A turnstile electron-spin entangler in semiconductors

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We propose a single-electron doped quantum dot in a field-effect structure as an optically triggered turnstile for spin-entangled electrons. A short laser pulse excites a charged exciton, whose quantum properties are transferred through tunneling and relaxation to the spin entanglement between electrons in the dot and contact. We identify the pertinent disentanglement mechanisms, and discuss experimental detection and possible application schemes. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1590425]

Devices based on single quantum systems can provide single quanta. This opens the possibility for the implementation of schemes based on the fundamental laws of quantum mechanics, for example, quantum cryptography[1] or quantum computation.[2,3] Within the field of semiconductors, it was soon realized that quantum dots (QDs),[4] sometimes referred to as *artificial atoms*, are ideal candidates for such challenging future applications, in particular in view of their high compatibility with existing semiconductor technology. Indeed, in the seminal work of Gérard and Gayral[5] the authors proposed a single QD embedded in a microcavity as a viable single-photon source; the applicability of this scheme was demonstrated experimentally soon after.[6–8] An important technological improvement is due to Yuan *et al.*[9] who replaced the optical triggering with an electrical one.

A reversed approach was recently pursued by Zrenner *et al.*,[10] where the authors used a QD photodiode as an optically triggered *single-electron* turnstile: a short laser pulse coherently excites an exciton in a QD embedded in a field-effect structure. If the structure is properly designed, such that tunneling occurs on a much shorter timescale than radiative decay, the electron–hole excitation of the QD decays into a separated electron and hole within the contacts, which is detected as the photocurrent. Within this scheme it thus becomes possible *to transfer optical excitations in a deterministic way to electrical currents*.

In this letter, we exploit this finding to propose a device that allows the optically triggered creation of a spin-entangled electron pair. The proposed structure [Fig. 1(a)] is identical to the one used by Zrenner *et al.*,[10,11] except that the dot is initially populated by a single surplus electron. This can be achieved by applying an external bias voltage such that an electron is transferred from a nearby *n*-type reservoir to the dot,[12,13] where further charging is prohibited because of the Coulomb blockade. Optical excitation of this structure then results in the excitation of a *charged exciton* (a complex consisting of two electrons and a single hole[12–14]), appropriate tuning of light polarization and frequency allows one to selectively excite the charged-exciton ground state, where the two electrons have opposite spin orientations. Since within the field-effect structure the charged exciton is not a stable configuration, in a consequent step, one electron and

hole will tunnel out from the dot to the nearby contacts. Here, the system can follow two pathways: either the electron in the dot has spin-up and the one in the reservoir spin-down orientation or vice versa. According to the laws of quantum mechanics, the total state of the system thus becomes a superposition of these two configurations. As will be proven subsequently, *in this state, the electron spins are maximally entangled*. Thus, the proposed device is an optically triggered turnstile for spin-entangled electrons, which could be used in future quantum information applications to establish entanglement between spatially separated sites.

In a sense, our scheme is similar to the proposal of Benson *et al.*[15] in which entangled photons are created in the cascade decay of a biexciton. However, in the system of our present concern, additional difficulties arise because the tunnel-generated electron and hole do not propagate freely (as photons would in the corresponding scheme), but are subject to interactions in the contact. The resulting scatterings of the entangled particles hamper a straightforward interpretation of the functionality of the proposed device and call for a careful theoretical analysis. It is the purpose of this letter to provide a comprehensible theory accounting for the complete cascade process of (1) the buildup of three-particle coherence through tunneling; (2) the swapping of quantum
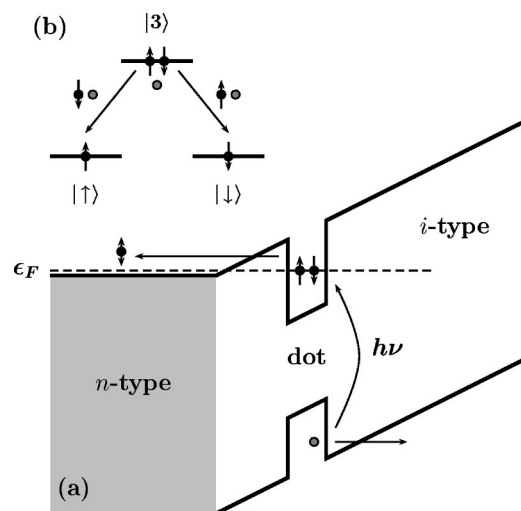

FIG. 1. (a) Schematic band diagram of the proposed structure. (b) Level scheme of the spin-degenerate electron states $|\sigma=\uparrow,\downarrow\rangle$ and the charged-exciton state $|3\rangle$ in the dot.

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coherence to spin entanglement through dephasing and relaxation in the reservoirs; and (3) the process of disentanglement through spin-selective scatterings. Since the main emphasis of our work is on the identification of the basic schemes underlying the buildup and decay of entanglement, we rely on a simplified description scheme of environment interactions, which will allow us to derive analytic expressions throughout.

Our model system comprises [Fig. 1(b)]: the spin-degenerate electron ground states $|\sigma\rangle$ and the charged-exciton ground state $|3\rangle$ in the dot (with energies E_σ and E_3, respectively); the electron and hole states in the reservoir, described by the usual field operators $c_{k\sigma}$ and $d_{k'}$ (energies $\epsilon_{k\sigma}^e$ and $\epsilon_{k'}^h$) with k labeling the quantum numbers (e.g., wave vector and band index), and we have explicitly indicated the electron spin. The Hamiltonian of the system without interactions (H_0) thus reads

$$H_0 = \sum_\sigma E_\sigma |\sigma\rangle\langle\sigma| + E_3 |3\rangle\langle3| + \sum_{k\sigma} \epsilon_{k\sigma}^e c_{k\sigma}^\dagger c_{k\sigma}$$
$$+ \sum_{k'} \epsilon_{k'}^h d_{k'}^\dagger d_{k'}. \tag{1}$$

Since we are dealing with an *open system* (i.e., system interacting with its environment) we have to adopt a density-matrix description.[16,17] Let us assume that initially, the electron spin direction is undetermined; that is, the corresponding density matrix is a mixture $\rho = 1/2\sum_\sigma |\sigma\rangle\langle\sigma|$. When at time $t=0$ the dot is subject to an unpolarized optical π-pulse,[10] it will be excited to state 3. Hence, the initial density matrix is $|3\rangle\langle3|$ (although the proposed scheme would also work for charged-exciton occupancies less than 1, as discussed later).

For the system's time evolution, we employ a master-equation framework of Lindblad form[16,17]

$$\dot\rho = -i[H_0,\rho] - \frac{1}{2}\sum_i (L_i^\dagger L_i \rho + \rho L_i^\dagger L_i) + \sum_i L_i \rho L_i^\dagger, \tag{2}$$

within which scatterings are described in the usual Markov and adiabatic approximations. In Eq. (2), the L_i are the Lindblad operators, which account for the different scattering channels.

Tunneling. For low temperatures and early times, we can safely neglect phonon processes and radiative decay in the dot, and tunneling becomes the only relevant scattering channel. Quite generally, the question as to whether combined electron–hole tunneling dominates over separate tunneling (as we will assume) depends on the design of the structure. In Ref. 11, the authors measured tunneling lifetimes between 29 and 330 ps, where the exact value strongly depends on the internal electric field;[18] alternatively, it might be advantageous to use type-II QDs,[19] where the hole is only Coulomb bound and much shorter tunneling lifetimes could be achieved. However, such details are not crucial to our study and the only relevant assumptions are: (1) all tunneling processes are independent of spin and (2) since the hole enters with a high excess energy into the contact, it immediately suffers an inelastic scattering, which guarantees that tunneling is an irreversible process. In our calculations, the latter

point is taken into account by tracing over the degrees of freedom of the hole in the reservoir and neglecting terms $tr_h \rho d^\dagger d$. Within this framework and assuming tunneling matrix elements $\hat{t}$ independent of k and k', we can solve Eq. (2) through an *unraveling of the master equation*,[20] to obtain[21]

$$\rho(t) \cong e^{-\lambda t}|3\rangle\langle3| + \lambda \int_0^t dt' e^{-\lambda t'} U(t,t')$$
$$\times |\Psi_1\rangle\langle\Psi_1| U(t',t), \tag{3}$$

with $\lambda = 2\pi|\hat{t}|^2 \Sigma_\sigma \int d\omega_e d\omega_h g_e(\omega_e) g_h(\omega_h) \delta(\omega_e + \omega_h + E_\sigma - E_3)$ the total tunneling rate, $g_{e,h}(\omega)$ the electron and hole density-of-states in the reservoir, $U(t,t')$ the time evolution operator in the reservoir, and $|\Psi_1\rangle\langle\Psi_1|$ the density matrix after tunneling. In the spirit of the quantum-jump approach,[20] in Eq. (3), the first term can be interpreted as the *conditional density matrix* for no tunneling (which decays with $e^{-\lambda t}$), whereas the second term is the conditional evolution after tunneling. The corresponding density matrix is obtained from[20] $L_i|3\rangle\langle3|L_i^\dagger/tr(\cdot)$, where L_i is the Lindblad operator for tunneling and the denominator ensures $tr\, \rho = 1$, which gives

$$|\Psi_1\rangle \propto \sum_\sigma \int_{\epsilon_F}^{\omega_c} d\omega C_\sigma^\dagger(\omega)|\bar\sigma\rangle. \tag{4}$$

Here, ϵ_F is the Fermi energy of the n-type reservoir, ω_c is a cutoff energy due to the kinematics of the tunneling process, $C_\sigma(\omega) = \Sigma_k c_{k\sigma}\delta(\omega - \epsilon_{k\sigma}^e)$, and $\bar\sigma$ a spin-orientation antiparallel to σ. Eq. (4) is an important and nontrivial result. First, it demonstrates that despite the incoherent nature of tunneling and hole relaxation, the electron system can be described in terms of wave functions; we note here in passing that the detection of the hole would even allow us to purify this wave function,[3] which might be of relevance when initially ρ is not equal to $|3\rangle\langle3|$. Second, a closer inspection of Eq. (4) reveals that the spin part $C_\uparrow^\dagger|\downarrow\rangle + C_\downarrow^\dagger|\uparrow\rangle$ is a *maximally entangled state* of the electrons in the dot and reservoir. We emphasize that this maximal entanglement is independent of the spin basis, which guarantees that our scheme is not deteriorated by possible polarization anisotropies of the dot states (fine-structure splittings).

Dephasing and relaxation. After tunneling, the system propagates in presence of scatterings, as described by $U(t,t')$ in Eq. (3). Quite generally, we assume that the orbital degrees of the reservoir electron are subject to much stronger interaction channels (e.g., phonons) than the spin degrees, as evidenced by the long measured spin lifetimes ($\sim$nanoseconds) in n-doped semiconductors.[22] For that reason, let us first consider an elastic electron scattering that does not depend on spin; that is, Lindblad operators of the form $\sqrt{\Gamma}\Sigma_\sigma C_\sigma^\dagger(\omega)C_\sigma(\omega)$, with Γ the scattering rate. Unraveling the corresponding master equation in an analogous fashion to Eq. (3), we again recover a conditional evolution for no scattering (that decays with $e^{-\Gamma t}$) and a remainder that describes the effects of scattering. Here, the density matrix after scattering becomes[21]

$$|\Psi_1\rangle\langle\Psi_1| \rightarrow \sum_{\sigma\sigma'} \int_{\epsilon_F}^{\omega_c} d\omega C_\sigma^\dagger(\omega)|\bar\sigma\rangle\langle\bar\sigma'|C_{\sigma'}(\omega). \tag{5}$$

In comparison to Eq. (4), the density matrix of Eq. (5) is diagonal in ω; that is, the elastic scattering has led to a destruction of the phase coherence (i.e., dephasing). However, *the spin part still shows the same degree of entanglement*, where similar conclusions would apply for inelastic but spin-independent scatterings. *Thus, the decay of an optically excited charged exciton indeed generates a robust spin entanglement between the electron in the dot and reservoir.*

Disentanglement. We finally comment on the process of disentanglement. In fact, any scattering channel that couples with unequal strength to the spins (or affects only one spin orientation) is responsible for such entanglement decay. Naively, one could expect that a preferential scattering of, for example, spin-up electrons in the reservoir would establish a stronger degree of spin-down population in the dot; however, this is not supported by our calculations, which show that any spin-selective scattering[23] forces the spins *with equal probability* to one of the two orientations. Thus, to experimentally detect spin entanglement in the proposed scheme, both electrons have to be monitored. This could be achieved by introducing a ferromagnetic contact at the interface of the n-doped region, which acts as a spin filter for the reservoir electron. Transmission across the interface corresponds to a spin measurement that also determines the spin orientation σ of the electron in the QD. The resulting state $|\sigma\rangle$ could be probed by a second, time-delayed optical π-pulse whose polarization is chosen such that it selectively excites the $\sigma-3$ transition. Thus, the transmission into the ferromagnet is accompanied by the optical excitation of a second charged exciton in the dot (that consecutively is transferred to an electric current). On the other hand, if the second optical pulse arrives before disentanglement, the dot density matrix is a mixture $1/2\Sigma_\sigma|\sigma\rangle\langle\sigma|$ and optical excitation occurs only with a 50% probability, which results in a distinctly different noise characteristics of the photocurrent.

In conclusion, we have proposed a scheme for an optically triggered spin entanglement of electrons in semiconductors. It consists of a single-electron doped quantum dot embedded in a field-effect structure. Optical excitation of an additional electron–hole pair (charged exciton) is transferred through tunneling to a photocurrent, where the spins of the electrons in the dot and reservoir are maximally entangled. We have discussed that this entanglement is robust against dephasing and relaxation processes that are not spin-selective, and thus benefits from the long spin lifetimes in semiconductors. The proposed device might be useful in future quantum information applications to establish entanglement between spatially separated sites; there, it might be advantageous to replace the n-type reservoir by quantum wires (for a natural realization of such combined dot-wire structures see, e.g., Ref. 14). Finally, in contrast to other proposal for spin entanglement in semiconductors,[24] our scheme allows the creation of spin-entangled electrons on demand (through optical triggering).

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