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Shaking the condensates: Optimal number squeezing in the dynamic splitting of a Bose–Einstein condensate

Julian Grond [a,*], Jörg Schmiedmayer [b], Ulrich Hohenester [a]

[a] Institut für Physik, Karl–Franzens–Universität Graz, Universitätsplatz 5, 8010 Graz, Austria
[b] Atominstitut der Österreichischen Universitäten, TU–Wien, Stadionallee 2, 1020 Wien, Austria

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ABSTRACT

We apply optimal control theory to the dynamic splitting process of a Bose–Einstein condensate (BEC). Number squeezing of two spatially separated BECs is important for interferometry applications and inhibits phase diffusion due to the nonlinear atom–atom interactions. We show how optimal number squeezing can be obtained on time scales much shorter compared to adiabatic splitting. The non-adiabatic time evolution of the condensates is controlled via the trap geometry, thus making our control schemes directly applicable to experiments. We find that the optimal solution for the trap is oscillatory, where a counterintuitive shaking during the ramp produces highly squeezed states. The underlying process can be identified as a parametric amplification.

1. Introduction

Matter wave interferometry with Bose–Einstein condensates (BECs) confined in microtraps offer new prospects for precision measurements [1]. Optical dipole traps [2], atom chips [3], and radio-frequency (RF) potentials [4,5] are versatile and powerful tools for the coherent manipulation and interference of matter waves [6–11]. Atom interferometers, where a BEC is continuously split in space into two separated parts, have been demonstrated [12,7]. This might allow for high-precision measurements of a relative phase difference, accumulated due to external potentials subject to one of the wells.

The problem inherent to atom interferometers is phase diffusion [13], caused by the nonlinear interaction of the atoms, resulting in dephasing on a timescale of 10−100 ms. Even without considering any further decoherence, a superposition state of atoms in the left and right well will become scrambled due to the different phases acquired by each term in the superposition. In principle, this could be overcome by tuning the interactions to zero with the help of a Feshbach resonance. Unfortunately this is experimentally difficult for Rb atoms, which are highly versatile for condensate splitting, but lack a broad Feshbach resonance.

In this paper we thus aim at using *number squeezing* as an alternative strategy [14–17], which inhibits phase diffusion due to the narrow number distribution. In experiments, number squeezing has been obtained using linear ramps [9,11], where the splitting has to be quasi-adiabatic to achieve a high degree of squeezing. This, however, has several disadvantages: The time scale for high squeezing is fairly long, and a linear improvement of squeezing requires an exponentially longer splitting time [18]. This could be prohibitive in view of experimental noise limitations. A more fundamental drawback is that phase diffusion is already present during splitting.

In this paper we identify, using optimal control theory [19–21], a novel control strategy by which squeezing can be obtained one order of magnitude faster [22]. Thus, our findings represent a substantial improvement in the preparation of states for atom interferometry.

In the following, we discuss the physics and the modeling of the dynamical splitting process. We describe our optimization procedure and discuss the results for optimal number squeezing. An intuitive interpretation of the underlying control mechanism is sketched. Finally, we discuss that atom interferometry could be substantially improved by employing our control schemes.

2. Dynamical splitting and number squeezing

In the splitting of a BEC, one can identify two time scales $\tau_c = 4/E_c N$ and $\tau_{\mathrm{trap}} = 2\pi/\omega_{\mathrm{trap}}$, which determine the breakdown of adiabaticity for the squeezing dynamics and the spatial dynamics, respectively [18]. Here, E_c is the energy associated with the nonlinear atom interaction, N is the total, fixed number of atoms, and ω_{trap} is the trap frequency.

* Corresponding author.
E-mail address: julian.grond@uni-graz.at (J. Grond).

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Usually, $\tau_c \gg \tau_{\mathrm{trap}}$, such that for a quasi-adiabatic splitting process with respect to the tunneling dynamics the spatial dynamics is negligible, and there is a fixed relationship between tunnel coupling and splitting distance. Then, a generic two-mode model [23,18]

$$\hat{H} = -\frac{2}{N}E_j\hat{J}_x + \frac{1}{2}E_c\hat{J}_z^2 \tag{1}$$

is applicable. This model is comprised of a term accounting for tunnel coupling at rate $\frac{2}{N}E_j$, and an interaction term. E_j and E_c are analogous to the Josephson energy and the charging energy in a superconducting Josephson junction. The pseudospin operator $\hat{J}_z$ counts the atom number difference between the left and right well, and $\hat{J}_x$ couples the two wells [23].

In the unsplit trap, we have approximately a single condensate with a wave function ϕ_g determined by the Gross–Pitaevskii (GP) equation, and an unpopulated one, ϕ_e, with ungerade symmetry. In contrast, in the split trap we have *localized* orbitals $\phi_{l,r} = (\phi_g \pm \phi_e)/\sqrt{2}$ in the left and right well, respectively, which are equally populated. As regarding the number fluctuations, initially tunneling dominates over the nonlinear interaction and the number distribution between the left and right well is binomial. When the trap is split, the ground state of the condensates is a squeezed state since the nonlinear interaction penalizes fluctuations. In optical lattices, the transition between these two extreme states (binomial versus squeezed) is termed the superfluid–Mott–insulator transition [24]. A quasi-adiabatic exponential splitting will be the reference for the optimized solutions to be compared with. As one is restricted in practice to a finite splitting time, at some point, depending on the time scale of the ramp relative to τ_c, adiabaticity breaks down and the fluctuations become frozen [18].

In contrast, for non-adiabatic splitting, i.e., when the trap changes much faster in time than τ_{trap}, the condensate dynamics is more complicated and the tunnel coupling strongly depends on its details. Hence, we have to use the trapping potential rather than the tunnel coupling itself as the control parameter.

A way to find the orbitals which describe the motion of the condensates are variational methods, such as the approach with Gaussian states by Menotti et al. [25], or the more general variational approach by Alon et al. [26], termed multi-configurational time dependent Hartree equations for bosons (MCTDHB). Within the latter approach, the atom number distribution vector **C** and the orbitals are determined self-consistently from a variational principle. **C** as well as the orbitals ϕ_g and ϕ_e then fulfill a set of coupled equations. The one- and two-body reduced densities depend on **C** and determine the nonlinear part of the orbitals equations. On the other hand, **C** fulfills a more complicated kind of two-mode equation as in Eq. (1) where the tunnel coupling and nonlinearity are functions of the orbitals.

For the unsplit trap, only the gerade orbital ϕ_g is populated, and hence its time evolution is governed by the GP equation. The weakly populated ungerade orbital ϕ_e is elevated in energy due to the interactions. Things are different for large splitting distances, where the energies of the two modes become degenerate due to the geometrical symmetry of the double well. Then both orbitals become equally populated and have a similar shape, thus giving rise to left and right localized orbitals. An example for the non-adiabatic time evolution of those orbitals is given in Fig. 1.

In our calculation we consider a BEC in an elongated trap, where the transverse frequency is $\omega_T = 2\pi \cdot 2\,\mathrm{kHz}$ and the aspect ratios are on the order of several hundreds. The chemical potential is such that we are in a 3d regime. The splitting is along a transverse direction, similar as in Ref. [7]. The potential determining the splitting distance depends on an RF-field $B_{rf} = (0.5 + 0.3\lambda)G$ [4] and we use λ as our control parameter. Typical values for $N = 100-1000$ are $\tau_{\mathrm{trap}} = 0.5\,\mathrm{ms}$ and $\tau_c \approx 10\,\mathrm{ms}$.

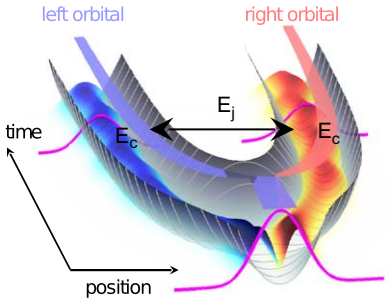

Fig. 1. Schematics of the dynamic splitting of a BEC by continuously transforming a single-well potential to a double-well potential. Shown are the orbitals which are localized in the left and right well.

3. Optimal control theory

For obtaining optimal number squeezing, we numerically optimize a cost functional consisting of a term proportional to the amount of squeezing between the left and right condensate at the final time. For that purpose, we introduce the pseudospin operator for the population in the localized orbitals $\hat{J}_z^{loc}(\phi_g(t), \phi_e(t))$, which accounts for a possible time-dependent relative phase between ϕ_g and ϕ_e. We additionally require that the final orbitals are close to some desired states ϕ_g^d and ϕ_e^d, which are stationary orbitals. A third term in the cost functional accounts for the smoothness of the control fields. Thus, it reads

$$J(\mathbf{C},\phi,\lambda) = \frac{\gamma_1}{2}\langle \mathbf{C}(T)|(\hat{J}_z^{loc})^2|\mathbf{C}(T)\rangle + \frac{\gamma_2}{2}(2 - |\langle \phi_g^d|\phi_g(T)\rangle|^2 - |\langle \phi_e^d|\phi_e(T)\rangle|^2) + \frac{\gamma}{2}\int_0^T (\dot{\lambda}(t))^2\,dt, \tag{2}$$

where all terms are weighted by appropriate factors γ_1, γ_2, and γ. Next, J is optimized subject to the constraint that **C**, ϕ_g and ϕ_e fulfill the equations of motion from MCTDHB. This can be achieved by introducing Lagrange multipliers. Variation of the Lagrange functional yields the so-called adjoint equations as well as the final conditions and the gradient. An optimization routine iteratively solves the equations of motion and the adjoint equations in order to calculate the gradient, which provides for a new search direction. The step size along this direction is then determined by a line search algorithm [20–22].

After this *squeezing optimization* we separate the orbitals further and bring them to a halt, using Eq. (2) with $\gamma_1 = 0$ as the cost functional, similar to Ref. [21]. This control sequence we term *trapping optimization* after which the condensates completely decouple and stay in a highly squeezed state.

4. Optimal number squeezing

A summary of our results is given in Fig. 2. The curves show the quasi-adiabatic exponential splitting, and display how much squeezing is possible using this control. Clearly, we have good squeezing only for very long splitting times. The squares ($N = 100$) and diamonds ($N = 500$) show results from our OCT calculations within MCTDHB, where squeezing is obtained approximately five times faster. Thus, our results demonstrate that with more refined splitting protocols, highly squeezed states can already be obtained on much shorter time scales.

In Fig. 3 we plot a typical OCT result. In the squeezing optimization, the fluctuations Δn (dark line) first oscillate strongly, until they significantly drop towards the end. The control field (bright line) shows rapid oscillations superimposed onto a bigger and a slower period. A similar behavior is exhibited

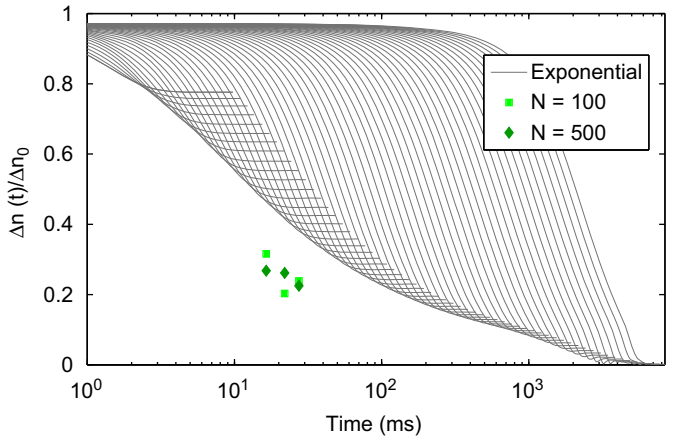

Fig. 2. Comparison of quasi-adiabatic exponential splitting (lines), obtained within the generic model (corresponding MCTDHB simulations, not shown, resemble closely), and OCT solutions (symbols), obtained within MCTDHB.

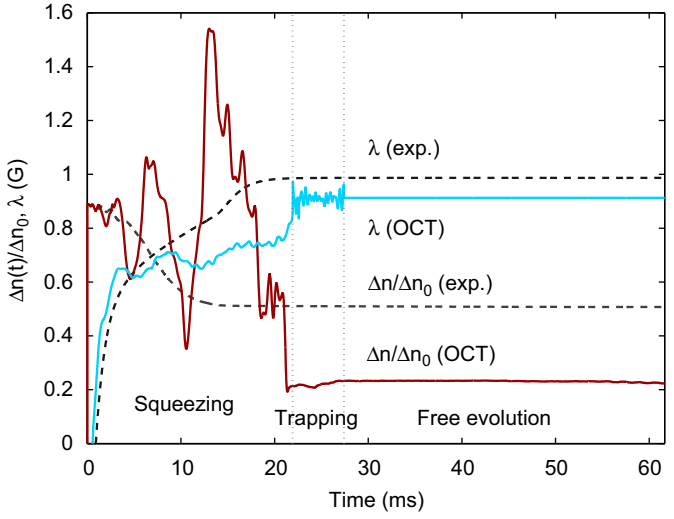

Fig. 3. Typical solution for optimal number squeezing. In the first stage, termed *squeezing optimization*, the fluctuations Δn (dark line) start to oscillate and then significantly drop for the optimized control λ (bright line). After the *trapping optimization*, the condensates remain in a highly squeezed state. This is compared to the quasi-adiabatic exponential splitting for the same splitting time (dashed lines). (For interpretation of the references to the color in this figure legend, the reader is referred to the web version of this article.)

by the tunnel coupling (not shown). The rapid oscillations are clearly connected to the oscillations of the condensates in the wells.

Understanding of the slower oscillation can be obtained by looking more closely to the simple two-mode Hamiltonian from Eq. (1), which underlies the atom tunnel dynamics. Taking the continuum limit $\mathbf{C}\rightarrow C(k)$ [18], where k is the atom number difference between the wells, and introducing a second quantized picture via the bosonic operator $\hat{a}$ [27], we obtain ($\tilde{E}_j/N = E_j/N + E_cN/8$ is the renormalized oscillator frequency)

$$\hat{H} = 2\frac{\tilde{E}_j}{N}\left[\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{E_cN^2}{16\tilde{E}_j}(\hat{a}^2 + \hat{a}^{\dagger 2})\right]. \quad (3)$$

Eq. (3) is similar to a parametric harmonic oscillator with a resonance frequency $\omega_{res} \approx 2E_j^0/N + E_cN/4$, where we have assumed the tunnel coupling to be of the form $E_j(t) = E_j^0 + E_j^1\cos(\omega t)$. Thus, as a property of the parametric oscillator, when E_j is oscillating on resonance the envelope of the number fluctuations Δn decays exponentially [28]. This behavior is very similar to that in Fig. 3 (red curve), where the oscillations drive the parametric amplifier.

5. Atom interferometry with optimal number squeezed states

In this last section we show how optimal number squeezing might improve the performance of atom interferometers [1]. When using time-of-flight (TOF) measurements, the condensates are released from the trap after a relative phase between the left and right condensate has been acquired due to a potential difference arising from an external force. The condensates expand, fall down and their interference pattern is recorded. From the fringe spacing, the phase can be deduced, as has been demonstrated in Refs. [7,12]. The sensitivity is related to the fringe visibility. Note that while in a single measurement a well defined interference picture might be obtained, when averaging over many measurements the fringes may average out. Therefore we plot the coherence factor defined as $\alpha = 2Re\langle\hat{a}_L^\dagger\hat{a}_R\rangle/N$ [29]. It is 1 for perfect fringe visibility and 0 for independent condensates with an undefined phase. In Fig. 4, we compare for different N the coherence for exponential and optimal splitting, showing that optimal squeezing brings a strong improvement in preserving the phase coherence. This allows for a longer accumulation time of the phase with better phase sensitivity. The coherence after splitting is around 0.9 for $N = 100$ and around 1 for $N = 500$ (τ_c is constant).

Furthermore we found a ξ_R factor [15,11] close to the Heisenberg limit [14]. This factor determines the phase sensitivity whenever phase readout is done by a number measurement, and provides a signature for N-body entanglement [30].

6. Summary

We have shown that high number squeezing can be obtained when the condensates are shaken in the traps with the right frequency. This was achieved by optimal control theory calculations. The underlying process has been identified as parametric amplification. Our results allow for generating squeezed states

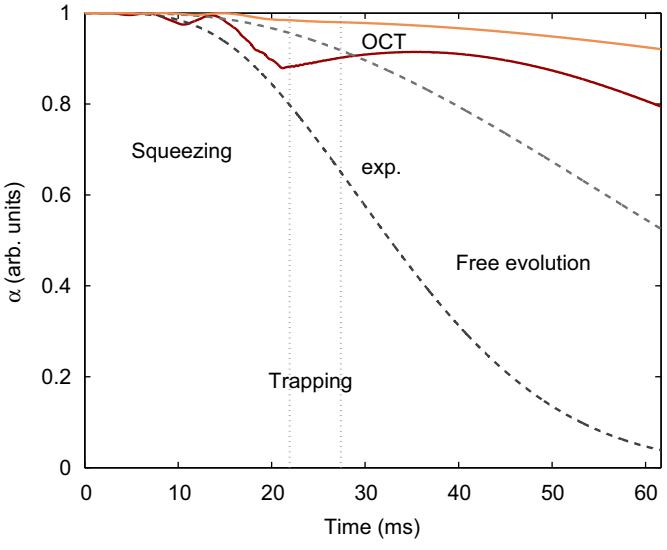

Fig. 4. Coherence factor α for exponential versus optimal splitting for $N = 100$ (bright lines) and $N = 500$ (dark lines). For the optimized solutions, the coherence is less reduced by phase diffusion due to number squeezing. (For interpretation of the references to the color in this figure legend, the reader is referred to the web version of this article.)

much faster than with quasi-adiabatic splitting, and may substantially improve the performance of atom interferometers.

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