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Quantum-Information Processing in Semiconductor Quantum Dots

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We propose an all-optical implementation of quantum-information processing in semiconductor quantum dots, where electron–hole excitations (excitons) serve as the computational degrees of freedom (qubits). The strong dot confinement leads to a strong renormalization of excitonic states, which, in analogy to NMR-based implementations of quantum-information processing, can be exploited for performing conditional and unconditional qubit operations.

1. Introduction Quantum information, quantum computation, quantum cryptography, and quantum teleportation represent exciting new arenas which exploit intrinsic quantum mechanical properties [1–3]. Basic elements to process quantum information are quantum bits (qubits), which, in analogy to classical bits, are defined as suitably chosen two-level systems. Much of the present excitement about quantum-information processing (QIP) originates from the seminal discoveries of Shor and others [4], who showed that – provided QIP can be successfully implemented for about 100–1000 qubits – quantum algorithms can perform some hard computations much faster than classical algorithms, and can allow the reduction of exponentially complex problems to polynomial complexity.

It is somewhat surprising that only a few basic requirements are needed for a successful implementation of QIP, which, according to DiVincenzo [3], can be summarized in the following five points: (i) a scalable physical system with well characterized qubits; (ii) the ability to initialize the state of the qubits; (iii) long relevant decoherence times, much longer than typical qubit-manipulation times; (iv) a "universal" set of quantum gates; and (v) a qubit-specific measurement capability. Apparently, the main difficulty for a successful implementation of QIP in a "real physical system" concerns the unavoidable coupling of qubits to their environment, which leads to the process of decoherence where some qubit or qubits of the computation become entangled with the environment, thus in effect "collapsing" the state of the quantum computer.

In this respect, semiconductor quantum dots (QDs) [5] are particularly promising candidates for a successful solid-state implementation of QIP, since in these nanoscopic structures carriers are strongly confined in all three space directions, leading in turn to a strongly suppressed environment coupling. For such dots we have recently proposed an all-optical implementation of QIP, where electron–hole excitations (excitons) serve as the computational degrees of freedom; quantum gates can be implemented by use of ultrashort laser pulses and coherent-carrier control [6]. Within the proposed scheme conditional qubit manipulations, which form a cornerstone for any implementation of

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QIP, naturally arise from the strong internal Coulomb interactions between electrons and holes. Indeed, optical single-dot spectroscopy [7] has recently revealed the importance of such Coulomb-induced renormalizations of exciton states in QDs (due to the strong quantum confinement) [8, 9]. The primary goal of the present contribution is to highlight the similarities of our proposed scheme with nuclear-magnetic-resonance (NMR) based implementations of QIP [10] and to provide a unified theoretical framework.

2. Theory The essence of our proposal is summarized in Fig. 1 (for a discussion of our detailed calculation see below). Figure 1a shows the absorption spectrum of an empty dot (i.e., no electrons and holes present): Two pronounced absorption peaks X_0 and X_1 can be identified whose energy splitting is of the order of the dot confinement (here ≈ 25 meV). As the central step within our proposal we (tentatively) ascribe the different exciton states to the computational degrees of freedom (qubits). Thus, for the specific case of Fig. 1 we have two qubits (X_0 and X_1), which have value one if exciton x is populated and zero otherwise ($x = X_0, X_1$).

Let us next consider the case where the first qubit (exciton X_0) is set equal to one (is populated). To simplify our analysis, we use the fact that in most semiconductors electron–hole pairs with given spin orientation can be selectively created by photons with a well-defined circular polarization. Throughout this paper, we shall only consider excitons with parallel spin orientations because of their strongly reduced available phase space and the resulting simplified optical density of states. For such polarizations, Fig. 1b reports the corresponding absorption spectrum: Due to state filling, the character of transition X_0 changes from absorption to gain (i.e., negative absorption); in addition, the higher-energetic transition is shifted to lower energy, which is attributed to the formation of a biexcitonic state B whose energy is reduced by an amount of Δ because of exchange interactions between the two electrons and holes, respectively [9].

We next formalize the above findings. To this end, we introduce: the exciton-operators I_x^+ (I_x^-) which create (annihilate) exciton x; and I_x^z which has eigenvalue 1 (0) if exciton x is populated (not populated). With these operators, our computational space is then given

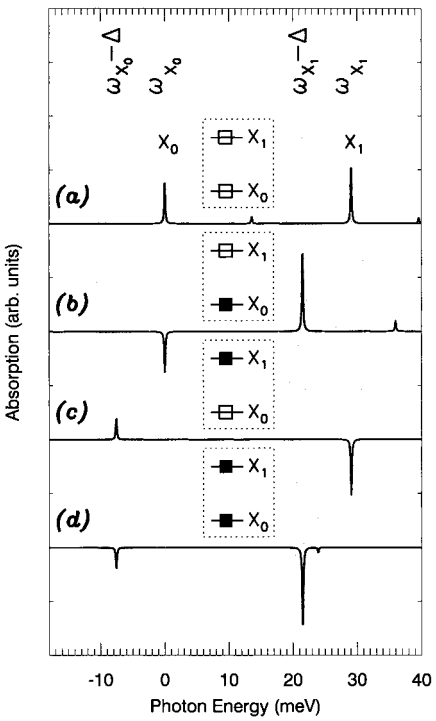

Fig. 1. Absorption spectra for semiconductor quantum dot described in the text, which is initially prepared (see insets) in a) vacuum state; b) exciton $|X_0\rangle$ state (exciton groundstate); c) exciton $|X_1\rangle$ state; d) biexciton $|B\rangle$ state. Photon energy zero is given by the groundstate exciton X_0

by: the vacuum state $|0\rangle$; the single-exciton states (where only one qubit is equal to one) $|x\rangle = I_x^+|0\rangle$; and $|11\rangle = I_{X_0}^+ I_{X_1}^+|0\rangle$, the state where both qubits are equal to one (note that, quite generally, it is not obvious that the biexciton state B corresponds to the state $|11\rangle$ since Coulomb interactions can mix different single-exciton states x; however, as will be shown below at the example of a prototypical dot confinement, in the strong confinement regime $|B\rangle$ is almost parallel to $|11\rangle$, thus providing the product-type Hilbert space required for QIP). Then, the Hamiltonian describing the exciton dynamics in a single dot can be written (in analogy to NMR-based QIP implementations [10])

$$\mathbf{H} = \sum_{x=X_0,\,X_1} E_x I_x^z - \Delta I_{X_0}^z I_{X_1}^z \tag{1}$$

with E_x the single-exciton energies. From related NMR work it is well known that a Hamiltonian of this form can account for the conditional and unconditional qubit operations required for QIP.

3. Results As a representative example for the Hamiltonian (1), in this paper we consider a prototypical dot confinement that is parabolic in the (x, y)-plane and box-like along z[2]). Following the procedure outlined in Ref. [6], we expand exciton and biexciton states within the subspaces of spin-selective electron–hole excitations, and obtain the excitonic eigenenergies and eigenstates through direct diagonalization of the two- and four-particle Schrödinger equations (accounting for all possible carrier–carrier Coulomb interactions). The single-exciton operators can then be expressed through $I_x^+ = \sum_{\mu\nu} \Psi_{\mu\nu}^x c_\mu^\dagger d_\nu^\dagger$, where $\Psi_{\mu\nu}^x$ is the exciton wavefunction and $c_\mu^\dagger$ ($d_\nu^\dagger$) creates an electron in state μ (hole in state ν). We checked that state $|11\rangle$ has more than 95% overlap with the true Coulomb-renormalized biexciton state B, which finally justifies the use of the Hamiltonian (1).

To understand how quantum gates can be implemented within our scheme, we first observe in Fig. 1 that the appearance and disappearance of peaks at the frequencies indicated by the (solid and dashed) shaded areas *conditionally* depends on the setting of specific qubits: e.g., the optical transition at $\omega_{X_0} - \Delta$ is *only* present if the second qubit is set equal to one (Figs. 1c and d), whereas the transition at ω_{X_0} *only* appears if the second qubit is set equal to zero (Figs. 1a and b); an analogous behaviour can be found for the other two transitions. Indeed, it is this conditional on- and off-switching of optical transitions that enables π-laserpulses to modify the state of one qubit or not, depending on the setting of the other qubit. For instance, as shown in Fig. 2, a π-pulse at frequency $\omega_{X_0} - \Delta$ changes the exciton population of X_0 only if exciton X_1 is populated: Such a transformation corresponds to a controlled-NOT (C-NOT) operation in which the second qubit (exciton X_1) acts as a control qubit. In addition, a combination of such a pulse with a π-pulse at frequency ω_{X_0} changes the state of exciton X_0 (target qubit) independently of the setting of the second qubit (NOT operation; see sequence of the two pulses in Fig. 2).

[2]) In our cylindrical QD the confinement energies due to the in-plane parabolic potential are $\omega_0^{(e)} = 20$ meV for electrons, and $\omega_0^{(h)} = 3.5$ meV for holes; with this choice, electron and hole wavefunctions have the same lateral extension. The quantum-well confinement along z is such that the intersubband splittings are much larger than $\omega_0^{(e,h)}$. Material parameters for GaAs are used. In our calculations we keep for electrons and holes, respectively, the ten single-particle states of lowest energy.

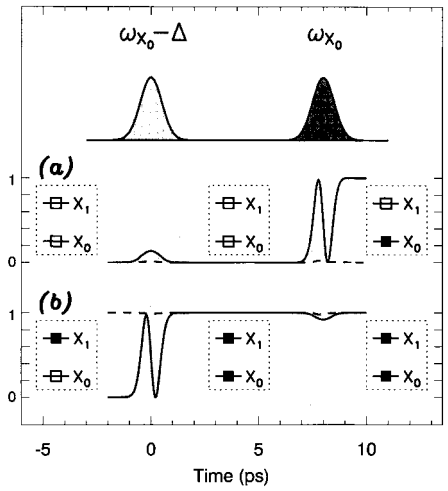

Fig. 2. Results of our simulations (neglecting dephasing) of qubit manipulations by means of coherent-carrier control: a) initial state $|00\rangle$; b) initial state $|01\rangle$. The solid (dashed) lines correspond to the transient population $\langle I^z_{X_0}\rangle_t$ of exciton X_0 ($\langle I^z_{X_1}\rangle_t$ of X_1). The sequence of pulses and the corresponding photon energies are indicated at the top of the figure (see also insets); for the envelopes of the laser pulses we use Gaussians $\propto \exp(-t^2/2\tau^2)$, with $\tau = 0.5$ ps. The first operation at time zero corresponds to a conditional operation where the second qubit acts as the control-qubit; the sequence of the two pulses corresponds to a NOT operation on the first qubit

4. Conclusion Let us finally comment to which extent DiVicenzo's five requirements for QIP are fulfilled within our scheme: (i) Although in this contribution we have only discussed a two-qubit implementation of QIP, one can expect that analogous schemes in arrays of coupled QDs (which arise naturally in dot fabrication) could allow implementation of a moderate number of qubits (see also [13]). (ii) Because of the nano-second electron–hole recombination times in semiconductors, "initialization" is not expected to cause major problems. (iii) Within our scheme, relevant decoherence times are of the order of several tens of picoseconds [12] which is not too much longer than the estimated sub-picosecond qubit-manipulation times [6]; use of excitonic groundstates in an array of QDs could help to improve such performance, because their decoherence times are of the order of nano-seconds. (iv) As shown in Figs. 1 and 2 conditional and unconditional qubit operations are implementable. (v) Finally, qubit-specific measurements could be performed in analogy to trapped-ion implementations of QIP [14], although more detailed strategies still have to be worked out. In conclusion, we expect the present proposal to be particularly promising for the first successful demonstration of QIP in this class of materials, and to evaluate whether optical excitations in QDs indeed can serve as reliable qubits.

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