Few-particle states and quantum-information processing in quantum dots

Ulrich Hohenester*, Filippo Troiani, and Elisa Molinari

Istituto Nazionale per la Fisica della Materia (INFM) and Dipartimento di Fisica, Università di Modena e Reggio Emilia, Via Campi 213A, I-41100 Modena, Italy

Abstract We present a consistent theoretical description of few-particle effects in the optical spectra of semiconductor quantum dots, based on a direct-diagonalization approach. Because of the strong dot confinement Coulomb interactions among carriers are strongly enhanced, and each electron-hole configuration of the confined few-particle system leads to its characteristic signature in the optical spectra. We discuss how these few-particle effects can be exploited for an all-optical solid-state implementation of quantum-information processing.

1 Introduction

The strong three-dimensional quantum confinement in semiconductor quantum dots (QDs) results in a discrete, atomic-like carrier density of states. In turn, *(i)* the coupling to the solid-state environment (e.g., phonons) is strongly suppressed and *(ii)* Coulomb correlations among charge carriers are strongly enhanced. Indeed, in the optical spectra of single dots spectrally narrow emission peaks have been observed (indicating a small environment coupling), which undergo discrete energy shifts when more carriers are added to the dot (indicating energy renormalizations due to additional Coulomb interactions) [1].

2 Results

2.1 Few-particle effects

We first discuss how these spectral changes result from few-particle interactions. In our calculations we use a prototypical dot confinement (parabolic in the (x,y)-plane and box-like along z [2]) and obtain the Coulomb-renormalized few-particle states through direct diagonalization of the Hamiltonian matrix (for details see Ref. [3]). In a typical single-dot experiment [1]: *(i)* A pump pulse creates electron-hole (e-h) pairs in continuum states (e.g., wetting layer) in the vicinity of the QD; *(ii)* some of the carriers become captured in the QD; *(iii)* due to environment coupling there is a fast subsequent carrier relaxation to the dot state of lowest energy; *(iv)* e-h pairs in the dot recombine by emitting photons. Thus, by varying in a steady-state experiment the pump intensity and by monitoring luminescence from the dot states one obtains information about the few-particle states for a varying number N of e-h pairs. From a theoretical point of view, luminescence involves a process where one e-h pair is removed from the interacting many-particle system and one photon is created:

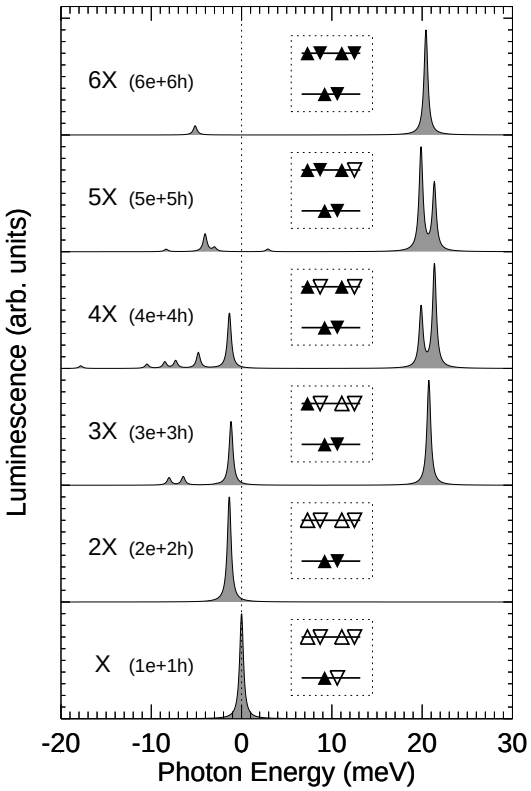

Fig. 1 Luminescence spectra for QD [2] and for different multi-exciton states (number of electron-hole pairs according to expression in parentheses). The insets sketch the respective electron-hole configurations before photon emission (spin orientation according to orientation of triangles). Photon energy zero is given by the groundstate exciton X_0.

$$I(h\nu) \propto \sum_{\ell} |\langle \ell; N-1|\hat{\pi}|0;N\rangle|^2 \, \mathcal{D}_\gamma(h\nu - E_0^N + E_\ell^{N-1}),$$

with $|\ell;N\rangle$ the ℓ-th excited state with energy E_ℓ^N, $\hat{\pi}$ the interband dipole operator, and $\mathcal{D}_\gamma$ a broadened delta function. Figure 1 shows luminescence spectra for a varying number of e-h pairs (see insets). Indeed, whenever one additional e-h pair is added to the dot the optical spectrum changes because of the additional Coulomb interactions. A comparison of such calculated luminescence spectra with experiment is presented for multi-excitons and multi-charged excitons in Refs. [4,5] (see Refs. [1,6,7] for related work).

* *E-mail:* hohenester@unimo.it

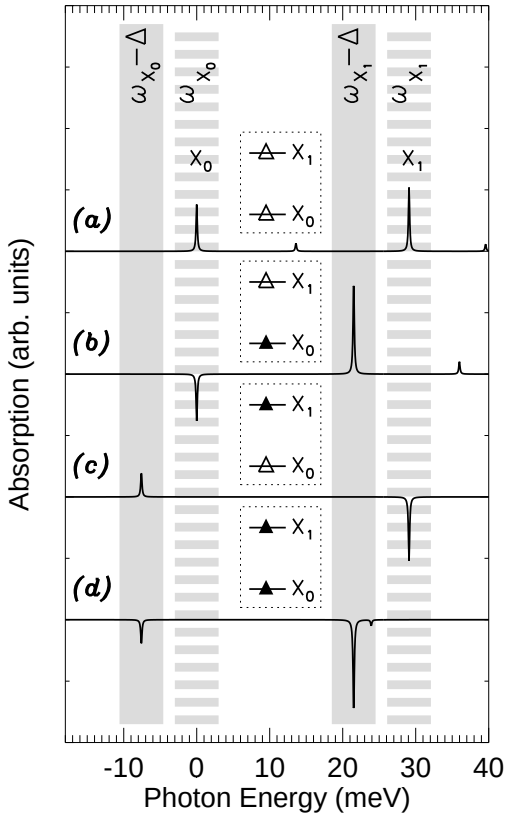

Fig. 2 Absorption spectra for same dot as in Fig. 1 [2], which is initially prepared in: *(a)* vacuum state; *(b)* exciton $|X_0\rangle$ state (exciton groundstate); *(c)* exciton $|X_1\rangle$ state; *(d)* biexciton $|B\rangle$ state.

2.2 Quantum-information processing

Recently, because of their discrete atomic-like density of states and the resulting suppressed environment coupling semiconductor QDs have been proposed for a solid-state implementation of quantum-information processing (QIP) [8]. Basic elements of QIP are properly chosen two-level systems (qubits) and coherent qubit manipulation on a timescale much shorter than dephasing (quantum gates). For QIP two types of gates have to be realized [9,10]: Unconditional gates, i.e., manipulation of one qubit independent of the setting of the others; conditional gates, i.e., manipulation of two qubits where one qubit is changed *only* if the second one has a specific setting. Recently, we have demonstrated [11] that exciton states in quantum dots can serve as qubits, where the requested quantum gates can be implemented by use of ultrafast spectroscopy and coherent-carrier control.

The essence of this proposal is summarized in Figure 2. Fig. 2*(a)* shows the absorption spectrum of an empty dot (i.e., no electrons and holes present): Two pronounced absorption peaks X_0 and X_1 can be identified whose energy splitting is of the order of the dot confinement (here ∼25 meV). As the central step within our proposal we ascribe the different exciton states to the computational degrees of freedom (qubits). Thus, for the specific case of Fig. 2 we have two qubits, which have value one if exciton x is populated, and zero otherwise. To simplify our analysis, we use the fact that in most semiconductors e-h pairs with given spin orientation can be selectively created by photons with a well-defined circular polarization. For such polarizations, Fig. 2*(b)* reports the corresponding absorption spectrum: Due to state filling, the character of transition X_0 changes from absorption to gain (i.e., negative absorption); in addition, the higher-energetic transition is shifted to lower energy, which is attributed to the formation of a biexcitonic state whose energy is reduced by an amount of Δ because of exchange interactions between the two electrons and holes, respectively.

To understand how quantum gates can be implemented, we observe in Fig. 2 that the appearance and disappearance of peaks at the frequencies indicated by the (solid and dashed) shaded areas *conditionally* depends on the setting of specific qubits: E.g., the optical transitions at $\omega_{X_0} - \Delta$ is *only* present if the second qubit is set equal to one (Figs. 1*(c)* and 1*(d)*), whereas the transition at ω_{X_0} *only* appears if the second qubit is set equal to zero (Figs. 1*(a)* and 1*(b)*); an analogous behaviour can be found for the other two transitions. Indeed, in analogy to NMR-based implementations of QIP [9] it is this conditional on- and off-switching of optical transitions that enables laserpulses at certain frequencies to modify the state of one qubit or not, depending on the setting of the other qubit. For details the reader is referred to Ref. [11].

This work was supported in part by INFM through PRA-99-SSQI, and by the EU under the TMR Network "Ultrafast Quantum Optoelectronics" and the IST programme "SQID".

References

1. For a review see: A. Zrenner, J. Chem. Phys. **112** (2000) 7790.
2. In our cylindrical QD the confinement energies due to the in-plane parabolic potential are $\omega_o^{(e)} = 20$ meV for electrons, and $\omega_o^{(h)} = 3.5$ meV for holes; with this choice, electron and hole wavefunctions have the same lateral extension. The quantum-well confinement along z is such that the intersubband splittings are much larger than $\omega_o^{(e,h)}$. Material parameters for GaAs are used.
3. U. Hohenester and E. Molinari, phys. stat. sol. (b) **221**, in press; cond-mat/0005253.
4. R. Rinaldi, S. Antonaci, M. DeVittorio, R. Cingolani, U. Hohenester, E. Molinari, H. Lipsanen, J. Tulkki, Phys. Rev. B **62** (2000) 1592.
5. A. Hartmann, Y. Ducommun, E. Kapon, U. Hohenester, and E. Molinari, Phys. Rev. Lett. **84** (2000) 5648.
6. M. Bayer *et al.*, Nature **405** (2000) 923.
7. R. Warburton *et al.*, Nature **405** (2000) 926.
8. L. Quiroga and N.F. Johnson, Phys. Rev. Lett. **83** (1999) 2270.
9. A. Steane. Rep. Prog. Phys. **61** (1998) 117.
10. D. P. DiVincenzo, arXiv:quant-ph/0002077.
11. F. Troiani, U. Hohenester, and E. Molinari, Phys. Rev. B **62** (2000) R2263.