left(\sum_n c_n |n\rangle\right) \left(\sum_m c_m^* \langle m|\right) = \underbrace{\sum_n |c_n|^2 |n\rangle \langle n|}_n + \underbrace{\sum_{m \neq n} c_n c_m^* |n\rangle \langle m|}_{\text{off-diagonal terms}}$$

Even more illustrative is the matrix representation of the density operator. For example for a two-state system in the form of

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle,$$

we get the density matrix

$$\begin{aligned} |\psi\rangle \langle\psi| &= |\alpha|^2 |0\rangle \langle 0| + \alpha\beta^* |0\rangle \langle 1| + \alpha^*\beta |1\rangle \langle 0| + |\beta|^2 |1\rangle \langle 1| \\ &= \begin{pmatrix} |\alpha|^2 & \alpha^*\beta \\ \alpha\beta^* & |\beta|^2 \end{pmatrix}. \end{aligned} \quad (10)$$

As we can see the diagonal elements of the density matrix can be interpreted as the probabilities of the basis state, while the off-diagonal elements represent the coherence of the basis states. These off-diagonal terms are the reason for the problem for the measurement process, since they apply equally to measurement outcomes as the definite diagonal terms.

3.1.2 The density operator for a mixed state

So far we considered an isolated system described by a simple wavefunction. Real systems can never be isolated perfectly, since even very small perturbations in the environment affect the system. But there are too many degrees of freedom in the environment to consider them all. So one has to introduce the mixed state which tells us the probability p_i to find the system in the state $|\psi_i\rangle$. This probability distribution on pure states is called mixed state: $\{(|\psi_1\rangle, p_1), (|\psi_2\rangle, p_2), \dots, (|\psi_d\rangle, p_d)\}$

The density matrix for such a mixed state is

$$\rho = \sum_{k=1}^d p_k |\psi_k\rangle \langle\psi_k| \quad (11)$$

We can demonstrate an analogy to the density operator of the pure state by showing its same physical and partly mathematical properties as we had before. Most importantly the expectation value for an observable is also given by the density matrix. For a pure state given by $|\psi\rangle = \sum_{\alpha} \underbrace{c_{\alpha}}_{\langle\phi_{\alpha}|\psi\rangle} |\phi_{\alpha}\rangle$ the expectation value of the observable B is given

by

$$b_i = \langle\psi_i| B |\psi_i\rangle = \sum_{\alpha, \alpha'} c_{\alpha}^* c_{\alpha} \langle\phi_{\alpha}| B |\phi_{\alpha'}\rangle$$

Since each of the states $|\psi_i\rangle$ occurs only with probability p_i we get the expectation value for the composite system as

$$\begin{aligned}
\langle B \rangle &= \sum_i p_i b_i = \sum_i p_i \sum_{\alpha, \alpha'} \underbrace{c_\alpha^{(i)*} c_\alpha^{(i)}}_{\langle \phi_\alpha | \psi \rangle \langle \psi | \phi_{\alpha'} \rangle} \langle \phi_\alpha | B | \phi_{\alpha'} \rangle \\
&= \sum_{\alpha'} \langle \phi_{\alpha'} | \underbrace{\sum_i p_i |\psi_i\rangle \langle \psi_i|}_{\rho} \underbrace{\sum_\alpha |\phi_\alpha\rangle \langle \phi_\alpha|}_{\mathbb{1}} | \phi_{\alpha'} \rangle \\
&= \sum_{\alpha'} \langle \phi_{\alpha'} | \rho B | \phi_{\alpha'} \rangle \\
&= \text{Tr}(\rho B)
\end{aligned} \tag{12}$$

As shown, one can extract physical observables from the density operator even for mixed states, so the state of a quantum mechanical system is completely defined by its density operator. Also it is independent of any specific probabilistic mixtures, thus states with identical density matrices are indistinguishable. But another very crucial fact is that the mixture setting up the density operator is not definite. In other words many different mixtures can lead to the same density operator.

3.1.3 The reduced density matrix

Now we found a mathematical tool to describe the composite system consisting of multiple subsystems. But what if we want to examine an observable of only one subsystem. In other words the observables of interest. Therefore we need to define the 'system observable':

$B_s = B^{(s)} \otimes \mathbb{1}^{(e)}$ where $B^{(s)}$ only acts in the Hilbertspace of system \mathcal{S} and $\mathbb{1}^{(e)}$ in the Hilbert space of the environment \mathcal{E} . Now we calculate the expectation value of this observable.

$$\begin{aligned}
\langle B_s \rangle &= \text{Tr}(\rho^{(s,e)} B^{(s)} \otimes \mathbb{1}^{(e)}) \\
&= \sum_{m,n} \langle m |^{(s)} \otimes \langle n |^{(e)} \rho^{(s,e)} B^{(s)} \otimes \mathbb{1}^{(e)} | m \rangle^{(s)} \otimes | n \rangle^{(e)} \\
&= \sum_m \langle m |^{(s)} \sum_n \langle n |^{(e)} \rho^{(s,e)} | n \rangle^{(e)} B^{(s)} | m \rangle^{(s)}
\end{aligned} \tag{13}$$

This can be written in the same form as the expectation value for the composite system:

$$\langle B_s \rangle = \text{Tr}_s(\rho^{(s)} B^{(s)}) \tag{14}$$

with the new defined reduced density operator for system observables:

$$\boxed{\rho^{(s)} = \text{Tr}_e(\rho^{(s,e)}) = \sum_n \langle n|^{(e)} \rho^{(s,e)} |n\rangle^{(e)}} \quad (15)$$

If we resubstitute $\rho^{(s,e)}$ in Eqs. (15) we get

$$\rho^{(s)} = \sum_n \sum_{kk'} c_k c_k'^* |k\rangle^{(s)} \langle k'|^{(s)} \langle n|^{(e)} |k\rangle^{(e)} \langle k'|^{(e)} |n\rangle^{(e)} \quad (16)$$

As we will see the decoherence effect will cancel the off-diagonal terms of the environmental system. So the environmental states $|e_n\rangle$ can be assumed as orthogonal.

$$\rightarrow \rho^{(s)} = \sum_k |c_k|^2 |k\rangle^{(s)} \langle k|^{(s)} \quad (17)$$

That the environmental states are assumed as orthogonal is reasonable since one expects macroscopic distinguishability. For measurements this means that the pointer is supposed to be found in a definite state. Therefore we refer to the environmental states as the pointer states from now on.

By evaluating the reduced density operator with a mixed state, containing states of the system as well as environmental states one sees that the state of the system of interest is dephased - the off-diagonal terms decay. The diagonal density matrix which is left looks like a classical ensemble of \mathcal{S} -states. It can be interpreted as a statistical mixture, where each of the definite states $|k\rangle^{(s)}$ is found with probability $|c_k|^2$ but the probability is zero to find a superposition of these states. It is important to distinguish between statistical mixture and coherent superposition.

Yet, the state of the system is not collapsed. It is just said to be in a definite state, but not in which one.

3.2 The measurement problem

The measurement problem arises from the apparent inconsistency between the results of quantum mechanics and our perception. This problem can be divided in two separate questions. (A) Why do we always see particular observables in definite states? - 'the preferred-basis problem'. (B) Why do we only measure a single value as an outcome? - 'the problem of outcomes'.[5]

3.2.1 The preferred-basis problem

So far we don't know how it comes to orthogonality, such that the conclusion from eqs. (16) to (17) is valid. And further if the pointer states get orthogonalized in any basis. There is no special assumption for the basis, so the given result is valid for any arbitrary basis. This leads to a problem with the physics that is described by this orthogonalization. Let us therefore consider an illustrative example. A system \mathcal{S} of spin-1/2 particles with spin states $|\uparrow_z\rangle_{\mathcal{S}}$ and $|\downarrow_z\rangle_{\mathcal{S}}$, which are the eigenstates of an operator σ_z that measures whether the spin points up or down along the z-axis. The system is measured by an apparatus \mathcal{A} with eigenstates $|\uparrow_z\rangle_{\mathcal{A}}$ and $|\downarrow_z\rangle_{\mathcal{A}}$. There is a \mathcal{SA} -entanglement such that if the system \mathcal{S} is in state $|\uparrow_z\rangle_{\mathcal{S}}$ then the apparatus \mathcal{A} ends up in $|\uparrow_z\rangle_{\mathcal{A}}$ and the system is in $|\downarrow_z\rangle_{\mathcal{S}}$. We now suppose that the state directly before the measurement is $|\psi\rangle_{\mathcal{S}} = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_{\mathcal{S}} - |\downarrow_z\rangle_{\mathcal{S}})$. Thus we get for the entangled system-apparatus-state

$$|\psi\rangle_{\mathcal{SA}} = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_{\mathcal{S}} |\uparrow_z\rangle_{\mathcal{A}} - |\downarrow_z\rangle_{\mathcal{S}} |\downarrow_z\rangle_{\mathcal{A}}) \quad (18)$$

So one can doubtlessly claim that the apparatus \mathcal{A} measured σ_z at the end. But the main point about the preferred basis problem is that the entangled state $|\psi\rangle_{\mathcal{SA}}$ can be written in any other basis $|\uparrow_n\rangle_{\mathcal{S}}, |\downarrow_n\rangle_{\mathcal{A}}$. For example

$$|\psi\rangle_{\mathcal{SA}} = \frac{1}{\sqrt{2}}(|\uparrow_x\rangle_{\mathcal{S}} |\uparrow_x\rangle_{\mathcal{A}} - |\downarrow_x\rangle_{\mathcal{S}} |\downarrow_x\rangle_{\mathcal{A}}) \quad (19)$$

Now one would claim that σ_x is apparently measured, thus the apparatus \mathcal{A} has measured σ_z as well as σ_x . But those two observables are non-commuting observables and Heisenberg's uncertainty does not allow to measure both of them simultaneously. The solution to this problem lies in decoherence. Decoherence is called the process in which the superpositions of a state are vanishing, such that only the diagonal terms of the density matrix as an classical statistical ensemble are left. Decoherence causes this orthogonalization only in certain bases - the 'preferred basis'.

Apparently it is not sufficient to consider the System-Aparatus entanglement \mathcal{SA} , since the state is decohering in any basis of the pointer. The key point that leads to decoherence is to include the environment \mathcal{E} to the System-Aparatus system. The main advantage of the idea of decoherence is that it is evolving from the deterministic Schrödinger equation, but not an a priori induced process. To understand the deterministic process, the evolution in time of the mixed states will be discussed in the next chapter.

3.2.2 The problem of outcomes

Later we will see that decoherence can reduce the possible outcomes of the \mathcal{SA} -system by dephasing the density matrix. The dephased matrix can be interpreted as a classical ensemble of potential measurement outcomes with definite probabilities. This interpretation is based on the probabilistic interpretation of the state vector, e.g. the state vector $|\Psi\rangle$ to be found in the state $|\phi\rangle$ with probability $|\langle\phi|\Psi\rangle|^2$, which already includes the assumption of a wavefunction 'collapse'. So decoherence does not solve the whole measurement problem. But it already reduces the number of possible outcomes. The possible outcomes which are left are definite states, thus it helps to understand the transition from a quantum to a classical state.

$$\underbrace{\begin{pmatrix} |\alpha|^2 & \alpha^*\beta & \alpha^*\gamma \\ \alpha\beta^* & |\beta|^2 & \beta^*\gamma \\ \alpha\gamma^* & \beta\gamma^* & |\gamma|^2 \end{pmatrix}}_{\text{Quantum state}} \xrightarrow{\text{Decoherence}} \underbrace{\begin{pmatrix} |\alpha|^2 & 0 & 0 \\ 0 & |\beta|^2 & 0 \\ 0 & 0 & |\gamma|^2 \end{pmatrix}}_{\text{Classical Ensemble}} \xrightarrow{\text{Collapse}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Single definite outcome}} \quad (20)$$

4 Decoherence in detail

Since the dephasing process is not a spontaneous process but is evolving continuously in time, in the following we are going to study the time evolution of the density operators which describe systems and composite system.

4.1 The Von Neumann-equation

To examine the time evolution we substitute the results of the Schrödinger equation in the definition of the density operator, thus we get the famous Von Neumann-equation.

$$\begin{aligned}
 i\hbar\dot{\rho}(t) &= \sum_i p_i [(H|\psi_i\rangle)\langle\psi_i| - |\psi_i\rangle\langle\psi_i|H] \\
 i\hbar\dot{\rho}(t) &= [H, \rho]
 \end{aligned} \quad (21)$$

If we want to be able to calculate the density operator at an arbitrary time, given by the initial density operator, we need to find an expression in the form $\rho(t) = f(\rho(t_0))$

From the Von Neumann-equation equation it is hard to extract this for a general subsystem. This leads to the Krauss-Representation

4.2 The Krauss representation

To understand the behaviour of a part of a bipartite system, we examine the evolution of the reduced density matrix. In our case the bipartite system consists of the system of interest \mathcal{S} and the environment \mathcal{E} . Now we let the initial density matrix of the composite system be a tensor product of the two subsystems.

$$\begin{aligned}\rho &= \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}} \\ \rho &= \rho_{\mathcal{S}} \otimes |0\rangle \langle 0|\end{aligned}\tag{22}$$

The time evolution of a density matrix is obtained by a unitary time operator $\rightarrow \rho(t) = U(t,0)(\rho_{\mathcal{S}} \otimes |0\rangle \langle 0|)U^\dagger(t,0)$. Furthermore the environment is 'traced out' as it was shown at the introduction of the reduced density matrix to finally get the time dependent evolution of the subsystem \mathcal{S} .

$$\rho_{\mathcal{S}}(t) = \sum_b \underbrace{\langle b|U(t,0)|0\rangle}_{\equiv M_b(t)} \rho_{\mathcal{S}} \underbrace{\langle 0|U^*(t,0)|b\rangle}_{\equiv M_b^\dagger(t)}\tag{23}$$

Such we get

$$\$(\rho_{\mathcal{S}}) \equiv \rho_{\mathcal{S}}(t) = \sum_b M_b(t)\rho_{\mathcal{S}}(0)M_b^\dagger(t).\tag{24}$$

This introduced map $\$$ is called a 'superoperator' since it maps linear operators to linear operators. Another important property about the Krauss operators M_b which we will need later is

$$\sum_b M_b^\dagger(t)M_b(t) = \mathbf{1}.\tag{25}$$

4.3 The Master equation

For isolated systems the Hamiltonian in the Schrödinger equation describes the evolution over an infinitesimally small time interval. Such one gets the state at an arbitrary time by intergrating the differential equation over the time. This leads to the well

known equation

$$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}Ht} |\Psi(t_0)\rangle. \quad (26)$$

The question is whether one can find a differential equation for non-isolated systems of the same form.

$$\rho(t) = e^{\mathcal{L}t} \rho(t_0), \quad (27)$$

where \mathcal{L} is called the *Liouville-operator* or *Lindbladian*.

4.3.1 The Markov approximation

Whether one can find a differential equation in the same form as the Schrödinger equation is (first order), depends on which process we are looking at. To be more specific, there is only such a differential equation if the evolution of the quantum system is "Markovian". Markovian processes are processes in which the state in the future $\rho(t+dt)$ only depends on the present state $\rho(t)$, but is independent on the states before (how it has come to the present state). In a bipartite system (e.g. System-Environment) in general one could expect other situations, where the present state also depends on earlier states. The environment can retain information of earlier states for a certain time and transform it back to the system. [11]

But if we chose the time scale of our examined process the right way, it becomes a Markovian process and we avoid the environmental feedback. Therefore following time scales are defined:

δt_D ... characteristic time scale of the description of the evolution

δt_E ... time scale in which the environmental reservoir 'forgets' the information it gained from the system

δt_S ... characteristic time scale in which the system interacts with the environment

So if we chose the time scale δt_D such that it is much larger than δt_D , the environment has no 'memory' about the earlier states of the system and the process is markovian. Furthermore we require the time scale to be much smaller than δt_S so the high frequency components of the motion of the systems are filtered.[6]

$$\delta t_E \ll \delta t_D \ll \delta t_S \quad (28)$$

4.3.2 Lindblad equation

The Lindbladian corresponds to the Hamiltonian of an open system. It describes Markovian but non-unitary evolution. The non-unitarity comes from the interaction with the environment. To derive the Lindbladian one applies the Markov approximation for the Krauss representation. Using this approximation is necessary to find an operator which fullfills eq. (27) .

Since from the Markovian approximation we get

$$\rho(\delta t) = \rho(0) + \mathcal{O}(\delta t), \quad (29)$$

we can conclude about the form of the terms of the Kraussrepresentation.

$$\begin{aligned} \implies M_0(\delta t) &= \mathbb{1} + \mathcal{O}(\delta t) \\ M_b(\delta t) &= \mathcal{O}(\sqrt{\delta t}) \end{aligned} \quad (30)$$

Further we are using new operators K and L to express this condition.

$$\begin{aligned} M_0(\delta t) &= \mathbb{1} + \left(\frac{i}{\hbar}H + K\right)\delta t \\ M_b(\delta t) &= L_b(\sqrt{\delta t}) \end{aligned} \quad (31)$$

These expressions for the Krauss operators can now be substituted in eq. (25) and one gets the relation between K and L and after further calculation steps finally the differential equation for non-unitary evolution in a bipartite system.

$$\boxed{\dot{\rho}(t) = \mathcal{L}\rho = -\frac{i}{\hbar}[\mathcal{H}, \rho(t)] + \sum_b (L_b\rho(t)L_b^\dagger - \frac{1}{2}L_b^\dagger L_b\rho(t) - \frac{1}{2}\rho(t)L_b^\dagger L_b)} \quad (32)$$

Lindbladian equation

In the Lindbladian equation one can clearly see that the first term is in the same form as the Von Neumann-equation, thus we clarify H as the Hamiltonian \mathcal{H} (the operator was already called H in the wise foresight of this result). This term apparently represents the unitary evolution of the system, while the other terms are representing the non-unitary evolution. The L_b operators are called Lindblad operator or quantum jump operator, because each of the $L_b\rho(t)L_b^\dagger$ -terms represents an environment induced quantum jump to the system \mathcal{S} . The two other terms $\frac{1}{2}L_b^\dagger L_b\rho(t)$ and $\frac{1}{2}\rho(t)L_b^\dagger L_b$ do normalize the case in which no jumps occur. [6]

4.4 Example of Decoherence

In this section an explicit example of decoherence will be examined.[6] The goal is to show that indeed the environment causes vanishing off-diagonal terms in the reduced density matrix of the observed system. Furthermore we will see how long it takes for a state to decohere.

The system of interest in the following example can be considered as a heavy "classical" particle, initially being in the superposition of position eigenstates $|x\rangle$ and $|-x\rangle$ (in our notation $|0\rangle_A$ and $|1\rangle_A$). The particle is imagined to be exposed to light, such the incident photons represent the interacting environment. Since we are not able to monitor all the scattering photons the quantum state is represented by a density operator. For evaluating the time evolution we use the Krauss-representation:

$$\rho_S = \sum_b M_b \rho_S M_b^\dagger \quad (33)$$

with $M_b = \langle b|U|0\rangle$, and U is a unitary evolution operator.

In this example the unitary evolution operator of the density operator is not given as usual in terms of time units, but in terms of the amount of interactions with the environment. It is given by

$$\begin{aligned} |0\rangle_A |0\rangle_E &\longrightarrow \sqrt{1-p} |0\rangle_A |0\rangle_E + \sqrt{p} |0\rangle_A |1\rangle_E \\ |1\rangle_A |0\rangle_E &\longrightarrow \sqrt{1-p} |1\rangle_A |0\rangle_E + \sqrt{p} |1\rangle_A |2\rangle_E \end{aligned} \quad (34)$$

Here the state of the system A (the position of the particle) is never disturbed by the environment. Only the environmental states are kicked into higher states with probability p with each interaction. The photon is kicked from its initial state $|0\rangle_E$, into $|1\rangle_E$ if the system A is in the state $|0\rangle_A$ and into $|2\rangle_E$ if the system A is in the state $|1\rangle_A$.

Krauss operators

Now given the unitary evolution operator we can write down the Krauss operators:

$$M_0 = \sqrt{1-p} \mathbb{1}, \quad M_1 = \sqrt{p} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad M_2 = \sqrt{p} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (35)$$

To evaluate the time evolution of the state the Krauss operators are inserted into eq. (33). Let the density operator thereby be in the initial state in the form of

$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$, thus we get

$$\$(\rho_S) = M_0\rho_A M_0 + M_1\rho_A M_1 + M_2\rho_A M_2 = \begin{pmatrix} \rho_{00} & (1-p)\rho_{01} \\ (1-p)\rho_{10} & \rho_{11} \end{pmatrix}. \quad (36)$$

Apparently the diagonal terms stay invariant while the off-diagonal terms vanish if the interaction between the system and the environment is repeated again and again. To translate this process into a time-dependent process we introduce Γ as the number of scattering events per unit time, such that $p = \Gamma\Delta t \ll 1$. The density matrix after time t is given by $\$(\rho_S)^n$, such that its off-diagonal terms are suppressed by $(1-p)^n = (1-\Gamma\Delta t)^{\frac{t}{\Delta t}} \xrightarrow{\Delta t \rightarrow 0} e^{-\Gamma t}$. After a long enough time ($\gamma t \gg 1$) the reduced density matrix takes the form describing a classical ensemble as sketched in eq. (20).

5 Information perspective of quantum physics

When studying the difference between the classical and quantum world we already found out that quantum entanglement is a critical property that distinguishes those two worlds. But entanglement is not the only phenomenon with this capability. Another phenomenon is gained from information theory and is called '*quantum discord*'. Therefore the next chapter is going to introduce the quantum entropy, an important measure to build up the field of quantum information. This is followed by the usage of information theory to find a new way to classify non-classical correlations.

5.1 Missing Information - Entropy

By applying the markovian approximation to derive the master equation one included that the concerned time scale is long enough that the environment 'forgets' information about the system \mathcal{S} . This implies a loss of information, at least for the subsystem. In this chapter this process is examined in a more formal way. In particular we will relate this loss of information with the classical concept of the thermodynamical entropy.

As we have seen the physical state of a system is completely defined by its density operator. But what also appeared is the fact that the mixture of states that build up a given mixed state is not definite - e.g. the same density operator can be obtained by two different mixture states.

The same property occurs in classical thermodynamics, where the temperature describes certain physical observations, independent on how the velocity distribution of the particles (microstates) is. There are many ways to actualize the same temperature by different velocity distributions. In classical thermodynamics the entropy is correlated with the number of possible distribution to actualize the same macrostate. The more ways there are, the higher the entropy.

The same concept can be applied for quantum systems. On the basis of classical definition of the entropy

$$S = -k_b \sum_i p_i \ln p_i, \quad (37)$$

one can write the generalized definition of the quantum entropy, also called the von Neumann entropy:

$$\boxed{S(\rho) = -\text{Tr}(\rho \ln \rho)} \quad (38)$$

Since the summands of the trace of the density operator are the eigenvalues of the density operator this further leads to

$$S(\rho) = - \sum_i \lambda_i \ln \lambda_i. \quad (39)$$

Now we examine the entropy of a mixed and a pure state.

- **Mixed state**

for a maximal mixed state the density operator is given by

$$\rho_{mix} = \frac{1}{d} \mathbb{1}_d.$$

So the corresponding quantum entropy is

$$S(\rho_{mix}) = -\text{Tr}(\rho_{mix} \ln \rho_{mix}) = -\text{Tr}\left(\frac{1}{d} \mathbb{1}_d \ln \frac{1}{d} \mathbb{1}_d\right) = - \sum_i \frac{1}{d} \ln \frac{1}{d} = \ln d.$$

The entropy can be normalized such that

$$0 \leq S(\rho) \leq 1.$$

- **Pure state**

The entropy for the pure state vanishes since the eigenvalues of its diagonalized density matrix always is 1 or 0.

$$\rightarrow S(\rho) = \sum_i \lambda_i \ln \lambda_i = 0$$

So we can classify following cases:

- $0 \leq S(\rho) \leq 1$ General state
- $S(\rho) = 1$ Maximal mixed
- $S(\rho) = 0$ Pure state

Since the process of decoherence is the transition from pure to mixed states an increase of entropy comes along with decoherence. To point out this increase of entropy from another point of view we once again consider the state as $\rho = \sum_i p_i |\phi_i\rangle \langle \phi_i|$ which is found in the state $|\phi_i\rangle \langle \phi_i|$ with probability p_i and $\sum_i p_i = 1$. Assuming the initial state to be in a pure state the density operator consists of only one term with $p_1 = 1$, such one knows the whole information about the system. If the state starts to decohere due to the environmental coupling, the pure state evolves to a mixed state

and the whole event space $\{p_i\}$ is spread to more and more p_i . The uncertainty about which state the system is found in increases, or in other words one loses information about the system.

5.2 Quantum discord

Systems with no entanglement still can have unique quantum correlation. To quantify this quantumness of correlations the measure $\delta(A|B)$, called 'discord', is introduced. It is defined by the difference between two different expressions for the mutual information (one representing general systems and one representing classical systems) for a bipartite system \mathcal{SA} .

$$\delta(A|B) = \mathcal{I}(\mathcal{A} : \mathcal{B}) - \mathcal{J}(\mathcal{A} : \mathcal{B}) \quad (40)$$

For the case of general mutual information $\mathcal{I}(\mathcal{A} : \mathcal{B})$ one can write down following expression:

$$\mathcal{I}(\mathcal{S} : \mathcal{B}) = S(\mathcal{A}) + S(\mathcal{B}) - S(\mathcal{A}, \mathcal{B}), \quad (41)$$

where $S(\mathcal{A})$ and $S(\mathcal{B})$ express the entropies for the single systems \mathcal{A} and \mathcal{B} , and $S(\mathcal{A}, \mathcal{B})$ expresses the joint entropy of the two systems. This formular is justified when looking at two special cases:

- There is no correlation between the two subsystems $S(\mathcal{A}, \mathcal{B}) = S(\mathcal{A}) + S(\mathcal{B})$ and the mutual information vanishes. $\Rightarrow \mathcal{I}(\mathcal{S} : \mathcal{B}) = 0$
- There is maximum correlation between the systems such $S(\mathcal{A}, \mathcal{B}) = S(\mathcal{A}) = S(\mathcal{B}) = \mathcal{I}(\mathcal{S} : \mathcal{B})$.

In the classical case the mutual entropy of a bipartite system can be written as

$$S(\mathcal{A}, \mathcal{B}) = S(\mathcal{A}) + S(\mathcal{A}|\mathcal{B}). \quad (42)$$

Where $S(\mathcal{A}|\mathcal{B})$ is the conditional entropy for a bipartite system - the entropy of the system \mathcal{A} given the state \mathcal{B} . Eq. (42) tells us that if one knows everything about state \mathcal{B} Information about \mathcal{A} is still missing. This relation for the classical mutual entropy also can be derived in a formal way. [7]

With this expression one can substitute $S(\mathcal{A}, \mathcal{B})$ in the formular for general mutual information, thus one obtains the expression for the classical mutual information.

$$\mathcal{J}(\mathcal{S} : \mathcal{B}) = S(\mathcal{A}) + S(\mathcal{B}) - (S(\mathcal{A}) + S(\mathcal{A}|\mathcal{B})) \quad (43)$$

For quantum physics the words ‘...given the state B...’ in the definition of the conditional entropy has crucial consequences. The state of an object can only be given after measurement. Since this measurement is basis-dependent in quantum physics the conditional entropy for quantum systems must also be basis dependent.

$$S_{|b_i\rangle}(\mathcal{A}, \mathcal{B}) = (S(\mathcal{A}) + S(\mathcal{A}|\mathcal{B}))_{|b_i\rangle} \quad (44)$$

Now there can be a discord between mutual information for general systems $\mathcal{I}(\mathcal{S} : \mathcal{B})$ and for pure classical systems $\mathcal{J}(\mathcal{S} : \mathcal{B})$. Such the difference can be considered as the non-classical, quantum part of mutual information. A measure for quantumness is found, which can be independent of the degree of entanglement.

$$\delta_{|b_i\rangle}(A|B) = \mathcal{I} - \mathcal{J} = (S(\mathcal{A}) + S(\mathcal{A}|\mathcal{B}))_{|b_i\rangle} - S(\mathcal{A}, \mathcal{B}) \quad (45)$$

There is also a way to express the discord in a basis-independent way by finding a basis-independent conditional entropy in the form of the von Neumann entropy $S(\rho)$ (introduced in the previous section). Therefore one calculates the von Neumann entropy for the conditional density operator $\rho_{\mathcal{A}|\Pi_j^{\mathcal{B}}}$, where the set of projection operators $\{\Pi_j^{\mathcal{B}}\}$ defines in which basis the measurement of \mathcal{B} was taken. By minimizing this entropy over the possible measurements on \mathcal{B} one can find the measurement that disturbs least the quantum state of \mathcal{A} . If the minimum discord with the corresponding basis is vanishing, a basis is found in which the system is classically accessible through the subsystem \mathcal{B} . [3] For pure states $\delta_{|b_i\rangle}(A|B) > 0$ in all bases, whereas for the decohered state $\delta_{|b_i\rangle}(A|B) = 0$ for the preferred basis. Thus we found another approach to show the classical aspects of a decohered state, namely the informational aspect.

Beside the boost of understanding the foundations of the quantum-classical border, quantum discord has an important meaning in quantum computation, where the ‘quantum-advantages’ are supposed to be used to achieve more efficient algorithms to solve problems. For a long time entanglement seemed to be the key-resource for quantum information processing (QIP). The problem about the entanglement-based quantum computing is that it is a very demanding task to create and manipulate entangled states. Especially because of their fragility concerning the environment induced decoherence, as seen in the section ‘Example of decoherence’. In contrast to quantum entanglement, discord is much more robust against environmental influences, thus it is a promising concept which can be used for quantum computation in the future.[8]

6 Alternative Interpretations

For a thorough examination of the interpretation of quantum mechanics different aspects and questions of the physical model have to be considered. Here are the most crucial aspects that distinguishes the different interpretations.

Determinism or randomness? Do processes happen deterministically or randomly? Is the so far observed randomness of nature due to ignorance or is it 'true' randomness. The discussion about the EPR-paradox and the Bell theorem is the historical basis for this question in quantum physics.

The measurement problem: This already introduced problem (section 3.2) is one of the key questions, especially because it raises the question how the classical world which we observe in our everyday life arises. Are there two separate worlds that cannot be combined in a unifying model? Or does classical mechanics and its properties (definite outcomes, classical measurement, collapse) arise from fundamental principles of quantum physics.

Locality: The principle of locality implies the impossibility of instantaneous action-at-distance. Is a physical model of reality satisfying the principle of locality?

In the following we give a brief overview over alternative ways of quantum interpretation and their relation to the above mentioned aspects. The sketched interpretations are chosen because of there completely different approaches to describe quantum physics.

6.1 The orthodox and the Copenhagen interpretation

The main element of the orthodox interpretation is the collapse postulate. It states that the measurement of a system leads to a collapse or projection of the state into an eigenstate of the measurement observable. This interpretation is the common version for standard textbooks, but it lacks the definition of what a measurement is fundamentally. Measurement is just introduced in words without specifying the exact physical process or the basis in which the state collapses. The preferred basis thus is just 'chosen' freely by the observer who sets up the experiment.

In the Copenhagen interpretation the measurement is more specified. According to its postulate of measurement, which is again declared in words, a quantum state is measured whenever it interacts with a classical system. Thereby the interpretation

never clarifies what 'classical' means and such implies a quantum-classical dualism, where two separate models are required to describe nature. Here is where decoherence takes its role of the interpretation: By clarifying the characteristic property of classical states to be the ones with no coherence, the process of environmental induced decoherence lets classical systems arise from quantum mechanical principles.

6.2 The relative-state interpretation

The motivation for the relative-state interpretation lies in the goal of physical models to be built up by one fundamental process, but not by two separate processes (unitary evolution and measurement) as it is done in the orthodox interpretation. Indeed the concept of environment induced decoherence does provide a description for vanishing superpositions, but it always concerns open systems, where degrees of freedom of a larger system are disregarded. Considering the whole universe, there is no larger system or environment, such the evolution must be unitary and superpositions of universe-states do exist.

In 1957 the English physicist Hugh Everett III introduced a completely new interpretation, in which quantum physics is universal valid - no extra 'collapse postulate' is necessary. The postulate was replaced by postulating 'that observer perceives the state of the rest of the Universe relative to his own state'. [12] To illustrate the idea behind this approach we consider the von Neumann measurement scheme.

The von Neumann measurement scheme models the (non-collapse) quantum measurement process of a system-apparatus interaction \mathcal{SA} , whereat the apparatus is not a classical device, but also considered as quantum. The initial state of \mathcal{SA} has the separable form

$$|\Psi\rangle = |\Psi_{\mathcal{S}}\rangle \otimes |a_0\rangle = \left(\sum_k \lambda_k |s_k\rangle \right) \otimes |a_0\rangle. \quad (46)$$

When the state evolves in time the \mathcal{S} and \mathcal{A} start to interact with each other, such they are in a nonseparable entangled state:

$$|\Psi(t)\rangle = \sum_k \lambda_k(t) |s_k\rangle \otimes |a_k(t)\rangle \quad (47)$$

This expression models the so-called '*premeasurement*'. The origin of this naming is the fact that it describes a state which is still 1) non-collapsed and 2) evolving unitarily. The relative-state interpretation states that the measurement is already described by eq. (47). Therefore one has to include the observer him/herself and ask the question "Why can't the observer see the whole range of states, but only one (collapsed) state?". The answer according to the interpretation is that the observer is perceiving

the universe that is consistent with the memory records. To put it another way, if the universe is in the state $|\mathcal{U}\rangle$ and the observer's state is $|\mathcal{O}_i\rangle$, after the measurement the state of the entire universe 'collapses' to one certain state. For example it collapses to $\langle\mathcal{O}_{42}|\mathcal{U}\rangle$ - a state which is consistent with the observer's state $|\mathcal{O}_{42}\rangle$. [12]

Even though the relative state interpretation provides a universal valid quantum framework, it leaves and launches some questions. First it does not explain, why the classical observer practically never superpositions. But this problem is again resolved by the environment induced superselection.

Another question is which role the probabilistic nature in quantum physics (summarized in Born's rule) plays in the interpretation. Since the publication of Everett many papers have occurred, in which Everett's idea was attempted to be reconstructed and extended in its approaches. Examples for most popular interpretations among those are the many-worlds, many-minds or many-histories theories.

6.3 Bohmian mechanics

The approach suggested by Bohm is a highly deterministic and non-local model of nature. It reinterpretes the particle-wave dualism in a completely new way by assigning the Schrödinger equation the role of a "guiding equation" which determines the dynamics of a particle. Such the motion of a particle has no probabilistic behaviour but follows a deterministic trajectory. One might ask how this is possible, when Bell proved that there can't be hidden variables that determine its deterministic path. But Bell was referring to *local* hidden variables, so the dynamics of Bohm's description still can imply non-local hidden variables. The main problem of Bohmian mechanics when applying it, is the explanation of quasiclassical trajectories thus this interpretation does not belong to the most supported interpretations among physicists.

Conclusion

In this thesis we have seen that even though the today most fundamental model of nature - quantum physics - seems to be a sufficient tool to predict a wide range of processes in nature, it still lacks in some of its interpretational perspectives. For example whether quantum physics is a theory of processes that are deterministic or if there is true randomness. Or how our classical world, consisting of only definite states, can arise from a fundamental law which allows superpositions. Within the Bell's inequality and the process of environment induced decoherence we have seen those two interpretational, philosophical problems can be turned into a problem which can be, at least partly, resolved within physical methods. Therefore an important step was to classify definitely the properties of classical systems, such as the entanglement and the quantum discord. Still we are left with open puzzles: Are there hidden non-local variables in quantum physics? Bohmian mechanics could give the answer in the future to this question. Whereas the relative-state theory can possibly reveal the mystery about the collapse and where all the potential states are.

Appendix

In the singlet state

$$|00\rangle = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

we next show that the spin correlation becomes

$$\langle S_A \cdot S_B \rangle = -\frac{\hbar^2}{4} \cos(\Theta)$$

with $S_{A/B}$ the spin of particle A/B and Θ the angle between measured components of S_A and S_B

To make the calculation easier, we calculate in spherical coordinates. We chose the coordinate system such that whereas $\vec{a} \parallel \vec{e}_z$ and \vec{b} are both in the $x-z$ -plane. This leads to $S_A = S_z$ and $S_B = \cos(\Theta)S_z + \sin(\Theta)S_x$

$$\begin{aligned} S_A S_B |00\rangle &= \frac{1}{\sqrt{2}} [S_z(\cos(\Theta)S_z + \sin(\Theta)S_x)(\uparrow\downarrow - \downarrow\uparrow)] \\ &= \frac{1}{\sqrt{2}} [(S_z \uparrow)(\cos(\Theta)S_z \downarrow) - (S_z \downarrow)(\cos(\Theta)S_z \uparrow + \sin(\Theta)S_x \uparrow)] \\ &= \frac{1}{\sqrt{2}} \left(\frac{\hbar}{2} \uparrow\right) [\cos(\Theta)(-\frac{\hbar}{2} \downarrow) + \sin(\Theta)((\frac{\hbar}{2} \uparrow) - (-(-\frac{\hbar}{2} \downarrow))] [\cos(\Theta)(-\frac{\hbar}{2} \uparrow) + \sin(\Theta)((\frac{\hbar}{2} \downarrow)] \\ &= \frac{\hbar^2}{4} [\cos(\Theta) \frac{1}{\sqrt{2}} (-\uparrow\downarrow + \downarrow\uparrow) + \sin(\Theta) \frac{1}{\sqrt{2}} (\uparrow\uparrow + \downarrow\downarrow)] \\ &= \frac{\hbar^2}{4} [-\cos(\Theta) \langle 00| + \sin(\Theta) \frac{1}{\sqrt{2}} (\langle 11| + \langle 1-1|)] \end{aligned} \tag{48}$$

So,

$$\begin{aligned} \langle S_a S_b \rangle &= \langle 00| S_a S_b |00\rangle \\ &= \frac{\hbar^2}{4} \langle 00| [-\cos(\Theta) |00\rangle + \sin(\Theta) \frac{1}{\sqrt{2}} (\langle 11| + \langle 1-1|)] \\ &= -\frac{\hbar^2}{4} \cos(\Theta) \langle 00| |00\rangle \\ &= -\frac{\hbar^2}{4} \cos(\Theta) \end{aligned} \tag{49}$$

Thus in units of $\frac{\hbar}{2}$ we get

$$\langle S_a^1 S_b^2 \rangle = -\cos(\Theta) = -\vec{a} \cdot \vec{b}$$

References

- [1] EINSTEIN, Albert; PODOLSKY, Boris; ROSEN, Nathan. Can quantum-mechanical description of physical reality be considered complete?. *Physical review*, 1935, 47. Jg., Nr. 10, S. 777.
- [2] BELL, John S., et al. On the einstein-podolsky-rosen paradox. *Physics*, 1964, 1. Jg., Nr. 3, S. 195-200.
- [3] ZUREK, Wojciech H. Decoherence and the transition from quantum to classical—REVISITED. arXiv preprint quant-ph/0306072, 2003.
- [4] GRIFFITHS, David Jeffrey; HARRIS, Edward G. *Introduction to quantum mechanics*. New Jersey: Prentice Hall, 1995.
- [5] SCHLOSSHAUER, Maximilian; FINE, Arthur. *Decoherence and the Foundations of Quantum Mechanics*. Springer Berlin Heidelberg, 2007.
- [6] PRESKILL, John. *Lecture notes for physics 229: Quantum information and computation*. California Institute of Technology, 1998.
- [7] COVER, Thomas M.; THOMAS, Joy A. *Elements of information theory* 2nd edition. Wiley-Interscience: NJ, 2006.
- [8] DAKIC, Borivoje, et al. Quantum discord as optimal resource for quantum communication. arXiv preprint arXiv:1203.1629, 2012.
- [9] GHIRARDI, GianCarlo. Does quantum nonlocality irremediably conflict with Special Relativity?. *Foundations of Physics*, 2010, 40. Jg., Nr. 9-10, S. 1379-1395.
- [10] TRISCH, M. *Inconsistency, asymmetry, and non-locality*. 2005.
- [11] CAPPELLARO, Prof, et al. *Quantum Theory of Radiation Interactions* (pdf). 2011.
- [12] ZUREK, Wojciech Hubert. Relative states and the environment: einselection, envariance, quantum Darwinism, and the existential interpretation. arXiv preprint arXiv:0707.2832, 2007.