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Optimal quantum control of electron–phonon scatterings in artificial atoms

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Abstract

We study the phonon-induced dephasing dynamics in optically excited semiconductor quantum dots within the frameworks of the independent Boson model and optimal control. It is shown that appropriate tailoring of laser pulses allows a complete control of the optical excitation despite the phonon dephasing, a finding in marked contrast to other environment couplings.
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1. Introduction

Quantum control in semiconductors has recently become a subject of enormous scientific and technological interest [1,2], motivated by the tremendous success of quantum optics in atomic and molecular systems [3], the emerging field of quantum information processing [4,5], and the high standards of semiconductor opto electronics. The primary goal in the application of quantum control is to fully exploit the quantum properties of quantum systems—e.g., atoms, molecules, or solids—, and to hereby bring the system into some highly non-classical state or to steer it through a sequence of desired states, the latter being a point of central importance for quantum computation applications. Quantum control is usually achieved by transferring the coherence from an external control, e.g., a laser pulse, to a *quantum coherence*, which allows for isolated systems to set deliberately the quantum mechanical state vector [6]. Isolated quantum systems are idealizations which cannot be realized since any system interacts with its environment. Through such environment couplings, the quantum system becomes entangled with the environment in an uncontrollable fashion,

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which leads to *decoherence*, a process within which the pureness of the quantum state and its controllability becomes degraded.

Within the field of *quantum control* one is seeking for control strategies that allow to steer the quantum state evolution despite the presence of decoherence. Efficient control strategies are known for simple systems, such as the celebrated stimulated Raman adiabatic passage control of a generic three-level system [7], which has found widespread applications in atomic systems. Although there are several proposals for related schemes in semiconductor nanostructures [8,9], it has become clear that, in many cases of interest, the description of a solid state system in terms of generic few-level systems is overly simplified and cannot account for the enhanced environment couplings in the solid state. A particularly interesting case is found in semiconductor quantum dots [10,11], often referred to as *artificial atoms*, where the optical excitation of an electron–hole pair in the state of lowest energy causes the deformation of the surrounding lattice but relaxation is completely inhibited because of the atomic-like carrier density of states. In coherent optical spectroscopy [1,2], which is sensitive to the optically induced coherence, this partial transfer of quantum coherence from the electron–hole state to the lattice degrees of freedom, i.e., phonons, results in *dephasing* [12–14]. It should, however, be noted that, contrary to other decoherence channels in solids where the system's wavefunction acquires an uncontrollable phase through environment coupling, in the independent Boson model the loss of phase coherence is due to the coupling of the electron–hole state to an ensemble of harmonic oscillators which all evolve with a coherent time evolution but different phase. This results in destructive interference and dephasing, and thus spoils the direct applicability of coherent carrier control. On the other hand, the coherent nature of the state-vector evolution suggests that more refined control strategies might allow to suppress dephasing losses. To address this problem, in this paper we examine phonon-assisted dephasing within the framework of *optimal control* [15–17] aiming at a most efficient control strategy to channel the system's wavefunction through a sequence of given states. We will find that appropriate tailoring of laser pulses allows to promote the system from the ground state through a sequence of excited states back to the ground state *without suffering significant dephasing losses*. Despite the widespread use of the independent Boson model, e.g., for the description of optical properties of localized states in solids or Mößbauer spectroscopy, to our best knowledge no such control strategy for suppression of environment losses has hitherto been reported in the literature.

2. Independent Boson model

In our theoretical approach we follow Refs. [13,14,18] and start with the usual independent Boson Hamiltonian. We describe the dot states in terms of a generic two-level system, with ground state 0 and excited state x, assuming a negligible contribution of excited exciton states due to the typically large energy splittings of several tens of meV [10] and of biexcitons, which, in optical experiments, can be achieved through appropriate polarization filtering. This two-level system is coupled to a reservoir of harmonic oscillators such that the interaction only occurs when the system is in the upper state [19]:

$$H = \sum_\lambda g_\lambda (a_\lambda + a_\lambda^\dagger)\,|x\rangle\langle x| + \sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda - \frac{1}{2}(\Omega|x\rangle\langle 0| + \Omega^*|0\rangle\langle x|). \quad (1)$$

Here, the bosonic degrees of freedom λ with energy ω_λ are described by the field operators a_λ and $a_\lambda^\dagger$, and g_λ is the coupling constant between x and λ. We describe the light-matter coupling within the usual dipole and rotating-wave approximations [20] with Ω the Rabi frequency, and consider a spherical dot model and acoustic deformation potential interactions as the only coupling mechanism. For the description of the time evolution of the two-level system in the presence of the phonon coupling (1), we adopt a density-matrix approach with $\boldsymbol{u} = \langle \boldsymbol{\sigma} \rangle$ the Bloch vector, $s_\lambda = \langle a_\lambda \rangle$ the coherent phonon amplitude, and $\boldsymbol{u}_\lambda = \langle \boldsymbol{\sigma}(a_\lambda - s_\lambda)\rangle$ the phonon-assisted density matrix as dynamic variables [8,18,21]. The resulting, somewhat lengthy, equations of motion for $\boldsymbol{u}$, s_λ, and $\boldsymbol{u}_\lambda$, as well as further details of our model can be found in Refs. [8,21].

While in the absence of phonon coupling $\Omega(t)$ would simply rotate $\boldsymbol{u}$ from $-\hat{\boldsymbol{e}}_3$ through a series of excited states back to $-\hat{\boldsymbol{e}}_3$, the phonon coupling of Eq. (1) entangles the two-level system with the lattice degrees of freedom and leads to dephasing. This can be seen in Fig. 1(b) which shows that after the action of the Gaussian 2π-pulse the Bloch vector remains in an excited state. From the inset of the figure we observe that the final deviation of $\boldsymbol{u}$ from the initial state $-\hat{\boldsymbol{e}}_3$ is not due to an incomplete rotation of $\boldsymbol{u}$ but to a loss of norm of $\|\boldsymbol{u}\|$, i.e., *the system has suffered dephasing losses*.

3. Optimal control theory

In the following we shall address the question whether such losses are inherent to the system under investigation or can be suppressed by more sophisticated control strategies. To this end, we employ the framework of *optimal control theory*, which has found widespread applications in engineering, economy, and medical sciences, but has received only little attention in the field of solid-state spectroscopy. It is a mathematical device that allows for a general determination of efficient control strategies [15–17]. Within the context of quantum optimal control, its key elements are the quantum system, described by the dynamic variables $x(t)$ (i.e., $\boldsymbol{u}$, s_λ, and $\boldsymbol{u}_\lambda$), the external control field $\Omega(t)$, and the *objective functional* $J(x, \varepsilon)$. The functional J expresses the objective of the control, which we quantify through

$$J(\boldsymbol{u}, \Omega) = \frac{1}{2}\left(\int_{-T}^{T} \mathrm{d}t\, \beta(t)\|\boldsymbol{u}(t) - \hat{\boldsymbol{e}}_3\|^2 + \|\boldsymbol{u}(T) + \hat{\boldsymbol{e}}_3\|^2 + \gamma \int_{-T}^{T} \mathrm{d}t\, \|\Omega(t)\|^2\right), \tag{2}$$

with β a Gaussian centered at time zero with a narrow full-width of half maximum, and γ a small constant [8,21]. In other words, we are seeking for solutions where $\boldsymbol{u}$ passes through the excited state $\hat{\boldsymbol{e}}_3$ at time zero and goes back to the ground state $-\hat{\boldsymbol{e}}_3$ at T. Optimal control theory accomplishes the task to determine a control which minimizes J, subject to the condition that x fulfills the dynamic equations describing the system propagation in the presence of the control and environment couplings. This is done by introducing Lagrange

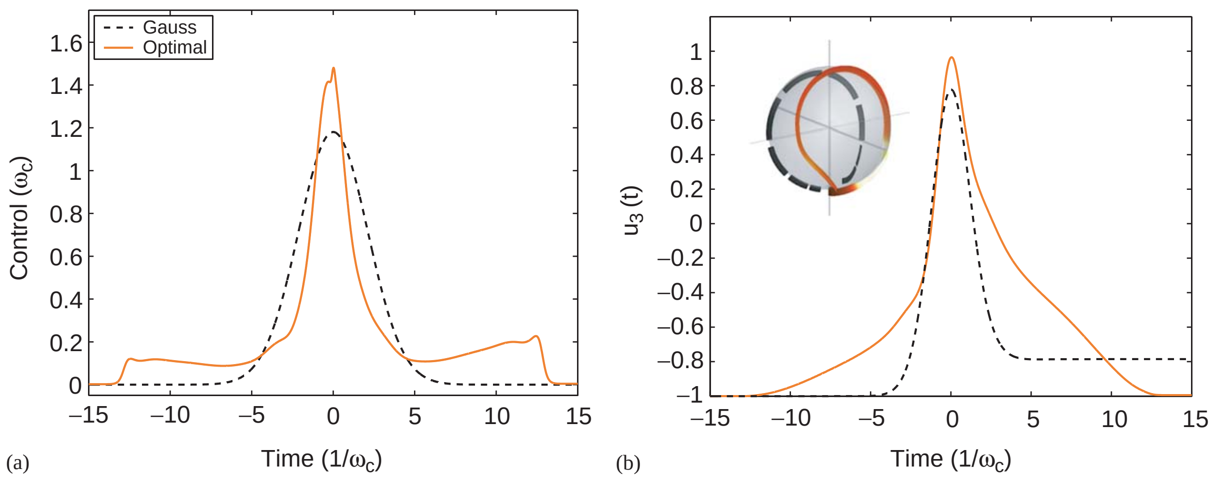

Fig. 1. Results of our calculations with a Gaussian 2π (dashed lines) and optimal-control (solid lines) laser pulse and for zero temperature and an electron–phonon coupling of $\alpha_p = 0.1$ [8]. Panel (a) shows $|\Omega(t)|$ and panel (b) the time evolution of $u_3(t)$, and the insets the trajectories of the Bloch vector $\boldsymbol{u}(t)$. For the Gaussian 2π-pulse Rabi flopping occurs but is damped due to electron–phonon interactions. For the optimal control decoherence losses are completely suppressed, and the system passes through the desired states of $\hat{\boldsymbol{e}}_3$ at time zero and $-\hat{\boldsymbol{e}}_3$ at T.

multipliers $\tilde{x}$ for the constraints, and turning the constrained minimization into an unconstrained one. Within this framework, we can derive a set of equations for x, $\tilde{x}$, and Ω, that has to be fulfilled simultaneously. In general, analytic solutions can be found only for highly simplified systems, whereas numerical calculation schemes have to be adopted for more realistic systems. A numerical algorithm for the solution of the optimality system was formulated in Ref. [17].

4. Results and conclusions

Results of our optimal-control calculations are shown in Fig. 1. Most remarkably, we can indeed obtain a control field for which $\boldsymbol{u}(t)$ passes through the desired states of $\hat{\boldsymbol{e}}_3$ at time zero and $-\hat{\boldsymbol{e}}_3$ at T. *Thus*, *appropriate pulse shaping allows to fully control the two-level system even in the presence of phonon couplings*. We emphasize that, with the exception of the somewhat pathological quantum "bang–bang" control [22] where the system is constantly flipped to suppress decoherence, no such simple control strategy for suppression of environment losses is known in the literature. Our surprising result is attributed to the fact that in the process of decoherence it takes some time for the system to become entangled with its environment. If during this entanglement buildup the system is acted upon by an appropriately designed control, it becomes possible to channel back quantum coherence from the environment to the system. Similar conclusion also hold for finite but low temperatures [21], which clearly highlights the strength and flexibility of optimal quantum control.

In conclusion, we have studied the phonon-induced dephasing dynamics in optically excited semiconductor quantum dots within the frameworks of the independent Boson model and optimal control. We have shown that appropriate tailoring of laser pulses allows to control the dot states without suffering significant dephasing losses, not only at the lowest but, though exceedingly difficult, also at elevated temperatures. The requirements for such laser-pulse shaping are well within the possibilities of presentday technology [15]. To highlight the applicability of quantum control, in this work we have focused on laser pulses with durations of a few picoseconds where the effects of dephasing losses are most pronounced. For other control objectives it might be advantageous to use shorter or longer laser pulses, for which control becomes substantially simplified, or to rely on more advanced control strategies. Besides their importance for future quantum-information processing applications, our findings might be also useful to address more fundamental questions regarding the detailed nature of decoherence in solids.

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