Preface to Special Issue on “Optimal Control of Nonlinear Differential Equations”

on the occasion of Fredi Tröltzsch’s 70th birthday

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Fredi Tröltzsch was born on November 30, 1951, in Dennheritz in the (then) German Democratic Republic. He grew up in Freiberg in the Saxonian "Erzgebirge", where he studied mathematics and received his PhD at the local Technical University, the *Bergakademie Freiberg*, in 1977. Five years later, he received his habilitation at the same university with a thesis titled *Necessary Optimality Conditions and Generalized Bang-Bang-Principles for Nonlinear Control Problems subject to Parabolic Differential Equations and Control and State Constraints.* Part of this work was carried out during a long stay at the Warsaw Technical University. After his habilitation, he moved to the Technical University Karl-Marx-Stadt (now Chemnitz) as an Associate Professor; where he was promoted to Full Professor in 1988. In 2000 he joined the Technical University Berlin as Full Professor of Numerical Partial Differential Equations and Nonlinear Optimization; a post he held until his retirement.

Starting from his PhD thesis, Fredi's work has been focused on optimal control problems for partial differential equations (PDEs), where he has made – and continues to make – significant analytical and numerical contributions, especially in the area of bang-bang principles (i.e., characterizing when an optimal control subject to control constraints only takes on the values of the bounds), existence and regularity results for Lagrange multipliers (as solutions of adjoint equations), second-order sufficient conditions for optimal controls under control and state constraints, as well as their use for showing stability properties, convergence of numerical methods, and finite element error estimates for approximations of solutions of nonlinear and nonconvex optimal control problems. Such a link of deep techniques from the analysis of PDEs, calculus of variations and infinite-dimensional optimization, and numerical methods is characteristic of Fredi's work. This is particularly evidenced in his seminal book *Optimal Control of Partial Differential Equations: Theory, Methods and Applications* (2010) which has influenced a whole generation of researchers in optimal control and has contributed significantly to this area becoming as active as it currently is. Besides abstract results, Fredi has been particularly interested in problems in fluid flow, crystal growth, heating and cooling, electromagnetic fields, and – more recently – pattern formation in biology. Often these problems arise from concrete industrial applications and are carried out in collaborations with companies interested in the results.

Fredi's influence is not only felt through his scientific output (zbMATH lists 160 publications at the time of writing, including four books) but also his strong engagement with the community. Outside his own network of co-authors (72 from all over the world) and his circle of students (19 according to the Math Genealogy Database, nine of which went on to become professors at research universities in Germany or abroad), he was visible through his leadership in the IFIP Technical Committee 7 "System Modeling and Optimization" of the International Federation for Information Processing, whose chair he served as from 2013 to 2018. As this body was founded in the Sixties to promote the interaction between mathematicians from countries east and west of the Iron Curtain, this is especially fitting given Fredi's work bridging East and West Germany as well as mathematical control and applications. Similarly influential was the series of Oberwolfach Workshops on optimal control of PDEs organized together with Irena Lasiecka, Günter Leugering, Karl Kunisch, and Jürgen Sprekels. Last but not least, he was a long-serving editor of the journal *Computational Optimization and Applications*, among others.

This special issue, prepared on the occasion of Fredi's 70th birthday, is comprised of six papers written by his colleagues, friends, and former students. These papers represent the work of thirteen international experts and researchers in the field of optimal control of nonlinear differential equations, a field in which Fredi has made seminal contributions. As befitting Fredi's broad interest, they span the whole gamut from analysis to numerical methods and concrete applications.

In [1] by P. Benner, J. Heiland, and S. Werner, output-feedback controls for stabilizing the solution of incompressible Navier–Stokes equations using low-dimensional $\mathcal{H}_\infty$-controllers are investigated. To deal with the challenging high dimensionality and differential-algebraic nature (due to the incompressibility constraint) of such problems, the authors derive a low-dimensional controller with numerically verifiable a priori robustness bounds. The performance and stability of this controller with respect to model and linearization errors is demonstrated in numerical examples.

Control of incompressible flows is also the topic of the paper [2] by E. Diehl, J. Haubner, M. Ulbrich, and S. Ulbrich. Here, differentiability results for the control-to-state mapping for incompressible two-phase Navier–Stokes equations are derived. Using L^p maximal regularity results for the corresponding linearizations, the Fréchet derivative of the control-to-state mapping is shown to exist and characterized as the solution of a suitable adjoint equation. These results are useful for obtaining numerically solvable optimality conditions for realistic flow control problems.

F. Hoppe and I. Neitzel in [3] consider model order reduction for quasilinear parabolic PDEs with non-monotone nonlinearities. Such PDEs appear in optimal control problems for heat conduction problems with temperature-dependent conductivities. Specifically, a posteriori error estimates for a reduced basis method with respect to the time discretization are derived that take into account errors arising from time discretization, empirical interpolation, and basis reduction. These estimates are validated in a numerical example.

The paper [4] by S. Kundu and K. Kunisch is concerned with the Hamilton–Jacobi–Bellman equation characterization of closed-loop feedback stabilization subject to control constraints. They demonstrate that these very high-dimensional PDEs can be solved algorithmically via so-called policy iteration by proving its convergence and illustrating it for judiciously chosen numerical examples.

In [5], C. Natemeyer and D. Wachsmuth consider a class of nonsmooth and nonconvex optimal control problems that cover problems with sparse or generalized bang-bang controls. Based on a Pontryagin Maximum Principle, a proximal gradient algorithm is derived and proved to produce iterates whose weak limit points satisfy a certain stationarity condition in a pointwise almost everywhere sense. The applicability of this algorithm is shown for model problems from the two above-mentioned problem classes.

Finally, the paper [6] by D. Wachsmuth studies optimal control problems with L^0 constraints (i.e., on the Lebesgue measure of the support of the control). Again, a Pontryagin Maximum Principle is established, which yields pointwise necessary optimality conditions and provides the basis for a proximal gradient algorithm whose convergence is established.

It is a pleasure to dedicate this issue to Fredi, to thank him for his scientific contributions and his personal friendship, and to wish him many more years of exciting mathematics and meetings with like-minded colleagues!

References

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