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Abstract

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JEL codes: G12, G11, C32.

Keywords: optimal portfolio selection, regime switching, Black–Litterman, risk measures.

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Portfolio Optimization: A Combined Regime-Switching and Black–Litterman Model

Abstract

Traditionally portfolios are optimized with the single-regime Markowitz model using the volatility as the risk measure and the historical return as the expected return. This study shows the effects that a regime-switching framework and alternative risk measures (modified value at risk and conditional value at risk) and return measures (CAPM estimates and Black–Litterman estimates) have on the asset allocation and on the absolute and relative performance of portfolios. It demonstrates that the combination of alternative risk and return measures within the regime-switching framework produces significantly better results in terms of performance and the modified Sharpe ratio. The usage of alternative risk and return measures is also shown to meet the need that asset returns very often are not distributed normally and serially correlated. To eliminate these empirical shortcomings of asset returns an unsmoothing algorithm is used in combination with the Cornish–Fisher expansion.

JEL: G12, G11, C32.
Keywords: Optimal Portfolio Selection, Regime Switching, Black–Litterman, Risk Measures

1 Introduction

How can we divide time series into good (bullish) and bad (bearish) markets? As early as 1989 Hamilton showed that economic numbers may follow a business cycle. With these numbers he described the business cycle in terms of up- and downswings. Ang and Bekaert (2002a) and (2002b) pointed out the advantages of dividing the whole time series into two regimes. The number of states in the regime-switching framework is three according to Guidolin and Timmermann (n.d.) and Graflund and Nilsson (2003), whereas Guidolin and Timmermann (2004) found that four states capture the joint distribution of returns the best. The fast-growing literature on regime-switching models (RSM) applied in empirical finance is documented by Guidolin (2011).

In recent years RSMs have been applied in portfolio optimization, extending the traditional single-regime model (SRM) of Markowitz with various risk measures other than volatility. Elliott and Miao (2009) and Guidolin and Timmermann (2004) combined the RSM with the value at risk (VaR) and conditional VaR (CVaR) and obtained significantly better performance. Bruder et al. (2011), Saunders et al. (2010) and Guidolin and Ria (2010) combined the Black–Litterman (BL) estimates with the RSM. Most studies focus on subcategories of only one asset class, like hedge funds, e.g. Saunders et al. (2010) and Bruder et al. (2011), or stocks, e.g. Zhao (2010) and Guidolin and Timmermann (n.d.).

Other problems with classical mean-variance optimization are the non-normality and the serial correlation in the asset class empirical return distributions. Empirical studies such as those by Levy (1969) and Samuelson (1970) showed that asset class returns are often skewed and fat tailed. As we know, Markowitz (1953) provided for his mean-variance framework the optimal solution only if the underlying sample is normally distributed and iid. Therefore we have to consider two main aspects: (1) we have to find a model that can handle non-normally distributed returns and (2) we have to eliminate the serial correlation. Firstly, for that reason and following Favre and Galeano (2002), it is necessary to take higher moments such as skewness and kurtosis into account. Instead of volatility, VaR and CVaR, the modified value at risk (mVaR) and modified conditional value at risk (mCVaR) with the expansion of Cornish–Fisher have the desired properties. Secondly, especially hedge funds and managed futures have high serial correlation in reported returns and this is taken as evidence of return smoothing (Okunev and White 2003). Such smoothing in asset returns underestimates the risk of an asset class. Therefore we have to determinate the ‘true’ returns to obtain the right estimation for the risk. Getmansky et al. (2004) presented an algorithm for unsmoothing such returns, in which the expected return stays the same, but the volatility rises to a ‘true’ risk level.

In our paper we combine several aspects discussed separately in the above-mentioned papers. We expand the SRM of the Markowitz portfolio optimization to an RSM with two regimes (bull and bear) in a broad investment universe (stocks, bonds, hedge funds, managed futures, commodities and real estate). We consider the possibility of skewness, kurtosis and serial correlations in the return data and apply besides the volatility two additional risk measures (mVaR and mCVaR) to the unsmoothed returns. Furthermore we investigate the optimal asset allocation not only for the historical returns but also for CAPM return estimates and BL return estimates. In contrast to Fischer and Lind-Braucher (2010) we not only combine various return and risk measures for an SRM, but also extend the model with the RSM to obtain the optimal portfolios empirically.

The structure of the paper is as follows. In Section 2 we describe the applied RSM, return and risk measures and unsmoothing algorithm for the serial correlated return data. Section 3 provides the algorithm used for the portfolio optimization. In Section 4 we examine the data used. In Section 5 we present the empirical results of the study and Section 6 concludes the paper.

2 Model

Following (2008) we suppose that a return time series until time $t = 1, 2, 3, \dots, t_0$ can be described by a first-order auto-regression process:

$$r_t = c_1 + \phi r_{t-1} + \varepsilon_t, \tag{1}$$

with $\epsilon_t \sim N(0, \sigma^2)$. At time t_0 a fundamental change occurs in the market such that from that time onwards the time series follows the dynamics

$$r_t = c + \phi r_{t-1} + \varepsilon_t. \tag{2}$$

For simplification the two equations can be combined:

$$r_t = c_{S_t} + \phi r_{t-1} + \varepsilon_t , \tag{3}$$

with $\varepsilon_t \sim N(0, \sigma^2_{S_t})$ and $c_{S_t} \in \{c_1, c_2\}$. The switching process from one regime to another can be modeled by a two-state Markov chain such that

$$P(S_t = j | S_{t-1} = i, S_{t-2} = k, \ldots, y_{t-1}, y_{t-2}, \ldots) = P\ (S_t = j | S_{t-1} = i) = p_{ij}. \tag{4}$$

Note that the regime at time t only depends on the regime at time $t-1$ and not on the whole history of the time series. Suppose that only y_t can be observed, while one can make an inference about S_t. This inference can be formulated based on (4) as

$$\xi_{j,t} = P\ (S_t = j | \Omega_t; \theta), \tag{5}$$

with $j = 1, 2$ and $\xi_{1,t} + \xi_{2,t} = 1$. Let $\Omega_t = \{y_t, y_t - 1, \ldots, y_1, y_0\}$ be the set of observations and let $\theta = (\sigma, \phi, c_1, c_2, p_{11}, p_{22})$ be the parameter vector of the model. The inference will be calculated iteratively as

$$\xi_{j,t-1} = P\ (S_{t-1} = j | \Omega_{t-1}; \theta), \tag{6}$$

for $j = 1, 2$. The main idea of the algorithm is that the conditional densities of the two regimes follow a normal distribution, i.e.

$$\eta_{jt} = f(y_t | S_t = j, \Omega_{t-1}; \theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y_t - c_j - \phi y_{t-1})^2}{2\sigma^2}\right), \tag{7}$$

for $j = 1, 2$. Using (6), the conditional density of the t-th observation is calculated by

$$f\left(y_t \middle| \Omega_t; \theta\right) = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{ij} \xi_{i,t-1} \eta_{jt} \tag{8}$$

to yield

$$\xi_{j,t} = \frac{\sum_{i=1}^{2} p_{ij} \xi_{i,t-1} \eta_{jt}}{f(y_t | \Omega_{t-1}; \theta)} P(S_t = j |\ \Omega_{t-1}; \theta). \tag{9}$$

Subsequent to the calculation of all the conditional densities for all the observations, θ can be calculated by maximizing the conditional log likelihood function

$$\log f(y_1, y_2, \ldots, y_T | y_0; \theta) = \sum_{t=1}^{T} \log f\ (y_t | \Omega_t; \theta). \tag{10}$$

Now the only additional requirement is the starting value for $\xi_{j,t}$. We apply the *matlab* packages provided by Perlin (2011) and, therefore, follow his method of using $\xi_{j,0} = \frac{1}{2}$ as a starting value.
The optimization problem for the minimum risk portfolio is then formulated by

$$\min Risk\ Measure_{S_t}(r_p) \tag{11}$$

subject to the two conditions for the portfolio weights w_i of the asset class i

$$\sum_{i}^{N_{S_t}} w_i = 1 \text{ and } w_i \geq 0 \text{ for all } i.$$

For the tangency portfolio we maximize the (modified) Sharpe ratio

$$max\ (modified)\ SR = \frac{\mu_{S_t}(r_p) - r}{Risk\ Measure_{S_t}(r_p)} \tag{12}$$

with

$$r = risk\ free\ return,$$

where the state-dependent expected portfolio return $\mu(r_p)$ and the state-dependent portfolio risk $Risk\ Measure_{s_t}(r_p)$ are calculated according to the chosen estimates for the return and risk measure. In the following we will show the formulas used.

2.1 Return and Risk Measures

• Historical mean. The historical mean of returns is calculated as follows:

$$\mu_{s_t}(r_p) = \sum_{j=1}^{N_{s_t}} E(r_j)\, x_j$$

• CAPM return estimates. To obtain the CAPM estimates we use the following formula:

$$\mu_{s_t}(r_p) = r + \ (Expected\ Market\ Return - r)\frac{Cov(Asset_{s_t}, Market_{s_t})}{Var\ (Market_{s_t})}.$$

• Black–Litterman return estimates. The BL will be the third and most widely used alternative return estimates. The idea of this approach is that we have one benchmark, like the market portfolio CAPM, as well as individual forecasts of the asset returns. The advantage of this approach is that we obtain more stable portfolio weights, which are also closer to the actual economy:

$$\mu_{s_t}(r_p) = \sum_j w_j^{views}\, E(r_j^{views}) + \sum_j w_j^{implicit}\, E(r_j^{implicit}) \tag{13}$$

with

$$\begin{aligned} w_{ij}^{views} &= \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} P'\Omega^{-1}P \\ w_{ij}^{implicit} &= \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1} (\tau\Omega)^{-1}P, \end{aligned}$$

where

$P = identity\ matrix,$
$\Omega = variance - covariance\ matrix\ of\ estimation\ error,$
$w_{ij}^{views} + w_{ij}^{implicit} = 1$ and
$\tau = \dfrac{1}{number\ of\ years\ in\ sample}.$

τ can be described as the trust in our return forecast (views). The higher τ, the more secure we are about our views.

• Volatility.

$$\sigma_{s_t}(r_p) = \sqrt{x'\sigma_{s_t}x}$$

with the regime-dependent variance–covariance matrix σ_{s_t} and the vector of portfolio weights x.

• Modified value at risk:

$$mVaR_{s_t}(r_p) = \mu_{s_t}(r_p) + z_{s_t,CF}\,\sigma_{s_t}(r_p)$$

with the Cornish–Fisher expansion

$$z_{s_t,CF} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S_{s_t} + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)K_{s_t} - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S_{s_t}^2,$$

where S_{s_t} and K_{s_t} stand for the portfolio's state-dependent skewness and kurtosis, repectively. z_α is the α-quantile of the standard normal function. Throughout our analysis we use α = 5%.

• Modified conditional value at risk:

$$mCVaR_{s_t}(r_p) = -\left[\mu_{s_t}(r_p) + \frac{\phi(z_{s_t,CF})}{N(z_{s_t,CF})}\sigma_{s_t}(r_p)\right],$$

where $\phi(.)$ is the standard normal density function and $N(.)$ the standard cumulative normal function.

2.2 Unsmoothing Algorithm

For the unsmoothing algorithm we follow the model of Okunev and White (2003), assuming that the reported return at time t ($r_{0,t}$) is a linear combination of the true return at time t, $r_{m,t}$, and the reported return $r_{0,[t-1,\dots,t-k]}$. The fund manager smooths the return in the following manner:

$$r_{0,t} = (1-\alpha)r_{m,t} + \sum_{i=1}^{m} \beta_i r_{0,t-i} \tag{14}$$

with $\sum_{i=1}^{m} \beta_i$ and m the number of adjustments of reported returns needed to obtain the true underlying return. This number has to be empirically determined by evaluating the serial correlation structure in the original smoothed return series. The methodology of Okunev and White, which is an extension of that of Geltner, allows the calculation to result in any desired level of serial correlation of any lag. We use the algorithm proposed by Geltner and modified by Okunev and White:

$$r_{m,t} = \frac{r_{m-1,t} - c_m r_{m-1,t-m}}{1-c_m} \tag{15}$$

with

$$a_{m,n} = \frac{\left(\alpha_{m-1,n}(1+c_m^2) - c_m(\alpha_{m-1,n-m} + \alpha_{m-1,n+m})\right)}{(1+c-2c_m\alpha_{m-1,n})} \tag{16}$$

and

$$c_m = \frac{(1+\alpha_{m-1,2m}) \pm \sqrt{(1+\alpha_{m-1,2m})^2 - 4\alpha_{m-1,m}^2}}{2\alpha_{m-1,m}}, \tag{17}$$

which requires

$$\alpha_{m-1,m}^2 \le \frac{(1+\alpha_{m-1,2m})^2}{4} \tag{18}$$

to obtain a real solution. $\alpha_{m,n}$ is the serial correlation of order n, after m adjustments.

3 Algorithm for the Portfolio Optimization

When applying the algorithm as provided by Perlin (2011), we fail to produce an estimate of the parameters via maximizing the likelihood function with five asset classes. This appears to be due to the large dimension of θ, as 5 equations and 3 explanatory variables over 80 parameters have to be estimated, such that the gradient descent optimization method, as implemented in the *fmincon* function of *matlab*, fails. For that reason and following Chen (2009) the choice of asset classes is restricted to stocks and bonds for the estimation of θ.
In the following a multivariate framework is chosen, with stock and bond returns as dependent variables. The focus is that regimes are driven mostly by stock and bond returns and therefore, after the estimation of the regimes, the model can easily be extended to five asset classes, as performed in Step 4 of the algorithm.
We simply model the state-dependent returns of stocks and bonds for each state $s_t = 1, 2$

$$r_{Stock,s_t} = \beta_{Stock,s_t} + \epsilon_{Stock,t} \text{ with } \epsilon_{Stock,t} \sim N(0, \sigma_{Stock,s_t}^2) \tag{19}$$

$$r_{Bond,s_t} = \beta_{Bond,s_t} + \epsilon_{Bond,t} \text{ with } \epsilon_{Bond,t} \sim N(0, \sigma_{Bond,s_t}^2).$$

To apply the portfolio optimization models, estimates of the returns and the risk measures of the asset classes are required. The risk and return measures are calculated as described in subsection 2.1. The following algorithm calculates the optimal portfolios conditional upon the regimes and the chosen return and risk measures:

- Step 1: Calculate θ by using the model as in (19) up to time t.
- Step 2: Take each ξ_{ik} for $k = 1, \ldots, t$ calculated by (9) and compare it with 0.5.
- Step 3: This divides the whole in-sample period into two parts, one per regime.
- Step 4: Expand the two asset classes used for the estimation of the regimes to five, adding the returns of hedge funds, real estate and commodities to the data set.
- Step 5: Estimate μ_i and Σ_i and calculate the optimal portfolio weights dependent on the regime.
- Step 6: Calculate the expected portfolio return by the product of the asset returns at time $t + 1$, times the unconditioned portfolio weights at time t.
- Step 7: Set $t = t + 1$.

4 Data

For the empirical investigation the data set contains time series with monthly data from October 1996 to May 2012. The investment universe consists of the major five asset classes. The following indices are considered to describe the returns of the different asset classes:

- MSCI World (stocks),
- JPM GLOBAL GOVT.BND (bonds),
- DJ CS HEDGE FUND (hedge funds),
- DJ CS HEDGE MANAGED FUT (managed futures),
- S&P GSCI Commodity Total Return (commodities) and the
- FTSE EPRA/NAREIT DEVELOPED (real estate),

with 187 price observations each. All the data were obtained from *Thomson Reuters Datastream*. For each index the rate of return is calculated as follows:

$$r_{it} = ln\left(\frac{p_{it}}{p_{it-1}}\right),$$

where p_{it} states the price of index i at time t.

The developments of the standardized indexes of our asset classes are shown in figure 1. For our empirical investigation we divide our data set into an in-sample period from October 1996 to September 2008 to obtain the required historical parameter estimates and an out-of-sample period from October 2008 to May 2012 to obtain monthly optimal portfolios and to calculate the resulting portfolio values, returns, risks and performances.

5 Empirical Results

5.1 Single-Regime and Regime-Switching Models for the In-Sample Period October 1996 to September 2008

For the regime estimation based on the algorithm of Perlin (2011) the MSCI WORLD and JPM GLOBAL GOVT.BND indices are selected as regime-dependent variables. In table 1 the empirical values of the regression for October 2008 are provided using the returns from the in-sample period. In figure 2 the standardized stock index in the in- and out-of-sample periods as well as the two regimes are mentioned. The background colors gray and white mark the underlying bull (regime 2) and bear (regime 1) regimes. In table 6 the regimes and scenarios in the in-sample period are counted and their resulting unconditional probabilities are shown. Thus we have the bear regime in 42 months of 144 and the bull regime in 102 months. This leads to the fact that with a probability of 30% we will be in the bear regime and with 70% we will be in the bull regime. The conditional and stationary probabilities for October 2008 can be seen in table 7. The left side is simply the solution of table 6, whereas on the right sight the values of the maximum likelihood estimation provided by the *matlab* program can be seen. In the first case we calculate the unconditional probabilities and in the second case we calculate the conditional probabilities. Table 7 also shows the resulting expected durations for the two regimes.

5.2 Data Analysis and Historical Return and Risk Estimates for the In-Sample Period October 1996 to September 2008

Table 2 shows the main statistics for the monthly return data of our asset classes. We can see that the p-value of the Jarque–Bera test of stocks, hedge funds and real estate is under 0.05, therefore the null hypothesis can be rejected. The analyzed data are therefore not normally distributed and higher moments should be taken into account. At the p-value of the Ljung–Box test at lag 1 significant serial correlation could be found. After the unsmoothing algorithm it can be seen that the volatility of hedge funds rises from 7.36 for the original return data (sd. orig. p.a.) to 9.12 for the unsmoothed return data (sd. us. p.a.). Thus it can be concluded that the unsmoothed data underestimate the risk in terms of volatility for the hedge fund class.

For the SRM we show the annualized mean returns and risk measures in the first column of table 3 and the correlation coefficients in table 4. For the RSM table 5 shows the annualized mean returns and risk measures for the two regimes and for the four scenarios, whereas table 5 informs us about the correlation coefficients for the two regimes (bear markets in the lower triangular matrix and bull markets in the upper one). Table 3 shows the enormous differences in the historical means and risk measures for all the asset classes between the two regimes. In bear markets only bonds, managed futures and commodities have a positive average return. The smallest average bear return can be observed for commodities with extremely high risk levels. In bull markets all the asset classes have positive average returns, with the highest value for stocks. For both regimes only bonds, managed futures and commodities show positive mean historical returns and remarkably higher values for the bear markets. Due to the higher risk measures for almost all the asset classes in the bear regime we can interpret this regime as the riskier regime. From table 4 it can be seen that stocks and bonds are negatively correlated. In table 5 the correlation between these two classes differs between the two regimes. In the bear regime the correlation is much more negative than in the bull regime. This is what we expected. However significant differences can be seen in the correlations of stocks with hedge funds and managed futures, respectively. In the bear regime we have almost no correlation between stocks and hedge funds and a high negative correlation of -0.27 between stocks and managed futures. In contrast, in the bull regime we have a high positive correlation of stocks of 0.28 (with hedge funds) and almost no correlation with managed futures.

5.3 Optimal Portfolios for the Beginning of the Out-of-Sample Period October 2008 to May 2012

The risk and return measures are estimated as explained in subsection 2.1 for the SRM and for the RSM conditioned on the two regimes. Subsequently (11) is optimized for the minimum risk portfolio. For the optimal tangency portfolio we maximize the modified Sharpe ratio as shown in equation (12). The calculated weights for the asset classes are taken as the optimal portfolios for the next month. Every month we calculate the new optimal portfolios in forecast for the next month. However first we have to show the results for our two alternative return estimates.

5.3.1 CAPM and Black–Litterman Return Estimates for October 2008

To calculate the expected asset returns from the CAPM the market portfolio has to be calculated. The data for the market capitalization, CAPM returns and βs are shown in table 9. The values of the market capitalization are taken from *Thomson Datastream*. The amount of commodities is calculated following Idzorek (2004). In order to obtain the desired excess stock returns in table 8, we calculate the stock market return minus the risk-free rate. In order to determine the expected market return, we calibrate the model under the condition that the assumed risk premium for the stock market is equal to the derived excess stock return.

$$ESR_{SRM} = \mu_{stocks,SRM} - r$$

For the two regimes we calculate:

$$ESR_{RS1} = \mu_{stocks,RS1} - r$$

$$ESR_{RS2} = (ESR_{SRM} - ESR_{RS1} * SP_{RS1})/SP_{RS2},$$

where $ESR = Excess\ Stock\ Return$ and $SP = Stationary\ Probability$. For the riskless rate we take the three months' US T-Bill with $r = 0.84\%\ p.a.$
Along these lines the excess stock returns will be calculated in pairs for scenario 11/21 and scenario 12/22. The resulting excess stock returns are shown in table 8 and the values of the stationary probabilities in table 7. To obtain the BL return estimates of (13) for τ = 1/12 in table 11 we take the CAPM return estimates as our equilibrium returns and, for the sake of simplicity,[1] the historical means of the periods starting twelve months prior to the portfolio decisions as views (see table 10). As for the historical returns the CAPM and BL return estimates for the asset classes bonds and managed futures are never negative in both regimes. Furthermore we find it notable that the only negative β value can be observed for managed futures for bear markets. It can be seen from tables 9 and 11 that both for the SRM and for the RSM the alternative return estimates for some asset classes significantly differ from their historical means. This will result in quite different optimal portfolios that will be discussed in the following subsections.

5.3.2 Analysis of the Minimum Risk Portfolios

Table 12 shows the optimal weights for all the asset classes and the expected returns and risk measures for the minimum risk portfolios for our various return estimates (rows) and risk measures (columns). For each risk estimate we present the optimal portfolio weights and portfolio return and risk measures for the SRM and for the RSM for the bear and bull regime and for all four scenarios i, j. The first number of the scenarios indicates the state of the market in the past month and the second number indicates the possible state of the market in the forthcoming month. Therefore scenarios $1,1$ and $2,2$ stand for two consecutive bear and bull periods, respectively, and $1,2$ and $2,1$ mean, in general, that commodities never play a significant role in the minimum risk portfolios and real estate only appears in the optimal solutions of scenario 2,1 (from bull to bear market). In the following we will interpret our results.

• The Impact of Return Estimates:

Volatility as the risk measure. As expected we see that the minimum variance portfolio (MVP) is independent of the expected returns and therefore also of the return estimates. Therefore the portfolio weights and volatility are all the same. The expected portfolio returns and the other risk measures are different because of the different return estimates. Therefore the portfolio for the SRM consists of about 12% stocks, 50% bonds, 23% hedge funds and 12% managed futures. In the two different regimes of the RSM the asset allocation does not differ very much from the SRMs' allocation as well as the optimal bear and bull portfolios: the bear regime (regime 1) has a higher amount of stocks and bonds, whereas the bull regime shows a higher amount of hedge funds and a lower amount of bonds. However the differences are not that great and therefore we obtain more or less the same values of volatility for the portfolios at an amount of about 1.3% per month.
mVaR as the risk measure. Looking at the historical mean and at the CAPM estimates as return measures it can be seen that the SRM does not differ very much from the solutions of the MVP. However for BL returns the portfolios of the SRM have a quite different profile in the amount of stocks and bonds, compared with the asset allocations of the two other return estimates. For the RSM the optimal asset allocation drastically depends on the assumed regime and differs greatly from the SRM solution: in the bear regime the amount of bonds increases up to about 65 or 70% and the weights of the managed futures also increase significantly (both depending on the applied return measure). Consequently the optimal weights for stocks are close to zero and for hedge funds they are much lower than in the SRM solution. In the bull regime stocks have an optimal share of about 20% and bonds about 25%. The optimal amounts of hedge funds and managed futures strongly depend on the applied return estimates and differ substantially from the solution of the SRM and of the bear regime. As expected, in bull markets the weights of more speculative products, such as stocks and hedge funds, are much higher than in bear markets, but the speculative managed futures have more weight in bear markets.

[1] We would like to point out that the investor's views are very simply modeled in our application of the BL return estimates just to demonstrate how they influence the optimal portfolios. However an enormous amount of literature exists on how to obtain much more realistic return forecasts from experts, but this is beyond the scope of our paper.

mCVaR as the risk measure. For the SRM the portfolios are almost the same as for the MVP, but they are very different for the two regimes of the RSM. For both types of models the effect of the return estimates is only marginal. The two regimes differ mostly in the four asset classes stocks, bonds and hedge funds: in bear regimes high amounts in bonds (more than 60%), less than 20% in hedge funds and managed futures and the rest in stocks; in bull regimes about 30% in bonds and hedge funds, less than 20% in stocks and about the rest in managed futures.

• The Impact of Risk Measures:

Historical return estimates. In the SRM the portfolio weights for the different risk measures are almost the same. Differences in the optimal portfolios can only be found in the RSM. In the bear regime the optimal amount of stocks is close to zero for mVaR and mCVaR and about 12% for volatility. The largest differences can be found for the amount of bonds from 52.90% up to 72.57%. Here it can be seen that mVaR and mCVaR favor bonds. We have the same differences as in the bull regime for hedge funds with a bandwidth from 26.79% for volatility up to 45.78% for mVaR.
CAPM return estimates. Again for the SRM it can be seen that there are no big differences. The differences are in the bear, and much bigger differences in the bull regime. In the bear regime the amount of bonds differs from 52.9% for volatility up to 64.3% for mVaR, whereas in the bull regime bonds have an optimal share of 27.4% for mVaR and 46.7% for volatility.
Black–Litterman return estimates. Here even for the SRM the optimal shares of stocks depend on the applied risk measure and are 1.4% for mVaR, 4.5% for mCVaR and 12% for volatility, resulting in slightly different amounts for the other asset classes. A similar impact of the risk measures can be seen for the two regimes of the RSM. For the volatility the bear (bull) market share of stocks is 13.66% (11.68%), whereas for the two other risk measures the weights of stocks are very different for the two regimes.

5.3.3 Analysis of the Tangency Portfolios

Table 13 shows the optimal weights for all the asset classes and the expected returns and risk measures for the tangency portfolios for our various return estimates (rows) and risk measures (columns).

• The Impact of Return Estimates:

Volatility as the risk measure. In contrast to the results for the minimum risk portfolios now the return estimates have a major influence on the optimal portfolio weights, even for the SRM. For example, for the SRM the amount of stocks lies between 0.00% for the BL estimates and almost 42% for the CAPM estimates. For the historical mean the SRM portfolio consists of all the asset classes except real estate. The reasons for that are the low return of about 0.31% per month and the relatively high standard deviation of about 4.36% per month (see table 2). Looking at the CAPM case, the SRM portfolio consists of all the asset classes except managed futures, whereas for the BL return the SRM only has bonds, managed futures and commodities. However the really interesting results are obtained for the RSM: due to the excellent return and risk values in the bear regime our portfolio only consists of bonds and managed futures for the historical means (75% and 25%) and for the BL returns (30% and 70%), and, additionally due to the negative β value, suggests investing all in managed futures for CAPM returns. For the bull regime the portfolios are much more diversified, containing at least four of the major asset classes, with weights depending on the return estimates.
mVaR as the risk measure. Again, the return estimates play an important role in the optimal solutions in both types of models. In the SRM we have well-diversified portfolios for the historical mean and for the CAPM returns, but with different weights, but for the BL returns the optimal investments are only in bonds (more than 60%), managed futures (30%) and the rest in commodities. For the RSM we see extreme solutions in the bear regime: only bonds (75%) and managed futures (25%) for historical means, full investment in managed futures for CAPM returns and only bonds and managed futures (about 40% and 60%) for BL returns. In the bull regime the solutions for the CAPM and BL returns are quite similar, investing in all asset classes except real estate, but for the historical means the optimal portfolio only consists of stocks (less more than 30%), bonds (less more than 60%) and the rest of commodities.
mCVaR as the risk measure. As in the previous two risk estimates the role of the return estimates is very important for both the SRM and the RSM. In the SRM for the historical means and for the CAPM returns all the asset classes except real estate appear in the optimal portfolios, but with quite different weights. For BL estimates the optimal portfolio only consists of bonds (about 60%), stocks

(less more than 30%) and the remainder commodities. For the bear regime the results are again very extreme and similar to the weights for mVaR as the risk measure. For the bull regime the portfolios are more diversified, with similar weights for CAPM and BL returns but different weights for historical means.

• The Impact of Risk Measures:

Historical return estimates. For the SRM and for the bear regime of the RSM the optimal portfolios are very similar for all the risk measures. However for the bull regime the impact of the risk measure has to be considered: all the portfolios consist of a little more than 30% of stocks, but they show quite different weights for bonds (between 0 and 14%), hedge funds (between 45 and 63%) and commodities. Neither managed futures nor real estate appear in the optimal bull portfolios.
CAPM return estimates. In contrast to the historical mean as a return measure we now have different portfolio weights even for the SRM. For our risk measures we obtain different portfolios with the major portfolio weights in stocks, bonds and hedge funds. In the RSM it is interesting that the portfolios for the bear and the bull market are more or less the same for the different risk measures. Interestingly optimal portfolios for the bear regime are always 100% managed futures. It can be said that the RSM compensates for the non-normality of the asset returns and therefore there is no need for risk measures with higher moments.
Black–Litterman return estimates. For the SRM the optimal portfolios only consist of bonds, managed futures and commodities (with different but similar weights), independent of the risk measure. In the RSM the portfolios in the bear regime consist of bonds and managed futures. In the bull regime we have four major asset classes (stocks, bonds, hedge funds and managed futures) and very small investments in commodities, where the weights depend on the risk measures.

5.4 Out-of-Sample Analysis: October 2008 to May 2012

In figure 3 the different efficient frontiers for the SRM and the RSM with the volatility as our risk measure can be seen for the beginning of our out-of-sample period. At a fixed level of volatility, let us say 5% p.a., the RSM generates a higher expected return. Figure 4 shows the corresponding efficient frontiers for the SRM for the complete out-of-sample period. At the beginning and the end of the in- and out-of-sample periods the expected return is much higher.
For the out-of-sample period we optimize our portfolios for the beginning of every month with updated estimates of all our variations for the expected returns and risk measures. The corresponding optimal weights for the asset classes over all the months of this period are exemplarily shown for the minimum risk portfolios in figure 7 for the SRM and in figure 8 for the RSM and for the tangency portfolios in figures 9 (SRM) and 10 (RSM). Then we calculate the mean returns, the risk measures and the performance for all the portfolios for the whole out-of-sample period (table 14) and determine the value of the portfolios at the end of each month (figures 5 and 6 for mVaR as the risk measure) at the end of the out-of-sample period and the assumption that the initial investment was 100 USD (table 15). The resulting portfolios can be ranked according to their absolute performance, which is the portfolio returns or, equivalently, the portfolio value at the end of the period, or according to their relative performance, which is the (modified) Sharpe ratio corresponding to the applied risk measure.

5.4.1 Minimum Risk Portfolio: October 2008 to May 2012

In the SRM the combination *BL estimates* and *mVaR* has the highest end-of-period value of 121.49 and the best relative performance for all three kinds of (modified) Sharpe ratios. In the RSM the highest portfolio returns and the best performance for all three kinds of (modified) Sharpe ratios are attained by the return and risk measure combination *historical mean* and *mVaR* (table 14), which results in a total end-of-period value of 130.04 (table 15). In figure 5 we can see that these model-specific best portfolios always dominate the others for the same type of model. Therefore the end of the investigated period does not affect the choice of these best portfolios. The main result, however, is that the results of the RSM are always better than the results of the corresponding SRM.
In figures 7 and 8 the differences between the SRM and the RSM in terms of portfolio weights can be seen. Figure 8 shows the regime-switching process in which stocks appear at a level of about 20%. This switching process is congruent with figure 5, which shows the two regimes as the background colors white and gray. Regime 1 (bear) consists only of three asset classes: bonds, hedge funds and managed futures. The second regime (bull) additionally consists of stocks and commodities. This leads us to the fact that the first regime will be the riskier one and an investor should be more

conservative, as evidenced by the amount of bonds, and the second regime will be the bullish one, in which riskier assets, like commodities and stocks, can be added.

5.4.2 Tangency Portfolio: October 2008 to May 2012

From figure 6 we can see that for the tangency portfolio we do not have such a straight end-of-period-independent dominance of portfolios as previously. For the SRM the return and risk measure combination *historical mean* and *volatility* has the highest end-of-period value of 118.97. For the RSM, however, the highest end-of-period value of 135.19 can be obtained for the return and risk measure combination *CAPM estimates* and *mCVaR*. The main result, however, is that the results of the RSM are again always better than the results of the corresponding SRM. If we choose the portfolios with the highest relative performance (for volatility the Sharpe ratio, for mVaR the SRmVaR and for mCVaR the SRmCVaR), we always obtain the SRM and RSM, respectively, with *historical mean* as the optimal portfolio.

In figure 9 we can see no abrupt changes in portfolio weights over time for the SRM, whereas figure 10 shows that the portfolio weights in the RSM change according to their regimes. The riskier period, which is the bear regime, has full investment in managed futures. The bullish market has a large amount of hedge funds, bonds and stocks (about 25% each at the beginning of the out-of-sample period and then changing over time), 12% managed futures and 8% commodities, which shows that if times go well, then riskier assets can be added to the portfolio.

6 Conclusion

This paper is concerned with several extensions of the traditional single-regime Markowitz mean-variance portfolio optimization to a regime-switching framework. We use a two-state regime-switching model with bull and bear markets, three different risk measures (volatility, modified value at risk and modified conditional value at risk), three different return estimates (historical, CAPM and Black–Litterman) and adjust our return data for non-normality and serial correlation. We estimate our parameters for an in-sample period and calculate the minimum risk portfolios and the tangency portfolios for an out-of-sample period.

Our results show that the large differences in portfolio performance are not between the different risk and return measures, but between the single and the regime-switching model. Overall the best results for the minimum risk portfolio as well as for the tangency portfolio are achieved by the regime-switching model. The difference in the average monthly performance for the minimum risk portfolio between the best (regime-switching model, historical mean, mVaR) and the worst (single-regime model, historical mean, mVaR) is about 0.2% and for the tangency portfolio between the best (regime-switching model, CAPM return, mCVaR) and the worst (single-regime model, CAPM return, volatility) portfolio about 0.7%. This leads us to the result that the non-normality in asset returns is better fitted by a regime-switching model than by different risk measures.

Further research could be addressed to the optimal number of states and months in the regime-switching model and the embedding of transaction costs, which may play an important role in the regime-switching model, when we have major changes in our portfolio weights in times of regime changes, whereas in the single-regime model we usually only have small adjustments in the optimal asset allocations over time.

	RSM	
	1 = Bear	2 = Bull
β Stock	-0.0208	0.0172
β Bond	0.0101	0.0018
σ Stock	0.0519	0.0269
σ Bond	0.0199	0.0186
ρ Stock,Bond	-0.0230	-0.0200

Table 1: State-dependent β, σ and ρ Values from Regressions

	Stocks	Bonds	Hedge.F	Man.F	Comm	Real.E
mean p.m.	0.42	0.46	0.76	0.60	0.57	0.31
mean p.a.	5.09	5.58	9.14	7.21	6.89	3.70
volatility orig p.m.	4.16	1.95	2.12	3.45	6.40	4.44
volatility orig p.a.	14.41	6.76	7.36	11.96	22.18	15.37
volatility us p.m.	4.25	1.89	2.63	3.06	6.75	4.36
volatility us p.a.	14.74	6.54	9.12	10.61	23.38	15.11
min p.m.	-14.84	-4.76	-7.85	-9.02	-14.99	-14.03
max p.m.	8.66	6.23	8.18	9.49	14.66	11.53
skewness p.m.	-0.86	0.10	-0.23	-0.01	-0.14	-0.79
kurtosis p.m.	3.85	3.30	6.47	2.75	2.63	3.94
JB chi-squared	23.06	0.85	73.80	0.39	1.34	21.49
p-value JB	0.00	0.67	0.00	0.83	0.51	0.00
normally distributed	no	yes	no	yes	yes	no
LB chi-squared	0.74	1.92	3.11	0.56	1.08	4.01
p-value LB	0.39	0.17	0.09	0.51	0.30	0.05
autocorrelated	yes	yes	yes	yes	yes	yes

Table 2: Data Analysis: October 1996 to September 2008

Asset Classes	Portfolio Return and Risk Measures in % p.m.	SRM	RSM					
			Regimes		Scenarios			
			1	2	1,1	1,2	2,1	2,2
Stocks	Return	0.42	-2.87	1.83	-2.49	3.20	-5.21	1.76
	Volatility	4.16	4.80	3.59	4.96	7.67	3.14	3.11
	mVaR	-7.01	-12.32	-2.70	-12.11	-6.07	-11.58	-2.18
	mCVaR	-8.67	-14.13	-4.42	-14.00	-9.79	-12.74	-3.66
Bonds	Return	0.46	1.22	0.14	1.12	0.53	1.85	0.12
	Volatility	1.89	1.88	1.88	1.92	1.37	1.71	1.86
	mVaR	-2.20	-1.18	-2.77	-1.31	-1.46	-0.38	-2.76
	mCVaR	-3.06	-2.07	-3.58	-2.23	-2.07	-1.18	-3.56
Hedge Funds	Return	0.76	-0.20	1.17	-0.07	-0.90	-1.01	1.28
	Volatility	2.60	2.69	2.43	2.26	2.47	4.65	2.38
	mVaR	-2.81	-5.55	-1.52	-4.28	-6.06	-10.72	-1.22
	mCVaR	-4.00	-6.56	-2.74	-5.16	-6.96	-12.41	-2.45
Managed Futures	Return	0.60	1.67	0.15	2.02	-1.43	-0.50	0.23
	Volatility	3.08	3.50	2.88	3.55	2.66	2.92	2.77
	mVaR	-4.03	-3.27	-4.64	-2.75	-6.63	-5.11	-4.27
	mCVaR	-5.38	-4.85	-5.84	-4.40	-7.64	-6.36	-5.44
Commodities	Return	0.57	0.93	0.42	0.83	-7.22	1.61	0.82
	Volatility	6.57	7.01	6.41	7.27	2.83	4.90	6.54
	mVaR	-10.06	-10.58	-9.86	-11.26	-12.95	-3.88	-9.27
	mCVaR	-12.83	-13.51	-12.57	-14.28	-14.00	-6.34	-12.10
Real Estate	Return	0.31	-1.84	1.22	-1.79	2.26	-2.13	1.17
	Volatility	4.20	4.03	4.16	4.16	5.85	3.43	4.04
	mVaR	-7.21	-9.9	-5.23	-10.18	-4.08	-8.69	-5.39
	mCVaR	-8.88	-11.40	-7.02	-11.73	-7.06	-10.00	-7.09

Table 3: Historical Return and Risk Estimates in % p.m.: October 1996 to September 2008

	Stocks	Bonds	Hedge.F	Man.F	Comm	Real.E
Stocks	1.00	-0.20	0.26	-0.10	0.03	0.59
Bonds	-0.20	1.00	-0.05	0.28	0.09	0.15
Hedge.F	0.26	-0.05	1.00	-0.02	0.01	0.23
Man.F	-0.10	0.28	-0.02	1.00	0.23	0.08
Comm	0.03	0.09	0.01	0.23	1.00	0.03
Real.E	0.59	0.15	0.23	0.08	0.03	1.00

Table 4: Correlation Matrix of the Single-Regime Model

	Stocks	Bonds	Hedge.F	Man.F	Comm	Real.E
Stocks	1.00	-0.14	0.28	0.04	-0.01	0.60
Bonds	-0.24	1.00	0.01	0.31	0.10	0.22
Hedge.F	0.06	-0.07	1.00	-0.06	-0.1	0.24
Man.F	-0.27	0.20	0.10	1.00	0.18	0.19
Comm	0.15	0.07	0.28	0.32	1.00	0.06
Real.E	0.53	0.07	0.07	-0.10	0.01	1.00

Table 5: Correlation Matrix of the Regime-Switching Model for Regime 1 (Lower Triangle) and Regime 2 (Upper Triangle)

	Counts of Scenarios and Regimes			Resulting Descriptive Unconditional Probabilities		
	1	2	∑	1	2	∑
1	37	5	42	0.26	0.04	0.30
2	6	96	102	0.03	0.67	0.70
∑	43	101	144	0.30	0.71	1.00

Table 6: Counts and Probabilities of Scenarios and Regimes

	Calculated by Counts				Calculated by Maximum Likelihood			
	1	2	Stationary Probabilities	Expected Duration in Months	1	2	Stationary Probabilities	Expected Duration in Months
1	0.86	0.14	0.26	7.17	0.87	0.13	0.37	7.56
2	0.05	0.95	0.74	20.20	0.08	0.92	0.63	13.15

Table 7: Transition Probabilities, Stationary Probabilities and Expected Duration for the In-Sample Period

Returns in % p.a.	SRM	RSM					
		Regimes		Scenarios			
		1	2	1,1	1,2	2,1	2,2
Excess Stock Return	4.3	-35.3	21.1	-30.8	37.6	-63.3	20.2
Market Return	3.9	-24.7	21.7	-21.7	39.7	-45.0	21.5

Table 8: Excess Stock and Market Returns in % p.a.

Asset Classes	SRM	RSM						Market Cap in Bn USD	in %
		Regimes		Scenarios					
		1	2	1,1	1,2	2,1	2,2		
Stocks	0.42	-2.87	2.10	-2.49	3.47	-5.21	2.08	24,666.73	52.61
	1.41	*1.38*	*1.19*	*1.36*	*1.05*	*1.38*	*1.20*		
Bonds	0.08	0.00	0.75	-0.04	1.12	-0.06	0.79	12,999.94	27.73
	0.04	*0.03*	*0.28*	*0.06*	*0.26*	*0.03*	*0.28*		
Hedge Funds	0.20	-0.85	1.16	-0.58	-1.65	-1.59	1.28	1,170.40	2.50
	0.50	*0.43*	*0.56*	*0.35*	*-0.67*	*0.43*	*0.63*		
Managed Futures	0.08	0.44	1.27	0.50	4.63	0.74	1.21	132.19	0.28
	0.04	*-0.17*	*0.63*	*-0.23*	*1.45*	*-0.17*	*0.58*		
Commodities	0.30	-2.27	2.09	-2.21	5.09	-4.14	2.01	7,869.35	16.78
	0.90	*1.10*	*1.19*	*1.21*	*1.60*	*1.10*	*1.16*		
Real Estate	0.32	-1.91	1.74	-1.63	5.57	-3.48	1.66	46.06	0.10
	0.99	*0.93*	*0.96*	*0.91*	*1.76*	*0.93*	*0.91*		

Table 9: CAPM Return and *Beta* Estimates for October 2008 in % p.m.

Asset Classes	Portfolio Return and Risk Measures	SRM	RSM					
			Regimes		Scenarios			
			1	2	1,1	1,2	2,1	2,2
Stocks	Return	-2.42	-2.42	1.75	-2.49	3.20	-5.21	1.76
	Volatility	0.17	0.23	0.13	0.25	0.59	0.10	0.10
Bonds	Return	0.52	0.52	0.37	1.12	0.53	1.85	0.12
	Volatility	0.04	0.04	0.04	0.04	0.02	0.03	0.03
Hedge Funds	Return	-0.67	-0.67	1.12	-0.07	-0.90	-1.01	1.28
	Volatility	0.07	0.07	0.06	0.05	0.06	0.22	0.06
Managed Futures	Return	0.82	0.82	0.88	2.02	-1.43	-0.50	0.23
	Volatility	0.09	0.12	0.08	0.13	0.07	0.09	0.08
Commodities	Return	0.96	0.96	-1.62	0.83	-7.22	1.61	0.82
	Volatility	0.43	0.49	0.41	0.53	0.08	0.24	0.43
Real Estate	Return	-3.49	-3.49	1.77	-1.79	2.26	-2.13	1.17
	Volatility	0.18	0.16	0.17	0.17	0.34	0.12	0.16

Table 10: Views as Historical Return Estimates for Twelve Months Prior to October 2008 in % p.m.

Asset Classes	SRM	RSM					
		Regimes		Scenarios			
		1	2	1,1	1,2	2,1	2,2
Stocks	-1.59	-2.95	2.03	-2.71	7.87	-4.83	1.92
Bonds	0.28	0.20	0.40	0.51	1.82	0.88	0.25
Hedge Funds	-0.47	-0.74	1.00	-0.34	-2.13	0.18	1.07
Managed Futures	0.47	0.77	0.84	1.51	2.82	-0.57	0.48
Commodities	0.65	-0.56	0.07	-0.40	2.09	-1.60	1.12
Real Estate	-1.85	-2.59	1.48	-1.59	5.97	-3.95	1.09

Table 11: BL Return Estimates for October 2008 in % p.m.

		Optimized Risk Measures																				
		Volatility							mVaR							mCVaR						
Return Estimates	Asset Class/*Portfolio Return and Risk Measures in % p.m.*	SRM	RSM						SRM	RSM						SRM	RSM					
			Regimes		Scenarios					Regimes		Scenarios					Regimes		Scenarios			
			1	2	1,1	1,2	2,1	2,2		1	2	1,1	1,2	2,1	2,2		1	2	1,1	1,2	2,1	2,2
Historical Mean	Stocks	12.02	13.66	11.68	9.04	2.07	-	13.99	7.91	-	19.79	-	15.36	-	26.06	9.12	2.34	17.79	1.89	14.22	-	22.90
	Bonds	51.46	52.90	46.75	53.23	61.76	54.32	46.17	47.16	72.57	23.25	65.06	82.48	70.63	14.35	48.29	66.51	32.25	60.79	79.84	64.65	25.89
	Hedge Funds	23.66	22.44	26.79	33.34	-	5.62	26.83	29.71	7.53	45.78	15.07	-	-	52.48	27.88	14.83	37.42	22.46	-	1.52	41.50
	Managed Futures	11.75	11.00	11.61	4.38	7.20	22.45	9.71	13.99	19.90	5.68	19.87	2.16	17.60	-	13.52	16.33	7.71	14.87	5.94	19.08	3.68
	Commodities	1.11	-	3.17	-	28.97	-	3.30	1.24	-	5.50	-	-	-	7.10	1.18	-	4.83	-	-	-	6.02
	Real Estate	-	-	-	-	-	17.60	-	-	-	-	-	-	11.77	-	-	-	-	-	-	14.75	-
	Return	*0.55*	*0.39*	*0.62*	*0.44*	*-1.80*	*0.46*	*0.70*	*0.57*	*1.20*	*0.96*	*1.12*	*0.90*	*0.97*	*1.21*	*0.56*	*0.99*	*0.84*	*0.92*	*0.79*	*0.77*	*1.02*
	Vola	*1.34*	*1.28*	*1.33*	*1.27*	*0.37*	*0.47*	*1.35*	*1.35*	*1.67*	*1.56*	*1.62*	*1.07*	*0.70*	*1.67*	*1.35*	*1.50*	*1.43*	*1.45*	*0.99*	*0.57*	*1.48*
	mVaR	*-1.31*	*-1.46*	*-1.17*	*-1.43*	*-2.56*	*-0.22*	*-1.10*	*-1.29*	*-0.84*	*-0.91*	*-0.85*	*-0.45*	*-0.00*	*-0.75*	*-1.29*	*-0.89*	*-0.95*	*-0.89*	*-0.46*	*-0.02*	*-0.79*
	mCVaR	*-1.92*	*-2.04*	*-1.78*	*-1.99*	*-2.69*	*-0.43*	*-1.73*	*-1.91*	*-1.65*	*-1.68*	*-1.64*	*-0.96*	*-0.32*	*-1.57*	*-1.90*	*-1.60*	*-1.63*	*-1.58*	*-0.93*	*-0.28*	*-1.51*
CAPM Return	Stocks	12.02	13.66	11.68	9.04	2.07	-	13.99	10.51	1.02	20.70	-	-	-	26.75	11.14	4.63	18.47	2.15	-	-	22.87
	Bonds	51.46	52.90	46.75	53.23	61.76	54.32	46.17	49.06	64.27	27.40	59.56	-	57.26	22.99	49.73	60.73	34.44	57.55	21.18	55.03	31.84
	Hedge Funds	23.66	22.44	26.79	33.34	-	5.62	26.83	25.80	17.39	33.26	26.75	-	5.51	35.94	24.99	18.63	29.35	29.05	-	6.41	31.60
	Managed Futures	11.75	11.00	11.61	4.38	7.20	22.45	9.71	13.16	17.32	11.38	13.70	49.66	37.23	6.38	12.73	16.01	11.66	11.25	34.72	30.81	7.17
	Commodities	1.11	-	3.17	-	28.97	-	3.30	1.47	-	7.26	-	37.61	-	7.94	1.40	-	6.09	-	36.50	-	6.53
	Real Estate	-	-	-	-	-	17.60	-	-	-	-	-	12.73	-	-	-	-	-	-	7.60	7.76	-
	Return	*0.09*	*-0.60*	*0.82*	*-0.48*	*2.39*	*-0.64*	*0.84*	*0.16*	*-0.17*	*1.38*	*-0.16*	*4.98*	*0.05*	*1.49*	*0.16*	*-0.30*	*1.33*	*-0.24*	*4.24*	*-0.28*	*1.41*
	Vola	*1.34*	*1.28*	*1.33*	*1.27*	*0.37*	*0.47*	*1.35*	*1.34*	*1.51*	*1.46*	*1.45*	*1.18*	*0.82*	*1.51*	*1.34*	*1.41*	*1.39*	*1.37*	*0.87*	*0.59*	*1.42*
	mVaR	*-1.77*	*-2.45*	*-0.97*	*-2.35*	*1.63*	*-1.32*	*-0.96*	*-1.69*	*-2.07*	*-0.44*	*-1.98*	*2.54*	*-1.07*	*-0.36*	*-1.69*	*-2.09*	*-0.46*	*-2.00*	*2.45*	*-1.14*	*-0.38*
	mCVaR	*-2.38*	*-3.03*	*-1.58*	*-2.91*	*1.50*	*-1.53*	*-1.58*	*-2.30*	*-2.79*	*-1.15*	*-2.67*	*2.11*	*-1.45*	*-1.08*	*-2.30*	*-2.76*	*-1.12*	*-2.65*	*2.13*	*-1.40*	*-1.05*
BL Return	Stocks	12.02	13.66	11.68	9.04	2.07	-	13.99	1.44	0.17	20.84	-	21.06	-	27.28	4.49	4.11	18.43	1.73	18.44	-	23.17
	Bonds	51.46	52.90	46.75	53.23	61.76	54.32	46.17	57.65	66.28	26.35	60.53	56.00	63.27	21.20	55.49	61.61	33.95	58.29	55.63	58.34	29.88
	Hedge Funds	23.66	22.44	26.79	33.34	-	5.62	26.83	22.48	14.97	37.08	20.77	-	6.73	42.34	23.01	17.84	32.03	25.65	-	7.04	36.24
	Managed Futures	11.75	11.00	11.61	4.38	7.20	22.45	9.71	15.64	18.58	12.31	18.70	22.93	29.99	1.79	14.67	16.44	12.37	14.33	25.93	26.92	4.63
	Commodities	1.11	-	3.17	-	28.97	-	3.30	2.78	-	3.42	-	-	-	7.39	2.35	-	3.22	-	-	-	6.08
	Real Estate	-	-	-	-	-	17.60	-	-	-	-	-	-	-	-	-	-	-	-	-	7.70	-
	Return	*-0.09*	*-0.38*	*0.79*	*-0.02*	*2.10*	*-0.33*	*0.75*	*0.16*	*0.15*	*1.15*	*0.53*	*3.62*	*0.38*	*1.27*	*0.10*	*-0.02*	*1.08*	*0.39*	*3.49*	*0.05*	*1.15*
	Vola	*1.34*	*1.28*	*1.33*	*1.27*	*0.37*	*0.47*	*1.35*	*1.44*	*1.57*	*1.47*	*1.54*	*1.09*	*0.83*	*1.56*	*1.39*	*1.43*	*1.39*	*1.42*	*0.96*	*0.60*	*1.44*
	mVaR	*-1.95*	*-2.24*	*-1.00*	*-1.89*	*1.35*	*-1.02*	*-1.04*	*-1.78*	*-1.80*	*-0.69*	*-1.38*	*2.18*	*-0.74*	*-0.61*	*-1.79*	*-1.83*	*-0.70*	*-1.41*	*2.17*	*-0.81*	*-0.64*
	mCVaR	*-2.56*	*-2.81*	*-1.62*	*-2.45*	*1.21*	*-1.22*	*-1.67*	*-2.45*	*-2.55*	*-1.39*	*-2.12*	*1.67*	*-1.12*	*-1.37*	*-2.43*	*-2.51*	*-1.36*	*-2.08*	*1.73*	*-1.07*	*-1.33*

Table 12: Minimum Risk Portfolio Weights for October 2008 (Current Regime = Regime 1)

		Optimized Risk Measures																				
		Volatility							mVaR							mCVaR						
Return	Asset Class/*Portfolio*	*SRM*	RSM						SRM	RSM						SRM	RSM					
Estimates	*Return and Risk*		Regimes		Scenarios					Regimes		Scenarios					Regimes		Scenarios			
	Measures in % p.m.		1	2	1,1	1,2	2,1	2,2		1	2	1,1	1,2	2,1	2,2		1	2	1,1	1,2	2,1	2,2
Historical	Stocks	*6.24*	-	35.79	-	15.86	-	41.57	4.47	-	31.04	-	16.67	-	33.09	5.11	-	33.23	-	16.32	-	36.64
Mean	Bonds	*39.56*	74.36	14.12	64.77	84.14	69.69	-	38.49	75.04	-	68.43	83.33	72.21	-	38.92	74.86	4.39	67.29	83.68	70.06	-
	Hedge Funds	*37.60*	-	45.79	-	-	-	51.16	40.28	-	62.98	-	-	-	59.60	39.17	-	56.69	-	-	-	55.98
	Managed Futures	*15.25*	25.64	-	35.23	-	18.57	-	15.70	24.96	-	31.57	-	15.49	-	15.62	25.14	-	32.71	-	18.10	-
	Commodities	*1.34*	-	4.30	-	-	-	7.28	1.06	-	5.98	-	-	-	7.31	1.17	-	5.69	-	-	-	7.38
	Real Estate	-	-	-	-	-	11.74	-	-	-	-	-	-	12.29	-	-	-	-	-	-	11.84	-
	Return	*0.60*	*1.33*	*1.23*	*1.43*	*0.95*	*0.95*	*1.44*	*0.61*	*1.33*	*1.33*	*1.40*	*0.98*	*1.00*	*1.40*	*0.60*	*1.33*	*1.30*	*1.41*	*0.97*	*0.95*	*1.42*
	Vola	*1.40*	*1.81*	*1.92*	*1.99*	*1.11*	*0.68*	*1.95*	*1.43*	*1.81*	*2.14*	*1.95*	*1.14*	*0.73*	*1.92*	*1.42*	*1.81*	*2.06*	*1.96*	*1.13*	*0.69*	*1.93*
	mVaR	*-1.31*	*-0.86*	*-1.05*	*-0.97*	*-0.45*	*0.00*	*-0.84*	*-1.33*	*-0.86*	*-1.07*	*-0.94*	*-0.46*	*0.00*	*-0.79*	*-1.32*	*-0.86*	*-1.06*	*-0.94*	*-0.46*	*0.00*	*-0.80*
	mCVaR	*-1.96*	*-1.74*	*-1.99*	*-1.93*	*-0.98*	*-0.31*	*-1.80*	*-1.99*	*-1.74*	*-2.14*	*-1.88*	*-1.01*	*-0.34*	*-1.74*	*-1.98*	*-1.74*	*-2.08*	*-1.90*	*-1.00*	*-0.31*	*-1.76*
CAPM	Stocks	*41.97*	-	23.51	-	2.46	-	29.24	29.11	-	24.34	-	0.03	-	29.63	31.98	-	24.48	-	0.04	-	30.07
Return	Bonds	*22.75*	-	28.83	-	56.87	38.98	27.86	31.55	-	23.95	-	-	-	20.21	31.71	-	27.53	-	-	-	24.54
	Hedge Funds	*23.81*	-	24.48	-	-	-	28.11	23.60	-	32.02	-	-	100.00	36.00	22.57	-	27.80	-	-	100.00	31.66
	Managed Futures	-	100.00	15.86	100.00	11.02	61.02	7.39	10.04	100.00	11.58	100.00	95.15	-	5.56	7.20	100.00	12.46	100.00	87.19	-	5.60
	Commodities	*9.94*	-	7.32	-	29.65	-	7.40	5.69	-	8.10	-	-	-	8.61	6.54	-	7.73	-	-	-	8.13
	Real Estate	*1.52*	-	-	-	-	-	-	-	-	-	-	4.83	-	-	-	-	-	-	12.78	-	-
	Return	*0.28*	*0.44*	*1.35*	*0.50*	*2.74*	*0.43*	*1.42*	*0.23*	*0.44*	*1.44*	*0.50*	*4.68*	*-1.60*	*1.54*	*0.25*	*0.44*	*1.43*	*0.50*	*4.76*	*-1.60*	*1.52*
	Vola	*2.14*	*3.50*	*1.47*	*3.55*	*0.38*	*1.47*	*1.49*	*1.63*	*3.50*	*1.53*	*3.55*	*2.41*	*4.65*	*1.56*	*1.70*	*3.50*	*1.49*	*3.55*	*2.08*	*4.65*	*1.52*
	mVaR	*-3.27*	*-4.49*	*-0.54*	*-4.27*	*1.96*	*-1.66*	*-0.44*	*-2.23*	*-4.49*	*-0.45*	*-4.27*	*0.00*	*-11.31*	*-0.36*	*-2.39*	*-4.49*	*-0.46*	*-4.27*	*0.80*	*-11.31*	*-0.36*
	mCVaR	*-4.16*	*-6.08*	*-1.23*	*-5.92*	*1.83*	*-2.32*	*-1.15*	*-2.94*	*-6.08*	*-1.19*	*-5.92*	*-0.92*	*-13.00*	*-1.12*	*-3.13*	*-6.08*	*-1.17*	*-5.92*	*0.00*	*-13.00*	*-1.10*
BL	Stocks	-	-	26.93	-	3.56	-	36.01	-	-	26.67	-	13.08	-	33.91	-	-	27.19	-	20.11	-	35.17
Return	Bonds	*55.74*	29.78	26.77	42.22	59.25	82.65	18.11	61.69	38.92	17.74	51.21	13.16	83.84	8.45	60.59	31.70	22.82	47.03	5.73	84.62	13.56
	Hedge Funds	-	-	28.86	-	-	1.49	38.41	-	-	38.97	-	13.07	-	49.15	-	-	33.51	-	15.09	-	43.20
	Managed Futures	*34.48*	70.22	17.44	57.78	10.29	15.86	-	30.70	61.08	14.27	48.79	34.55	16.16	-	31.27	68.30	14.99	52.97	50.34	15.38	-
	Commodities	*9.78*	-	-	-	26.89	-	7.47	7.61	-	2.36	-	13.07	-	8.49	8.14	-	1.48	-	4.36	-	8.08
	Real Estate	-	-	-	-	-	-	-	-	-	-	-	13.07	-	-	-	-	-	-	4.36	-	-
	Return	*0.41*	*0.63*	*1.19*	*1.11*	*2.37*	*0.71*	*1.33*	*0.40*	*0.55*	*1.26*	*1.00*	*3.04*	*0.59*	*1.43*	*0.41*	*0.60*	*1.24*	*1.05*	*3.20*	*0.60*	*1.40*
	Vola	*1.93*	*2.63*	*1.53*	*2.40*	*0.37*	*1.18*	*1.66*	*1.85*	*2.40*	*1.62*	*2.21*	*1.56*	*1.20*	*1.77*	*1.87*	*2.58*	*1.56*	*2.30*	*1.43*	*1.22*	*1.71*
	mVaR	*-2.18*	*-2.87*	*-0.78*	*-1.91*	*1.60*	*-0.82*	*-0.68*	*-2.06*	*-2.56*	*-0.72*	*-1.73*	*0.00*	*-0.95*	*-0.65*	*-2.08*	*-2.81*	*-0.72*	*-1.81*	*0.56*	*-0.96*	*-0.64*
	mCVaR	*-3.07*	*-4.09*	*-1.50*	*-3.05*	*1.46*	*-1.38*	*-1.49*	*-2.93*	*-3.69*	*-1.50*	*-2.79*	*-0.59*	*-1.51*	*-1.52*	*-2.95*	*-4.02*	*-1.47*	*-2.91*	*0.00*	*-1.54*	*-1.47*

Table 13: Tangency Portfolio Weights for October 2008 (Current Regime = Regime 1)

		Minimum Risk Portfolios						Tangency Portfolios					
Risk Estimates	Portfolio Return and Risk Measures	Historical Mean		CAPM Return		BL Return		Historical Mean		CAPM Return		BL Return	
		SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM
Volatility	Return	0.434	0.460	0.434	0.460	0.434	0.460	0.385	0.663	0.004	0.433	0.177	0.575
	Vola	1.891	1.978	1.891	1.978	1.891	1.978	1.771	2.011	4.423	2.396	2.137	2.190
	SR	0.230	0.232	0.230	0.232	0.230	0.232	0.217	0.330	0.001	0.181	0.083	0.262
mVaR	Return	0.413	0.605	0.416	0.557	0.435	0.567	0.382	0.671	0.228	0.571	0.194	0.594
	mVaR	-2.740	-2.530	-2.832	-2.587	-2.734	-2.493	-2.641	-2.801	-5.265	-3.883	-3.166	-3.477
	SRmVaR	0.151	0.239	0.147	0.215	0.159	0.227	0.145	0.240	0.043	0.147	0.061	0.171
mCVaR	Return	0.420	0.567	0.422	0.520	0.425	0.533	0.381	0.665	0.221	0.698	0.193	0.589
	mCVaR	-3.511	-3.401	-3.604	-3.436	-3.514	-3.394	-3.367	-3.639	-6.778	-4.407	-4.066	-4.397
	SRmCVaR	0.120	0.167	0.117	0.151	0.121	0.157	0.113	0.183	0.033	0.158	0.047	0.134

Table 14: Out-of Sample Performance and Risk Measures in % p.m.

	Minimum Risk Portfolios						Tangency Portfolios					
Risk	Historical Mean		CAPM Return		BL Return		Historical Mean		CAPM Return		BL Return	
Estimates	SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM	SRM	RSM
Volatility	121.26	122.50	121.26	122.50	121.26	122.50	118.97	132.91	99.96	120.71	109.31	128.09
mVaR	120.33	130.04	120.42	127.43	121.49	128.04	118.85	133.43	110.27	128.06	110.16	128.94
mCVar	120.68	128.00	120.69	125.49	120.91	126.28	118.77	133.08	109.83	135.19	110.07	128.68

Table 15: Out-of-Sample Analysis: End-of-Period Value

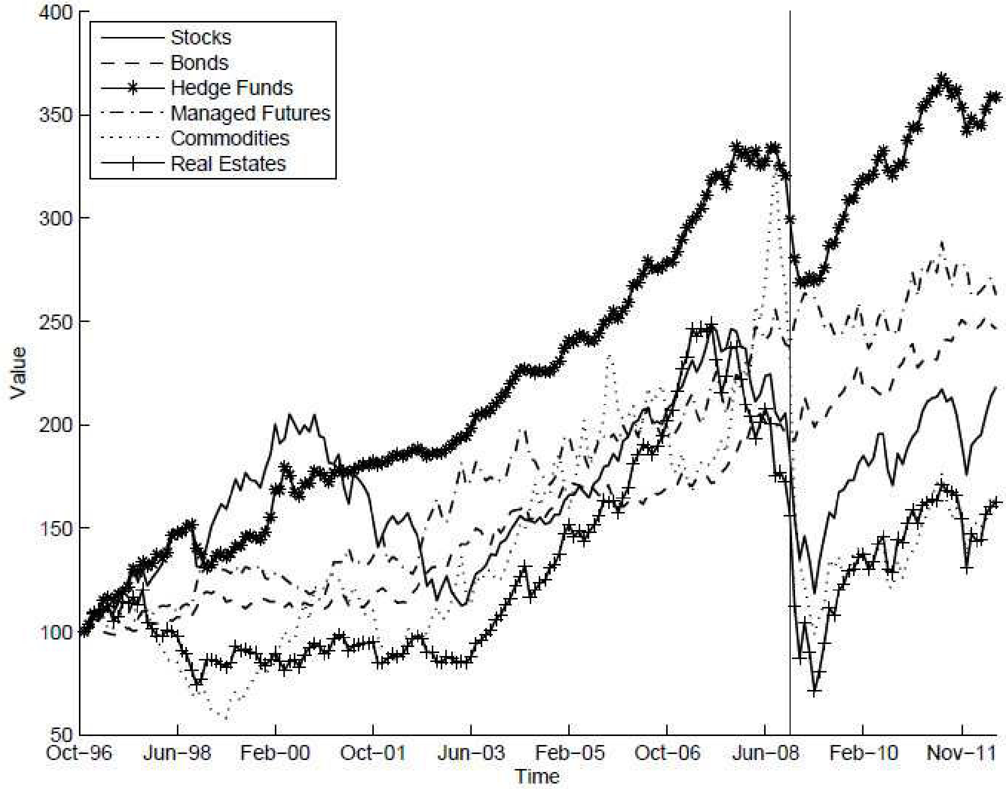


Figure 1: Performances of Asset Classes, In- and Out-of-Sample

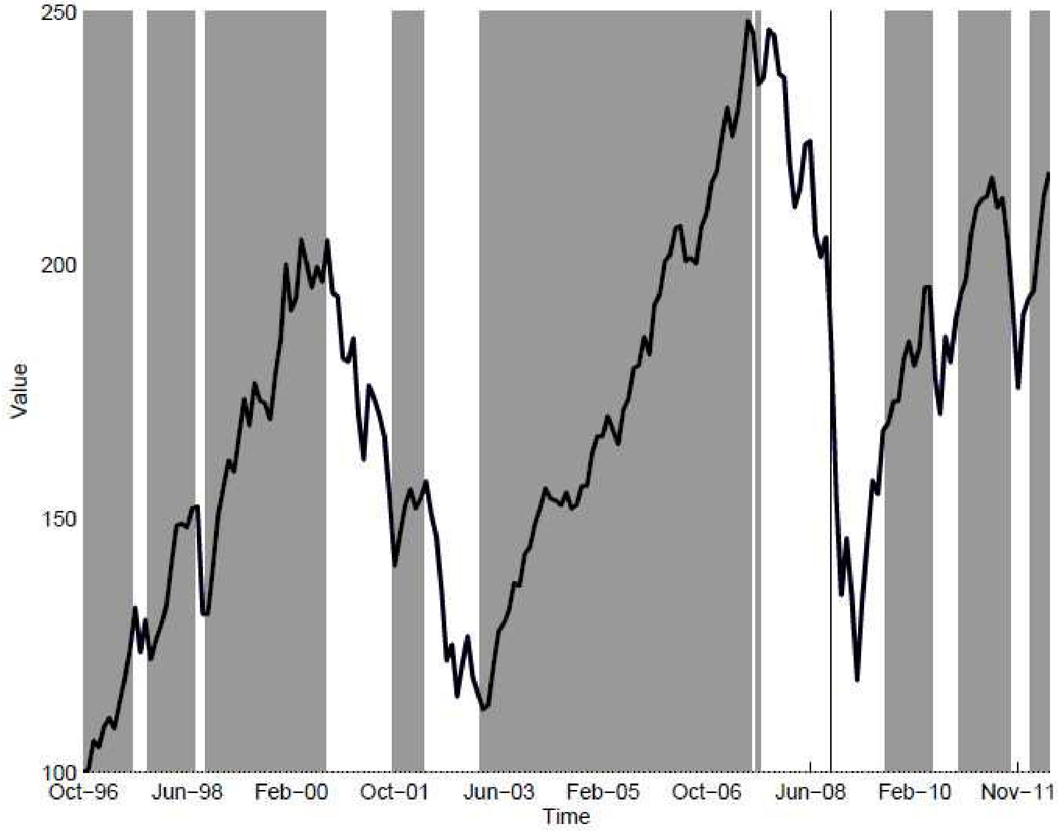


Figure 2: Stocks' Performance, In- and Out-of-Sample: White = Bear Regime, Gray = Bull Regime

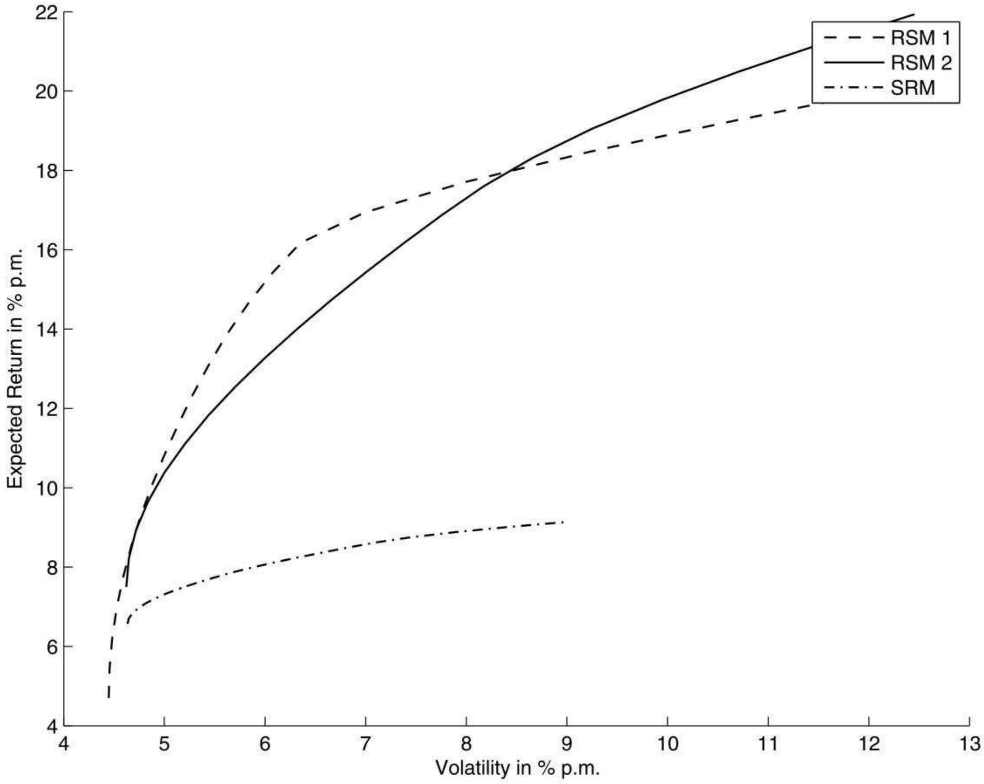


Figure 3: Efficient Frontiers (October 2008):
Return Measure = Historical Mean, Optimized Risk Measure = Volatility

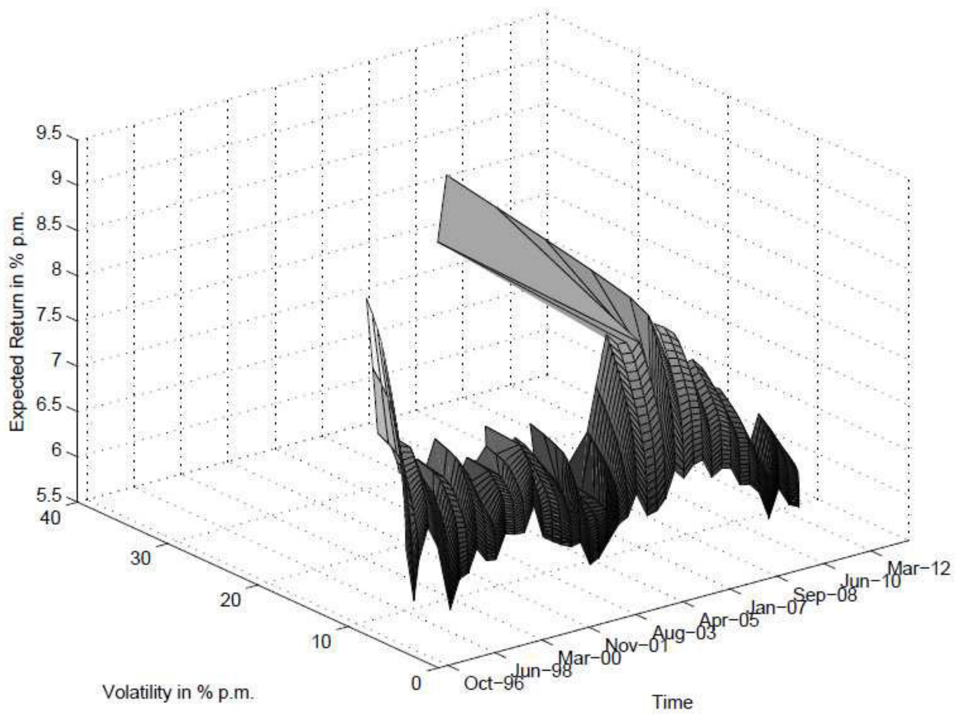
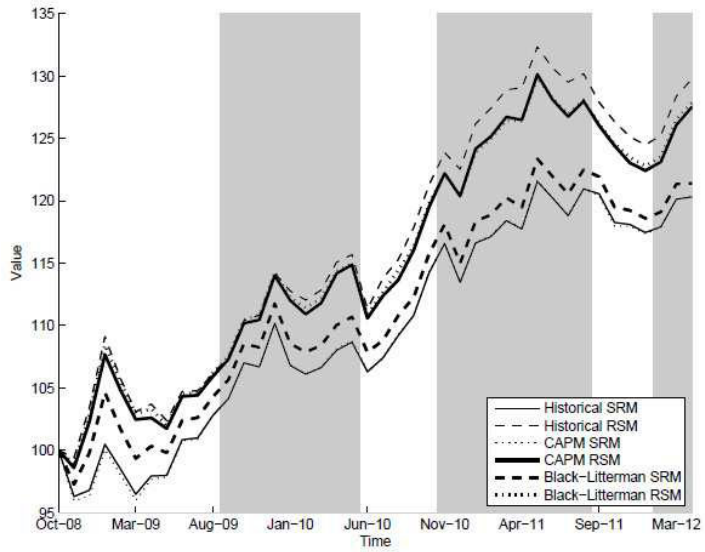


Figure 4: Efficient Frontiers for the Regime-Switching Model (October 2008–October 2011):
Return Measure = Historical Mean, Optimized Risk Measure = Volatility



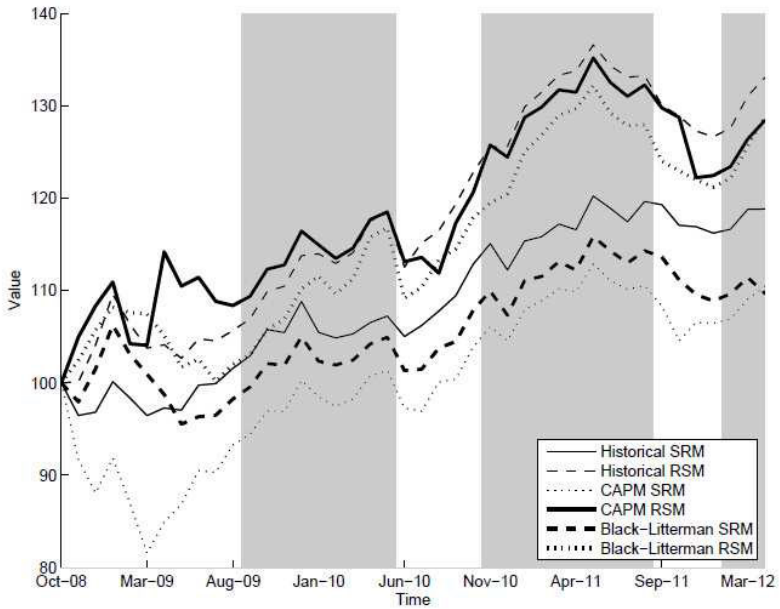
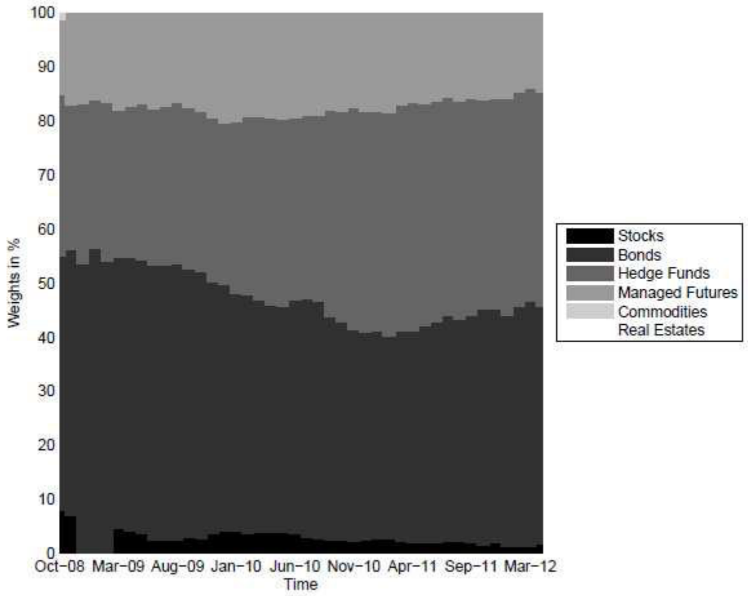


Figure 5: Out-of-Sample Performance of Minimum Risk Portfolios: Optimized Risk Measure = mVaR

Figure 6: Out-of-Sample Performance of Tangency Portfolios: Optimized Risk Measure = mVaR



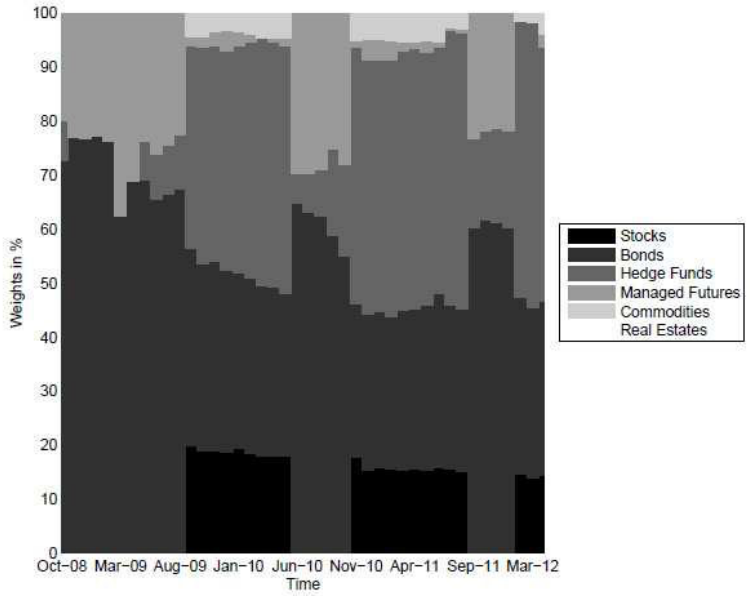


Figure 7: Out-of-Sample Minimum Risk Portfolio Weights of SRM: Return Measure = Historical Mean, Optimized Risk Measure = mVaR

Figure 8: Out-of-Sample Minimum Risk Portfolio Weights of RSM Return Measure = Historical Mean, Optimized Risk Measure = mVaR

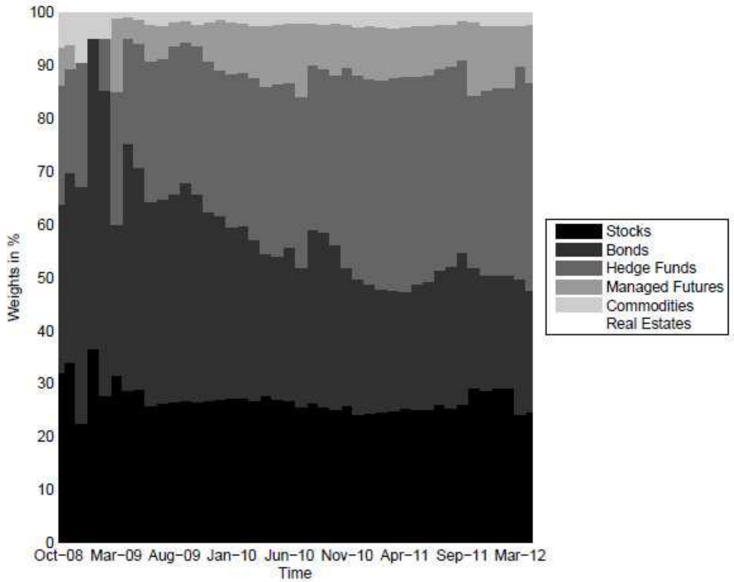


Figure 9: Out-of-Sample Tangency Portfolio Weights for SRM:
Return Measure = CAPM Return, Optimized Risk Measure = mCVaR

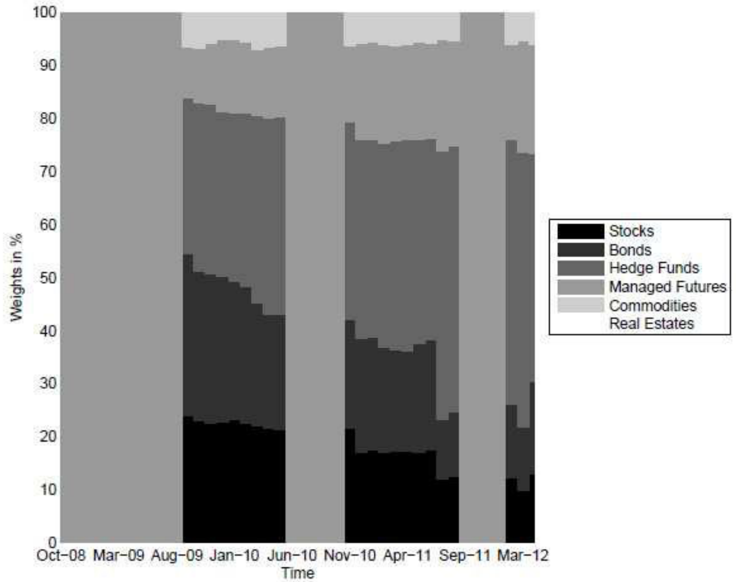


Figure 10: Out-of-Sample Tangency Portfolio Weights for RSM:
Return Measure = CAPM Return, Optimized Risk Measure = mCVaR

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